

Absolute Velocity and Total Stellar Aberration

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Abstract

It is generally accepted that stellar annual or secular aberration is attributed to the changes in velocity of the detector. We can say it in a slightly different way. By means of the all known experiments, stellar aberration is directly or indirectly detectable and measurable, only if a detector changes its velocity. Our presumption is that stellar aberration is not caused by the changes in the velocity of the detector. It exists due to the movement of the detector regarding to an absolute inertial frame. Therefore it is just the question of how to choose such a frame. In this paper it is proposed a method to detect and measure instantaneous stellar aberration due to absolute velocity. We can call it an “absolute” stellar aberration. Combining an “annual” and an “absolute” we can define a “total” stellar aberration.

Keywords

Annual Stellar Aberration, “Absolute” Stellar Aberration, “Total” Stellar Aberration, Absolute Velocity, One-Way Velocity of Light

1. Introduction

It is commonly assumed that due to the aberration the observed position of a star is displaced of about 150” toward the direction of the instantaneous velocity of the observer with respect to an inertial reference frame at rest. But, for an observer located at the barycenter of the solar system, the instantaneous effect of the relativistic aberration due to the galactic motion of the solar system (220 km/s) is not directly observable because the velocity-induced aberration pattern is constant [1] [2] [3].

Our hypothesis is in contradiction to the relativistic view on stellar aberration, because according to this theory an absolute frame does not exist nor a measurement of the absolute aberration is possible.

We will start a discussion by the classical explanation of the annual stellar ab-

erration.

A star (Z) is stationary, and the light travels from the star at velocity c constant speed of light in the stationary frame. A telescope is in motion at velocity v regarding to the heliocentric-ecliptic coordinate system.

Suppose that AB represents a median line of the telescope at the instant t_0 and $A'B'$ represents a median line of the telescope at the instant t_1 . In the reference frame of the telescope AB is identical to $A'B'$. But in the sun's reference frame median lines AB and $A'B'$ are represented by two different positions.

In referring to **Figure 1**, the following definitions apply

Δt —a time required for light to traverse the length of the telescope

θ —an angle between the earth velocity about the sun and light ray from the star

$\Delta\theta$ —an angle at which a telescope should be tilted in the direction of motion in order for the photons move along the median line of the telescope. Actually this angle is obtained by the two measurements at six months intervals.

$$\Delta t = t_1 - t_0 \tag{1.1}$$

$$\Delta\theta = \angle(ABA') = \angle(B'A'B) \tag{1.2}$$

$$\theta = \angle(SA'B) \tag{1.3}$$

$$\theta - \Delta\theta = \angle(A'B'B) \tag{1.4}$$

$$BB' = \Delta t * v \tag{1.5}$$

$$BA' = \Delta t * c \tag{1.6}$$

$$\frac{v * \Delta t}{\sin(\Delta\theta)} = \frac{c * \Delta t}{\sin(\theta - \Delta\theta)} \tag{1.7}$$

$$c * \sin(\Delta\theta) = v * \sin(\theta) * \cos(\Delta\theta) - v * \cos(\theta) * \sin(\Delta\theta) \tag{1.8}$$

$$\tan(\Delta\theta) = \frac{\frac{v}{c} * \sin(\theta)}{1 + \frac{v}{c} * \cos(\theta)} \tag{1.9}$$

for $\theta = \Pi/2$, Equation (1.9) is being reduced to the equation

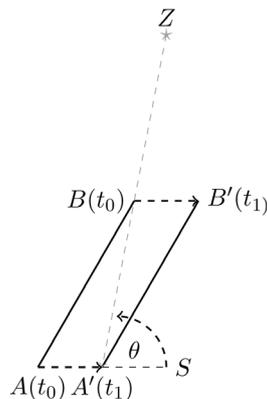


Figure 1. Stellar aberration according to Bradley.

$$\tan(\Delta\theta) = \frac{v}{c} \quad (1.10)$$

assuming that $|\Delta\theta| \ll 1$ and after neglecting terms higher than the second order with the respect to $\Delta\theta$ we have that

$$\left| \tan(x) = x + \frac{x^3}{3} + \frac{2x^5}{15} + \dots \right| \quad (1.11)$$

$$\frac{v}{c} = \Delta\theta + \frac{\Delta\theta^3}{3} + \frac{2 * \Delta\theta^5}{15} + \dots \quad (1.12)$$

we can say that an approximate value for the stellar aberration $\Delta\theta$ is equal to

$$\Delta\theta \approx \frac{v}{c} \quad (1.13)$$

Its maximal value is approximately the same for all stars. The accepted value is 20.49552 arc seconds.

The problems related to the stellar aberration that are being treated in this paper, had been already defined and mentioned in the numerous works for example [4].

2. Description and Role of the “Telescope”

In this paper we will use a term “telescope”, although a standard telescope is not suitable for this experiment. In order to perform this experiment instead of using a such telescope one have to design and build a new apparatus. Because of that it will be given its description, but just in few words.

Point **S** represents a center of the top “plane” while **S’** represents a center of the bottom “plane” of the telescope. Photons enter the telescope at the point **S** and their direction is perpendicular to the top plane. This means that there is a some part of the telescope who has a role to point the “telescope” in such direction that median line SS' becomes parallel to the starlight.

At the bottom plane of the telescope it should be installed a camera in order to take an image of the star (**Z**) at the point **A**.

It will be assumed that a star (**Z**) is extremely far away, so that a parallax may be ignored. For now we can assume that extra-galactic stars are being observed only. Starlight moves in straight line and will remain in the same direction regarding to the ecliptic plane. Photons enter in a perpendicular direction to the top plane of the telescope.

The starlight represents an inertial frame of reference marked by (L) and the telescope represents a moving frame of reference that is marked by (T). We will assume that

- P_1 — speed of light c is constant and equal in all inertial frames (L)
- P_2 — there is a one common time for the all frames (L) and the moving frame (T)
- P_3 — frame (T) is moving uniformly in a straight line regarding the frame (L)

Now we will discuss the three cases that are depicted on the **Figure 2**.

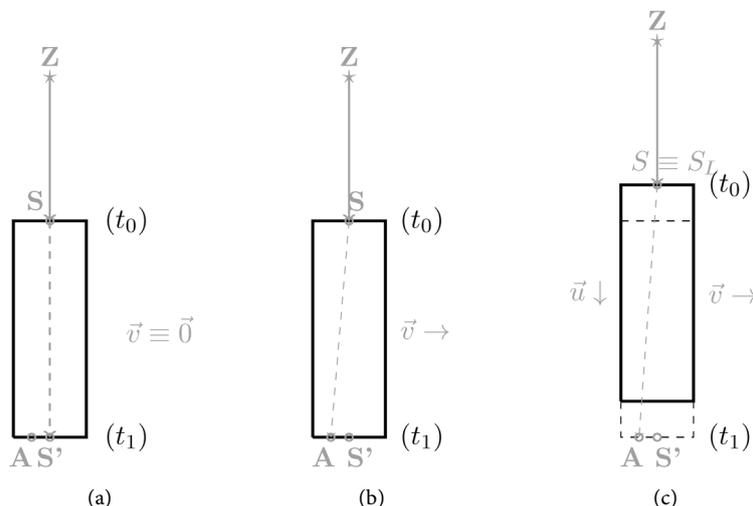


Figure 2. (a) A frame of the telescope is stationary regarding to the starlight; (b) A frame of the telescope is moving perpendicular regarding to the starlight; (c) A frame of the telescope is moving by some arbitrary velocity regarding to the starlight. The effect of the velocity v on the position of the telescope is not depicted on the last two figures.

