The Laws of Interaction of Currents and Charges and the Determination of Magnetic Field

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Different versions of the interaction of current elements and moving charges, written in vector form, starting from the Ampère theory, are considered. It is shown that from the point of view of the classical two-body problem the most adequate is the Gauss-Grassmann-Neumann theory. An expression is obtained for magnetic induction created by a moving current element, which, in addition to usual Biot-Savart-Laplace law, includes additional terms due to both the electron spin and the mutual displacement of elements.

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1. Introduction

The problem of the interaction of moving objects is one of the main issues in physics. Despite its long history, it still does not have a final, or better, more or less acceptable solution. The currently known gravitational, electromagnetic, strong and weak interactions are described by various theories, attempts to combine which are carried out in numerous theoretical schemes. In our opinion, all these interactions have a unified nature, with what most physicists agree, and one law, which accordingly manifests itself at different scales of distances, should describe them.

Since, from the generally accepted point of view, all matter consists of positively charged, negatively charged, and neutral particles, then, above all, this law should be established for the interaction of stable elementary particles such as an electron and a proton. In classical mechanics and electrodynamics, the interaction is considered to be known, if we know the forces acting on the physical objects and determining the change of the dynamic momentum with time. Therefore, the interaction between two moving stable particles should be determined by the force acting from one particle to another. If such a system is in equilibrium, then, apparently, Newton's third law should be satisfied, which may be violated for a nonequilibrium system.

In electrodynamics of Maxwell, who considered a magnetic field as a collection of vortices in an incompressible ethereal fluid, the force acting on a moving charged particle consists of the electric and magnetic Lorentz forces

$$\mathbf{F} = e\mathbf{E} + e[\mathbf{v}\times\mathbf{B}], \tag{1.1}$$

determined by the electric field strength,

$$\mathbf{E} = -\frac{\partial\varphi}{\partial\mathbf{R}} - \frac{\partial\mathbf{A}}{\partial t}, \tag{1.2}$$

and magnetic induction,

$$\mathbf{B} = \mathrm{rot}\,\mathbf{A}, \tag{1.3}$$

where φ and $\mathbf{A}$ are scalar and vector potentials, depending only on the coordinates and time. The values of fields (1.2) and (1.3) should be taken at the point where the charge e is located. Thus, expression (1.1) is essentially applicable only for point charges.[1])

Four Maxwell equations, including the electric field (1.2) created by the distribution of charges, and the magnetic field (1.3) created by the distribution of currents, are usually used in well-known classical electron theories. The first two equations are actually equations for electric and magnetic forces, and the next two ones, expressed by the Gauss theorem, are the conditions that these forces should satisfy, and, in essence, they hold for stationary distributions of charges and currents. Spin of charge carriers does not take into account in any way. Taking account of

[1]) More precisely, Maxwell is talking about the density of force acting on an object in which a charge is distributed with a certain density.

spin, as well as possible dependence of electric and magnetic forces on the state of motion of charge carriers, should inevitably lead to equations that may differ from the standard Maxwell equations. These new equations must take into account not only the nature of motion, but also the structure of charge carriers. In order to construct such equations, it is necessary to redefine the concepts of electric and magnetic field strengths within Maxwell equations, and set the ratio between them.

It was shown in [1]-[4] that electric charge should not be considered as a physical quantity characterizing the electromagnetic interaction, but as a consequence of the presence of a spin of a particle. Then the electromagnetic interaction can be interpreted as the interaction of the spin with an external field. If we abandon the concept of charge and use instead the spin polarization (helicity) of the particle, then in (1.1) we should put either $e = +1$ for positively charged particles, and $e = -1$ for negatively charged particles, or $e = +1$ for any particles. If in the first case the helicity *e* (or charge) is strictly fixed (the tangential component of the spin $s_\tau = es$ for free particles, [6]; the binormal component of the spin $s_\mathrm{b} = es$ for particles in a constant uniform magnetic field, $\mathbf{B} = B_\mathrm{b}\mathbf{e}_\mathrm{b}$, where $\mathbf{e}_\mathrm{b}$ is the binormal vector, [7]), then the second case admits the existence of solutions in which the spin polarization is different from ± 1 (for example, in the case of particles in a constant uniform electric field, [8]). It follows that the classical concept of charge is strictly defined only in the case of freely moving particles.

It was shown in [6] that free spinning particle moves along a helical trajectory, which can be interpreted as *Zitterbewegung*. If such a particle creates an electromagnetic field, then at each point in surrounding space the values of components of this field will periodically change in accordance with particle motion, which can be considered as passing through this point of electromagnetic wave. Thus, the quantum concepts of *Zitterbewegung* and the wave-particle dualism get interpretation from the classical point of view.

In classical electrodynamics, the law of interaction between moving charges is usually obtained from the interaction force of two current elements (*rheophores* in Ampère terminology), considered as a flux of charge carriers. Dividing this force by the number of charge carriers in both current elements, we obtain an expression for *the interaction force between moving charges*. However, the force thus obtained will in fact be *the average force per pair of interacting charged particles*.

There exist several variants of deriving this force. The first variant, as is known, was proposed by Ampère in 1820, who proceeded from four experimentally established cases of equilibrium [2]) and the assumption that this force acts along the line connecting the rheophores ([9]; [10], pp. 323, 374). Let hereinafter $\mathbf{R}_1$, $\mathbf{R}_2$ denote the coordinates of the elements $\delta\mathbf{R}_1$, $\delta\mathbf{R}_2$, along which currents with intensities $i_1 = i$ and $i_2 = i'$ flow, respectively, $\mathbf{r} \equiv \mathbf{r}_{21} = \mathbf{R}_2 - \mathbf{R}_1$ is the radius vector drawn from the first rheophore to the second one, $r = |\mathbf{r}_{21}| = |\mathbf{R}_2 - \mathbf{R}_1|$ is the distance between them. Then, if we represent an infinitesimal force acting on the second element by the first element in the form

$$d^2\mathbf{F}_{21} = i_1 i_2 d^2\mathbf{f}_{21}, \tag{1.4}$$

where $\mathbf{f}_{21}$ is a dimensionless vector function, then the formula obtained by Ampère will look as follows

[2]) 1) The effect of a current is reversed when the direction of the current is reversed. 2) The effect of a current flowing in a circuit twisted into small sinuosities is the same as if the circuit were smoothed out. 3) The force exerted by a closed circuit on an element of another circuit is at right angles to the latter. 4) The force between two elements of circuits is unaffected when all linear dimensions are increased proportionately, the current-strengths remaining unaltered ([11], p. 85).

$$d^2\mathbf{f}_{21}^{\text{Ampère}} = \frac{1}{r^3}\left[(\delta\mathbf{R}_1\cdot\delta\mathbf{R}_2) - \frac{3}{2r^2}(\mathbf{r}\cdot\delta\mathbf{R}_1)(\mathbf{r}\cdot\delta\mathbf{R}_2)\right]\mathbf{r} = -d^2\mathbf{f}_{12}^{\text{Ampère}}, \tag{1.5}$$

which means that the second element repels from the first, if $i_1 i_2 > 0$.

