The Seiberg-Witten equations for spin 3/2

A.Balan

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Abstract

We define the Seiberg-Witten equations for spin 3/2 with help of the Rarita-Schwinger operator.

1 The Seiberg-Witten equations

For a four-manifold with riemannian metric (M, g), we may define the Seiberg-Witten equations which are written for spin 1/2 particules [F]:

$$\mathcal{D}_A \psi = 0$$

$$F(A)_+ = \omega(\psi)$$

with (ψ, A), a spinor and a connection for the line bundle of the spin-c structure.

$$\omega(\psi) = < (XY - YX).\psi, \psi >$$

2 The Seiberg-Witten equations for spin 3/2

For a particule of spin 3/2, we may define the Seiberg-Witten equations with help of the Rarita-Schwinger operator $\mathcal{D}_A^{RS}$ ([BT] p.296), with the connection:

$$\tilde{\nabla}^A = \nabla^A \otimes 1 + 1 \otimes \nabla$$

over the fiber bundle $\Sigma \otimes TM$:

$$\tilde{\psi} = \sum_a \psi^a \otimes e^a$$

$$\sum_a e^a.\psi^a = 0$$

with (e^a), an orthonormal basis of the tangent fiber bundle and ψ^a, 1/2 spinors.

$$\omega(\tilde{\psi}) = \sum_a < (XY - YX).\psi^a, \psi^a >$$

this definition doesn't depend on the choice of the basis (e^a). The spin 3/2 Seiberg-Witten equations are:

$$\mathcal{D}_A^{RS}\tilde{\psi} = 0$$

$$F(A)_+ = \omega(\tilde{\psi})$$

References

[BT] I.M.Benn & R.W.Tucker, "An Introduction to Spinors and Geometry with Applications in Physics", Adam Hilger, London, 1987.

[F] T.Friedrich, "Dirac Operators in Riemannian Geometry", GSM vol.25, AMS, Rhode Island, 2000.