The mass, radius, and magnetic moment of electrons and protons

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Abstract

The electron-proton scattering experiment by the PRad (proton radius) team at Jefferson Lab measured the root mean square (rms) charge radius of the proton as $r_p = 0.831 \pm 0.007_{stat} \pm 0.012_{syst}$ fm.[1] We offer a theoretical explanation of the new measurement based on a ring current model of a proton. This model further builds on older ring current and/or *Zitterbewegung* models for an electron and, hence, we will also highlight those results when relevant. We obtain a theoretical radius that is equal to four times the range parameter ($\hbar/m_p c$) in Yukawa's formula:

$$r_p = 4\hbar/m_p c \approx 0.841 \text{ fm}$$

The 1/4 factor stems from the energy equipartition theorem: using Wheeler's 'mass without mass' idea, we effectively assume half of the energy of a proton is explained by the electromagnetic, while the other half is attributed to the strong force, which we do not model but isolate from the analysis using the energy equipartition theorem.

As for the small difference between the theoretical and measured radius, we attribute this to the mathematical idealizations that underpin ring current models. While useful and necessary as a concept, we think pointlike electric charges with zero rest mass and/or zero dimension that, therefore, move at lightspeed, do not exist: they must have some (very) small dimension which explains the anomaly. We think mathematical idealization also explains the anomalous magnetic moment of an electron.

We think the calculations may offer a model of matter-particles in general.

[1] https://www.jlab.org/prad/collaboration.html

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1. The *recommended* CODATA values for the magnetic moment (mm) and the mass of a proton are the following:

$$\mu = 1.41060679736\times10^{-26}\ \text{J}\cdot\text{T}^{-1} \pm 0.00000000060\ \text{J}\cdot\text{T}^{-1}$$
$$m = 1.67262192369\times10^{-27}\ \text{kg} \pm 0.00000000051\times10^{-27}\ \text{kg}$$

We also have the following defined (or *exact*) values for the elementary charge, the velocity of light and Planck's constant[2]:

$$q_e = 1.602176634\times10^{-19}\ \text{C}$$
$$c = 299\,792\,458\ \text{m/s}$$
$$h = 6.62607015\ \text{J}\cdot\text{s}$$

From a mathematical point of view, we have a set of exact values (the physical constants) and a set of variables (mass, magnetic moment, radius, angular momentum of a proton, etcetera) that depend on them. The constants are related to the variables through a number of physical laws and theorems we accept to be valid.[3] The laws and theorems that we will use in this article are:

- The energy equipartition theorem
- The Planck-Einstein relation: $E = h\cdot f = \hbar\cdot\omega$
- The principle of relativity and the energy-mass equivalence relation: $E = m\cdot c^2$
- The force law, which states that a force acts upon a charge and changes its state of motion
- Maxwell's laws of electromagnetism

The system is completely determined – possible over-determined – and, hence, it is easy to derive the relations we seek: from the energy (or equivalent mass) of the particle (electron or proton), we can

[2] As part of the 2019 revision of SI units, *exact* numerical values were set for Planck's constant (h), the elementary electric charge (q_e), the Boltzmann constant (k_B), and Avogadro's constant (N_A). The fine-structure constant has now also been *defined* as:

$$\alpha = \frac{q_e^2}{4\pi\varepsilon_0\hbar c}$$

Its value still has an uncertainty of 1.5×10^{-19} on it, which it shares with the electric and magnetic constants because of the $c^2 = 1/\varepsilon_0\mu_0$ relation.

[3] We adhere to a Popperian view here: we accept them to be valid because they have resisted falsification (Popper, 1959).

calculate all other observables using the three above-mentioned constants (Planck's quantum of action, the elementary charge and the speed of light).

2. We believe a realist interpretation of quantum mechanics is possible, which means the structure of the laws and theorems should not only reflect some structure in our mind, but in Nature as well. We, therefore, imagine the magnetic moment of a proton to be created by a circular current of the elementary charge. It is, therefore, equal to the current times the area of the loop:

$$\mu = \mathrm{I}\pi a^2 = \mathrm{q_e} f \pi a^2 = \frac{\mathrm{q_e}\omega a^2}{2} \Leftrightarrow a = \sqrt{\frac{2\mu}{\mathrm{q_e}\omega}}$$

The frequency is equal to the velocity of the charge (v) divided by the circumference of the loop ($2\pi a$). However, for a reason the reader will readily understand after reading this article, we prefer to use the Planck-Einstein relation for the frequency. We believe the Planck-Einstein relation ($\mathrm{E} = \mathrm{h}\cdot f = \hbar\cdot\omega$) reflects a fundamental *cycle* in Nature. It, therefore, makes sense to also apply it to the ring current idea of a proton.[4] Hence, we write:

$$a = \sqrt{\frac{2\mu}{\mathrm{q_e}\omega}} = \sqrt{\frac{2\mu\hbar}{\mathrm{q_e E}}}$$