In 1833, Gauss derived a different expression, but it became known only in 1867 after his death. Instead of currents $i = i_1$ and $i' = i_2$ flowing in rheophores $\delta\mathbf{R}_1$ and $\delta\mathbf{R}_2$, Gauss used elements of electricity $e = e_1$ and $e' = e_2$, located in them and moving with velocities $\mathbf{V}_1 = \delta\mathbf{R}_1 / dt$ and $\mathbf{V}_2 = \delta\mathbf{R}_2 / dt$, respectively, so that relations $i_{1,2}\delta\mathbf{R}_{1,2} = e_{1,2}\mathbf{V}_{1,2}$, $i_{1,2} = e_{1,2} / dt$ are valid. An action of the first element on the second one is expressed by a dimensionless vector function

$$d^2\mathbf{f}_{21}^{\text{Gauss}} = \frac{1}{r^3}[\delta\mathbf{R}_2\times[\delta\mathbf{R}_1\times\mathbf{r}]]. \tag{1.6}$$

Apparently, Gauss was not sure of this expression, possibly due to the failure of Newton's third law, i. e. $d^2\mathbf{f}_{21}^{\text{Gauss}} \neq -d^2\mathbf{f}_{12}^{\text{Gauss}}$, and believed that "the effect of *two galvanic elements on each other* has yet to be further studied" ([12], S. 604).

If we are talking about the interaction of electrical elements e_1 and e_2, rather than current elements, then laws (1.5) and (1.6) can describe only part of this interaction associated with the *absolute* motion of these elements. Meanwhile, the total force of interaction in the case of relative rest of the elements should be reduced to the Coulomb law. In July 1835, Gauss found another expression for *the basic law for all interactions of galvanic currents* in the form of a repulsive force that satisfies Newton's third law and takes into account the Coulomb interaction,[3]) which can be represented as

$$\begin{aligned}\mathbf{F}_{21}^{\text{G}} &= e_1 e_2 \frac{d^2\mathbf{f}_{21}^{\text{G}}}{dt^2} = \frac{e_1 e_2}{r^3}\left\{1 + k\left(v^2 - \frac{3}{2}\left(\frac{dr}{dt}\right)^2\right)\right\}\mathbf{r} = \\ &= \frac{e_1 e_2}{r^3}\left\{1 + \frac{2\mathbf{v}^2}{c^2} - \frac{3(\mathbf{r}\cdot\mathbf{v})^2}{c^2 r^2}\right\}\mathbf{r} = \frac{e_1 e_2}{r^3}\left\{1 - \frac{\mathbf{v}^2}{c^2} + \frac{3[\mathbf{r}\times\mathbf{v}]^2}{c^2 r^2}\right\}\mathbf{r},\end{aligned} \tag{1.7}$$

where $\sqrt{1/k}$ is some velocity, which after the experiments of Weber-Kohlrausch was associated with the velocity of light ($k = 2/c^2$),

$$\mathbf{v} \equiv \mathbf{v}_{21} = \frac{d\mathbf{r}}{dt} = \mathbf{V}_2 - \mathbf{V}_1 \tag{1.8}$$

is the velocity of the second charge relative to first one. Gauss decided not to publish his research until he found a mechanism for the transmission of electrical action ([11], p. 240).

Later, expressions for the interaction force between moving charges were obtained in 1845 by H. Grassmann, [16], and F. Neumann, [17], who independently came to the law (1.6), obtained earlier by Gauss ([18], [19]; [20], pp. 215-220). Grassmann noted in several points of his article that the validity of the Ampère law (1.5) or the Gauss-Grassmann-Neumann law (1.6) can be established only by doing experiments with open circuits. Neumann proposed to use the potential of ponderomotive forces acting according to Ampère's theory between the circuit and the magnet to construct the theory of induction currents.

To explain the nature of electric current, G. T. Fechner in 1845 proposed the hypothesis that electric current consists of equal flows of positive (*vitrious*) and negative (*resinous*) electricity moving in opposite directions. In addition, charges of the same nature moving in one direction are attracted to each other, while moving in opposite directions are repelled ([21], p. 338). In 1846, W. Weber, based on the Fechner hypothesis and Ampère's law (1.5), obtained *the fundamental law of electrical action*, [22], in which the interaction force depends on both the relative position

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[3]) **[13].** Grundgesetz für alle Wechselwirkungen galvanischer Ströme. (Gefunden im Juli 1835). – [12], S. 616-617; [13], Ch. 6; [14], p. 183; [15], p. 508.}

and the state of the relative motion of charge carriers, and acts along the line connecting the electrical elements.[4]) He "proved that the formula of Newton's forces is completely insufficient to characterize electrical actions, that attractive and repulsive forces of electric fluids are not independent on movements, as Newton's ideas require, and that they depend not only on speed, but also on acceleration of motion" ([28], S. 330-331).

Weber's law can be written as ([22], S. 316, where $a = 1/c$)

$$\mathbf{F}_{21}^{\mathrm{W}} = e_1 e_2 \frac{d^2 \mathbf{f}_{21}^{\mathrm{W}}}{dt^2} = \frac{e_1 e_2}{r^3}\left\{1 - \frac{1}{c^2}\left(\frac{dr}{dt}\right)^2 + \frac{2r}{c^2}\frac{d^2 r}{dt^2}\right\}\mathbf{r} = \\ = \frac{e_1 e_2}{r^3}\left\{1 - \frac{\mathbf{v}^2}{c^2} + \frac{3[\mathbf{r}\times\mathbf{v}]^2}{c^2 r^2} + \frac{2(\mathbf{r}\cdot\mathbf{w})}{c^2}\right\}\mathbf{r}\,, \tag{1.9}$$

wherefrom it can be seen that it reduces to the Gauss law (1.7) at a constant relative velocity of charges ($\mathbf{w} = \mathbf{0}$).[5]) Weber showed that the force (1.9) can be obtained from the potential ([27], S. 229-230)

$$U^{\mathrm{W}} = \frac{e}{R}(1 - [R]^2), \tag{1.10}$$

where $[R]$ denotes the derivative dR/dt, depending on both *R*, and time *t* (in this formula, Weber's *R* should be replaced by our *r*).

Another version of the interaction law of charges was proposed in 1858 by Riemann, who "established that electrodynamic effects of galvanic currents can be explained on the basis of the assumption that the action of electric mass on others is not instantaneous, but propagates towards them at a constant velocity (within the limits of possible observation errors equal to the velocity of light). With this assumption, the differential equation for the propagation of electric force is the same as the equation for the propagation of light and radiant heat" ([32], S. 237). Riemann showed that the interaction potential of two current elements

$$U = \frac{ee' r^2}{C^2}\frac{dd'\left[\dfrac{1}{r}\right]}{dt^2}, \tag{1.11}$$

established from observations, can be obtained as a solution of the equation

$$\frac{\partial^2 U}{\partial t^2} - \alpha^2(\Delta U - 4\pi\rho) = 0\,, \tag{1.12}$$

where $C = c\sqrt{2}$ is Weber-Kohlrausch constant, $\rho = \rho(x,y,z)$ is electrical mass density, $\alpha^2 = C^2/2 = c^2$,

$$\frac{d}{dt} = \frac{ds}{dt}\frac{d}{ds}, \quad \frac{d'}{dt} = \frac{ds'}{dt}\frac{d}{ds'}, \tag{1.13}$$

$$ds = ds_1 = |\,\delta\mathbf{R}_1\,| = \sqrt{\delta X_1^2 + \delta Y_1^2 + \delta Z_1^2} = \sqrt{\mathbf{V}_1^2}\,dt\,,$$

$$ds' = ds_2 = |\,\delta\mathbf{R}_2\,| = \sqrt{\delta X_2^2 + \delta Y_2^2 + \delta Z_2^2} = \sqrt{\mathbf{V}_2^2}\,dt\,. \tag{1.14}$$

If we express the function (1.11) in terms of the vectors $\mathbf{R}_1$, $\mathbf{R}_2$ and their derivatives with respect to time, we obtain

[4]) Weber's electrodynamic studies are most fully described by Reiff and Sommerfeld, [23], and Whittaker, [11], and in recent time in the papers of Assis, [24], [25]. See also [13], Ch. 6; [26].

