FROM CLASSICAL LOGIC TO NEBULOUS LOGIC

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May, 2017

ABSTRACT

This article presents a small digression that begins with the presentation of deductive logic and advances with the expansion of the valid states of propositions (until then only true or false) thus uncovering the multivalue logic and reaching the fuzzy logic, also known as nebula or fuzzy.

Key words: classic logic, multivalue, fuzzy, fuzzy.

I. INTRODUCTION

This work presents an analytical escalation that begins in classical logic, passes through multivalued logic, and reaches the nebulous logic. In the first part we present some considerations about classical logic and its importance in the evolution of mathematics from Aristotle and Euclid to the beginning of the 20th century where several advances, especially in mathematics and related areas, opened new horizons of research. In the second part we present the multivalued logic as an intermediate position between the classical logic and the diffuse one through the ternary logic of Klenne and the tetravalence of Belnap. We also quote Shramko's sixteen-state logic showing that multivalue logic is not limited to three or four states. Finally, we come to the nebulous logic (also called fuzzy or fuzzy) where we present some of its main features and some notes about its applications.

I. CLASSICAL LOGIC

The logic can be defined, according to Bishop [1], as *"the science that studies the principles and methods that establish the conditions of validity and invalidity of arguments."* According to Aristotle [2] logic studies thought, the laws and rules that control it so that it may prove to be true.

The classical logic is based on 3 basic principles: the *principle of identity* (every proposition is identical to itself), the principle of *non-contradiction* (a proposition cannot be true and false at the same time) and the *principle of the third excluded* (every proposition is either true or false, and there is no third value that it can assume) [1].

An example of the practical application of classical logic are the axioms of Euclidean geometry. From a finite set of assumed truths many theorems are proven using only deductive logic. Through it is possible to conclude whether a given sentence is true or false from its premises, starting from the veracity of the latter. *"Deduction only organizes and specifies the knowledge one already possesses"* [3], although the result of the theorems of Euclidean geometry originating from its axioms are often against intuitive. It is hard to imagine that so many truths can come from such a small set of initial axioms.

The classical logic guides the basis of certainty in the acquisition (revelation of "hidden" truths in the premises) and organization of knowledge and it is it that has propitiated the development of the concept of mathematical proof used for centuries and allowed the development of computers in the middle of the 20th century. The monumental and almost superhuman work *Principia Mathematica* by A. N. Whitehead (1861-1947) and Bertrand Russell (1872-1970), published in 3 volumes between the years 1910 and 1913, and other works such as *on the consistency of arithmetic* by Frances Jacques Herbrand (1908-1931), transformed logic into a new science [1]:

> "*... the formal linguistic approach is imposed, that is, logic is seen as a linguistic structure. From this point of view, logic (...) presupposes a syntax (rules or laws of combinations of signs) and a semantics (interpretation and meaning of signs). Since 1930 (...) the logic moves in a direction of greater integration to mathematics, reaching a high technical complexity and considerably extending its domain with applications in the most diverse areas.* [1]

Bishop [1] discusses the logical consequence or formal deduction:

> *"... the concept of logical consequence can be associated not only with the notion of mathematical proof but also with the application of inference rules, with which some transformations of a purely structural character are carried out on the axioms or theorems already proven of a certain formalized theory, so that the propositions or formulas resulting from this process of transformation are also considered proven".* [1]

Classical logic is undoubtedly of paramount importance for science and man's development, but there are issues it cannot address in an efficient way.

> *"What's all the "tall men"? Does number three belong to the "small numbers" set? Questions like these, seemingly simple, carry a different mathematical language than is normally encountered in schools. (...) Dealing with factors such as ambiguity, uncertainty and vague information for problem solving is a characteristic of human thought, articulated through previous knowledge and experiences".* [4]

To answer these questions and deal with problems that cannot be solved with the binomial "right or wrong" we need to use a logic that exceeds the "true or false" conclusion. In this way the non-classic logics were born.

II. MULTIVALUE LOGIC

The logical and mathematical Stephen C. Kleene (1909 - 1994) proposed a trivalent logic that opposes the classical logic whose bivalence says that a certain proposition has to be true or false. In Kleene's logic, true, false or undetermined values are considered for a proposition [5].

Such a reality may astonish us at first because in this case our new logic does not allow us to completely elucidate our propositional statements, often arriving at the verdict "undetermined" or "undecidable".

The logical Kurt Godel (1906-1978) had already proved in 1931 that many mathematical questions could not be proved regardless of the nature and number of axioms a formal logical system might contain. This is the famous incompleteness theorem [6]. In this case it seems necessary to deal with "mathematical uncertainty" and Kleene's logic contributes to this by considering a state of indetermination of a proposition.

> "*... are multivalue logics of 3 true values: Kleene Logic; Lukasiewiccz Logic; Bochvar Logic. The first arises from the need to represent the true value of undecidable mathematical operations; the second arises from the existence of contingent propositions about the future; and the third (...) arises for the formal treatment of semantic paradoxes. These logics use, besides V and F, according to a notion very close to that of classical logic, values such as I and C, the former can be read as undecidable or undetermined, the latter as contradictory or paradoxical".* [7]

Another interesting logic is the tetravalence of Belnap. It introduced a logic designed to deal usefully with inconsistencies and incomplete information. This logic is based on a structure called *four,* which has four truth values: the classics, **V** and **F, and** two new ones: **I**, which intuitively indicates lack of information (without knowledge), and **C** which indicates inconsistency (contradiction). Belnap gave quite convincing arguments as to why *"the way a computer should think"* should be based on these four values [8].

