# Can Maxwell's equations be reduced to a more general form

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**Abstract:** The original Maxwell equations consisted of more than a dozen equations, which have been continuously simplified into four current equations. If the existence of virtual space-time is taken into account, the existing Maxwell equations will increase to eight. This article attempts to further simplify Maxwell's equations based on virtual space-time, and finds that four of them can be derived from the other four equations. Therefore, Maxwell's equations based on virtual space-time can be simplified into four more general forms. Using the simplified Maxwell equations, the inverse square relationship of the electrostatic field strength can be derived.

**Keywords:** Maxwell's equations; Virtual space-time; Inverse square

# 1 Introduction

The basic form of Maxwell's equations:

$$\begin{cases} \nabla \cdot \mathbf{E} = \dfrac{\rho}{\varepsilon} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{H} = \mathbf{J} + \varepsilon \dfrac{\partial \mathbf{E}}{\partial t} \end{cases} \quad (1)$$

According to some conventions of virtual space-time physics [1], it can be changed into a more general form

$$\begin{cases} \nabla \cdot \mathbf{F} = \rho_e \\ \nabla \cdot \mathbf{G} = 0 \\ \nabla \times \mathbf{F} = -\dfrac{\partial \mathbf{G}}{\partial y} \\ \nabla \times \mathbf{G} = \mathbf{J_e} + \dfrac{\partial \mathbf{F}}{\partial y} \end{cases} \quad (2)$$

Among them:

Generalized electric and magnetic field intensities

$$\mathbf{F} = \sqrt{\varepsilon}\mathbf{E}$$

$$\mathbf{G} = \sqrt{\mu}\mathbf{H}$$

Generalized charge density

$$\rho_e = \frac{\rho}{\sqrt{\varepsilon}}$$

Generalized current density

$$\mathbf{J}_e = \sqrt{\mu}\mathbf{J}$$

Generalized time

$$y = ct$$

Then according to the requirements of symmetry, another set of Maxwell equations in virtual space-time can be established. The specific method is to symmetrically transform these physical parameters symbols.

The symbols to be replaced are as follows

$$\begin{gathered} \mathbf{F} \leftrightarrow \mathbf{G} \\ \rho_e \to \rho_m \\ \mathbf{J_e} \to \mathbf{J_m} \\ \nabla \to \nabla_y \\ \frac{\partial}{\partial y} \to \frac{\partial}{\partial x} \end{gathered}$$

Among them, $\rho_m$ is the monopolar charge density. $\mathbf{J_m}$ is the monopolar current density. And $x$ is the generalized time of virtual space-time. $\nabla_y$ is the differential operator of the space component in the virtual space-time. In this way, a new set of Maxwell equations based on virtual space-time is established.

$$\begin{cases} \nabla_y \cdot \mathbf{G} = \rho_m \\ \nabla_y \cdot \mathbf{F} = 0 \\ \nabla_y \times \mathbf{G} = -\dfrac{\partial \mathbf{F}}{\partial x} \\ \nabla_y \times \mathbf{F} = \mathbf{J_m} + \dfrac{\partial \mathbf{F}}{\partial x} \end{cases} \qquad (3)$$

# 2 Maxwell equations without charges

If there is no electric charge or magnetic charge of the magnetic monopole, then equations (2) and (3) can be simplified into four equations

$$\begin{cases} \nabla \times \mathbf{F} = -\dfrac{\partial \mathbf{G}}{\partial y} \\ \nabla \times \mathbf{G} = \dfrac{\partial \mathbf{F}}{\partial y} \\ \nabla_y \times \mathbf{G} = -\dfrac{\partial \mathbf{F}}{\partial x} \\ \nabla_y \times \mathbf{F} = \dfrac{\partial \mathbf{F}}{\partial x} \end{cases} \quad (4)$$

It can be seen that these four equations are highly symmetrical and very concise. For the solutions of the equations, please refer to the "Foundations of Virtual Spacetime Physics" [1]

# 3 Plane wave solution

The solutions to this equation include real space-time and virtual space-time electromagnetic wave solutions, respectively. It also contains an interesting wave equation that spans virtual space time and real space time:

$$\left(\nabla \cdot \nabla_y + \frac{\partial^2}{\partial x \partial y}\right)\mathbf{F} = 0 \quad (5)$$

Equation (5) can be solved from the perspective of electric field quantization and classical electrodynamics.