3. When applying this formula to an electron, we get the Compton radius of an electron ($a = \hbar/\mathrm{m}c$).[5] When applying the $a = \hbar/\mathrm{m}c$ radius formula to a proton, we get a value which is about 1/4 of the *measured* proton radius. We, therefore, need to consider using the same fraction of the proton energy to calculate the frequency:

$$\omega = \frac{1}{4}\frac{\mathrm{E}}{\hbar}$$

We should motivate the 1/4 factor, of course. We think the huge value of the proton mass and its tiny size – as compared to the mass and size of an electron – lend credibility to the assumption of another force (or another charge) inside of the proton.[6] Hence, the 1/4 factor combines (1) the energy

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[4] There is a long tradition of thinking of an electron in terms of a current ring. We may refer to Parson (1915), Schrödinger (1930) and, more recently, Hestenes (1990). It has been suggested it may also apply to protons (Consa, 2018) but, based on quick feedback from sympathetic researchers, we think this paper may be the first fully consistent theory in this regard. Alexander Burinskii, whose work on an integrated theory of the electron we admire greatly, drew our attention to earlier work of M.E. Shulman but Shulman's work seems to focus on leptons only (https://www.scirp.org/journal/paperinformation.aspx?paperid=78086). Giorgio Vassallo also sent useful references we will further examine over the coming months. We thank both for their quick feedback on our 'back-of-the-envelope' calculations.

[5] The reader who is not familiar with ring current and/or *Zitterbewegung* models of an electron may also not be familiar with the concept of a Compton *radius*. It is, of course, the reduced form of the Compton wavelength. We think of it as an actual (electric) charge radius (see Annex I and II).

[6] We use a model explaining mass as the equivalent mass of energy here, i.e. Wheeler's idea of "mass without mass". Energy is force over a distance and, hence, we can distinguish between electromagnetic energy (and the equivalent mass) and some new strong energy or mass, which is defined in terms of some strong force and the related strong charge. Our interpretation of Wheeler's "mass without mass" theory is explained in a previous paper (https://vixra.org/abs/2001.0453).

equipartition theorem (half of the energy or mass of the electron is to be explained by the strong force) and (2) Hestenes' interpretation of Schrödinger's *Zitterbewegung* interpretation of an electron.[7] We can, finally, do an actual calculation now:

$$a = \sqrt{\frac{2\mu}{q_e\omega}} = \sqrt{\frac{4 \cdot 2\mu\hbar}{q_e E}} = 2 \cdot \sqrt{\frac{2\mu\hbar}{q_e mc^2}} \approx 2 \cdot 0.35146 \ldots \times 10^{-15} \approx 0.703 \text{ fm}$$

The gap between the 0.831 and 0.703 values suggests we are missing a √2 factor:

$$a = \sqrt{\frac{\sqrt{2} \cdot 2\mu}{q_e\omega}} = 2 \cdot \sqrt{\frac{2\sqrt{2}\mu\hbar}{q_e E}} \approx 0.8359278 \text{ fm}$$

The difference between this calculated value (which used all of the precision of the CODATA values) and the PRad result is only about 0.005 fm[8], which is well within the statistical standard error of the measurement. Hence, it is a good result.

4. We now need to motivate the insertion of the √2 factor. We think there is some *real* magnetic moment here, which we denote as μ_L:

$$\mu_L = \sqrt{2} \cdot (1.41060679736 \times 10^{-26} \text{ J} \cdot \text{T}^{-1}) \approx 1.995 \times 10^{-26} \text{ J} \cdot \text{T}^{-1}$$

The subscript L in the μ_L notation stands for (orbital) angular momentum. A magnetic dipole will *precess* when placed in a magnetic field—which is what is being done when measuring the magnetic moment of a proton. We refer to Feynman[9] for an easy and very meaningful explanation of the relation between the magnitude of the actual – or imagined?[10] – angular momentum of a precessing magnet (L) and L_z (the *measured* quantum value) as:

$$\frac{L}{L_z} = \frac{\sqrt{j(j+1)} \cdot \hbar}{j \cdot \hbar} = \frac{\sqrt{j(j+1)}}{j}$$

For j = ½, we get:

$$= \frac{\sqrt{1/2(1/2+1)}}{1/2} = 2 \cdot \sqrt{\frac{3}{4}} = \sqrt{3}$$

[7] Hestenes summarizes his various papers as follows: "The electron is nature's most fundamental superconducting current loop. Electron spin designates the orientation of the loop in space. The electron loop is a superconducting LC circuit. The mass of the electron is the energy in the electron's electromagnetic field. Half of it is magnetic (potential) energy and half is kinetic." (email from Dr. David Hestenes to the author dated 17 March 2019)

[8] 0.831 – 0.836 = 0.005. We showed a result with seven digits to show the difference between this calculation and another value we will get out of another calculation (see Section 5).

