The proton radius puzzle solved

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Summary

The electron-proton scattering experiment by the PRad (proton radius) team at Jefferson Lab measured the root mean square (rms) charge radius of the proton as $r_p = 0.831 \pm 0.007_{stat} \pm 0.012_{syst}$ fm.

Assuming all of the electric charge in the proton is packed into a single pointlike (elementary) charge and applying the ring current model to a proton, one gets a radius for the circular current that is equal to $a = 2\mu_p/q_e c \approx 0.58736$ fm. Using CODATA values for all variables and constants in this equation, and applying a $\sqrt{2}$ form factor to, somehow, account for the envelope of the magnetic field around the ring current, yields an electric charge radius of 0.8065 fm. The difference between the PRad point estimate and this theoretical value is 0.00035 fm, which represents 5% of the standard error (0.007 fm) of PRad's point estimate. It is, therefore, hard to argue this is a mere coincidence.

We can also calculate a proton radius based on the idea of a strong charge. This radius corresponds to the range parameter in Yukawa's equation and is equal to $a = \hbar/m_p c \approx 0.2103$, which is about 1/4 of the PRad point estimate. This 1/4 factor is, obviously, far more mysterious, and the difference between 0.831 and this strong charge radius multiplied by 4 is 0.01 fm, which is about 50% of the *combined* statistical and systematic error (0.007 + 0.012 = 0.019). We, therefore, think that, while being somewhat less precise, the 1/4 factor cannot be a coincidence.

We, therefore, feel the new measurement of the proton radius by JLAB's PRad team may lend credibility to attempts to extend the *Zitterbewegung* hypothesis from electrons to also include protons and other elementary particles. In contrast, the measurement is hard to fit into a model of oscillating quarks that have partial charge only.

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The new measurement

Anyone who follows the weird world of quantum physics with some interest, must have heard the latest good news: the 'puzzle' of the charge radius of the proton has been solved. We think that is a rather grand statement to make. A more sober way of stating what happened is this: a very precise electron-proton scattering experiment by the PRad (proton radius) team using the Continuous Electron Beam Accelerator Facility (CEBAF) at Jefferson Lab has now measured the *root mean square* (rms) charge radius of the proton as[1]:

$$r_p = 0.831 \pm 0.007_{stat} \pm 0.012_{syst} \text{ fm}$$

Most commentators[2] interpret the measurement as putting an end to various divergent measurements from past experiments – not only using nuclear scattering but also spectroscopy techniques[3] – which typically yielded a value centered in the range of 0.87 or 0.88 fm. In light of the precision of these experiments, which is expressed in the statistical and systematic errors mentioned above, this discrepancy was – and, according to many, still is – very worrying. Indeed, claims that "the discrepancy was likely due to measurement errors" work in both ways.

The illustration below, for example, was taken from a March 2019 article on the issue which, based on the previous measurement data, established a (statistical) *lower bound* on the proton's radius equal to 0.848 fm. To be precise, these researchers claimed – just a few months before the result of the new measurements came out[4] – that the *actual* charge radius of a proton, based on common definitions and a decade of high-precision measurements, should be larger than 0.848 fm. To be precise, applying common statistical concepts, they said so *with 95% confidence*.[5]

However, the newly measured radius (0.831) is 0.017 fm *smaller* than what these researchers think is the lower bound of the proton's radius. If 0.007 is the standard error of the new measurement, then a difference of 0.017 is about 2.43 times that value. The difference may, therefore, be considered to be quite significant.[6] So who is right, and who is wrong here?

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[1] See: https://www.nature.com/articles/s41586-019-1721-2. See also: https://www.jlab.org/prad/collaboration.html and https://www.jlab.org/experiment-research.

[2] See, for example, the *Physics Today* article on it: https://physicstoday.scitation.org/do/10.1063/PT.6.1.20191106a/full/.

