The metaphysics of physics (II)

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Summary: This paper complements an earlier (Nov 2018) paper on the metaphysics of physics. The earlier paper focused on an alternative (realist) interpretation of quantum mechanics. While inspired by the previous one, this follow-up paper is a much simpler one: we just offer very basic thoughts on the most fundamental physical concepts, which are the idea of force, energy and mass.

Based on the recent precision measurements of the (electric) charge radius of a proton, we also argue previous reflections on the *zbw radius* of a proton make more sense now.

Needless to say, we also offer our usual reflections on the concept of fields and messenger particles. We think the first is very useful – even if fields are relative. In contrast, we argue that the latter (the idea of virtual photons, gluons or other messenger particles) is purely metaphysical. In other words, we continue to think the idea of messenger particles is a non-scientific successor to *aether* theories.

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The idea of a force

Newton's force law tells us a force changes the state of motion of an object, and Maxwell's equations tell us a force does so by acting on a *charge*. The force we know best is the electromagnetic force: it acts on an electric charge and we usually analyze it as the sum of two components: an electrostatic force and a magnetic force. Adding the concept of relativistic mass, we can write:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\mathrm{m}_0 \cdot \boldsymbol{v}}{\sqrt{1 - v^2/c^2}}\right] = \boldsymbol{F} = \boldsymbol{F_E} + \boldsymbol{F_B}$$

$$= \mathrm{q}\boldsymbol{E} + \mathrm{q}\boldsymbol{v}\times\boldsymbol{B} = \mathrm{q}(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})$$

The separation between electric (or electrostatic) and magnetic forces comes with the frame of reference: what charges move (and in which direction and how fast) and what charges do not, depends on your own state of motion. That's what led Einstein to look at Maxwell's equations and develop his theory of (special) relativity. In reality – whatever that may be – there is only *one* combined electromagnetic force and – as shown in the force law above – it changes the state of motion of an object. To be precise, it changes its momentum $\mathbf{p} = \mathrm{m}_v\boldsymbol{v} = \gamma\mathrm{m}_0\boldsymbol{v}$. The m_0 is the equivalent mass of the *rest* energy of the object, i.e. its energy in its own *inertial* frame of reference.[1] The γ in the $\mathrm{m}_v = \gamma\mathrm{m}_0$, of course, the ubiquitous Lorentz factor.

The corollary of the idea of a force being that what changes the state of motion of an object is that the state of motion of an object does *not* change in the absence of a force: such resistance to change is referred to as *inertia* and is captured by the concept of mass. Let us look at the force law again and make sure we use correct definitions.

$$\boldsymbol{F} = \mathrm{m}_v \cdot \boldsymbol{a} = \frac{\mathrm{d}(\mathrm{m}_v \cdot \boldsymbol{v})}{\mathrm{d}t} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}$$

$$\mathrm{m}_v = \gamma \cdot \mathrm{m}_0 = \frac{1}{\sqrt{1 - v^2/c^2}} \cdot \mathrm{m}_0$$

The m_v factor in the $\mathrm{m}_v{\cdot}\boldsymbol{a}$ product is a scalar. Note that we can only bring it inside – or outside – of the brackets in the $\mathrm{d}(\mathrm{m}_v{\cdot}\boldsymbol{a})/\mathrm{d}t$ expression because we assume it is *not* a function of *t*: it is a function of *v*, its velocity, *only*. Because it is a scalar quantity – as opposed to a vector quantity – mass is the same in every direction. That sounds trivial – even plain stupid, perhaps – but it is less intuitive or trivial than you may think: from our experience in daily life, it looks much easier to *change the direction* of a massive object than its velocity[2] and, hence, we may think there is *less* inertia sideways. In fact, Einstein himself

[1] Of course, if we think it does not move, then its own inertial frame of reference is the same as ours.

[2] Think of all the movies involving some asteroid threatening to crash into our planet: the hero in his rocket will not try to stop it, but he will try to, somehow, change its direction—or, else, destroy it (or some combination of both). Bruce Willis is my favorite action hero, so I guess you know what movie I am thinking of right now. 😊

used velocity-dependent concepts of mass in his seminal 1905 article introducing the principle of relativity, distinguishing between the "longitudinal" and "transverse" mass of a moving charge.[3]

We now know that a *scalar* mass concept will do: the directional aspect is taken care of by the acceleration vector $\boldsymbol{a}$: the same force, for example, will be associated with *half* the acceleration if the mass doubles—because it is an object with a much larger rest mass or, else, because its velocity is equal to 0.866 times the velocity of light.[4]

Having said that, if the nature of a force is defined by the charge it acts upon, then it might still make sense to define the concept of mass in terms of the force or the charge it acts upon. The concept of *electromagnetic* mass may, therefore, still be useful, and we will, therefore, come back to it later.

The idea of a field

The concept of a field is, perhaps, much more mysterious than the concept of a field. At the same time, it's not all that difficult either: we should just try to imagine ***a force in the absence of a charge to act on***. That's all. The $\boldsymbol{E}$ and $\boldsymbol{B}$ vectors in the $\boldsymbol{F} = \text{q}\boldsymbol{E} + \text{q}(\boldsymbol{v}\times\boldsymbol{B})$ expression are vectors. We can replace both by introducing a combined *electromagnetic* vector $\boldsymbol{\mathcal{E}}$, so we write the electromagnetic force as[5]:

$$\boldsymbol{F} = \text{q}(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}) = \text{q}\boldsymbol{\mathcal{E}}$$

Can we do that? The $\boldsymbol{v}$ in the $\boldsymbol{v}\times\boldsymbol{B}$ expression is there for a reason, right? It's the velocity of the charge on which the (magnetic) force is supposed to act. Can we make abstraction of it? Of course, we can. It is, in fact, a very logical thing to do: if we make abstraction of the charge (as we do when we think of a field), we should also make abstraction of its velocity.