[9] See: https://www.feynmanlectures.caltech.edu/II_34.html#Ch34-S7

[10] The difference between actual and imagined here depends on one's *interpretation* of quantum-mechanical laws. From what we present in this article, it should be obvious to the reader that we like to think this magnitude is something real. However, such metaphysical questions should not be the concern of the reader: he or she should just check our calculations so as to *verify* them. The *interpretation* of the results is a different matter.

We need a √2 factor. Hence, the spin number must be one:

$$\frac{L}{L_z} = \frac{\sqrt{j(j+1)} \cdot \hbar}{j \cdot \hbar} = \frac{\sqrt{1(1+1)}}{1} = \sqrt{2}$$

We know this assumption relates to the theoretical distinction between fermions and bosons. However, we will show the $j = 1$ assumption makes sense.

5. Because of the apparent randomness of this √2 factor, we must try the simpler approach to calculating the magnetic moment, which calculates the frequency from the $f = c/2\pi a$ formula:

$$\mu_L = I\pi a^2 = q_e f \pi a^2 = q_e \frac{c}{2\pi a} \pi a^2 = \frac{q_e c}{2} a$$

$$\Leftrightarrow a = \frac{2\mu_L}{q_e c} = \sqrt{2} \cdot \frac{2\mu}{q_e c} = \sqrt{2} \cdot 0.587 \times 10^{-15} \approx 0.83065344 \ldots \text{fm}$$

The result differs – slightly but significantly – from the result we obtained from using the Planck-Einstein relation for the frequency calculation (see Section 3). It is a *very* small difference. To be precise, it is, again, of the order of 0.005 fm. At the same time, this result is closer to the 0.831 PRad value: the difference is 0.000346656… fm only, which is less than 5% of the standard error of the PRad point estimate (0.007 fm).

6. In our calculations, we used the CODATA value for the magnetic moment of a proton in two different formulas for the radius, and we found the result is slightly different. While the two values do not differ significantly from the experimentally measured value for the proton radius – and, thereby, may be seen as a confirmation of the relevance of the PRad experiment – the two different values suggest we may think of some *unique* or *absolute* theoretical value for the magnetic moment. Indeed, because we have two equations for the radius a – and both of them involve μ_L – we can just equate them:

$$a = 2 \cdot \sqrt{\frac{2\mu_L \hbar}{q_e E}} = \frac{2\mu_L}{q_e c} \Leftrightarrow \sqrt{\frac{2\mu_L \hbar \cdot q_e^2 c^2}{q_e E \cdot \mu_L^2}} = 1$$

$$\Leftrightarrow \mu_L = \frac{2q_e}{m} \hbar \approx 2.02035 \times 10^{-26} \text{ J} \cdot \text{T}^{-1}$$

We get a value that is *almost* 2, but not quite. We think of this as a coincidence. We can now calculate an exact theoretical value for the proton radius:

$$a = \frac{2\mu_L}{q_e c} = \frac{2}{q_e c} \cdot \frac{2q_e \hbar}{m} = 4 \cdot \frac{\hbar}{mc} \approx 4 \cdot (0.21 \ldots \text{fm}) \approx 0.8413564 \ldots \text{ fm}$$

This value is not within the 0.831 ± 0.007 fm interval, but it is well within the wider $r_p = 0.831 \pm 0.007_{stat} \pm 0.012_{syst}$ fm interval.[11]

[11] We readily admit the insertion of the √2 factor needs further examination. We have a $\mu_L = 2q_e\hbar/m_p \approx 2.02\ldots$ J/T value for the magnetic proton which, we argue, differs from the CODATA value with a √2 factor because of precession. In contrast, the formula for the magnetic moment of an electron ($\mu_e = q_e\hbar/2m_e \approx 9.274$ J/T) gives us the

It may be noted that the $\hbar/m_p c$ factor is equal to the range parameter in Yukawa's formula for the nuclear potential.[12] As such, it is equivalent to the concept of the Compton radius for an electron.

