On some Ramanujan expressions: mathematical connections with ϕ and various formulas concerning several sectors of Cosmology and Black Holes Physics. XII

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Abstract

In this paper we have described some Ramanujan formulas and obtained some mathematical connections with ϕ and various equations concerning different sectors of Cosmology and Black Holes Physics.

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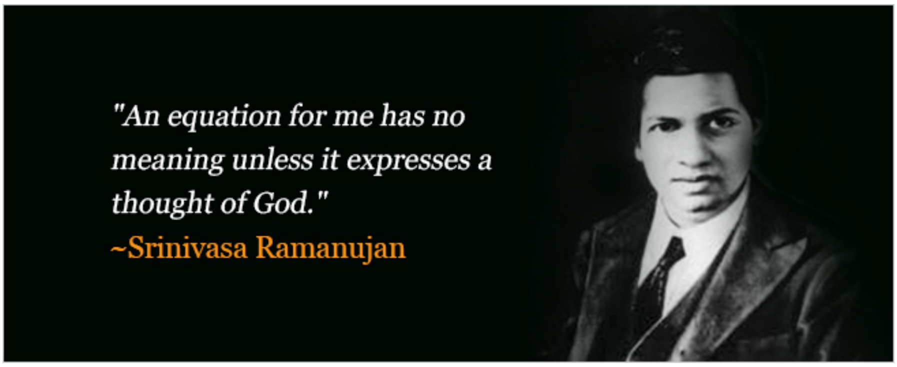
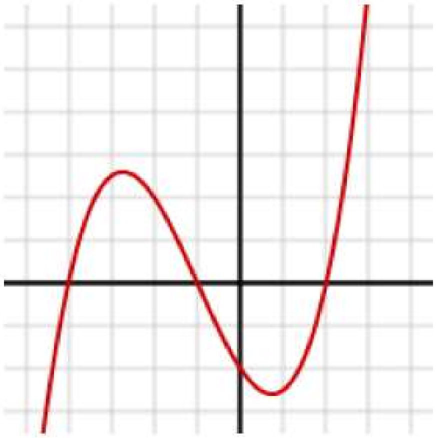
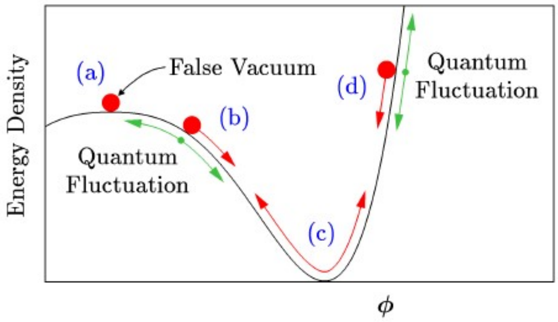


http://www.aicte-india.org/content/srinivasa-ramanujan





(arXiv:2002.01291v1 [astro-ph.GA] 4 Feb 2020)

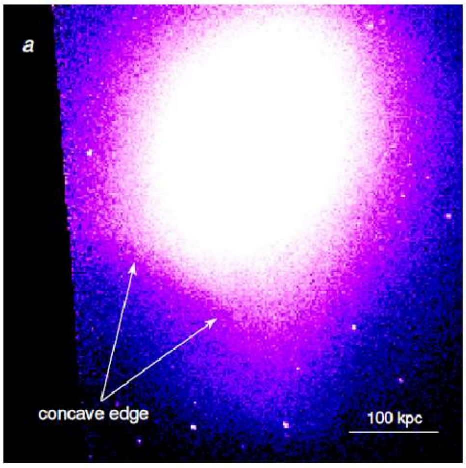


Figure 9. Chandra X-ray image of the Ophiuchus cluster in the 0.5-4 keV band, binned to 4'' pixels. (a) The concave edge, first reported by W16, is shown by arrows.

From:

Hawking radiation power equations for black holes - *Ravi Mistry, Sudhaker Upadhyay, Ahmed Farag Ali, Mir Faizal*
Nuclear Physics B 923 (2017) 378–393 - Received3 July 2017; accepted14 August 2017 - Available online19 September 2017 - Editor:Hubert Saleur

For greybody factor in the low frequency limit as given in (2), the Hawking radiation power equation for asymptotically flat black holes is given by

$$P_{low}^{(d+1)} = \frac{T_H}{2\pi}\int_0^\infty d\omega \frac{4\pi\omega^{d-1}R_H^{d-1}}{2^{d-1}[\Gamma(\frac{d}{2})]^2}\frac{\omega}{T_H(e^{\frac{\omega}{T_H}}-1)},$$

where we have utilized relation (1). After further simplification, this reduces to the following expression:

$$P_{low}^{(d+1)} = \frac{R_H^{d-1}}{2^{d-2}[\Gamma(\frac{d}{2})]^2}\int_0^\infty d\omega \frac{\omega^d}{e^{\frac{\omega}{T_H}}-1}. \tag{3}$$

In order to solve above equation, we recall Riemann Zeta function, which is given by,

$$\zeta(s) = \frac{1}{\Gamma(s)}\int_0^\infty dy \frac{y^{s-1}}{e^y-1} \quad \text{for} \ \ Re(s) > 1. \tag{4}$$

[2] Throughout the paper $d \to (d+1)$ represents that d is being replaced by $(d+1)$, where d stands for an arbitrary number of dimensions.

To match this Zeta function with the desired integral, we replace $y = \frac{\omega}{T_H}$ in above equation to get,

$$\zeta(s) = \frac{1}{T_H^s\Gamma(s)}\int_0^\infty d\omega \frac{\omega^{s-1}}{e^{\frac{\omega}{T_H}}-1} \quad \text{for} \ \ Re(s) > 1. \tag{5}$$

Now, exploiting (3) and (5), the Hawking radiation power equation for asymptotically flat black holes is given by

$$P_{low}^{(d+1)} = C_{low}^{(d+1)} T_H^{d+1} R_H^{d-1}, \tag{6}$$

where explicit form of $C_{low}^{(d+1)}$ is $C_{low}^{(d+1)} = \frac{\zeta(d+1)\Gamma(d+1)}{2^{d-2}[\Gamma(\frac{d}{2})]^2}$. Here we see that the Hawking radiation power equation depends on both the Hawking temperature and horizon radius with different power law. For example, for a black hole in four dimensional spacetime, the Hawking radiation power equation is proportional to T_H^4 and R_H^2. The behavior of Hawking radiation power with respect to Hawking temperature and horizon radius can been seen from Fig. 1.

From

$$C_{low}^{(d+1)} = \frac{\zeta(d+1)\Gamma(d+1)}{2^{d-2}[\Gamma(\frac{d}{2})]^2}.$$

we obtain:

(((zeta (3+1) gamma (3+1)))) / (((2(gamma (3/2))^2)))

Input:

$$\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}$$

$\zeta(s)$ is the Riemann zeta function

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{2\,\pi^3}{15}$$

Decimal approximation:

4.134170224039976023396842008946852693630038475451347692552...

4.13417022403…

Property:

$\frac{2\,\pi^3}{15}$ is a transcendental number

Alternative representations:

$$\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2} = \frac{e^{-\log(2)+\log(12)}\,\zeta(4,1)}{2\left(e^{-\log G(3/2)+\log G(5/2)}\right)^2}$$

$$\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2} = \frac{(1)_3\,\zeta(4,1)}{2\left((1)_{\frac{1}{2}}\right)^2}$$

$$\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2} = \frac{e^{-\log(2)+\log(12)}\,\zeta\left(4,\frac{1}{2}\right)}{\left(-1+2^4\right)\left(2\left(e^{-\log G(3/2)+\log G(5/2)}\right)^2\right)}$$

Series representations:

$$\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2} = -\frac{64}{15}\sum_{k=1}^{\infty}\frac{(-1)^k}{(-1+2k)^3}$$

$$\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2} = \frac{128}{15}\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^3$$

$$\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2} = \frac{2}{15}\left(\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^3$$

Integral representations:

$$\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2} = \frac{16}{15}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^3$$

$$\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2} = \frac{128}{15}\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)^3$$

$$\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2} = \frac{16}{15}\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^3$$

And:

[(((zeta (3+1) gamma (3+1)))) / (((2(gamma (3/2))^2)))]^1/3 + 13/10^3

Input:

$$\sqrt[3]{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}}+\frac{13}{10^3}$$

$\zeta(s)$ is the Riemann zeta function

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{13}{1000}+\sqrt[3]{\frac{2}{15}}\,\pi$$

Decimal approximation:

1.6179547221227671773676210303536923910414728863373806737 15...

1.617954722122…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Property:

$\frac{13}{1000} + \sqrt[3]{\frac{2}{15}}\,\pi$ is a transcendental number

Alternate forms:

$$\frac{39 + 200\sqrt[3]{2}\; 15^{2/3}\,\pi}{3000}$$

$$\frac{13 + 200\sqrt[3]{\frac{2}{3}}\; 5^{2/3}\,\pi}{1000}$$

Alternative representations:

$$\sqrt[3]{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}} + \frac{13}{10^3} = \frac{13}{10^3} + \sqrt[3]{\frac{(1)_3\,\zeta(4,1)}{2\left((1)_{\frac{1}{2}}\right)^2}}$$

$$\sqrt[3]{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}} + \frac{13}{10^3} = \frac{13}{10^3} + \sqrt[3]{\frac{e^{-\log(2)+\log(12)}\,\zeta(4,1)}{2\left(e^{-\log G(3/2)+\log G(5/2)}\right)^2}}$$

$$\sqrt[3]{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}} + \frac{13}{10^3} = \frac{13}{10^3} + \sqrt[3]{\frac{e^{\log\Gamma(4)}\,\zeta(4,1)}{2\left(e^{\log\Gamma(3/2)}\right)^2}}$$

Series representations:

$$\sqrt[3]{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}} + \frac{13}{10^3} = \frac{13}{1000} + 4\sqrt[3]{\frac{2}{15}}\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt[3]{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}}+\frac{13}{10^3}=$$

$$\frac{13}{1000}+\sum_{k=0}^{\infty}-\frac{4\,(-1)^k\sqrt[3]{\frac{2}{3}}\;5^{-4/3-2k}\times 239^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt[3]{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}}+\frac{13}{10^3}=\frac{13}{1000}+\sqrt[3]{\frac{2}{15}}\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations:

$$\sqrt[3]{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}}+\frac{13}{10^3}=\frac{13}{1000}+4\sqrt[3]{\frac{2}{15}}\int_0^1\sqrt{1-t^2}\,dt$$

$$\sqrt[3]{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}}+\frac{13}{10^3}=\frac{13}{1000}+2\sqrt[3]{\frac{2}{15}}\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt[3]{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}}+\frac{13}{10^3}=\frac{13}{1000}+2\sqrt[3]{\frac{2}{15}}\int_0^\infty\frac{1}{1+t^2}\,dt$$

[(((zeta (3+1) gamma (3+1)))) / (((2(gamma (3/2))^2)))]^1/2 -(18+11)/10^2-11/10^3

Input:

$$\sqrt{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}}-\frac{18+11}{10^2}-\frac{11}{10^3}$$

$\zeta(s)$ is the Riemann zeta function

$\Gamma(x)$ is the gamma function

Exact result:

$$\sqrt{\frac{2}{15}}\,\pi^{3/2}-\frac{301}{1000}$$

Decimal approximation:

1.7322659009681876882063124673837713424456033201873001 68139...

1.73226590096…. ≈ $\sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3})\, M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

Property:

$-\frac{301}{1000} + \sqrt{\frac{2}{15}}\ \pi^{3/2}$ is a transcendental number

Alternate forms:

$$\frac{200\sqrt{30}\ \pi^{3/2} - 903}{3000}$$

$$\frac{200\sqrt{\frac{10}{3}}\ \pi^{3/2} - 301}{1000}$$

Alternative representations:

$$\sqrt{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}} - \frac{18+11}{10^2} - \frac{11}{10^3} = -\frac{29}{10^2} - \frac{11}{10^3} + \sqrt{\frac{(1)_3\,\zeta(4,1)}{2\left((1)_{\frac{1}{2}}\right)^2}}$$

$$\sqrt{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}} - \frac{18+11}{10^2} - \frac{11}{10^3} = -\frac{29}{10^2} - \frac{11}{10^3} + \sqrt{\frac{e^{-\log(2)+\log(12)}\,\zeta(4,1)}{2\left(e^{-\log G(3/2)+\log G(5/2)}\right)^2}}$$

$$\sqrt{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}} - \frac{18+11}{10^2} - \frac{11}{10^3} = -\frac{29}{10^2} - \frac{11}{10^3} + \sqrt{\frac{e^{\log\Gamma(4)}\,\zeta(4,1)}{2\left(e^{\log\Gamma(3/2)}\right)^2}}$$

Series representations:

$$\sqrt{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}} - \frac{18+11}{10^2} - \frac{11}{10^3} = \frac{-301 + 500\sqrt{2}\sqrt{\frac{\exp\left(-\sum_{k=1}^{\infty}\log\left(1-\frac{1}{(p_k)^4}\right)\right)\sum_{k=0}^{\infty}\frac{(4-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^2}}}{1000} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\sqrt{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}} - \frac{18+11}{10^2} - \frac{11}{10^3} = \frac{-301 + 500\sqrt{2}\sqrt{\frac{\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(4-z_0)^{k_2}\,\Gamma^{(k_2)}(z_0)}{k_2!\,k_1^4}}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^2}}}{1000}$$

for $(z_0 \notin \mathbb{Z}$ or $z_0 > 0)$

$$\sqrt{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}} - \frac{18+11}{10^2} - \frac{11}{10^3} = \frac{-301 + 500\sqrt{2}\sqrt{\frac{\exp\left(\sum_{k=1}^{\infty}\frac{P(4k)}{k}\right)\sum_{k=0}^{\infty}\frac{(4-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^2}}}{1000} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

Integral representations:

$$\sqrt{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}} - \frac{18+11}{10^2} - \frac{11}{10^3} = -\frac{301}{1000} + \frac{\sqrt{\frac{\Gamma(4)}{2!\,\Gamma\left(\frac{3}{2}\right)^2}\int_0^1\frac{\log^4(1-t^3)}{t^4}\,dt}}{2\sqrt{2}}$$

$$\sqrt{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}} - \frac{18+11}{10^2} - \frac{11}{10^3} = -\frac{301}{1000} + \frac{1}{4}\sqrt{\frac{\left(\oint_L\frac{e^t}{t^{3/2}}\,dt\right)^2}{2\,i\,\pi\oint_L\frac{e^t}{t^4}\,dt}\int_0^1\frac{\log^4(1-t^3)}{t^4}\,dt}$$

$$\sqrt{\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}} - \frac{18+11}{10^2} - \frac{11}{10^3} = -\frac{301}{1000} + \frac{1}{4}\sqrt{\frac{i\left(\oint_L \frac{e^{-t}}{(-t)^{3/2}}\,dt\right)^2}{2\pi\oint_L \frac{e^{-t}}{t^4}\,dt}\int_0^1 \frac{\log^4(1-t^3)}{t^4}\,dt}$$

2*exp[(((zeta (3+1) gamma (3+1)))) / (((2(gamma (3/2))^2)))]+1-1/golden ratio

Input:

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right) + 1 - \frac{1}{\phi}$$

$\zeta(s)$ is the Riemann zeta function

$\Gamma(x)$ is the gamma function

ϕ is the golden ratio

Exact result:

$$-\frac{1}{\phi} + 1 + 2\,e^{(2\pi^3)/15}$$

Decimal approximation:

125.2574864452811602983398846289192301847917314302005528092...

