On some Ramanujan expressions: mathematical connections with various equations concerning some sectors of Cosmology and Black Holes/Wormholes Physics. VI

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Abstract

In this paper we have described several Ramanujan's expressions and obtained some mathematical connections with various equations concerning different sectors of Cosmology and Black Holes/Wormholes Physics.

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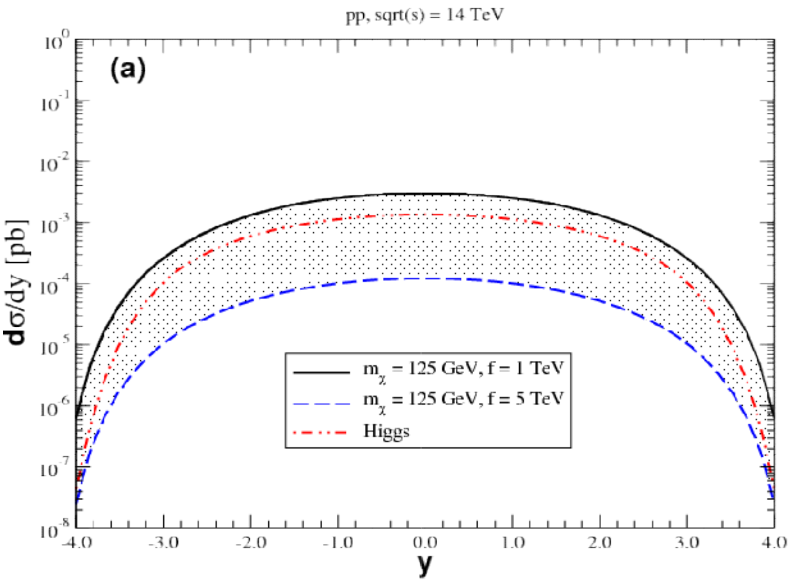
https://www.pinterest.it/pin/742319951051634216/?lp=true

http://inspirehep.net/record/1341042/plots



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Rapidity distribution for the dilaton production in \pom\pom interactions considering (a) pp and (b) PbPb collisions at LHC energies. The corresponding predictions for the SM Higgs production are also presented for comparison.

From:

Black Hole Dynamics in Einstein-Maxwell-Dilaton Theory
Eric W. Hirschmann, Luis Lehner, Steven L. Liebling and Carlos Palenzuela
arXiv:1706.09875v1 [gr-qc] 29 Jun 2017

We have:

values. This behavior can be extracted analytically from the solution presented in Appendix § A (neglecting, for the moment, the asymptotic value of the dilaton) and from which the scalar charge can be calculated as

$$\phi_1 = \frac{\alpha_0 Q_e^2}{M} \frac{1}{1+\sqrt{1+(\alpha_0^2-1)Q_e^2/M^2}} . \qquad (38)$$

The behavior at small α_0 extracted from Eq. (38) is $\phi_1 \approx \alpha_0 Q_e^2/(2M)$ while for large values $\phi_1 \to |Q_e|$. The numerical solutions obtained for $\alpha_0 \lesssim 5000$ are in excellent agreement with this expression while a lower than expected scalar charge is obtained above this value of α_0. We note however that numerical simulations be-

We calculate Q, for M = 13.12806e+39 and $\alpha_0 = 5000$, from

$$\phi_1 = \frac{\alpha_0 Q_e^2}{M} \frac{1}{1+\sqrt{1+(\alpha_0^2-1)Q_e^2/M^2}} .$$

We obtain:

((5000*(x)^2))/(13.12806e+39) * 1/ ((1+sqrt((((((1+(5000^2-1)
*(x)^2))/(13.12806e+39)^2)))))) = 4.82e-7

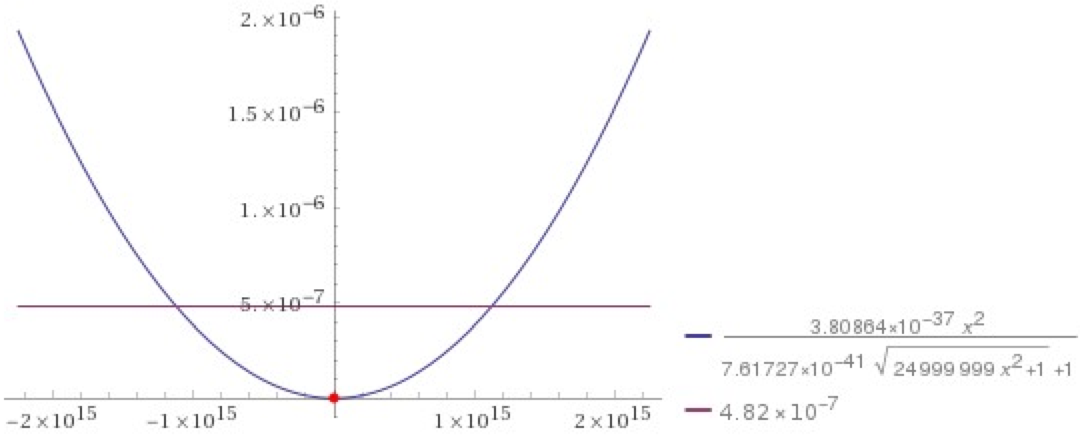
Input interpretation:

$$\frac{5000\, x^2}{13.12806 \times 10^{39}} \times \frac{1}{1+\sqrt{\frac{1+(5000^2-1)\, x^2}{(13.12806 \times 10^{39})^2}}} = 4.82 \times 10^{-7}$$

Result:

$$\frac{3.80864 \times 10^{-37}\, x^2}{7.61727 \times 10^{-41} \sqrt{24\,999\,999\, x^2+1}+1} = 4.82 \times 10^{-7}$$

Plot:



Alternate form:

$$\frac{3.80864 \times 10^{-37}\, x^2}{7.61727 \times 10^{-41} \sqrt{24\,999\,999\, x^2+1}+1} = 4.82 \times 10^{-7}$$

Alternate form assuming x is positive:

$$3.80864 \times 10^{-37}\, x^2 = 3.67152 \times 10^{-47} \sqrt{24\,999\,999\, x^2+1} + 4.82 \times 10^{-7}$$

Solutions:

$x \approx -1.12496 \times 10^{15}$

$x \approx 1.12496 \times 10^{15}$

$1.12496*10^{15}$

Thence Q = 1.12496e+15

Inserting the value of Q in the following expression, we obtain:

((5000*(1.12496e+15)^2))/(13.12806e+39) * 1/ ((1+sqrt((((((1+(5000^2-1) *(1.12496e+15)^2))/(13.12806e+39)^2))))))

Input interpretation:

$$\frac{5000\,(1.12496\times 10^{15})^2}{13.12806\times 10^{39}}\times\frac{1}{1+\sqrt{\frac{1+(5000^2-1)(1.12496\times 10^{15})^2}{(13.12806\times 10^{39})^2}}}$$

Result:

$4.81996\ldots\times 10^{-7}$

$4.81996\ldots * 10^{-7} = \phi_1$ (scalar charge)

pling and scalar charge. The coupling value is straightforward, but the black hole charge is chosen as the charge of individual black holes in isolation. Thus the charges for equal mass binaries are chosen as: $\phi_1 = \{-4.8\times 10^{-7}, -4\times 10^{-4}, -6.9\times 10^{-4}\}$ and for unequal mass binaries (m_1, m_2): $\phi_1 = \{(-3,-2)\times 10^{-7}, (-2.4,-1.6)\times 10^{-4}, (-4.2,-2.7)\times 10^{-4}\}$ for $\alpha_0 = \{1, 10^3, 3\times 10^3\}$ respectively (which are well approximated by the analytical expression Eq. 38).

From:

Higgs Inflation
Javier Rubio - Institut für Theoretische Physik, Ruprecht-Karls-Universität Heidelberg, Heidelberg, Germany – REVIEW - published: 22 January 2019 - doi: 10.3389/fspas.2018.00050

We have that:
For $\xi_h = 3$; $\xi_\chi = 5$, $\alpha = 2$, $\chi = 4$, $h = 8$, $\gamma = 0.40160966445....$

$\bar{a} = 0.10526315789473$ $a = -0.1578947368421....$ $\Theta = 1.0151802656$

$\lambda \leq 10^{-9}$.

$$M_P \equiv 1/\sqrt{8\pi G_N} = 2.435 \times 10^{18} \text{ GeV}$$

$$U(\Theta) = U_0(1-\Theta)^2\,, \qquad U_0 \equiv \frac{\lambda M_P^4}{4}\left(\frac{1+6\bar{a}}{\bar{a}}\right)^2\,, \quad (4.16)$$

(10^-9 (2.435e+18)^4)/4 ((1+6*0.10526315789473)/(0.10526315789473))^2

Input interpretation:

$$\left(\frac{1}{4}\times\frac{(2.435\times10^{18})^4}{10^9}\right)\left(\frac{1+6\times0.10526315789473}{0.10526315789473}\right)^2$$

Result:

$2.1115397778564573045403646928026305245177978552851464... \times 10^{66}$

$2.111539777856...*10^{66}$

$(2.111539777856457304540364 \times 10^{66})*(1-1.0151802656)^2$

Input interpretation:

$2.111539777856457304540364 \times 10^{66}\,(1-1.0151802656)^2$

Result:

486 584 205 501 822 782 661 239 277 904 216 756 183 040 000 000 000 000 000 000 000 000

Scientific notation:

$4.86584205501822782661239277904216756183040 \times 10^{62}$

$4.86584205501822 * 10^{62} = U(\Theta)$

Now, we have that:

$$\exp\left[\frac{2\gamma\Phi}{M_P}\right] \equiv \frac{a}{\bar{a}}\frac{(1+6\xi_h)h^2+(1+6\xi_\chi)\chi^2}{M_P^2}\,, \quad (4.13)$$

For $\xi_h = 3$; $\xi_\chi = 5$, $\alpha = 2$, $\chi = 4$, $h = 8$, $\gamma = 0.40160966445....$
$\bar{a} = 0.10526315789473$ $a = -0.1578947368421$ $\Theta = 1.0151802656$

exp(2*0.40160966445*x)/(2.435e+18) = (-0.1578947368421 / 0.10526315789473) *((1+6*3)*8^2+(1+6*5)*4^2))/(2.435e+18)^2

Input interpretation:

$$\frac{\exp(2\times 0.40160966445\,x)}{2.435\times 10^{18}} = \frac{0.1578947368421}{0.10526315789473}\times\frac{(1+6\times 3)\times 8^2+(1+6\times 5)\times 4^2}{(2.435\times 10^{18})^2}$$

Result:

$$4.10678\times 10^{-19}\,e^{0.80321932890\,x} = 4.33109\times 10^{-34}$$

Alternate form:

$$e^{0.80321932890\,x} = 1.05462\times 10^{-15}$$

Alternate form assuming x is real:

$$4.10678\times 10^{-19}\,e^{0.80321932890\,x}+0 = 4.33109\times 10^{-34}$$

Real solution:

$x \approx -42.9342$

-42.9342 = Φ

exp(2*0.40160966445*(-42.9342))/(2.435e+18)

Input interpretation:

$$\frac{\exp(2\times 0.40160966445\times(-42.9342))}{2.435\times 10^{18}}$$

Result:

$4.33116\ldots\times 10^{-34}$

$4.33116\ldots*10^{-34}$

(0.1578947368421 / 0.10526315789473) *((1+6*3)*8^2+(1+6*5)*4^2)/(2.435e+18)^2

Input interpretation:

$$\frac{0.1578947368421}{0.10526315789473} \times \frac{(1+6\times 3)\times 8^2+(1+6\times 5)\times 4^2}{(2.435\times 10^{18})^2}$$

Result:

$4.331088801656340111903326328965083969659345997158144 6\ldots \times 10^{-34}$

$4.331088801656\ldots*10^{-34}$

From which:

21*5(((-(-0.1578947368421 / 0.10526315789473)
*((1+6*3)*8^2+(1+6*5)*4^2)/(2.435e+18)^2)))^1/4

Input interpretation:

$$21\times 5\sqrt[4]{-\left(-\frac{0.1578947368421}{0.10526315789473}\right)\times \frac{(1+6\times 3)\times 8^2+(1+6\times 5)\times 4^2}{(2.435\times 10^{18})^2}}$$

Result:

$4.7900337249846\ldots \times 10^{-7}$

$4.7900337\ldots*10^{-7}$ result very near to the value ($4.81996\ldots*10^{-7} = \phi_1$) of the scalar charge obtained from the previous expression

$$\phi_1 = \frac{\alpha_0 Q_e^2}{M} \frac{1}{1+\sqrt{1+(\alpha_0^2-1)Q_e^2/M^2}}\,.$$

and:

1/(((-(-0.1578947368421 / 0.10526315789473)
*((1+6*3)*8^2+(1+6*5)*4^2)/(2.435e+18)^2)))^1/160+2*1/10^3

Input interpretation:

$$\frac{1}{\sqrt[160]{-\left(-\frac{0.1578947368421}{0.10526315789473}\right)\times \frac{(1+6\times 3)\times 8^2+(1+6\times 5)\times 4^2}{(2.435\times 10^{18})^2}}}+2\times\frac{1}{10^3}$$

Result:

$1.61829737721863\ldots$

1.61829737721863… result that is a very good approximation to the value of the golden ratio 1,618033988749...

Furthermore, from the ratio between the two expressions

$$2.1115397778564573045403646 \times 10^{66} (1 - 1.0151802656)^2$$

and:

$$\frac{\exp(2 \times 0.40160966445 \times (-42.9342))}{2.435 \times 10^{18}}$$

we obtain:

(((2.111539777856457 × 10^66)*(1-1.0151802656)^2))
*(((exp(2*0.40160966445*(-42.9342))/(2.435e+18))))

Input interpretation:

$$\left(2.111539777856457 \times 10^{66} (1 - 1.0151802656)^2\right) \times \frac{\exp(2 \times 0.40160966445 \times (-42.9342))}{2.435 \times 10^{18}}$$

Result:

$2.10747\ldots \times 10^{29}$

$2.10747\ldots * 10^{29}$

And performing the 140th root, we obtain:

((((((2.111539777856457 × 10^66)*(1-1.0151802656)^2))
*(((exp(2*0.40160966445*(-42.9342))/(2.435e+18))))))))^1/140

Input interpretation:

$$\left(\left(2.111539777856457 \times 10^{66} (1 - 1.0151802656)^2\right) \times \frac{\exp(2 \times 0.40160966445 \times (-42.9342))}{2.435 \times 10^{18}}\right) \wedge (1/140)$$

Result:

1.619778...

1.619778… result that is a good approximation to the value of the golden ratio 1,618033988749...

From:

$$\phi_{\rm E} = \frac{M_P}{2\sqrt{a}} \, \text{arcsinh}(\sqrt{32a}) \,, \tag{4.9}$$

the inflaton value at the end of inflation ($\epsilon(\phi_{\rm end}) \equiv 1$), with $\phi_{\rm E} = \sqrt{3/2}\, \text{arcsinh}(4/\sqrt{3})\, M_P$ corresponding to the $\xi \to \infty$ limit and $\phi_{\rm E} = 2\sqrt{2}\, M_P$ to the end of inflation in a minimally coupled $\lambda\phi^4$ theory. A relation between the non-minimal coupling ξ, the

$$\phi_{\rm E} = \frac{M_P}{2\sqrt{a}} \, \text{arcsinh}(\sqrt{32a}) \,,$$

a = -0.1578947368421; M_P = 2.435e+18

(2.435e+18)/(2*sqrt(0.1578947368421)) asinh(sqrt(32*0.1578947368421))

Input interpretation:

$$\frac{2.435 \times 10^{18}}{2\sqrt{0.1578947368421}} \sinh^{-1}\left(\sqrt{32 \times 0.1578947368421}\right)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Result:

$4.746913025937... \times 10^{18}$

$4.746913025937... * 10^{18} = \phi_E$

From which:

(((1/10^25*1/32 (567 Catalan - 12 + 192 π - 121 π^2 + 28 π log(3) + π log(512)))))((((2.435e+18)/(2*sqrt(0.1578947368421)) asinh(sqrt(32*0.1578947368421))))))

Input interpretation:

$$\left(\frac{1}{10^{25}} \times \frac{1}{32}\left(567\,C - 12 + 192\,\pi - 121\,\pi^2 + 28\,\pi\log(3) + \pi\log(512)\right)\right)$$
$$\left(\frac{2.435 \times 10^{18}}{2\sqrt{0.1578947368421}}\sinh^{-1}\left(\sqrt{32 \times 0.1578947368421}\right)\right)$$

$\log(x)$ is the natural logarithm

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

C is Catalan's constant

Result:

$4.829006130149... \times 10^{-7}$

4.82900613….*10^-7 result very near to the value (4.81996...*10^-7 = ϕ_1) of the scalar charge obtained from the previous expression

$$\phi_1 = \frac{\alpha_0 Q_e^2}{M}\,\frac{1}{1+\sqrt{1+(\alpha_0^2-1)Q_e^2/M^2}}\,.$$

and:

$$\sqrt{3/2}\,\mathrm{arcsinh}(4/\sqrt{3})\,M_P$$

sqrt(3/2)asinh(4/sqrt3)*(2.435e+18)

Input interpretation:

$$\sqrt{\frac{3}{2}}\sinh^{-1}\left(\frac{4}{\sqrt{3}}\right) \times 2.435 \times 10^{18}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Result:

$4.694128762427170044... \times 10^{18}$

4.6941287624…*10^18

and again:

2sqrt2 * (2.435e+18)

Input interpretation:

$$2\sqrt{2}\times 2.435\times 10^{18}$$

Result:

$6.887220048756972888\ldots\times 10^{18}$

$6.887220048\ldots*10^{18}$

The difference between the two results is:

2sqrt2 * (2.435e+18) - ((sqrt(3/2)asinh(4/sqrt3)*(2.435e+18)))

Input interpretation:

$$2\sqrt{2}\times 2.435\times 10^{18}-\sqrt{\frac{3}{2}}\sinh^{-1}\left(\frac{4}{\sqrt{3}}\right)\times 2.435\times 10^{18}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Result:

$2.19309128632980284\ldots\times 10^{18}$

$2.1930912\ldots*10^{18}$

From which:

(((2sqrt2 * (2.435e+18) - ((sqrt(3/2)asinh(4/sqrt3)*(2.435e+18))))))^1/88 + 2*1/10^3

Input interpretation:

$$\sqrt[88]{2\sqrt{2}\times 2.435\times 10^{18}-\sqrt{\frac{3}{2}}\sinh^{-1}\left(\frac{4}{\sqrt{3}}\right)\times 2.435\times 10^{18}}+2\times\frac{1}{10^3}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Result:

1.6179246241723494051...