7. We will now come back to the question of the spin number. Quantum-mechanical spin is expressed in units of $\hbar/2$ and, according to the Copenhagen interpretation of quantum mechanics, we should *not* try to think of it as a classical property—as something that has some *physical* meaning. We obviously disagree with this point of view. We think we can just use the classical $L = I \cdot \omega$ expression and substitute I and ω for the angular mass and the angular frequency.[13] To calculate the angular mass, one must assume some form factor: a hoop, a disk, a sphere or a shell are associated with different form factors. Our electron model[14] assumes that the effective mass of the electron is spread over a circular disk. We can, therefore, calculate the angular momentum as:

$$\mathrm{L} = I \cdot \omega = \frac{\mathrm{m}a^2}{2}\frac{c}{a} = \frac{\mathrm{m}c}{2} \cdot a = \frac{mc}{2} \cdot \frac{\hbar}{mc} = \frac{\hbar}{2}$$

Hence, we may effectively refer to an electron as a spin-1/2 particle. However, we do not think of this property as some obscure 'intrinsic' property of an equally obscure 'pointlike' particle: we think of the electron as an actual disk-like structure with some *torque* on it. Its angular momentum is, therefore, *real*.[15] Likewise, we think of the magnetic moment as being equally real[16]:

$$\mu = \mathrm{I} \cdot \pi a^2 = \frac{\mathrm{q_e} c \cdot \pi a^2}{2\pi a} = \frac{\mathrm{q_e} c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar \approx 9.274 \times 10^{-24}\ \mathrm{J} \cdot \mathrm{T}^{-1}$$

We think there is a confusion in regard to spin numbers and g-factors because we cannot directly measure the angular momentum: in real-life experiments, we measure the *magnetic moment*. Having

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CODATA value (apart from the anomaly, of course) *without the need for any correction factor because of precession*. If an electron is some ring current as well, then it must precess as well. We looked on the NIST site, but could not find much in terms of methodology. We sent an email to the NIST Public Affairs section with a request to guide us to the necessary materials in this regard. Annex I offers a full discussion of the perceived issue.

[12] We calculated this range parameter in previous papers. See, for example, our *Metaphysics of Physics* paper (https://vixra.org/abs/2001.0453).

[13] The reader should not confuse the *I* and I symbols. The first (*I* in *italics*) stands for angular mass (expressed in $\mathrm{kg \cdot m^2}$), while the second (I, normal type) is the symbol for current (expressed in C/s). We could have used different symbols, but we wanted to stick to the usual conventions. The reader will, of course, also not confuse the concepts of angular mass (*I*), also known as the moment of inertia, and angular momentum (L).

[14] See: https://vixra.org/abs/1905.0521.

[15] We will not engage in philosophical discussions here. We hope the reader understands what we want him/her to understand.

[16] The CODATA value for the magnetic moment includes the anomaly and is, therefore, slightly different from the theoretical value: $\mu_e \approx 9.285$ J/T. We think the difference between the theoretical and measured value is to be explained by a form factor: the circular point charge must have some (tiny) dimension and/or must have some (very tiny) non-zero rest mass. We believe the two letters of Gregory Breit to Gregory Breit to Isaac Rabi can easily be interpreted as Breit defending the idea that an intrinsic magnetic moment "of the order of $\alpha\mu_B$" is not anomalous at all. For more details on this conversation, see: Silvan S. Schweber, *QED and the Men Who Made It: Dyson, Feynman, Schwinger, and Tomonaga* , p. 222-223.

said that, it is true we can combine the two formulas to get the g-factor that is usually associated with the spin of an electron[17]:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g} = 2$$

We should now apply these ideas to the proton. The idea of a current ring – and the idea of precession, of course – strongly suggests we should, once again, think of the proton as a disk-like structure. However, not all of the mass is in the electromagnetic oscillation: we think half of it remains to be explained by what is referred to as the strong force (or, what amounts to the same, the idea of a strong charge).[18] We will, therefore, use a 1/4 rather than a 1/2 factor in the angular mass formula. This yields the following result:

$$\mathrm{L_p} = I_\mathrm{p} \cdot \omega = \frac{\mathrm{m_p} r_\mathrm{p}^2}{4} \cdot \frac{c}{r_\mathrm{p}} = \frac{\mathrm{m_p} c}{4} \cdot r_\mathrm{p} = \frac{\mathrm{m_p} c}{4} \cdot \frac{4\hbar}{\mathrm{m_p} c} = \hbar$$

Hence, our 'spin number' is equal to one. Most academics will cry wolf here: we cannot possibly believe a proton is a spin-one particle, can we? We think we can. We think there is no need for the concept of a spin number and a g-factor in a *realist* interpretation of quantum mechanics. We think of the angular momentum and the magnetic moment as being *real* and, hence, whatever else is being calculated – be it a spin number or a g-factor – is not very relevant. Worse, we think it *confuses* rather than clarifies the analysis. We, therefore, think our calculation of L_p is consistent. We also think it is consistent with the use of the √2 factor – as opposed to a √3 factor – to calculate what we think of as a *real* magnetic moment of a proton (μ_p).