If equation (5) is viewed from the perspective of electric field quantization, each electric field quanta should be a plane wave. However, equation (5) involves a problem of real space-time and virtual space-time vector directions. Considering the requirements of the symmetry of two space-times, it is a simpler method to set the unit vectors of the two space-times to be completely opposite, which is:

$$\hat{x}_i = -\hat{y}_i$$

Then equation (5) can be transformed into

$$\left(-\sum_{i=1}^{3} \frac{\partial^2}{\partial x_i \partial y_i} + \frac{\partial^2}{\partial x \partial y}\right)\mathbf{F} = 0 \quad (5)$$

Notice that these plane wave solutions contain

$$\mathbf{F} = \mathbf{F_0} e^{\left(\mathbf{k}\cdot\mathbf{X} + \frac{\omega}{c} y\right)} e^{\left(\mathbf{k}\cdot\mathbf{Y} + \frac{\omega}{c} x\right)} \quad (6)$$

Or

$$\mathbf{F} = \mathbf{F_0} e^{-\left(\mathbf{k}\cdot\mathbf{X}+\frac{\omega}{c}y\right)} e^{-\left(\mathbf{k}\cdot\mathbf{Y}+\frac{\omega}{c}x\right)} \qquad (7)$$

Where

$$\mathrm{k}^2 = \left(\frac{\omega}{c}\right)^2$$

Of course, there are other special solutions.

Analyze the solution (7) here. The solution is an electric field without an oscillating phase. So over time, the final result of this solution is that the electric field strength is 0 or infinity.

In order to avoid the problem that the electric field strength is 0 or infinity, only the conditions need to be met

$$\begin{cases} \mathbf{X} = -\mathbf{Y} \\ x = -y \end{cases} \qquad (8)$$

In this case, the result of this solution is

$$\mathbf{F} = \mathbf{F_0} \qquad (9)$$

It can be seen that in this case, the electric field strength is a constant value and no longer changes with time and distance.

Combining plane wave solutions of classical electrodynamics, for a wave function of an electromagnetic wave, the frequency therein represents the energy of a photon, and the amplitude $\mathbf{F}_0$ is the number of photons. Solution (9) reflects that the quantum number of the electric field remains constant. Combined with the symmetry requirements of the three-dimensional space, in order to achieve the conservation of the electric field flux, the inverse square relationship of the electric field strength needs to be satisfied, so that the Gauss law of the electrostatic field can be obtained.

$$\mathbf{F} = \frac{\mathbf{F_0} \cdot \mathbf{S}_0}{S} \hat{\mathbf{S}} \qquad (10)$$

# 4 Spherical wave solution

The other method is to solve the problem by classical electrodynamics method. The wave function solution of the spherical wave is solved. Here we consider the case of spherical symmetry. For spherical polar coordinates, there is

$$\nabla = \sum_i \frac{\partial}{\partial x_i}\hat{x}_i = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta} + \frac{1}{rsin\theta}\frac{\partial}{\partial \varphi}\hat{\varphi}$$

$$\nabla_y = \sum_i \frac{\partial}{\partial y_i}\hat{y}_i = \frac{\partial}{\partial r_y}\hat{r}_y + \frac{1}{r_y}\frac{\partial}{\partial \theta_y}\hat{\theta}_y + \frac{1}{r_y sin\theta_y}\frac{\partial}{\partial \varphi_y}\hat{\varphi}_y$$

We then consider the requirements of virtual space-time and real space-time symmetry.

$$\hat{r}_y = -\hat{r}$$

$$\hat{\varphi}_y = -\hat{\varphi}$$

$$\hat{\theta}_y = -\hat{\theta}$$

Under this condition of symmetry, the radial directions of real space-time and virtual space-time are exactly opposite. Due to the symmetry, the directions between the other two angles are exactly opposite. The advantage of this is that when the radius of a sphere in real space-time becomes smaller and smaller, less than a certain value, you can enter virtual space-time. The radius between virtual space-time and real space-time can form a reciprocal relationship. Figure 1 shows the relationship between real space-time and virtual space-time radial direction.

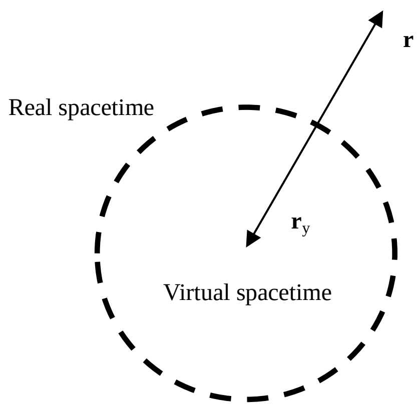


Figure 1. Radial direction of virtual spacetime and real spacetime

Then

$$\nabla \cdot \nabla_y = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r_y}\right) - \frac{1}{rsin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{1}{r_y}\frac{\partial}{\partial \theta_y}\right) - \frac{1}{rsin\theta}\frac{\partial}{\partial \varphi}\left(\frac{1}{r_y sin\theta_y}\frac{\partial}{\partial \varphi_y}\right)$$

Considering the requirements of spherical symmetry, there are

$$\frac{\partial}{\partial \theta}\mathbf{F} = 0$$

$$\frac{\partial}{\partial \varphi}\mathbf{F} = 0$$

Then

$$(\nabla \cdot \nabla_y)\mathbf{F} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r_y}\right)\mathbf{F} = -\left(\frac{2}{r}\frac{\partial}{\partial r_y} + \frac{\partial^2}{\partial r \partial r_y}\right)\mathbf{F}$$