-In the first case **Figure 2(a)**, the relative velocity of the telescope \mathbf{v} regarding to the frame of the starlight is equal to the $\mathbf{0}$. At some instant t_0 photon hits the top “plane” at the point S and at some instant t_1 hits the bottom “plane” at the point S' .

-In the second case **Figure 2(b)**, under the same circumstances except that $\mathbf{v} \neq \mathbf{0}$ contrary to our expectations photon does not hit the bottom plane at the point S but rather at a some point A . Referring to the **Figure 2(b)** it follows

$$d = SS' \quad (2.1)$$

$$\Delta t = t_1 - t_0 = \frac{d}{c} \quad (2.2)$$

$$S'A = -\Delta t * \mathbf{v} = \frac{d}{c} * \mathbf{v} \quad (2.3)$$

$$c_{(T)} = \frac{SA}{\Delta t} = \sqrt{v^2 + c^2} \quad \text{speed of light in the frame (T)} \quad (2.4)$$

$$\tan(\Delta\theta) = \frac{S'A}{d} = -\frac{d * v}{c * d} = -\frac{v}{c} \quad (2.5)$$

$$\Delta\theta \approx -\frac{v}{c} \quad (v \ll c) \quad (2.6)$$

-In the third case **Figure 2(c)**, velocity at which a telescope moves relative to the starlight is decomposed to the two components. The first component noted by \mathbf{v} is perpendicular to the starlight and the second one noted by \mathbf{u} is parallel to starlight. Referring to the **Figure 2(c)**, it follows

$$d = SS' \quad (2.7)$$

$$\frac{d+x}{c} = \frac{x}{u} \Rightarrow \frac{d}{c} = \frac{x}{u} - \frac{x}{c} \Rightarrow x = d * \frac{u}{c-u} \quad (2.8)$$

$$x + d = d + d * \frac{u}{c - u} = d * \frac{c}{c - u} \quad (2.9)$$

$$\Delta t = t_1 - t_0 = \frac{d}{c - u} \quad (2.10)$$

$$S'A = -\Delta t * v = -\frac{d}{c - u} * v \quad (2.11)$$

$$SA = \sqrt{S'A^2 + d^2} = \sqrt{\left(\frac{d}{c - u} * v\right)^2 + d^2} = \frac{d * \sqrt{(c - u)^2 + v^2}}{c - u} \quad (2.12)$$

$$c_{(T)} = \frac{SA}{\Delta t} = \sqrt{(c - u)^2 + v^2} \quad \text{speed of light in the frame (T)} \quad (2.13)$$

$$\tan(\Delta\theta) = \frac{S'A}{d} = \frac{d * v}{d * (c - u)} = \frac{v}{c - u} \quad (2.14)$$

$$\Delta\theta \approx \frac{v}{c - u} \quad (u \ll c, v \ll c) \quad (2.15)$$

An additional explanation will be given for the third case.

-From the point view of a spectator at the frame (L) a photon hits top plane at the point S (this point is identical to the fixed point S_L at the frame (L)). The telescope is moving parallel with the starlight by velocity u , therefore a distance between the point S_L and the bottom plane changes. A total distance that photon travels from the instant t_0 to the instant t_1 is equal to $(d * c)/(c - u)$. Time that it takes for the photon to travel from the top to the bottom plane of the telescope is equal to $d/(c - u)$.

-From the point view of a spectator at the frame (T) photon traveled a distance equal to the SA . Time is common for the both frames therefore the speed of the photon in the frame (T) is given by Equation (2.13). An Equation (2.15) represents modified equation for the stellar aberration.

On the basis of these observations the point S' will be used as a referential origin for measuring the drift caused by the movement of the frame (T) relative to the frame (L).

Contrary to the classical experiments when the telescope must be tilted, thus the detection and measuring of displacement is possible, in this experiment the "telescope" will be pointed to the star at the beginning and fixed at the same position until the end of the experiment.

3. Coordinate Systems

We have already defined starlight as a referential inertial frame. In this section are given the descriptions of the three coordinate systems that will be used in a further discussion.

Let the $S(x'', y'', z'')$ represents "The Heliocentric-Ecliptic Coordinate System" **Figure 3(a)**. Its origin S is at the center of the sun and the fundamental plane $S(x'', y'')$ coincides with the "ecliptic", plane of the Earth's revolution about the sun. The line of intersection of the ecliptic plane and the earth's

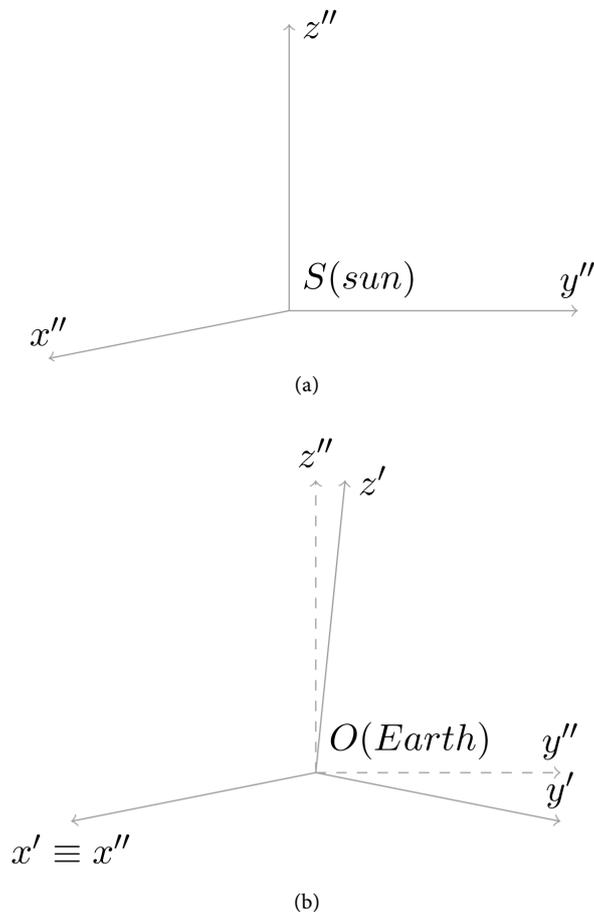


Figure 3. (a) The heliocentric-ecliptic coordinate system; (b) Geocentric equatorial coordinate system.

equatorial plane defines the x'' -axis. On the first day of Spring a line joining the center of the Earth and the center of the sun points in the direction of positive x'' -axis. This is called a vernal equinox direction [5].

Let $\varphi = 23.43693 * \pi/180$ denotes Earth's axial tilt (**Figure 3(b)**).

Let a $O(x'y'z')$ represents "The Geocentric-Equatorial Coordinate System" (**Figure 3** and **Figure 4**). Its origin O is at the center of the Earth, the fundamental plane is the equator and the positive x' -axis points in the vernal equinox direction. The z' -axis points in the direction of the north pole. By the definition the coordinate system $O(x'y'z')$ is non-rotating with the respect to the stars [5].