1.6179246241723494051…. result that is a very good approximation to the value of the golden ratio 1,618033988749...

Now, we have that:

maximum value 1/6. This effective limit simplifies considerably the expression for the critical scale ϕ_C separating the low- and high-energy regimes,

$$\phi_C \simeq \sqrt{\frac{2}{3}} \frac{M_P}{\xi}, \qquad (2.43)$$

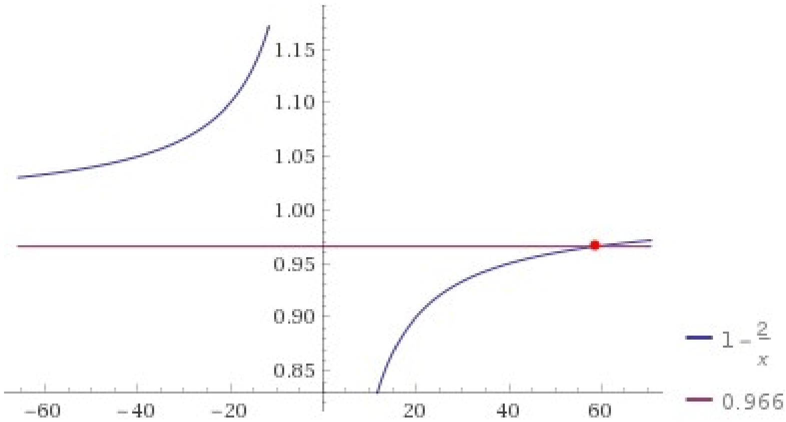
From

$$n_s \simeq 1 - \frac{2}{\bar{N}_*} \simeq 0.966,$$

Input:

$$1 - \frac{2}{x} = 0.966$$

Plot:



Alternate form assuming x is real:

$$\frac{58.8235}{x} = 1$$

Alternate form:

$$\frac{x-2}{x} = 0.966$$

Alternate form assuming x is positive:

$x = 58.8235$ (for $x \neq 0$)

Solution:

$x \approx 58.8235$

58.8235

$$\xi \simeq 3.8 \times 10^6 \bar{N}_*^2 \lambda \,. \qquad (4.10)$$

A simple inspection of Equation (4.7) reveals that the predicted tensor-to-scalar ratio in Palatini Higgs inflation is within the reach of current or future experiments (Matsumura et al., 2016) only if $\xi \lesssim 10$, which, assuming $\bar{N} \simeq 59$, requires a very small coupling $\lambda \lesssim 10^{-9}$. For a discussion of unitarity violations in the Palatini formulation, see Bauer and Demir (2011).

N = 58.8235

3.8e+6 * 58.8235^2 *1/10^8

Input interpretation:

$3.8 \times 10^6 \times 58.8235^2 \times \frac{1}{10^8}$

Result:

131.4877577855

131.4877577855

Or:

3.8e+6 * 59^2 *1/10^8

Input interpretation:

$3.8 \times 10^6 \times 59^2 \times \frac{1}{10^8}$

Result:

132.278

132.278

From

$$\phi_C \simeq \sqrt{\frac{2}{3}} \frac{M_P}{\xi},$$

we obtain:

sqrt(2/3) * (2.435e+18)/131.4877577855

Input interpretation:

$$\sqrt{\frac{2}{3}} \times \frac{2.435 \times 10^{18}}{131.4877577855}$$

Result:

$1.512056489550... \times 10^{16}$

$1.512056489550... * 10^{16} = \phi_C$

We have also:

(6Pi^2)/(((sqrt(2/3) * (2.435e+18)/131.4877577855)))^1/2

Input interpretation:

$$\frac{6\pi^2}{\sqrt{\sqrt{\frac{2}{3}} \times \frac{2.435 \times 10^{18}}{131.4877577855}}}$$

Result:

$4.815783867537... \times 10^{-7}$

$4.815783867537... * 10^{-7} = \phi_1$ (scalar charge)

We note that from the ratio between

$$\phi_E = \frac{M_P}{2\sqrt{a}} \operatorname{arcsinh}(\sqrt{32a}),$$

And

$$\phi_C \simeq \sqrt{\frac{2}{3}}\frac{M_P}{\xi},$$

we obtain:

((((2.435e+18)/(2*sqrt(0.1578947368421)) asinh(sqrt(32*0.1578947368421))))) 1/(((sqrt(2/3) * (2.435e+18)/131.4877577855)))

Input interpretation:

$$\left(\frac{2.435\times10^{18}}{2\sqrt{0.1578947368421}}\sinh^{-1}\left(\sqrt{32\times0.1578947368421}\right)\right)\times\frac{1}{\sqrt{\frac{2}{3}}\times\frac{2.435\times10^{18}}{131.4877577855}}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Result:

313.9375452402...

313.9375452402... = ϕ_E/ϕ_C

From which:

10^2/((3^3*8)) * ((((2.435e+18)/(2*sqrt(0.1578947368421)) asinh(sqrt(32*0.1578947368421))))) 1/(((sqrt(2/3) * (2.435e+18)/131.4877577855))) – 8

Input interpretation:

$$\frac{10^2}{3^3\times8}\left(\frac{2.435\times10^{18}}{2\sqrt{0.1578947368421}}\sinh^{-1}\left(\sqrt{32\times0.1578947368421}\right)\right)\times\frac{1}{\sqrt{\frac{2}{3}}\times\frac{2.435\times10^{18}}{131.4877577855}}-8$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Result:

137.3414561297...

137.3414561297… result practically equal to the golden angle value 137.5 and very near to the inverse of fine-structure constant 137.035

and:

10^2/((3^3*8)) * ((((2.435e+18)/(2*sqrt(0.1578947368421)) asinh(sqrt(32*0.1578947368421))))) 1/(((sqrt(2/3) * (2.435e+18)/131.4877577855))) - 7-3

Input interpretation:

$$\frac{10^2}{3^3 \times 8}\left(\frac{2.435 \times 10^{18}}{2\sqrt{0.1578947368421}} \sinh^{-1}\left(\sqrt{32 \times 0.1578947368421}\right)\right) \times \frac{1}{\sqrt{\frac{2}{3}} \times \frac{2.435 \times 10^{18}}{131.4877577855}} - 7 - 3$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Result:

135.3414561297...

135.3414561297… result practically equal to the rest mass of Pion meson 134.9766 MeV

10^2/((3^3*8)) * ((((2.435e+18)/(2*sqrt(0.1578947368421)) asinh(sqrt(32*0.1578947368421))))) 1/(((sqrt(2/3) * (2.435e+18)/131.4877577855))) - 18-2

Input interpretation:

$$\frac{10^2}{3^3 \times 8}\left(\frac{2.435 \times 10^{18}}{2\sqrt{0.1578947368421}} \sinh^{-1}\left(\sqrt{32 \times 0.1578947368421}\right)\right) \times \frac{1}{\sqrt{\frac{2}{3}} \times \frac{2.435 \times 10^{18}}{131.4877577855}} - 18 - 2$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Result:

125.3414561297...

125.3414561297… result very near to the Higgs boson mass 125.18 GeV

1/2(((10^2/((3^3*8)) * ((((2.435e+18)/(2*sqrt(0.1578947368421)) asinh(sqrt(32*0.1578947368421))))) 1/(((sqrt(2/3) * (2.435e+18)/131.4877577855))) -18+1/golden ratio)))

Input interpretation:

$$\frac{1}{2}\left(\frac{10^2}{3^3\times 8}\left(\frac{2.435\times 10^{18}}{2\sqrt{0.1578947368421}}\sinh^{-1}\left(\sqrt{32\times 0.1578947368421}\right)\right)\times \frac{1}{\sqrt{\frac{2}{3}}\times\frac{2.435\times 10^{18}}{131.4877577855}}-18+\frac{1}{\phi}\right)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

ϕ is the golden ratio

Result:

63.97974505923...

63.97974505923… ≈ 64

27*1/2(((10^2/((3^3*8)) * ((((2.435e+18)/(2*sqrt(0.1578947368421)) asinh(sqrt(32*0.1578947368421))))) 1/(((sqrt(2/3) * (2.435e+18)/131.4877577855))) -18+1/golden ratio)))+8/5

Input interpretation:

$$27\times\frac{1}{2}\left(\frac{10^2}{3^3\times 8}\left(\frac{2.435\times 10^{18}}{2\sqrt{0.1578947368421}}\sinh^{-1}\left(\sqrt{32\times 0.1578947368421}\right)\right)\times \frac{1}{\sqrt{\frac{2}{3}}\times\frac{2.435\times 10^{18}}{131.4877577855}}-18+\frac{1}{\phi}\right)+\frac{8}{5}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

ϕ is the golden ratio

Result:

1729.053116599...

1729.053116599…

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(((27*1/2(((10^2/((3^3*8)) * ((((2.435e+18)/(2*sqrt(0.1578947368421)) asinh(sqrt(32*0.1578947368421))))) 1/(((sqrt(2/3) * (2.435e+18)/131.4877577855))) -18+1/golden ratio)))+8/5)))^1/15 - (21+5)/10^3

Input interpretation:

$$\left(27\times\frac{1}{2}\left(\frac{10^2}{3^3\times 8}\left(\frac{2.435\times 10^{18}}{2\sqrt{0.1578947368421}}\sinh^{-1}\left(\sqrt{32\times 0.1578947368421}\right)\right)\times \frac{1}{\sqrt{\frac{2}{3}\times\frac{2.435\times 10^{18}}{131.4877577855}}}-18+\frac{1}{\phi}\right)+\frac{8}{5}\right)\hat{}\,(1/15)-\frac{21+5}{10^3}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

ϕ is the golden ratio

Result:

1.6178185953430...

1.617818595343… result that is a very good approximation to the value of the golden ratio 1,618033988749...

For

$$a \equiv -\frac{\xi}{1+6\xi} < 0\,, \qquad (2.8)$$

And

$$\phi_C \equiv \frac{2M_P(1-6|a|)}{\sqrt{|a|}}\,. \qquad (2.22)$$

From (2.8), we obtain:

-131.4877577855/(1+6*131.4877577855)

Input interpretation:

$$-\frac{131.4877577855}{1+6\times 131.4877577855}$$

Result:

−0.16645567658491768498555268277476136012980015768130988672...

a = -0.166455676584917684985

For a = -0.166455676584917684985 from (2.22), we obtain:

(((2*(2.435e+18)*(1-6*(-0.166455676584917684985)))))/(sqrt(-0.166455676584917684985))

Input interpretation:

$$\frac{2\times 2.435\times 10^{18}\,(1-6\times(-0.166455676584917684985))}{\sqrt{-0.166455676584917684985}}$$

Result:

$-2.385803488014150983... \times 10^{19}\, i$

Polar coordinates:

$r = 2.3858\times 10^{19}$ (radius), $\theta = -90°$ (angle)

$2.3858*10^{19} = \phi_C$

From which:

Pi^(2/7)*1/((((((2*(2.435e+18)*(1-6*(-0.16645567658)))))/(sqrt(-0.16645567658)))))^1/3

Input interpretation:

$$\pi^{2/7} \times \frac{1}{\sqrt[3]{\frac{2\times 2.435\times 10^{18}\,(1-6\times(-0.16645567658))}{\sqrt{-0.16645567658}}}}$$

Result:

$4.1721690404\ldots \times 10^{-7} +$
$2.4088029186\ldots \times 10^{-7}\, i$

Polar coordinates:

$r = 4.81761\times 10^{-7}$ (radius), $\theta = 30.°$ (angle)

$4.81761*10^{-7} = \phi_1$ (scalar charge)

and:

(7*1/10^2)i - ((((((2*(2.435e+18)*(1-6*(-0.16645567658)))))/(sqrt(-0.16645567658)))))^1/93

Input interpretation:

$$\left(7\times\frac{1}{10^2}\right) i - \sqrt[93]{\frac{2\times 2.435\times 10^{18}\,(1-6\times(-0.16645567658))}{\sqrt{-0.16645567658}}}$$

i is the imaginary unit

Result:

$-1.615473030310\ldots +$
$0.09728839180483\ldots\, i$

Polar coordinates:

$r = 1.6184$ (radius), $\theta = 176.554°$ (angle)

1.6184 result that is a very good approximation to the value of the golden ratio 1,618033988749...

From:

Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? *Ramil N. Izmailov and Eduard R. Zhdanov† Amrita Bhattacharya,‡ Alexander A. Potapov, K.K. Nandi* - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019

We have that:

$$b_R = \int_0^1 g(z, x_m) dz = -\frac{8q \log(q)}{\sqrt{k^2+q^2}} \quad (41)$$

$$= -\left[8\left(\frac{q}{R_s}\right)\ln\left(\frac{q}{R_s}\right)\right]/\sqrt{\left(\frac{\Sigma}{R_s}\right)^2 + 2\left(\frac{q}{R_s}\right)^2}, \quad (42)$$

For numerical estimates of the strong lensing signatures, we choose as Schwarzschild black hole the SgrA* residing in our galactic center[2] but add that any other black hole would be good enough for comparison with wormhole signatures provided they share the same u_m. If any other black hole is chosen, then only u_m^{Sch}/R_s would numerically change as would the corresponding values of q/R_s for EMD wormhole and m/R_s for EMS wormhole. We adopt the latest observed data pertaining to SgrA* from [28]: Mass $M_s = 4.2 \times 10^6 M_\odot$, $D_{\mathrm{OL}} = 7.6$ kpc, which imply $R_s = 2M_s$,

For a correct comparison, the minimum impact parameter u_m of rays in the Schwarzschild black hole and EMD wormhole spacetime should be the same, which implies

$$u_m^{\mathrm{Sch}} = \left(\frac{3\sqrt{3}}{2}\right) R_s = \left(3\sqrt{3}\right) M_s = u_m^{\mathrm{EMD}} = 2q \quad (43)$$

$$\rightarrow \frac{q}{R_s} - \frac{3\sqrt{3}}{4}. \quad (44)$$

The last equation yields a formal identification of the Wheelerian mass q with the BH mass M_s as

$$q = \frac{(3\sqrt{3})\, M_s}{2}. \quad (45)$$

The only variable in the Eqs.(38,39) now is the adimensionalized dilatonic charge $\frac{\Sigma}{R_s}$, and by varying it, we shall tabulate below the observables for massless EMD wormhole.

