Kaluza in Four Dimensions (with Complex Time)

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Abstract

A number of researchers have employed complex time. If time is complex, it is reasonable to allow the time components of the metric tensor to also be complex. The four imaginary quantities in the metric tensor can be shown to be consistent with the values of the magnetic vector potential. The (four-dimensional complex) line element is then shown to be functionally identical to the Kaluza 5-dimentional line element. The four dimensional (complex) metric therefore inherits the results Kaluza obtained when he included the magnetic vector potential in a five dimensional metric.

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I. INTRODUCTION

Various authors (e.g. Hawking[1, 2], Yousef[3], Fujii[4], and Frederick[5] building on a previous paper[6]) have explored the idea of complex time. In particular, complex time is required in Frederick's model of quantum mechanics. In that model, the imaginary time component is 'rolled up (as is the fifth-dimension in the Kaluza model).

If the time component is complex, one might then expect that the metric tensor $g_{0(0,1,2,\text{ and }3)}$ and $g_{(0,1,2,\text{ and }3)0}$ components would also be complex. This then gives four additional variables in the metric tensor. The Kaluza 5-dimensional model, incidentally, has five additional variables. But one of those variables, g_{44}, is problematic; It's often taken to be superfluous and set equal to one[7].

Those four additional variables in the 4-dimensional model are shown to be the magnetic vector potential components. Thus it is shown that the 4-dimensional complex metric reproduces the properties of the 5-dimensional Kaluza metric. So one then has all the Kaluza results in four rather than five dimensions. This then supports the use of complex time.

II. 5-DIMENSIONS: KALUZA STANDARD MODEL

About a hundred years ago, Theodor Kaluza proposed adding a fifth-dimension in order to bring the magnetic vector into geometry[8, 9]. So that the dimension not affect observable geometry, the fifth dimension was assumed to be 'rolled up' at the Planck scale. This feature[10], the 'cylinder condition', says to drop all derivatives with respect to the fifth dimension coordinate.

The Kaluza Metric Tensor (which we'll refer to as KMT) is,

$$\text{KMT}\equiv\left\{\begin{matrix} g_{00} & g_{01} & g_{02} & g_{03} & A_0 \\ g_{10} & g_{11} & g_{12} & g_{13} & A_1 \\ g_{20} & g_{21} & g_{22} & g_{23} & A_2 \\ g_{30} & g_{31} & g_{32} & g_{33} & A_3 \\ A_0 & A_1 & A_2 & A_3 & 1 \end{matrix}\right\}$$

where A_0, A_1, A_2, and A_3 are the components of the electromagnetic 4-potential.

We'll call the fifth-dimension 'w'

The line element $\equiv g_{AB}dx^A dx^B$ (where as per convention, Roman letters range from zero through four).

The Kaluza Line Element (KLE) then, is,

KLE$\equiv g_{00}dt^2+2g_{01}dtdx+2g_{02}dtdy+2g_{03}dtdz+2A_0dtdw+g_{11}dx^2+2g_{12}dxdy+2g_{13}dxdz+2A_1dxdw+g_{22}dy^2+2g_{23}dydz+2A_2dydw+g_{33}dz^2+2A_3dzdw+dw^2$

III. 4–DIMENSIONS WITH COMPLEX TIME: KALUZA EQUIVALENT

The (somewhat immodestly named) Frederick Metric Tensor (which we'll refer to as FMT) is,

$$\text{FMT}\equiv\left\{\begin{matrix} g_{00}+i\alpha_0 & g_{01}+i\alpha_1 & g_{02}+i\alpha_2 & g_{03}+i\alpha_3 \\ g_{10}+i\alpha_1 & g_{11} & g_{12} & g_{13} \\ g_{20}+i\alpha_2 & g_{21} & g_{22} & g_{23} \\ g_{30}+i\alpha_3 & g_{31} & g_{32} & g_{33} \end{matrix}\right\}$$

where α_0,α_1,α_2,and,α_3 are quantities which we will show to be identifiable with the electromagnetic 4-potential (A_0,A_1,A_2,A_3).

Analogously to the KLE, the Frederick Line Element (FLE) is,

FLE$\equiv g_{00}dt_c^2+i\alpha_0dt_c^2+2g_{01}dt_cdx+2i\alpha_1dt_cdx+2g_{02}dt_cdy+2i\alpha_2dt_cdy+2g_{03}dt_cdz+2i\alpha_3dt_cdz+g_{11}dx^2+2g_{12}dxdy+2g_{13}dxdz+g_{22}dy^2+2g_{23}dydz+g_{33}dz^2$

where t_c represents complex time; $t_c=t_r+it_i$ where t_r and t_i represent real and imaginary time respectively, and the αs are a yet to determined function of the As.