The position of the star is described by two angles called right ascension and declination (**Figure 4**). The right ascension α is measured eastward in the plane of equator from the vernal equinox direction. The declination δ is measured northward from the equator to the line of sight, we would say that is an angle between the plane of equator and the direction of the starlight [5]. Unlike longitude, right ascension is usually measured in hours, minutes, and seconds with 24 hours being a full circle, but in this experiment it will be assumed that it

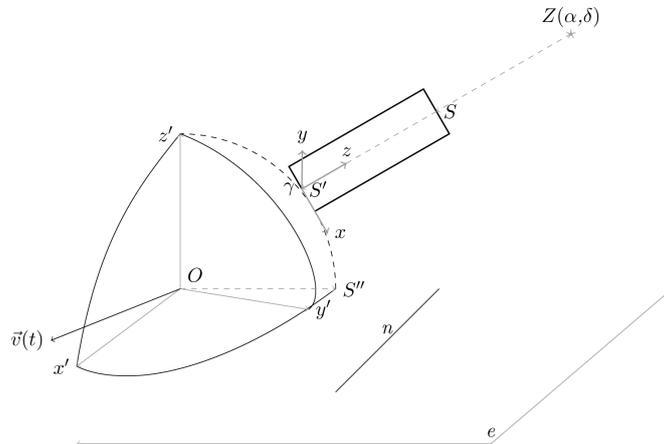


Figure 4. The geocentric-equatorial coordinate system and the frame of the “telescope”.

is measured in radians.

Referring to the **Figure 4** we have

$$\alpha = \angle(x'OS'') \tag{3.1}$$

$$\delta = \angle(S''OZ) \tag{3.2}$$

$$\gamma(\text{plane}) = (Oz', SS') \tag{3.3}$$

Now we will define a coordinate system $S'(xyz)$ that is attached to the telescope in the following way (**Figure 4**).

Let suppose that a telescope is positioned in such way that points S and S' lie in the same meridian plane (a plane that passes through the Earth’s axis of rotation) (γ) . The plane (γ) is rotating about z' -axis but at a fixed sidereal time that will be marked as s_0 ($\sim a$) this plane is parallel to the photons who are coming from a distant star. Just to mention that sidereal time has the same value as the right ascension of any celestial body that is crossing the local meridian at that same moment. At that same moment the telescope has to be tilted so starlight is perpendicular to the top plane of the telescope. That means that at that instant the starlight is perpendicular to the bottom plane as well. Let the point S' represents origin of the $S'(xyz)$ coordinate system and direction $\overline{S'S}$ represents positive z -axis. A x -axis is determined by a intersection between the plane (γ) and the bottom plane of the telescope. Positive y -axis is perpendicular to the plane (γ) and eastward directed. Positive x -axis is chosen so as to form a right-handed coordinate system.

The meridian plane (γ) is perpendicular to the earth equatorial plane $O(x'y')$, and y -axis is perpendicular to the plane (γ) , therefore y -axis is parallel to the earth equatorial plane $O(x'y')$.

Plane (e) represents “ecliptic” plane and a line (n) represents an intersection between “ecliptic” and $S'xy$ plane. The measurements will be taken daily during the year at the fixed sidereal time s_0 , when the top plane of the telescope is perpendicular to the starlight.

4. Coordinate Transformations

To each of these coordinate systems we are going to join an orthonormal basis. It means that they are all unit vectors and orthogonal to each other.

The triplet $(\hat{i}_2, \hat{j}_2, \hat{k}_2)$ represents an orthonormal basis for the coordinate system $S(x'', y'', z'')$ $(\hat{i}_1, \hat{j}_1, \hat{k}_1)$ represents the orthonormal basis for the coordinate system $O(x', y', z')$ $(\hat{i}, \hat{j}, \hat{k})$ represent the orthonormal basis for the coordinate system $S'(x, y, z)$.

Now we will derive a matrix of the transformation B , from the basis $(\hat{i}_2, \hat{j}_2, \hat{k}_2)$ to the basis $(\hat{i}_1, \hat{j}_1, \hat{k}_1)$ and a matrix of the transformation A from the basis $(\hat{i}_1, \hat{j}_1, \hat{k}_1)$ to the basis $(\hat{i}, \hat{j}, \hat{k})$.

Referring to **Figure 3** it follows that we have to rotate the coordinate system $S(x''y''z'')$ about the positive x'' -axis through an angle $-\varphi$, to transform it to the coordinate system $O(x', y', z')$.

The corresponding transformation matrix B is equal to

$$B[x'', -\varphi] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(-\varphi) & \sin(-\varphi) \\ 0 & -\sin(-\varphi) & \cos(-\varphi) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{pmatrix}$$

Now we are going to derive a transformation matrix A . As shown in **Figure 4**, it follows that

$$\angle(Ox', S'x) = \alpha \quad (4.1)$$

$$\angle(Oz', S'z) = \Pi/2 - \delta \quad (4.2)$$

First we are going to rotate the coordinate system $O(x'y'z')$ about the positive z' -axis through an angle α , to a some temporary coordinate system K . The corresponding transformation matrix A_1 is equal to

$$A_1[z', \alpha] = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

After that we are going to rotate the coordinate system K about its positive y -axis through an angle $\Pi/2 - \delta$, to the coordinate system K' . The corresponding transformation matrix A_2 is equal to

$$A_2[y, \Pi/2 - \delta] = \begin{pmatrix} \cos(\Pi/2 - \delta) & 0 & -\sin(\Pi/2 - \delta) \\ 0 & 1 & 0 \\ \sin(\Pi/2 - \delta) & 0 & \cos(\Pi/2 - \delta) \end{pmatrix} \\ = \begin{pmatrix} \sin(\delta) & 0 & -\cos(\delta) \\ 0 & 1 & 0 \\ \cos(\delta) & 0 & \sin(\delta) \end{pmatrix}$$

In that way the coordinate frame $O(x'y'z')$ is transformed to the coordinate frame $S'(xyz)$. The corresponding transformation matrix A is equal to the product of the matrices A_2, A_1

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = A_2 * A_1 \tag{4.3}$$

$$A = \begin{pmatrix} \sin(\delta) & 0 & -\cos(\delta) \\ 0 & 1 & 0 \\ \cos(\delta) & 0 & \sin(\delta) \end{pmatrix} \begin{pmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4.4}$$

$$= \begin{pmatrix} \sin(\delta)\cos(\alpha) & \sin(\delta)\sin(\alpha) & -\cos(\delta) \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ \cos(\delta)\cos(\alpha) & \cos(\delta)\sin(\alpha) & \sin(\delta) \end{pmatrix}$$

$$\begin{pmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{pmatrix} = \begin{pmatrix} \sin(\delta)\cos(\alpha) & \sin(\delta)\sin(\alpha) & -\cos(\delta) \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ \cos(\delta)\cos(\alpha) & \cos(\delta)\sin(\alpha) & \sin(\delta) \end{pmatrix} \begin{pmatrix} \hat{i}_1 \\ \hat{j}_1 \\ \hat{k}_1 \end{pmatrix} \tag{4.5}$$

In a different form we can write that

$$\begin{pmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \hat{i}_1 \\ \hat{j}_1 \\ \hat{k}_1 \end{pmatrix} \tag{4.6}$$

A corresponding matrix of the transformation from the basis $(\hat{i}, \hat{j}, \hat{k})$ to the basis $(\hat{i}_1, \hat{j}_1, \hat{k}_1)$ is the matrix A^T obtained by exchanging rows and columns of the matrix A .

$$A^T = \begin{pmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \\ a'_{31} & a'_{32} & a'_{33} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \tag{4.7}$$

$$= \begin{pmatrix} \sin(\delta)\cos(\alpha) & -\sin(\alpha) & \cos(\delta)\cos(\alpha) \\ \sin(\delta)\sin(\alpha) & \cos(\alpha) & \cos(\delta)\sin(\alpha) \\ -\cos(\delta) & 0 & \sin(\delta) \end{pmatrix}$$

The proof is simple.