From:

$$q = \frac{(3\sqrt{3})\, M_s}{2}.$$

For M_s = 8.35422e+36, we obtain:

((3sqrt3)*(4.2*10^6 * 1.9891*10^30))/2

Input interpretation:

$$\frac{1}{2}\left(\left(3\sqrt{3}\right)\left(4.2\times 10^6\times 1.9891\times 10^{30}\right)\right)$$

Result:

$2.17049\ldots \times 10^{37}$

$2.17049\ldots*10^{37} = q$

From

$$u_m^{\text{Sch}} = \left(\frac{3\sqrt{3}}{2}\right) R_s = \left(3\sqrt{3}\right) M_s = u_m^{\text{EMD}} = 2q$$
$$\Rightarrow \frac{q}{R_s} = \frac{3\sqrt{3}}{4}.$$

we obtain:

(2.17049e+37)/x = (3sqrt3)/4

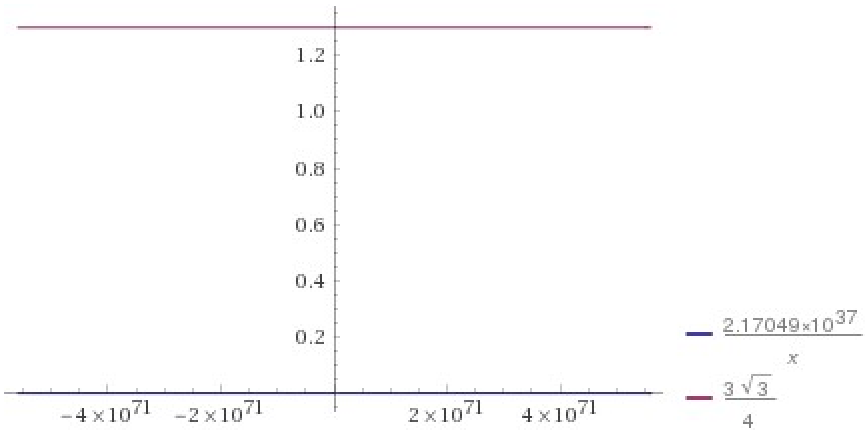
Input interpretation:

$$\frac{2.17049 \times 10^{37}}{x} = \frac{1}{4}\left(3\sqrt{3}\right)$$

Result:

$$\frac{2.17049 \times 10^{37}}{x} = \frac{3\sqrt{3}}{4}$$

Plot:



Alternate form assuming x is real:

$$\frac{1.67084 \times 10^{37}}{x} = 1$$

Alternate form assuming x is positive:

$x = 1.67084 \times 10^{37}$ (for $x \neq 0$)

$1.67084 * 10^{37} = R_s$

Solution:

$x = 16\,708\,439\,810\,311\,879\,274\,437\,516\,082\,225\,872\,896$

Integer solution:

$x = 16\,708\,439\,810\,311\,879\,274\,437\,516\,082\,225\,872\,896$

For $\Sigma = 0.001$, $q = 2.17049e+37$, $R_s = 1.67084e+37$, we obtain:

-2,71890834931952985 6 / 1,837121684299621192

$$b_R = \int_0^1 g(z, x_m)dz = -\frac{8q\log(q)}{\sqrt{k^2+q^2}}$$
$$= -\left[8\left(\frac{q}{R_s}\right)\ln\left(\frac{q}{R_s}\right)\right]/\sqrt{\left(\frac{\Sigma}{R_s}\right)^2+2\left(\frac{q}{R_s}\right)^2},$$

-[8((2.17049e+37)/(1.67084e+37)) ln ((2.17049e+37)/(1.67084e+37))] * 1 / sqrt((((((0.001)/(1.67084e+37))^2+2((2.17049e+37)/(1.67084e+37))^2))))

Input interpretation:

$$-\left(8\times\frac{2.17049\times10^{37}}{1.67084\times10^{37}}\log\left(\frac{2.17049\times10^{37}}{1.67084\times10^{37}}\right)\right)\times\frac{1}{\sqrt{\left(\frac{0.001}{1.67084\times10^{37}}\right)^2+2\left(\frac{2.17049\times10^{37}}{1.67084\times10^{37}}\right)^2}}$$

$\log(x)$ is the natural logarithm

Result:

$-1.47998...$

$-1.47998... = b_R$ this result is very near to the following sum of two Ramanujan mock theta functions values:

1.1424432422 + 0.346471936 = 1.4889151782

Indeed:

$$2\phi(-q^2) - f(q) = \frac{1-2q+2q^4-2q^9+\ldots}{(1-q)(1-q^4)(1-q^6)(1-q^9)\ldots},$$

$$\frac{1+2\times(-0.449329)+2\times 0.449329^4-2\times 0.449329^9}{(1-0.449329)(1-0.449329^4)(1-0.449329^6)(1-0.449329^9)}$$

0.346471936078199831796528233455818281740464490507361168330...

$2\phi(-q^2) - f(q)$ = 0.34647193607819....

Mock ϑ-functions (of 5th order).

$$f(q)=1+\frac{q^2}{1+q}+\frac{q^6}{(1+q)(1+q^2)}+\frac{q^{12}}{(1+q)(1+q^2)(1+q^3)}+\ldots,$$

$$\phi(q)=q+q^4(1+q)+q^9(1+q)(1+q^3)+\ldots,$$

$$\psi(q)=1+q(1+q)+q^3(1+q)(1+q^2)+q^6(1+q)(1+q^2)(1+q^3)+\ldots,$$

$$\chi(q)=\frac{1}{1-q}+\frac{q}{(1-q^2)(1-q^3)}+\frac{q^2}{(1-q^3)(1-q^4)(1-q^5)}+\ldots,$$

$$F(q)=\frac{1}{1-q}+\frac{q^4}{(1-q)(1-q^3)}+\frac{q^{12}}{(1-q)(1-q^3)(1-q^5)}+\ldots$$

From the first expression, we obtain:

$$\left(1+\frac{0.449329^2}{1+0.449329}+\frac{0.449329^6}{(1+0.449329)+(1+0.449329^2)}\right)+\frac{0.449329^{12}}{(1+0.449329)(1+0.449329^2)(1+0.449329^3)}$$

1.142443242201380904097917635488946328383797361320962332093...

f(q) = 1.1424432422...

we have that:

The weak field deflection of light by this wormhole has been verified by three independent methods in [26]. The meaning of $\pm m$ as Wheelerian masses, up to an unimportant constant factor of $\sqrt{2}$, is evident from Eq.(56). These masses can also be called scalar charges, since the total integrated energy of the ghost scalar field ϕ is $\pm m$. The mass $+m$ is responsible for inward bending of light on the positive side of the massless EMS wormhole. We find that

$$\log\left(\frac{2\beta_m}{y_m}\right) = \log\left(\frac{\pi^2}{2}\right), \tag{57}$$

and a much simplified expression

$$g(z,0,0) = \frac{\pi z - \sqrt{2}\sqrt{1-\cos(\pi z)}}{z\sin\left(\frac{\pi z}{2}\right)}, \tag{58}$$

leading to

$$b_R = \int_0^1 g(z,0,0)dz = \log(16) - 2\log\pi. \tag{59}$$

Collecting the values from Eqs.(52), (57) and (59), we find

$$\bar{b} = -\pi + b_R + \bar{a}\log\frac{2\beta_m}{y_m} = -\pi + 3\log(2), \tag{60}$$

$$\bar{a} = 1. \tag{61}$$

ln((Pi^2)/2)

Input:

$$\log\left(\frac{\pi^2}{2}\right)$$

log(x) is the natural logarithm

Decimal approximation:

1.596312591138855038869622581247940855219089491470367888906...

1.596312591138855…. = $\log\left(\frac{2\beta_m}{y_m}\right)$

Alternate form:

$2\log(\pi) - \log(2)$

Alternative representations:

$$\log\left(\frac{\pi^2}{2}\right) = \log_e\left(\frac{\pi^2}{2}\right)$$

$$\log\left(\frac{\pi^2}{2}\right) = \log(a)\log_a\left(\frac{\pi^2}{2}\right)$$

$$\log\left(\frac{\pi^2}{2}\right) = -\text{Li}_1\left(1-\frac{\pi^2}{2}\right)$$

Series representations:

$$\log\left(\frac{\pi^2}{2}\right) = \log\left(\frac{1}{2}(-2+\pi^2)\right) - \sum_{k=1}^{\infty}\frac{\left(-\frac{2}{-2+\pi^2}\right)^k}{k}$$

$$\log\left(\frac{\pi^2}{2}\right) = 2 i \pi \left\lfloor \frac{\arg(\pi^2 - 2x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k (\pi^2 - 2x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log\left(\frac{\pi^2}{2}\right) = 2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k (\pi^2 - 2z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$\log\left(\frac{\pi^2}{2}\right) = \int_1^{\frac{\pi^2}{2}} \frac{1}{t}\,dt$$

$$\log\left(\frac{\pi^2}{2}\right) = -\frac{i}{2\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(\frac{2}{-2+\pi^2}\right)^s \Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1 < \gamma < 0$$

-Pi+3ln2

Input:

$-\pi + 3\log(2)$

$\log(x)$ is the natural logarithm

Decimal approximation:

−1.062151111909957310210947018904973179970668996294340058 61...

-1.06215111190995731….. = $\bar{b}$

Alternate form:

$\log(8) - \pi$

Alternative representations:

$$-\pi + 3\log(2) = -\pi + 3\log_e(2)$$

$$-\pi + 3\log(2) = -\pi + 3\log(a)\log_a(2)$$

$$-\pi + 3\log(2) = -\pi + 6\coth^{-1}(3)$$

Series representations:

$$-\pi + 3\log(2) = -\pi + 6\,i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor + 3\log(x) - 3\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$-\pi + 3\log(2) = -\pi + 3\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right) + 3\log(z_0) + 3\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0) - 3\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}$$

$$-\pi + 3\log(2) = -\pi + 6\,i\,\pi\left\lfloor\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi}\right\rfloor + 3\log(z_0) - 3\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}$$

Integral representations:

$$-\pi + 3\log(2) = -\pi + 3\int_1^2 \frac{1}{t}\,dt$$

$$-\pi + 3\log(2) = -\pi - \frac{3\,i}{2\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1 < \gamma < 0$$

ln(16)-2lnPi

Input:

$$\log(16) - 2\log(\pi)$$

$\log(x)$ is the natural logarithm

Decimal approximation:

0.4831289505409808893820737831265888490074109116103978734 55...

0.4831289505… = b_R

Alternate forms:

$$\log\left(\frac{16}{\pi^2}\right)$$

$$-2\,(\log(\pi) - 2\log(2))$$

$$4\log(2) - 2\log(\pi)$$

Alternative representations:

$$\log(16) - 2\log(\pi) = \log_e(16) - 2\log_e(\pi)$$

$$\log(16) - 2\log(\pi) = \log(a)\log_a(16) - 2\log(a)\log_a(\pi)$$

$$\log(16) - 2\log(\pi) = -\text{Li}_1(-15) + 2\,\text{Li}_1(1-\pi)$$

Series representations:

$$\log(16) - 2\log(\pi) = \log(15) - 2\log(-1+\pi) + \sum_{k=1}^{\infty} \frac{(-1)^{1+k}\,15^{-k} + 2\left(\frac{1}{1-\pi}\right)^k}{k}$$

$$\log(16) - 2\log(\pi) = -2\,i\,\pi\left\lfloor\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\,\pi}\right\rfloor - \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^{1+k}\left((16-z_0)^k - 2\,(\pi - z_0)^k\right) z_0^{-k}}{k}$$

$$\log(16) - 2\log(\pi) = 2\,i\,\pi\left\lfloor\frac{\arg(16-x)}{2\,\pi}\right\rfloor - 4\,i\,\pi\left\lfloor\frac{\arg(\pi-x)}{2\,\pi}\right\rfloor - \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^{1+k}\left((16-x)^k - 2\,(\pi-x)^k\right)x^{-k}}{k} \quad \text{for } x < 0$$

Integral representations:

$$\log(16) - 2\log(\pi) = \int_1^{16}\left(\frac{2 - 2\,\pi}{16 + \pi\,(-1+t) - t} + \frac{1}{t}\right)dt$$

$$\log(16) - 2\log(\pi) = \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{i\,(2\times 15^s - (-1+\pi)^s)\,(15\,(-1+\pi))^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{2\,\pi\,\Gamma(1-s)}\,ds \quad \text{for } -1 < \gamma < 0$$

For the strong field deflection α in the EMS wormhole metric (46), we find that the integrand $g(z, x_m)$ has a formidable expression that has to be integrated only numerically. Thus, we can in general write, using $x_m = 2M = 2m\gamma$,

choosing $\gamma = -i$

From:

2M = 2m*-i ;   for M_s = 8.35422e+36 , we obtain:

2*(8.35422e+36) = 2*x(-i)

Input interpretation:

$2 \times 8.35422 \times 10^{36} = 2\,x\,(-i)$

i is the imaginary unit

Result:

$1.67084 \times 10^{37} = -2\,i\,x$

Alternate form:

$1.67084 \times 10^{37} + 2\,i\,x = 0$

Complex solution:

$x = 8\,354\,220\,000\,000\,000\,025\,204\,024\,079\,080\,751\,104\,i$

8.35422e+36 *i* = m

From:

$$\phi(x) = \pm\sqrt{\frac{1}{2}}\left[\frac{\pi}{2} - 2\tan^{-1}\left(\frac{x}{m}\right)\right] \simeq \pm\phi_0 \pm \frac{\sqrt{2}m}{x} + O\left(\frac{m^3}{x^3}\right)$$

For m = 8.35422e+36*i* ; x = 2M = 2*8.35422e+36 , we obtain:

sqrt(1/2)*(Pi/2 – 2 tan^-1((2*(8.35422e+36))/(8.35422e+36i)))

Input interpretation:

$$\sqrt{\frac{1}{2}}\left(\frac{\pi}{2} - 2\tan^{-1}\left(\frac{2 \times 8.35422 \times 10^{36}}{8.35422 \times 10^{36}\,i}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

i is the imaginary unit

Result:

3.33216... +
0.776836... i

(result in radians)

Polar coordinates:

$r = 3.42152$ (radius), $\theta = 13.1231°$ (angle)

3.42152 = $\phi(x)$ = exotic scalar field

Now, we have that:

The story with the EMD wormhole is quite different since there is now a freely specifiable dilatonic charge Σ/R_s. Specifying different values to it, we find that the equation, with the Wheelerian mass $q/R_s = \frac{3\sqrt{3}}{4}$, viz.,

$$\alpha(\Delta, \Sigma/R_s) - 2\pi = 0 \tag{69}$$

corresponds to different sets Δ and of (θ_∞, s, r) for EMD wormholes, the difference signalling the presence of dilatonic charge in the lens. These sets of values are markedly different from those of EMS wormhole and black hole. The

From:

$$\frac{q}{R_s} = \frac{3\sqrt{3}}{4}.$$

We obtain:

((2.17049e+37)/(1.67084e+37))

Input interpretation:

$$\frac{2.17049 \times 10^{37}}{1.67084 \times 10^{37}}$$

Result:

1.2990412008331138828373752124679801776352014555552895549 54...

1.2990412008331138……

((3sqrt3)/4)

Input:

$$\frac{1}{4}\left(3\sqrt{3}\right)$$

Exact result:

$$\frac{3\sqrt{3}}{4}$$

Decimal approximation:

1.299038105676657970145584756129404275207103940357785471041...

1.29903810567665797……

For

$$\bar{a} = \left(\frac{q}{R_s}\right) / \sqrt{\left(\frac{\Sigma}{R_s}\right)^2 + 2\left(\frac{q}{R_s}\right)^2}$$

((2.17049e+37)/(1.67084e+37))* 1 / sqrt((((((0.001)/(1.67084e+37))^2+2((2.17049e+37)/(1.67084e+37))^2))))

Input interpretation:

$$\frac{2.17049\times 10^{37}}{1.67084\times 10^{37}} \times \frac{1}{\sqrt{\left(\frac{0.001}{1.67084\times 10^{37}}\right)^2 + 2\left(\frac{2.17049\times 10^{37}}{1.67084\times 10^{37}}\right)^2}}$$

Result:

0.707106781186547524400844362104849039284835937688474036588...

0.7071067811865.... = $\bar{a}$

For -1.47998… = b_R; $\bar{a}$ = 0.7071067811865....

$$\bar{b} = -\pi + b_R + \bar{a}\log 2,$$

-Pi-1.47998+0.7071067811865 ln(2)

Input interpretation:

$-\pi - 1.47998 + 0.7071067811865 \log(2)$

$\log(x)$ is the natural logarithm

Result:

-4.13144...