Now replacing dt_c with dt_r+idt_i,

FLE$\equiv g_{00}(dt_r+idt_i)^2+i\alpha_0(dt_r+idt_i)^2+2g_{01}(dt_r+idt_i)dx+2i\alpha_1(dt_r+idt_i)dx+2g_{02}(dt_r+idt_i)dy+2i\alpha_2(dt_r+idt_i)dy+2g_{03}(dt_r+idt_i)dz+2i\alpha_3(dt_r+idt_i)dz+g_{11}dx^2+2g_{12}dxdy+2g_{13}dxdz+g_{22}dy^2+2g_{23}dydz+g_{33}dz^2$

Expanding.

$FLE=g_{00}dt_r^2-g_{00}dt_i^2+2ig_{00}dt_rdt_i+i\alpha_0dt_r^2-i\alpha_0dt_i^2-2\alpha_0dt_rdt_i+2g_{01}dt_rdx+2ig_{01}dt_idx+2i\alpha_1dt_rdx-2\alpha_1dt_idx+2g_{02}dt_rdy+2ig_{02}dt_idy+2i\alpha_2dt_rdy-2\alpha_2dt_idy+2g_{03}dt_rdz+2ig_{03}dt_idz+2i\alpha_3dt_rdz-2\alpha_3dt_idz+g_{11}dx^2+2g_{12}dxdy+2g_{13}dxdz+g22dy^2+2g_{23}dydz+g_{33}dz^2$

t_r is equivalent to KLE's t, so in FLE, we'll replace t_r with t.

$FLE=g_{00}dt^2-g_{00}dt_i^2+2ig_{00}dtdt_i:+i\alpha_0dt^2-i\alpha_0dt_i^2-2\alpha_0dtdt_i+2g_{01}dtdx+2ig_{01}dt_idx:+2i\alpha_1dtdx-2\alpha_1dt_idx:+2g_{02}dtdy+2ig_{02}dt_idy:+2i\alpha_2dtdy-2\alpha_2dt_idy:+2g_{03}dtdz+2ig_{03}dt_idz:+2i\alpha_3dtdz-2\alpha_3dt_idz:+g_{11}dx^2+2g_{12}dxdy+2g_{13}dxdz+g_{22}dy^2+2g_{23}dydz+g_{33}dz^2$

w is the Kaluza fifth-dimension. It is 'rolled up'.

t_i is the Frederick imaginary component of time. It is also 'rolled up'. Hawking has said that the imaginary time component is orthogonal to the real time component, and therefore acts as if it were a space coordinate. Accordingly, we are justified in associating it_i with w. I.e. $t_i = -iw$.

$FLE = g_{00}dt^2 - g_{00}(-idw)(-idw) + 2ig_{00}dt(-idw) + i\alpha_0 dt^2 - i\alpha_0(-idw)(-idw) - 2\alpha_0 dt(-idw) + 2g_{01}dtdx + 2ig_{01}(-idw)dx + 2i\alpha_1 dtdx - 2\alpha_1(-idw)dx + 2g_{02}dtdy + 2ig_{02}(-idw)dy + 2i\alpha_2 dtdy - 2\alpha_2(-idw)dy + 2g_{03}dtdz + 2ig_{03}(-idw)dz + 2i\alpha_3 dtdz - 2\alpha_3(-idw)dz + g_{11}dx^2 + 2g_{12}dxdy + 2g_{13}dxdz + g_{22}dy^2 + 2g_{23}dydz + g_{33}dz^2$

$FLE = g_{00}dt^2 + g_{00}dw^2 + 2g_{00}dtdw + i\alpha_0 dt^2 + i\alpha_0 dw^2 + 2i\alpha_0 dtdw + 2g_{01}dtdx + 2g_{01}dwdx + 2i\alpha_1 dtdx + 2i\alpha_1 dwdx + 2g_{02}dtdy + 2g_{02}dwdy + 2i\alpha_2 dtdy + 2i\alpha_2 dwdy + 2g_{03}dtdz + 2g_{03}dwdz + 2i\alpha_3 dtdz + 2i\alpha_3 dwdz + g_{11}dx^2 + 2g_{12}dxdy + 2g_{13}dxdz + g_{22}dy^2 + 2g_{23}dydz + g_{33}dz^2$

Now we'll collect real and imaginary terms,

$FLE = g_{00}dt^2 + g_{00}dw^2 + 2g_{00}dtdw + 2g_{01}dtdx + 2g_{01}dwdx + 2g_{02}dtdy + 2g_{02}dwdy + 2g_{03}dtdz + 2g_{03}dwdz + g_{11}dx^2 + 2g_{12}dxdy + 2g_{13}dxdz + g_{22}dy^2 + 2g_{23}dydz + g_{33}dz^2 + i(\alpha_0 dt^2 + \alpha_0 dw^2 + 2\alpha_0 dtdw + 2\alpha_1 dtdx + 2\alpha_1 dwdx + 2\alpha_2 dtdy + 2\alpha_2 dwdy + 2\alpha_3 dtdz + 2\alpha_3 dwdz)$

The line element, $ds^2 = g_{AB}dx^A dx^b$, must be real, so the imaginary components must sum to zero:

$i(\alpha_0 dt^2 + \alpha_0 dw^2 + 2\alpha_0 dtdw + 2\alpha_1 dtdx + 2\alpha_1 dwdx + 2\alpha_2 dtdy + 2\alpha_2 dwdy + 2\alpha_3 dtdz + 2\alpha_3 dwdz) = 0$

If the above is zero, then -i times the above is also zero. We can then replace the above by -i times the above in the FLE.