125.2574864… result very near to the Higgs boson mass 125.18 GeV

Alternate forms:

$$\frac{\phi + 2\,e^{(2\pi^3)/15}\,\phi - 1}{\phi}$$

$$1 - \frac{2}{1+\sqrt{5}} + 2\,e^{(2\pi^3)/15}$$

$$\frac{1}{2}\left(3-\sqrt{5}\right) + 2\,e^{(2\pi^3)/15}$$

Alternative representations:

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+1-\frac{1}{\phi}=1+2\exp\left(\frac{e^{-\log(2)+\log(12)}\,\zeta(4,1)}{2\left(e^{-\log G(3/2)+\log G(5/2)}\right)^2}\right)-\frac{1}{\phi}$$

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+1-\frac{1}{\phi}=1+2\exp\left(\frac{(1)_3\,\zeta(4,1)}{2\left((1)_{\frac{1}{2}}\right)^2}\right)-\frac{1}{\phi}$$

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+1-\frac{1}{\phi}=1+2\exp\left(\frac{(1)_3\,\zeta\left(4,\frac{1}{2}\right)}{\left(-1+2^4\right)\left(2\left((1)_{\frac{1}{2}}\right)^2\right)}\right)-\frac{1}{\phi}$$

Series representations:

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+1-\frac{1}{\phi}=1+2\,e^{128/15\left(\sum_{k=0}^{\infty}(-1)^k/(1+2k)\right)^3}-\frac{1}{\phi}$$

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+1-\frac{1}{\phi}=1-\frac{1}{\phi}+2\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{(2\pi^3)/15}$$

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+1-\frac{1}{\phi}=1-\frac{1}{\phi}+2\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{(2\pi^3)/15}$$

Integral representations:

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+1-\frac{1}{\phi}=1+2\,e^{16/15\left(\int_0^{\infty}1/(1+t^2)\,dt\right)^3}-\frac{1}{\phi}$$

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+1-\frac{1}{\phi}=1+2\,e^{128/15\left(\int_0^1\sqrt{1-t^2}\,dt\right)^3}-\frac{1}{\phi}$$

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+1-\frac{1}{\phi}=1+2\,e^{16/15\left(\int_0^1 1/\sqrt{1-t^2}\,dt\right)^3}-\frac{1}{\phi}$$

2*exp[(((zeta (3+1) gamma (3+1)))) / (((2(gamma (3/2))^2)))]+13+golden ratio

Input:

$$2 \exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right) + 13 + \phi$$

$\zeta(s)$ is the Riemann zeta function

$\Gamma(x)$ is the gamma function

ϕ is the golden ratio

Exact result:

$$\phi + 13 + 2\,e^{(2\pi^3)/15}$$

Decimal approximation:

139.4935544227809499947490582976505064202323497898120785334...

139.493554422… result practically equal to the rest mass of Pion meson 139.57 MeV

Alternate forms:

$$\frac{1}{2}\left(27 + \sqrt{5} + 4\,e^{(2\pi^3)/15}\right)$$

$$\frac{27}{2} + \frac{\sqrt{5}}{2} + 2\,e^{(2\pi^3)/15}$$

$$\frac{1}{2}\left(27 + \sqrt{5}\right) + 2\,e^{(2\pi^3)/15}$$

Alternative representations:

$$2 \exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right) + 13 + \phi = 13 + \phi + 2 \exp\left(\frac{e^{-\log(2)+\log(12)}\,\zeta(4,1)}{2\left(e^{-\log G(3/2)+\log G(5/2)}\right)^2}\right)$$

$$2 \exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right) + 13 + \phi = 13 + \phi + 2 \exp\left(\frac{(1)_3\,\zeta(4,1)}{2\left((1)_{\frac{1}{2}}\right)^2}\right)$$

$$2 \exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right) + 13 + \phi = 13 + \phi + 2 \exp\left(\frac{(1)_3\,\zeta\left(4,\frac{1}{2}\right)}{\left(-1+2^4\right)\left(2\left((1)_{\frac{1}{2}}\right)^2\right)}\right)$$

Series representations:

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+13+\phi = 13+2\,e^{128/15\left(\sum_{k=0}^{\infty}(-1)^k/(1+2k)\right)^3}+\phi$$

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+13+\phi = 13+\phi+2\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{(2\pi^3)/15}$$

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+13+\phi = 13+\phi+2\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{(2\pi^3)/15}$$

Integral representations:

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+13+\phi = 13+2\,e^{16/15\left(\int_0^{\infty}1/(1+t^2)\,dt\right)^3}+\phi$$

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+13+\phi = 13+2\,e^{128/15\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)^3}+\phi$$

$$2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+13+\phi = 13+2\,e^{16/15\left(\int_0^{1}1/\sqrt{1-t^2}\,dt\right)^3}+\phi$$

27*1/2*(((2*exp[(((zeta (3+1) gamma (3+1)))) / (((2(gamma (3/2))^2)))]+4-1/golden ratio)))-5/2

Input:

$$27\times\frac{1}{2}\left(2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+4-\frac{1}{\phi}\right)-\frac{5}{2}$$

$\zeta(s)$ is the Riemann zeta function

$\Gamma(x)$ is the gamma function

ϕ is the golden ratio

Exact result:

$$\frac{27}{2}\left(-\frac{1}{\phi}+4+2\,e^{(2\pi^3)/15}\right)-\frac{5}{2}$$

Decimal approximation:

1728.976067011295664027588442490409607494688374307707462924...

1728.976067011… ≈ 1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

Alternate forms:

$$-\frac{27}{2\,\phi}+\frac{103}{2}+27\,e^{(2\pi^3)/15}$$

$$\frac{103\,\phi+54\,e^{(2\pi^3)/15}\,\phi-27}{2\,\phi}$$

$$\frac{233}{4}-\frac{27\sqrt{5}}{4}+27\,e^{(2\pi^3)/15}$$

Alternative representations:

$$\frac{27}{2}\left(2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+4-\frac{1}{\phi}\right)-\frac{5}{2}=$$
$$-\frac{5}{2}+\frac{27}{2}\left(4+2\exp\left(\frac{e^{-\log(2)+\log(12)}\,\zeta(4,1)}{2\left(e^{-\log G(3/2)+\log G(5/2)}\right)^2}\right)-\frac{1}{\phi}\right)$$

$$\frac{27}{2}\left(2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+4-\frac{1}{\phi}\right)-\frac{5}{2}=-\frac{5}{2}+\frac{27}{2}\left(4+2\exp\left(\frac{(1)_3\,\zeta(4,1)}{2\left((1)_{\frac{1}{2}}\right)^2}\right)-\frac{1}{\phi}\right)$$

$$\frac{27}{2}\left(2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+4-\frac{1}{\phi}\right)-\frac{5}{2}=$$
$$-\frac{5}{2}+\frac{27}{2}\left(4+2\exp\left(\frac{(1)_3\,\zeta\left(4,\frac{1}{2}\right)}{\left(-1+2^4\right)\left(2\left((1)_{\frac{1}{2}}\right)^2\right)}\right)-\frac{1}{\phi}\right)$$

Series representations:

$$\frac{27}{2}\left(2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+4-\frac{1}{\phi}\right)-\frac{5}{2}=\frac{103}{2}+27\,e^{128/15\left(\sum_{k=0}^{\infty}(-1)^k/(1+2k)\right)^3}-\frac{27}{2\,\phi}$$

$$\frac{27}{2}\left(2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+4-\frac{1}{\phi}\right)-\frac{5}{2}=\frac{233}{4}-\frac{27\sqrt{5}}{4}+27\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{(2\pi^3)/15}$$

$$\frac{27}{2}\left(2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+4-\frac{1}{\phi}\right)-\frac{5}{2}=\frac{103}{2}-\frac{27}{2\,\phi}+27\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{(2\pi^3)/15}$$

Integral representations:

$$\frac{27}{2}\left(2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+4-\frac{1}{\phi}\right)-\frac{5}{2}=\frac{103}{2}+27\,e^{16/15\left(\int_0^{\infty}1/(1+t^2)\,dt\right)^3}-\frac{27}{2\,\phi}$$

$$\frac{27}{2}\left(2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+4-\frac{1}{\phi}\right)-\frac{5}{2}=\frac{103}{2}+27\,e^{128/15\left(\int_0^1\sqrt{1-t^2}\,dt\right)^3}-\frac{27}{2\,\phi}$$

$$\frac{27}{2}\left(2\exp\left(\frac{\zeta(3+1)\,\Gamma(3+1)}{2\,\Gamma\left(\frac{3}{2}\right)^2}\right)+4-\frac{1}{\phi}\right)-\frac{5}{2}=\frac{103}{2}+27\,e^{16/15\left(\int_0^1 1/\sqrt{1-t^2}\,dt\right)^3}-\frac{27}{2\,\phi}$$

Now, we have that:

Finally, using (15) and (16), we get simplified expression for the radiation power equation corresponding to greybody factor in low frequency limit as

$$P_{low}^{(d+1)}=\sum_{n=1}^{\infty}C_{low}^{(d+1)}\frac{T_H^{nd-n+2}}{k^{2n(d-1)}R_H^{n(d-1)}}, \tag{17}$$

where, $C_{low}^{(d+1)}=\frac{2}{\pi}\left[(-1)^{n+1}\frac{n\pi^n}{2^{n(d-1)}\left[\Gamma\left(\frac{d}{2}\right)\right]^{2n}}\zeta(nd-n+2)\Gamma(nd-n+2)\right]$. Here, the radiation power equation has an infinite sum series with terms depending on Hawking temperature and horizon radius differently. Also, we notice that, contrary to flat spacetime case, the radiation power equation depends on horizon radius with inverse power law.

$$C_{low}^{(d+1)}=\frac{2}{\pi}\left[(-1)^{n+1}\frac{n\pi^n}{2^{n(d-1)}\left[\Gamma\left(\frac{d}{2}\right)\right]^{2n}}\zeta(nd-n+2)\Gamma(nd-n+2)\right].$$

For n = 3 and d = 3

2/Pi [((((3*Pi^3))) / (((2^(3*(3-1))(gamma (3/2))^6))) zeta(3*3-3+2) gamma (3*3-3+2)]

Input:

$$\frac{2}{\pi}\left(\frac{3\,\pi^3}{2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6}\,\zeta(3\times3-3+2)\,\Gamma(3\times3-3+2)\right)$$

$\Gamma(x)$ is the gamma function

$\zeta(s)$ is the Riemann zeta function

Exact result:

$$\frac{16\pi^7}{5}$$

Decimal approximation:

9664.938328885734616045460777830533862135783920945287240813...

9664.93832888…

Property:

$\frac{16\pi^7}{5}$ is a transcendental number

Alternative representations:

$$\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi} = \frac{6(1)_7\pi^3\zeta(8,1)}{\pi\left(2^6\left((1)_{\frac{1}{2}}\right)^6\right)}$$

$$\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi} = \frac{6e^{-\log(24883200)+\log(125411328000)}\pi^3\zeta(8,1)}{\pi\left(2^6\left(e^{-\log G(3/2)+\log G(5/2)}\right)^6\right)}$$

$$\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi} = \frac{6e^{\log\Gamma(8)}\pi^3\zeta(8,1)}{\pi\left(2^6\left(e^{\log\Gamma(3/2)}\right)^6\right)}$$

Series representations:

$$\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi} = \frac{262144}{5}\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^7$$

$$\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi} =$$
$$\frac{16}{5}\left(\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^7$$

$$\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi} = \frac{16}{5}\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^7$$

Integral representations:

$$\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi} = \frac{2048}{5}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^7$$

$$\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi} = \frac{2048}{5}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^7$$

$$\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi} = \frac{262144}{5}\left(\int_0^1 \sqrt{1-t^2}\,dt\right)^7$$

-233-34 + 2/Pi [(((3*Pi^3))) / (((2^(3*(3-1))(gamma (3/2))^6))) zeta(3*3-3+2) gamma (3*3-3+2)]

Input:

$$-233-34+\frac{2}{\pi}\left(\frac{3\pi^3}{2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6}\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2)\right)$$

$\Gamma(x)$ is the gamma function

$\zeta(s)$ is the Riemann zeta function

Exact result:

$$\frac{16\pi^7}{5}-267$$

Decimal approximation:

9397.938328885734616045460777830533862135783920945287240813...

9397.93832888…. result practically equal to the rest mass of Bottom eta meson 9398

Property:

$-267+\frac{16\pi^7}{5}$ is a transcendental number

Alternate form:

$$\frac{1}{5}(16\pi^7-1335)$$

Alternative representations:

$$-233-34+\frac{\left(\left(3\pi^3\right)\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)2}{\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)\pi} = -267+\frac{6\,(1)_7\,\pi^3\,\zeta(8,1)}{\pi\left(2^6\left((1)_{\frac{1}{2}}\right)^6\right)}$$

$$-233-34+\frac{\left(\left(3\pi^3\right)\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)2}{\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)\pi} =$$
$$-267+\frac{6\,e^{-\log(24\,883\,200)+\log(125\,411\,328\,000)}\,\pi^3\,\zeta(8,1)}{\pi\left(2^6\left(e^{-\log G(3/2)+\log G(5/2)}\right)^6\right)}$$

$$-233-34+\frac{\left(\left(3\pi^3\right)\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)2}{\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)\pi} = -267+\frac{6\,e^{\log\Gamma(8)}\,\pi^3\,\zeta(8,1)}{\pi\left(2^6\left(e^{\log\Gamma(3/2)}\right)^6\right)}$$

Series representations:

$$-233-34+\frac{\left(\left(3\pi^3\right)\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)2}{\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)\pi} = -267+\frac{262\,144}{5}\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^7$$

$$-233-34+\frac{\left(\left(3\pi^3\right)\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)2}{\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)\pi} =$$
$$-267+\frac{16}{5}\left(\sum_{k=0}^{\infty}-\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^7$$

$$-233-34+\frac{\left(\left(3\pi^3\right)\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)2}{\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)\pi} =$$
$$-267+\frac{16}{5}\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^7$$

Integral representations:

$$-233-34+\frac{\left(\left(3\pi^3\right)\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)2}{\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)\pi} = -267+\frac{2048}{5}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^7$$

$$-233-34+\frac{\left(\left(3\pi^3\right)\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)2}{\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)\pi} = -267+\frac{2048}{5}\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^7$$

$$-233-34+\frac{\left(\left(3\pi^3\right)\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)2}{\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)\pi}=$$

$$-267+\frac{262\,144}{5}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^7$$

(((2/Pi [(((3*Pi^3))) / (((2^(3*(3-1))(gamma (3/2))^6))) zeta(3*3-3+2) gamma (3*3-3+2)])))^1/2+29+11+sqrt2

Input:

$$\sqrt{\frac{2}{\pi}\left(\frac{3\pi^3}{2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6}\,\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)}+29+11+\sqrt{2}$$

$\Gamma(x)$ is the gamma function

$\zeta(s)$ is the Riemann zeta function

Exact result:

$$40+\sqrt{2}+\frac{4\pi^{7/2}}{\sqrt{5}}$$

Decimal approximation:

139.72463177369866672619483912793142674705907874126441098 00...