Let us think about this some more. We know a field is generally defined as the force per unit charge. What is the unit charge? It is *not* the charge of the electron (or the proton) – that is the *elementary* charge, which is something else – but the two concepts are, obviously, closely related. To be precise, the unit charge is the *coulomb* (C), and under the 2019 redefinition of the SI base units, the coulomb is now defined in terms of the elementary charge, which is the charge of the proton (or, what amounts to the same, its negative: the charge of the electron). Under this redefinition, which took effect on 20 May 2019, the elementary charge has now effectively been *defined* as $1.602176634\times10^{-19}$ coulombs, *exactly*—and vice versa: the coulomb is the charge of $1/1.602176634\times10^{-19}$ protons, *exactly*.

This value is, obviously, not relative. It is a fundamental constant of Nature, just like c or h. To be precise, its value does *not* depend on the reference frame. In contrast, the values we'll measure for $\boldsymbol{E}$ and $\boldsymbol{B}$ and, hence, for $\boldsymbol{F}$ and $\boldsymbol{\mathcal{E}}$ will be relative and dependent on our motion relative to the charge. We

[3] See p. 21 of the English translation of Einstein's article on special relativity, which can be downloaded from: http://hermes.ffn.ub.es/luisnavarro/nuevo_maletin/Einstein_1905_relativity.pdf. The two concepts may be associated with the equally relative distinction between the electrostatic and magnetic forces. While the concept of mass has been reduced to something scalar (which is quite consistent with the Higgs theory of mass), we feel the concept of energy is still ambiguous: both the concepts of kinetic and potential energy imply the idea of direction (kinetic energy comes from velocity, which is directional, and potential energy is calculating by bringing charges together – which also involves the idea of a trajectory).

[4] The Lorentz factor is equal to 2 for $\beta = v/c \approx 0.866$.

[5] The symbol is a so-called *Latin* epsilon, also known as an open e (majuscule: Ɛ, minuscule: ɛ) and, yes, you have not seen it before.

do want to introduce four-vector algebra here but just note that (electric) charge is *not* relative and that, therefore, the ***F*/*ℰ* =** q *ratio* should also *not* depend on the reference frame: it should be the same in *any* reference frame. This may sound trivial again but it is, in fact, a very interesting observation.

As part of this discussion on the field concept, I should probably say a few words about how a force actually *works*. However, I won't do that. Both physicists and philosophers alike tend to write volumes about that, usually noting that what is relative can, somehow, not be real and that we, therefore, should try to find "some kind of gear wheels", or "something that can transmit the force."[6] In fact, I find most physicists will initially dismiss such discussions as rubbish but, unfortunately, then proceed to discuss what might or might not be going on inside of the nucleus of an atom and suddenly bring back the whole idea of 'gear wheels' through the back door: think of 'messenger particles' such as virtual photons or gluons here. I'll be clear: I think of that as rubbish.

The idea of a strong force

We introduced the electromagnetic force above. Let us now think about other forces, charges and masses that may or may not exist. There is the gravitational force, of course—but Einstein did not think of it as a force, and so we will not dwell on it either here.

We may also think of some kind of strong force. Indeed, because protons stay together in multi-proton nuclei, physicists also believe some kind of short-range strong force must exist: this strong force must act on some strong charge whose nature is not well understood. The idea here is rather simple: because protons carry positive charge, the electrostatic repulsive force between them should push them apart. Hence, some other – stronger – force must keep them together. This inspired Hideki Yukawa to propose a potential function for a *nuclear* force—some new force that, supposedly, holds nucleons together: protons as well as neutrons. The Yukawa potential has the following shape:

$$\mathrm{U}(r) = -\frac{\mathrm{g_N^2}}{4\pi}\frac{e^{-r/a}}{r}$$

This formula reflects the formula for the electrostatic potential:

$$\mathrm{V}(r) = \frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r} = \mathrm{e}^2\frac{1}{r}$$

Yukawa's formula differs from Coulomb's formula because of the minus sign (but that is because the electrostatic and strong forces are opposite) and, most importantly, because of the $e^{-r/a}$ factor, which is there to ensure that, at distances smaller than the *range* parameter *a*, the strong *attractive* force would, effectively, be stronger than the electrostatic *repulsive* force.

Yukawa's formula also misses the equivalent of the electric constant (ε_0). This is an oft-missed point and we do not think of it as a minor detail. In fact, we think it is a grave mistake: if there is something like a strong force, then there must something like a strong charge and, hence, Yukawa should have inserted some kind of nuclear constant. Because Yukawa had the freedom to choose a *unit* for this new hypothetical strong charge, its numerical value could be one, but it would still have some *physical*

[6] I quote from the interesting and, at the same time, fairly concise treatment of fields by Richard Feynman: see his *Lectures*, II-1-5, *What are the fields?*

dimension to ensure dimensional consistency on both sides of his U(r) equation. The discussion warrants a small digression to highlight the point.

Electric charge is measured in units of *coulomb*, and it is a fundamental unit: the electron charge is the electron charge—regardless of the reference frame. As such, it is as fundamental as c or h.[7] If the strong force would be as fundamental as the electromagnetic force, then the charge it acts upon should be as fundamental as the electric charge. In one of our previous papers, we invented a temporary unit for it: the *dirac*, which we abbreviated as Y to not only honor Dirac but Yukawa as well.[8] Hence, if ε_0 is expressed in $C^2/N{\cdot}m^2$, then our nuclear constant (which we will denote as υ_0) will be expressed in $Y^2/N{\cdot}m^2$. It is, then, easy to calculate the value of the nucleon charge as[9]:

$$\mathrm{g_N} = \sqrt{e \cdot \alpha \cdot h \cdot c \cdot \upsilon_0} = 6.27723 \ldots \times 10^{-14}\ \mathrm{Y}$$

It is a weird and, therefore, interesting formula: its key purpose in the context of this paper is to demonstrate a philosophical point only.[10] It consists of a mathematical constant (Euler's number) which is there because of the exponential function in Yukawa's formula[11], three natural constants (α, h and c)[12] and a *physical* proportionality constant whose only function is to ensure dimensional consistency and whose *numerical* value is, therefore, unity.