We should, of course, relate this to the usual conventions. We will, therefore, do some calculations involving a g-factor. Instead of the *Bohr magneton* $\mu_B = q_e\hbar/2m_e$, we should use the *nuclear magneton* $\mu_N = q_e\hbar/2m_p$. We get the following result:

$$\boldsymbol{\mu}_\mathbf{L} = \mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m_p}}\right)\mathbf{L} \Leftrightarrow \frac{2\mathrm{q_e}}{\mathrm{m_p}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m_p}}\hbar \Leftrightarrow \mathrm{g} = 4$$

That is, of course, a strange number: the CODATA value is about 5.5857. However, this result depends on the use of a theoretical $\hbar/2$ value for the angular momentum. It also uses the CODATA value for the magnetic moment—as opposed to our μ_L value, which is the CODATA value corrected for precession. Hence, the CODATA calculation of the g-factor is this:

$$\mu_\mathrm{p} = \mathrm{g_p}\frac{\mathrm{q_e}}{2\mathrm{m_p}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g_p} = \frac{4\mu_\mathrm{p}\mathrm{m_p}}{\hbar\mathrm{q_e}} = 5.58569\ldots$$

We get a slightly different value when we insert our newly found theoretical value for the magnetic moment:

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[17] We used vector notation (**boldface**) to draw attention, once again, to our *physical* interpretation of what might be going on: the *minus* sign (–) is there because, in the case of an electron, the magnetic moment and angular momentum vectors have opposite directions.

[18] See our paper on the idea of a strong force and/or a strong charge: https://vixra.org/abs/2001.0453.

$$\mu_p = g_p \frac{q_e}{2m_p}\frac{\hbar}{2} \Leftrightarrow g_p = \frac{4\mu_p m_p}{\hbar q_e} = \frac{4}{\sqrt{2}} \cdot \frac{\mu_L m_p}{\hbar q_e} = \frac{4}{\sqrt{2}} \cdot \frac{2q_e\hbar}{m_p} \cdot \frac{m_p}{\hbar q_e} = \frac{8}{\sqrt{2}} = 5.65685 \ldots$$

How can we explain the difference?

8. The difference of about 0.071 (about 1.2%) is not surprising: the difference is of the same order of magnitude as the difference between our theoretical value for the radius – which is based on the assumption of a pointlike charge – and the actually measured radius. We think this difference confirms both the theory as well as the PRad measurement. We anticipate theorists and experimenters to argue about the next digit of the anomalous magnetic moment of a proton in pretty much the same way as they have been arguing about the anomalous magnetic moment of an electron. We think both 'anomalies' are there because of the mathematical idealization in our assumptions: the pointlike charge may have zero rest mass (or some value *very* close to zero), but we should not assume it has no dimension whatsoever.[19]

Why not? We can only give a philosophical answer here: something that has no dimension whatsoever probably exists in our mind only. Something real – like a charge – *must* have *some* dimension.

From what we write above, the reader will understand that we think some of the generalizations in quantum physics – most notably, the concept of bosons – are not necessary to understand Nature.

Jean Louis Van Belle, 4 February 2020

[19] See our paper on Consa's calculations of the anomalous magnetic moment (https://vixra.org/abs/2001.0264), which also references our approach to the matter (https://vixra.org/abs/1906.0007).

Annex I: Precession of electrons and protons

When inserting the CODATA value for the magnetic moment of an electron in the two formulas that we have used to calculate the theoretical magnetic moment of a proton, we get the electron's Compton radius. Calculating the frequency using the geometric formula (*f* = *c*/2π*a*), we get:

$$\mu_e = I\pi a^2 = q_e f \pi a^2 = q_e \frac{c}{2\pi a} \pi a^2 = \frac{q_e c}{2} a \Leftrightarrow a = \frac{2\mu_e}{q_e c} \approx 0.3866607 \ldots \text{pm}$$

Calculating the frequency using the Planck-Einstein relation (*f* = E/*h*), we get the same value:

$$\mu_e = I\pi a^2 = q_e f \pi a^2 = \frac{q_e \omega a^2}{2} \Leftrightarrow a = \sqrt{\frac{2\mu_e}{q_e \omega}} = \sqrt{\frac{2\mu_e \hbar}{q_e E}} \approx 0.3863831 \ldots \text{pm}$$

There is, once again, a small difference between the two values and we can, therefore, equate the two formulas to calculate a theoretical value for the magnetic moment of an electron[20]:

$$a = \sqrt{\frac{2\mu_e \hbar}{q_e m c^2}} = \frac{2\mu_e}{q_e c} \Leftrightarrow \sqrt{\frac{\mu_e \hbar \cdot q_e^2 c^2}{2 q_e m c^2 \cdot \mu_e^2}} = 1$$

$$\Leftrightarrow \mu_e = \frac{q_e}{2m} \hbar \approx 9.274 \ldots \times 10^{-24} \text{ J} \cdot \text{T}^{-1}$$

The reader should note this differs slightly from the CODATA recommended value – which is equal to about 9.284 $J \cdot T^{-1}$, which is based on experimental measurement. Again, we think this difference confirms both the theory as well as the measurement: we think the 'anomaly' is there because of the mathematical idealization in our assumptions: the pointlike charge may have zero rest mass (or some value *very* close to zero), but we should not assume it has no dimension whatsoever. We may also assume its velocity is, perhaps, *nearly* lightspeed but not quite. We, therefore, think it can be explained using classical physics.