139.72463177… result practically equal to the rest mass of Pion meson 139.57 MeV

Property:

$40+\sqrt{2}+\frac{4\pi^{7/2}}{\sqrt{5}}$ is a transcendental number

Alternate forms:

$$\frac{1}{5}\left(200+5\sqrt{2}+4\sqrt{5}\,\pi^{7/2}\right)$$

$$\frac{\sqrt{5}\,(40+\sqrt{2})+4\pi^{7/2}}{\sqrt{5}}$$

Alternative representations:

$$\sqrt{\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi}}+29+11+\sqrt{2}=$$

$$40+\sqrt{\frac{6(1)_7\pi^3\zeta(8,1)}{\pi\left(2^6\left((1)_{\frac{1}{2}}\right)^6\right)}}+\sqrt{2}$$

$$\sqrt{\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi}}+29+11+\sqrt{2}=$$

$$40+\sqrt{\frac{6e^{\log\Gamma(8)}\pi^3\zeta(8,1)}{\pi\left(2^6\left(e^{\log\Gamma(3/2)}\right)^6\right)}}+\sqrt{2}$$

$$\sqrt{\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi}}+29+11+\sqrt{2}=$$

$$40+\sqrt{\frac{6e^{-\log(24\,883\,200)+\log(125\,411\,328\,000)}\pi^3\zeta(8,1)}{\pi\left(2^6\left(e^{-\log G(3/2)+\log G(5/2)}\right)^6\right)}}+\sqrt{2}$$

Series representations:

$$\sqrt{\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi}}+29+11+\sqrt{2}=$$

$$\frac{1}{8}\left(320+8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$\left.\sqrt{6}\sqrt{\frac{\pi^2\exp\left(-\sum_{k=1}^{\infty}\log\left(1-\frac{1}{(p_k)^8}\right)\right)\sum_{k=0}^{\infty}\frac{(8-z_0)^k\Gamma^{(k)}(z_0)}{k!}}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)^6}}\right)$$

for ($x\in\mathbb{R}$ and ($z_0\notin\mathbb{Z}$ or $z_0>0$) and $x<0$)

$$\sqrt{\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi}+29+11+\sqrt{2}} =$$

$$\frac{1}{8}\left(320+8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+\right.$$

$$\left.\sqrt{6}\sqrt{\frac{\pi^2\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(8-z_0)^{k_2}\Gamma^{(k_2)}(z_0)}{k_2!k_1^8}}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)^6}}\right)$$

for ($x \in \mathbb{R}$ and ($z_0 \notin \mathbb{Z}$ or $z_0 > 0$) and $x < 0$)

$$\sqrt{\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi}+29+11+\sqrt{2}} =$$

$$\frac{1}{8}\left(320+8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+\right.$$

$$\left.\sqrt{6}\sqrt{\frac{\pi^2\exp\left(\sum_{k=1}^{\infty}\frac{P(8k)}{k}\right)\sum_{k=0}^{\infty}\frac{(8-z_0)^k\Gamma^{(k)}(z_0)}{k!}}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)^6}}\right)$$

for ($x \in \mathbb{R}$ and ($z_0 \notin \mathbb{Z}$ or $z_0 > 0$) and $x < 0$)

Integral representations:

$$\sqrt{\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi}+29+11+\sqrt{2}} =$$

$$40+\frac{1}{16}\sqrt{3}\sqrt{\frac{\pi^2\Gamma(8)}{6!\,\Gamma\left(\frac{3}{2}\right)^6}\int_0^1\frac{\log^8(1-t^7)}{t^8}\,dt}+\sqrt{2}$$

$$\sqrt{\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi}}+29+11+\sqrt{2}=$$

$$40+\frac{1}{64}\sqrt{\frac{3}{2}}\sqrt{\frac{\left(\oint_L \frac{e^t}{t^{3/2}}\,dt\right)^6}{720\,i\,\pi^3\oint_L \frac{e^t}{t^8}\,dt}\int_0^1 \frac{\log^8(1-t^7)}{t^8}\,dt}+\sqrt{2}$$

$$\sqrt{\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi}}+29+11+\sqrt{2}=$$

$$40+4\sqrt{\frac{3}{127}}\sqrt{\frac{\left(\oint_L \frac{e^t}{t^{3/2}}\,dt\right)^6}{i\,\pi^3\oint_L \frac{e^t}{t^8}\,dt}\int_0^\infty \frac{\cos(8\tan^{-1}(t))}{(1+t^2)^4\cosh\left(\frac{\pi t}{2}\right)}\,dt}+\sqrt{2}$$

(((2/Pi [(((3*Pi^3))) / (((2^(3*(3-1))(gamma (3/2))^6))) zeta(3*3-3+2) gamma (3*3-3+2)])))^1/2+29-2

Input:

$$\sqrt{\frac{2}{\pi}\left(\frac{3\pi^3}{2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6}\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2)\right)}+29-2$$

$\Gamma(x)$ is the gamma function

$\zeta(s)$ is the Riemann zeta function

Exact result:

$$27+\frac{4\pi^{7/2}}{\sqrt{5}}$$

Decimal approximation:

125.3104182113255716773931504037217286684894068658874629068...

125.310418211… result very near to the Higgs boson mass 125.18 GeV

Property:

$27+\frac{4\pi^{7/2}}{\sqrt{5}}$ is a transcendental number

Alternate forms:

$$\frac{1}{5}\left(135+4\sqrt{5}\,\pi^{7/2}\right)$$

$$\frac{27\sqrt{5}+4\pi^{7/2}}{\sqrt{5}}$$

Alternative representations:

$$\sqrt{\frac{((3\pi^3)\zeta(3\times3-3+2)\Gamma(3\times3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi}}+29-2=27+\sqrt{\frac{6(1)_7\pi^3\zeta(8,1)}{\pi\left(2^6\left((1)_{\frac{1}{2}}\right)^6\right)}}$$

$$\sqrt{\frac{((3\pi^3)\zeta(3\times3-3+2)\Gamma(3\times3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi}}+29-2=27+\sqrt{\frac{6e^{\log\Gamma(8)}\pi^3\zeta(8,1)}{\pi\left(2^6\left(e^{\log\Gamma(3/2)}\right)^6\right)}}$$

$$\sqrt{\frac{((3\pi^3)\zeta(3\times3-3+2)\Gamma(3\times3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi}}+29-2=$$
$$27+\sqrt{\frac{6e^{-\log(24883200)+\log(125411328000)}\pi^3\zeta(8,1)}{\pi\left(2^6\left(e^{-\log G(3/2)+\log G(5/2)}\right)^6\right)}}$$

Series representations:

$$\sqrt{\frac{((3\pi^3)\zeta(3\times3-3+2)\Gamma(3\times3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi}}+29-2=$$
$$\frac{1}{8}\left(216+\sqrt{6}\sqrt{\frac{\pi^2\exp\left(-\sum_{k=1}^{\infty}\log\left(1-\frac{1}{(p_k)^8}\right)\right)\sum_{k=0}^{\infty}\frac{(8-z_0)^k\Gamma^{(k)}(z_0)}{k!}}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)^6}}\right)$$
for $(z_0\notin\mathbb{Z}$ or $z_0>0)$

$$\sqrt{\frac{((3\pi^3)\zeta(3\times3-3+2)\Gamma(3\times3-3+2))2}{\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)\pi}}+29-2=$$
$$\frac{1}{8}\left(216+\sqrt{6}\sqrt{\frac{\pi^2\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(8-z_0)^{k_2}\Gamma^{(k_2)}(z_0)}{k_2!k_1^8}}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)^6}}\right)$$ for $(z_0\notin\mathbb{Z}$ or $z_0>0)$

$$\sqrt{\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{(2^{3(3-1)}\Gamma(\frac{3}{2})^6)\pi}+29-2} =$$

$$\frac{1}{8}\left(216+\sqrt{6}\sqrt{\frac{\pi^2\exp\left(\sum_{k=1}^{\infty}\frac{P(8k)}{k}\right)\sum_{k=0}^{\infty}\frac{(8-z_0)^k\Gamma^{(k)}(z_0)}{k!}}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)^6}}\right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

Integral representations:

$$\sqrt{\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{(2^{3(3-1)}\Gamma(\frac{3}{2})^6)\pi}+29-2} =$$

$$27+\frac{1}{16}\sqrt{3}\sqrt{\frac{\pi^2\Gamma(8)}{6!\,\Gamma(\frac{3}{2})^6}\int_0^1\frac{\log^8(1-t^7)}{t^8}\,dt}$$

$$\sqrt{\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{(2^{3(3-1)}\Gamma(\frac{3}{2})^6)\pi}+29-2} =$$

$$27+\frac{1}{64}\sqrt{\frac{3}{2}}\sqrt{\frac{\left(\oint_L\frac{e^t}{t^{3/2}}\,dt\right)^6}{720\,i\,\pi^3\oint_L\frac{e^t}{t^8}\,dt}\int_0^1\frac{\log^8(1-t^7)}{t^8}\,dt}$$

$$\sqrt{\frac{((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2))2}{(2^{3(3-1)}\Gamma(\frac{3}{2})^6)\pi}+29-2} =$$

$$27+4\sqrt{\frac{3}{127}}\sqrt{\frac{\left(\oint_L\frac{e^t}{t^{3/2}}\,dt\right)^6}{i\,\pi^3\oint_L\frac{e^t}{t^8}\,dt}\int_0^\infty\frac{\cos(8\tan^{-1}(t))}{(1+t^2)^4\cosh\left(\frac{\pi t}{2}\right)}\,dt}$$

27*1/2*((((((2/Pi [(((3*Pi^3))) / (((2^(3*(3-1))(gamma (3/2))^6))) zeta(3*3-3+2) gamma (3*3-3+2)]))))^1/2+29+1/golden ratio)))+2

Input:

$$27\times\frac{1}{2}\left(\sqrt{\frac{2}{\pi}\left(\frac{3\pi^3}{2^{3(3-1)}\Gamma(\frac{3}{2})^6}\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2)\right)}+29+\frac{1}{\phi}\right)+2$$

$\Gamma(x)$ is the gamma function

$\zeta(s)$ is the Riemann zeta function

ϕ is the golden ratio

Exact result:

$$\frac{27}{2}\left(\frac{1}{\phi}+29+\frac{4\pi^{7/2}}{\sqrt{5}}\right)+2$$

Decimal approximation:

1729.034104701018798095569452714179451613831166616858547881...

1729.034104701…

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Property:

$2+\frac{27}{2}\left(29+\frac{1}{\phi}+\frac{4\pi^{7/2}}{\sqrt{5}}\right)$ is a transcendental number

Alternate forms:

$$\frac{27}{2\phi}+\frac{787}{2}+\frac{54\pi^{7/2}}{\sqrt{5}}$$

$$\frac{787}{2}+\frac{27}{1+\sqrt{5}}+\frac{54\pi^{7/2}}{\sqrt{5}}$$

$$2+\frac{27}{2}\left(\frac{1}{2}\left(57+\sqrt{5}\right)+\frac{4\pi^{7/2}}{\sqrt{5}}\right)$$

Alternative representations:

$$\frac{27}{2}\left(\sqrt{\frac{2\left(\left(3\pi^3\right)\zeta(3\times3-3+2)\,\Gamma(3\times3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2=$$

$$2+\frac{27}{2}\left(29+\frac{1}{\phi}+\sqrt{\frac{6\,(1)_7\,\pi^3\,\zeta(8,1)}{\pi\left(2^6\left((1)_{\frac{1}{2}}\right)^6\right)}}\right)$$

$$\frac{27}{2}\left(\sqrt{\frac{2\left(\left(3\pi^3\right)\zeta(3\times3-3+2)\,\Gamma(3\times3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2=$$

$$2+\frac{27}{2}\left(29+\frac{1}{\phi}+\sqrt{\frac{6\,e^{\log\Gamma(8)}\,\pi^3\,\zeta(8,1)}{\pi\left(2^6\left(e^{\log\Gamma(3/2)}\right)^6\right)}}\right)$$

$$\frac{27}{2}\left(\sqrt{\frac{2\left(\left(3\pi^3\right)\zeta(3\times3-3+2)\,\Gamma(3\times3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2=$$

$$2+\frac{27}{2}\left(29+\frac{1}{\phi}+\sqrt{\frac{6\,e^{-\log(24883200)+\log(125411328000)}\,\pi^3\,\zeta(8,1)}{\pi\left(2^6\left(e^{-\log G(3/2)+\log G(5/2)}\right)^6\right)}}\right)$$

Series representations:

$$\frac{27}{2}\left(\sqrt{\frac{2\left(\left(3\pi^3\right)\zeta(3\times3-3+2)\,\Gamma(3\times3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2=$$

$$\frac{216+6296\,\phi+27\sqrt{6}\,\phi\sqrt{\frac{\pi^2\exp\left(-\sum_{k=1}^{\infty}\log\left(1-\frac{1}{(p_k)^8}\right)\right)\sum_{k=0}^{\infty}\frac{(8-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^6}}}{16\,\phi}$$

for $(z_0 \notin \mathbb{Z}$ or $z_0 > 0)$

$$\frac{27}{2}\left(\sqrt{\frac{2\left(\left(3\pi^3\right)\zeta(3\times3-3+2)\,\Gamma(3\times3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2=$$

$$\frac{216+6296\,\phi+27\sqrt{6}\,\phi\sqrt{\frac{\pi^2\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(8-z_0)^{k_2}\,\Gamma^{(k_2)}(z_0)}{k_2!\,k_1^8}}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^6}}}{16\,\phi}$$ for $(z_0 \notin \mathbb{Z}$ or $z_0 > 0)$

$$\frac{27}{2}\left(\sqrt{\frac{2\left(\left(3\pi^3\right)\zeta(3\times3-3+2)\,\Gamma(3\times3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2=$$

$$\frac{216+6296\phi+27\sqrt{6}\,\phi\sqrt{\frac{\pi^2\exp\left(\sum_{k=1}^{\infty}\frac{P(8\,k)}{k}\right)\sum_{k=0}^{\infty}\frac{(8-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^6}}}{16\,\phi}\quad\text{for } (z_0\notin\mathbb{Z}\text{ or } z_0>0)$$

Integral representations:

$$\frac{27}{2}\left(\sqrt{\frac{2\left(\left(3\pi^3\right)\zeta(3\times3-3+2)\,\Gamma(3\times3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2=$$

$$\frac{787}{2}+\frac{27}{2\,\phi}+\frac{27}{32}\sqrt{3}\sqrt{\frac{\pi^2\,\Gamma(8)}{6!\,\Gamma\left(\frac{3}{2}\right)^6}\int_0^1\frac{\log^8\left(1-t^7\right)}{t^8}\,dt}$$

$$\frac{27}{2}\left(\sqrt{\frac{2\left(\left(3\pi^3\right)\zeta(3\times3-3+2)\,\Gamma(3\times3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2=$$

$$\frac{787}{2}+\frac{27}{2\,\phi}+\frac{27}{128}\sqrt{\frac{3}{2}}\sqrt{\frac{\left(\oint_L\frac{e^t}{t^{3/2}}\,dt\right)^6}{720\,i\,\pi^3\oint_L\frac{e^t}{t^8}\,dt}\int_0^1\frac{\log^8\left(1-t^7\right)}{t^8}\,dt}$$

$$\frac{27}{2}\left(\sqrt{\frac{2\left(\left(3\pi^3\right)\zeta(3\times3-3+2)\,\Gamma(3\times3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2=$$

$$\frac{787}{2}+\frac{27}{2\,\phi}+54\sqrt{\frac{3}{127}}\sqrt{\frac{\left(\oint_L\frac{e^t}{t^{3/2}}\,dt\right)^6}{i\,\pi^3\oint_L\frac{e^t}{t^8}\,dt}\int_0^\infty\frac{\cos\left(8\tan^{-1}(t)\right)}{\left(1+t^2\right)^4\cosh\left(\frac{\pi t}{2}\right)}\,dt}$$

(((((27*1/2*((((((2/Pi [(((3*Pi^3))) / (((2^(3*(3-1))(gamma (3/2))^6))) zeta(3*3-3+2) gamma (3*3-3+2)])))^1/2+29+1/golden ratio)))+2)))))^1/15-(21+5)/10^3

Input:

$$\sqrt[15]{27\times\frac{1}{2}\left(\sqrt{\frac{2}{\pi}\left(\frac{3\pi^3}{2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6}\zeta(3\times3-3+2)\Gamma(3\times3-3+2)\right)}+29+\frac{1}{\phi}\right)+2}-\frac{21+5}{10^3}$$

$\Gamma(x)$ is the gamma function

$\zeta(s)$ is the Riemann zeta function

ϕ is the golden ratio

Exact result:

$$\sqrt[15]{\frac{27}{2}\left(\frac{1}{\phi}+29+\frac{4\pi^{7/2}}{\sqrt{5}}\right)+2}-\frac{13}{500}$$

Decimal approximation:

1.617817390357012742197753313879591628021082948251445863877...