[5]) We do not consider here various objections (mainly Helmholtz, [29], and Maxwell, [31], ch. XXIII) against the Weber's law related to energy conservation. In the end, these objections turned out to be erroneous.

$$U^{\mathrm{R}} = \frac{ee'r^2}{C^2}\frac{ds}{dt}\frac{d}{ds}\frac{ds'}{dt}\left[\frac{d}{ds'}\frac{1}{r}\right] = -\frac{ee'r^2}{C^2}\frac{d}{dt}\left[\frac{(\mathbf{r}\cdot\mathbf{v})}{r^3}\right] =$$
$$= \frac{ee'}{C^2r^3}\left[3(\mathbf{r}\cdot\mathbf{v})^2 - r^2\mathbf{v}^2 - r^2(\mathbf{r}\cdot\mathbf{w})\right] = -\frac{ee'}{C^2r^3}\left[3[\mathbf{r}\times\mathbf{v}]^2 - 2r^2\mathbf{v}^2 + r^2(\mathbf{r}\cdot\mathbf{w})\right]. \tag{1.15}$$

As can be seen, this function depends not only on the relative radius vector, **r**, but also on relative velocity, **v**, and relative acceleration, **w**. Therefore, the force acting on the second element from the side of the first one will be described by

$$\mathbf{F}_{21}^{\mathrm{R}} = -\frac{\partial U^{\mathrm{R}}}{\partial \mathbf{R}_2} + \frac{d}{dt}\frac{\partial U^{\mathrm{R}}}{\partial \mathbf{V}_2} - \frac{d^2}{dt^2}\frac{\partial U^{\mathrm{R}}}{\partial \mathbf{W}_2} = \frac{2ee'}{C^2r^5}\left(2\mathbf{r}^2\mathbf{v}^2 - 3(\mathbf{r}\cdot\mathbf{v})^2 + 2\mathbf{r}^2(\mathbf{r}\cdot\mathbf{w})\right)\mathbf{r} =$$
$$= \frac{e_1e_2}{r^3}\left\{-\frac{\mathbf{v}^2}{c^2} + \frac{3[\mathbf{r}\times\mathbf{v}]^2}{c^2r^2} + \frac{2(\mathbf{r}\cdot\mathbf{w})}{c^2}\right\}\mathbf{r}. \tag{1.16}$$

To obtain the action of two current elements, Riemann sums the force (1.16) over all electric masses $e = e_1$ and $e' = e_2$ of both current conductors.

Comparison of the Riemann formula (1.16) with the Weber formula (1.9) shows their complete identity with the only difference that the Coulomb force $\mathbf{F}_{21}^{\mathrm{C}} = \frac{e_1e_2}{r^3}\mathbf{r}$ is not taken into account in the Riemann formula.[6]) Thus, the Riemann law (1.16) is the magnetic part of Weber's law,

$$\mathbf{F}_{21}^{\mathrm{W}} = \mathbf{F}_{21}^{\mathrm{C}} + \mathbf{F}_{21}^{\mathrm{R}}. \tag{1.17}$$

In the course of lectures on the mathematical theory of gravitation, electricity and magnetism, which Riemann gave in Göttingen in 1861, recorded by E. Schulze and published by K. Hattendorf in 1876, [33], he proposed for the electrokinetic energy of two electric particles ε and ε', located at points with coordinates $\mathbf{R} = (x, y, z)$ and $\mathbf{R}' = (x', y', z')$, the expression

$$D = \frac{\varepsilon\varepsilon'}{c^2r}\left\{\left(\frac{\partial(x-x')}{\partial t}\right)^2 + \left(\frac{\partial(y-y')}{\partial t}\right)^2 + \left(\frac{\partial(z-z')}{\partial t}\right)^2\right\} = \frac{\varepsilon\varepsilon'}{c^2r}\mathbf{v}^2, \tag{1.18}$$

where $\mathbf{r} = \mathbf{R} - \mathbf{R}'$ ([33], S. 326, formula (II)),[7]) that he deduced from the assumption that all elementary work arising from the interaction of two galvanic currents is the total differential of the interaction potential ([33], S. 315)

$$D_1 = -\iint \frac{dS\cdot dS'}{r}(\mathbf{i}\cdot\mathbf{i}'). \tag{1.19}$$

The associated law of interaction in vector form looks like ([33], S. 327, formulæ (4)-(6))

$$\mathbf{F}_{21}^{\mathrm{Riem}} = e_1e_2\frac{d^2\mathbf{f}_{21}^{\mathrm{Riem}}}{dt^2} = \frac{\varepsilon\varepsilon'}{r^3}\left(1 + \frac{\mathbf{v}^2}{c^2}\right)\mathbf{r} + \frac{\varepsilon\varepsilon'}{c^2}\frac{d\left(\frac{2}{r}\mathbf{v}\right)}{dt} =$$
$$= \frac{\varepsilon\varepsilon'}{r^3}\left\{\left(1 + \frac{\mathbf{v}^2}{c^2}\right)\mathbf{r} - \frac{2(\mathbf{r}\cdot\mathbf{v})}{c^2}\mathbf{v} + \frac{2\mathbf{r}^2}{c^2}\mathbf{w}\right\} =$$
$$= \frac{\varepsilon\varepsilon'}{r^3}\left[\left(1 - \frac{\mathbf{v}^2}{c^2}\right)\mathbf{r} + \frac{2[\mathbf{v}\times[\mathbf{r}\times\mathbf{v}]]}{c^2} + \frac{2\mathbf{r}^2}{c^2}\mathbf{w}\right] = -\mathbf{F}_{12}^{\mathrm{Riem}}. \tag{1.20}$$

In contrast to the law (1.16), (1.17) the force (1.20) is not collinear to the radius vector **r**.

[6] Gauss, Weber and Riemann, based on Fechner's assumption, believed that two particles of electric fluids of opposite signs moving in opposite directions through point 1, both act on a particle located in point 2, what led to a doubling of the force.

[7]) According to Whittaker, the coefficient in front of braces for some reason is equal to $-\frac{ee'}{2r}$ ([10], p. 231). This seems to be a mistake.

In 1863-1864, Enrico Betti proposed a theory, [34]-[36] in which “supposes the closed circuits in which the electric currents flow to consist of elements each of which is polarized periodically, that is, at equidistant intervals of time. These polarized elements act on one another as if they were little magnets whose axes are in the direction of the tangent to the circuits. The periodic time of this polarization is the same in all electric circuits. Betti supposes the action of one polarized element on another at a distance to take place, not instantaneously, but after a time proportional to the distance between the elements. In this way he obtains expressions for the action of one electric circuit on another, which coincide with those which are known to be true. Clausius, however, has, in this case also, criticized some parts of the mathematical calculations into which we shall not here enter” ([31], pp. 436-437).