We can now re-insert the theoretical magnetic moment in our formulas for the radius to calculate the theoretical radius of the electron:

$$a = \sqrt{\frac{2\mu_e \hbar}{q_e m c^2}} = \sqrt{\frac{2 q_e \hbar^2}{2m q_e m c^2}} = \frac{2\mu_e}{q_e c} = \frac{2 q_e \hbar}{2m q_e c} = \frac{\hbar}{mc} \approx 0.3861592 \ldots \text{pm}$$

This is a nice result, but let us explore it some more. Do we assume precession and, if so, what factor should we use? The formulas do not suggest so. Our equality no longer holds. Indeed, if we would denote the so-called *real* magnetic moment as $\mu_J = \sqrt{n} \cdot \mu_e$[21] and then our equality becomes this:

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[20] We should add a *minus* sign because of the opposite direction of magnetic moment and angular momentum. However, here we are only calculating magnitudes.

[21] We use a *J* instead of *L* so as to avoid confusion with the μ_L symbol which we used for the angular momentum of a proton. The physical meaning of μ_L and μ_J is, therefore, exactly the same. Note that *n* is not necessarily a whole number. For example, when inserting *j* = 3/2 in the formula for the L/L_z or J/J_z ratio, we get $\sqrt{n} = \sqrt{15}/3$.

$$a = \sqrt{\frac{2\sqrt{n}\mu_e\hbar}{q_e mc^2}} = \frac{2\sqrt{n}\mu_e}{q_e c} \Leftrightarrow \frac{\sqrt{n}\mu_e\hbar \cdot q_e^2 c^2}{2q_e mc^2 \cdot n\mu_e^2} = \frac{\sqrt{n}q_e\hbar}{2m \cdot n\mu_e} = 1 \Leftrightarrow \frac{\sqrt{n}}{n} \cdot \frac{q_e\hbar}{2m} \cdot \frac{2m}{q_e\hbar} = 1 \Leftrightarrow \sqrt{n} = n$$

This equality only holds for n = 1. It is very puzzling: should we assume that the CODATA value for the magnetic moment of an electron has already been corrected to include the idea of precession?

While contemplating this possibility, we should also note we did not use a 1/2 factor in our f = E/h formula. If we used such factor for our proton calculations – arguing half of the energy is kinetic and the other half is electromagnetic – then we should use such factor for our electron calculations as well. Let us see if this gets us anywhere. Substituting ω for E/2$\hbar$ instead of E/$\hbar$, we get this formula for the electron radius:

$$\mu = \mathrm{I}\pi a^2 = \mathrm{q}f\pi a^2 = \frac{\mathrm{q}\omega a^2}{2} \Leftrightarrow a = \sqrt{\frac{2\mu}{\mathrm{q}\omega}} = \sqrt{\frac{4\mu\hbar}{\mathrm{qE}}} = 2 \cdot \sqrt{\frac{\mu\hbar}{\mathrm{qE}}}$$

The other way to calculate μ was like this:

$$\mu = \mathrm{I}\pi a^2 = \mathrm{q}f\pi a^2 = \mathrm{q}\frac{c}{2\pi a}\pi a^2 = \frac{\mathrm{q}c}{2}a \Leftrightarrow a = \frac{2\mu}{\mathrm{q}c}$$

Equating both equations for a gives us this:

$$a = 2 \cdot \sqrt{\frac{\mu\hbar}{\mathrm{q}mc^2}} = \frac{2\mu}{\mathrm{q}c} \Leftrightarrow \sqrt{\frac{\mu\hbar}{\mathrm{q}mc^2} \cdot \frac{\mathrm{q}^2c^2}{\mu^2}} = 1 \Leftrightarrow \mu = \frac{\mathrm{q}}{\mathrm{m}}\hbar \approx 18.548 \times 10^{-24}\ \mathrm{J} \cdot \mathrm{T}^{-1}$$

This is, unsurprisingly, *twice* the CODATA value for the magnetic moment. The corresponding radius is, unsurprisingly, *twice* the Compton radius:

$$a = 2 \cdot \sqrt{\frac{\mu\hbar}{\mathrm{q}mc^2}} = 2 \cdot \sqrt{\frac{\mathrm{q}\hbar^2}{\mathrm{q}m^2c^2}} = 2\frac{\hbar}{mc} = \frac{2\mu}{\mathrm{q}c} = \frac{2\mathrm{q}\hbar}{\mathrm{q}mc} = 2\frac{\hbar}{mc}$$