Substituting into equation (5), we get

$$\left(\frac{2}{r}\frac{\partial}{\partial r_y} + \frac{\partial^2}{\partial r \partial r_y} - \frac{\partial^2}{\partial x \partial y}\right)\mathbf{F} = 0 \tag{11}$$

Considering

$$\frac{\partial^2}{\partial r \partial r_y}(r^2\mathbf{F}) = \frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r_y}\mathbf{F}\right) = 2r\frac{\partial}{\partial r_y}\mathbf{F} + r^2\frac{\partial^2}{\partial r \partial r_y}\mathbf{F}$$

Then the equation can be transformed into

$$\frac{\partial^2}{\partial r \partial r_y}(r^2\mathbf{F}) = \frac{\partial^2}{\partial x \partial y}(r^2\mathbf{F})$$

Therefore, a special solution of equation (11) is

$$\mathbf{F} = \frac{\mathbf{A_0}}{r^2}e^{kr + kr_y + \frac{\omega}{c}y + \frac{\omega}{c}x} \tag{12}$$

It can be seen from the solution (12) that if

$$\begin{cases} r = -r_y \\ x = -y \end{cases} \tag{13}$$

Then solution (12) will form a steady state. In this steady state, the electric field strength is inversely proportional to the square of the distance. This is

$$\mathbf{F} = \frac{\mathbf{A_0}}{r^2} \tag{14}$$

This is consistent with Gauss's law of electrostatic fields.

# 5 Conclusions

In this paper, the in-depth analysis of the virtual space-time Maxwell equations in vacuum is performed, and a solution without oscillation terms is obtained. This article analyzes the corresponding plane wave solution and spherical wave solution. The results show that as long as conditions (8) and (13) are met, plane wave solutions (9) and spherical wave solutions (14) can be obtained in a steady state.

Taking into account the requirements of the quantization of the electric field, corresponding to the quantization of classical electromagnetic waves, the amplitude of the electromagnetic waves corresponds to the number of optical quanta. Therefore, the amplitude $\mathbf{F}_0$ in solution (9) can be regarded as the number of electric field quanta. In order to maintain the conservation of the electric field quantum flux in three-dimensional space, the requirement of flux conservation must be met. In this way, Gauss's law of the inverse square relationship of the electric field strength distance can be obtained (10)

In addition, the equation (5) can also be solved by the classical electrodynamic spherical wave solution method. It is through solving the spherical wave solution of equation (5) to obtain the change law of the electric field intensity. The solution results in this paper show that if we consider spherically symmetric solutions, we can obtain a spherically symmetric solution (14) without oscillatory terms after making some symmetry rules for the unit vector direction of virtual space-time spherical polar coordinates. If condition (13) is satisfied, the solution is stationary, and the electric field strength is inversely proportional to the square of the distance.

From the comparison of the above two solutions, the advantage of the plane wave solution lies in the clear physical meaning. Considering the inevitable requirements of the quantization of the micro world, the electric field should also be quantized. This quantization of the electric field reflects that the energy carried by each electric field quantum is the same. This corresponds to a classical wave of a specific frequency. The electric field strength is related to the number of electric field quanta. In this way, Gauss' law reflects the conservation of electric field quantum flux. Therefore, the inverse square relationship of electromagnetic interaction is essentially determined by the characteristics of three-dimensional space. The perfect dimensional relationship in the three-dimensional space we are in now determines the exact establishment of the inverse square law. If we consider the gravitational interaction, near the very large mass, the space-time is bent, resulting in the inverse square relationship is not valid. Perhaps we can associate the electromagnetic interaction with the gravitational interaction at the level of space-time.

From the perspective of the classical spherical wave solution, it can automatically obtain the inverse square relationship of the electric field strength, which in turn shows that the wave equation (5) can explain the existence of the electrostatic field. For this reason, the divergence relationship between electric and magnetic fields in Maxwell's equations is superfluous. Therefore equations (4) can already be used to explain all the laws of electromagnetic interaction. This is

$$\begin{cases} \nabla \times \mathbf{F} = -\dfrac{\partial \mathbf{G}}{\partial y} \\ \nabla \times \mathbf{G} = \dfrac{\partial \mathbf{F}}{\partial y} \\ \nabla_y \times \mathbf{G} = -\dfrac{\partial \mathbf{F}}{\partial x} \\ \nabla_y \times \mathbf{F} = \dfrac{\partial \mathbf{F}}{\partial x} \end{cases} \qquad (4)$$

# Reference

[1] Cheng, Z. Foundations of Virtual Spacetime Physics. LAP LAMBERT Academic Publishing. Brivibas gatve 197, LV-1039, Riga. Latvia, European Union