Rudolf Clausius, having “criticized some parts of mathematical calculations”, also suggested that the electrodynamic forces depend on the state of motion, but he believed that the electric current is caused by the flow of only one electric fluid. Weber’s law turned out to be incompatible with this idea, and therefore in 1875 Clausius tried to derive a new law for the interaction of moving electric particles, [37]-[41]. As a result, the interaction potential of electric masses is as follows ([39], S. 127; [40], p. 270; [41], S. 277)

$$U^{\text{Claus}} = \frac{ee'}{r}[1 + k(\mathbf{V}_1 \cdot \mathbf{V}_2)], \tag{1.21}$$

where $\mathbf{V}_1$ and $\mathbf{V}_2$ are absolute velocities under which Clausius meant the velocities of electric particles relative to the stationary medium in which they move. Accordingly, in our notation the forces acting on particles are equal

$$\mathbf{F}_{21}^{\text{Claus}} = -\frac{\partial U^{\text{Claus}}}{\partial \mathbf{R}_2} + \frac{d}{dt}\frac{\partial U^{\text{Claus}}}{\partial \mathbf{V}_2} = \frac{ee'}{r^3} k\left[\mathbf{W}_1 - (\mathbf{r}\cdot\mathbf{v})\mathbf{V}_1 + [k^{-1} + (\mathbf{V}_1 \cdot \mathbf{V}_2)]\mathbf{r}\right], \tag{1.22}$$

$$\mathbf{F}_{12}^{\text{Claus}} = -\frac{\partial U^{\text{Claus}}}{\partial \mathbf{R}_1} + \frac{d}{dt}\frac{\partial U^{\text{Claus}}}{\partial \mathbf{V}_1} = \frac{ee'}{r} k\left[\mathbf{W}_2 - (\mathbf{r}\cdot\mathbf{v})\mathbf{V}_2 - [k^{-1} + (\mathbf{V}_1 \cdot \mathbf{V}_2)]\mathbf{r}\right] \neq -\mathbf{F}_{21}^{\text{Claus}}. \tag{1.23}$$

The laws of interaction between current elements or moving charges, presented above, were analyzed by Maxwell in Ch. XXIII of his “A Treatise on Electricity and Magnetism“, [30]-[31]. According to the words of L. Boltzmann, “fruitless debates about the various elementary laws of electrodynamics”,[8]) were overcome by Maxwell’s theory, which instead of description of the interaction of electric corpuscles (that is, long-range theory), preferred a hydrodynamic analogy of the electromagnetic field, the force lines of which were represented by current tubes of an ideal incompressible ethereal fluid, the speed and the direction of motion of which is determined by the forces acting on the particles. In particular, magnetic induction **B** can be represented as the velocity of this fluid.

Another formula was obtained in 1910 by Edmund Whittaker, who noted that “The weakness of Ampère’s work evidently lies in the assumption that the force is directed along the line joining the two elements; for in the analogous case of the action between two magnetic molecules, we know that the force is not directed along the line joining the molecules. It is therefore of interest to find the form of **F** when this restriction is removed” ([11], pp. 86-87). As a result, in our notation, the Whittaker formula takes the form

$$d^2\mathbf{f}_{21}^{\text{Whit}} = \frac{1}{r^3}\left[(\mathbf{r}\cdot\delta\mathbf{R}_2)\delta\mathbf{R}_1 + (\mathbf{r}\cdot\delta\mathbf{R}_1)\delta\mathbf{R}_2 - (\delta\mathbf{R}_1\cdot\delta\mathbf{R}_2)\mathbf{r}\right], \tag{1.24}$$

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[8]) [42], S. 99. Studies of the fundamental law of electric long-range action continued in works of physicists par excellence of the German school: Wilhelm Gottlieb Hankel (Leipzig, [43]-[45]), Hermann von Helmholtz (Heidelberg, [29], [46], [47]), his student Isidore Fröhlich (Budapest, [48]), Eric Edlund (Stockholm, [49]-[50]), Carl Neumann (Leipzig, [51]-[54]), Peter Guthry Tait (Edinburgh, [55]), Johann Karl Friedrich Zöllner (Leipzig, [56], [57]), Albrecht Ludolf Hermann Lorberg (Strasbourgh, [58]-[60]), Eduard Rikke (Göttingen, [61]), Emil Arnold Budde (Berlin, [62], [63]), Diderik Johannes Korteweg (Breda, [64]). British school physicists abandoned long-range theory and began to develop theory of Maxwell.

leading to the Biot-Savart-Laplace law and the Ampère law for the ponderomotive force acting on a current element placed in a magnetic field.

In 1888, Heaviside noted that, in practice, not the law of force between two current elements (having in mind the Ampère's law (1.5), which is considered the basic formula of electrodynamics) is applying, but the law "expressing the mechanical force on an element of a conductor supporting current in any magnetic field; the vector product of current and induction" ([65], p. 502; [11], p. 88). If in macroscopic electrodynamics, such as Maxwell's theory, *the fundamental law of electrical action* does not really matter much due to the statistical nature of this theory, then its correct formulation is absolutely necessary when considering the interaction of elementary particles. The choice between the various formulas given above cannot be solved by macroscopic experiments. In this connection, we note that the laws of Ampère (1.5), Gauss-Grassmann-Neumann (1.6) and Whittaker (1.24) represent the interaction laws of elements of currents flowing in conductors that are at rest relative to each other, while the laws of Gauss (1.7), Weber (1.9), Riemann (1.16), (1.20) and Clausius (1.22)-(1.23) are the laws of interaction of moving electric particles. Accordingly, we obtain the problem of obtaining the correct interaction law of current elements and the problem of obtaining the correct interaction law of moving charged particles. Below we will give the derivation of the first law from the point of view of the theory developed in [1]-[8].

2. The interaction of currents and charges as a problem of two bodies

Let us consider the interaction of two elementary objects with spins $\mathbf{s}_1$ and $\mathbf{s}_2$, which we call particles. Let $\mathbf{R}_1$, $\mathbf{R}_2$ be the coordinates of centers of mass of particles in the absolute reference frame (r. f.), $\mathbf{V}_1$, $\mathbf{V}_2$ the corresponding velocities, $\mathbf{W}_1$, $\mathbf{W}_2$ the accelerations, $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{w}_1$, $\mathbf{w}_2$ the coordinates, velocities and accelerations of the centers of mass of the particles relative to the center of mass of the system. The general equations of motion for a system of two particles located in an external field, according to [5] have the form

$$\frac{d\mathbf{P}_1}{dt} = \mathbf{F}_1, \ \frac{d\mathbf{P}_2}{dt} = \mathbf{F}_2, \tag{2.1}$$

where

$$\mathbf{P}_1 = m_1\mathbf{V}_1 = m_{01}\mathbf{V}_1 - \frac{\partial(U_1^{\text{ext}} + U^{\text{int}})}{\partial \mathbf{V}_1} + [(\varsigma_1\mathbf{s}_1 + \mathbf{S}_1^{\text{ext}} + \mathbf{S}_{12}^{\text{int}}) \times \mathbf{W}_1], \tag{2.2}$$

$$\mathbf{P}_2 = m_2\mathbf{V}_2 = m_{02}\mathbf{V}_2 - \frac{\partial(U_2^{\text{ext}} + U^{\text{int}})}{\partial \mathbf{V}_2} + [(\varsigma_2\mathbf{s}_2 + \mathbf{S}_2^{\text{ext}} + \mathbf{S}_{21}^{\text{int}}) \times \mathbf{W}_2] \tag{2.3}$$

are the dynamic momenta, m_{01} and m_{02} are bare masses of particles without of taking into account interactions and internal structure, m_1 and m_2 are effective masses determined from dynamic momenta,

$$\mathbf{F}_1 = -\frac{\partial(U_1^{\text{ext}} + U^{\text{int}})}{\partial \mathbf{R}_1} + [(-\varsigma_1\Omega_{01}^2\mathbf{s}_1 + \mathbf{C}_1^{\text{ext}} + \mathbf{C}_{12}^{\text{int}}) \times \mathbf{V}_1], \tag{2.4}$$

$$\mathbf{F}_2 = -\frac{\partial(U_2^{\text{ext}} + U^{\text{int}})}{\partial \mathbf{R}_2} + [(-\varsigma_2\Omega_{02}^2\mathbf{s}_2 + \mathbf{C}_2^{\text{ext}} + \mathbf{C}_{21}^{\text{int}}) \times \mathbf{V}_2] \tag{2.5}$$

are forces acting on the first and second particles, respectively.