-4.13144… = $\bar{b}$

Alternative representations:

$$-\pi - 1.47998 + 0.70710678118650000 \log(2) = -1.47998 - \pi + 0.70710678118650000 \log_e(2)$$

$$-\pi - 1.47998 + 0.70710678118650000 \log(2) = -1.47998 - \pi + 0.70710678118650000 \log(a) \log_a(2)$$

$$-\pi - 1.47998 + 0.70710678118650000 \log(2) = -1.47998 - \pi + 1.4142135623730000 \coth^{-1}(3)$$

Series representations:

$$-\pi - 1.47998 + 0.70710678118650000 \log(2) = -1.47998 - \pi + 1.4142135623730000\, i\, \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + 0.70710678118650000 \log(x) - 0.70710678118650000 \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$-\pi - 1.47998 + 0.70710678118650000 \log(2) = -1.47998 - \pi + 0.70710678118650000 \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + 0.70710678118650000 \log(z_0) + 0.70710678118650000 \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \log(z_0) - 0.70710678118650000 \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}$$

$$-\pi - 1.47998 + 0.70710678118650000 \log(2) = -1.47998 - \pi + 1.4142135623730000\, i\, \pi \left\lfloor -\frac{-\pi + \arg\left(\frac{2}{z_0}\right) + \arg(z_0)}{2\pi} \right\rfloor + 0.70710678118650000 \log(z_0) - 0.70710678118650000 \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$-\pi - 1.47998 + 0.70710678118650000 \log(2) = -1.47998 - \pi + 0.70710678118650000 \int_1^2 \frac{1}{t}\, dt$$

$$-\pi - 1.47998 + 0.70710678118650000 \log(2) = -1.47998 - \pi + \frac{0.35355339059325000}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1 < \gamma < 0$$

From:

$$\alpha(b) = \frac{\pi}{4b^2}\left(3q^2 - \Sigma^2\right), \qquad (75)$$

We obtain:

Pi/(4*-4.13144^2) * (3*(2.17049e+37)^2 – 0.001^2)

Input interpretation:

$$\frac{\pi}{4\times(-1)\times 4.13144^2}\left(3\,(2.17049\times 10^{37})^2 - 0.001^2\right)$$

Result:

$-6.50315... \times 10^{73}$

$-6.50315...*10^{73}$

the effective gravitating mass is $M_0 = \sqrt{3q^2 - \Sigma^2}$,

sqrt(((3*(2.17049e+37)^2 – 0.001^2)))

Input interpretation:

$$\sqrt{3\,(2.17049\times 10^{37})^2 - 0.001^2}$$

Result:

$3.75940... \times 10^{37}$

$3.75940...*10^{37} = M_0$

All 2nd roots of 1.41331×10^75:

$3.7594\times 10^{37}\, e^{0} \approx 3.7594\times 10^{37}$ (real, principal root)

$3.7594\times 10^{37}\, e^{i\pi} \approx -3.7594\times 10^{37}$ (real root)

(the leading order deflection by the massless EMD wormhole obtained in [24] using the Gauss-Bonnet method (75). which reveals, following Schwarzschild formula, that the effective gravitating mass is M_0 and not merely q.)

$2.17049 * 10^{37} = q$

(when dilaton is switched off, Σ = 0, the metric (29-31) reduces to the famous Einstein-Rosen bridge [32] and in this case, the mass is proportional to just q).

The ratio between M_0 and q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3})\, M_s}{2}.$$

is equal to:

sqrt(((3*(2.17049e+37)^2 – 0.001^2))) / ((3sqrt3)*(4.2*10^6 * 1.9891*10^30))/2

Input interpretation:

$$\frac{\sqrt{3\,(2.17049\times 10^{37})^2 - 0.001^2}}{\frac{1}{2}\left((3\sqrt{3})(4.2\times 10^6\times 1.9891\times 10^{30})\right)}$$

Result:

1.7320507879051944207039476256710181600835665488020824605 20...

Input interpretation:

1.7320507879

$1.7320507879 \approx \sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q of the wormhole

Rational approximation:

$$\frac{310070}{179019} = 1 + \frac{131051}{179019}$$

Possible closed forms:

$\sqrt{3} \approx 1.73205080756$

$\mathcal{T}_T \approx 1.73205080756$

$\frac{2\,\mathcal{T}_{20VE}}{3} \approx 1.73205080756$

$\frac{3-2\,\mathcal{T}_T}{\mathcal{T}_T-2} \approx 1.73205080756$

$\frac{75757\,\pi}{137408} \approx 1.73205078785807$

$-\frac{e!}{9}+9-\frac{16}{3\,e}-\frac{16\,e}{9} \approx 1.73205078780076$

$\sqrt[3]{\frac{1}{7}(7-78\,e+70\,\pi+31\log(2))} \approx 1.73205078780433$

π (root of $10\,x^4+87\,x^3-61\,x^2-9\,x+8$ near $x = 0.551329$) ≈ 1.73205078779476

π (root of $197\,x^3+82\,x^2-67\,x-21$ near $x = 0.551329$) ≈ 1.73205078787978

$\sqrt{\frac{1}{46}(-65+24\,e+54\,\pi-46\log(2))} \approx 1.73205078793372$

$\log\left(\frac{1}{5}\left(-4+22\sqrt{2}+5\,e+7\,e^2-11\,\pi-3\,\pi^2\right)\right) \approx 1.73205078789848$

$\frac{1}{7}\left(-3+e+6\sqrt{1+e}-5\,\pi+\pi^2+13\sqrt{1+\pi}-6\sqrt{1+\pi^2}\right) \approx 1.73205078791912$

$\frac{1}{6}(-C-16+14\,\pi+\pi^2-14\,\pi\log(3)+2\,\pi\log(32)) \approx 1.73205078784673$

π (root of $37\,x^5+14\,x^4+9\,x^3-8\,x^2+34\,x-21$ near $x = 0.551329$) $\approx$
1.73205078790007

$\frac{13\log^2(2)\log(3)+15\log(2)\log^2(3)-18\log(2)+38\log(3)-57\log(2)\log(3)}{\log(3)\log(16)} \approx$
1.73205078794961

We note that:

(-1/2+i/2(sqrt3))-(-1/2-i/2(sqrt3))

Input:

$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)$

i is the imaginary unit

Result:

$i\sqrt{3}$

Decimal approximation:

1.7320508075688772935274463415058723669428052538103806280558... i

Polar coordinates:

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

1.73205

This result is very near to the ratio between M_0 and q, that is equal to 1.7320507879 $\approx \sqrt{3}$

With regard $\sqrt{3}$, we note that is a fundamental value of the formula structure that we need to calculate a Cubic Equation

In algebra, a **cubic equation** in one variable is an equation of the form

$$ax^3 + bx^2 + cx + d = 0$$

in which *a* is nonzero

We note that, from the above equation, applying the following substitution,

$$x = y - \frac{b}{3a}$$

we obtain the following form

$$y^3 + py + q = 0$$

Where

$$p = \frac{c}{a} - \frac{b^2}{3a^2} \quad q = \frac{d}{a} - \frac{bc}{3a^2} + \frac{2b^3}{27a^3}$$

We obtain an equation where the solutions are

$$y = u + v$$

and where u and v are the roots

$$u = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}; \quad v = \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

From which we have that:

$$y = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

Thence, the formula for calculate the solutions of a cubic equation, is:

$$x = -\frac{b}{3a} + \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

For the fundamental theorem of algebra a cubic equation must have 3 solutions, therefore we must also evaluate the complex results of the roots. Now it is necessary to calculate whether the quantity under the square roots, which we will call Δ_{III} , is positive or negative.

If

$$\Delta_{III} > 0$$

We calculate the two real numbers u and v that are equals to:

$$u = \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta_{III}}}; \quad v = \sqrt[3]{-\frac{q}{2} - \sqrt{\Delta_{III}}}$$

and the solutions of equation are:

$$\begin{aligned} y_1 &= u + v \\ y_2 &= u \cdot \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) + v \cdot \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) \\ y_3 &= u \cdot \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) + v \cdot \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \end{aligned}$$

Thence, there are two real numbers:

$$u_1 = \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta_{III}}}$$
$$v_1 = \sqrt[3]{-\frac{q}{2} - \sqrt{\Delta_{III}}}$$

and six results

$$u_1 = \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta_{III}}}, \quad u_2 = u_1 \cdot \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right), \quad u_3 = u_1 \cdot \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$
$$v_1 = \sqrt[3]{-\frac{q}{2} - \sqrt{\Delta_{III}}}, \quad v_2 = v_1 \cdot \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right), \quad v_3 = v_1 \cdot \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$

and the solutions of cubic equation that are

$$y_1 = u_1 + v_1$$
$$y_2 = u_2 + v_3 = u_1 \cdot \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) + v_1 \cdot \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$
$$y_3 = u_3 + v_2 = u_1 \cdot \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) + v_1 \cdot \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

We note that in the previous analyzed expressions, we have obtained

$$\left(-\frac{1}{2} + \frac{i}{2}\sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2}\sqrt{3}\right) = i\sqrt{3} =$$

$$= 1.7320508075688772935274463415058723669428052538103806280 55\ldots\, i$$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

that is

$$y_2 = u_2 + v_3 = u_1 \cdot \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) + v_1 \cdot \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$

with $u_1 = 1$ and $v_1 = -1$.

With regard the cubic equations, we have the following section.

Cubic Equation

The most general cubic equation (equation of third degree):

$$x^3 + rx^2 + px + q = 0 \quad (1)$$

can be reduced in the following form:

$$x^3 + px + q = 0 \quad (2)$$

Indeed:

$$(x-u)(x-v)(x-z) = (x^2 - vx - ux + uv)(x-z) =$$

$$= [x^2 - (u+v)x + uv](x-z) = x^3 - (u+v)x^2 + uvx - zx^2 + z(u+v)x - zuv =$$

$$= x^3 - (u+v+z)x^2 + [uv + z(u+v)]x - zuv\,. \quad (2b)$$

Thence:

$$x^3 - (u+v+z)x^2 + [uv + z(u+v)]x - zuv = 0 \quad (3)$$

Putting:

$$\begin{cases} u + v + z = -r \\ z(u+v) + uv = p \\ uvz = -q \end{cases} \quad (4)$$

because the term r become null, is necessary and sufficient that $z = -(u+v)$. Thence:

$$x^3-(u+v+z)x^2+[uv+z(u+v)]x-zuv=0;$$

$$x^3-[u+v-(u+v)]x^2+[uv-(u+v)(u+v)]x-[-(u+v)uv]=0;$$

$$x^3-[uv-(u+v)^2]x-[-(u+v)uv]=0;$$

$$x^3-[uv-(u^2+2uv+v^2)]x+u^2v+uv^2=0;$$

$$x^3-[uv-u^2-2uv-v^2]x+u^2v+uv^2=0;$$

$$x^3-(-uv-u^2-v^2)x+uv(u+v)=0;$$

$$x^3-(u^2+uv+v^2)x+uv(u+v)=0 \qquad (5)$$

Putting:

$$\begin{cases} u^2+uv+v^2=-p \\ uv(u+v)=q \end{cases} \qquad (6)$$

the $x^3-(u^2+uv+v^2)x+uv(u+v)=0$, become:

$$x^3+px+q=0 \qquad (7)$$

From the $u^2+uv+v^2=0$, we obtain two quadratic equations (of second degree):

1) $u^2+uv+v^2=0$;

$$u=\frac{-v\pm\sqrt{v^2-4v^2}}{2}=\frac{-v\pm\sqrt{-3v^2}}{2}=-\frac{1}{2}v\pm v\frac{\sqrt{3i^2}}{2}=-v\left(\frac{1}{2}\pm\frac{\sqrt{3i^2}}{2}\right)=-v\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right);$$

$$2)\ v^2 + uv + u^2 = 0;$$

$$v = \frac{-u \pm \sqrt{u^2 - 4u^2}}{2} = \frac{-u \pm \sqrt{-3u^2}}{2} = -\frac{1}{2}u \pm u\frac{\sqrt{3i^2}}{2} = -u\left(\frac{1}{2} \pm \frac{\sqrt{3i^2}}{2}\right) = -u\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right).$$

Now, we take the eq. (6). We have that:

$$\begin{cases} u^2 + uv + v^2 = -p \\ u = -v\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right) \\ v = -u\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right) \end{cases} \quad (7a) \qquad \begin{cases} uv = -uv\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right)^2 \\ u^2 = u \cdot u = -uv\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right)^2 \\ v^2 = v \cdot v = -uv\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right)^2 \end{cases}$$

thence:

$$-\left[uv\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right)^2 + uv\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right)^2 + uv\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right)^2\right] = -(uv + uv + uv) = p;$$

$$3uv = -p; \quad uv = -\frac{p}{3} \quad (8)$$

Indeed, we have, for example:

$$-\left[uv\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^2 + uv\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^2 + uv\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^2\right] = p$$

$$-\left[uv\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\right]=$$

$$=-\left[uv\left(\frac{1}{4}+\frac{i\sqrt{3}}{2}-\frac{3}{4}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(\frac{1}{4}+\frac{i\sqrt{3}}{2}-\frac{3}{4}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(\frac{1}{4}+\frac{i\sqrt{3}}{2}-\frac{3}{4}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\right]=$$

$$=-\left[uv\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\right]=$$

$$=-\left[uv\left(\frac{1}{4}+\frac{3}{4}\right)+uv\left(\frac{1}{4}+\frac{3}{4}\right)+uv\left(\frac{1}{4}+\frac{3}{4}\right)\right]=$$

$$=-(uv+uv+uv)=p\,;\quad 3uv=-p\,;\quad uv=-\frac{p}{3}\,.$$

Now:

$$\begin{cases} u^2v+uv^2=q \\ u=-v\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right) \\ v=-u\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right) \end{cases} \quad \text{(8b)}$$

$$u^2(-u)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)+v^2(-v)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)=q\,;$$

that for the (8b) and for sign + , for example, become:

$$v^2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\cdot v\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)+u^2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\cdot u\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q$$

$$v^2\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-v)+u^2\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-u)=q$$

$$-v^3\left(\frac{1}{4}+\frac{3}{4}\right)-u^3\left(\frac{1}{4}+\frac{3}{4}\right)=q\,; \quad -v^3-u^3=q\,;$$

$$u^3+v^3=-q \qquad (9)$$

Finally, we have:

$$\begin{cases} u^3+v^3=-q \\ uv=-\dfrac{p}{3} \end{cases} \qquad \text{thence:} \qquad \begin{cases} u^3+v^3=-q \\ u^3v^3=-\dfrac{p^3}{27} \end{cases} \qquad (10)$$

We obtain the same result putting in the (2) $x=u+v$. Indeed, we have:

$$x^3+px+q=0\,; \qquad x=u+v$$

$$(u+v)^3+p(u+v)+q=0\,; \quad u^3+3u^2v+3uv^2+v^3+p(u+v)+q=0\,;$$

$$u^3+v^3+3uv(u+v)+p(u+v)+q=0\,;$$

from this equation, we obtain:

$u^3+v^3=-q\,; \quad 3uv(u+v)=-p(u+v);$ thence, in conclusion:

$$\begin{cases} u^3 + v^3 = -q \\ 3uv = -p \end{cases} \qquad \begin{cases} u^3 + v^3 = -q \\ uv = -\dfrac{p}{3} \end{cases} \qquad \begin{cases} u^3 + v^3 = -q \\ u^3 v^3 = -\dfrac{p^3}{27} \end{cases} \qquad (11)$$

From the relations (10) and (11), we obtain the sum and the product of u^3 and v^3. We have that u^3 and v^3 must be roots of the following quadratic equation:

$$t^2 + qt - \frac{p^3}{27}; \quad (12)$$

This equation is defined the "resolving" of the cubic equation (2). Thence, we have:

$$t^2 + qt - \frac{p^3}{27}$$

$$\frac{-q \pm \sqrt{q^2 - 4(1)\left(-\frac{p^3}{27}\right)}}{2} = \frac{-q \pm \sqrt{q^2 + \frac{4p^3}{27}}}{2} = -\frac{q}{2} \pm \sqrt{\frac{1}{4}q^2 + \frac{1}{4} \cdot \frac{4p^3}{27}} =$$

$$= -\frac{q}{2} \pm \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}.$$

Thence, we have that:

$$\begin{matrix} t_1 = u^3 = -\dfrac{q}{2} + \sqrt{\dfrac{q^2}{4} + \dfrac{p^3}{27}} \\ t_2 = v^3 = -\dfrac{q}{2} - \sqrt{\dfrac{q^2}{4} + \dfrac{p^3}{27}} \end{matrix} \quad ; \qquad (13)$$

from this, we obtain:

$$u = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \;;$$

if $uv = -\frac{p}{3}$; $v = -\frac{p}{3u}$, we obtain:

$$v = -\frac{p}{3\sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}} \,.$$

From the $x = u + v$, we obtain:

$$x = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} - \frac{p}{3\sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}} \,. \quad (14)$$

The expression (14) is the “resolving” formula of the cubic equation (2), from this we have the three roots in correspondence of the three values of the cubic radical. If $p = 0$ the eq. (2) become the binomial equation $x^3 + q = 0$, solved from the formula $x = \sqrt[3]{-q}$.