$$(a_{11} = \hat{i} * \hat{i}_1, a'_{11} = \hat{i}_1 * \hat{i}, \hat{i} * \hat{i}_1 = \hat{i}_1 * \hat{i}) \Rightarrow (a_{11} = a'_{11}) \tag{4.8}$$

$$(a_{12} = \hat{i} * \hat{j}_1, a'_{21} = \hat{j}_1 * \hat{i}, \hat{i} * \hat{j}_1 = \hat{j}_1 * \hat{i}) \Rightarrow (a_{12} = a'_{21}) \tag{4.9}$$

...etc.

we can write it in a different form

$$\begin{pmatrix} \hat{i}_1 \\ \hat{j}_1 \\ \hat{k}_1 \end{pmatrix} = \begin{pmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \\ a'_{31} & a'_{32} & a'_{33} \end{pmatrix} \begin{pmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{pmatrix} \tag{4.10}$$

A corresponding matrix of the transformation marked by C , from the basis $(\hat{i}_2, \hat{j}_2, \hat{k}_2)$ to the basis $(\hat{i}, \hat{j}, \hat{k})$ is equal to the product of the matrices A and B

$$C = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} = A * B \tag{4.11}$$

$$C = \begin{pmatrix} \sin(\delta)\cos(\alpha) & \sin(\delta)\sin(\alpha) & -\cos(\delta) \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ \cos(\delta)\cos(\alpha) & \cos(\delta)\sin(\alpha) & \sin(\delta) \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{pmatrix} \quad (4.12)$$

$$= \begin{pmatrix} \sin(\delta)\cos(\alpha) & \sin(\delta)\sin(\alpha)\cos(\varphi) - \cos(\delta)\sin(\varphi) & -\sin(\delta)\sin(\alpha)\sin(\varphi) - \cos(\delta)\cos(\varphi) \\ -\sin(\alpha) & \cos(\alpha)\cos(\varphi) & -\cos(\alpha)\sin(\varphi) \\ \cos(\delta)\cos(\alpha) & \cos(\delta)\sin(\alpha)\sin(\varphi) + \sin(\delta)\sin(\varphi) & \cos(\delta)\sin(\alpha)\sin(\varphi) + \sin(\delta)\cos(\varphi) \end{pmatrix}$$

$$\begin{pmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{pmatrix} = \begin{pmatrix} \sin(\delta)\cos(\alpha) & \sin(\delta)\sin(\alpha)\cos(\varphi) - \cos(\delta)\sin(\varphi) & -\sin(\delta)\sin(\alpha)\sin(\varphi) - \cos(\delta)\cos(\varphi) \\ -\sin(\alpha) & \cos(\alpha)\cos(\varphi) & -\cos(\alpha)\sin(\varphi) \\ \cos(\delta)\cos(\alpha) & \cos(\delta)\sin(\alpha)\sin(\varphi) + \sin(\delta)\sin(\varphi) & \cos(\delta)\sin(\alpha)\sin(\varphi) + \sin(\delta)\cos(\varphi) \end{pmatrix} \begin{pmatrix} \hat{i}_2 \\ \hat{j}_2 \\ \hat{k}_2 \end{pmatrix} \quad (4.13)$$

The latest equation can be written in following way

$$\begin{pmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \begin{pmatrix} \hat{i}_2 \\ \hat{j}_2 \\ \hat{k}_2 \end{pmatrix} \quad (4.12)$$

5. Experiment

The absolute motion of the Earth may be presumed to be resultant of the three components. One of these \mathbf{v} is the Earth's orbital motion about the sun, the second component is the motion of the sun about the center of the Milky Way and the third one is the motion of our Galaxy regarding to other galaxies in the Universe. The sum of the second and third component will be marked by \mathbf{v}_0 .

An absolute earth velocity vector $\mathbf{V}(t)$ is given by equation:

$$\mathbf{V}(t) = \mathbf{v}(t) + \mathbf{v}_0 \quad (5.1)$$

During the period of one year we can assume that $\mathbf{v}(t)$ changes continuously in direction and magnitude, whereby vector \mathbf{v}_0 remains invariable. The coordinate system $S'(xyz)$ is moving relatively to the starlight by the velocity $\mathbf{W}(t)$. Here we assume a starlight as a straight line that is not moving along $S'z$ -axis in contrast to the photons who are moving along a starlight by constant velocity c . Plane $S'(xy)$ is rotating about Earth's axis so this relation is considered at the instant when plane $S'(xy)$ is perpendicular to the beams of photons, only. Similarly it may be presumed that velocity $\mathbf{W}(t)$ was resultant of the two components, the first one that is changing in direction and magnitude and second one that is invariable.

$$\mathbf{W}(t) = \mathbf{w}(t) + \mathbf{w}_0 \quad (5.2)$$

Our task is to find out a relation between the Equations (5.2) and (5.1) in other words to find out relation between the absolute velocity $\mathbf{V}(t)$ of the frame of the telescope and relative velocity $\mathbf{W}(t)$ of the frame of the telescope regarding to the starlight.

For now instead of the coordinate system $S'(xyz)$, plane $S'(xy)$ will be taken into the consideration, only.

$$\mathbf{W}_{xy}(t) = \text{proj}_{xy} \mathbf{W}(t) \quad (5.3)$$

$$\mathbf{w}_{xy}(t) = \text{proj}_{xy} \mathbf{w}(t) \tag{5.4}$$

$$\mathbf{w}_1 = \text{proj}_{xy} \mathbf{w}_0 \tag{5.5}$$

In that way Equation (5.2) is being reduced to the form

$$\mathbf{W}_{xy}(t) = \mathbf{w}_{xy}(t) + \mathbf{w}_1 \tag{5.6}$$

where $\mathbf{w}_{xy}(t)$ represents a normal projection of the velocity $\mathbf{w}(t)$ on the plane xy and \mathbf{w}_1 represent normal projection of the velocity \mathbf{w}_0 on the plane xy . One component of the vector $\mathbf{w}_{xy}(t)$ is a normal projection of the vector $\mathbf{v}(t)$ on the plane xy . A second component of the vector $\mathbf{w}_{xy}(t)$ eventually could be some vector $\mathbf{v}_z(t)$ that starlight has inherited from the orbital motion of the star (Z), if as a such exist.

$$\mathbf{w}_{xy}(t) = \text{proj}_{xy} \mathbf{v}(t) + \text{proj}_{xy} \mathbf{v}_z(t) \tag{5.7}$$

Let the (Z) and (Z') represents a pair of binary stars. For the simplicity, we can assume that they circle about a center of the mass by velocities $\mathbf{v}'_z(t)$ and $\mathbf{v}''_z(t)$ respectively, with the same magnitude but opposite directions. At the same time the center of the mass is moving by some constant uniform velocity marked by \mathbf{u}_0 regarding to the absolute frame of reference.

$$\mathbf{w}'_{xy}(t) = \text{proj}_{xy} \mathbf{v}(t) + \text{proj}_{xy} \mathbf{v}'_z(t) \tag{5.8}$$

$$\mathbf{w}''_{xy}(t) = \text{proj}_{xy} \mathbf{v}(t) + \text{proj}_{xy} \mathbf{v}''_z(t) \tag{5.9}$$

$$\mathbf{W}'_{xy}(t) = \mathbf{w}'_{xy}(t) + \mathbf{w}_1 \tag{5.10}$$

$$\mathbf{W}''_{xy}(t) = \mathbf{w}''_{xy}(t) + \mathbf{w}_1 \tag{5.11}$$

$$\mathbf{w}'_1 = \mathbf{w}''_1 = \mathbf{v}_0 - \mathbf{u}_0 \tag{5.12}$$

$$\mathbf{v}'_z(t) = -\mathbf{v}''_z(t) \tag{5.13}$$

Let the Z' and Z'' represent their images at the some instant T on the plane $S'(xy)$. Replacing A' by Z' and A'' by Z'' we have got a situation similar to the that one shown in **Figure 5**.