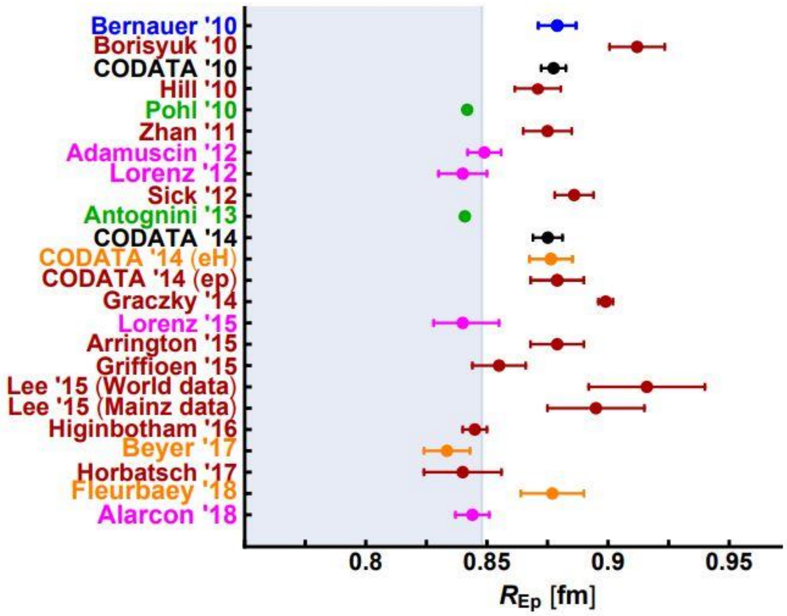
[3] The Wikipedia article on the proton radius puzzle offers a very good non-technical introduction to what's at stake. See: https://en.wikipedia.org/wiki/Proton_radius_puzzle, accessed on 26 January 2020.

[4] The article on the new proton radius was published in *Nature* in November 2019 (https://www.nature.com/articles/s41586-019-1721-2), but preliminary results had been shared with researchers by one of the authors of the referenced article at the occasion of the ELBA Conference, which was held from 23 to 28 June 2019. The presentation for the ELBA Conference participants is interesting and, surprisingly, quite readable: https://agenda.infn.it/event/17166/contributions/85329/attachments/64938/78815/Gasparian.pdf.

[5] Franziska Hagelstein and Vladimir Pascalutsa, 25 March 2019, *Lower bound on the proton charge radius from electron scattering data*, (https://arxiv.org/pdf/1812.02028.pdf).

[6] This statement assumes, naturally, that 0.007 is the standard error of the mean (SEM) of the new measurement, *not* the standard deviation of the distribution of measurements (σ). There is also the systematic error, of course,

Figure 1: Historical measurements of the proton radius



Source: Hagelstein and Pascalutsa, 25 March 2019

Looking at Figure 1, we think Hagelstein and Pascalutsa should make a better case for their rather high cut-off value. The colors indicate the source and/or technique that was used. CODATA values are in black, so these should not count because they are based on other experiments. Values measured in hydrogen and deuterium spectroscopy are in yellow-orange. Values based on electron-proton scattering experiments – like the new experiments – are in red-brown. Finally, muonic-hydrogen spectroscopy results are in green but, for some reason we do not quite understand, seem to have been *excluded as being valid* – because they are outside of the calculated lower bound, which is given by the light blue-grey band in the image. Finally, the magenta values, which are based on "electron-proton scattering fits within a dispersive framework" do also *not* seem to be acceptable to these two researchers[7] because – well – we must our reader to the article itself because we could not quite understand their reason for excluding the results of those scientific experiments in their calculations of this so-called lower bound (0.848 fm) for acceptable measurements.

The point is this: the new measurement result will, most likely, not solve the controversy. Indeed, the conclusion that "the puzzle seems to be resolved" because "the discrepancy was likely due to measurement errors"[8] seems to be premature: the latter statement works in both ways. If the PRad team would be convinced that previous experiments were wrong because of "measurement errors", then they should explain these. Otherwise, the experiment may be subjected to the same conjecture:

which – added to the measured SEM – would bring the difference of 0.017 within less than one *sigma* of the estimated lower bound. We will come back to statistical definitions in a few seconds.