The expression (14) is written generally in the “Cardano” or “Tartaglia” form:

$$x = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \,. \quad (15)$$

The solution that Cardano gives of the cubic equation of type $x^3 + px = q$, leads to the following formula:

$$x=\sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2}+\frac{q}{2}}-\sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2}-\frac{q}{2}} \; ; \quad (15b)$$

while for the cubic equation of type $x^3+px^2=q$, we have the following formula:

$$x=\sqrt[3]{\sqrt{\left(\frac{q}{2}\right)^2-\left(\frac{p}{3}\right)^3}+\frac{q}{2}}-\sqrt[3]{\sqrt{\left(\frac{q}{2}\right)^2-\left(\frac{p}{3}\right)^3}-\frac{q}{2}} \; . \quad (15c)$$

We have that the above result

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) = i\sqrt{3} =$$

$$= 1.7320508075688772935274463415058723669428052538103806280558\ldots i$$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

can be related with:

$$u^2(-u)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)+v^2(-v)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)=q$$

Considering:

$$(-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q$$

$$= i\sqrt{3} = 1.7320508075688772935274463415058723669428052538103806280558\ldots i$$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

Thence:

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) \Rightarrow$$

$$\Rightarrow(-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q=1.73205\approx\sqrt{3}$$

Now, we have that:

From

$$m=\left(3\sqrt{3}\right)M_s.$$

Input interpretation:

$3\sqrt{3}\times 8.35422\times 10^{36}$

Result:

$4.34098...\times 10^{37}$

4.34098...*10^{37}

From:

$$V''(r_c) = -\frac{\left[L^2-m^2\left(1-E^2\right)\right]^3}{2L^4m^4}. \qquad (88)$$

for L = 0.5 , m = 4.34098e+37 , E = 8 , we obtain:

-1/(2*0.5^4*(4.34098e+37)^4)*((0.5^2-(4.34098e+37)^2(1-(8)^2)))^3

Input interpretation:

$$-\frac{(0.5^2-(4.34098\times 10^{37})^2(1-8^2))^3}{2\times 0.5^4(4.34098\times 10^{37})^4}$$

Result:

$-3.7695300105167510400000000000000000000000000000000...\times 10^{81}$

-3.769530010516…*10^{81}

From which, we obtain:

[-(((-1/(2*0.5^4*(4.34098e+37)^4)*((0.5^2-(4.34098e+37)^2(1-(8)^2)))^3)))]^((((3 (2/(7*11))^(1/3) e)/(5*7*11 π^(2/3)))))

Input interpretation:

$$\left(-\left(-\frac{(0.5^2-(4.34098\times10^{37})^2(1-8^2))^3}{2\times0.5^4(4.34098\times10^{37})^4}\right)\right)^{\left(3\sqrt[3]{\frac{2}{7\times11}}\,e\right)/\left(5\times7\times11\,\pi^{2/3}\right)}$$

Result:

1.7320508424581498681212724237881454665516057979097485731 45...

1.7320508424.....≈ $\sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q

Possible closed forms:

$\sqrt{3} \approx 1.732050807568$

$\mathcal{T}_T \approx 1.732050807568$

$\frac{3-2\mathcal{T}_T}{\mathcal{T}_T-2} \approx 1.732050807568$

$\frac{32132\pi}{58281} \approx 1.73205084238683$

$\sqrt{\frac{1}{51}(22+46e+3\pi-5\log(2))} \approx 1.732050842476412$

From:

$$V(r)=\frac{1}{2}+\frac{L^2+m^2\left(E^2-1\right)}{2r^2}-\frac{L^2m^2}{2r^4},$$

for R_s = 1.67084e+37 , L = 0.5 , m = 4.34098e+37 , E = 8 , we obtain:

1/2+(((0.5^2+(4.34098e+37)^2*(8^2-1)))) / ((((2*(1.67084e+37)^2)))) – (0.5^2*(4.34098e+37)^2)/(2*(1.67084e+37)^4)

Input interpretation:

$$\frac{1}{2}+\frac{0.5^2+(4.34098\times10^{37})^2(8^2-1)}{2(1.67084\times10^{37})^2}-\frac{0.5^2(4.34098\times10^{37})^2}{2(1.67084\times10^{37})^4}$$

Result:

213.126013224204253128080193279196639573618047997290736 2121...

213.1260132242....

From which:

((((1/2+(((0.5^2+(4.34098e+37)^2*(8^2-1)))) / ((((2*(1.67084e+37)^2)))) – (0.5^2*(4.34098e+37)^2)/(2*(1.67084e+37)^4)))))^(((11/96 + 19/(96 e) - e/32)))

Input interpretation:

$$\left(\frac{1}{2}+\frac{0.5^2+(4.34098\times10^{37})^2(8^2-1)}{2(1.67084\times10^{37})^2}-\frac{0.5^2(4.34098\times10^{37})^2}{2(1.67084\times10^{37})^4}\right)^{11/96+19/(96e)-e/32}$$

Result:

1.732050908642439404012916883468346569013439664780178117286...

1.732050908.....≈ $\sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q

Possible closed forms:

$\sqrt{3} \approx 1.73205080756$

$\mathcal{T}_T \approx 1.73205080756$

$\frac{2\mathcal{T}_{20VE}}{3} \approx 1.73205080756$

$e^{-10+2e-1/\pi+\pi}\pi^2 \approx 1.73205089709$

$\frac{7841\pi}{14222} \approx 1.7320509068202$

$\frac{2(1+e)(2+3e)}{-10+17e+e^2} \approx 1.7320509074646$

$\frac{3\sqrt{3}}{(2e)^{5/6}\log^3(2)\log(9)} \approx 1.7320509087838 7$

$-\frac{37 e!}{18} + \frac{35}{9} + \frac{19}{6 e} + 2 e \approx 1.73205090811603$

$\log\left(\frac{1}{5}\left(9 - 2\sqrt{2} + 6 e - 13 e^2 + \pi + 10 \pi^2\right)\right) \approx 1.73205090856337$

$\frac{1}{21}\left(6 e^{\pi} - 30 \pi - 39 \log(\pi) + 26 \log(2 \pi) - 9 \tan^{-1}(\pi)\right) \approx 1.7320509079232$

π root of $40 x^4 + 41 x^3 + 18 x^2 + 9 x - 21$ near $x = 0.551329$ ≈ 1.7320509079393

$\frac{149}{66} + \frac{20}{11 e} - \frac{29 e}{66} \approx 1.7320509078069$

π root of $49 x^3 - 216 x^2 + 184 x - 44$ near $x = 0.551329$ ≈ 1.7320509093977

$\frac{287 \pi}{212} - \frac{1679}{212 \pi} \approx 1.73205090885701$

$\frac{15 \pi \pi! - 69 + 62 \pi - 41 \pi^2}{11 \pi} \approx 1.73205090825249$

From:

Cubic Polynomials, Linear Shifts, and Ramanujan Cubics.
Gregory Dresden, Prakriti Panthi, Anukriti Shrestha, Jiahao Zhang
September 6, 2017 - arXiv:1709.00534v2 [math.NT] 5 Sep 2017

We have that:

Theorem 1. *For $p_B(x) = x^3 - \left(\frac{3+B}{2}\right) x^2 - \left(\frac{3-B}{2}\right) x + 1$ the Ramanujan simple cubic defined earlier,*

1. *The roots r_1, r_2, r_3 of $p_B(x)$ are always permuted by the order-three map $n(x) = \frac{1}{1-x}$.*
2. *The roots r_1, r_2, r_3 satisfy*

$$\sqrt[3]{r_1} + \sqrt[3]{r_2} + \sqrt[3]{r_3} = \sqrt[3]{\left(\frac{3+B}{2}\right) - 6 + 3\sqrt[3]{\frac{27+B^2}{4}}} \tag{5}$$

so long as, for complex arguments, we choose the appropriate values for the cube roots.

3. *If we define the elements of the set $\{s_1, s_2, \ldots, s_6\}$ as*

$$s_k = \frac{1}{3}\left(\left(\frac{3+B}{2}\right) + \sqrt{27+B^2}\cos\left(\frac{k\pi}{3} + \frac{1}{3}\arctan\frac{3\sqrt{3}}{B}\right)\right) \tag{6}$$

then for $B \geq 0$ the roots of $p_B(x)$ are $\{s_2, s_4, s_6\}$ and for $B \leq 0$ the roots of $p_B(x)$ are $\{s_1, s_3, s_5\}$.

Theorem 2. *Let $f(x) = x^3 + Px^2 + Qx + R$ have non-repeated roots t_1, t_2, t_3, and let a and c be as defined above.*

1. *If $c = 0$, then there exists h and k such that $f(x) = (x-h)^3 + k$. In other words, $f(x)$ is a translation of x^3 (by h units horizontally and k units vertically).*
2. *If $c \neq 0$, then $f\left(\frac{a-x}{c}\right) \cdot (-c)^3$ equals the Ramanujan simple cubic $p_B(x) = x^3 - \left(\frac{3+B}{2}\right)x^2 - \left(\frac{3-B}{2}\right)x + 1$, with $B = 6a + 2cP - 3$. In particular, the set of roots of $p_B(x)$ are $\{a - c \cdot t_1,\ a - c \cdot t_2,\ a - c \cdot t_3\}$.*

Corollary 1. *Let $f(x) = x^3 + Px^2 + Qx + R$ have non-repeated roots t_1, t_2, t_3, and let a, B, and c be as defined in Theorem 2, with $c \neq 0$. Then,*

1. *The order-three map $n(x) = \frac{1}{1-x}$ permutes the set $\{a - c \cdot t_1,\ a - c \cdot t_2,\ a - c \cdot t_3\}$.*
2. *We have the Ramanujan-style equation*

$$\sqrt[3]{a - c \cdot t_1} + \sqrt[3]{a - c \cdot t_2} + \sqrt[3]{a - c \cdot t_3} = \sqrt[3]{\left(\frac{3+B}{2}\right) - 6 + 3\sqrt[3]{\frac{27+B^2}{4}}}, \tag{7}$$

so long as, for complex arguments, we choose the appropriate values for the cube roots.

3. *If we define the elements of the set $\{u_1, u_2, \ldots, u_6\}$ as*

$$u_k = \frac{a}{c} - \frac{1}{3c}\left(\left(\frac{3+B}{2}\right) + \sqrt{27 + B^2}\,\cos\left(\frac{k\pi}{3} + \frac{1}{3}\arctan\frac{3\sqrt{3}}{B}\right)\right) \tag{8}$$

then for $B \geq 0$ the roots of $f(x)$ are $\{u_2, u_4, u_6\}$ and for $B \leq 0$ the roots of $f(x)$ are $\{u_1, u_3, u_5\}$.

Example 4. We can do similar calculations for $2\cos\pi/18$. This has a minimal polynomial of degree 6, but it factors in $\mathbb{Q}(\sqrt{3})[x]$ and we choose the degree-three factor $g(x) = x^3 - 3x - \sqrt{3}$. One root of $g(x)$ is indeed $2\cos\pi/18$, and the other two roots are $2\cos 11\pi/18$ and $2\cos 13\pi/18$. Calculating a, c as defined in Section 3, we get $a = 2$ and $c = -\sqrt{3}$. Thus, by Theorem 2, we have $g(\frac{a-x}{c}) \cdot (-c)^3 = x^3 - 6x^2 + 3x + 1$ which is a particularly nice Ramanujan simple cubic with $B = 9$. (We will return to this cubic in Example 6.) By Corollary 1, we get a nice identity:

$$\sqrt[3]{2 + 2\sqrt{3}\cos\frac{\pi}{18}} + \sqrt[3]{2 + 2\sqrt{3}\cos\frac{11\pi}{18}} + \sqrt[3]{2 + 2\sqrt{3}\cos\frac{13\pi}{18}} = \sqrt[3]{9}. \tag{9}$$

Furthermore, by Theorem 1, we know the roots of $x^3 - 6x^2 + 3x + 1$ are permuted by $1/(1-x)$. Therefore, by choosing our roots carefully, we get

$$2 + 2\sqrt{3}\cos\frac{13\pi}{18} = \frac{1}{1 - \left(2 + 2\sqrt{3}\cos\frac{\pi}{18}\right)}$$

and this simplifies to

$$2\cos\frac{\pi}{18} + \cos\frac{13\pi}{18} + \sqrt{3}\cos\frac{14\pi}{18} = 0$$

which reduces to

$$\cos\frac{5\pi}{18} = 2\cos\frac{\pi}{18} - \sqrt{3}\cos\frac{4\pi}{18}. \tag{10}$$

From (10), we obtain:

cos((5Pi)/(18))

Input:

$\cos\left(\frac{5\pi}{18}\right)$

Exact result:

$\sin\left(\frac{2\pi}{9}\right)$

Decimal approximation:

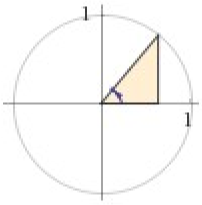
0.642787609686539326322643409907263432907559884205681790324...

0.642787609...

Conversion from radians to degrees:

$\cos(50°)$

Reference triangle for angle (5 π)/18 radians:



width	$\cos\left(\frac{5\pi}{18}\right) = \sin\left(\frac{2\pi}{9}\right) \approx 0.642788$
height	$\sin\left(\frac{5\pi}{18}\right) = \cos\left(\frac{2\pi}{9}\right) \approx 0.766044$

Alternate forms:

$2\sin\left(\frac{\pi}{9}\right)\cos\left(\frac{\pi}{9}\right)$

$-\frac{1}{2}(-1)^{5/18}\left((-1)^{4/9}-1\right)$

$\frac{1}{2}\sqrt{3}\cos\left(\frac{\pi}{9}\right)-\frac{1}{2}\sin\left(\frac{\pi}{9}\right)$

Alternative representations:

$\cos\left(\frac{5\pi}{18}\right) = \cosh\left(-\frac{5i\pi}{18}\right)$

$$\cos\left(\frac{5\pi}{18}\right) = \cosh\left(\frac{5 i \pi}{18}\right)$$

$$\cos\left(\frac{5\pi}{18}\right) = \frac{1}{\sec\left(\frac{5\pi}{18}\right)}$$

Series representations:

$$\cos\left(\frac{5\pi}{18}\right) = \sum_{k=0}^{\infty} \frac{(-1)^{3k} \left(\frac{5\pi}{18}\right)^{2k}}{(2k)!}$$

$$\cos\left(\frac{5\pi}{18}\right) = 2 \sum_{k=0}^{\infty} (-1)^k J_{1+2k}\left(\frac{2\pi}{9}\right)$$

$$\cos\left(\frac{5\pi}{18}\right) = \sum_{k=0}^{\infty} \frac{(-1)^k 9^{-1-2k} (2\pi)^{1+2k}}{(1+2k)!}$$

Integral representations:

$$\cos\left(\frac{5\pi}{18}\right) = \frac{2\pi}{9} \int_0^1 \cos\left(\frac{2\pi t}{9}\right) dt$$

$$\cos\left(\frac{5\pi}{18}\right) = -\frac{i\sqrt{\pi}}{18} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-\pi^2/(81 s)+s}}{s^{3/2}} \, ds \quad \text{for } \gamma > 0$$

$$\cos\left(\frac{5\pi}{18}\right) = -\frac{i}{2\sqrt{\pi}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(\frac{\pi}{9}\right)^{1-2s} \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} \, ds \quad \text{for } 0 < \gamma < 1$$

Multiple-argument formulas:

$$\cos\left(\frac{5\pi}{18}\right) = -\frac{1}{2} (-1)^{5/18} \left(-1 + (-1)^{4/9}\right)$$

$$\cos\left(\frac{5\pi}{18}\right) = 3 \sin\left(\frac{2\pi}{27}\right) - 4 \sin^3\left(\frac{2\pi}{27}\right)$$

$$\cos\left(\frac{5\pi}{18}\right) = 2 \cos\left(\frac{\pi}{9}\right) \sin\left(\frac{\pi}{9}\right)$$

$$\cos\left(\frac{5\pi}{18}\right) = 3 \cos^2\left(\frac{2\pi}{27}\right) \sin\left(\frac{2\pi}{27}\right) - \sin^3\left(\frac{2\pi}{27}\right)$$

and:

2cos((Pi)/(18))-sqrt3 cos((4Pi)/(18))

Input:

$$2\cos\left(\frac{\pi}{18}\right)-\sqrt{3}\cos\left(\frac{4\pi}{18}\right)$$

Exact result:

$$2\cos\left(\frac{\pi}{18}\right)-\sqrt{3}\cos\left(\frac{2\pi}{9}\right)$$

Decimal approximation:

0.6427876096865393263226434099072634329075598842056817903 24...