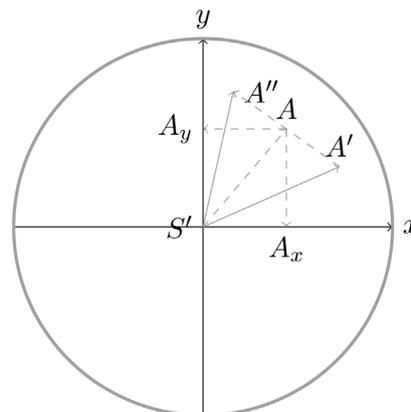


Figure 5. Two measurements that have been taken at the instants T_0 and T_1 six months apart.

$$\mathbf{S}'\mathbf{Z}' = -\Delta t * \mathbf{W}'_{xy}(T) \quad (5.14)$$

$$\mathbf{S}'\mathbf{Z}'' = -\Delta t * \mathbf{W}''_{xy}(T) \quad (5.15)$$

$$\mathbf{Z}'\mathbf{Z}'' = -\Delta t * (\mathbf{W}''_{xy}(T) - \mathbf{W}'_{xy}(T)) \quad (5.16)$$

$$\mathbf{Z}'\mathbf{Z}'' = -2 * \Delta t * \text{proj}_{xy} \mathbf{v}'_z(T) \quad (5.17)$$

$$\tan(\Delta\theta) = \frac{|\mathbf{Z}'\mathbf{Z}''|}{SS'} = \frac{2 * |\text{proj}_{xy} \mathbf{v}'_z(T)| * \Delta t}{c * \Delta t} \quad (5.18)$$

$$\Delta\theta \approx \frac{2 * |\text{proj}_{xy} \mathbf{v}'_z(T)|}{c} \quad (5.19)$$

What is not true. Because it has never been observed any major aberration between the two binary stars. That means that variable component of the vector $\mathbf{w}_{xy}(t)$ depends on the $\mathbf{v}(t)$ earth velocity about the sun only. The Equation (5.7) has to be reduced to the following form

$$\mathbf{w}_{xy}(t) = \text{proj}_{xy} \mathbf{v}(t) \quad (5.20)$$

In that way it has been proved that the first component of the vectors $\mathbf{V}(t)$ and $\mathbf{W}(t)$ are identical and equal to the $\mathbf{v}(t)$. Still it is has been left out to find out a relation between the second invariant components \mathbf{v}_0 and \mathbf{w}_0 of the vectors $\mathbf{V}(t)$ and $\mathbf{W}(t)$ respectively.

Suppose that we are taking measurements during one year period. For any two observations that have been taken at the instant T_0 and T_1 six months apart we will have situation shown in the **Figure 5**. The point A' corresponds to the first and the point A'' to the second measurement.

Referring to **Figure 5** it follows

$$\Delta t \approx \frac{d}{c} \quad (5.21)$$

$$\mathbf{S}'\mathbf{A}'' = -\Delta t * \mathbf{W}_{xy}(T_1) = -\Delta t * (\mathbf{w}_{xy}(T_1) + \mathbf{w}_1) \quad (5.22)$$

$$\mathbf{S}'\mathbf{A}' = -\Delta t * \mathbf{W}_{xy}(T_0) = -\Delta t * (\mathbf{w}_{xy}(T_0) + \mathbf{w}_1) \quad (5.23)$$

$$\mathbf{A}'\mathbf{A}'' = -\Delta t * (\mathbf{w}_{xy}(T_1) - \mathbf{w}_{xy}(T_0)) \quad (5.24)$$

$$\mathbf{A}'\mathbf{A}'' = -\Delta t * (\text{proj}_{xy} \mathbf{v}(T_1) - \text{proj}_{xy} \mathbf{v}(T_0)) \quad (5.25)$$

Times T_0 and T_1 are chosen in a such way that $\mathbf{v}(T_0)$ and $\mathbf{v}(T_1)$ have approximately the same magnitude but opposite directions. Equation (5.25) can be written in the form

$$\mathbf{A}'\mathbf{A}'' \approx -2 * \Delta t * \text{proj}_{xy} \mathbf{v}(T_1) \quad (5.26)$$

In that way it has been proved that $\mathbf{A}'\mathbf{A}''$ depends on the earth velocity about the sun, only.

$$\mathbf{S}'\mathbf{A} = \frac{\mathbf{S}'\mathbf{A}' + \mathbf{S}'\mathbf{A}''}{2} = -\Delta t * \frac{\mathbf{w}_{xy}(T_0) + \mathbf{w}_{xy}(T_1)}{2} - \Delta t * \mathbf{w}_1 \quad (5.27)$$

$$\mathbf{S}'\mathbf{A} = -\frac{\Delta t * (\text{proj}_{xy} \mathbf{v}(T_1) + \text{proj}_{xy} \mathbf{v}(T_0))}{2} - \Delta t * \mathbf{w}_1 \quad (5.28)$$

$$w_1 = -\frac{S'A}{\Delta t} = -\frac{S'A * c}{d} \tag{5.29}$$

6. Transformation of the Vector $v(t)$ from the Ecliptic Plane to the Frame of Telescope

Vector $w(t)$ the variable component of the vector $W(t)$ is equal to the vector $v(t)$ Earth's orbital velocity about the sun.

$$w(t) = v(t) \tag{6.1}$$

With the respect to the basis $(\hat{i}_2, \hat{j}_2, \hat{k}_2)$ vector $v(t)$ is given by the equation:

$$v(t) = v_x(t) * \hat{i}_2 + v_y(t) * \hat{j}_2 \tag{6.2}$$

Magnitude of the vector $v(t)$ would hereafter be marked by $v(t)$.

Using the matrix of transformation C we will transform vector $v(t)$ from the basis $(\hat{i}_2, \hat{j}_2, \hat{k}_2)$ to the basis $(\hat{i}, \hat{j}, \hat{k})$. That transformed vector is noted by $V(t)$.

$$V(t)_{[\hat{i}, \hat{j}, \hat{k}]} = v(t)_{[\hat{i}_2, \hat{j}_2, \hat{k}_2]} \tag{6.3}$$

$$V(t) = V_x(t) * \hat{i} + V_y(t) * \hat{j} + V_z(t) * \hat{k} \tag{6.4}$$

$$V(t) = \begin{pmatrix} V_x(t) \\ V_y(t) \\ V_z(t) \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} * \begin{pmatrix} v_x(t) \\ v_y(t) \\ 0 \end{pmatrix} = \begin{pmatrix} c_{11} * v_x(t) + c_{12} * v_y(t) \\ c_{21} * v_x(t) + c_{22} * v_y(t) \\ c_{31} * v_x(t) + c_{32} * v_y(t) \end{pmatrix} \tag{6.5}$$

$$V_x(t) = v_x(t) * c_{11} + v_y(t) * c_{12} \tag{6.6}$$

$$V_y(t) = v_x(t) * c_{21} + v_y(t) * c_{22} \tag{6.7}$$

$$V_z(t) = v_x(t) * c_{31} + v_y(t) * c_{32} \tag{6.8}$$

$$V(t)^2 = v(t)^2 = V_x^2(t) + V_y^2(t) + V_z^2(t) \tag{6.9}$$

$$V_{xy}(t) = V_x(t) + V_y(t) = V_x(t) * \hat{i} + V_y(t) * \hat{j} \tag{6.10}$$

$$V_{xy}(t) = \sqrt{V_x^2(t) + V_y^2(t)} = \sqrt{v(t)^2 - V_z(t)^2} \tag{6.11}$$

In the special case when $V_z(t) = 0$, in other words when the vector $V(t)$ (Earth velocity about the sun) is perpendicular to the $S'z$ -axis (direction of the starlight) it follows that

$$V_z(t) = v_x(t) * c_{31} + v_y(t) * c_{32} = 0 \tag{6.12}$$

$$\frac{v_y(t)}{v_x(t)} = -\frac{c_{31}}{c_{32}} \tag{6.13}$$

$$\beta = \arctan\left(\frac{\cos(\delta)\cos(\alpha)}{\cos(\delta)\sin(\alpha)\sin(\varphi) + \sin(\delta)\sin(\varphi)}\right) \tag{6.14}$$

where β represents an angle between Ox'' -axis (vernal equinox direction) and vector $v(t)$. Obviously there are two instants during the year when this happens.