1.6178173903…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Property:

$$-\frac{13}{500}+\sqrt[15]{2+\frac{27}{2}\left(29+\frac{1}{\phi}+\frac{4\pi^{7/2}}{\sqrt{5}}\right)}$$ is a transcendental number

Alternate forms:

$$\sqrt[15]{2+\frac{27}{2}\left(\frac{1}{2}\left(57+\sqrt{5}\right)+\frac{4\pi^{7/2}}{\sqrt{5}}\right)}-\frac{13}{500}$$

$$\sqrt[15]{2+\frac{27}{2}\left(29+\frac{2}{1+\sqrt{5}}+\frac{4\pi^{7/2}}{\sqrt{5}}\right)}-\frac{13}{500}$$

$$\frac{\sqrt[15]{\frac{\left(787\sqrt{5}+108\pi^{7/2}\right)\phi+27\sqrt{5}}{\phi}}}{\sqrt[15]{2}\sqrt[30]{5}}-\frac{13}{500}$$

Alternative representations:

$$\sqrt[15]{\frac{27}{2}\left(\sqrt{\frac{2\left((3\pi^3)\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2}-\frac{21+5}{10^3}=$$

$$-\frac{26}{10^3}+\sqrt[15]{2+\frac{27}{2}\left(29+\frac{1}{\phi}+\sqrt{\frac{6\,(1)_7\,\pi^3\,\zeta(8,1)}{\pi\left(2^6\left((1)_{\frac{1}{2}}\right)^6\right)}}\right)}$$

$$\sqrt[15]{\frac{27}{2}\left(\sqrt{\frac{2\left((3\pi^3)\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2}-\frac{21+5}{10^3}=$$

$$-\frac{26}{10^3}+\sqrt[15]{2+\frac{27}{2}\left(29+\frac{1}{\phi}+\sqrt{\frac{6\,e^{\log\Gamma(8)}\,\pi^3\,\zeta(8,1)}{\pi\left(2^6\left(e^{\log\Gamma(3/2)}\right)^6\right)}}\right)}$$

$$\sqrt[15]{\frac{27}{2}\left(\sqrt{\frac{2\left((3\pi^3)\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2}-\frac{21+5}{10^3}=$$

$$-\frac{26}{10^3}+\sqrt[15]{2+\frac{27}{2}\left(29+\frac{1}{\phi}+\sqrt{\frac{6\,e^{-\log(24\,883\,200)+\log(125\,411\,328\,000)}\,\pi^3\,\zeta(8,1)}{\pi\left(2^6\left(e^{-\log G(3/2)+\log G(5/2)}\right)^6\right)}}\right)}$$

Series representations:

$$\sqrt[15]{\frac{27}{2}\left(\sqrt{\frac{2\left((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2)\right)}{\pi\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2}-\frac{21+5}{10^3}=$$

$$\frac{1}{500}\left(-13+250\times 2^{11/15}\sqrt[15]{\frac{216+6296\phi+27\sqrt{6}\,\phi\sqrt{\frac{\pi^2\exp\left(-\sum_{k=1}^{\infty}\log\left(1-\frac{1}{(p_k)^8}\right)\right)\sum_{k=0}^{\infty}\frac{(8-z_0)^k\Gamma^{(k)}(z_0)}{k!}}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)^6}}}{\phi}}\right)$$

for $(z_0 \notin \mathbb{Z}$ or $z_0 > 0)$

$$\sqrt[15]{\frac{27}{2}\left(\sqrt{\frac{2\left((3\pi^3)\zeta(3\times 3-3+2)\Gamma(3\times 3-3+2)\right)}{\pi\left(2^{3(3-1)}\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2}-\frac{21+5}{10^3}=\frac{1}{500}$$

$$\left(-13+250\times 2^{11/15}\sqrt[15]{\frac{216+6296\phi+27\sqrt{6}\,\phi\sqrt{\frac{\pi^2\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(8-z_0)^{k_2}\Gamma^{(k_2)}(z_0)}{k_2!\,k_1^8}}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)^6}}}{\phi}}\right)$$

for $(z_0 \notin \mathbb{Z}$ or $z_0 > 0)$

$$\sqrt[15]{\frac{27}{2}\left(\sqrt{\frac{2\left(\left(3\pi^3\right)\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2}-\frac{21+5}{10^3}=$$

$$\frac{1}{500}\left(-13+250\times 2^{11/15}\sqrt[15]{\frac{216+6296\,\phi+27\sqrt{6}\,\phi\sqrt{\frac{\pi^2\exp\left(\sum_{k=1}^{\infty}\frac{P(8\,k)}{k}\right)\sum_{k=0}^{\infty}\frac{(8-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)^6}}}{\phi}}\right)$$

for $(z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$

Integral representations:

$$\sqrt[15]{\frac{27}{2}\left(\sqrt{\frac{2\left(\left(3\pi^3\right)\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2}-\frac{21+5}{10^3}=$$

$$-\frac{13}{500}+\sqrt[15]{2+\frac{27}{2}\left(29+\frac{1}{\phi}+\frac{1}{16}\sqrt{3}\sqrt{\frac{\pi^2\,\Gamma(8)}{6!\,\Gamma\left(\frac{3}{2}\right)^6}\int_0^1\frac{\log^8(1-t^7)}{t^8}\,dt}\right)}$$

$$\sqrt[15]{\frac{27}{2}\left(\sqrt{\frac{2\left((3\pi^3)\,\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2}-\frac{21+5}{10^3}=$$

$$-\frac{13}{500}+\sqrt[15]{2+\frac{27}{2}\left(29+\frac{1}{\phi}+\frac{1}{64}\sqrt{\frac{3}{2}}\sqrt{\frac{\left(\oint_L \frac{e^t}{t^{3/2}}\,dt\right)^6}{720\,i\,\pi^3\oint_L \frac{e^t}{t^8}\,dt}\int_0^1 \frac{\log^8(1-t^7)}{t^8}\,dt}\right)}$$

$$\sqrt[15]{\frac{27}{2}\left(\sqrt{\frac{2\left((3\pi^3)\,\zeta(3\times 3-3+2)\,\Gamma(3\times 3-3+2)\right)}{\pi\left(2^{3(3-1)}\,\Gamma\left(\frac{3}{2}\right)^6\right)}}+29+\frac{1}{\phi}\right)+2}-\frac{21+5}{10^3}=$$

$$-\frac{13}{500}+\sqrt[15]{2+\frac{27}{2}\left(29+\frac{1}{\phi}+4\sqrt{\frac{3}{127}}\sqrt{\frac{\left(\oint_L \frac{e^t}{t^{3/2}}\,dt\right)^6}{i\,\pi^3\oint_L \frac{e^t}{t^8}\,dt}\int_0^\infty \frac{\cos(8\tan^{-1}(t))}{(1+t^2)^4\cosh\left(\frac{\pi t}{2}\right)}\,dt}\right)}$$

Now, we have that:

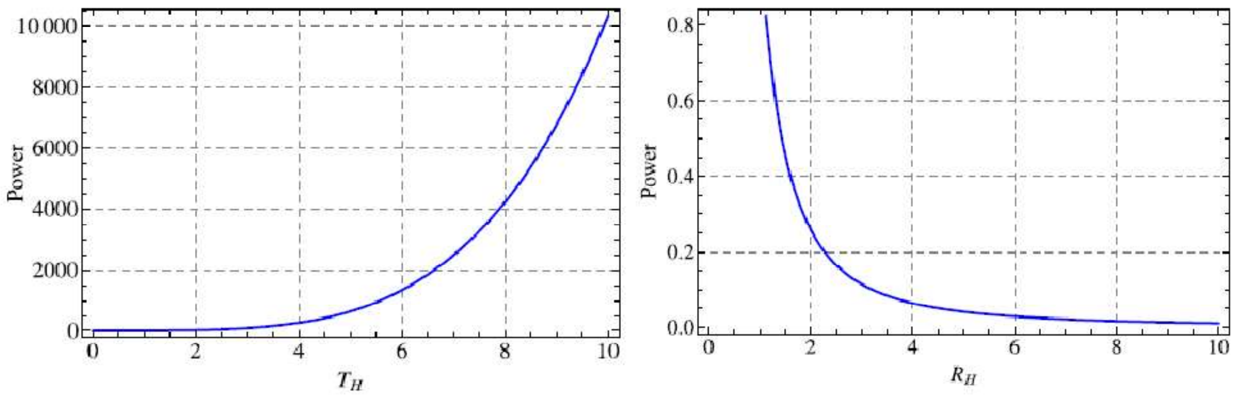


Fig. 2. Left: Hawking radiation power versus Hawking temperature for $d = 3$ and $k = R_H = 1$. Right: Hawking radiation power versus horizon radius for $d = 3$ and $k = T_H = 1$.

From

$$\hat{\omega}_c = \frac{2\left[\Gamma\left(\frac{d-1}{2}\right)\right]^{\frac{2}{d-2}}}{\pi^{\frac{1}{d-2}}} k R_H .$$

We have:

((2[gamma (1)]^2))/(Pi)

Input:

$$\frac{2\,\Gamma(1)^2}{\pi}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{2}{\pi}$$

Decimal approximation:

0.6366197723675813430755350534900574481378385829618257949906...

0.636619772367581343….

Property:

$\frac{2}{\pi}$ is a transcendental number

Alternative representations:

$$\frac{2\,\Gamma(1)^2}{\pi} = \frac{2 \times 1^2}{\pi}$$

$$\frac{2\,\Gamma(1)^2}{\pi} = \frac{2\,(e^0)^2}{\pi}$$

$$\frac{2\,\Gamma(1)^2}{\pi} = \frac{2\,(0!)^2}{\pi}$$

Series representations:

$$\frac{2\,\Gamma(1)^2}{\pi} = \frac{1}{2\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}$$

$$\frac{2\,\Gamma(1)^2}{\pi} = \frac{1}{\sum_{k=0}^{\infty} -\frac{2(-1)^k\, 1195^{-1-2k}\left(5^{1+2k} - 4 \times 239^{1+2k}\right)}{1+2k}}$$

$$\frac{2\,\Gamma(1)^2}{\pi} = \frac{2}{\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)}$$

Integral representations:

$$\frac{2\,\Gamma(1)^2}{\pi} = \frac{1}{\int_0^{\infty}\frac{1}{1+t^2}\,dt}$$

$$\frac{2\,\Gamma(1)^2}{\pi} = \frac{1}{\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt}$$

$$\frac{2\,\Gamma(1)^2}{\pi} = \frac{1}{2\int_0^{1}\sqrt{1-t^2}\,dt}$$

From which:

$$\gamma(\hat{\omega}) = \frac{\pi}{2^{d-1}\left[\Gamma\left(\frac{d}{2}\right)\right]^2}\frac{\omega^{d-1}}{k^{2d-2}R_H^{d-1}}. \tag{18}$$

Pi/(2^2*(((gamma (3/2))))^2) * (0.636619772367581343)^2

Input interpretation:

$$\frac{\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2} \times 0.636619772367581343^2$$

$\Gamma(x)$ is the gamma function

Result:

0.405284734569351085679343635721683649

0.405284734569351085679343635721683649

Repeating decimal:

0.4052847345693510856793436357216836490

Alternative representations:

$$\frac{0.6366197723675813430000^2\,\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2} = \frac{\pi\,0.6366197723675813430000^2}{4\left(\frac{1}{2}!\right)^2}$$

$$\frac{0.6366197723675813430000^2\,\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2} = \frac{\pi\,0.6366197723675813430000^2}{4\,\Gamma\left(\frac{3}{2},\,0\right)^2}$$

$$\frac{0.6366197723675813430000^2\,\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2} = \frac{\pi\,0.6366197723675813430000^2}{4\left((1)_{\frac{1}{2}}\right)^2}$$

Series representations:

$$\frac{0.6366197723675813430000^2\,\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2} = \frac{0.101321183642337771419\,8\,\pi}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^2}$$

for $(z_0 \notin \mathbb{Z}$ or $z_0 > 0)$

$$\frac{0.6366197723675813430000^2\,\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2} = \frac{1}{\pi}0.1013211836423377714198 \left(\sum_{k=0}^{\infty}\left(\frac{3}{2}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\,\sin\left(\frac{1}{2}\,\pi\,(-j+k+2\,z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^2$$

Integral representations:

$$\frac{0.6366197723675813430000^2\,\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2} = \frac{0.1013211836423377714198\,\pi}{\left(\int_0^{\infty}e^{-t}\,\sqrt{t}\,dt\right)^2}$$

$$\frac{0.6366197723675813430000^2\,\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2} = \frac{0.1013211836423377714198\,\pi}{\left(\int_0^{1}\sqrt{\log\left(\frac{1}{t}\right)}\,dt\right)^2}$$

$$\frac{0.6366197723675813430000^2\,\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2} = 0.10132118364233777144198\,\exp\left(-2\int_0^1 \frac{\frac{1}{2}-\frac{3x}{2}+x^{3/2}}{(-1+x)\log(x)}\,dx\right)\pi$$

From which:

4 * Pi/(2^2*(((gamma (3/2))))^2) * (0.636619772367581343)^2 - 3/10^3

Input interpretation:

$$4\times\frac{\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2}\times 0.636619772367581343^2-\frac{3}{10^3}$$

$\Gamma(x)$ is the gamma function

Result:

1.618138938277404342717374542886734596

Repeating decimal:

1.618138938277404342717374542886734596

1.61813893827740…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternative representations:

$$\frac{\left(4\times 0.6366197723675813430000^2\right)\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2}-\frac{3}{10^3} = -\frac{3}{10^3}+\frac{4\,\pi\,0.6366197723675813430000^2}{4\left(\frac{1}{2}!\right)^2}$$

$$\frac{\left(4\times 0.6366197723675813430000^2\right)\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2}-\frac{3}{10^3} = -\frac{3}{10^3}+\frac{4\,\pi\,0.6366197723675813430000^2}{4\,\Gamma\left(\frac{3}{2},0\right)^2}$$

$$\frac{\left(4\times 0.6366197723675813430000^2\right)\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2}-\frac{3}{10^3} = -\frac{3}{10^3}+\frac{4\,\pi\,0.6366197723675813430000^2}{4\left((1)_{\frac{1}{2}}\right)^2}$$