This is very weird, of course, even if the math here are very simple.[22] Let us quickly examine if this strange result respects conventional wisdom in regard to spin numbers and g-factors. The formula to be used depends, once again, on our assumption in regard to the form factor:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L}$$

Do we think of the electron as a loop or a hoop, or do we think its mass is effectively spread out over a disk? The formulas below show that we only get the conventional g-factor (g = 2) if we assume, once

[22] It is especially weird because most *Zitterbewegung* theorists, including Hestenes (1990), Burinskii (2008, 2016) and Gauthier (2019), etcetera arrive at the conclusion that the radius of the oscillation must be equal to *half* the Compton radius. Oliver Consa (2018) is one of the few physicists who also equate the electron radius with the reduced Compton wavelength. As for the math, one should note that the current is inversely proportional to the radius (f = c/2πa) but that the surface of the loop (πa^2) is proportional to the *square* of the radius. The magnetic moment (μ)is the product of both. Hence, the radius (a) will be proportional to μ.

again, that the mass of the electron is spread out over a disk, which allows us to insert the necessary ½ factor [23]:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\hbar \Leftrightarrow \mathrm{g} = 2$$

$$\Leftrightarrow \mathrm{L} = I \cdot \omega = \frac{\mathrm{m}a^2}{2}\frac{c}{a} = \frac{\mathrm{m}c}{2} \cdot a = \frac{mc}{2} \cdot \frac{2\hbar}{mc} = \hbar$$

We are not sure how to make sense of the 1/2 factor and the thorny question of quantum-mechanical precession. Perhaps they are related to two different concepts of the radius: while we can calculate the radius of a loop of a pointlike charge, our model suggests the electromagnetic field will extend beyond the current ring. This may result in an *effective* charge radius which is larger than the Compton radius. It may also explain the 1/2 factor we used for the energy: if we do *not* include the energy of the magnetic field, then we get a radius that is only half the Compton radius. We welcome suggestions as to how to improve on this rather sloppy answer.

As for the methodology used to calculate the CODATA value of the magnetic moment of an electron, we have requested NIST to provide us with more details. This may or may not lead to future revisions of some of the remarks we presented in this paper.

[23] This allows us to insert a 1/2 factor in the formula for the angular mass.

Annex II: The calculation of the Compton wavelength from scattering

The reader who is not familiar with ring current and/or *Zitterbewegung* models of an electron may also not be familiar with the concept of a Compton *radius*. It is, of course, the reduced form of the Compton wavelength. From our paper, it is obvious we think of it as an actual (electric) charge radius. However, for the convenience of the reader, we remind him how one gets the Compton wavelength from more standard (mainstream) calculations.[24] The reader should note that the calculations do *not* involve any quantum-mechanical weirdness: the uncertainty principle, for example, is *not* being invoked. In fact, we wanted to add this annex to illustrate how classical basic quantum-mechanical calculations can actually be.[25]

Compton scattering is referred to as inelastic because the frequency of the incoming and outgoing photon are different. We, therefore, suspect an electron first absorbs the photon, before re-emitting a new photon and – using the energy difference between the two photons – acquiring some linear momentum. Hence, before the photon hits it, the electron is thought of as being stationary. Two classical laws govern the process: (1) energy conservation, and (2) momentum conservation.

1. The energy conservation law tells us that the total (relativistic) energy of the electron ($E = m_e c^2$) and the incoming photon must be equal to the total energy of the outgoing photon and the electron, which is now moving and, hence, includes the kinetic energy from its (linear) motion. We use a prime (') to designate variables measured after the interaction. Hence, $E_{e'}$ and $E_{\gamma'}$ are the energy of the moving electron (e') and the outgoing photon (γ') in the state after the event. We write:

$$\mathrm{E_e} + \mathrm{E_\gamma} = \mathrm{E_{e'}} + \mathrm{E_{\gamma'}}$$

We can now use (i) the mass-energy equivalence relation ($E = mc^2$), (ii) the Planck-Einstein relation for a photon ($E = h \cdot f$) and (iii) the relativistically correct relation ($E^2 - p^2c^2 = m^2c^4$) between energy and momentum for *any* particle – charged or non-charged, matter-particles or photons or whatever other distinction one would like to make[26] – to re-write this as[27]:

$$\mathrm{m_e}c^2 + hf = \sqrt{\mathrm{p_{e'}^2}c^2 + \mathrm{m_e^2}c^4} + hf' \Leftrightarrow \mathrm{p_{e'}^2}c^2 = (hf - hf' + \mathrm{m_e^2}c^4)^2 - \mathrm{m_e^2}c^4 \ (1)$$

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[24] The presentation in this annex reflects standard analysis, but we relied on Prof. Dr. Barton Zwiebach's introduction to quantum mechanics in MIT's edX course on quantum mechanics (8.01.1*x*).