Let these instants are marked by t_0 and t_2 . Beside these two we can define new times t_1 and t_3 in that way that a difference between t_1 and t_0 and difference between t_3 and t_2 are approximately equal to three months. We must keep in mind that corresponding sidereal times for the instants t_0, t_1, t_2, t_3 are the same and equal to the s_0 . In this way we can make sure that at least two of these four instants fall in the nighttime.

Note that vectors $\mathbf{v}(t_0)$ and $\mathbf{v}(t_1)$ are parallel to the line (n) as shown in **Figure 4**.

7. Analysis

Now we will analyze the vector \mathbf{w}_1 normal projection of the vector \mathbf{w}_0 on the plane xy .

Depending on the vector $S'A$ (5.29) there are two possibilities:

1) ($S'A \neq 0$)

In this case we assume that measurements have been taken for the different stars and at least in one case $S'A \neq 0$.

We can claim that the outcome of the experiment is positive, because some stellar aberration different from the Bradley's stellar aberration has been detected.

Let, with the respect to the basis $(\hat{i}, \hat{j}, \hat{k})$ an invariable component \mathbf{w}_0 of the vector $W(t)$ is given by the equation:

$$\mathbf{w}_0 = U_x * \hat{i} + U_y * \hat{j} + U_z * \hat{k} \tag{7.1}$$

Referring to **Figure 6** we obtain

$$\Delta t = \frac{d}{c - (U_z + V_z)} \tag{7.2}$$

$$S'A'_x = S'A_x + A_x A'_x = \Delta t * (U_x + V_x) \tag{7.3}$$

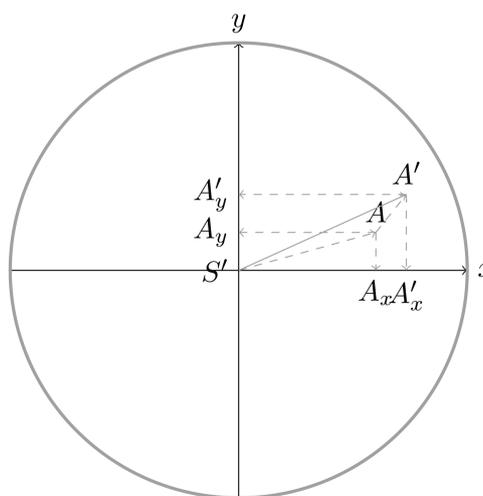


Figure 6. Bottom plane of the telescope, where A' represents image of the star at the instant $T_0(T_1)$.

$$(U_x + V_x) * \Delta t = S'A'_x \quad (7.4)$$

$$a = S'A'_x \quad (7.5)$$

$$b = S'A'_y \quad (7.6)$$

$$U_x + V_x = \frac{c - (U_z + V_z)}{d} * a \quad (7.7)$$

$$U_x - \frac{a}{d} * (c - U_z) = -\frac{V_z}{d} * a - V_x \quad (7.8)$$

Analogously we can get following expression

$$U_y - \frac{b}{d} * (c - U_z) = -\frac{V_z}{d} * b - V_{y,x} \quad (7.9)$$

Let suppose that the two measurements have been made at the times T_0 and T_1 . A difference between the times T_0 and T_1 is equal to the six (or the three) months.

$$U_x - \frac{a(T_1)}{d} * (c - U_z) = -\frac{V_z(T_1)}{d} * a(T_1) - V_x(T_1) \quad (7.10)$$

$$U_x - \frac{a(T_0)}{d} * (c - U_z) = -\frac{V_z(T_0)}{d} * a(T_0) - V_x(T_0) \quad (7.11)$$

We get the linear system of two equations in two unknowns U_x and $(c - U_z)$. The solution is given by expression

$$U_x = \frac{a(T_1) * a(T_0) * (V_z(T_0) - V_z(T_1)) + d * (a(T_0) * V_x(T_1) - a(T_1) * V_x(T_0))}{d * (a(T_1) - a(T_0))} \quad (7.12)$$

$$(a(T_1) \neq a(T_0))$$

$$c - U_z = \frac{a(T_0) * V_z(T_0) - a(T_1) * V_z(T_1) + d * (V_x(T_1) - V_x(T_0))}{a(T_1) - a(T_0)} \quad (7.13)$$

$$U_z = c - \frac{a(T_0) * V_z(T_0) - a(T_1) * V_z(T_1) + d * (V_x(T_1) - V_x(T_0))}{a(T_1) - a(T_0)} \quad (7.14)$$

Analogously to the Equation (7.12) we can get value for the component U_y .

$$U_y = \frac{b(T_1) * b(T_0) * (V_z(T_0) - V_z(T_1)) + d * (b(T_0) * V_y(T_1) - b(T_1) * V_y(T_0))}{d * (b(T_1) - b(T_0))} \quad (7.15)$$

$$(b(T_1) \neq b(T_0))$$

The vector \mathbf{w}_0 is given by the following equation

$$\mathbf{w}_0(\alpha, \delta) = \mathbf{U}(\alpha, \delta) = U_x(\alpha, \delta) * \hat{\mathbf{i}} + U_y(\alpha, \delta) * \hat{\mathbf{j}} + U_z(\alpha, \delta) * \hat{\mathbf{k}} \quad (7.16)$$

Using the transformation matrix A^T vector $\mathbf{U}(\alpha, \delta)$ will be transformed from the basis $(\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}})$ to the basis $(\hat{\mathbf{i}}_1, \hat{\mathbf{j}}_1, \hat{\mathbf{k}}_1)$. The transformed vector is noted by $\mathbf{u}(\alpha, \delta)$.

$$\mathbf{u}(\alpha, \delta)_{[i, j, k]} = \mathbf{U}(\alpha, \delta)_{[i, j, k]} \quad (7.17)$$

$$\mathbf{u}(\alpha, \delta) = u_x(\alpha, \delta) * \hat{\mathbf{i}}_1 + u_y(\alpha, \delta) * \hat{\mathbf{j}}_1 + u_z(\alpha, \delta) * \hat{\mathbf{k}}_1 \quad (7.18)$$

$$\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = \begin{pmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \\ a'_{31} & a'_{32} & a'_{33} \end{pmatrix} * \begin{pmatrix} U_x \\ U_y \\ U_z \end{pmatrix} = \begin{pmatrix} a'_{11} * U_x + a'_{12} * U_y + a'_{13} * U_z \\ a'_{21} * U_x + a'_{22} * U_y + a'_{23} * U_z \\ a'_{31} * U_x + a'_{32} * U_y + a'_{33} * U_z \end{pmatrix} \quad (7.19)$$

In that way a vector $\mathbf{w}_0 = (U_x(\alpha, \delta), U_y(\alpha, \delta), U_z(\alpha, \delta))$ from the coordinate system $S'xyz$ (the frame of telescope) has been transformed to the vector $\mathbf{u}(\alpha, \delta)$ at the coordinate system $Ox'y'z'$ (frame of the Earth).