Series representations:

$$\frac{(4\times 0.63661977236758134300000^2)\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2}-\frac{3}{10^3}=$$
$$-\frac{3}{1000}+\frac{0.405284734569351085679\,\pi}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)^2}\quad\text{for } (z_0\notin\mathbb{Z}\text{ or } z_0>0)$$

$$\frac{(4\times 0.63661977236758134300000^2)\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2}-\frac{3}{10^3}=-\frac{1}{\pi}0.0030000000000000000000$$
$$\left(1.0000000000000000000\,\pi-135.09491152311702856\right.$$
$$\left.\left(\sum_{k=0}^{\infty}\left(\frac{3}{2}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^2\right)$$

$\mathbb{Z}$ is the set of integers

Integral representations:

$$\frac{(4\times 0.63661977236758134300000^2)\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2}-\frac{3}{10^3}=$$
$$-\frac{3}{1000}+\frac{0.405284734569351085679\,\pi}{\left(\int_0^{\infty}e^{-t}\sqrt{t}\,dt\right)^2}$$

$$\frac{(4\times 0.63661977236758134300000^2)\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2}-\frac{3}{10^3}=$$
$$-\frac{3}{1000}+\frac{0.405284734569351085679\,\pi}{\left(\int_0^1\sqrt{\log\left(\frac{1}{t}\right)}\,dt\right)^2}$$

$$\frac{(4\times 0.63661977236758134300000^2)\pi}{2^2\,\Gamma\left(\frac{3}{2}\right)^2}-\frac{3}{10^3}=$$
$$-\frac{3}{1000}+0.405284734569351085679\exp\left(-2\int_0^1\frac{\frac{1}{2}-\frac{3x}{2}+x^{3/2}}{(-1+x)\log(x)}\,dx\right)\pi$$

$\log(x)$ is the natural logarithm

3.1.1. Case I: when $\hat{\omega} << \hat{\omega}_c$

In this case, the greybody factor for $\hat{\omega} << \hat{\omega}_c$ is given as follows,

$$\gamma(\hat{\omega}) = 4z(\hat{\omega}) = \frac{\pi}{2^{d-2}\left[\Gamma\left(\frac{d-1}{2}\right)\right]^2}\frac{\hat{\omega}^{d-2}}{(kR_H)^{d-2}}.$$

Pi/ [2((gamma (1)))^2] * 1/12

Input:

$$\frac{\pi}{2\,\Gamma(1)^2} \times \frac{1}{12}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{\pi}{24}$$

Decimal approximation:

0.130899693899574718269276807636645953508215391640629409207...

0.130899693899…

Property:

$\frac{\pi}{24}$ is a transcendental number

Alternative representations:

$$\frac{\pi}{12\,(2\,\Gamma(1)^2)} = \frac{\pi}{12\,(2 \times 1^2)}$$

$$\frac{\pi}{12\,(2\,\Gamma(1)^2)} = \frac{\pi}{12\,(2\,(e^0)^2)}$$

$$\frac{\pi}{12\left(2\,\Gamma(1)^2\right)} = \frac{\pi}{12\left(2\,(0!)^2\right)}$$

Series representations:

$$\frac{\pi}{12\left(2\,\Gamma(1)^2\right)} = \frac{1}{6}\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\frac{\pi}{12\left(2\,\Gamma(1)^2\right)} = \sum_{k=0}^{\infty} -\frac{(-1)^k\,1195^{-1-2k}\left(5^{1+2k} - 4\times 239^{1+2k}\right)}{6\,(1+2k)}$$

$$\frac{\pi}{12\left(2\,\Gamma(1)^2\right)} = \frac{1}{24}\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

Integral representations:

$$\frac{\pi}{12\left(2\,\Gamma(1)^2\right)} = \frac{1}{6}\int_0^1\sqrt{1-t^2}\,dt$$

$$\frac{\pi}{12\left(2\,\Gamma(1)^2\right)} = \frac{1}{12}\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt$$

$$\frac{\pi}{12\left(2\,\Gamma(1)^2\right)} = \frac{1}{12}\int_0^{\infty}\frac{1}{1+t^2}\,dt$$

From which:

1+ 5 Pi/ [2((gamma (1)))^2] * 1/12 - (29+7)1/10^3

Input:

$$1 + 5\times\frac{\pi}{2\,\Gamma(1)^2}\times\frac{1}{12} - (29+7)\times\frac{1}{10^3}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{241}{250} + \frac{5\pi}{24}$$

Decimal approximation:

1.618498469497873591346384038183229767541076958203147046036...

1.61849846949….. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Property:

$\frac{241}{250} + \frac{5\pi}{24}$ is a transcendental number

Alternate form:

$\frac{2892 + 625\pi}{3000}$

Alternative representations:

$$1 + \frac{5\pi}{12(2\Gamma(1)^2)} - \frac{29+7}{10^3} = 1 + \frac{5\pi}{12(2\times 1^2)} - \frac{36}{10^3}$$

$$1 + \frac{5\pi}{12(2\Gamma(1)^2)} - \frac{29+7}{10^3} = 1 - \frac{36}{10^3} + \frac{5\pi}{12(2(e^0)^2)}$$

$$1 + \frac{5\pi}{12(2\Gamma(1)^2)} - \frac{29+7}{10^3} = 1 - \frac{36}{10^3} + \frac{5\pi}{12(2(0!)^2)}$$

$n!$ is the factorial function

Series representations:

$$1 + \frac{5\pi}{12(2\Gamma(1)^2)} - \frac{29+7}{10^3} = \frac{241}{250} + \frac{5}{6}\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$1 + \frac{5\pi}{12(2\Gamma(1)^2)} - \frac{29+7}{10^3} = \frac{241}{250} + \sum_{k=0}^{\infty}\frac{(-1)^k\left(956\times 5^{-2k} - 5\times 239^{-2k}\right)}{1434(1+2k)}$$

$$1 + \frac{5\pi}{12(2\Gamma(1)^2)} - \frac{29+7}{10^3} = \frac{241}{250} + \frac{5}{24}\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

Integral representations:

$$1+\frac{5\pi}{12\left(2\,\Gamma(1)^2\right)}-\frac{29+7}{10^3}=\frac{241}{250}+\frac{5}{6}\int_0^1\sqrt{1-t^2}\,dt$$

$$1+\frac{5\pi}{12\left(2\,\Gamma(1)^2\right)}-\frac{29+7}{10^3}=\frac{241}{250}+\frac{5}{12}\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt$$

$$1+\frac{5\pi}{12\left(2\,\Gamma(1)^2\right)}-\frac{29+7}{10^3}=\frac{241}{250}+\frac{5}{12}\int_0^\infty\frac{1}{1+t^2}\,dt$$

3.1.2. Case II: when $\hat{\omega} >> \hat{\omega}_c$

In this case of frequencies much higher than the critical frequency, the greybody factor for $d \to (d+1)$ is given by,

$$\gamma\left(\hat{\omega}\right)=\frac{2^{d-1}\left[\Gamma\left(\frac{d}{2}\right)\right]^2}{\pi}\frac{R_H^{d-1}k^{2d-2}}{\omega^{d-1}}. \tag{20}$$

(2^2*(gamma(3/2))^2) / (Pi) *1/(8^2)

Input:

$$\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{\pi}\times\frac{1}{8^2}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{1}{64}$$

Decimal form:

0.015625

0.015625

Alternative representations:

$$\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{8^2\,\pi}=\frac{4\left(\frac{1}{2}!\right)^2}{\pi\,8^2}$$

$$\frac{2^2 \Gamma\left(\frac{3}{2}\right)^2}{8^2 \pi} = \frac{4\left(e^{-\log G(3/2)+\log G(5/2)}\right)^2}{\pi 8^2}$$

$$\frac{2^2 \Gamma\left(\frac{3}{2}\right)^2}{8^2 \pi} = \frac{4\,\Gamma\left(\frac{3}{2}, 0\right)^2}{\pi 8^2}$$

Series representations:

$$\frac{2^2 \Gamma\left(\frac{3}{2}\right)^2}{8^2 \pi} = \frac{\left(\sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}-z_0\right)^k \Gamma^{(k)}(z_0)}{k!}\right)^2}{16 \pi} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{2^2 \Gamma\left(\frac{3}{2}\right)^2}{8^2 \pi} = \frac{\pi}{16\left(\sum_{k=0}^{\infty}\left(\frac{3}{2}-z_0\right)^k \sum_{j=0}^{k} \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}\pi(-j+k+2 z_0)\right)\Gamma^{(j)}(1-z_0)}{j!(-j+k)!}\right)^2}$$

Integral representations:

$$\frac{2^2 \Gamma\left(\frac{3}{2}\right)^2}{8^2 \pi} = \frac{\left(\int_0^{\infty} e^{-t} \sqrt{t}\, dt\right)^2}{16 \pi}$$

$$\frac{2^2 \Gamma\left(\frac{3}{2}\right)^2}{8^2 \pi} = \frac{\left(\int_0^1 \sqrt{\log\left(\frac{1}{t}\right)}\, dt\right)^2}{16 \pi}$$

$$\frac{2^2 \Gamma\left(\frac{3}{2}\right)^2}{8^2 \pi} = \frac{\exp\left(2\int_0^1 \frac{\frac{1}{2}-\frac{3x}{2}+x^{3/2}}{(-1+x)\log(x)}\, dx\right)}{16 \pi}$$

Occurrence in convergents:

$$\frac{\pi}{201} \approx 0, \frac{1}{63}, \frac{1}{64}, \frac{50}{3199}, \frac{51}{3263}, \ldots$$

$$\frac{e^{-\gamma}}{36} \approx 0, \frac{1}{64}, \frac{8}{513}, \frac{17}{1090}, \ldots$$

(simple continued fraction convergent sequences)

γ is the Euler-Mascheroni constant

From which:

27/((((2^2*(gamma(3/2))^2) / (Pi) *1/(8^2))))+1

Input:

$$\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{\pi}\times\frac{1}{8^2}}+1$$

$\Gamma(x)$ is the gamma function

Exact result:

1729

1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternative representations:

$$\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{8^2\,\pi}}+1=1+\frac{27}{\frac{4\left(\frac{1}{2}!\right)^2}{\pi\,8^2}}$$

$$\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{8^2\,\pi}}+1=1+\frac{27}{\frac{4\,\Gamma\left(\frac{3}{2},0\right)^2}{\pi\,8^2}}$$

$$\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{8^2\,\pi}}+1=1+\frac{27}{\frac{4\left((1)_{\frac{1}{2}}\right)^2}{\pi\,8^2}}$$

Integral representations:

$$\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{8^2\,\pi}}+1=1+\frac{432\,\pi}{\left(\int_0^\infty e^{-t}\sqrt{t}\,dt\right)^2}$$

$$\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{8^2\,\pi}}+1=1+\frac{432\,\pi}{\left(\int_0^1\sqrt{\log\left(\frac{1}{t}\right)}\,dt\right)^2}$$

$$\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{8^2\,\pi}}+1=1+432\exp\left(-2\int_0^1\frac{\frac{1}{2}-\frac{3x}{2}+x^{3/2}}{(-1+x)\log(x)}\,dx\right)\pi$$

log(x) is the natural logarithm

((((27/((((2^2*(gamma(3/2))^2) / (Pi) *1/(8^2))))+1))))^1/15 - (29-3)1/10^3

Input:

$$\sqrt[15]{\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{\pi}\times\frac{1}{8^2}}+1}-(29-3)\times\frac{1}{10^3}$$

Γ(x) is the gamma function

Exact result:

$$\sqrt[15]{1729}-\frac{13}{500}$$

Decimal approximation:

1.617815228748728130580088031324769514329283143699940172645...

1.6178152287487…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms:

$$\frac{1}{500}\left(500\sqrt[15]{1729}-13\right)$$

$$\frac{1}{500}\left(\left(500\left(\text{root of } 31\,250\,000\,000\,000\,x^5+686\,562\,500\,000\,000\,x^4+6\,033\,511\,250\,000\,000\,x^3+26\,511\,248\,432\,500\,000\,x^2+58\,245\,212\,806\,202\,500\,x-52\,764\,892\,578\,124\,999\,999\,999\,999\,948\,814\,106\,985\,909\,243 \text{ near } x=1.11045\times10^6\right)+2197\right)\hat{}\,(1/3)-13\right)$$

Minimal polynomial:

30 517 578 125 000 000 000 000 000 000 000 000 000 000 x^{15} +
11 901 855 468 750 000 000 000 000 000 000 000 000 000 x^{14} +
2 166 137 695 312 500 000 000 000 000 000 000 000 000 x^{13} +
244 051 513 671 875 000 000 000 000 000 000 000 000 x^{12} +
19 036 018 066 406 250 000 000 000 000 000 000 000 x^{11} +
1 088 860 233 398 437 500 000 000 000 000 000 000 x^{10} +
47 183 943 447 265 625 000 000 000 000 000 000 x^{9} +
1 577 291 823 808 593 750 000 000 000 000 000 x^{8} +
41 009 587 419 023 437 500 000 000 000 000 x^{7} +
829 304 990 029 140 625 000 000 000 000 x^{6} +
12 937 157 844 454 593 750 000 000 000 x^{5} +
152 893 683 616 281 562 500 000 000 x^{4} + 1 325 078 591 341 106 875 000 000 x^{3} +
7 950 471 548 046 641 250 000 x^{2} + 29 530 322 892 744 667 500 x −
52 764 892 578 124 999 999 999 999 948 814 106 985 909 243

Alternative representations:

$$\sqrt[15]{\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{8^2\,\pi}}+1}-\frac{29-3}{10^3}=-\frac{26}{10^3}+\sqrt[15]{1+\frac{27}{\frac{4\left(\frac{1}{2}!\right)^2}{\pi\,8^2}}}$$

$$\sqrt[15]{\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{8^2\,\pi}}+1}-\frac{29-3}{10^3}=-\frac{26}{10^3}+\sqrt[15]{1+\frac{27}{\frac{4\,\Gamma\left(\frac{3}{2},0\right)^2}{\pi\,8^2}}}$$

$$\sqrt[15]{\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{8^2\,\pi}}+1}-\frac{29-3}{10^3}=-\frac{26}{10^3}+\sqrt[15]{1+\frac{27}{\frac{4\left((1)_{\frac{1}{2}}\right)^2}{\pi\,8^2}}}$$

Series representations:

$$\sqrt[15]{\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{8^2\,\pi}}+1}-\frac{29-3}{10^3}=-\frac{13}{500}+\sqrt[15]{1+\frac{432\,\pi}{\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}\right)^2}}$$

for ($z_0 \notin \mathbb{Z}$ or $z_0 > 0$)

$$\sqrt[15]{\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{8^2\,\pi}}+1}-\frac{29-3}{10^3}=$$

$$\frac{1}{500}\left(-13+500\sqrt[15]{\frac{\pi+432\left(\sum_{k=0}^{\infty}\left(\frac{3}{2}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi\,(-j+k+2\,z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^2}{\pi}}\right)$$