0.642787609…

Alternate forms:

root of $64x^6-96x^4+36x^2-3$ near $x=0.642788$

$$-\frac{3}{2}\sin\left(\frac{\pi}{9}\right)+2\cos\left(\frac{\pi}{18}\right)-\frac{1}{2}\sqrt{3}\cos\left(\frac{\pi}{9}\right)$$

$$\frac{1}{2}(-1)^{7/9}\left(-2\sqrt[6]{-1}-2(-1)^{5/18}+\sqrt{3}+(-1)^{4/9}\sqrt{3}\right)$$

Minimal polynomial:

$$64x^6-96x^4+36x^2-3$$

Alternative representations:

$$2\cos\left(\frac{\pi}{18}\right)-\sqrt{3}\cos\left(\frac{4\pi}{18}\right)=2\cosh\left(\frac{i\pi}{18}\right)-\cosh\left(\frac{4i\pi}{18}\right)\sqrt{3}$$

$$2\cos\left(\frac{\pi}{18}\right)-\sqrt{3}\cos\left(\frac{4\pi}{18}\right)=2\cosh\left(-\frac{i\pi}{18}\right)-\cosh\left(-\frac{4i\pi}{18}\right)\sqrt{3}$$

$$2\cos\left(\frac{\pi}{18}\right)-\sqrt{3}\cos\left(\frac{4\pi}{18}\right)=\frac{2}{\sec\left(\frac{\pi}{18}\right)}-\frac{\sqrt{3}}{\sec\left(\frac{4\pi}{18}\right)}$$

Series representations:

$$2\cos\left(\frac{\pi}{18}\right)-\sqrt{3}\cos\left(\frac{4\pi}{18}\right)=\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{324}\right)^k\left(2-\sqrt{3}\,16^k\right)\pi^{2k}}{(2k)!}$$

$$2\cos\left(\frac{\pi}{18}\right)-\sqrt{3}\cos\left(\frac{4\pi}{18}\right)=\sum_{k=0}^{\infty}\frac{(-1)^{3k}\,2^{-1-2k}\times 3^{-5/2-4k}\left(\sqrt{3}\,4^{2+3k}-3\times 5^{1+2k}\right)\pi^{1+2k}}{(1+2k)!}$$

$$2\cos\left(\frac{\pi}{18}\right)-\sqrt{3}\cos\left(\frac{4\pi}{18}\right)=\sum_{k=0}^{\infty}\frac{3^{-5/2-4k}\left(\sqrt{3}\,4^{2+3k}-3\times 5^{1+2k}\right)e^{3ik\pi}\left(\frac{2}{\pi}\right)^{-1-2k}}{(1+2k)!}$$

Integral representations:

$$2\cos\left(\frac{\pi}{18}\right)-\sqrt{3}\cos\left(\frac{4\pi}{18}\right)=2-\sqrt{3}+\int_0^1-\frac{1}{9}\pi\left(\sin\left(\frac{\pi t}{18}\right)-2\sqrt{3}\sin\left(\frac{2\pi t}{9}\right)\right)dt$$

$$2\cos\left(\frac{\pi}{18}\right)-\sqrt{3}\cos\left(\frac{4\pi}{18}\right)=\int_{\frac{\pi}{2}}^{\frac{\pi}{18}}\left(-2\sin(t)+\frac{5}{8}\sqrt{3}\sin\left(\frac{9\left(\frac{\pi^2}{12}+\frac{5\pi t}{18}\right)}{4\pi}\right)\right)dt$$

$$2\cos\left(\frac{\pi}{18}\right)-\sqrt{3}\cos\left(\frac{4\pi}{18}\right)=\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{i\,e^{-\pi^2/(81s)+s}\left(\sqrt{3}-2\,e^{(5\pi^2)/(432s)}\right)}{2\sqrt{\pi}\sqrt{s}}\,ds \quad \text{for } \gamma>0$$

$$2\cos\left(\frac{\pi}{18}\right)-\sqrt{3}\cos\left(\frac{4\pi}{18}\right)=\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{i\,81^s\left(-2^{1+4s}+\sqrt{3}\right)\pi^{-1/2-2s}\,\Gamma(s)}{2\,\Gamma\left(\frac{1}{2}-s\right)}\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

Multiple-argument formulas:

$$2\cos\left(\frac{\pi}{18}\right)-\sqrt{3}\cos\left(\frac{4\pi}{18}\right)=\frac{1}{4}\sqrt[18]{-1}\left(1+2(-1)^{2/9}-i\sqrt{3}\right)$$

$$2\cos\left(\frac{\pi}{18}\right)-\sqrt{3}\cos\left(\frac{4\pi}{18}\right)=2\left(-1+2\cos^2\left(\frac{\pi}{36}\right)\right)-\sqrt{3}\left(-1+2\cos^2\left(\frac{\pi}{9}\right)\right)$$

$$2\cos\left(\frac{\pi}{18}\right)-\sqrt{3}\cos\left(\frac{4\pi}{18}\right)=2\left(1-2\sin^2\left(\frac{\pi}{36}\right)\right)-\sqrt{3}\left(1-2\sin^2\left(\frac{\pi}{9}\right)\right)$$

From which, we obtain also:

-((((0.6427876096865393)-2cos((Pi)/(18))))) * 1/(((cos((4Pi)/(18)))))

Input interpretation:

$$-\left(0.6427876096865393-2\cos\left(\frac{\pi}{18}\right)\right)\times\frac{1}{\cos\left(\frac{4\pi}{18}\right)}$$

Result:

1.732050807568877...

1.732050807568877 ≈ $\sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q

Alternative representations:

$$-\frac{0.64278760968653930000-2\cos\left(\frac{\pi}{18}\right)}{\cos\left(\frac{4\pi}{18}\right)}=-\frac{0.64278760968653930000-2\cosh\left(\frac{i\pi}{18}\right)}{\cosh\left(\frac{4i\pi}{18}\right)}$$

$$-\frac{0.64278760968653930000-2\cos\left(\frac{\pi}{18}\right)}{\cos\left(\frac{4\pi}{18}\right)}=-\frac{0.64278760968653930000-2\cosh\left(-\frac{i\pi}{18}\right)}{\cosh\left(-\frac{4i\pi}{18}\right)}$$

$$-\frac{0.64278760968653930000-2\cos\left(\frac{\pi}{18}\right)}{\cos\left(\frac{4\pi}{18}\right)}=-\frac{0.64278760968653930000-\frac{2}{\sec\left(\frac{\pi}{18}\right)}}{\frac{1}{\sec\left(\frac{4\pi}{18}\right)}}$$

Series representations:

$$-\frac{0.64278760968653930000-2\cos\left(\frac{\pi}{18}\right)}{\cos\left(\frac{4\pi}{18}\right)}=\frac{1}{\sum_{k=0}^{\infty}\frac{\left(-\frac{4}{81}\right)^k\pi^{2k}}{(2k)!}}2.0000000000000000\left(-0.321393804843269650+1.0000000000000000\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{324}\right)^k\pi^{2k}}{(2k)!}\right)$$

$$-\frac{0.64278760968653930000-2\cos\left(\frac{\pi}{18}\right)}{\cos\left(\frac{4\pi}{18}\right)}=\left(2.0000000000000000\left(-0.321393804843269650+1.0000000000000000\,\pi\sum_{k=0}^{\infty}\frac{(-1)^k\,4^{1+2k}\times 9^{-1-2k}(-\pi)^{2k}}{(1+2k)!}\right)\right)\Big/\left(\pi\sum_{k=0}^{\infty}\frac{(-1)^k\,5^{1+2k}\times 18^{-1-2k}(-\pi)^{2k}}{(1+2k)!}\right)$$

$$-\frac{0.64278760968653930000-2\cos\left(\frac{\pi}{18}\right)}{\cos\left(\frac{4\pi}{18}\right)} = \left(2.0000000000000000\left(-0.3213938048432696500+1.0000000000000000000 \sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left(\frac{\pi}{18}-z_0\right)^k}{k!}\right)\right) \Big/ \left(\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left(\frac{2\pi}{9}-z_0\right)^k}{k!}\right)$$

Integral representations:

$$-\frac{0.64278760968653930000-2\cos\left(\frac{\pi}{18}\right)}{\cos\left(\frac{4\pi}{18}\right)} = \frac{1}{\int_{\frac{\pi}{2}}^{\frac{2\pi}{9}}\sin(t)\,dt}2.0000000000000000000\left(0.3213938048432696500+1.0000000000000000000\int_{\frac{\pi}{2}}^{\frac{\pi}{18}}\sin(t)\,dt\right)$$

$$-\frac{0.64278760968653930000-2\cos\left(\frac{\pi}{18}\right)}{\cos\left(\frac{4\pi}{18}\right)} = \left(2.000000000000000\left(-0.6427876096865393\,i\,\pi+1.0000000000000000\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(1296 s)+s}}{\sqrt{s}}\,ds\right)\right) \Big/ \left(\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(81 s)+s}}{\sqrt{s}}\,ds\right) \quad \text{for } \gamma > 0$$

$$-\frac{0.64278760968653930000-2\cos\left(\frac{\pi}{18}\right)}{\cos\left(\frac{4\pi}{18}\right)} = \left(2.000000000000000\left(-0.6427876096865393\,i\,\pi+1.0000000000000000\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{1296^s\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\,ds\right)\right) \Big/ \left(\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{81^s\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\,ds\right) \quad \text{for } 0 < \gamma < \frac{1}{2}$$

Multiple-argument formulas:

$$-\frac{0.64278760968653930000 - 2\cos\left(\frac{\pi}{18}\right)}{\cos\left(\frac{4\pi}{18}\right)} = \frac{4.0000000000000000000\left(-0.66069690242163482500 + \cos^2\left(\frac{\pi}{36}\right)\right)}{-1.0000000000000000000 + 2.0000000000000000000\cos^2\left(\frac{\pi}{9}\right)}$$

$$-\frac{0.64278760968653930000 - 2\cos\left(\frac{\pi}{18}\right)}{\cos\left(\frac{4\pi}{18}\right)} = \frac{4.0000000000000000000\left(-0.33930309757836517500 + \sin^2\left(\frac{\pi}{36}\right)\right)}{-1.0000000000000000000 + 2.0000000000000000000\sin^2\left(\frac{\pi}{9}\right)}$$

$$-\frac{0.64278760968653930000 - 2\cos\left(\frac{\pi}{18}\right)}{\cos\left(\frac{4\pi}{18}\right)} = \frac{2.0000000000000000000\left(-0.32139380484326965000 + T_{\frac{1}{18}}(\cos(\pi))\right)}{T_{\frac{2}{9}}(\cos(\pi))}$$

1.732050807568877

Input interpretation:

1.732050807568877

Possible closed forms:

$\sqrt{3} \approx 1.73205080756887729352$

$\frac{9\,\mathcal{L}_{si}}{8} \approx 1.73205080756887729352$

$\frac{2\,\mathcal{T}_{20VE}}{3} \approx 1.73205080756887729352$

$\frac{8}{3\,\mathcal{L}_{si}} \approx 1.73205080756887729352$

$\mathcal{T}_T \approx 1.73205080756887729352$

$\frac{6418716\,\pi}{11642263} \approx 1.73205080756887757100$

$$\frac{-47+70e+80e^2}{2(4+14e+23e^2)} \approx 1.73205080756887655545$$

$$\log\left(\frac{1}{150}\left(-27+\sqrt{2}-25e+7e^2+35\pi+79\pi^2\right)\right) \approx 1.73205080756887704873$$

From: **Manuscript Book 2 of Srinivasa Ramanujan**

Now, we have that:

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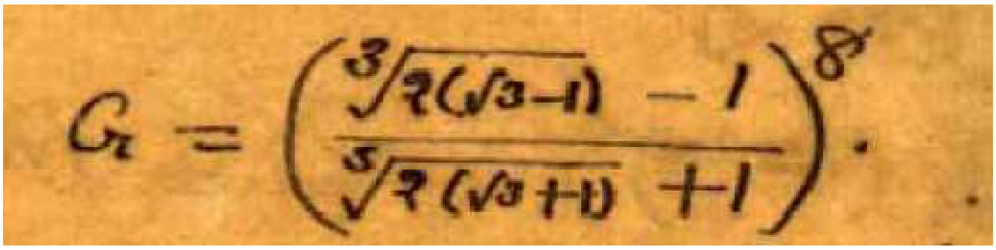


[(((((2(sqrt3-1))^1/3 – 1)))/((((2(sqrt3+1))^1/3 + 1))))]^8

Input:

$$\left(\frac{\sqrt[3]{2(\sqrt{3}-1)}-1}{\sqrt[3]{2(\sqrt{3}+1)}+1}\right)^8$$

Exact result:

$$\frac{\left(\sqrt[3]{2(\sqrt{3}-1)}-1\right)^8}{\left(1+\sqrt[3]{2(1+\sqrt{3})}\right)^8}$$

Decimal approximation:

$3.3635105126017025642101935069568301602502922669138748... \times 10^{-11}$

$3.363510512601...*10^{-11}$

Alternate forms:

$$\text{root of } x^6 - 29730842118x^5 + 109557553167x^4 - 643038569492x^3 + 109557553167x^2 - 29730842118x + 1 \text{ near } x = 3.36351\times10^{-11}$$

$$\frac{\left(\text{root of } x^6 + 4x^3 - 8 \text{ near } x = 1.13551 - 1\right)^8}{\left(\text{root of } x^6 - 4x^3 - 8 \text{ near } x = 1.76133 + 1\right)^8}$$

$$\frac{561}{\left(1+\sqrt[3]{2(1+\sqrt{3})}\right)^8} - \frac{336\sqrt{3}}{\left(1+\sqrt[3]{2(1+\sqrt{3})}\right)^8} + \frac{140\sqrt[3]{2}(\sqrt{3}-1)^{4/3}}{\left(1+\sqrt[3]{2(1+\sqrt{3})}\right)^8} - \frac{112\times2^{2/3}(\sqrt{3}-1)^{5/3}}{\left(1+\sqrt[3]{2(1+\sqrt{3})}\right)^8} - \frac{32\sqrt[3]{2}(\sqrt{3}-1)^{7/3}}{\left(1+\sqrt[3]{2(1+\sqrt{3})}\right)^8} + \frac{4\times2^{2/3}(\sqrt{3}-1)^{8/3}}{\left(1+\sqrt[3]{2(1+\sqrt{3})}\right)^8} - \frac{8\sqrt[3]{2(\sqrt{3}-1)}}{\left(1+\sqrt[3]{2(1+\sqrt{3})}\right)^8} + \frac{28(2(\sqrt{3}-1))^{2/3}}{\left(1+\sqrt[3]{2(1+\sqrt{3})}\right)^8}$$

Minimal polynomial:

$$x^6 - 29730842118x^5 + 109557553167x^4 - 643038569492x^3 + 109557553167x^2 - 29730842118x + 1$$

From which:

[((((((2(x-1))^1/3 – 1)))/((((2(sqrt3+1))^1/3 + 1))))]^8 = 3.363510512601*10^-11

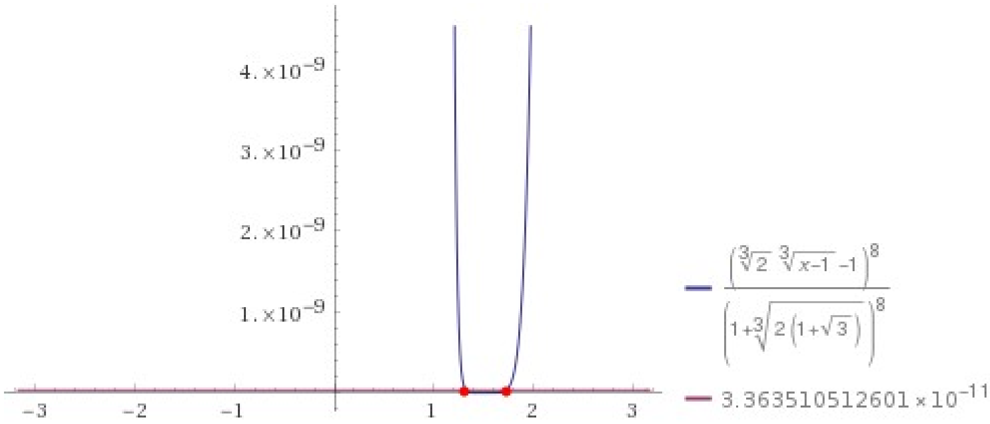
Input interpretation:

$$\left(\frac{\sqrt[3]{2(x-1)} - 1}{\sqrt[3]{2(\sqrt{3}+1)} + 1}\right)^8 = 3.363510512601\times10^{-11}$$

Result:

$$\frac{\left(\sqrt[3]{2}\sqrt[3]{x-1} - 1\right)^8}{\left(1+\sqrt[3]{2(1+\sqrt{3})}\right)^8} = 3.363510512601\times10^{-11}$$

Plot:

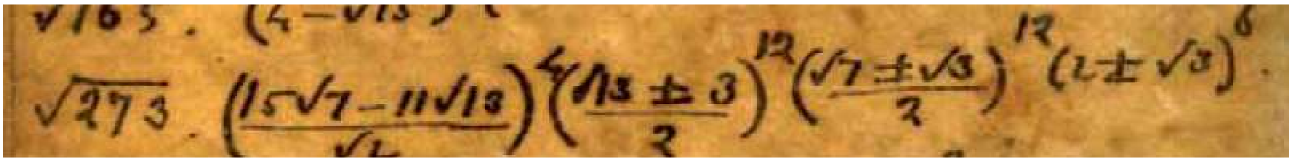


Numerical solutions:

$x \approx 1.32303688937029...$

$x \approx 1.73205080756887...$

1.73205080756887... = $\sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q



sqrt273 ((15sqrt7-11sqrt13)/sqrt2)^4 ((sqrt13+3)/2)^12 ((sqrt7+sqrt3)/2)^12 (2+sqrt3)^6

Input:

$$\sqrt{273}\left(\frac{15\sqrt{7}-11\sqrt{13}}{\sqrt{2}}\right)^4\left(\frac{1}{2}\left(\sqrt{13}+3\right)\right)^{12}\left(\frac{1}{2}\left(\sqrt{7}+\sqrt{3}\right)\right)^{12}\left(2+\sqrt{3}\right)^6$$

Result:

$$\frac{\sqrt{273}\left(2+\sqrt{3}\right)^6\left(\sqrt{3}+\sqrt{7}\right)^{12}\left(15\sqrt{7}-11\sqrt{13}\right)^4\left(3+\sqrt{13}\right)^{12}}{67108864}$$

Decimal approximation:

$9.1824737518144176470814807511555873056750416394159784 1... \times 10^7$

9.1824737518…*10^7

Alternate forms:

$$\left(-110605294800 - 63858014220\sqrt{3} + 11594585340\sqrt{91} + 6694138801\sqrt{273}\right)\left(842401 + 233640\sqrt{13}\right)\left(6049 + 1320\sqrt{21}\right)$$

$$146694653305200 - 93587690976780\sqrt{3} - 55445658840000\sqrt{7} + 40685996763720\sqrt{13} + 35372653996680\sqrt{21} - 25956428097600\sqrt{39} - 15377775662340\sqrt{91} + 9810657124849\sqrt{273}$$

root of

$$x^8 - 1173557226441600\,x^7 - 76106152301529269554935956292\,x^6 - 234780099116432216051255469348513283200\,x^5 + 22200332401524008582319892402796733330586546374\,x^4 - 64094967058785994981992743132144126313600\,x^3 - 5672115424880674930659821886486468\,x^2 - 23877684702544219747200\,x + 5554571841$$ near $x = 9.18247\times10^7$

Minimal polynomial:

$$x^8 - 1173557226441600\,x^7 - 76106152301529269554935956292\,x^6 - 234780099116432216051255469348513283200\,x^5 + 22200332401524008582319892402796733330586546374\,x^4 - 64094967058785994981992743132144126313600\,x^3 - 5672115424880674930659821886486468\,x^2 - 23877684702544219747200\,x + 5554571841$$

From which:

sqrt273 ((15sqrt7-11sqrt13)/sqrt2)^4 ((sqrt13+3)/2)^12 ((sqrt7+x)/2)^12 (2+sqrt3)^6 = 9.1824737518e+7

Input interpretation:

$$\sqrt{273}\left(\frac{15\sqrt{7}-11\sqrt{13}}{\sqrt{2}}\right)^4\left(\frac{1}{2}\left(\sqrt{13}+3\right)\right)^{12}\left(\frac{1}{2}\left(\sqrt{7}+x\right)\right)^{12}\left(2+\sqrt{3}\right)^6 = 9.1824737518\times10^7$$

Result:

$$\frac{\sqrt{273}\,(2+\sqrt{3})^6\,(15\sqrt{7}-11\sqrt{13})^4\,(3+\sqrt{13})^{12}\,(x+\sqrt{7})^{12}}{67108864} = 9.1824737518\times10^7$$

Alternate forms:

False

True

$$\left(x+\sqrt{7}\right)^{12} \quad \text{root of } 79228162514264337593543950336\,x^8 + 14417979846175060652082807098872414234214400\,x^7 - 359383147830304277882284079638636683052644021829632\,x^6 - 22374227584662575959928373602252589114673325093683200\,x^5 + 42694708757978294316727008613728287255985036139167744\,x^4 - 364074953234963609997060656155047227639306649600\,x^3 - 95157540956750876852825449856922380402688\,x^2 + 62120174559093257292074188800\,x + 5554571841 \text{ near } x = 1.85305 \quad = 9.1824737518\times 10^{7}$$

Real solutions:

$x = -7$

$x = 0$

$x \approx 1.7320508076$

1.7320508076 = $\sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q

Complex solutions:

$x = -4.834652 - 3.791288\,i$

$x = -4.834652 + 3.791288\,i$

$x = -2.6457513111 + 4.3778021186\,i$

$x = -2.6457513111 - 4.3778021186\,i$

$x = -0.456850252 - 3.791287847\,i$

and:

(((sqrt273 ((15sqrt7-11sqrt13)/sqrt2)^4 ((sqrt13+3)/2)^12 ((sqrt7+sqrt3)/2)^12 (2+sqrt3)^6))^(((-1208 - 1009 π + 497 π^2)/(223 + 1187 π + 1383 π^2)))

Input:

$$\left(\sqrt{273}\left(\frac{15\sqrt{7}-11\sqrt{13}}{\sqrt{2}}\right)^4\left(\frac{1}{2}\left(\sqrt{13}+3\right)\right)^{12}\left(\frac{1}{2}\left(\sqrt{7}+\sqrt{3}\right)\right)^{12}\left(2+\sqrt{3}\right)^6\right)^{\left(-1208-1009\pi+497\pi^2\right)/\left(223+1187\pi+1383\pi^2\right)}$$

Exact result:

$$273^{(-1208-1009\pi+497\pi^2)/(2(223+1187\pi+1383\pi^2))} \times 67108864^{-(-1208-1009\pi+497\pi^2)/(223+1187\pi+1383\pi^2)} \left(2+\sqrt{3}\right)^{(6(-1208-1009\pi+497\pi^2))/(223+1187\pi+1383\pi^2)} \left(15\sqrt{7}-11\sqrt{13}\right)^{(4(-1208-1009\pi+497\pi^2))/(223+1187\pi+1383\pi^2)} \left(\left(\sqrt{3}+\sqrt{7}\right)\left(3+\sqrt{13}\right)\right)^{(12(-1208-1009\pi+497\pi^2))/(223+1187\pi+1383\pi^2)}$$

Decimal approximation:

1.732050425564708016810454420238044085829531944862141494140...

1.7320504255... ≈ $\sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q

Alternate form:

$$273^{-604/(223+1187\pi+1383\pi^2)-(1009\pi)/(2(223+1187\pi+1383\pi^2))+(497\pi^2)/(2(223+1187\pi+1383\pi^2))} \times 67108864^{1208/(223+1187\pi+1383\pi^2)+(1009\pi)/(223+1187\pi+1383\pi^2)-(497\pi^2)/(223+1187\pi+1383\pi^2)} \left(2+\sqrt{3}\right)^{-7248/(223+1187\pi+1383\pi^2)-(6054\pi)/(223+1187\pi+1383\pi^2)+(2982\pi^2)/(223+1187\pi+1383\pi^2)} \left(15\sqrt{7}-11\sqrt{13}\right)^{-4832/(223+1187\pi+1383\pi^2)-(4036\pi)/(223+1187\pi+1383\pi^2)+(1988\pi^2)/(223+1187\pi+1383\pi^2)} \left(\left(\sqrt{3}+\sqrt{7}\right)\left(3+\sqrt{13}\right)\right)^{-14496/(223+1187\pi+1383\pi^2)-(12108\pi)/(223+1187\pi+1383\pi^2)+(5964\pi^2)/(223+1187\pi+1383\pi^2)}$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\Gamma(-a)} \quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

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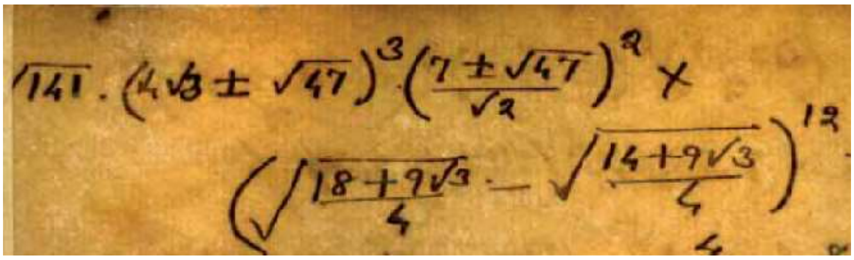

sqrt141(4sqrt3+sqrt47)^3((7+sqrt47)/sqrt2)^2(((((18+9sqrt3)/4)^1/2-((14+9sqrt3)/4)^1/2)))^12

Input:

$$\sqrt{141}\left(4\sqrt{3}+\sqrt{47}\right)^3\left(\frac{7+\sqrt{47}}{\sqrt{2}}\right)^2\left(\sqrt{\frac{1}{4}\left(18+9\sqrt{3}\right)}-\sqrt{\frac{1}{4}\left(14+9\sqrt{3}\right)}\right)^{12}$$

Result:

$$\frac{1}{2}\sqrt{141}\left(7+\sqrt{47}\right)^2\left(4\sqrt{3}+\sqrt{47}\right)^3\left(\frac{1}{2}\sqrt{18+9\sqrt{3}}-\frac{1}{2}\sqrt{14+9\sqrt{3}}\right)^{12}$$

Decimal approximation:

0.003022671022197832307683321004063382185339616341869234335...

0.003022671022…

Alternate forms:

$$\frac{1}{16\,777\,216}\left(746\,172+430\,896\sqrt{3}+108\,864\sqrt{47}+62\,839\sqrt{141}\right)\left(-2\sqrt{14+9\sqrt{3}}+3\sqrt{2}+3\sqrt{6}\right)^{12}$$

$$\frac{1}{128}\sqrt{141}\left(7+\sqrt{47}\right)^2\left(4\sqrt{3}+\sqrt{47}\right)^3\left(16+9\sqrt{3}-3\sqrt{55+32\sqrt{3}}\right)^6$$

root of $x^8-2\,947\,821\,237\,008\,928\,x^7-136\,895\,253\,836\,531\,896\,500\,x^6+28\,294\,648\,826\,644\,112\,163\,552\,x^5-14\,424\,132\,395\,908\,316\,146\,312\,458\,x^4-3\,989\,545\,484\,556\,819\,815\,060\,832\,x^3-2\,721\,614\,541\,524\,090\,634\,316\,500\,x^2+8\,263\,394\,395\,829\,404\,157\,088\,x+395\,254\,161$ near $x=0.00302267$

Minimal polynomial:

$$x^8-2\,947\,821\,237\,008\,928\,x^7-136\,895\,253\,836\,531\,896\,500\,x^6+28\,294\,648\,826\,644\,112\,163\,552\,x^5-14\,424\,132\,395\,908\,316\,146\,312\,458\,x^4-3\,989\,545\,484\,556\,819\,815\,060\,832\,x^3-2\,721\,614\,541\,524\,090\,634\,316\,500\,x^2+8\,263\,394\,395\,829\,404\,157\,088\,x+395\,254\,161$$

From which:

(((sqrt141(4sqrt3+sqrt47)^3((7+sqrt47)/sqrt2)^2(((((18+9sqrt3)/4)^1/2-((14+9sqrt3)/4)^1/2)))^12))) * e^(2Pi)

Input:

$$\left(\sqrt{141}\left(4\sqrt{3}+\sqrt{47}\right)^3\left(\frac{7+\sqrt{47}}{\sqrt{2}}\right)^2\left(\sqrt{\frac{1}{4}\left(18+9\sqrt{3}\right)}-\sqrt{\frac{1}{4}\left(14+9\sqrt{3}\right)}\right)^{12}\right)e^{2\pi}$$

Exact result:

$$\frac{1}{2}\sqrt{141}\left(7+\sqrt{47}\right)^2\left(4\sqrt{3}+\sqrt{47}\right)^3\left(\frac{1}{2}\sqrt{18+9\sqrt{3}}-\frac{1}{2}\sqrt{14+9\sqrt{3}}\right)^{12}e^{2\pi}$$

Decimal approximation:

1.618615109783450122352859022360730463420491031120326504140...

1.61861510978…. result that is a very good approximation to the value of the golden ratio 1,618033988749...

Property:

$$\frac{1}{2}\sqrt{141}\left(7+\sqrt{47}\right)^2\left(4\sqrt{3}+\sqrt{47}\right)^3\left(-\frac{1}{2}\sqrt{14+9\sqrt{3}}+\frac{1}{2}\sqrt{18+9\sqrt{3}}\right)^{12}e^{2\pi}$$

is a transcendental number

Alternate forms:

$$e^{2\pi}\left(\text{root of } x^8-2947821237008928x^7-136895253836531896500x^6+28294648826644112163552x^5-14424132395908316146312458x^4-3989545484556819815060832x^3-2721614541524090634316500x^2+8263394395829404157088x+395254161 \text{ near } x=0.00302267\right)$$

$$\frac{1}{524288}\sqrt{141}\left(7+\sqrt{47}\right)^2\left(4\sqrt{3}+\sqrt{47}\right)^3\left(-3-3\sqrt{3}+\sqrt{14-i\sqrt{47}}+\sqrt{i\left(\sqrt{47}+-14i\right)}\right)^{12}e^{2\pi}$$

$$368477654626116\, e^{2\pi} + 212740673088720\sqrt{3}\, e^{2\pi} + 53747989960896\sqrt{47}\, e^{2\pi} + 31031416472329\sqrt{141}\, e^{2\pi} -$$
$$\frac{99136598463}{256}\left(2+\sqrt{3}\right)^{11/2}\sqrt{14+9\sqrt{3}}\, e^{2\pi} -$$
$$\frac{201944922795}{256}\left(2+\sqrt{3}\right)^{9/2}\left(14+9\sqrt{3}\right)^{3/2} e^{2\pi} -$$
$$\frac{29154558015}{64}\sqrt{3}\left(2+\sqrt{3}\right)^{9/2}\left(14+9\sqrt{3}\right)^{3/2} e^{2\pi} -$$
$$\frac{1841443065}{16}\sqrt{47}\left(2+\sqrt{3}\right)^{9/2}\left(14+9\sqrt{3}\right)^{3/2} e^{2\pi} -$$
$$\frac{68027302035\sqrt{141}\left(2+\sqrt{3}\right)^{9/2}\left(14+9\sqrt{3}\right)^{3/2} e^{2\pi}}{1024} -$$
$$\frac{40388984559}{128}\left(2+\sqrt{3}\right)^{7/2}\left(14+9\sqrt{3}\right)^{5/2} e^{2\pi} -$$
$$\frac{5830911603}{32}\sqrt{3}\left(2+\sqrt{3}\right)^{7/2}\left(14+9\sqrt{3}\right)^{5/2} e^{2\pi} -$$
$$\frac{368288613}{8}\sqrt{47}\left(2+\sqrt{3}\right)^{7/2}\left(14+9\sqrt{3}\right)^{5/2} e^{2\pi} -$$
$$\frac{13605460407}{512}\sqrt{141}\left(2+\sqrt{3}\right)^{7/2}\left(14+9\sqrt{3}\right)^{5/2} e^{2\pi} -$$
$$\frac{4487664951}{128}\left(2+\sqrt{3}\right)^{5/2}\left(14+9\sqrt{3}\right)^{7/2} e^{2\pi} -$$
$$\frac{647879067}{32}\sqrt{3}\left(2+\sqrt{3}\right)^{5/2}\left(14+9\sqrt{3}\right)^{7/2} e^{2\pi} -$$
$$\frac{40920957}{8}\sqrt{47}\left(2+\sqrt{3}\right)^{5/2}\left(14+9\sqrt{3}\right)^{7/2} e^{2\pi} -$$
$$\frac{1511717823}{512}\sqrt{141}\left(2+\sqrt{3}\right)^{5/2}\left(14+9\sqrt{3}\right)^{7/2} e^{2\pi} -$$
$$\frac{277016355}{256}\left(2+\sqrt{3}\right)^{3/2}\left(14+9\sqrt{3}\right)^{9/2} e^{2\pi} -$$
$$\frac{39992535}{64}\sqrt{3}\left(2+\sqrt{3}\right)^{3/2}\left(14+9\sqrt{3}\right)^{9/2} e^{2\pi} -$$
$$\frac{2525985}{16}\sqrt{47}\left(2+\sqrt{3}\right)^{3/2}\left(14+9\sqrt{3}\right)^{9/2} e^{2\pi} -$$
$$\frac{93315915\sqrt{141}\left(2+\sqrt{3}\right)^{3/2}\left(14+9\sqrt{3}\right)^{9/2} e^{2\pi}}{1024} -$$
$$\frac{1678887}{256}\sqrt{2+\sqrt{3}}\left(14+9\sqrt{3}\right)^{11/2} e^{2\pi} -$$
$$\frac{242379}{64}\sqrt{3\left(2+\sqrt{3}\right)}\left(14+9\sqrt{3}\right)^{11/2} e^{2\pi} -$$
$$\frac{15309}{16}\sqrt{47\left(2+\sqrt{3}\right)}\left(14+9\sqrt{3}\right)^{11/2} e^{2\pi} -$$
$$\frac{565551\sqrt{141\left(2+\sqrt{3}\right)}\left(14+9\sqrt{3}\right)^{11/2} e^{2\pi}}{1024} -$$
$$\frac{14312237571}{64}\left(2+\sqrt{3}\right)^{11/2}\sqrt{3\left(14+9\sqrt{3}\right)}\, e^{2\pi} -$$
$$\frac{903981141}{16}\left(2+\sqrt{3}\right)^{11/2}\sqrt{47\left(14+9\sqrt{3}\right)}\, e^{2\pi} -$$
$$\frac{33395220999\left(2+\sqrt{3}\right)^{11/2}\sqrt{141\left(14+9\sqrt{3}\right)}\, e^{2\pi}}{1024}$$

and:

(((sqrt141(4sqrt3+sqrt47)^3((7+sqrt47)/sqrt2)^2(((((18+9sqrt3)/4)^1/2-((14+9sqrt3)/4)^1/2)))^12)))*24^2-(3^2)/10^3

Input:

$$\left(\sqrt{141}\left(4\sqrt{3}+\sqrt{47}\right)^3\left(\frac{7+\sqrt{47}}{\sqrt{2}}\right)^2\left(\sqrt{\frac{1}{4}\left(18+9\sqrt{3}\right)}-\sqrt{\frac{1}{4}\left(14+9\sqrt{3}\right)}\right)^{12}\right)\times 24^2-\frac{3^2}{10^3}$$

Exact result:

$$288\sqrt{141}\left(7+\sqrt{47}\right)^2\left(4\sqrt{3}+\sqrt{47}\right)^3\left(\frac{1}{2}\sqrt{18+9\sqrt{3}}-\frac{1}{2}\sqrt{14+9\sqrt{3}}\right)^{12}-\frac{9}{1000}$$

Decimal approximation:

1.732058508785951409225592898340508138755619012916678977143...