[7] Descriptions of the various techniques and/or measurements are quoted from the referenced article (https://arxiv.org/pdf/1812.02028.pdf), so we should not be suspected of any bias here.

[8] See: https://physicstoday.scitation.org/do/10.1063/PT.6.1.20191106a/full/.

perhaps it is the PRad experiment which suffers from “measurement errors”, rather than the previous experiments?

The ring current radius

Having said that, we actually do like the new measurement of the PRad team. Why? Because we immediately see some remarkable relations here. The first, and most obvious, relation is the relation between the new radius and the theoretical ring current radius of a proton. The second is with the range parameter that comes out of Yukawa’s potential formula for the nuclear (strong) force, which we will look at in the next section.

Let us start with the ring current model.

If a proton would, somehow, have a pointlike elementary (electric) charge in it, and if it is in some kind of circular motion (as we presume in *Zitterbewegung* models of elementary particles[9]), then we can establish a simple relation between the magnetic moment (μ) and the radius (*a*) of the circular current.

Indeed, the magnetic moment is the current (I) times the surface area of the loop (πa^2), and the current is just the product of the elementary charge (q_e) and the frequency (*f*), which we can calculate as $f = c/2\pi a$, i.e. the velocity of the charge divided by the circumference of the loop. We write:

$$\mu = \mathrm{I} \cdot \pi a^2 = \mathrm{q_e} c \frac{\pi a^2}{2\pi a} = \mathrm{q_e} c \frac{a}{2} \approx 0.24 \ldots \times 10^{-10} \cdot a$$

Using the Compton radius of an electron ($a_e = \hbar/m_e c$), this yields the correct magnetic moment for the electron[10]:

$$\mu_e = (0.24 \ldots \times 10^{-10} \cdot 0.386 \ldots \times 10^{-12}) \approx 9.2847647043 \times 10^{-24}\ \mathrm{J/T}$$

When applying the $a = \mu/0.24\ldots\times10^{-10}$ relation to the (experimentally measured) magnetic moment of a proton, we get the following value for the *ring current radius* of a proton:

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[9] The *Zitterbewegung* model assumes an electron consists of a pointlike charge whizzing around some center. The *rest* mass of the pointlike charge is zero, which is why its velocity is equal to the speed of light. However, because of its motion, it acquires an *effective* mass – pretty much like a photon, which has mass because of its motion. One can show the effective mass of the pointlike charge – which is a relativistic mass concept – is half the rest mass of the electron: $m_\gamma = m_e/2$. The concept goes back to Alfred Lauck Parson (1915) and Erwin Schrödinger, who stumbled upon the idea while exploring solutions to Dirac’s wave equation for free electrons. It’s always worth quoting Dirac’s summary of it: “The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. *It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment.*” (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

[10] The calculations do away with the niceties of the + or – sign conventions as they focus on the *values* only. We also invite the reader to add the SI units so as to make sure all equations are consistent from a *dimensional* point of view. For the values themselves, see the CODATA values on the NIST website (https://physics.nist.gov/cuu/Constants/index.html).

$$a = \frac{1.41 \ldots \times 10^{-26}}{0.24 \ldots \times 10^{-10}} = 0.587 \times 10^{-15}\ \mathrm{m}$$

When we multiply this with √2, we get a value which fits into the 0.831 ± 0.007 interval:

$$(0.587 \ldots \times 10^{-15}\ \mathrm{m}) \cdot \sqrt{2} \approx 0.83065 \times 10^{-15}\ \mathrm{m}$$

The √2 factor is puzzling, of course. We have no real explanation for it but we venture it is some form factor that should have a very logical explanation. The magnetic field of the current ring, for example, will envelop the current ring itself. We would, therefore, expect the measured charge radius to be larger than the radius of the current ring.