Potential functions $U_1^{\text{ext}} = U_1^{\text{ext}}(t;\mathbf{R}_1,\mathbf{V}_1,...)$, $U_2^{\text{ext}} = U_2^{\text{ext}}(t;\mathbf{R}_2,\mathbf{V}_2,...)$ and pseudovectors $\mathbf{S}_1^{\text{ext}} = \mathbf{S}_1^{\text{ext}}(t;\mathbf{R}_1,\mathbf{V}_1,...)$, $\mathbf{S}_2^{\text{ext}} = \mathbf{S}_2^{\text{ext}}(t;\mathbf{R}_2,\mathbf{V}_2,...)$, $\mathbf{C}_1^{\text{ext}} = \mathbf{C}_1^{\text{ext}}(t;\mathbf{R}_1,\mathbf{V}_1,...)$ and $\mathbf{C}_2^{\text{ext}} = \mathbf{C}_2^{\text{ext}}(t;\mathbf{R}_2,\mathbf{V}_2,...)$ determine an interaction of particles with external fields and objects, whereas the interaction between particles is determined by the function $U^{\text{int}} = U_0(\mathbf{r}, \mathbf{v}, \mathbf{w}, \ldots) + u(\mathbf{v}, \mathbf{w}, \ldots)$ and pseudovectors

$$\mathbf{S}_{12}^{\text{int}} = \frac{1}{2}\varsigma_{12}(\mathbf{s}_1 + \mathbf{s}_2) - \varsigma_1 \mathbf{s}_1 + \varsigma_{12}\frac{m_1 m_2}{2m}[\mathbf{r} \times \mathbf{v}], \tag{2.6}$$

$$\mathbf{S}_{21}^{\text{int}} = \frac{1}{2}\varsigma_{21}(\mathbf{s}_1 + \mathbf{s}_2) - \varsigma_2 \mathbf{s}_2 + \varsigma_{21}\frac{m_1 m_2}{2m}[\mathbf{r} \times \mathbf{v}], \tag{2.7}$$

$$\mathbf{C}_{12}^{\text{int}} = -\frac{1}{2}\varsigma_{12}\Omega_0^2(\mathbf{s}_1 + \mathbf{s}_2) + \varsigma_1 \Omega_{01}^2 \mathbf{s}_1 - \varsigma_{12}\frac{m_1 m_2 \Omega_0^2}{2m}[\mathbf{r} \times \mathbf{v}], \tag{2.8}$$

$$\mathbf{C}_{21}^{\text{int}} = -\frac{1}{2}\varsigma_{21}\Omega_0^2(\mathbf{s}_1 + \mathbf{s}_2) + \varsigma_2 \Omega_{02}^2 \mathbf{s}_2 - \varsigma_{21}\frac{m_1 m_2 \Omega_0^2}{2m}[\mathbf{r} \times \mathbf{v}], \tag{2.9}$$

where constants ς_1 and ς_2 are related to polarizations of the particles, while ς_{12} and ς_{21} belong to the particle system, Ω_{01}, Ω_{02} and Ω_0 are some functions of relative variables, which in the case of free non-interacting particles are constant frequencies of *Zitterbewegung* of particles, determined from the equations of spin motion

$$\frac{d\mathbf{s}_1}{dt} = [\boldsymbol{\Omega}_1 \times \mathbf{s}_1] + \mathbf{m}_1(t), \; \boldsymbol{\Omega}_1 = \Omega_{10}\mathbf{N}_1, \tag{2.10}$$

$$\frac{d\mathbf{s}_2}{dt} = [\boldsymbol{\Omega}_2 \times \mathbf{s}_2] + \mathbf{m}_2(t), \; \boldsymbol{\Omega}_2 = \Omega_{20}\mathbf{N}_2, \tag{2.11}$$

$\mathbf{N}_1$, $\mathbf{N}_2$ are unit vectors of directions along which *Zitterbewegung* occurs, $\mathbf{m}_1(t)$, $\mathbf{m}_2(t)$ are pseudovectors depending on external fields and determining the behavior of the spin orientation. Spin constancy modulo means $(\mathbf{m}_1 \cdot \mathbf{s}_1) = 0$, $(\mathbf{m}_2 \cdot \mathbf{s}_2) = 0$.

The transformation to the center-of-mass and relative coordinates is given by

$$\mathbf{R} = \frac{m_1 \mathbf{R}_1 + m_2 \mathbf{R}_2}{m}, \; \mathbf{r} = \mathbf{r}_{12} = \mathbf{r}_2 - \mathbf{r}_1 = \mathbf{R}_2 - \mathbf{R}_1; \tag{2.12}$$

$$\mathbf{R}_1 = \mathbf{R} + \mathbf{r}_1 = \mathbf{R} - \frac{m_2}{m}\mathbf{r}, \; \mathbf{R}_2 = \mathbf{R} + \mathbf{r}_2 = \mathbf{R} + \frac{m_1}{m}\mathbf{r}; \tag{2.13}$$

$$\frac{\partial U^{\text{int}}}{\partial \mathbf{R}_1} = -\frac{\partial U^{\text{int}}}{\partial \mathbf{R}_2} = -\frac{\partial U^{\text{int}}}{\partial \mathbf{r}}, \; \frac{\partial U^{\text{int}}}{\partial \mathbf{V}_1} = -\frac{\partial U^{\text{int}}}{\partial \mathbf{V}_2} = -\frac{\partial U^{\text{int}}}{\partial \mathbf{v}}. \tag{2.14}$$

Equations (2.2)-(2.5) in variables (2.12) in view of (2.6)-(2.9) take the form

$$\mathbf{P}_1 = m_1 \mathbf{V}_1 = m_{01}\mathbf{V}_1 - \frac{\partial U_1^{\text{ext}}}{\partial \mathbf{V}_1} + [\mathbf{S}_1^{\text{ext}} \times \mathbf{W}_1] + \frac{\partial U^{\text{int}}}{\partial \mathbf{v}} + \frac{1}{2}\varsigma_{12}[\mathbf{s} \times \mathbf{W}_1], \tag{2.15}$$

$$\mathbf{P}_2 = m_2 \mathbf{V}_2 = m_{02}\mathbf{V}_2 - \frac{\partial U_2^{\text{ext}}}{\partial \mathbf{V}_2} + [\mathbf{S}_2^{\text{ext}} \times \mathbf{W}_2] - \frac{\partial U^{\text{int}}}{\partial \mathbf{v}} + \frac{1}{2}\varsigma_{21}[\mathbf{s} \times \mathbf{W}_2]; \tag{2.16}$$

$$\mathbf{F}_1 = -\frac{\partial U_1^{\text{ext}}}{\partial \mathbf{R}_1} + [\mathbf{C}_1^{\text{ext}} \times \mathbf{V}_1] + \frac{\partial U^{\text{int}}}{\partial \mathbf{r}} - \frac{1}{2}\varsigma_{12}\Omega_0^2[\mathbf{s} \times \mathbf{V}_1], \tag{2.17}$$

$$\mathbf{F}_2 = -\frac{\partial U_2^{\text{ext}}}{\partial \mathbf{R}_2} + [\mathbf{C}_2^{\text{ext}} \times \mathbf{V}_2] - \frac{\partial U^{\text{int}}}{\partial \mathbf{r}} - \frac{1}{2}\varsigma_{21}\Omega_0^2[\mathbf{s} \times \mathbf{V}_2], \tag{2.18}$$

where velocities $\mathbf{V}_1$, $\mathbf{V}_2$ and accelerations $\mathbf{W}_1$, $\mathbf{W}_2$ are calculated from (2.13) taking account of possible dependence of effective masses m_1, m_2 on time,

$$\mathbf{s} = \mathbf{j}_1 + \mathbf{j}_2 = \mathbf{s}_1 + \mathbf{s}_2 + \mathbf{l} \tag{2.19}$$

is the total angular momentum of the particle system relative to the center of mass, actually representing the spin of the system,

$$\mathbf{l} = \mathbf{l}_1 + \mathbf{l}_2 = \frac{m_1 m_2}{m}[\mathbf{r} \times \mathbf{v}], \tag{2.20}$$

is the total orbital momentum of the particle system relative to the center of mass,

$$\mathbf{l}_1 = m_1[\mathbf{r}_1 \times \mathbf{v}_1],\ \mathbf{l}_2 = m_2[\mathbf{r}_2 \times \mathbf{v}_2] \tag{2.21}$$

are orbital momenta of particles relative to the center of mass,

$$\mathbf{j}_1 = \mathbf{l}_1 + \mathbf{s}_1,\ \mathbf{j}_2 = \mathbf{l}_2 + \mathbf{s}_2 \tag{2.22}$$

are total momenta of particles relative to the center of mass.