Multiplying the above by -i, and then factoring,

$\alpha_0(dt^2 + dw^2 + 2dtdw) + 2\alpha_1(dtdx + dwdx) + 2\alpha_2(dtdy + dwdy) + 2\alpha_3(dtdz + dwdz) = 0$

$\alpha_0(dt + dw)^2 + 2\alpha_1 dx(dt + dw) + 2\alpha_2 dy(dt + dw) + 2\alpha_3 dz(dt + dw) = 0$

Dividing by dt+dw,

$\alpha_0(dt + dw) + 2\alpha_1 dx + 2\alpha_2 dy + 2\alpha_3 dz = 0$, and multiplying by dw,

$\alpha_0(dt + dw)dw + 2\alpha_1 dxdw + 2\alpha_2 dydw + 2\alpha_3 dzdw = 0$.

t and w are both real time coordinates. It seems reasonable to take dt to be equal to dw so the above becomes,

$2\alpha_0 dtdw + 2\alpha_1 dxdw + 2\alpha_2 dydw + 2\alpha_3 dzdw = 0$.

Note now the KLE with terms containing A collected,

KLE$\equiv g_{00}dt^2 + 2g_{01}dtdx + 2g_{02}dtdy + 2g_{03}dtdz + g_{11}dx^2 + 2g_{12}dxdy + 2g_{13}dxdz + g_{22}dy^2 +$

$2g_{23}dydz + g_{33}dz^2 + dw^2 + (2A_0dtdw + 2A_1dxdw + 2A_2dydw + 2A_3dzdw)$

Notice now that the A terms in the KLE are identical to the α terms in the FLE. So we are justified in presuming that the FLE α terms are indeed the magnetic vector potential terms of the KLE. So then, the two line elements are equivalent. And that shows that we get the full Kaluza formalism in four (rather than five) dimensions, but with complex time. And again, note that the problematic g_{44} component of the Kaluza metric is absent in the Frederick metric.

IV. INTERPRETATION OF COMPLEX TIME AND SPECULATIONS

Steven Hawking has written that we should not worry about the interpretation of complex time as long as the concept has been found useful. And in an earlier paper of mine[5] it is more than useful. It is necessary to, among other things, provide a mechanism whereby fixed-volume space-time granules can be created or annihilated as the universe, or parts thereof expand or contract. Still, it would be good to provide a meaning for complex time.

Time has a particular difference from space; time moves forward (or backwards). We then consider time made up of two parts; a conventional real coordinate going from minus infinity (or from the beginning of time) to plus infinity (or to the end of time). And a second part represented by the 'rolled-up' imaginary component (acting sort of like a phase) giving the relative speed and direction (clockwise or counterclockwise) of time. Complex time, therefore, allows a more nuanced idea about dimensions.

However, insofar as we'd like all dimensions to be treated similarly, a complex time (with other dimensions being real) is problematic.

The following is highly speculative:

Quantum mechanics does have stationary states, but, due to he uncertainty principle, no stationary masses. Perhaps, as with the case of time, motion is an intrinsic property of space-time (and not a property of mass). In that case, the other three dimensions could well have imaginary components perhaps determining the local arrangement of space-time granules.

That would give another twelve free variables. As imaginary time provides a geometric representation of the magnetic (and/or electroweak) force, Perhaps the twelve variables might represent three additional forces. The hypothesized fifth force[11] might be one of them. The strong force is not monotonic. We wonder if the strong force might in reality be two forces; a sharply peaked attractive force decreasing as one moves away from a mass, and a gradually increasing repulsive force.

The three space dimensions are generally assumed to be interchangeable. That might not be the case. We could assume a privileged global reference frame, namely where the cosmic black-body radiation is isotropic. And because of parity, the other two dimensions would not be interchangeable.

It has been said that in General Relativity, 'mass tells space how to bend, and space tells mass how to move'. In light of the above, perhaps with quantum mechanics, mass tells space how to organize and space tells mass how to propagate.

We note that Richard Hutchin[12] has proposed a complex *Hermitian* metric tensor and has indeed generated the electromagnetic field as well as three more fields. (Our metric is *symmetric*.)

V. CONCLUSION

A four-dimensional space-time with complex time can replicate the Kaluza five-dimensional model. As a result, Kaluza theory has been brought into the Frederick discrete stochastic space-time model. And this work perhaps adds additional justification for complex time in the model. Further, the four (rather than five) dimensional metric might obviate the need for the Klein modification[13] of the Kaluza metric.

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