There are, of course, also the intricacies related to the definition of a *root mean square* (rms) radius. We could invent some crackpot theory, for example, in which the measured value (0.831 fm) would be the largest value of a sinusoidal distribution.[11] However, we readily admit the concept of a sinusoidal distribution sounds rather non-sensical. We may also think of some kind of randomness in the motion of the pointlike charge[12] but we admit that sounds equally *ad hoc*.

In short, a simple form factor related to the magnetic field of the proton – or to the electrons that scatter off it[13] – is much more probable.

Of course, the argument is entirely heuristic—simplistic, even. At the same time, we feel the √2 factor can*not* be a coincidence: the difference between the 'theoretical' 0.83065 fm value and the 0.831 fm measurement is only 0.000346656… fm, which is less than 5% of the standard error of the PRad point estimate (0.007 fm).

The strong charge radius

Our particular interpretation of the *Zitterbewegung* model of an electron allows us to calculate another theoretical radius of the proton. We've explained the idea elsewhere[14] and, hence, we will not elaborate too much here. We refer to it as the oscillator model, and it involves a direct calculation of the Compton radius combining the E = ħ·ω, $c = a \cdot \omega$ and E = m·c^2 relations. When using the mass for an electron, we get:

$$a = \frac{c}{\omega} = \frac{c \cdot \hbar}{\mathrm{m} \cdot c^2} = \frac{\hbar}{\mathrm{m} \cdot c} = \frac{\lambda_C}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

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[11] See the annex on statistics to this paper. The peak value of a sinusoidal wave and its rms value are, effectively, related through a √2 factor but, we admit, this is a very poor argument.

[12] The concept of the random walk, as modeled by Einstein, involves a mean *squared* distance. See: https://www.feynmanlectures.caltech.edu/I_06.html#Ch6-S3.

[13] One should note that both the classical as well as the *Compton* radius of electron – about 2.8 fm and 386 fm respectively – are both much larger than the proton radius. Unfortunately, we have not seen any easy or comprehensible explanation of how these electron-proton scattering experiments account for the rather large size of electrons as compared to the target: even the smallest electron radius (2.8 fm) is almost 3.5 times the estimated size of the proton!

[14] See, for example, our previous paper: *the Metaphysics of Physics*, http://vixra.org/abs/2001.0453.

When applying the E = ħ·ω, $c = a\cdot\omega$ and E = m·c^2 relations to the mass/energy of proton (or a neutron[15]), we get this:

$$a_{\mathrm{p}} = \frac{\hbar}{\mathrm{m_p}\cdot c} = \frac{\hbar}{\mathrm{E_p}/c} = \frac{(6.582\times10^{-16}\ \mathrm{eV}\cdot s)\cdot(3\times10^{8}\ m/s)}{938\times10^{6}\ \mathrm{eV}} \approx 0.21\times10^{-15}\ \mathrm{m}$$

The result that we obtain here is about 1/4 of the experimentally measured value. This distance is exactly the same as the distance that we get for the range parameter *a* in Yukawa's formula.[16] In fact, we can equate the range parameter *a* and the distance *r* with the $a_p = \hbar/m_p c$ value in the force formulas we get from the potential formulas and we'll see the electrostatic and nuclear force – which we'll denote as F_C and F_N respectively – are, effectively the same[17]:

$$\mathrm{F_C} = -\frac{\mathrm{dV}}{\mathrm{d}r} = -\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r^2} = -\frac{\alpha\hbar c}{r^2} = -\frac{\alpha \mathrm{m_p^2}c^2}{\hbar}$$

$$\mathrm{F_N} = -\frac{\mathrm{dU}}{\mathrm{d}r} = -\frac{\mathrm{g_N^2}}{4\pi}\cdot\frac{\left(\frac{r}{a}+1\right)\cdot e^{-\frac{r}{a}}}{r^2} = -\frac{\mathrm{g_N^2}}{4\pi}\cdot\frac{2e^{-1}}{r^2} = -\frac{e\alpha\mathrm{hc}}{4\pi}\cdot\frac{2e^{-1}\mathrm{m_p^2}c^2}{\hbar^2} = -\frac{\alpha\mathrm{m_p^2}c^2}{\hbar}$$