$\mathbb{Z}$ is the set of integers

Integral representations:

$$\sqrt[15]{\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{8^2\,\pi}}+1}-\frac{29-3}{10^3}=-\frac{13}{500}+\sqrt[15]{1+\frac{432\,\pi}{\left(\int_0^1\sqrt{\log\left(\frac{1}{t}\right)}\,dt\right)^2}}$$

$$\sqrt[15]{\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{8^2\,\pi}}+1}-\frac{29-3}{10^3}=-\frac{13}{500}+\sqrt[15]{1+\frac{432\,\pi}{\left(\int_0^\infty e^{-t}\sqrt{t}\,dt\right)^2}}$$

$$\sqrt[15]{\frac{27}{\frac{2^2\,\Gamma\left(\frac{3}{2}\right)^2}{8^2\,\pi}}+1}-\frac{29-3}{10^3}=-\frac{13}{500}+\sqrt[15]{1+432\exp\left(-2\int_0^1\frac{\frac{1}{2}-\frac{3x}{2}+x^{3/2}}{(-1+x)\log(x)}\,dx\right)\pi}$$

$\log(x)$ is the natural logarithm

Now, we have that:

From: 0.636619772367581343…. and d = 5

$$h(\hat{\omega})=\frac{\pi\,\hat{\omega}}{2}\coth\left(\frac{\pi\,\hat{\omega}}{2}\right)\prod_{n=1}^{\frac{(d+1)-3}{2}}\left(1+\frac{\hat{\omega}^2}{(2n)^2}\right)=\frac{\pi\,\hat{\omega}}{2}\coth\left(\frac{\pi\,\hat{\omega}}{2}\right)\prod_{n=1}^{\frac{d-2}{2}}\left(1+\frac{\hat{\omega}^2}{(2n)^2}\right)$$

1/2*Pi*(0.636619772367581343) coth (1/2*Pi*(0.636619772367581343)) * product (1+((0.636619772367581343)^2) / (2n)^2), n=1..1.5

Input interpretation:

$$\frac{1}{2}\pi\times 0.636619772367581343\left(\coth\left(\frac{1}{2}\pi\times 0.636619772367581343\right)\prod_{n=1}^{1.5}\left(1+\frac{0.636619772367581343^2}{(2n)^2}\right)\right)$$

coth(x) is the hyperbolic cotangent function

Result:

1.446073574790278 46

1.446073574790278 46

From which:

1+1/2((((1/2*Pi*(0.63661977) coth (1/2*Pi*(0.63661977)) * product (1+((0.63661977)^2) / (2n)^2), n=1..1.5))))^1/2+(11+4+2)1/10^3

Input interpretation:

$$1+\frac{1}{2}\sqrt{\frac{1}{2}\pi\times 0.63661977\left(\coth\left(\frac{1}{2}\pi\times 0.63661977\right)\prod_{n=1}^{1.5}\left(1+\frac{0.63661977^2}{(2n)^2}\right)\right)}+(11+4+2)\times\frac{1}{10^3}$$

coth(x) is the hyperbolic cotangent function

Result:

1.61826

1.61826 result that is a very good approximation to the value of the golden ratio 1.618033988749...

Now, we have that:

Now, exploiting relation (1), we write the power radiation equation for four-dimensional dS black hole as follows:

$$P^{(3+1)}_{low-even} = \frac{2}{\pi} R_H^2 \left[k^2 \int_0^\infty d\omega \frac{\omega}{e^{\frac{\omega}{T_H}} - 1} + \int_0^\infty d\omega \frac{\omega^3}{e^{\frac{\omega}{T_H}} - 1} \right]. \quad (28)$$

The integrals of above expression can be solved very easily with the help of Riemann Zeta function (5). After simplification the expression (28) reduces to

$$P^{(3+1)}_{low-even} = \frac{2}{\pi} R_H^2 \left(\frac{1}{6}\pi^2 k^2 T_H^2 + \frac{6}{90}\pi^4 T_H^4 \right),$$
$$= \frac{1}{3}\pi k^2 R_H^2 T_H^2 + \frac{2}{15}\pi^3 R_H^2 T_H^4. \quad (29)$$

2/Pi (1/6*Pi^2+6/90*Pi^4) = 1/3*Pi+2/15*Pi^3

Input:

$$\frac{2}{\pi}\left(\frac{1}{6}\pi^2 + \frac{6}{90}\pi^4\right) = \frac{1}{3}\pi + \frac{2}{15}\pi^3$$

Result:

True

Left hand side:

$$\frac{2\left(\frac{\pi^2}{6} + \frac{6\pi^4}{90}\right)}{\pi} = \frac{1}{15}\pi\left(5 + 2\pi^2\right)$$

Right hand side:

$$\frac{\pi}{3} + \frac{2\pi^3}{15} = \frac{1}{15}\pi\left(5 + 2\pi^2\right)$$

1/3*Pi+2/15*Pi^3

Input:

$$\frac{1}{3}\pi + \frac{2}{15}\pi^3$$

Result:

$$\frac{\pi}{3} + \frac{2\pi^3}{15}$$

Decimal approximation:

5.181367775236573769551056470040020321695761608576382966210...

5.18136777523657....

Property:

$\frac{\pi}{3} + \frac{2\pi^3}{15}$ is a transcendental number

Alternate form:

$$\frac{1}{15}\pi\left(5+2\pi^2\right)$$

Alternative representations:

$$\frac{\pi}{3}+\frac{\pi^3\,2}{15} = 60\,° + \frac{2}{15}\left(180\,°\right)^3$$

$$\frac{\pi}{3}+\frac{\pi^3\,2}{15} = \frac{1}{3}\cos^{-1}(-1) + \frac{2}{15}\cos^{-1}(-1)^3$$

$$\frac{\pi}{3}+\frac{\pi^3\,2}{15} = -\frac{1}{3}\,i\log(-1) + \frac{2}{15}\left(-i\log(-1)\right)^3$$

Integral representations:

$$\frac{\pi}{3}+\frac{\pi^3\,2}{15} = \frac{2}{15}\left(5+8\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2\right)\int_0^\infty \frac{1}{1+t^2}\,dt$$

$$\frac{\pi}{3}+\frac{\pi^3\,2}{15} = \frac{4}{15}\left(5+32\left(\int_0^1 \sqrt{1-t^2}\,dt\right)^2\right)\int_0^1 \sqrt{1-t^2}\,dt$$

$$\frac{\pi}{3}+\frac{\pi^3\,2}{15} = \frac{2}{15}\left(5+8\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2\right)\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

(((1/3*Pi+2/15*Pi^3)))^1/3+2/10^3

Input:

$$\sqrt[3]{\frac{1}{3}\pi + \frac{2}{15}\pi^3} + \frac{2}{10^3}$$

Exact result:

$$\frac{1}{500} + \sqrt[3]{\frac{\pi}{3} + \frac{2\pi^3}{15}}$$

Decimal approximation:

1.732406508461850402011231976950381644277957865877659496408...

1.73240650846…. $\approx \sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3})\,M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

Property:

$\frac{1}{500} + \sqrt[3]{\frac{\pi}{3} + \frac{2\pi^3}{15}}$ is a transcendental number

Alternate forms:

$$\frac{1}{500} + \sqrt[3]{\frac{1}{15}\pi(5 + 2\pi^2)}$$

$$\frac{3 + 100 \times 15^{2/3}\sqrt[3]{\pi(5 + 2\pi^2)}}{1500}$$

$$\frac{\sqrt[3]{15} + 500\sqrt[3]{\pi(5 + 2\pi^2)}}{500\sqrt[3]{15}}$$

Alternative representations:

$$\sqrt[3]{\frac{\pi}{3}+\frac{\pi^3 2}{15}}+\frac{2}{10^3}=\frac{2}{10^3}+\sqrt[3]{60°+\frac{2}{15}(180°)^3}$$

$$\sqrt[3]{\frac{\pi}{3}+\frac{\pi^3 2}{15}}+\frac{2}{10^3}=\frac{2}{10^3}+\sqrt[3]{\frac{1}{3}\cos^{-1}(-1)+\frac{2}{15}\cos^{-1}(-1)^3}$$

$$\sqrt[3]{\frac{\pi}{3}+\frac{\pi^3 2}{15}}+\frac{2}{10^3}=\frac{2}{10^3}+\sqrt[3]{\frac{2E(0)}{3}+\frac{2}{15}(2E(0))^3}$$

Series representations:

$$\sqrt[3]{\frac{\pi}{3}+\frac{\pi^3 2}{15}}+\frac{2}{10^3}=\frac{1}{500}+\sqrt[3]{\frac{\pi}{3}-\frac{64}{15}\sum_{k=1}^{\infty}\frac{(-1)^k}{(-1+2k)^3}}$$

$$\sqrt[3]{\frac{\pi}{3}+\frac{\pi^3 2}{15}}+\frac{2}{10^3}=\frac{3+100\times 30^{2/3}\sqrt[3]{\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)\left(5+32\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^2\right)}}{1500}$$

$$\sqrt[3]{\frac{\pi}{3}+\frac{\pi^3 2}{15}}+\frac{2}{10^3}=\frac{1}{1500}\left(3+100\times 15^{2/3}\left(\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)\left(5+2\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^2\right)\right)\hat{}\,(1/3)\right)$$

Integral representations:

$$\sqrt[3]{\frac{\pi}{3}+\frac{\pi^3 2}{15}}+\frac{2}{10^3}=\frac{3+100\sqrt[3]{2}\,15^{2/3}\sqrt[3]{5+8\left(\int_0^{\infty}\frac{1}{1+t^2}dt\right)^2\int_0^{\infty}\frac{1}{1+t^2}dt}}{1500}$$

$$\sqrt[3]{\frac{\pi}{3}+\frac{\pi^3 2}{15}}+\frac{2}{10^3}=\frac{3+100\times 30^{2/3}\sqrt[3]{5+32\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2\int_0^1\sqrt{1-t^2}\,dt}}{1500}$$

$$\sqrt[3]{\frac{\pi}{3}+\frac{\pi^3 2}{15}}+\frac{2}{10^3}=\frac{3+100\sqrt[3]{2}\,15^{2/3}\sqrt[3]{5+8\left(\int_0^\infty\frac{\sin(t)}{t}\,dt\right)^2\int_0^\infty\frac{\sin(t)}{t}\,dt}}{1500}$$

1+1/(((1/3*Pi+2/15*Pi^3)))^1/4-(47-2)/10^3

Input:

$$1+\frac{1}{\sqrt[4]{\frac{1}{3}\pi+\frac{2}{15}\pi^3}}-\frac{47-2}{10^3}$$

Exact result:

$$\frac{191}{200}+\frac{1}{\sqrt[4]{\frac{\pi}{3}+\frac{2\pi^3}{15}}}$$

Decimal approximation:

1.617809760303757329354839567366503599494221326594189131033...

1.617809760303757…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Property:

$\frac{191}{200}+\frac{1}{\sqrt[4]{\frac{\pi}{3}+\frac{2\pi^3}{15}}}$ is a transcendental number

Alternate forms:

$$\frac{191}{200}+\sqrt[4]{\frac{15}{5\pi+2\pi^3}}$$

$$\frac{191}{200}+\sqrt[4]{\frac{15}{\pi(5+2\pi^2)}}$$

$$\frac{200\sqrt[4]{15}+191\sqrt[4]{\pi(5+2\pi^2)}}{200\sqrt[4]{\pi(5+2\pi^2)}}$$

Alternative representations:

$$1+\frac{1}{\sqrt[4]{\frac{\pi}{3}+\frac{\pi^3 2}{15}}}-\frac{47-2}{10^3}=1-\frac{45}{10^3}+\frac{1}{\sqrt[4]{60^\circ+\frac{2}{15}(180^\circ)^3}}$$

$$1+\frac{1}{\sqrt[4]{\frac{\pi}{3}+\frac{\pi^3 2}{15}}}-\frac{47-2}{10^3}=1-\frac{45}{10^3}+\frac{1}{\sqrt[4]{\frac{1}{3}\cos^{-1}(-1)+\frac{2}{15}\cos^{-1}(-1)^3}}$$

$$1+\frac{1}{\sqrt[4]{\frac{\pi}{3}+\frac{\pi^3 2}{15}}}-\frac{47-2}{10^3}=1-\frac{45}{10^3}+\frac{1}{\sqrt[4]{\frac{2E(0)}{3}+\frac{2}{15}(2E(0))^3}}$$

Series representations:

$$1+\frac{1}{\sqrt[4]{\frac{\pi}{3}+\frac{\pi^3 2}{15}}}-\frac{47-2}{10^3}=\frac{191}{200}+\frac{1}{\sqrt[4]{\frac{\pi}{3}-\frac{64}{15}\sum_{k=1}^{\infty}\frac{(-1)^k}{(-1+2k)^3}}}$$

$$1+\frac{1}{\sqrt[4]{\frac{\pi}{3}+\frac{\pi^3 2}{15}}}-\frac{47-2}{10^3}=\frac{100\sqrt{2}\sqrt[4]{15}+191\sqrt[4]{\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)\left(5+32\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^2\right)}}{200\sqrt[4]{\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)\left(5+32\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^2\right)}}$$

$$1+\frac{1}{\sqrt[4]{\frac{\pi}{3}+\frac{\pi^3 2}{15}}}-\frac{47-2}{10^3}=$$

$$\left(200\sqrt[4]{15}+191\left(\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)\left(5+2\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^2\right)\right)^\wedge(1/4)\right)\Big/$$
$$\left(200\left(\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)\left(5+2\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^2\right)\right)^\wedge(1/4)\right)$$

Now, we have that:

Once the expression for the greybody factor is known, it is matter of calculation to obtain the radiation power equation which utilizes relation (1). For the greybody factor (30), the expression for the radiation power equation is given by

$$P_{low-even}^{(5+1)} = \frac{2}{\pi} R_H^4 \left[k^4 \int_0^\infty d\omega \frac{\omega}{e^{\frac{\omega}{T_H}} - 1} + \frac{10}{9} k^2 \int_0^\infty d\omega \frac{\omega^3}{e^{\frac{\omega}{T_H}} - 1} + \frac{1}{9} \int_0^\infty d\omega \frac{\omega^5}{e^{\frac{\omega}{T_H}} - 1} \right]. \quad (31)$$

In order to solve the integrals in above expression, we utilize the Zeta function (5). After doing so, the expression for the radiation power equation simplified to

$$\begin{aligned} P_{low-even}^{(5+1)} &= \frac{2}{\pi} R_H^4 \left[k^4 \zeta(2)\Gamma(2) T_H^2 + \frac{10k^2}{9} \zeta(4)\Gamma(4) T_H^4 + \frac{1}{9} \zeta(6)\Gamma(6) T_H^6 \right], \\ &= \frac{1}{3} \pi k^4 R_H^4 T_H^2 + \frac{4}{27} \pi^3 k^2 R_H^4 T_H^4 + \frac{16}{567} \pi^5 R_H^4 T_H^6. \end{aligned} \quad (32)$$

Here, we see that the expression for the radiation power equation depends on both the horizon radius and Hawking temperature. In particular, it depends on the sum of different powers of Hawking radiation. In order to see behavior of the radiation power equation with respect to horizon radius and Hawking temperature, we plot Fig. 4.

1/3*Pi+4/27*Pi^3+16/567*Pi^5

Input:

$$\frac{1}{3}\pi + \frac{4}{27}\pi^3 + \frac{16}{567}\pi^5$$

Decimal approximation:

14.276196136222502767219243431695437294153440144736232619 62...

14.2761961362225….