When a system of particles forms a bound state, it has a spin (2.19), satisfying the equation of motion

$$\frac{d\mathbf{s}}{dt} = [\mathbf{\Omega}(t) \times \mathbf{s}] + \mathbf{m}(t) = \Omega_0[\mathbf{N} \times \mathbf{s}] + \mathbf{m}(t), \tag{2.23}$$

where **N** is the unit vector tangent to the instantaneous direction along which the *Zitterbewegung* of the system occurs. The pseudovector $\mathbf{m}(t)$ according to (2.10), (2.11), (2.19) and (2.23) should satisfy the relation

$$\mathbf{m} = \mathbf{m}_1 + \mathbf{m}_2 + [(\mathbf{\Omega}_1 - \mathbf{\Omega}) \times \mathbf{s}_1] + [(\mathbf{\Omega}_2 - \mathbf{\Omega}) \times \mathbf{s}_2] + \frac{d\mathbf{l}}{dt} - [\mathbf{\Omega} \times \mathbf{l}]. \tag{2.24}$$

Assuming that pseudovectors $\mathbf{m}_1(t)$, $\mathbf{m}_2(t)$, $\mathbf{m}(t)$ have a similar structure in the form

$$\mathbf{m}_{1,2} = -\frac{d\mathbf{l}}{dt} + \mathbf{A}_{1,2},\ \mathbf{m} = -\frac{d\mathbf{l}}{dt} + \mathbf{A}, \tag{2.25}$$

where $\mathbf{A}_1$, $\mathbf{A}_2$ and **A** should contain only terms related to the first particle, second particle, and system of particles, respectively, and substituting (2.25) into (2.24), we obtain

$$\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2 + [\mathbf{\Omega}_1 \times \mathbf{s}_1] + [\mathbf{\Omega}_2 \times \mathbf{s}_2] - [\mathbf{\Omega} \times \mathbf{s}], \tag{2.26}$$

i. e.

$$\mathbf{A}_1 = -[\mathbf{\Omega}_1 \times \mathbf{s}_1],\ \mathbf{A}_2 = -[\mathbf{\Omega}_2 \times \mathbf{s}_2],\ \mathbf{A} = -[\mathbf{\Omega} \times \mathbf{s}]. \tag{2.27}$$

Then

$$\mathbf{m}_1 = -\frac{d\mathbf{l}}{dt} - [\mathbf{\Omega}_1 \times \mathbf{s}_1],\ \mathbf{m}_2 = -\frac{d\mathbf{l}}{dt} - [\mathbf{\Omega}_2 \times \mathbf{s}_2],\ \mathbf{m} = -\frac{d\mathbf{l}}{dt} - [\mathbf{\Omega} \times \mathbf{s}], \tag{2.28}$$

and equations of motion (2.10), (2.11) и (2.23) become

$$\frac{d\mathbf{s}_1}{dt} = -\frac{d\mathbf{l}}{dt},\ \frac{d\mathbf{s}_2}{dt} = -\frac{d\mathbf{l}}{dt},\ \frac{d\mathbf{s}}{dt} = -\frac{d\mathbf{l}}{dt}, \tag{2.29}$$

which implies the conservation of spin directions relative to each other,

$$\frac{d\Delta\mathbf{s}}{dt} = \frac{d(\mathbf{s}_2 - \mathbf{s}_1)}{dt} = 0, \tag{2.30}$$

and the absence of precession, i. e. we can put $\mathbf{\Omega}_1 = \mathbf{\Omega}_2 = \mathbf{\Omega} = \mathbf{0}$, while the third equation (2.29) is a consequence of the first two.

In addition to (2.27), a representation is also admissible of $\mathbf{A}_1$, $\mathbf{A}_2$ and **A** in the form

$$\mathbf{A}_1 = [(\mathbf{\Omega} - \mathbf{\Omega}_1) \times \mathbf{s}_1],\ \mathbf{A}_2 = [(\mathbf{\Omega} - \mathbf{\Omega}_2) \times \mathbf{s}_2],\ \mathbf{A} = -[\mathbf{\Omega} \times \mathbf{l}]. \tag{2.31}$$

Then

$$\mathbf{m}_1 = -\frac{d\mathbf{l}}{dt} + [(\mathbf{\Omega} - \mathbf{\Omega}_1) \times \mathbf{s}_1],\ \mathbf{m}_2 = -\frac{d\mathbf{l}}{dt} + [(\mathbf{\Omega} - \mathbf{\Omega}_2) \times \mathbf{s}_2],\ \mathbf{m} = -\frac{d\mathbf{l}}{dt} - [\mathbf{\Omega} \times \mathbf{l}], \tag{2.32}$$

and equations of motion (2.10), (2.11) and (2.23) take the form

$$\frac{d\mathbf{s}_1}{dt} = [\mathbf{\Omega} \times \mathbf{s}_1] - \frac{d\mathbf{l}}{dt},\ \frac{d\mathbf{s}_2}{dt} = [\mathbf{\Omega} \times \mathbf{s}_2] - \frac{d\mathbf{l}}{dt}, \tag{2.33}$$

$$\frac{d\mathbf{s}}{dt} = [\mathbf{\Omega} \times \mathbf{s}] - \frac{d\mathbf{l}}{dt} - [\mathbf{\Omega} \times \mathbf{l}]. \tag{2.34}$$

It follows from (2.33)

$$\frac{d\Delta\mathbf{s}}{dt} = [\mathbf{\Omega} \times \Delta\mathbf{s}], \tag{2.35}$$

i. e. $\Delta\mathbf{s}$ precesses with an angular velocity **Ω**, so that $|\Delta\mathbf{s}| = \mathrm{const}$.

If the system of particles does not form a bound state, as can be assumed for free electrons scattered on each other or electrons moving along different conductors under the influence of external forces, then the concept of spin of the system loses meaning. Furthermore, as it is clear from (2.15)-(2.18), the spins of particles are not included in the equation of motion, regardless of whether they are free or not. In accordance with the foregoing, the interaction forces of two current elements, the current and charge element and two moving charges should be described by different formulas.