As mentioned, the formula currently only serves to demonstrate a philosophical point: the formulas does not *prove* such strong force effectively exists. Other reasons may explain why nucleons tend to stick together inside of the nucleus. Indeed, considering electrostatic repulsion alone, as Yukawa and other theorists usually do, narrows the perspective considerably. If we think of the *electric* charges inside of the nucleus as, somehow, moving around, the *magnetic* forces between them might act as counterbalancing the electrostatic repulsive force. You should think of the *attraction* between two wires carrying current in the same direction or – more relevant in this context – between two *loops* of current acting as magnetic dipoles.[13]

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[7] The 2019 revision of SI units defines the coulomb in terms of the elementary charge. To be precise, the coulomb is the charge of $1/1.602176634{\times}10^{-19}$ protons, *exactly*.

[8] The choice of Y is also consistent with our choice of an *upsilon* symbol (υ) for the nuclear constant.

[9] See, for example, my paper with some thoughts on the nature of protons and neutrons (http://vixra.org/abs/2001.0104).

[10] While downplaying the (non-)importance of the formula, I insist on earning 20 points on the John Baez 'crackpot index' (see: http://math.ucr.edu/home/baez/crackpot.html) here for naming something after myself: this is Van Belle's formula. And, yes, I've requested my kids to put it on my tombstone. *Dirac* or *Yukawa* can be the unit, but the formula is mine. 😊 Its theoretical value may be limited but its *practical* value – in terms of reminding academic physicists they are supposed to *explain* stuff, not complicate it even more – may or may not be as significant as Yukawa's.

[11] It would be tempting to try other functional forms but these would result in very complicated calculations and, in any case, in the lack of other good reasons, Yukawa's choice of the natural exponential function is, *a priori*, as good or as bad as any other choice he could have made.

[12] While the fine-structure constant has no physical dimension (it is a scalar), it is obviously a *physical* – rather than mathematical – constant. The fine-structure constant has many meanings, but we primarily think of it as a scaling constant in a layered model of electron motion (http://vixra.org/abs/1812.0273).

[13] For a non-technical discussion of the idea, see the *Encyclopædia Britannica* article on it: https://www.britannica.com/science/magnetism/Repulsion-or-attraction-between-two-magnetic-dipoles. We like this article because it effectively also discusses *nuclear* magnetic moment. It does so in the context of technology (magnetic resonance imaging) but it serves to illustrate the point we're trying to make here.

In fact, we are very much intrigued by calculations in the context of the forces between charged particles in accelerator beams here. One author – in the context of a very interesting article on relativity – actually claims that the charges on the surface of the beam and inside the beam would experience zero radial force *if the charged particles would move at the speed of light*.[14]

One may also want to wonder why electron orbitals consist of electron *pairs* or – why in the context of superconduction – Cooper pairs of like charges emerge. In short, considering electrostatic forces alone and then argue some strong force must counter these is, obviously, a bit of a flawed argument.

We'd like to conclude this section with a critical remark or question: the range parameter a in Yukawa's formula is about 0.2 fm.[15] That is much smaller than the measured charge radius of a proton, which is about 0.83 fm. I find it bizarre Yukawa hoped to explain some strong attractive force (stronger than the electrostatic repulsive force) *between protons* introducing a force whose range is much smaller than the proton's size.

The strong(er) force hypothesis

Despite our skepticism above, we actually do believe some kind of strong force inside of the nucleus exists. However, the reason has got nothing to do with the idea some other force should counter the electromagnetic forces inside of a nucleus. The most compelling reason to believe some enormous force must govern the nucleus is the extraordinarily *large mass* and the equally extraordinary *small size* of protons and neutrons as compared to electrons. The energy *density* inside of protons and neutrons is, effectively, *massive* as compared to electrons.

We have elaborated the *Zitterbewegung* model of an electron elsewhere and, hence, will not repeat ourselves here.[16] We just note it does what it is designed to do: it yields an elegant explanation of both the mass and the *Compton* radius of an electron in terms of a local oscillation of a pointlike electric charge.[17] By now, the skeptical reader may be inclined to stop reading all of this, so we will try to revive his or her interest by noting the *Zitterbewegung* hypothesis goes back to Erwin Schrödinger. Schrödinger stumbled upon the idea while exploring solutions to Dirac's wave equation for free electrons, and it's probably worth quoting Dirac's summary of it once more:

> "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of

[14] See: Oleg D. Jefimenko, 1998, *On the Experimental Proofs of Relativistic Length Contraction and Time Dilation*, Z. Naturforsch. 53a, 977-982 (https://www.degruyter.com/downloadpdf/j/zna.1998.53.issue-12/zna-1998-1208/zna-1998-1208.pdf).

[15] See Aitchison and Hey's introduction to *Gauge Theories in Particles Physics*, Vol. 1, Chapter 1 ((*The Particles and Forces of the Standard Model*) , p. 16. To be precise, Aitchison and Hey there write the range parameter as ~ 2 fm. They don't explain this result. We will do in a moment. Because they get the same value (more or less, at least), we must assume they went through the same calculations. We are not sure why they don't mention them.

