The spinorial Dirac operator

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Abstract

We define a Dirac type operators called the spinorial Dirac operator

1 The Dirac operator

Let (M, g) be a spin manifold, then we can define the Dirac operator D with help of the Levi-Civita connection ∇ [F].

$$D(\psi) = \sum_i e_i.\nabla_{e_i}(\psi)$$

(e_i) is an orthonormal basis.

$$D = \mu \circ \nabla$$

with μ the Clifford multiplication.

2 The spinorial Clifford algebra

We define the spinorial Clifford algebra as the free algebra over the space of spinors with relations:

$$\psi.\psi' + \psi'.\psi = 2g(\psi, \psi')$$

$$(X.\psi).\psi' = -\psi.(X.\psi')$$

with X a vector and ψ, ψ' two spinors. We have:

$$\psi^{-1} = \frac{\psi}{||\psi||^2}$$

The double spinorial group $Spin_2(n)$ is defined as the products of an even number of spinors of unit norm.

$$Spin_2(n) = \{\prod_i \psi_i \psi'_i, ||\psi_i|| = ||\psi'_i|| = 1\}$$

3 The spinorial derivations

A spinor is a derivation of the space of smooth functions by the formula:

$$\psi(f) = (df)^*.\psi$$

4 The spinorial connection

A spinorial connection can be defined for a module over the spinorial Clifford algebra:

$$\nabla_\psi(fs) = \psi(f).s + f\nabla_\psi(s)$$
$$\nabla_{f\psi}(s) = f\nabla_\psi(s)$$

5 The spinorial Dirac operator

The spinorial Dirac operator is defined by the formula:

$$\mathcal{D}_\psi = \sum_i \psi_i.\nabla_{\psi_i}$$

with the ψ_i an orthogonal basis of the spinors.

References

[F] T.Friedrich, "Dirac Operators in Riemannian Geometry", vol 25, AMS, 2000.

[GHL] S.Gallot, D.Hulin, J.Lafontaine, "Riemannian geometry", 3ed., Springer, Berlin, 2004.