> *"The four values of truth in Belnap's Tetravalent Logic are read as the unknown, the contradictory, the absolutely true and the absolutely false. Belnap proposed this logic in 1977 motivated by the need to implement in an expert system of questions and answers the ability to continue generating dialogue even under contradictions and ignorance.* [7]

Belnap's four-value logic found numerous applications in various fields, such as deductive database theory, distributed logic, programming and other areas [9].

> *"According to a strategy of semantic analysis devised by J. Michael Dunn, a sentence can be rationally considered not only true or simply false, but also true and false. This can be made explicit by developing an appropriate evaluation procedure that generalizes the notion of a classical truth value function, allowing for under-determined and over-determined evaluations".* [9]

There are also other types of multivalue logic (with a finite set of propositional states). The logic presented by Shramko for example has "16 states". To outline it we will take the states {V, **F**, **I**, **C}** of Belnap (true, false, undetermined and contradictory, respectively). Shramko is part of a generalization of this logic:

> *"There is an interesting question about the four-value semantics of Dunn and Belnap, namely: why should we stop the 'generalization procedure' only at the four-value stage and not proceed to consider, say, combinations of V and C, I and C, etc.? If we can obtain a 'useful logic of four values', taking the power set of 2, why shouldn't we consider a logic of sixteen values based on the generalized truth set of values previously obtained? Would that logic be 'useful'?"* [9]

The multivalue logic was motivated in part by philosophical objectives (among others) and in the early years of its development this caused some doubts about its usefulness. But interesting applications have been found in various fields such as linguistics, hardware design, artificial intelligence and mathematics [10].

III. FUZZY LOGIC

The great generalization of multivalue logic is nebulous logic, also called fuzzy logic. This logic allows to instantiate each interval through which a proposition can pass from the false to the true state or vice versa.

Nebulous logic was developed in the 1960s by Lofti A. Zadeh. It combines multivalue logic, probabilistic theory and computational intelligence in order to process human knowledge [12].

In nebulous logic, truth-values are expressed linguistically, (true, very true, not true, false, very false, ...) [11] in counterpoint to the classical system of being only true or false.

> *"Other features of fuzzy logic can be summarized as follows: in binary logic systems, the predicates are accurate (e.g.: pair, larger than), whereas in fuzzy logic the predicates are nebulous (e.g.: high, low, ...). In classical logical systems, the most used modifier is denial while in fuzzy logic a variety of predicate modifiers are possible (e.g.: much, more or less, ...). These modifiers are essential in the generation of linguistic terms (e.g.: very high, more or less close, etc.). In classical logical systems there are only existential (**3**) and universal (**i**) quantifiers. Fuzzy logic admits, in addition, a wide variety of quantifiers (e.g.: few, several, usually around five, etc.)".* [11]

What differentiates fuzzy logic from Boolean logic is the ability to provide answers beyond extremes. This makes it possible to approach the exact solution of a complex problem

[12] . The real world is not exact and unfortunately, we don't have an algorithm to solve each and every problem (this is impossible [6]). It is then necessary to create a path of possible solutions (even if not optimal) that can circumvent the problem and provide us with a way out. Nebulous logic helps us to find this path by giving up the accuracy of classical logic, but at the same time offering an acceptable conclusion where otherwise we would only find indecisiveness.

> *"When a certain problem presents a great degree of uncertainty it is necessary to use a mathematical model that contemplates this specificity and does not disregard aspects that can be ignored in the application of traditional logics. (...) One of the great objectives inherent to fuzzy logic is to approach in its logic the way that human reasoning relates information seeking approximate answers to problems, so the great focus of this logic is the solution of problems whose present information is uncertain".* [13]

Fuzzy logic has applications in artificial intelligence, such as automatic control of aircraft flights, didactic games, various specialist systems, pattern recognition, robotics, intelligent control systems, decision support systems, genetic algorithms, data mining, etc.

McNeil and Thro (1994) relate some characteristics of systems where the application of fuzzy logic is necessary or beneficial:

> *"1. complex systems that are difficult or impossible to model.*
> *2. Systems controlled by [human] specialists.*
> *3. Systems with complex and continuous inputs and outputs.*
> *4. Systems that use human observation as inputs or as a basis for rules.*
> *5. Systems that are naturally "vague", such as those involving social and behavioural sciences, whose description is extremely complex". [14]*

IV. CONCLUSION

The path taken by the human ingenuity from Aristotle's formulation of classical logic (384 B.C. - 322 B.C.) to our days moved in unimaginable directions, especially from the end of the 19th century and beginning of the 20th century where mathematics was trying to become a complete science in which everything could be proved and rigorously explained. Godel [6] showed the deception of this goal led by David Hilbert (18621943) in the 1900s, but opened doors to other forms of reasoning that revolutionized logical thinking.

Godel's incompleteness theorem (1931) inaugurated a new period where limits were set for deductive axiomatic methodology. This frustrated part of the mathematical community of the time, but simultaneously opened the field to find other types of logic different from classical, other forms of reasoning to prove theorems and solve issues that, if analyzed only with the deductive eye, were irremediably undecidable.

The 20th century, especially since its second half, has brought an incomparable evolution to today's world. With the advent of computers complex problems were solved and other horizons were uncovered. As far as fuzzy logic is concerned, its importance for artificial intelligence is undeniable. The problems addressed by AI transcend the limits of classical logic, as they cannot be summarized in two states, due to their complexity and the ambiguity of the data available for possible solutions.

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