3. The interaction of two current elements

In the case of two current elements, to determine the interaction forces, one should use equations (2.17)-(2.18). Assuming that electric current caused by an orderly motion of electrons under the action of an external field strength, it should be assumed that electron spins $\mathbf{s}_1$ and $\mathbf{s}_2$ are directed antiparallel to their velocities $\mathbf{V}_1$ and $\mathbf{V}_2$. We note here that these velocities (as well as accelerations $\mathbf{W}_1$ and $\mathbf{W}_2$) are the velocities (accelerations) of electrons through conductors only when they are in rest relative to each other. In the case of moving conductors, $\mathbf{V}_1$ and $\mathbf{V}_2$ consist of conduction electron velocities, $\mathbf{v}_{e1} = \delta\mathbf{R}_1 / dt$, $\mathbf{v}_{e2} = \delta\mathbf{R}_2 / dt$, and current element velocities, $\mathbf{u}_1 = d\mathbf{x}_1 / dt$, $\mathbf{u}_2 = d\mathbf{x}_2 / dt$. Therefore, in general, we should put

$$d\mathbf{R}_1 = \delta\mathbf{R}_1 + d\mathbf{x}_1,\ d\mathbf{R}_2 = \delta\mathbf{R}_2 + d\mathbf{x}_2, \tag{3.1}$$

$$\mathbf{V}_1 = d\mathbf{R}_1 / dt = \mathbf{v}_{e1} + \mathbf{u}_1,\ \mathbf{V}_2 = d\mathbf{R}_2 / dt = \mathbf{v}_{e2} + \mathbf{u}_2, \tag{3.2}$$

where $d\mathbf{x}_1$ and $d\mathbf{x}_2$ are absolute displacements of elements $\delta\mathbf{R}_1$ and $\delta\mathbf{R}_2$. Then the condition of collinearity (antiparallelism) of spins and current elements may be expressed by

$$[\mathbf{v}_{e2} \times \mathbf{s}_1] = [\mathbf{s}_2 \times \mathbf{v}_{e1}] = -[\mathbf{v}_{e1} \times \mathbf{s}_2],\ [\mathbf{v}_{e1} \times \mathbf{s}_1] = [\mathbf{v}_{e2} \times \mathbf{s}_2] = \mathbf{0}. \tag{3.3}$$

If the current elements are not in an external field, then $U_1^{\mathrm{ext}} = U_2^{\mathrm{ext}} = 0$, $\mathbf{S}_1^{\mathrm{ext}} = \mathbf{S}_2^{\mathrm{ext}} = \mathbf{0}$, $\mathbf{C}_1^{\mathrm{ext}} = \mathbf{C}_2^{\mathrm{ext}} = \mathbf{0}$. Then the forces (2.17)-(2.18) with (2.13), (2.19), (2.20) can be considered as the sum of electric and magnetic forces

$$\mathbf{F}_1 \equiv \mathbf{F}_{12} = \mathbf{F}_{12}^{e} + \mathbf{F}_{12}^{m},\ \mathbf{F}_2 \equiv \mathbf{F}_{21} = \mathbf{F}_{21}^{e} + \mathbf{F}_{21}^{m}, \tag{3.4}$$

where

$$\mathbf{F}_{21}^{e} = \mathbf{E}_2 = -\mathbf{E}_1 = -\mathbf{F}_{12}^{e} = -\frac{\partial U^{\mathrm{int}}}{\partial \mathbf{r}} \tag{3.5}$$

is electric force (electric field strength) acting on the second charge from the first charge,

$$\mathbf{F}_{12}^{m} = -\frac{1}{2}\varsigma_{12}\Omega_0^2[\mathbf{s} \times \mathbf{V}_1] = \frac{1}{2}\varsigma_{12}\Omega_0^2 m_e[\frac{d\mathbf{R}_1}{dt} \times \left(\frac{1}{m_e}\mathbf{s}_1 + \frac{1}{m_e}\mathbf{s}_2 + \frac{1}{2}[\mathbf{r} \times \mathbf{v}]\right)], \tag{3.6}$$

$$\mathbf{F}_{21}^{m} = -\frac{1}{2}\varsigma_{21}\Omega_0^2[\mathbf{s} \times \mathbf{V}_2] = \frac{1}{2}\varsigma_{21}\Omega_0^2 m_e[\frac{d\mathbf{R}_2}{dt} \times \left(\frac{1}{m_e}\mathbf{s}_1 + \frac{1}{m_e}\mathbf{s}_2 + \frac{1}{2}[\mathbf{r} \times \mathbf{v}]\right)] \tag{3.7}$$

are magnetic forces acting on the first and second charges, respectively, $m_1 = m_2 = m_e$ is effective electron mass in the conductor, determined from equations (2.15)-(2.16).

Addition of $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ in view of (3.3) gives

$$\begin{gathered}\mathbf{F}_{12} + \mathbf{F}_{21} = \frac{1}{2}\Omega_0^2\big[(\varsigma_{12} - \varsigma_{21})[\mathbf{v}_{e1} \times \mathbf{s}_2] + \varsigma_{12}[\mathbf{u}_1 \times \mathbf{s}_2] + \varsigma_{21}[\mathbf{u}_2 \times \mathbf{s}_1]\big] + \\ + \frac{1}{4} m_e \Omega_0^2 [(\varsigma_{12}\mathbf{V}_1 + \varsigma_{21}\mathbf{V}_2) \times [\mathbf{r} \times \mathbf{v}]] \,, \end{gathered} \quad . \tag{3.8}$$

whence it follows that for conductors being in mutual rest, $\mathbf{u}_1 = \mathbf{u}_2 = \mathbf{0}$, Newton's third law $\mathbf{F}_{21} + \mathbf{F}_{12} = \mathbf{0}$ is satisfied in the center-of-mass reference frame at $\varsigma_{21} = \varsigma_{12} = \varsigma$. Then equations (2.15)-(2.16) reduce to

$$m_e \mathbf{V} = m_{0e} \mathbf{V} + \frac{1}{2}\varsigma[\mathbf{s} \times \mathbf{W}], \tag{3.9}$$

$$m_e \mathbf{v} = m_{0e} \mathbf{v} - 2\frac{\partial U^{\mathrm{int}}}{\partial \mathbf{v}} + \frac{1}{2}\varsigma[\mathbf{s} \times \mathbf{w}], \tag{3.10}$$

and lead to the dependence of effective mass on the state of relative motion of electrons. It follows from (3.8) that in absolute reference frame the total orbital angular momentum **l** of the particle system relative to the center of mass is collinear to the velocity **V** of the center of mass.

Expressions (3.6)-(3.7) are actually a generalization of the Ampère's law, which Heaviside spoke of ([65], p. 502), determining the force acting on the current element in magnetic field. In our case, the force acting on the element $i_2 \delta \mathbf{R}_2$ by the element $i_1 \delta \mathbf{R}_1$ will be equal to

$$d^2\mathbf{F}_{21}^m = i_1 i_2 d^2 \mathbf{f}_{21}^m = i_2 [\delta \mathbf{R}_2 \times d\mathbf{B}_1]. \tag{3.11}$$

where $d\mathbf{B}_1$ is usually determined from the Biot-Savart-Laplace law

$$d\mathbf{B}_1 = d\mathbf{B}_1^{\mathrm{BSL}} = \frac{i_1}{r^3}[\delta \mathbf{R}_1 \times \mathbf{r}] = \frac{\mu_0 I_1}{4\pi r^3}[\delta \mathbf{R}_1 \times \mathbf{r}], \tag{3.12}$$

if I_1 is measured in amperes. In this article $d\mathbf{B}_1$ can be determined from (3.7). Taking into account relations (2.12) and the condition (3.3), or

$$\mathbf{V}_1 = \frac{d\mathbf{R}_1}{dt} = \mathbf{V} - \frac{1}{2}\mathbf{v}, \ \mathbf{V}_2 = \frac{d\mathbf{R}_2}{dt} = \mathbf{V} + \frac{1}{2}\mathbf{v}, \tag{3.13}$$

$$\mathbf{s}_1 = -\frac{s}{v_{e1}}\mathbf{v}_{e1} = -\frac{\hbar}{2v_{e1}}\mathbf{v}_{e1}, \ \mathbf{s}_2 = -\frac{s}{v_{e2}}\mathbf{v}_{e2} = -\frac{\hbar}{2v_{e2}}\mathbf{v}_{e2}, \tag{3.14}$$