Property:

$\frac{\pi}{3} + \frac{4\pi^3}{27} + \frac{16\pi^5}{567}$ is a transcendental number

Alternate form:

$$\frac{1}{567}\pi\left(189 + 84\pi^2 + 16\pi^4\right)$$

Alternative representations:

$$\frac{\pi}{3}+\frac{\pi^3\,4}{27}+\frac{\pi^5\,16}{567}=60\,°+\frac{4}{27}\,(180\,°)^3+\frac{16}{567}\,(180\,°)^5$$

$$\frac{\pi}{3}+\frac{\pi^3\,4}{27}+\frac{\pi^5\,16}{567}=\frac{1}{3}\cos^{-1}(-1)+\frac{4}{27}\cos^{-1}(-1)^3+\frac{16}{567}\cos^{-1}(-1)^5$$

$$\frac{\pi}{3}+\frac{\pi^3\,4}{27}+\frac{\pi^5\,16}{567}=\frac{2\,E(0)}{3}+\frac{4}{27}\,(2\,E(0))^3+\frac{16}{567}\,(2\,E(0))^5$$

Integral representations:

$$\frac{\pi}{3}+\frac{\pi^3\,4}{27}+\frac{\pi^5\,16}{567}= \frac{2}{567}\left(189+336\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2+256\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^4\right)\int_0^\infty \frac{1}{1+t^2}\,dt$$

$$\frac{\pi}{3}+\frac{\pi^3\,4}{27}+\frac{\pi^5\,16}{567}= \frac{2}{567}\left(189+336\left(\int_0^\infty \frac{\sin(t)}{t}\,dt\right)^2+256\left(\int_0^\infty \frac{\sin(t)}{t}\,dt\right)^4\right)\int_0^\infty \frac{\sin(t)}{t}\,dt$$

$$\frac{\pi}{3}+\frac{\pi^3\,4}{27}+\frac{\pi^5\,16}{567}= \frac{4}{567}\left(189+1344\left(\int_0^1 \sqrt{1-t^2}\,dt\right)^2+4096\left(\int_0^1 \sqrt{1-t^2}\,dt\right)^4\right)\int_0^1 \sqrt{1-t^2}\,dt$$

1+1/((1/3*Pi+4/27*Pi^3+16/567*Pi^5))^1/6-(21+3)/10^3

Input:

$$1+\frac{1}{\sqrt[6]{\frac{1}{3}\,\pi+\frac{4}{27}\,\pi^3+\frac{16}{567}\,\pi^5}}-\frac{21+3}{10^3}$$

Exact result:

$$\frac{122}{125}+\frac{1}{\sqrt[6]{\frac{\pi}{3}+\frac{4\,\pi^3}{27}+\frac{16\,\pi^5}{567}}}$$

Decimal approximation:

1.618043690372115989565430650630851437735554047625500329444...

1.61804369037211…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Property:

$$\frac{122}{125}+\frac{1}{\sqrt[6]{\frac{\pi}{3}+\frac{4\pi^3}{27}+\frac{16\pi^5}{567}}} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{122}{125}+3^{2/3}\sqrt[6]{\frac{7}{\pi\left(189+84\pi^2+16\pi^4\right)}}$$

$$\frac{125\times 3^{2/3}\sqrt[6]{7}+122\sqrt[6]{\pi\left(189+84\pi^2+16\pi^4\right)}}{125\sqrt[6]{\pi\left(189+84\pi^2+16\pi^4\right)}}$$

Alternative representations:

$$1+\frac{1}{\sqrt[6]{\frac{\pi}{3}+\frac{\pi^3 4}{27}+\frac{\pi^5 16}{567}}}-\frac{21+3}{10^3}=1-\frac{24}{10^3}+\frac{1}{\sqrt[6]{60\,°+\frac{4}{27}(180\,°)^3+\frac{16}{567}(180\,°)^5}}$$

$$1+\frac{1}{\sqrt[6]{\frac{\pi}{3}+\frac{\pi^3 4}{27}+\frac{\pi^5 16}{567}}}-\frac{21+3}{10^3}=$$
$$1-\frac{24}{10^3}+\frac{1}{\sqrt[6]{\frac{1}{3}\cos^{-1}(-1)+\frac{4}{27}\cos^{-1}(-1)^3+\frac{16}{567}\cos^{-1}(-1)^5}}$$

$$1+\frac{1}{\sqrt[6]{\frac{\pi}{3}+\frac{\pi^3 4}{27}+\frac{\pi^5 16}{567}}}-\frac{21+3}{10^3}=1-\frac{24}{10^3}+\frac{1}{\sqrt[6]{\frac{2E(0)}{3}+\frac{4}{27}(2E(0))^3+\frac{16}{567}(2E(0))^5}}$$

Series representations:

$$1+\frac{1}{\sqrt[6]{\frac{\pi}{3}+\frac{\pi^3 4}{27}+\frac{\pi^5 16}{567}}}-\frac{21+3}{10^3}=\frac{122}{125}+\frac{1}{\sqrt[6]{\frac{\pi}{3}+\frac{16\pi^5}{567}-\frac{128}{27}\sum_{k=1}^{\infty}\frac{(-1)^k}{(-1+2k)^3}}}$$

$$1+\frac{1}{\sqrt[6]{\frac{\pi}{3}+\frac{\pi^3 4}{27}+\frac{\pi^5 16}{567}}}-\frac{21+3}{10^3}=$$
$$\frac{125\times 6^{2/3}\sqrt[6]{7}+244\sqrt[6]{\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)\left(189+1344\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^2+4096\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^4\right)}}{250\sqrt[6]{\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)\left(189+1344\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^2+4096\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^4\right)}}$$

$$1+\frac{1}{\sqrt[6]{\frac{\pi}{3}+\frac{\pi^3 4}{27}+\frac{\pi^5 16}{567}}}-\frac{21+3}{10^3}=$$
$$\left(125\times 3^{2/3}\sqrt[6]{7}+122\left(\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)\left(189+84\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^2+16\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^4\right)\right)^{\wedge}(1/6)\right)\Big/$$
$$\left(125\left(\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)\left(189+84\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^2+16\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^4\right)\right)^{\wedge}(1/6)\right)$$

Now, from

A few properties of Ramanujan cubic polynomials and Ramanujan cubic polynomials of the second kind
Beata BAJORSKA-HARAPINSKA, Mariusz PLESZCZYNSKI, Damian SLOTA and Roman WITULA - 2010 Mathematics Subject Classification: 11C08, 11B83, 33B10.

we have that:

$$\sin\left(\frac{1}{3}\arctan\frac{3\sqrt{15}}{11}\right) = \frac{\sqrt{15}-\sqrt{3}}{8} = \frac{\sqrt{3}}{4\varphi},$$

sin(1/3 atan (((3sqrt15)/11)))

Input:

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{11}\left(3\sqrt{15}\right)\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)$$

(result in radians)

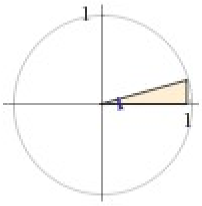
Decimal approximation:

0.267616567329817448956477382284565905486264556435151274816...

(result in radians)

0.2676165673298…

Reference triangle for angle 0.2709 radians:



width	$\cos\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) \approx 0.963525$
height	$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) \approx 0.267617$

Alternate forms:

$$\frac{1}{4}\sqrt{\frac{3}{2}\left(3-\sqrt{5}\right)}$$

$$\frac{1}{2}\, i \exp\left(\frac{1}{6}\left(\log\left(1-\frac{3\, i\sqrt{15}}{11}\right)-\log\left(1+\frac{3\, i\sqrt{15}}{11}\right)\right)\right)-\frac{1}{2}\, i \exp\left(\frac{1}{6}\left(\log\left(1+\frac{3\, i\sqrt{15}}{11}\right)-\log\left(1-\frac{3\, i\sqrt{15}}{11}\right)\right)\right)$$

log(x) is the natural logarithm

Alternative representations:

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=\cos\left(\frac{\pi}{2}-\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)$$

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=-\cos\left(\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)$$

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=\frac{-e^{-1/3\, i \tan^{-1}\left(\left(3\sqrt{15}\right)/11\right)}+e^{1/3\, i \tan^{-1}\left(\left(3\sqrt{15}\right)/11\right)}}{2\, i}$$

Series representations:

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=\sum_{k=0}^{\infty}\frac{(-1)^k\, 3^{-1-2k}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)^{1+2k}}{(1+2k)!}$$

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)^{2k}}{(2k)!}$$

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \frac{1}{6}\sqrt{\pi}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{36^s\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}$$

Integral representations:

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\int_0^1\cos\left(\frac{1}{3}t\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)dt$$

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = -\frac{i\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)}{12\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)^2/(36s)}}{s^{3/2}}\,ds \quad \text{for } \gamma > 0$$

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = -\frac{i}{2\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(\frac{1}{6}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)^{1-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds \quad \text{for } 0 < \gamma < 1$$

Continued fraction representations:

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \sin\left(\frac{\sqrt{15}}{11\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{135k^2}{121}}{1+2k}\right)}\right) =$$

$$\sin\left(\frac{\sqrt{15}}{11\left(1+\frac{135}{121\left(3+\frac{540}{121\left(5+\frac{1215}{121\left(7+\frac{2160}{121(9+\ldots)}\right)}\right)}\right)}\right)}\right)$$

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \sin\left(\frac{\sqrt{15}}{11\left(1 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{\frac{135}{121}(1-2k)^2}{-\frac{4}{121}(-64+7k)}\right)}\right) =$$

$$\sin\left(\frac{\sqrt{15}}{11\left(1+\frac{135}{121\left(\frac{228}{121}+\frac{1215}{121\left(\frac{200}{121}+\frac{3375}{121\left(\frac{172}{121}+\frac{6615}{121\left(\frac{144}{121}+\ldots\right)}\right)}\right)}\right)}\right)}\right)$$

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \sin\left(\frac{1}{3}\left(\frac{3\sqrt{15}}{11} - \frac{405\sqrt{15}}{1331\left(3 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{\frac{135}{121}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}\right)}\right)\right) =$$

$$\sin\left(\frac{1}{3}\left(\frac{3\sqrt{15}}{11} - \frac{405\sqrt{15}}{1331\left(3 + \frac{1215}{121\left(5+\frac{540}{121\left(7+\frac{3375}{121\left(9+\frac{2160}{121\,(11+\ldots)}\right)}\right)}\right)}\right)}\right)\right)$$

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \sin\left(\frac{11\sqrt{15}}{256+121\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{270}{121}\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1+\frac{135}{242}\left(1+(-1)^k\right)\right)(1+2k)}\right)}\right) =$$

$$\sin\left(\frac{11\sqrt{15}}{256+121\left(-\frac{270}{121\left(3-\frac{270}{121\left(\frac{1280}{121}-\frac{1620}{121\left(7-\frac{1620}{121\left(\frac{2304}{121}+\ldots\right)}\right)}\right)}\right)}\right)}\right)$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

Multiple-argument formulas:

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \sin\left(\frac{1}{3}\cos^{-1}\left(\frac{11}{16}\right)\right)$$

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = 2\cos\left(\frac{1}{6}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)\sin\left(\frac{1}{6}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)$$

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = 3\sin\left(\frac{1}{9}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) - 4\sin^3\left(\frac{1}{9}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)$$

sqrt3/(4golden ratio)

Input:

$$\frac{\sqrt{3}}{4\phi}$$

ϕ is the golden ratio

Decimal approximation:

0.2676165673298174489564773822845659054862645564351512748 16...

0.2676165673298…

Alternate forms:

$$\frac{1}{8}\sqrt{3}\left(\sqrt{5}-1\right)$$

$$\frac{\sqrt{3}}{2\left(1+\sqrt{5}\right)}$$

$$\frac{1}{4}\sqrt{\frac{3}{2}\left(3-\sqrt{5}\right)}$$

Minimal polynomial:

$$256x^4 - 144x^2 + 9$$

Series representations:

$$\frac{\sqrt{3}}{4\phi} = \frac{\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}}{4\phi}$$

$$\frac{\sqrt{3}}{4\phi} = \frac{\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}{4\phi}$$

$$\frac{\sqrt{3}}{4\phi} = \frac{\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{8\phi\sqrt{\pi}}$$

Thence, we have the following equations:

sin(1/3 atan (((3sqrt15)/11))) = sqrt3/(4x)

from which:

Input:

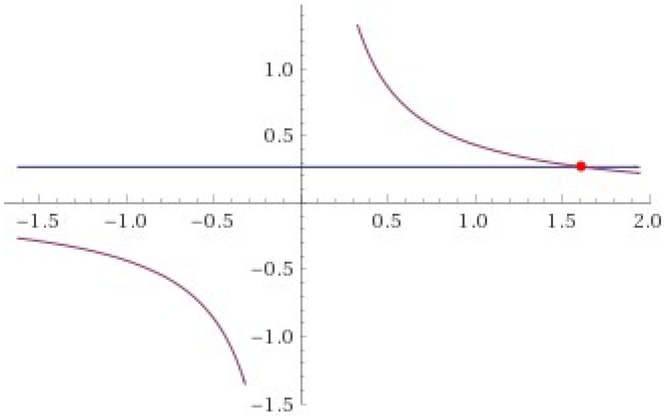
$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{11}\left(3\sqrt{15}\right)\right)\right) = \frac{\sqrt{3}}{4x}$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact result:

$$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \frac{\sqrt{3}}{4x}$$

Plot:



$\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)$

$\frac{\sqrt{3}}{4x}$

Alternate form assuming x is real:

$$\frac{2}{x}+1=\sqrt{5}$$

Alternate forms:

$$\frac{1}{4}\sqrt{\frac{3}{2}\left(3-\sqrt{5}\right)}=\frac{\sqrt{3}}{4x}$$

$$\frac{1}{2}\,i\exp\left(\frac{1}{6}\left(\log\left(1-\frac{3\,i\sqrt{15}}{11}\right)-\log\left(1+\frac{3\,i\sqrt{15}}{11}\right)\right)\right)-\frac{1}{2}\,i\exp\left(\frac{1}{6}\left(\log\left(1+\frac{3\,i\sqrt{15}}{11}\right)-\log\left(1-\frac{3\,i\sqrt{15}}{11}\right)\right)\right)=\frac{\sqrt{3}}{4x}$$

$\log(x)$ is the natural logarithm

Alternate form assuming x is positive:

$$\sqrt{5}\,x=x+2 \quad (\text{for } x\neq 0)$$

Solution:

$$x=\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)$$

that is equal to:

Input:

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{11}\left(3\sqrt{15}\right)\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

$\csc(x)$ is the cosecant function

Exact Result:

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)$$

(result in radians)

Decimal approximation:

1.6180339887498948482045868343656381177203091798057628621354...