1.73205850878… ≈ $\sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q

Alternate forms:

$$\frac{9}{128}\sqrt{141}\left(7+\sqrt{47}\right)^2\left(4\sqrt{3}+\sqrt{47}\right)^3\left(\sqrt{14+9\sqrt{3}}-3\sqrt{2+\sqrt{3}}\right)^{12}-\frac{9}{1000}$$

root of $1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,x^8 - 1\,697\,945\,032\,517\,142\,527\,928\,000\,000\,000\,000\,000\,000\,000\,000\,x^7 - 45\,418\,559\,843\,839\,743\,541\,763\,979\,261\,732\,000\,000\,000\,000\,000\,000\,000\,x^6 + 5\,407\,189\,143\,044\,369\,248\,190\,934\,987\,156\,559\,912\,824\,000\,000\,000\,000\,000\,x^5 - 1\,587\,740\,661\,872\,260\,896\,149\,694\,287\,399\,831\,617\,611\,601\,460\,730\,000\,000\,000\,000\,x^4 - 253\,007\,830\,904\,258\,865\,285\,686\,070\,414\,982\,657\,406\,675\,624\,413\,973\,256\,000\,000\,000\,x^3 - 99\,401\,139\,083\,276\,030\,894\,524\,066\,673\,354\,125\,860\,235\,822\,209\,011\,838\,431\,652\,000\,000\,x^2 + 172\,037\,292\,750\,801\,220\,817\,160\,100\,123\,401\,080\,405\,773\,117\,567\,660\,068\,478\,327\,752\,000\,x + 1\,556\,386\,947\,379\,791\,386\,642\,776\,086\,854\,211\,932\,265\,155\,418\,886\,204\,024\,037\,414\,721$ near $x = 1.73206$

$$\frac{1}{1000}9\left(23\,582\,569\,896\,071\,423\,999+13\,615\,403\,077\,678\,080\,000\sqrt{3}+3\,439\,871\,357\,497\,344\,000\sqrt{47}+1\,986\,010\,654\,229\,056\,000\sqrt{141}\right)-82\,944\sqrt{\left(2+\sqrt{3}\right)\left(14+9\sqrt{3}\right)}\left(243\,508\,278\,024+140\,589\,569\,868\sqrt{3}+35\,519\,332\,905\sqrt{47}+20\,507\,096\,414\sqrt{141}\right)$$

Minimal polynomial:

$$1\,000\,000\,000\,000\,000\,000\,000\,000\,x^8-1\,697\,945\,032\,517\,142\,527\,928\,000\,000\,000\,000\,000\,000\,000\,x^7-45\,418\,559\,843\,839\,743\,541\,763\,979\,261\,732\,000\,000\,000\,000\,000\,000\,x^6+5\,407\,189\,143\,044\,369\,248\,190\,934\,987\,156\,559\,912\,824\,000\,000\,000\,000\,000\,x^5-1\,587\,740\,661\,872\,260\,896\,149\,694\,287\,399\,831\,617\,611\,601\,460\,730\,000\,000\,000\,000\,x^4-253\,007\,830\,904\,258\,865\,285\,686\,070\,414\,982\,657\,406\,675\,624\,413\,973\,256\,000\,000\,000\,x^3-99\,401\,139\,083\,276\,030\,894\,524\,066\,673\,354\,125\,860\,235\,822\,209\,011\,838\,431\,652\,000\,000\,x^2+172\,037\,292\,750\,801\,220\,817\,160\,100\,123\,401\,080\,405\,773\,117\,567\,660\,068\,478\,327\,752\,000\,x+1\,556\,386\,947\,379\,791\,386\,642\,776\,086\,854\,211\,932\,265\,155\,418\,886\,204\,024\,037\,414\,721$$

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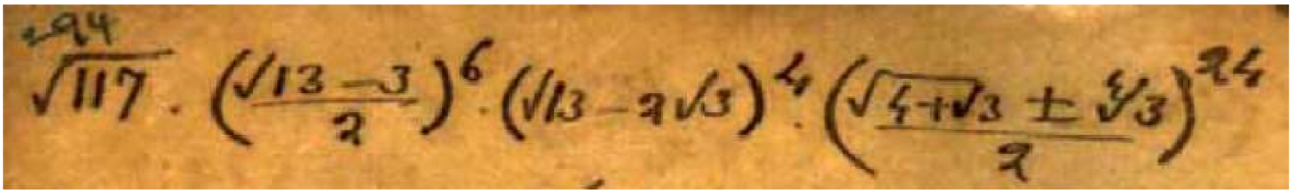

sqrt117((sqrt13-3)/2)^6(sqrt13-2sqrt3)^4((((sqrt(4+sqrt3)+(3)^1/4))/2)^24

Input:

$$\sqrt{117}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^6\left(\sqrt{13}-2\sqrt{3}\right)^4\left(\frac{1}{2}\left(\sqrt{4+\sqrt{3}}+\sqrt[4]{3}\right)\right)^{24}$$

Result:

$$\frac{3\sqrt{13}\left(\sqrt{13}-3\right)^6\left(\sqrt{13}-2\sqrt{3}\right)^4\left(\sqrt[4]{3}+\sqrt{4+\sqrt{3}}\right)^{24}}{1\,073\,741\,824}$$

Decimal approximation:

9.2080160378908244588555702543046888456194589534370529783 89...

9.20801603789…

Alternate forms:

$$\frac{3\left(649\sqrt{13}-2340\right)\left(1249-200\sqrt{39}\right)\left(\sqrt{4+\sqrt{3}}+\sqrt[4]{3}\right)^{24}}{16777216}$$

root of $x^8 + 96805418168160\,x^7 + 461446278882732\,x^6 + 34031477760639840\,x^5 + 128439125155493334\,x^4 - 3981682897994861280\,x^3 + 6316738111625718348\,x^2 - 155044816208561242080\,x + 187388721$ near $x = 9.20802$

$$-12\,100\,677\,271\,020 - 6\,986\,329\,279\,800\sqrt{3} + 3\,356\,124\,028\,347\sqrt{13} +$$
$$1\,937\,659\,111\,200\sqrt{39} - \frac{1\,383\,878\,925\sqrt[4]{3}\sqrt{4+\sqrt{3}}}{262\,144} -$$
$$\frac{1\,597\,964\,355\times 3^{3/4}\sqrt{4+\sqrt{3}}}{524\,288} - \frac{134\,761\,660\,605\sqrt[4]{3}\,(4+\sqrt{3})^{3/2}}{524\,288} -$$
$$\frac{38\,902\,374\,225\times 3^{3/4}\,(4+\sqrt{3})^{3/2}}{262\,144} + \frac{149\,504\,816\,637\sqrt[4]{3}\sqrt{13}\,(4+\sqrt{3})^{3/2}}{2\,097\,152} +$$
$$\frac{2\,697\,391\,125\times 3^{3/4}\sqrt{13}\,(4+\sqrt{3})^{3/2}}{65\,536} - \frac{816\,949\,858\,725\sqrt[4]{3}\,(4+\sqrt{3})^{5/2}}{262\,144} -$$
$$\frac{943\,331\,624\,235\times 3^{3/4}\,(4+\sqrt{3})^{5/2}}{524\,288} + \frac{56\,645\,213\,625\sqrt[4]{3}\sqrt{13}\,(4+\sqrt{3})^{5/2}}{65\,536} +$$
$$\frac{1\,046\,533\,716\,459\times 3^{3/4}\sqrt{13}\,(4+\sqrt{3})^{5/2}}{2\,097\,152} - \frac{7\,681\,414\,654\,485\sqrt[4]{3}\,(4+\sqrt{3})^{7/2}}{524\,288} -$$
$$\frac{2\,217\,435\,330\,825\times 3^{3/4}\,(4+\sqrt{3})^{7/2}}{262\,144} + \frac{8\,521\,774\,548\,309\sqrt[4]{3}\sqrt{13}\,(4+\sqrt{3})^{7/2}}{2\,097\,152} +$$
$$\frac{153\,751\,294\,125\times 3^{3/4}\sqrt{13}\,(4+\sqrt{3})^{7/2}}{65\,536} - \frac{4\,188\,488\,958\,225\sqrt[4]{3}\,(4+\sqrt{3})^{9/2}}{131\,072} -$$
$$\frac{4\,836\,446\,263\,935\times 3^{3/4}\,(4+\sqrt{3})^{9/2}}{262\,144} + \frac{290\,419\,111\,125\sqrt[4]{3}\sqrt{13}\,(4+\sqrt{3})^{9/2}}{32\,768} +$$
$$\frac{5\,365\,561\,752\,639\times 3^{3/4}\sqrt{13}\,(4+\sqrt{3})^{9/2}}{1\,048\,576} - \frac{9\,233\,215\,594\,785\sqrt[4]{3}\,(4+\sqrt{3})^{11/2}}{262\,144} -$$
$$\frac{2\,665\,402\,064\,325\times 3^{3/4}\,(4+\sqrt{3})^{11/2}}{131\,072} + \frac{10\,243\,345\,164\,129\sqrt[4]{3}\sqrt{13}\,(4+\sqrt{3})^{11/2}}{1\,048\,576} +$$
$$\frac{184\,812\,161\,625\times 3^{3/4}\sqrt{13}\,(4+\sqrt{3})^{11/2}}{32\,768} - \frac{2\,665\,402\,064\,325\sqrt[4]{3}\,(4+\sqrt{3})^{13/2}}{131\,072} -$$
$$\frac{3\,077\,738\,531\,595\times 3^{3/4}\,(4+\sqrt{3})^{13/2}}{262\,144} + \frac{184\,812\,161\,625\sqrt[4]{3}\sqrt{13}\,(4+\sqrt{3})^{13/2}}{32\,768} +$$
$$\frac{3\,414\,448\,388\,043\times 3^{3/4}\sqrt{13}\,(4+\sqrt{3})^{13/2}}{1\,048\,576} - \frac{1\,612\,148\,754\,645\sqrt[4]{3}\,(4+\sqrt{3})^{15/2}}{262\,144} -$$
$$\frac{465\,387\,662\,025\times 3^{3/4}\,(4+\sqrt{3})^{15/2}}{131\,072} + \frac{1\,788\,520\,584\,213\sqrt[4]{3}\sqrt{13}\,(4+\sqrt{3})^{15/2}}{1\,048\,576} +$$
$$\frac{32\,268\,790\,125\times 3^{3/4}\sqrt{13}\,(4+\sqrt{3})^{15/2}}{32\,768} - \frac{246\,381\,703\,425\sqrt[4]{3}\,(4+\sqrt{3})^{17/2}}{262\,144} -$$
$$\frac{284\,496\,839\,055\times 3^{3/4}\,(4+\sqrt{3})^{17/2}}{524\,288} + \frac{17\,083\,477\,125\sqrt[4]{3}\sqrt{13}\,(4+\sqrt{3})^{17/2}}{65\,536} +$$
$$\frac{315\,621\,279\,567\times 3^{3/4}\sqrt{13}\,(4+\sqrt{3})^{17/2}}{2\,097\,152} -$$
$$\frac{34\,938\,208\,305\sqrt[4]{3}\,(4+\sqrt{3})^{19/2}}{524\,288} - \frac{10\,085\,800\,725\times 3^{3/4}\,(4+\sqrt{3})^{19/2}}{262\,144} +$$
$$\frac{38\,760\,508\,017\sqrt[4]{3}\sqrt{13}\,(4+\sqrt{3})^{19/2}}{2\,097\,152} + \frac{699\,323\,625\times 3^{3/4}\sqrt{13}\,(4+\sqrt{3})^{19/2}}{65\,536} -$$
$$\frac{480\,276\,225\sqrt[4]{3}\,(4+\sqrt{3})^{21/2}}{262\,144} - \frac{554\,574\,735\times 3^{3/4}\,(4+\sqrt{3})^{21/2}}{524\,288} +$$
$$\frac{33\,301\,125\sqrt[4]{3}\sqrt{13}\,(4+\sqrt{3})^{21/2}}{65\,536} + \frac{615\,246\,159\times 3^{3/4}\sqrt{13}\,(4+\sqrt{3})^{21/2}}{2\,097\,152} -$$
$$\frac{6\,575\,985\sqrt[4]{3}\,(4+\sqrt{3})^{23/2}}{524\,288} - \frac{1\,898\,325\times 3^{3/4}\,(4+\sqrt{3})^{23/2}}{262\,144} +$$
$$\frac{7\,295\,409\sqrt[4]{3}\sqrt{13}\,(4+\sqrt{3})^{23/2}}{2\,097\,152} + \frac{131\,625\times 3^{3/4}\sqrt{13}\,(4+\sqrt{3})^{23/2}}{65\,536} +$$
$$\frac{95\,954\,625\sqrt[4]{3}\sqrt{13\,(4+\sqrt{3})}}{65\,536} + \frac{1\,772\,784\,387\times 3^{3/4}\sqrt{13\,(4+\sqrt{3})}}{2\,097\,152}$$

Minimal polynomial:

$$x^8 + 96\,805\,418\,168\,160\,x^7 + 461\,446\,278\,882\,732\,x^6 + 34\,031\,477\,760\,639\,840\,x^5 + 128\,439\,125\,155\,493\,334\,x^4 - 3\,981\,682\,897\,994\,861\,280\,x^3 + 6\,316\,738\,111\,625\,718\,348\,x^2 - 155\,044\,816\,208\,561\,242\,080\,x + 187\,388\,721$$

From which:

((sqrt117((sqrt13-3)/2)^6(sqrt13-2sqrt3)^4((((sqrt(4+sqrt3)+(3)^1/4))/2)^24)))^1/4-(8+2)*1/10^3

Input:

$$\sqrt[4]{\sqrt{117}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^6\left(\sqrt{13}-2\sqrt{3}\right)^4\left(\frac{1}{2}\left(\sqrt{4+\sqrt{3}}+\sqrt[4]{3}\right)\right)^{24}} - (8+2)\times\frac{1}{10^3}$$

Result:

$$\frac{\sqrt[4]{3}\,\sqrt[8]{13}\,\left(\sqrt{13}-3\right)^{3/2}\left(\sqrt{13}-2\sqrt{3}\right)\left(\sqrt[4]{3}+\sqrt{4+\sqrt{3}}\right)^6}{128\sqrt{2}} - \frac{1}{100}$$

Decimal approximation:

1.731973390905548085350460297098014377889571036414137464397...

1.7319733909… ≈ $\sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q

Alternate forms:

$$\frac{1}{1600}\left(450\sqrt[4]{\left(649\sqrt{13}-2340\right)\left(1249-200\sqrt{39}\right)}\sqrt{3\left(4+\sqrt{3}\right)} + 1500\sqrt{100+51\sqrt{3}}\sqrt[4]{\left(649\sqrt{13}-2340\right)\left(1249-200\sqrt{39}\right)} + 150\sqrt{3}\sqrt{3124+1769\sqrt{3}}\sqrt[4]{\left(649\sqrt{13}-2340\right)\left(1249-200\sqrt{39}\right)} + 9600\times 3^{3/4}\sqrt[4]{\left(649\sqrt{13}-2340\right)\left(1249-200\sqrt{39}\right)} + 16\,000\sqrt[4]{3\left(649\sqrt{13}-2340\right)\left(1249-200\sqrt{39}\right)} - 16\right)$$

$$\frac{1}{100}\left(-1 + 5950\sqrt[4]{3}\sqrt[8]{13}\sqrt{2\left(\sqrt{13}-3\right)} + 3450\times 3^{3/4}\sqrt[8]{13}\sqrt{2\left(\sqrt{13}-3\right)} - 1650\sqrt[4]{3}\,13^{5/8}\sqrt{2\left(\sqrt{13}-3\right)} - 950\times 3^{3/4}\times 13^{5/8}\sqrt{2\left(\sqrt{13}-3\right)}\right) -$$

$$\frac{3}{2}\sqrt[8]{13}\sqrt{\frac{1}{2}\left(4+\sqrt{3}\right)\left(\sqrt{13}-3\right)}\left(-44-25\sqrt{3}+12\sqrt{13}+7\sqrt{39}\right)$$

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64 g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276 e^{-\pi\sqrt{22}} - \cdots, \\ 64 g_{22}^{-24} &= \qquad\qquad 4096 e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64 g_{22}^{-24} = 4096 e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the *golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.*

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every *quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals*

We note how the following three values: 137.508 (golden angle), 139.57-134.9766 (masses of the two Pions – Pi mesons) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

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