Using the *exact* value for a_p, we can calculate the ratio between the new experimental value of the proton and the ratio as calculated above more exactly as:

$$\frac{a_{\mathrm{p}}}{r_{\mathrm{p}}} = \frac{0.21\ldots}{0.831} \approx 0.25308$$

Hence, the ratio differs from the ¼ ratio (0.25) by about 1.2% only. Is this *good enough*?

The systematic and statistical variance of the measured radius add up to 0.012 + 0.007 = 0.019 fm, which is about 2.3% of the point estimate (0.019/0.831) so, yes, we think it is significant. Indeed, the difference between 0.831 fm and this strong charge radius multiplied by 4 is 0.01 fm, so that's about 50% of the mentioned *combined* statistical and systematic error . We, therefore, think that, while being somewhat less precise, the 1/4 factor can also *not* be a coincidence

[15] The mass of a neutron is about 939,565,413 eV/c^2 and about 938,272,081 eV/c^2 for the proton. Hence, the energy *difference* is a bit less than 1.3 MeV. It is, therefore, very tempting to think a neutron might, somehow, combine a proton and an electron: the electron mass is about 0.511 MeV/c^2 and, hence, we may think of the remaining difference as some kind of binding energy—the attractive force between the positive and a negative charge, perhaps? These thoughts are, obviously, very speculative. We did explore some of these, however, in our paper on the nature of protons and neutrons (http://vixra.org/abs/2001.0104), and we very much welcome comments.

[16] See Aitchison and Hey's introduction to *Gauge Theories in Particles Physics*, Vol. 1, Chapter 1 ((*The Particles and Forces of the Standard Model*) , p. 16. To be precise, Aitchison and Hey there write the range parameter is ~ 2 fm. They do not explain this result and we wonder why they do not calculate some more precise value, which is easy enough based on the idea of calculating and equating the forces involved.

[17] As for the theoretical model that we use – and the reference to the *strong force radius* in the title of this section – see the above-mentioned paper (http://vixra.org/abs/2001.0104) as well as our *Metaphysics of Physics* paper (http://vixra.org/abs/2001.0453). Note that we left the nuclear constant (υ_0) out because its numerical value is one. You can, of course, calculate the *exact* value of the force using the CODATA values for the various constants. We leave it as a teaser for the interested reader.

Conclusions

The concluding comments of *Physics Today*[18] on the very precise measurement of the proton's *rms* charge radius were this:

> "The PRad radius result, about 0.83 fm, agrees with the smaller value from muonic and now electronic hydrogen spectroscopy measurements. With that, it seems the puzzle is resolved, and the discrepancy was likely due to measurement errors. *Unfortunately, the conclusion requires no new physics*." (my italics)

We wonder what kind of new physics they are talking about. We get two different theorical radii of the proton from 'new physics' here, and their relation with the measured radius is strangely perfect:

1. The charge radius, which relates to the measured radius by a factor equal to $\sqrt{2}$; and
2. The 'oscillator' or *strong force* radius, which is 1/4 of the measured value.