where $s = \hbar / 2$ is electron spin, expressions (3.6)-(3.7) will look like

$$\mathbf{F}_{12}^m = -\frac{1}{2}\varsigma\Omega_0^2[\mathbf{s} \times \mathbf{V}_1] = -\frac{1}{4}\varsigma\Omega_0^2 m_e [\frac{d\mathbf{R}_1}{dt} \times \left(\frac{\hbar}{m_e v_{e1}}\mathbf{v}_{e1} + \frac{\hbar}{m_e v_{e2}}\mathbf{v}_{e2} + [\mathbf{v} \times \mathbf{r}] \right)], \tag{3.15}$$

$$\mathbf{F}_{21}^m = -\frac{1}{2}\varsigma\Omega_0^2[\mathbf{s} \times \mathbf{V}_2] = -\frac{1}{4}\varsigma\Omega_0^2 m_e [\frac{d\mathbf{R}_2}{dt} \times \left(\frac{\hbar}{m_e v_{e1}}\mathbf{v}_{e1} + \frac{\hbar}{m_e v_{e2}}\mathbf{v}_{e2} + [\mathbf{v} \times \mathbf{r}] \right)], \tag{3.16}$$

or

$$d^2\mathbf{F}_{12}^m = [d\mathbf{R}_1 \times \left(-\frac{\varsigma\Omega_0^2\hbar}{4v_{e2}}\delta\mathbf{R}_2 - \frac{1}{4}\varsigma\Omega_0^2 m_e [d\mathbf{R}_2 \times \mathbf{r}] + \frac{1}{4}\varsigma\Omega_0^2 m_e [d\mathbf{R}_1 \times \mathbf{r}] + \frac{\varsigma\Omega_0^2\hbar}{4v_{e1}} d\mathbf{x}_1 \right)], \tag{3.17}$$

$$d^2\mathbf{F}_{21}^m = [d\mathbf{R}_2 \times \left(-\frac{\varsigma\Omega_0^2\hbar}{4v_{e1}}\delta\mathbf{R}_1 + \frac{1}{4}\varsigma\Omega_0^2 m_e [d\mathbf{R}_1 \times \mathbf{r}] - \frac{1}{4}\varsigma\Omega_0^2 m_e [d\mathbf{R}_2 \times \mathbf{r}] + \frac{\varsigma\Omega_0^2\hbar}{4v_{e2}} d\mathbf{x}_2 \right)]. \tag{3.18}$$

The magnetic field at the point where the moving current element $i_2\delta\mathbf{R}_2$ is located is determined by expression

$$d\mathbf{B}_1 = \frac{\varsigma\Omega_0^2 m_e}{4i_2}\left(-\frac{\hbar}{m_e v_{e1}}\delta\mathbf{R}_1 - [d\mathbf{r}\times\mathbf{r}] + \frac{\hbar}{m_e v_{e2}}d\mathbf{x}_2\right)] = \\ = \frac{\varsigma\Omega_0^2 m_e}{2i_2}\left([d\mathbf{R}_1\times\mathbf{r}] - \frac{\hbar}{2m_e v_{e1}}\delta\mathbf{R}_1 + \frac{\hbar}{2m_e v_{e2}}d\mathbf{x}_2 - [d\mathbf{R}\times\mathbf{r}]\right), \tag{3.19}$$

which contains four terms. The first of them is the Biot-Savart-Laplace law

$$d\mathbf{B}_1^{\mathrm{BSL}} = \frac{\varsigma m_e\Omega_0^2}{4i_2}[d\mathbf{R}_1\times\mathbf{r}], \tag{3.20}$$

corresponding to the Gauss-Grassmann-Neumann interaction law (1.6). Comparing (3.20) with (3.12) and taking into account that $d\mathbf{R}_1 = \delta\mathbf{R}_1$ for a conductor at rest, we obtain that Ω_0 should depend on *r* according to

$$\Omega_0 = \sqrt{\frac{2i_1i_2}{\varsigma m_e}}r^{-3/2} = \frac{\mu_0}{4\pi}\sqrt{\frac{2I_1I_2}{\varsigma m_e}}r^{-3/2}, \tag{3.21}$$

where the sign of the constant ς coincides with the sign of the product i_1i_2 of the current intensities when the currents are collinear.

The second term in (3.19) is the magnetic induction of spinning electron of the first current element,

$$d\mathbf{B}_1^{\mathrm{s}_1} = -\frac{\varsigma\Omega_0^2\hbar}{4i_2v_{e1}}\delta\mathbf{R}_1 = -\frac{\hbar i_1}{2m_e v_{e1}r^3}\delta\mathbf{R}_1 = \frac{\varsigma\Omega_0^2}{2i_2}\mathbf{s}_1 dt = \frac{i_1}{m_e r^3}\mathbf{s}_1 dt. \tag{3.22}$$

Assuming that the current of the first element is created by one electron, i. e. $i_1 dt = e$, we find from here the magnetic field of spinning electron [9])

$$\mathbf{B}^{\mathrm{s}} = \frac{e}{m_e r^3}\mathbf{s}. \tag{3.23}$$

The third term in (3.19) is determined by the displacement $d\mathbf{x}_2$ of the second current element

$$d\mathbf{B}_1^{\mathrm{s}_2} = \frac{\varsigma\Omega_0^2\hbar}{4i_2v_{e2}}d\mathbf{x}_2 = \frac{i_1\hbar}{2m_e v_{e2}r^3}d\mathbf{x}_2. \tag{3.24}$$

Finally, the fourth contribution,

$$d\mathbf{B}_1^{\mathbf{R}} = \frac{\varsigma\Omega_0^2 m_e}{2i_2}[d\mathbf{R}\times\mathbf{r}] = \frac{i_1}{r^3}[d\mathbf{R}\times\mathbf{r}], \tag{3.25}$$

is due to the absolute motion of the center of mass of the elements $i_1\delta\mathbf{R}_1$ and $i_2\delta\mathbf{R}_2$, which always belong to the currents flowing through closed conductors L_1 and L_2, respectively.

Obviously, the action of the first element on the second one is determined only by the first two terms, (3.20) and (3.22), whereas the terms (3.24) and (3.25) are associated only with the second element. Therefore, if we talk about the magnetic field created by the first current at some point, we should integrate the expression (3.19) along the contour L_1 without last two terms, i. e.

[9]) In the SI system, where $I_1 dt = e$, this expression should be multiplied by $\mu_0 / 4\pi$.

$$\mathbf{B}_1 = \frac{\varsigma m_e}{2i_2} \oint_{L_1} \Omega_0^2 \left([d\mathbf{R}_1 \times \mathbf{r}] - \frac{\hbar}{2m_e v_{e1}} \delta \mathbf{R}_1 \right). \quad (3.26)$$

Consider examples with long straight conductors. In the case of parallel conductors, the spin part of the magnetic field does not affect the interaction forces of current elements, since the first term in parentheses in formulas (3.17)-(3.18) makes no contribution and for magnetic induction we obtain the Biot-Savart-Laplace law (3.20) with condition (3.21). If the conductors are perpendicular, then the usual magnetic forces defined by law (3.20) are supplemented by spin forces determined by addition (3.22) and additionally by additions (3.25) if the conductors move. If relations (3.17)-(3.18) are valid, then the influence of these spin forces can be detected using simple but precision experiments. In this regard, one should pay attention to the Graneau effect, which can be explained by the action of the longitudinal Ampère force, [66].

The problem which remains is the dependence of $\Omega_0(r)$, obtained not using independent inference, but only by comparing (3.20) with (3.12). Moreover, taking into account the retardation associated with the finite propagation velocity of the interaction should lead to a dependence of Ω_0 also on the relative velocity and to subsequent modification of the law (3.12).

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