(result in radians)

1.6180339887…. = golden ratio

Alternate forms:

$$\frac{1}{2}\left(1+\sqrt{5}\right)$$

$$\sqrt{\frac{2}{3-\sqrt{5}}}$$

$$\frac{\sqrt{3}}{4\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)}$$

Alternative representations:

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=\frac{\sqrt{3}}{4\cos\left(\frac{\pi}{2}-\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)}$$

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=\frac{2\,i\,e^{1/3\,i\tan^{-1}\left(\left(3\sqrt{15}\right)/11\right)}\sqrt{3}}{4\left(-1+e^{2/3\,i\tan^{-1}\left(\left(3\sqrt{15}\right)/11\right)}\right)}$$

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=\frac{2\,i\sqrt{3}}{4\left(-e^{-1/3\,i\tan^{-1}\left(\left(3\sqrt{15}\right)/11\right)}+e^{1/3\,i\tan^{-1}\left(\left(3\sqrt{15}\right)/11\right)}\right)}$$

Series representations:

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=-\frac{1}{2}\,i\sqrt{3}\sum_{k=1}^{\infty}q^{-1+2k}\quad\text{for } q=e^{1/3\,i\tan^{-1}\left(\left(3\sqrt{15}\right)/11\right)}$$

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=\frac{3}{4}\sqrt{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\sum_{k=-\infty}^{\infty}\frac{(-1)^k}{-9k^2\pi^2+\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)^2}$$

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \frac{3\sqrt{3}}{4\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)} - \frac{3}{2}\sqrt{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\sum_{k=1}^{\infty}\frac{(-1)^k}{9k^2\pi^2 - \tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)^2}$$

Integral representation:

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \frac{\sqrt{3}}{4\pi}\int_0^{\infty}\frac{t^{\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)/(3\pi)}}{t+t^2}\,dt$$

Continued fraction representations:

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \frac{1}{4}\sqrt{3}\csc\left(\frac{\sqrt{15}}{11\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{135k^2}{121}}{1+2k}\right)}\right) =$$

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{\sqrt{15}}{11\left(1+\frac{135}{121\left(3+\frac{540}{121\left(5+\frac{1215}{121\left(7+\frac{2160}{121(9+\ldots)}\right)}\right)}\right)}\right)}\right)$$

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=\frac{1}{4}\sqrt{3}\csc\left(\frac{\sqrt{15}}{11\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{135}{121}(1-2k)^2}{-\frac{4}{121}(-64+7k)}\right)}\right)=$$

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{\sqrt{15}}{11\left(1+\frac{135}{121\left(\frac{228}{121}+\frac{1215}{121\left(\frac{200}{121}+\frac{3375}{121\left(\frac{172}{121}+\frac{6615}{121\left(\frac{144}{121}+\ldots\right)}\right)}\right)}\right)}\right)}\right)$$

$$\frac{1}{4}\sqrt{3}\,\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=$$

$$\frac{1}{4}\sqrt{3}\,\csc\left(\frac{1}{3}\left(\frac{3\sqrt{15}}{11}-\frac{405\sqrt{15}}{1331\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{135}{121}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}\right)}\right)\right)=$$

$$\frac{1}{4}\sqrt{3}\,\csc\left(\frac{1}{3}\left(\frac{3\sqrt{15}}{11}-\frac{405\sqrt{15}}{1331\left(3+\frac{1215}{121\left(5+\frac{540}{121\left(7+\frac{3375}{121\left(9+\frac{2160}{121\,(11+\ldots)}\right)}\right)}\right)}\right)}\right)\right)$$

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \frac{1}{4}\sqrt{3}\csc\left(\frac{11\sqrt{15}}{256+121\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{270}{121}\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1+\frac{135}{242}\left(1+(-1)^{k}\right)\right)(1+2k)}\right)}\right) =$$

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{11\sqrt{15}}{256+121\left(-\frac{270}{121\left(3-\frac{270}{121\left(\frac{1280}{121}-\frac{1620}{121\left(7-\frac{1620}{121\left(\frac{2304}{121}+\ldots\right)}\right)}\right)}\right)}\right)}\right)$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

Multiple-argument formulas:

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\cos^{-1}\left(\frac{11}{16}\right)\right)$$

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \frac{1}{8}\sqrt{3}\csc\left(\frac{1}{6}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)\sec\left(\frac{1}{6}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)$$

$$\frac{1}{4}\sqrt{3}\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \frac{\sqrt{3}\csc\left(\frac{1}{9}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)}{4+8\cos\left(\frac{2}{9}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)}$$

And:

1/4 x csc(1/3 tan^(-1)((3 sqrt(15))/11)) = ((sqrt5+1)/2)

From which:

Input:

$$\frac{1}{4}x\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{11}\left(3\sqrt{15}\right)\right)\right) = \frac{1}{2}\left(\sqrt{5}+1\right)$$

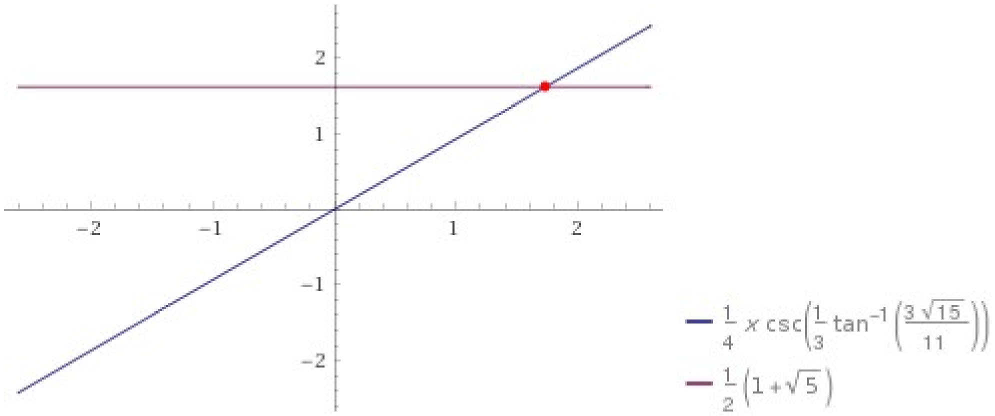
$\tan^{-1}(x)$ is the inverse tangent function

$\csc(x)$ is the cosecant function

Exact result:

$$\frac{1}{4}x\csc\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \frac{1}{2}\left(1+\sqrt{5}\right)$$

Plot:



Alternate forms:

$$\sqrt{\frac{1}{6}\left(3+\sqrt{5}\right)}\ x = \frac{1}{2}\left(1+\sqrt{5}\right)$$

$$\sqrt{\frac{2}{3\left(3-\sqrt{5}\right)}}\ x = \frac{1}{2}\left(1+\sqrt{5}\right)$$

$$\frac{1}{4} x \csc\left(\frac{1}{3} \tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) - \frac{\sqrt{5}}{2} - \frac{1}{2} = 0$$

Expanded form:

$$\frac{1}{4} x \csc\left(\frac{1}{3} \tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \frac{1}{2} + \frac{\sqrt{5}}{2}$$

Alternate form assuming x is real:

$$-\frac{x \sin\left(\frac{1}{3} \tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)}{2\left(\cos\left(\frac{2}{3} \tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) - 1\right)} = \frac{1}{2} + \frac{\sqrt{5}}{2}$$

Solution:

$$x = 2\left(1+\sqrt{5}\right) \sin\left(\frac{1}{3} \tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)$$

that is equal to:

Input:

$$2\left(1+\sqrt{5}\right) \sin\left(\frac{1}{3} \tan^{-1}\left(\frac{1}{11}\left(3\sqrt{15}\right)\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)$$

(result in radians)

Decimal approximation:

1.732050807568877293527446341505872366942805253810380628055...

(result in radians)

1.732050807…. = $\sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3})\, M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

Alternate forms:

$$\sqrt{3}$$

$$\left(2+2\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)$$

$$\frac{1}{2}\sqrt{\frac{3}{2}\left(3-\sqrt{5}\right)}\left(1+\sqrt{5}\right)$$

Alternative representations:

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = -2\cos\left(\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)\left(1+\sqrt{5}\right)$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = 2\cos\left(\frac{\pi}{2}-\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)\left(1+\sqrt{5}\right)$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \frac{2\left(-e^{-1/3\, i\tan^{-1}\left(\left(3\sqrt{15}\right)/11\right)}+e^{1/3\, i\tan^{-1}\left(\left(3\sqrt{15}\right)/11\right)}\right)\left(1+\sqrt{5}\right)}{2i}$$

Series representations:

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=2\left(1+\sqrt{5}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)^{1+2k}}{(1+2k)!}$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=2\left(1+\sqrt{5}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)^{2k}}{(2k)!}$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=$$
$$\frac{1}{3}\left(1+\sqrt{5}\right)\sqrt{\pi}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{36^s\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}$$

Integral representations:

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=$$
$$\frac{2}{3}\left(1+\sqrt{5}\right)\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\int_0^1\cos\left(\frac{1}{3}\,t\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)dt$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=$$
$$-\frac{i\left(1+\sqrt{5}\right)\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)}{6\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)^2/(36s)}}{s^{3/2}}\,ds\quad\text{for }\gamma>0$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=$$
$$-\frac{i\left(1+\sqrt{5}\right)}{\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(\frac{1}{6}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)^{1-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\quad\text{for }0<\gamma<1$$

Continued fraction representations:

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=2\left(1+\sqrt{5}\right)\sin\left(\frac{\sqrt{15}}{11\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{135k^2}{121}}{1+2k}\right)}\right)=$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{\sqrt{15}}{11\left(1+\frac{135}{121\left(3+\frac{540}{121\left(5+\frac{1215}{121\left(7+\frac{2160}{121(9+\ldots)}\right)}\right)}\right)}\right)}\right)$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=2\left(1+\sqrt{5}\right)\sin\left(\frac{\sqrt{15}}{11\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{135}{121}(1-2k)^2}{-\frac{4}{121}(-64+7k)}\right)}\right)=$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{\sqrt{15}}{11\left(1+\frac{135}{121\left(\frac{228}{121}+\frac{1215}{121\left(\frac{200}{121}+\frac{3375}{121\left(\frac{172}{121}+\frac{6615}{121\left(\frac{144}{121}+\ldots\right)}\right)}\right)}\right)}\right)}\right)$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\left(\frac{3\sqrt{15}}{11}-\frac{405\sqrt{15}}{1331\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{135}{121}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}\right)}\right)\right)=$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\left(\frac{3\sqrt{15}}{11}-\frac{405\sqrt{15}}{1331\left(3+\frac{1215}{121\left(5+\frac{540}{121\left(7+\frac{3375}{121\left(9+\frac{2160}{121(11+\ldots)}\right)}\right)}\right)}\right)}\right)\right)$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)=$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{11\sqrt{15}}{256+121\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{270}{121}\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1+\frac{135}{242}\left(1+(-1)^k\right)\right)(1+2k)}\right)}\right)=$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{11\sqrt{15}}{256+121\left(-\frac{270}{121\left(3-\frac{270}{121\left(\frac{1280}{121}-\frac{1620}{121\left(7-\frac{1620}{121\left(\frac{2304}{121}+\ldots\right)}\right)}\right)}\right)}\right)}\right)$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

Multiple-argument formulas:

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = 2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\cos^{-1}\left(\frac{11}{16}\right)\right)$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = 4\left(1+\sqrt{5}\right)\cos\left(\frac{1}{6}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)\sin\left(\frac{1}{6}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right)$$

$$2\left(1+\sqrt{5}\right)\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{3\sqrt{15}}{11}\right)\right) = \frac{i\left(1+\sqrt{5}\right)\left(\sqrt[3]{11-3i\sqrt{15}} - \sqrt[3]{11+3i\sqrt{15}}\right)}{2\sqrt[3]{2}}$$

Observations

Figs.

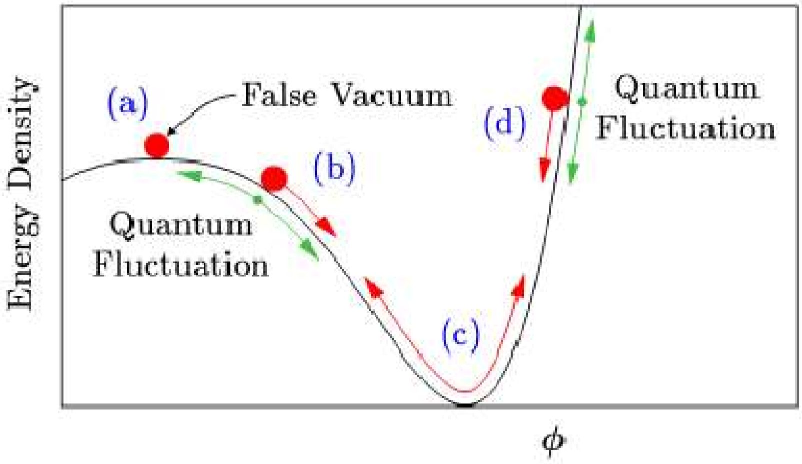
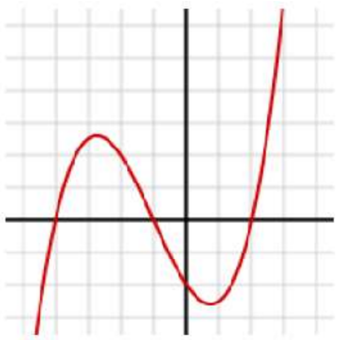


FIG. 1: In simple inflationary models, the universe at early times is dominated by the potential energy density of a scalar field, ϕ. Red arrows show the classical motion of ϕ. When ϕ is near region (a), the energy density will remain nearly constant, $\rho \cong \rho_f$, even as the universe expands. Moreover, cosmic expansion acts like a frictional drag, slowing the motion of ϕ: Even near regions (b) and (d), ϕ behaves more like a marble moving in a bowl of molasses, slowly creeping down the side of its potential, rather than like a marble sliding down the inside of a polished bowl. During this period of "slow roll," ρ remains nearly constant. Only after ϕ has slid most of the way down its potential will it begin to oscillate around its minimum, as in region (c), ending inflation.



Graph of a cubic function with 3 real roots (where the curve crosses the horizontal axis at $y = 0$). The case shown has two critical points. Here the function is $f(x) = (x^3 + 3x^2 - 6x - 8)/4$.

The ratio between M_0 and q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3})\, M_s}{2}.$$

i.e. the gravitating mass M_0 and the Wheelerian mass q of the wormhole, is equal to:

$$\frac{\sqrt{3\,(2.17049\times 10^{37})^2 - 0.001^2}}{\frac{1}{2}\left((3\sqrt{3})\,(4.2\times 10^6\times 1.9891\times 10^{30})\right)}$$

1.7320507879051944207039476256710181600835665488020824 60520...

1.7320507879

$1.7320507879 \approx \sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q of the wormhole

We note that:

$$\left(-\frac{1}{2} + \frac{i}{2}\sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2}\sqrt{3}\right)$$

i is the imaginary unit

$i\sqrt{3}$

1.7320508075688772935274463415058723669428052538103806280 55... i

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

1.73205

This result is very near to the ratio between M_0 and q, that is equal to 1.7320507879 $\approx \sqrt{3}$

With regard $\sqrt{3}$, we note that is a fundamental value of the formula structure that we need to calculate a Cubic Equation

We have that the previous result

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) = i\sqrt{3} =$$

$$= 1.7320508075688772935274463415058723669428052538103806280 55\ldots i$$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

can be related with:

$$u^2(-u)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)+v^2(-v)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)=q$$

Considering:

$$(-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q$$

$$= i\sqrt{3} = 1.7320508075688772935274463415058723669428052538103806280 55\ldots i$$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

Thence:

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) \Rightarrow$$

$$\Rightarrow (-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q=1.73205\approx\sqrt{3}$$

We observe how the graph above, concerning the cubic function, is very similar to the graph that represent the scalar field (in red). It is possible to hypothesize that cubic functions and cubic equations, with their roots, are connected to the scalar field.

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= \qquad\qquad 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = \qquad 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n*, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.*
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

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