Suppose that the measurements have been made for n stars.

There are two possibilities:

a) Vector $\mathbf{u}(\alpha, \delta)$ has a constant value.

By this we mean that $\mathbf{u}(\alpha_i, \delta_i) = \mathbf{u}(\alpha_j, \delta_j)$, for each $(i, j < n + 1)$.

The velocity \mathbf{w}_0 given by (5.2) is equal to the velocity \mathbf{v}_0 given by the Equation (5.1). We can make conclusion that the starlight represents absolute stationary frame and the velocity at which the Earth moves relative to starlight depends on Earth absolute velocity and an angle between the ecliptic and starlight.

By the time, because of the star movement through the space an angle between the ecliptic plane and the starlight will change. In long run it will affect stellar aberration but the velocity of the star is irrelevant for stellar aberration instant measuring.

If we choose any two different stars Z and Z' we will get

$$(\mathbf{w}_0)^2 = U_x(Z)^2 + U_y(Z)^2 + U_z(Z)^2 = U_x(Z')^2 + U_y(Z')^2 + U_z(Z')^2 \quad (7.20)$$

$$(\mathbf{w}_0)^2 = U_x(Z)^2 + U_y(Z)^2 + (c - R(Z))^2 = U_x(Z')^2 + U_y(Z')^2 + (c - R(Z'))^2 \quad (7.21)$$

$$c = \frac{(U_x(Z')^2 + U_y(Z')^2 + R(Z')^2) - (U_x(Z)^2 + U_y(Z)^2 + R(Z)^2)}{2 * (R(Z') - R(Z))} \quad (7.22)$$

$(R(Z') \neq R(Z))$

In this case for the each pair of stars (Z_i, Z_j) we are able to determine speed of light $c_{i,j}$ and compare it to the already known speed of light c . Beside that the constancy of the one way speed of light can be tested.

b) Vector $\mathbf{u}(\alpha, \delta)$ does not have a constant value.

By this we mean that $\mathbf{u}(\alpha_i, \delta_i) \neq \mathbf{u}(\alpha_j, \delta_j)$, for some $(i, j < n + 1)$.

In this case we are not able to determine the absolute velocity nor the speed of light. Instead of that one can derive relative velocity at which the telescope (the Earth) moves regarding to the starlight only.

2) ($S'A = 0$)

We assume that measurements have been taken for the different stars and in each case $S'A = 0$.

This means that vector $\mathbf{W}_{xy}(t)$ does not depend on the vector \mathbf{w}_1 , in other

words we can say that the sun is stationary regarding to starlight and the only movement that can be detected is Earth's revolution about the Sun. In this case we can say that the instantaneous effect of the aberration due to solar movement through the space is not directly measurable.

The Equation (5.6) could be rewritten in the following form

$$\mathbf{W}_{xy}(t) = \vec{\mathbf{w}}_{xy}(t) = \text{proj}_{xy} \mathbf{v}(t) \quad (7.23)$$

$$S'A' = \frac{d}{c - V_z} * V_{xy} \quad [\text{Equation (2.14)}] \quad (7.24)$$

$$\tan(\Delta\theta) = \frac{S'A'}{SS'} = \frac{V_{xy}}{c - V_z} \quad (7.25)$$

$$\Delta\theta \approx \frac{V_{xy}}{c - V_z} \quad (7.26)$$

$$c = \frac{d * V_{xy}}{S'A'} + V_z \quad (7.27)$$

Equation (7.26) represents modified Bradley's equation for stellar aberration. This equation can be written in a different form. First let us define:

- Φ an angle between the plane $S'xy$ and vector $\mathbf{V}(t)$

$$\Phi' = \arccos\left(\frac{V_z(t)}{V(t)}\right) = \arccos\left(\frac{v_x(t) * c_{31} + v_y(t) * c_{32}}{v(t)}\right) \quad (7.28)$$

$$\Phi = \Pi/2 - \Phi' \quad (7.29)$$

Referring to (6.9) we obtain that

$$V_{xy}(t) = V(t) * \cos(\Phi) = v(t) * \cos(\Phi) \quad (7.30)$$

$$V_z(t) = V(t) * \sin(\Phi) = v(t) * \sin(\Phi) \quad (7.31)$$

$$\tan(\Delta\theta) = \frac{V_{xy}}{c - V_z} = \frac{v(t) * \cos(\Phi)}{c - v(t) * \sin(\Phi)} = \frac{v(t) * \cos(\Phi)}{c * \left(1 - \frac{v(t) * \sin(\Phi)}{c}\right)} \quad (7.32)$$

$$= \frac{v(t) * \cos(\Phi)}{c} * \left(1 + \frac{v(t) * \sin(\Phi)}{c} + \left(\frac{v(t) * \sin(\Phi)}{c}\right)^2 + \dots\right)$$

$$\tan(\Delta\theta) \approx \frac{v(t) * \cos(\Phi)}{c} + \frac{v(t)^2 * \cos(\Phi) \sin(\Phi)}{c^2} \quad (7.33)$$

And finally the annual aberration $\Delta\theta$ as a function of time is given by the equation

$$\Delta\theta(t) \approx \frac{v(t) * \cos(\Phi(t))}{c} + \frac{v(t)^2 * \sin(2\Phi(t))}{2 * c^2} \quad (7.34)$$

We must declare the experiment failed, and the definition of the "absolute" stellar aberration must be discarded, because aberration as such doesn't exist.

3) Stellar aberration in case when $S'A \neq 0$.

In this section we will find formulas for Bradley's, "absolute" and "total" stel-

lar aberration.

$$\Delta t = \frac{d}{c - U_z - V_z} \quad (7.36)$$

$$S'A = \Delta t * U_{xy} \quad (7.37)$$

$$AA' = \Delta t * v_{xy} \quad (7.38)$$

$$SS' = d \quad (7.39)$$

$$\Delta\theta = \angle(SA, SA') \quad (7.40)$$

$$\beta = \angle(AA', AB) \quad (7.41)$$

$$\gamma = \angle(AA', AS) \quad (7.42)$$

$$\Delta\varepsilon = \angle(SS', SA) \quad (7.43)$$

$$\Delta\tau = \angle(SS', SA') \quad (7.44)$$

$$AB = A'A * \cos(\beta) \quad (7.45)$$

$$A'B = A'A * \sin(\beta) \quad (7.46)$$

First we will derive a formula for the classical Bradley's aberration. In other words our task is to find an angle $\Delta\theta$ as shown in **Figure 7**.

$$A'S^2 = S'S^2 + S'B^2 + A'B^2 \quad (7.47)$$

$$AS^2 = S'S^2 + S'A^2 = S'S^2 + (S'B + BA)^2 = d^2 + S'B^2 + AB^2 + 2 * S'B * AB \quad (7.48)$$

$$A'A^2 = AB^2 + A'B^2 \quad (7.49)$$

$$A'S^2 = A'A^2 + AS^2 - 2 * AS * A'A * \cos(\gamma) \quad (7.50)$$

$$\begin{aligned} \cos(\gamma) &= \frac{A'A^2 + AS^2 - A'S^2}{2 * AS * A'A} \\ &= \frac{A'A^2 + d^2 + S'B^2 + AB^2 + 2 * S'B * AB - d^2 - S'B^2 - A'B^2}{2 * AS * A'A} \end{aligned} \quad (7.51)$$