[16] For a brief overview, see our (critical) discussion of Oliver Consa's classical calculations of the anomalous magnetic moment (http://vixra.org/abs/2001.0264). We also have more comprehensive papers on the topic (see, for example, http://vixra.org/abs/1905.0521).

[17] The oscillation is usually thought of as a ring current, and the pointlike charge may be associated as having some small but non-zero radius itself, which is supposed to explain Thomson scattering, as opposed to Compton scattering (see below).

small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

David Hestenes revived the *Zitterbewegung* (often abbreviated as *zbw*) interpretation in the 1970s and 1980s, basically reviving an earlier ring electron model (Parson, 2015[18]), which thinks of the electron as a unitary charge orbiting *at the speed of light* around some center, thereby generating a strong magnetic field which keeps the current going. As such, it is a rather nice example of a *perpetuum mobile* or a self-sustaining oscillation. More importantly, the model does allow us to explain the two different radii we get from elastic versus inelastic scattering experiments (Thomson versus Compton scattering).[19]

The point is: the electric current and the associated electromagnetic force allow us to *calculate* the Compton radius of an electron (r_C), and its formula effectively corresponds to what is *measured* in those scattering experiments:

$$r_{\mathrm{C}} = \frac{\hbar}{\mathrm{m_e} c} = \frac{\hbar c}{\mathrm{E}} = 0.386 \ldots \times 10^{-12}\, m$$

We used Einstein's mass-energy equivalence formula above: $E = m_e \cdot c^2$. It is, effectively, very important to underline that, in our particular model of the *zbw* electron, *all of the electron mass is explained as the equivalent mass of the energy in the (two-dimensional) oscillation of the pointlike charge*. The pointlike charge itself has zero rest mass: all of its mass is derived from its motion. As such, it reminds us of a photon which, supposedly, also has zero rest mass but which can be associated with some *effective* mass as well, which it derives from its motion.[20]

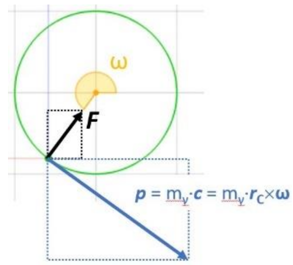
The formula for the Compton radius establishes an inverse proportionality between the radius (or size) of our particle (the electron, in this case) and its energy. Now, we said all of the energy of the electron is electromagnetic: the *mass* of the electron – as measured in experiments (about 0.511 MeV/c^2) – is the equivalent mass of the energy in the oscillation. The oscillation is electromagnetic and we can, therefore, calculate the energy from the electromagnetic force that drives the pointlike charge. The basic assumptions are depicted in Figure 1.

[18] See: https://en.wikipedia.org/wiki/Alfred_Lauck_Parson. Lucknow (India) lacks monuments, so I'll recommend erecting one in his honor.

[19] Thomson scattering is referred to as *elastic* scattering because the energy of the photons remains unchanged. In contrast, Compton scattering involves some interaction between the photon and the electron. We think of the photon as being briefly absorbed, before the electron emits another photon of *lower* energy, and the energy difference between the incoming and outgoing photon gets added to the *kinetic* energy of the electron.

[20] Some authors refer to the pointlike charge as a toroidal or electron photon but we find this term misplaced because we think one should not associate a photon with electric charge, and vice versa. In fact, we think this distinguishes photons from matter: photons do not carry charge. Matter does—even neutrons, as evidenced by the fact they have a magnetic moment. As for the mysterious neutrinos, we may say a few words about them later.

Figure 1: The ring current model of an electron



We distinguish between the *effective* mass of the pointlike charge, which we denote as m_γ[21], and the mass of the electron as a whole, which we denote as m_e. Based on the geometry of the situation, it is easy to show that $m_\gamma = m_e/2$. One can also show that the ratio between the force *F* and the momentum *p* must be equal to the ration between the speed of light and the radius $a = r_C$, so we can write: $F/p = c/a$. To make a long story short, we can relate the force and the energy as follows:

$$F = \frac{p \cdot c}{a} = \frac{\mathrm{m}_\gamma \cdot c^2}{a} = \frac{\mathrm{m_e} \cdot c^2}{2a} = \frac{\mathrm{E}}{2a} \Leftrightarrow a = \frac{\mathrm{E}}{2F}$$

This shows the radius is *inversely* proportional to the strength of the force. In other words, if we'd find ourselves in some other universe, where the electromagnetic force would – for some totally random reason – be much *stronger*, the electrons there would be smaller than our electrons here.

Of course, you'll immediately note the obvious mistake in this argument: if the force would be stronger, the energy would be much larger as well, wouldn't it? That is correct. Let us, therefore, try to develop a more general argument. Let us *not* make any assumption about the strength of the force. However, we will assume its *structure* is the one we presented above: a circular current of the *charge* it acts on will produce a field which keeps that charge in the orbit which it happens to be in. We can now *derive* the radius of the oscillation in another way. For some reason we do not understand, the angular frequency of the motion respects the Planck-Einstein relation:

$$a = \frac{\hbar}{\mathrm{m}c} = \frac{\hbar}{\mathrm{E}/c} \Leftrightarrow \mathrm{E} = \frac{\hbar c}{a} = \hbar\omega = \mathrm{h}f = \mathrm{h/T}$$

These calculations are *not* mere entertainment. We get fantastic but not necessarily impossible values for the cycle time and the current here[22]:

[21] The *gamma* (γ) in the subscript refers to the Lorentz factor. However, one should not think of the charge as a photon. Photons do not carry charge. For our photon model, see our other papers (e.g. http://vixra.org/abs/2001.0345). At the same time, we do not mind the association with a photon because, as we noted above, the pointlike charge is photon-like in the sense that it (also) travels at the speed of light. Alexander Burinskii, a Russian physicist who specializes in physical electron models, wrote me the following when I first contacted him (December 2018): "I know many people who considered the electron as a toroidal photon and do it up to now. I also started from this model about 1969 and published an article in JETP in 1974 on it: "Microgeons with spin". Editor E. Lifschitz prohibited me then to write there about Zitterbewegung [because of ideological reasons], but there is a remnant on this notion. There was also this key problem: what keeps [the pointlike charge] in its circular orbit?" We think we managed to answer his question.