Ratios like this suggest it should *not* be difficult to connect the numbers but then, somehow, it is. Hopefully, some researchers smarter than us[19] will be able to connect the dots and come up with a realist interpretation of quantum mechanics combining the idea of an electromagnetic and a 'strong' force.[20] Till that day, the words which Mr. Dirac wrote back in 1958, as the last paragraph in the last edition of his Principles of Quantum Mechanics, will continue to ring true:

> "Now there are other kinds of interactions, which are revealed in high-energy physics and are important for the description of atomic nuclei. These interactions are not at present sufficiently well understood to be incorporated into a system of equations of motion. Theories of them have been set up and much developed and useful results obtained from them. But in the absence of equations of motion these theories cannot be presented as a logical development of the principles set up in this book. We are effectively in the pre-Bohr era with regard to these other interactions. It is to be hoped that with increasing knowledge a way will eventually be found for adapting the high-energy theories into a scheme based on equations of motion, and so unifying them with those of low-energy physics." (Principles of Quantum Mechanics, 4th edition, p. 312)

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[18] See: https://physicstoday.scitation.org/do/10.1063/PT.6.1.20191106a/full/.

[19] Needless to say, we did contact the PRad team at JLAB through their spokesperson (Prof. Dr. Ashot Gasparian). Mr. Gasparian was kind enough to react almost immediately to our email, stating he thought "the approach and numbers were interesting" and that he would share them with the students and postdocs in the team. We look forward to future comments.

[20] The weak force is supposed to explain why things fall apart, or why particles are unstable, rather than stable. We prefer to not think of decay or disintegration as a force. It is, in fact, the exact opposite of the idea of a force: a force is supposed to keep things together. In the same vein, we like to add we do not want to entertain the idea of messenger particles or force carriers – virtual photons, gluons, or whatever other bosons or metaphysical constructs that have been invented since Yukawa first presented these ideas. Indeed, it is unfortunate that – instead of realizing he was actually proposing the existence of a new charge – he used his formula to derive a hypothetical nuclear force quantum.

Statistical annex

To understand anything of the articles explaining how one actually arrives at a *root mean square* (rms) charge radius from experiments, one seems to need not one but three PhDs: not only in physics, but also in math and statistics. It starts with the definition of the concept of a root mean square radius, which is basically this[21]:

$$R_{\mathrm{p}} = \sqrt{\langle r_{\mathrm{p}} \rangle^2}$$

At first, this looks non-sensical: angle brackets usually denote an average – which makes sense – but why would you first square it, and then take a square root again? You would think that $\sqrt{\langle r_{\mathrm{p}} \rangle^2}$ would equal r_{p}, right? Of course, it is not. A *root mean square* value of a function is defined as follows:

$$x_{\mathrm{RMS}} = \sqrt{\frac{x_1^2 + x_2^2 + \cdots + x_n^2}{n}} = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}}$$

There is also an equivalent for a continuous function. For example, if x is a continuous function of *time* (t), then the *rms* value as measured over some time interval $[T_1, T_2]$ is equal to:

$$x_{\mathrm{RMS}} = \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} [x(t)]^2 dt}$$

If we obtain the *same* value – or very nearly the same value – for *every* measurement, such as in distribution of the c_i and d_i variables in Table 1, then the *rms* value will be very close or equal to the average value. However, if that is *not* the case, then the *rms* value will diverge from it.

Table 1: Average versus *rms* value of a distribution

i	a_i	b_i	c_i	d_i
1	1	3	4.6	5
2	2	4	4.7	5
3	3	4	4.8	5
4	4	5	4.9	5
5	5	5	5	5
6	6	5	5.1	5
7	7	6	5.2	5
8	8	6	5.3	5
9	9	7	5.4	5
average	5.00	5.00	5.00	5.00
rms value	5.63	5.13	5.01	5.00

The relations between the *rms* value, the average and the peak value of a sine wave are interesting:

[21] See, for example, A. Vorobyev, 22 April 2019, https://arxiv.org/ftp/arxiv/papers/1905/1905.03181.pdf.

Table 2: Average, *rms* and peak value of a sinusoidal function

function	***average***	***rms value***	***peak value***
$x = \sin\alpha$	$\bar{x} = \frac{2}{\pi} \approx 0.637$	$x_{rms} = \frac{1}{\sqrt{2}} \approx 0.707$	1

The question is: why would want to calculate an *rms* value for a radius, rather than just the more straightforward average? We did not find any convincing answer to this question but we must also admit that – because of time constraints[22] – we did not look very hard. We assume it has to do with the same statistical conventions that lead scientists to use the standard error rather than the mean absolute deviation as a measure of the accuracy of an estimate.