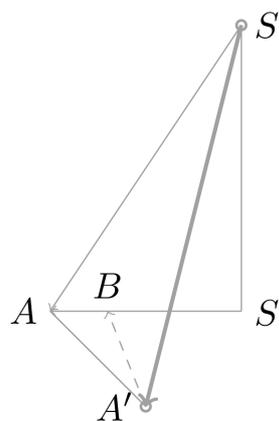


Figure 7. An angle $\angle(SS', SA')$ represents a “total” aberration.

$$\cos(\gamma) = \frac{AB^2 + S'B * AB}{A'A * A'S} = \frac{AB}{A'A} * \frac{AB + BS'}{AS} = \cos(\beta) * \frac{S'A}{AS} \tag{7.52}$$

$$\sin(\gamma) = \sqrt{1 - \cos^2(\gamma)} \approx 1 - \frac{\cos^2(\gamma)}{2} \quad (\cos(\gamma) \ll 1) \tag{7.53}$$

using the law sines for the $\Delta(AA'S)$ it follows that

$$\frac{A'A}{\sin(\Delta\theta)} = \frac{A'S}{\sin(\gamma)} \tag{7.54}$$

$$\sin(\Delta\theta) = \frac{A'A * \sin(\gamma)}{A'S} \approx \frac{A'A}{A'S} * \left(1 - \frac{\cos^2(\gamma)}{2}\right) = \frac{A'A}{A'S} - \frac{A'A}{A'S} * \frac{S'A^2}{AS^2} * \frac{\cos^2(\beta)}{2} \tag{7.55}$$

$$\left| \frac{A'A}{A'S} * \frac{S'A^2}{AS^2} * \frac{\cos^2(\beta)}{2} \right| < \left| \frac{A'A}{S'S} * \frac{S'A^2}{S'S^2} \right| \leq \frac{(v_{xy} * \Delta t) * (U_{xy} * \Delta t)^2}{((c - U_z - V_z) * \Delta t)^3} \leq \frac{v * U^2}{(c - U_z - V_z)^3} \tag{7.56}$$

$$\sin(\Delta\theta) \approx \frac{A'A}{A'S} \tag{7.57}$$

$$\frac{A'S^2}{(\Delta t)^2} = (c - U_z - V_z)^2 + (U_{xy} - V_{xy} * \cos(\beta))^2 + (V_{xy} * \sin(\beta))^2 \tag{7.58}$$

$$\frac{A'S^2}{(\Delta t)^2} = c^2 + U^2 + v^2 - 2 * c * U_z - 2 * c * V_z + 2 * U_z * V_z - 2 * U_{xy} * V_{xy} * \cos(\beta) \tag{7.59}$$

$$\frac{A'A}{A'S} = \frac{V_{xy}}{\sqrt{c^2 + U^2 + v^2}} * \frac{1}{\sqrt{1 - 2 * \frac{c * U_z + c * V_z - U_z * V_z + U_{xy} * V_{xy} * \cos(\beta)}{c^2 + U^2 + v^2}}} \tag{7.60}$$

$$\frac{A'A}{A'S} \approx \frac{V_{xy}}{\sqrt{c^2 + U^2 + v^2}} * \left(1 + \frac{c * U_z + c * V_z - U_z * V_z + U_{xy} * V_{xy} * \cos(\beta)}{c^2 + U^2 + v^2}\right) \tag{7.61}$$

$$\frac{A'A}{A'S} \approx \frac{V_{xy}}{\sqrt{c^2 + U^2 + v^2}} + \frac{c * U_z * V_{xy} + c * V_z * V_{xy}}{\sqrt{(c^2 + U^2 + v^2)^3}} \tag{7.62}$$

$$\Delta\theta(t) \approx \frac{V_{xy}(t)}{\sqrt{c^2 + U^2(\alpha, \delta) + v^2(t)}} + \frac{c * U_z(\alpha, \delta) * V_{xy}(t) + c * V_z(t) * V_{xy}(t)}{\sqrt{(c^2 + U^2(\alpha, \delta) + v^2(t))^3}} \tag{7.63}$$

in the special case for $V_z(T) = 0$ we obtain

$$\Delta\theta(T) \approx \frac{v(T)}{\sqrt{c^2 + U^2(\alpha, \delta) + v^2(T)}} + \frac{c * U_z(\alpha, \delta) * v(T)}{\sqrt{(c^2 + U^2(\alpha, \delta) + v^2(T))^3}} \tag{7.64}$$

Now we will derive a formula for “absolute” stellar aberration that will be noted by $\Delta\varepsilon$

$$\tan(\Delta\varepsilon) = \frac{S'A}{SS'} = \frac{U_{xy} * \Delta t}{d} = \frac{U_{xy} * d}{d * (c - U_z - V_z)} = \frac{U_{xy}}{c - U_z - V_z} \tag{7.65}$$

$$\Delta\varepsilon(t, \alpha, \delta) \approx \frac{U_{xy}(\alpha, \delta)}{c - U_z(\alpha, \delta) - V_z(t)} \quad (7.66)$$

for $V_z(T) = 0$ we obtain that

$$\Delta\varepsilon(\alpha, \delta) \approx \frac{U_{xy}(\alpha, \delta)}{c - U_z(\alpha, \delta)} \quad (7.67)$$

and finally we can get a formula for “total” stellar aberration that will be noted by $\Delta\tau$

$$\tan(\Delta\tau) = \frac{S'A'}{SS'} \quad (7.68)$$

$$\begin{aligned} \left(\frac{S'A'}{\Delta t}\right)^2 &= (V_{xy} * \sin(\beta))^2 + (U_{xy} - V_{xy} * \cos(\beta))^2 \\ &= U_{xy}^2 + V_{xy}^2 - 2 * U_{xy} * V_{xy} * \cos(\beta) \end{aligned} \quad (7.69)$$

$$S'A' = \sqrt{U_{xy}^2 + V_{xy}^2 - 2 * U_{xy} * V_{xy} * \cos(\beta)} * \Delta t \quad (7.70)$$

$$\frac{S'A'}{SS'} = \frac{\sqrt{U_{xy}^2 + V_{xy}^2 - 2 * U_{xy} * V_{xy} * \cos(\beta)}}{c - U_z - V_z} \quad (7.71)$$

$$\Delta\tau(t, \alpha, \delta) \approx \frac{\sqrt{U_{xy}^2(\alpha, \delta) + V_{xy}^2(t) - 2 * U_{xy}(\alpha, \delta) * V_{xy}(t) * \cos(\beta)}}{c - U_z(\alpha, \delta) - V_z(t)} \quad (7.72)$$

Obviously in these formulas have not been taken into account Earth’s rotation on its axis nor the changes in axial precession.

8. Discussion

An annual stellar aberration is related to the orbital revolution of the Earth about the sun. It is already known that a secular or galactic aberration is related to the changes in the movement of the solar system inside the Galaxy [3]. Our assumption is that there exists stellar aberration due to the Galaxy’s movement through the space.

By means of Bradley’s experiment it is possible to measure stellar aberration only if the velocity of the telescope frame is changing. The goal of the experiment presented in this paper is to overcome these limitations and measure a “total” stellar aberration caused by the uniform velocity of the solar system regarding the frame (L).

It has been experimentally proved that a circular motion of a star about some other star does not affect stellar aberration; consequently we can make assumption that all other kinds of the movements that are attributed to the star do not affect stellar aberration.

In that way we can finally predict that outcome of the experiment should be with the accordance to the case (1.a) from the precedent section. If it is not so then at least one of the propositions P_1 or P_2 is not valid or our methodology is wrong.

This experiment can be repeated, but instead of extra-galactic stars this time we can observe the stars in the Milky Way Galaxy and make comparison between the results that are obtained from the two experiments.

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