[22] These values are also obtained by other authors, even if some of the other calculations differ. See our above-mentioned *critique* of Consa's calculations.

$$\mathrm{T} = \frac{h}{\mathrm{E}} \approx \frac{6.626 \times 10^{-34}\ \mathrm{J \cdot s}}{8.187 \times 10^{-14}\ \mathrm{J}} \approx 0.8 \times 10^{-20}\ \mathrm{s}$$

$$\mathrm{I} = \mathrm{q_e} f = \mathrm{q_e}\frac{\mathrm{E}}{h} \approx (1.6 \times 10^{-19}\ \mathrm{C})\frac{8.187 \times 10^{-14}\ \mathrm{J}}{6.626 \times 10^{-34}\ \mathrm{Js}} \approx 19.8\ \mathrm{A}$$

The point is: one obtains the Compton radius most easily from combining the E = ħ·ω, $c = a\cdot\omega$ and E = m·c^2 relations, as shown below.

$$a = \frac{c}{\omega} = \frac{c \cdot \hbar}{\mathrm{m} \cdot c^2} = \frac{\hbar}{\mathrm{m} \cdot c} = \frac{\lambda_\mathrm{C}}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

Let us now apply the E = ħ·ω, $c = a\cdot\omega$ and E = m·c^2 relations to the mass/energy of proton (or a neutron[23]), we get this:

$$a_\mathrm{p} = \frac{\hbar}{\mathrm{m_p} \cdot c} = \frac{\hbar}{\mathrm{E_p}/c} = \frac{(6.582 \times 10^{-16}\ \mathrm{eV} \cdot s)\cdot(3 \times 10^{8}\ m/s)}{938 \times 10^{6}\ \mathrm{eV}} \approx 0.21 \times 10^{-15}\ \mathrm{m}$$

The result that we obtain here is about 1/4 of the experimentally measured value. Indeed, the radius of a proton is thought to be around 0.83 or 0.84 fm.[24] Hence, the *order of magnitude* is right, at least! More importantly, the distance we have obtained above matches the range parameter *a* that is usually associated with Yukawa's formula, which is about 2 fm.[25] In fact, we can equate the range parameter *a* and the distance *r* with the $a_\mathrm{p} = \hbar/\mathrm{m_p}c$ value in the force formulas we get from the potential formulas and we'll see the electrostatic and nuclear force – which we'll denote as $\mathrm{F_C}$ and $\mathrm{F_N}$ respectively – are, effectively the same[26]:

$$\mathrm{F_C} = -\frac{\mathrm{dV}}{\mathrm{d}r} = -\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r^2} = -\frac{\alpha\hbar c}{r^2} = -\frac{\alpha \mathrm{m_p^2} c^2}{\hbar}$$

[23] The mass of a neutron is about 939,565,413 eV/c^2 and about 938,272,081 eV/c^2 for the proton. Hence, the energy *difference* is a bit less than 1.3 MeV. It is, therefore, very tempting to think a neutron might, somehow, combine a proton and an electron: the electron mass is about 0.511 MeV/c^2 and, hence, we may think of the remaining difference as some kind of binding energy—the attractive force between the positive and a negative charge, perhaps? These thoughts are, obviously, very speculative. We did explore some of these, however, in our paper on the nature of protons and neutrons (http://vixra.org/abs/2001.0104), and we very much welcome comments.

[24] We refer to Wikipedia for a very readable account of the experiments and their results (https://en.wikipedia.org/wiki/Proton_radius_puzzle). Earlier measurements were somewhat inconclusive because they yielded a radius between 0.84 and 0.90. However, recent research suggests the so-called proton radius puzzle has been solved (see: https://physicstoday.scitation.org/do/10.1063/PT.6.1.20191106a/full/). For those who would wonder, we may, perhaps, also note the same calculations do work very well for the muon-electron. We've done these calculations in another speculative paper (http://vixra.org/abs/1908.0430).

[25] See Aitchison and Hey's introduction to *Gauge Theories in Particles Physics*, Vol. 1, Chapter 1 ((*The Particles and Forces of the Standard Model*) , p. 16. To be precise, Aitchison and Hey there write the range parameter is ~ 2 fm. They don't explain this result. Hence, we must assume they use the same formulas.

[26] For the detailed calculations in regard to force formulas, see: http://vixra.org/abs/2001.0104. Note that we left the nuclear constant (υ_0) out because its numerical value is one. You can, of course, calculate the *exact* value using the CODATA values for the various constants. You should find a pretty decent value: about 0.0000174 N, if we are not mistaken.

$$\mathrm{F_N} = -\frac{\mathrm{dU}}{\mathrm{d}r} = -\frac{\mathrm{g_N^2}}{4\pi}\cdot\frac{\left(\frac{r}{a}+1\right)\cdot e^{-\frac{r}{a}}}{r^2} = -\frac{\mathrm{g_N^2}}{4\pi}\cdot\frac{2e^{-1}}{r^2} == -\frac{e\alpha\mathrm{hc}}{4\pi}\cdot\frac{2e^{-1}\mathrm{m_p^2}c^2}{\hbar^2} = \frac{\alpha\mathrm{m_p^2}c^2}{\hbar}$$

Eureka ! Have we discovered the strong force? Is it just a stronger *variant* of the electromagnetic force?

