The nuclear self-energy and the strong equivalence principle

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Abstract

In the present work I discuss whether the gravito-electric self-energy is a valid approach to study the nuclear structure and the nuclear forces.

In particular I investigate the validity of the strong equivalence principle (SEP) in the atomic nucleus, by assuming that in the nucleus the gravito-electric force $\left(F_{ge} = \frac{GKMm}{R^2}\right)$ to be operating and that the potential related to this force to be "self-energy", namely depends on the mass of the nucleons squared (M^2).

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- **The nuclear radius and the gravito-electric force**

We know from Einstein's theory of relativity that the energy contained in the atomic nucleus is equal to $E = Mc^2$, where M is the mass of the nucleus.

The mass, in this formula, is understood as the inertial mass, namely it is considered as the inertial resistance to acceleration.

Now, one of the cornerstones of the theory of relativity is the strong equivalence principle (SEP), namely the equivalence between inertial mass and gravitational mass.

One way to theoretically demonstrate this equivalence is to hypothesize that the gravitational mass gives rise to a self-energy, namely a potential energy which depends on the mass of the body squared (M^2).

In the reference [1] the author tries to demonstrate the existence of the self-energy in the celestial body, by resorting to the PNN formalism, namely a modification of Newtonian potential energy, and the result is that, for the Sun, the ratio $\frac{E}{Mc^2}$ is equal to 3.52×10^{-6}, where E is the self-energy of the Sun, obtained by means of the PNN parameter.

In this paper we propose a different way to demonstrate the existence of the self-energy within the atomic nucleus.

As it's known, the gravitational potential energy of a body subjected to the attractive force of gravity is:

$$U = F_g * R \qquad (1)$$

where F_g is the force of gravity $\frac{GMm}{R^2}$:

Therefore the eq. (1) becomes:

$$U = \frac{GMm}{R^2} * R$$

$$U = \frac{GMm}{R}$$

If we consider the mass m as negligible with respect to the mass M, we have that the potential energy of a massless point orbitating about a greater body with mass M, will be:

$$U = \frac{GM}{R}$$

The reason of the direct proportionality between the potential energy and the distance — which we have seen in the equation (1) — rather than the inverse proportionality — which instead we have in the equation of force of gravity — is explained by the fact that in the first case we observe the phenomenon of gravitational attraction in terms of the potentiality of the body subjected to a given gravitational force, located at a certain height and free to fall, to affect the surrounding reality, in particular by impacting the ground.

It is obvious that the higher up the body is located, the greater its gravitational potential will be, because the damage it will cause to the Earth's soil is the greater, the greater the height from which it begins to fall is (in this case, in fact, a body would reach the Earth's soil with the greater speed, the greater the distance from the Earth).

But if we suppose that in the atomic nucleus there exists an attractive-repulsive field generated by the nucleus itself, and that this field gives rises to a pendulum, in particular to a harmonic oscillator in which the center of the nucleus would be the fixed point (*fulcrum*) of the pendulum, and in which $g = \frac{GM}{l^2}$ where l is the length of the wire, it would follow that, by increasing the distance from the center of the nucleus, the gravity acceleration g decreases, and consequently the formula of potential energy has to change.

If we admit, indeed, that the effect of the attractive-repulsive field is not to make the bodies fall towards the central attractor-repulsor, but to make them move around it at decreasing speed as the distance from the central body increases, according to the formulae of a pendulum in which g is inversely proportional to the square of the length of the wire (l^2), then it would follow that the formula of the gravitational potential energy (E) would be as follows:

$$E = \frac{F_g}{2\,\pi R} \qquad (2)$$

This time, differently from the eq. (1), the distance R is in the denominator, because, the greater is the distance, the lower will be the linear velocity produced by the attractive-repulsive field, then, in the final analysis, the lower will be the energy of the orbitating mass body m.

In fact, the period T of the pendulum harmonic oscillator is directly proportional to the length (l) of the wire $\left(T = 2\,\pi * \sqrt{\frac{l}{g}}\right)$, so that it increases if the length increases, and in this case not only the angular velocity of the pendulum, but also its linear velocity (more precisely the tangential velocity) decreases, because above we have assumed that in such a particular type of pendulum, the gravity acceleration g decreases with the increase of the square of the wire's lenght ($g = \frac{GM}{l^2}$).