[25] We repeat: the presentation in this annex is mainstream analysis. For a more speculative – but also more *intuitive* – theory of what might actually be going on, we refer to our own classical analysis of Compton scattering (https://vixra.org/abs/1912.0251).

[26] This is, once again, a standard textbook equation but – if the reader would require a reminder of how this formula comes out of special relativity theory – we may refer him to the online *Lectures* of Richard Feynman. Chapters 15 and 16 offer a concise but comprehensive overview of the basics of relativity theory and section 5 of Chapter 6 (https://www.feynmanlectures.caltech.edu/I_16.html#Ch16-S5) gives the reader the formula he needs here. It should be noted that we dropped the 0 subscript for the rest mass or energy: $m_0 = m$. The *prime* symbol (') takes care of everything here and so you should carefully distinguish between primed and non-primed variables.

[27] We realize we are cutting some corners. We trust the reader will be able to *google* the various steps in-between.

2. This looks rather monstrous but things will fall into place soon enough because we will now derive another equation based on the momentum conservation law. Momentum is a vector, and so we have a vector equation here[28]:

$$\vec{p}_\gamma = \vec{p}_{\gamma'} + \vec{p}_{e'} \Leftrightarrow \vec{p}_{e'} = \vec{p}_\gamma - \vec{p}_{\gamma'}$$

For reasons that will be obvious later – it is just the usual thing: ensuring we can combine two equations into one, as we did with our formulas for the radius – we square this equation and multiply with Einstein's constant c^2 to get this[29]:

$$\vec{p}_{e'}^{2} = \vec{p}_\gamma^{2} + \vec{p}_{\gamma'}^{2} - 2\vec{p}_\gamma\vec{p}_{\gamma'} \Leftrightarrow p_{e'}^{2}c^2 = p_\gamma^2 c^2 + p_{\gamma'}^2 c^2 - 2(p_\gamma c)(p_{\gamma'} c)\cdot\cos\theta$$

$$\Leftrightarrow p_{e'}^{2}c^2 = h^2f^2 + h^2f'^2 - 2(hf)(hf')\cdot\cos\theta \;(2)$$

3. We are now ready for the *punch line*, as Prof. Dr. Zwiebach refers to it. We can combine equations (1) and (2) to get this:

$$p_{e'}^{2}c^2 = (Eq.1) = (Eq.1) = (hf - hf' + m_e^2c^4)^2 - m_e^2c^4 = h^2f^2 + h^2f'^2 - 2(hf)(hf')\cdot\cos\theta$$

The reader will be able to do the horrible stuff of actually squaring the expression between the brackets and verifying only cross-products remain. We get:

$$(hf - hf')m_ec^2 = h(f - f')m_ec^2 = h^2ff'(1 - \cos\theta)$$

Multiplying both sides of the equation by the $1/hm_eff'$ constant yields the formula we were looking for:

$$\frac{(f - f')m_ec}{f\cdot f'} = \frac{fm_ec - f'm_ec}{f\cdot f'} = \frac{h}{m_ec}(1-\cos\theta) \Leftrightarrow \frac{c}{f'} - \frac{c}{f} = \lambda' - \lambda = \frac{h}{m_ec}(1-\cos\theta)$$

The factor on the left-hand side of the right-hand side of the equation is, effectively, a wavelength. Not a radius. We trust the reader will be able to connect the dots here. If not, we refer him or her to our photon model.[30]

[28] We could have used **boldface** to denote vectors, but the calculations make the arrow notation more convenient here. So as to make sure our reader stays awake, we note that the objective of the step from the first to the second equation is to derive a formula for the (linear) momentum of the electron *after* the interaction. As mentioned, the linear momentum of the electron *before* the interaction is zero, because its (linear) velocity is zero: $\mathbf{p}_e = \mathbf{0}$.

[29] We do not want to sound disrespectful when referring to c^2 as Einstein's constant. It has a deep meaning, in fact. Einstein does not have any fundamental constant or unit named after him. Nor does Dirac. We think c^2 would be an appropriate 'Einstein constant'. Also, in light of Dirac's remarks on the nature of the strong force, we would suggest naming the unit of the strong charge after him. More to the point, note these steps – finally ! – incorporated the directional aspect we needed for the analysis. When everything is said and done, we don't only want some value for the Compton wavelength ($\lambda_C = h/mc$), but for the scattering angle (θ) as well! Note that we also use the rather obvious E = pc relation for photons in the transformation of formulas here.

[30] See our classical theory of light: https://vixra.org/abs/2001.0345.