Maybe, but probably not. Again, this calculation only serves to demonstrate a philosophical point: if the energy (and equivalent mass) of nucleons is the energy of some oscillating *strong* charge, then the energy density of protons and neutrons suggest it is going to be a very strong force, indeed!

To illustrate the point, the above F = E/2*a* formula yields a force of 0.115 N for the electron: such force gives a mass of about 115 *gram* (1 g = 10^{-3} kg) an acceleration of 1 m/s per second, which is humongous on the *pico*meter scale that we are talking about here. However, terms such as massive or humongous suddenly become very relative when using the same formulas to calculate the value of the presumed strong(er) version of the oscillatory force inside a proton. We will let you go through them and, to keep the exercise somewhat interesting, you may want to think of they'd imply in terms of spacetime curvature.

> Note: In contrast to the enormity of the presumed forces inside of an electron (or, even larger, inside of a proton), you should note that the force *between* two protons at the calculated cut-off value (0.21 fm) is quite reasonable: you can, effectively, calculate the exact value for $F_N = F_C$ using the formulas above (and the CODATA values for the various constants). You should find a pretty decent value: about 0.0000174 N, if we are not mistaken.

To conclude this section, we'd like to repeat the remark we made above: the effective range of Yukawa's force is much smaller than the measured charge radius of a proton: one fourth (1/4), to be precise. Hence, it is really bizarre Yukawa hoped to explain some strong attractive force (stronger than the electrostatic repulsive force) *between protons*. It is one of those weird things that just doesn't make sense.

The magnetic moment of electrons, protons and neutrons

Before we move on, we should quickly insert some other obvious calculations. If a proton, somehow, combines electric and strong charge, we could, for the time being, think of the electric charge alone and ask some questions in regard to the *electric current* inside of the proton. A circular electric current creates a magnetic moment which, for the electron, we calculated as:

$$\mu = \mathrm{I}\cdot\pi a^2 = \mathrm{q_e}f\cdot\pi a^2 = \mathrm{q_e}\frac{\mathrm{m}c^2}{h}\cdot\pi a^2 = \mathrm{q_e}c\frac{\pi a^2}{2\pi a} = \frac{\mathrm{q_e}c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

It is also consistent with the presumed angular momentum of an electron, which is that of a spin-1/2 particle. Here we must make some assumption as to how the effective mass of the electron will be spread over the disk. If we assume it is spread uniformly over the whole disk[27], then we can use the 1/2 form factor for the moment of inertia (*I*). We write:

[27] This is a very essential point. It is also very deep and philosophical. We say the energy of the electron is in the motion of the pointlike charge, but it's also in the magnetic field inside of the ring. Our two-dimensional oscillator model models this as the energy of two independent (perpendicular) oscillations. The mass factor in the formula is the *effective* mass of the electron which, based on a geometric argument (http://vixra.org/abs/1905.0521), we can calculate as ½ of the (rest) mass of the electron. Hence, we write: $m_\gamma = m_e/2$. We should then use the relativistically

$$L = I \cdot \omega = \frac{ma^2}{2}\frac{c}{a} = \frac{mc}{2}\frac{\hbar}{mc} = \frac{\hbar}{2}$$

We now get the correct g-factor for the pure spin moment of an electron:

$$\boldsymbol{\mu} = -g\left(\frac{q_e}{2m}\right)\mathbf{L} \Leftrightarrow \frac{q_e}{2m}\hbar = g\frac{q_e}{2m}\frac{\hbar}{2} \Leftrightarrow g = 2$$

However, let us look at the formula for the magnetic moment once more. If the proton, somehow, has a pointlike elementary charge in it and if it – as we presume in a *Zitterbewegung* model – is in some kind of circular motion, then we can establish a simple relation between the magnetic moment and radius of the circular current. Indeed, the current I is just the product of the elementary charge and the frequency, which we can calculate as f = E/h (Planck-Einstein relation) or – more prosaically – as f = c/2πa, i.e. the velocity of the charge divided by the circumference of the loop. We write:

$$\mu = I \cdot \pi a^2 = q_e c\frac{\pi a^2}{2\pi a} = q_e c\frac{a}{2}$$

$$= (1.602176634{\times}10^{-19}) \cdot (299{,}792{,}458) \cdot \frac{a}{2} \approx 0.24 \ldots{\times}10^{-10} \cdot a$$

Using the Compton radius, this yields the correct magnetic moment for the electron[28]:

$$\mu_e = (0.24 \ldots{\times}10^{-10} \cdot 0.386 \ldots \times 10^{-12}) \approx 9.2847647043 \times 10^{-24}\ \mathrm{J/T}$$

What radius do we get when applying the a = μ/0.24…×10^{-10} relation to the (experimentally measured) magnetic moment of a proton? I invite the reader to verify the next calculation using CODATA values:

$$a = \frac{1.41 \ldots{\times}10^{-26}}{0.24 \ldots{\times}10^{-10}} = 0.587{\times}10^{-15}\ m$$

Again, the *order of magnitude* is right, at least! So… Is this rubbish, or is it *good enough*?

Wait a minute! The *measured* value of the *charge* radius of a proton radius is 0.83 fm, right? Correct. And the exact *value* of a charge radius also depends on statistical definitions and all that, right? Correct. Then look at this:

$$(0.587 \ldots{\times}10^{-15}\ \mathrm{m}) \cdot \sqrt{2} \approx 0.83{\times}10^{-15}\ \mathrm{m}$$

We get the *exact* value of the charge radius of a proton based our theoretical *zbw* model, don't we? Hence, our model makes perfectly sense, doesn't it?[29]

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correct energy equation for an oscillator (see p. 66-67 of my manuscript: http://vixra.org/pdf/1901.0105vG.pdf, which we also copied in annex 2):

$$\frac{dE}{dt} = \frac{d}{dt}\left[\frac{1}{2}kx^2 + mc^2\right] = 0$$

[28] The calculations do away with the niceties of the + or – sign conventions as they focus on the *values* only. We also invite the reader to add the SI units so as to make sure all equations are consistent from a *dimensional* point of view. For the values themselves, see the CODATA values on the NIST website (https://physics.nist.gov/cuu/Constants/index.html).