In fact, the formula of the tangential maximum velocity of pendulum is $v = \omega * l$, and, by knowing that the angular velocity of harmonic oscillator is $\omega = \sqrt{\frac{g}{l}}$, its tangential velocity will be $v = \sqrt{\frac{g}{l} * l^2} = \sqrt{\frac{GM}{l^3} * l^2} = \sqrt{\frac{GM}{l}}$ which demonstrates that, in such a particular pendulum, the increase of the wire implies the decrease of the tangential velocity of the pendulum.

In essence, if the linear velocity of pendulum decreases with the distance from the center of the nucleus, it means that its energy, in particular the kinetic energy, decreases, therefore, by assuming that the attractive-repulsive field generates a pendulum, in particular a harmonic oscillator, we can infer that the potential energy of a body inserted in such a field decreases as the distance from the central body increases, so that this energy can be mathematically expressed as inversely proportional to the circumference ($2\pi R$) descripted by the orbitating body.

The term π is extremely important because from it one can deduce that it's not the case of an exclusively repulsive field, in which the potential energy should be inversely proportional to the distance, not to the circumference.

But the equation (2) must still be modified if to be applied to the atomic nucleus.

Here, in fact, even if we admit that gravity operates, it would not be the only operating force, because it is not possible to neglect the electrostatic one.

Therefore I have supposed that in the atom the force of gravity and the electrostatic force were merged, giving rise to the *gravito-electric* force F_{ge} (or, if one prefers, *electro-gravitational* force) having this magnitude:

$$F_{ge} = \frac{GKMm}{R^2} \tag{3}$$

where K is the Coulomb's constant and G is the gravitational constant, so the eq. (2) becomes:

$$E = \frac{GKMm}{R^2} * \frac{1}{2\,\pi R} \tag{4}$$

Let's assume that in the nucleus there exists the gravito-electric **self**-energy, so we have to replace in eq. (4) m with M, i.e. with the mass of the nucleus itself, so that the eq. (4) becomes:

$$E = \frac{GKM^2}{2\,\pi R^3} \tag{5}$$

where R is the nuclear radius (for medium and heavy atoms, $R = 1.21 * \sqrt[3]{A}\ fm$, see references [2])

Now, in order to demonstrate the respect of the strong equivalence principle within the nucleus, we have to verify if the energy expressed in eq. (5) is equal to Mc^2, i.e. the total mass-energy, so we can write:

$$\frac{GKM^2}{2\,\pi R^3} = Mc^2 \tag{6}$$

Let's test now the eq. (6), considering the nucleus of bromum atom (^{79}Br), which contains 35 protons and 44 neutrons, whose radius — according to the empirical formula $R = 1.21151 * \sqrt[3]{A}\ fm$ — is 5.1983 $femtometers$:

$$\frac{(6.67433*10^{-11})*(8.9975*10^{9})*\{[(35*1.6726)+(44*1.6749)]*10^{-27}\}^2}{2*3.1415*(5.1983*10^{-15})^3} = [(35*1.6726)+(44*1.6749)]*10^{-27}*c^2$$

where c is the speed of light in vacuum: $299{,}792{,}458\ m/sec$

$$1.1884*10^{-8}\ joule\ = 1.1884*10^{-8}\ joule$$

$$\frac{E}{mc^2} = \frac{1.1884*10^{-8}}{1.1884*10^{-8}} = 1$$

- **Conclusions**

This study has revealed that the self-energy approach is a valid way to study the nuclear structure and the nuclear forces.

In particular the demonstration of the validity of strong equivalence principle even within the atomic nucleus confirms that the Einstein's theory of relativity can work even at this scale.

Further studies will verify whether this approach is always useful, or if it is valid only for the nucleus of the atom.

References

[1] S. G. Turyshev, *Experimental Tests of General Relativity*, Ann. Rev. Nucl. Part. Sci. 58 (2008) 207–248. arXiv:0806.1731, doi: http://doi.org/10.1146/annurev.nucl.58.020807.111839

[2] Bogdan Povh, Klaus Rith, Christoph Scholz, Frank Zetsche, Martin Lavelle, *Particles and Nuclei*. ISBN: 978-3-662-46321-5. DOI: https://doi.org/10.1007/978-3-662-46321-5