Indeed, we should double-check but we must assume that the 0.007_{stat} in the $r_p = 0.831 \pm 0.007_{stat} \pm 0.012_{syst}$ fm equation refers to the standard error of the mean (SEM) – also known as *root mean squared error* (RMSE) – of the new measurement ($\sigma_{\bar{x}}$), *not* the standard deviation of the distribution of measurements (σ_x). We may remind the reader of the difference between the two concepts by jotting down the formula for the RMSE:

$$\mathrm{RMSE} = \mathrm{SEM} = \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{\sqrt{\frac{\sum_{i=1}^{n}(x_i - x_n)^2}{n-1}}}{\sqrt{n}} = \frac{\sqrt{\sum_{i=1}^{n}(x_i - x_n)^2}}{\sqrt{n(n-1)}} \approx \frac{\sqrt{\sum_{i=1}^{n}(x_i - x_n)^2}}{n}$$

The RMSE or SEM statistic[23] is used mainly because of computational convenience: it is much easier to do variance analysis and other gimmicks with the RMSE than with the alternative statistic, which is the mean absolute deviation or mean absolute error (MAD or MAE)[24]:

$$\mathrm{MAD} = \mathrm{MAE} = |\sigma|_{\bar{x}} = \frac{\sum_{i=1}^{n}|x_i - x_n|}{n}$$

Where do we want to go with this statistical annex? Not very far. In fact, we'll stop here. For the rather bewildering detail of how a charge radius is actually calculated, we refer to the referenced papers (see, for example, Hagelstein and Pascalutsa, 2019 or, Vorobyev, 2019) which, to be honest, we find totally impenetrable.

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[22] As an amateur physicist, I need to attend to my real job from time to time.

[23] The reader can check the rapid convergence between the $1/\sqrt{n(n-1)}$ and $1/\sqrt{n^2} = 1/n$ formulas for any meaningful sample size (say, more than five or ten measurements, for example). As for the abbreviation, the SEM (*standard error of the mean*) is probably more common, but we find the RMSE abbreviation (*root mean squared error*) more scientific, if only because it reminds us of the formula that is used to calculate this value. To be fully transparent here, some authors distinguish SEM and RMSE based on the use of $\sqrt{n-1}$ or $\sqrt{n}$ in the calculation of the average error from the calculated mean of the observations. In light of the rapid convergence between the two, we think the distinction is purely academic and, hence, we treat the two concepts as being more or less the same.

[24] When simplifying expressions, it is easier to deal with quadratic or square root functions, as opposed to absolute values. This is probably the most important reason why scientific model builders and statisticians stick to the SEM or RMSE definition when calculating an average error from the mean. To be complete, one may also find a distinction between the MAD and MAE definitions but we also merge them for the sake of practicality.

One would expect, for example, some kind of explanation of the fact that the charge radius of electrons is actually supposed to be much larger than the charge radius of the target. Indeed, both the classical as well as the *Compton* radius of electron – about 2.8 fm and 386 fm respectively – are both much larger than the proton radius. Unfortunately, we have not seen any easy or comprehensible explanation of how these electron-proton scattering experiments account for that. Even the smallest electron radius (2.8 fm) is almost 3.5 times the estimated size of the proton!

We must assume the answer to this obvious question is somewhere hidden in the rather abstruse arguments on the various form factors that are used in the methodologies and calculations. If our approach and numbers make sense, then we may get into those in the future.[25]

[25] We contacted the PRad team at JLAB through their spokesperson (Prof. Dr. Ashot Gasparian). Mr. Gasparian was kind enough to react almost immediately to our email, stating he thought "the approach and numbers were interesting" and that he would share them with the students and postdocs in the team. We look forward to future comments, based on which we may do further research within our own rather limited means and time.