[29] In case you wonder how a √2 factor can be related to statistics, see Feynman's analysis of Einstein's treatment of the random walk (https://www.feynmanlectures.caltech.edu/I_06.html).

Maybe. Maybe not. These are, largely, tautological calculations. We only modeled an electric current based on the assumption that the *idea* of a proton must include the idea of a circulating elementary charge, *somehow*. Nothing more. Nothing less. But I admit it is nice we get very meaningful results.

What about the weak force?

To conclude this rather philosophical introduction, we should probably say a few words about the weak force. The weak force is supposed to explain why things fall apart, or why particles are *unstable*, rather than stable. We prefer to *not* think of decay or disintegration as a force. It is, in fact, the exact *opposite* of the idea of a force: a force is supposed to keep things together.[30]

In the same vein, we like to add we do not want to entertain the idea of messenger particles or force carriers – virtual photons, gluons, or whatever other bosons or metaphysical constructs that have been invented since Yukawa first presented these ideas. Indeed, it is unfortunate that – instead of realizing he was actually proposing the existence of a new *charge* – he used his formula to derive a hypothetical *nuclear force quantum*.[31]

It is now time to turn to the concept of mass—or to the concepts (*plural*) of mass, we should say.

Kinetic, electromagnetic and other masses

We should probably not remind the reader of the *classical* concept of electromagnetic mass. If so, we will refer him or her to an equally classic textbook, such as Feynman's Lectures.[32] These classical calculations are usually based on the assembly of a spherical shell or sphere of charge. Another, more intricate, argument involves the concept of *field momentum*.[33] However, they all involve the idea of *naked* charge, i.e. electric charge stripped of *any other attribute or idea*. Hence, the basic idea, here too, is that charge is just charge, *with zero rest mass*. As such, these models are not entirely dissociated from our modern-day *zbw* model of an electron.[34]

We should highlight the key differences and issues, however. First, these classical calculations do usually *not* use Compton radius, but the (classical) Thomson radius, which we can write as[35]:

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[30] You will wonder how we think about unstable particles, then. We think of them as wavicles that do *not quite* respect the Planck-Einstein relation. Hence, we think of them as non-harmonic oscillations: after a while, they just break down in stuff that does respect the Planck-Einstein relation..

[31] For a brief but accessible treatment of this matter, see the above-mentioned introduction to Aitchison and Hey's *Gauge Theories in Particle Physics* (2013). I am quoting this textbook rather than any other because it also incorporates the Higgs mechanism: the 'missing' scalar field that is supposed to explain mass and is now thought of as being real. Why? Because some CERN data might be interpreted as some 'signature' of it and, more importantly, because the current Nobel Prize Committee thinks such 'signals' or 'signatures' give the hypothesis sufficient credibility.

[32] See: Feynman's Lectures, Volume II, Chapter 28, on *Electromagnetic mass.*

[33] See section 2 in Feynman's above-mentioned *Lecture* (the field momentum of a moving charge).

[34] Note that calculations of electromagnetic mass never revolve around protons because their mass is inexplicably large as compared to that of an electron, as we pointed out already.

[35] The formula uses the fine-structure constant $\alpha = q_e^2/4\pi\varepsilon_0\hbar c = e^2/\hbar c \approx 0.0073$, which relates all of the three radii of the electron (Thomson, Compton and Bohr radius). The fine-structure constant has several meanings but, as mentioned before, we primarily think of it as a scaling constant in a layered model of electron motion. It is surely

$$r_e = \alpha \cdot r_C = \frac{q_e^2}{4\pi\varepsilon_0 \hbar c}\frac{\hbar}{m_e c} = \frac{e^2}{m_e c^2} = \frac{e^2}{E_e}$$

For example, if we assume all of the electron charge is to be assembled in a spherical shell with radius *a* = r_e, then the energy needed to do so, will be equal to[36]:

$$U = \frac{1}{2}\frac{e^2}{a} = \frac{1}{2}\frac{e^2 E}{e^2} = \frac{1}{2}E$$

If the *form factor* is a proper sphere instead of a *shell*, then we get:

$$U = \frac{3}{5}\frac{e^2}{a} = \frac{3}{5}E$$

The more advanced idea of using the idea of field momentum – an argument which takes some time to explain and, hence, which we won't elaborate here – gives us a value of 0.75 (3/4) times the actual electron energy:

$$U = \frac{3}{4}\frac{e^2}{a} = \frac{3}{4}E$$

Are we getting there? Can we *assemble* an electron, somehow, so as to make sure the energy of the assembly adds up to the total electron mass? No. Feynman writes the following about that:

> "It is impossible to get all the mass to be electromagnetic in the way we hoped. It is not a legal theory if we have nothing but electrodynamics. Something else has to be added. Whatever you call them—"rubber bands," or "Poincaré stresses," or something else—there have to be other forces in nature to make a consistent theory of this kind. Clearly, as soon as we have to put forces on the inside of the electron, the beauty of the whole idea begins to disappear. Things get very complicated. You would want to ask: How strong are the stresses? How does the electron shake? Does it oscillate? What are all its internal properties? And so on. It might be possible that an electron does have some complicated internal properties. If we made a theory of the electron along these lines, it would predict odd properties, like modes of oscillation, which haven't apparently been observed. We say "apparently" because we observe a lot of things in nature that still do not make sense. We may someday find out that one of the things we don't understand today (for example, the muon) can, in fact, be explained as an oscillation of the Poincaré stresses. It doesn't seem likely, but no one can say for sure. There are so many things about fundamental particles that we still don't understand. Anyway, the complex structure implied by this theory is undesirable, and the attempt to explain all mass in terms of electromagnetism—at least in the way we have described—has led to a blind alley."[37]

What rubbish! Doing some more thinking about the equivalent mass of *magnetic* forces resulting from the *motion* of charge would have solved the problem!

