<u>**Derivation of the θ-parameter in Quantum Chromodynamics**</u>

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Abstract

An ongoing challenge of the Standard Model is to explain the nearly vanishing magnitude of CP symmetry breaking in Quantum Chromodynamics (QCD). This challenge goes by the name of the *strong CP problem* and is quantified by the θ-parameter of the QCD Lagrangian. It is known that *axions* are hypothetical particles conjectured to offset the contribution of the θ-parameter and restore the CP symmetry of QCD. Starting from the near equilibrium interpretation of gluon dynamics, here we bypass the axion conjecture and derive the numerical value of the θ-parameter in close agreement with experimental bounds. A surprising finding of this brief report is that the strong CP and the baryon asymmetry problems appear to be related to each other.

Key words: Quantum Chromodynamics, strong CP problem, axions, fractional dynamics, non-equilibrium field theory.

The violation of parity (P) and time-reversal (T) symmetries in QCD follows from the nontrivial structure of the quantum vacuum and is embodied in the so-called θ-term entering the Lagrangian [1-3, 10]

$$\Delta L_{QCD} = L_{QCD} - L_{QCD}^{(\theta=0)} = \theta \frac{g_s^2}{64\pi^2} G_{\mu\nu}^a \overline{G}^{a\mu\nu} \tag{1}$$

in which g_s is the strong coupling charge, $G_{\mu\nu}^a$ the gluon field strength and $\overline{G}^{a\mu\nu}$ its dual. The θ-term can be written as

$$G^a_{\mu\nu}\overline{G}^{a\mu\nu} = \partial_\mu K^\mu \tag{2}$$

where K_μ represents a current built from the components of the gluon field,

$$\mathbf{A}_\mu = A^a_\mu \frac{\boldsymbol{\lambda}^a}{2} \tag{3}$$

Consider a gluon field configuration that starts off at $t = -\infty$ and ends up at $t = +\infty$. It can be shown that the following integral is non-vanishing

$$\frac{g_s^2}{64\pi^2}\int d^4x G^a_{\mu\nu}\overline{G}^{a\mu\nu} \neq 0 \tag{4}$$

In particular [3]

$$\frac{g_s^2}{64\pi^2}\int d^4x\, G^a_{\mu\nu}\overline{G}^{a\mu\nu} = \frac{g_s^2}{64\pi^2}\int d^3x\, K_0 \Big|^{t=+\infty}_{t=-\infty} = 1 \tag{5}$$

We proceed below with the assumption that the dynamics of the gluon field is in *near equilibrium* conditions and that the integral (5) can be well approximated by

$$\frac{g_s^2}{64\pi^2}\left\langle G^a_{\mu\nu}\overline{G}^{a\mu\nu}\right\rangle \cdot V_4 = 1 \Rightarrow \left\langle G^a_{\mu\nu}\overline{G}^{a\mu\nu}\right\rangle = O(V_4^{-1}) \tag{6}$$

where V_4 is the finite four-dimensional integration volume, commensurate with the QCD scale via

$$V_4 = \int d^4x = O(\Lambda_{QCD}^{-4}) \tag{7}$$

Because the product of a field Lagrangian with the four-dimensional volume defining its range yields the field action, by (1), (6) and (7) we obtain

$$\theta = \left\langle \Delta L_{QCD} \right\rangle \cdot V_4 = \left\langle S_{QCD} - S_{QCD}^{(\theta=0)} \right\rangle \tag{8}$$

It is known that the field action can be specified up to an arbitrary constant [7]. Since gluon dynamics is considered in near equilibrium conditions, we choose an infinitesimal departure of the action from its equilibrium value S_{ech} and rewrite (8) as

$$\theta = \left\langle (S_{QCD} - S_{ech}) - (S_{QCD}^{(\theta=0)} - S_{ech}) \right\rangle = \left\langle \delta S_{QCD} - \delta S_{QCD}^{(\theta=0)} \right\rangle \tag{9}$$

Appealing to [4-6] motivates the plausible assumption that both δS_{QCD} and $\delta S_{QCD}^{(\theta=0)}$ scale linearly with the dimensional deviation from four spacetime dimensions described by $\varepsilon = 4 - D << 1$. As a result, (9) leads to

$$\theta = O(\varepsilon) - O(\varepsilon) = O(\varepsilon^2) \tag{10}$$

Furthermore, since ε flows with the energy scale, it likely reaches its uppermost observable value close to the formation of the cosmic microwave background (CMB) [8]. It is therefore reasonable to conjecture that the maximal dimensional deviation is given by

$$\varepsilon_{\max} = O(10^{-5}) \tag{11}$$

We recall that this scenario falls in line with the requirements set by the Sakharov conditions for baryogenesis. Given the long-range effects carried by dimensional deviation ε throughout all energy scales, it makes sense to assume that (11) persists as a relic effect at the Standard Model scale. With these considerations in mind and replacing (11) in (10) yields

$$\boxed{\theta \le O(\varepsilon_{\max}^{2}) = O(10^{-10})} \qquad (12)$$

in close agreement with current experimental data on the neutron dipole moment [9-10].

References

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