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not some "magical" or "God-given number." Its meaning is perfectly comprehensible. See our paper on the meaning of the fine-structure constant (http://vixra.org/abs/1812.0273).

[36] For the formulas of the energy, we refer to Feynman's *Lecture* on electromagnetic mass (Volume II, Chapter 28). Needless to say, with E we mean the actual *total* energy of the electron, i.e. about 0.511 MeV/c^2).

[37] Feynman's *Lectures*, section II-28-4 (https://www.feynmanlectures.caltech.edu/II_28.html).

Richard Feynman was a clever man, and the ring electron model had been around for quite a while already. Consa offers a short but interesting history of the idea, and it goes all the way back to 1915.[38] It begs the question: why are (were?) gems like this hidden from common sight for so long?

To conclude this paper, we should say something about other masses—because that's what I suggested in the title of this section (electromagnetic and other masses). However, we did that already. We suggested the *rest* mass of a particle is determined by the force(s) inside. The only other mass that is relevant, then, is the concept of kinetic or dynamic mass: the extra energy we get from the *motion* of our particle.

Now that's the concept of relativistic mass. It's not any different. We've added an annex on that, which does not have any advantage to the treatment you'll find in a textbook except we've tried to make it somewhat wittier. Hence, we think we've exhausted the topic—for the time being, that is.

Jean Louis Van Belle, 24 January 2020

[38] Consa's paper can be found on: http://www.ptep-online.com/2018/PP-53-06.PDF. We should mention David Hestenes also refers to earlier calculations by Antonio F. Rañada. We found the link (https://link.springer.com/article/10.1007/BF00401864) but have not examined this paper in detail.

Annex 1: Relativity

A force is that what changes the state of motion of an object. How do we define an object, and how do we model its state of motion? In the previous sections, we argued the *rest* mass of an electron may be explained by the oscillatory motion of a pointlike charge which – by itself – is not supposed to have any other attributes. To be precise, we assumed the pointlike charge had *zero* rest mass.

The idea of a zero-mass object is problematic. Newton's force law (F = m·*a*) tells us that, if m is zero, even the smallest force will give it an infinitely large acceleration and, therefore, its velocity should also be infinitely large. It would, therefore, be impossible to calculate its position along its direction of motion at any point in time. The idea of infinity is a wonderful mathematical concept but, *in reality*, it is a rather inconvenient thing to work with. If mass would *not* depend on velocity, then any mass could be accelerated to humongous velocities by the tiniest of forces: all that is needed is time.

Infinity is, perhaps, just some *Platonic idea*: an idealization that cannot be real—whatever that means. To explain relativity and the *absolute* speed of light to someone who has *not* had the luxury to study physics, I'd use the following story.[39] Because of the above-mentioned inconveniences related to the idea of infinite velocity (a particle with infinite velocity is everywhere and, therefore, nowhere at the same time), you'll agree there should be some (absolute) speed *cap* in the Universe. Now, if you would be God, and you'd have to regulate the Universe by putting a cap on speed, how would you do that?

You would probably want to benchmark speed against the fastest thing in the Universe, which is a light photon. Why is a photon the fastest thing in the Universe? Because it has no rest mass and, hence, it can effectively travel at the speed of light: *c*. It's the speed of the fastest-traveling *signal*.[40] So now you want to put a speed limiter on everything else, so it can only travel at some *fraction* of the speed of light.

That fraction (v/c) is just a ratio between 0 and 1, of course. Now, because you're God, you do *not* want to police around so you want something mechanical: you want to *burden* everything with an intricate friction device, so as to make sure the friction goes up *progressively* as v/c goes to 0 to 1. You do *not* want something linear because you want the friction to become infinite as v/c goes to 1, so that's when v approaches c. So that's one thing you have figured out in your design.

Of course, you'll also want a device that can cope with *every*thing: electrons, bicycles, spaceships, solar systems—whatever you can think of. The speed limit applies to all. But then you don't need too much force to accelerate a proton as compared to, say, that new spaceship that was just built on planet X. Hence, you think about brakes and engines and all that but, after a while, you realize it's probably better to just ask some of your best engineers to finalize your design.

[39] I borrow this story from my e-book on a realist interpretation of quantum physics (http://vixra.org/abs/1901.0105).

[40] If you know anything about quantum mechanics, you will know that the *phase* velocity of a composite wave packet may be superluminal. In fact, it usually *is*. However, this phase velocity is just a mathematical concept. It is not something *real* that is traveling through space. In other words, it can*not* carry any information. Only the *shape* of the wave can carry information and, therefore, can qualify as a proper *signal*. Now, the shape of the wave travels with the *group* velocity of the wave packet, which is always smaller than *c*. This is probably confusing you, but I just wanted to be correct—especially because I must assume you have already done quite a lot of homework when you are reading papers such as this one.

So you all sit together and you explain your problem and the design requirements. One of them, *Newton*, will tell you that, when applying a force to an object, its acceleration will be proportional to its mass. So he goes to the blackboard and writes it down like this: $\boldsymbol{F} = \text{m}\cdot\boldsymbol{a}$. Of course, you tell him you know that already, and that this is *exactly* your problem: even the smallest force can accelerate the very massive object to crazy speeds—to *infinite* speeds, in fact! You just need to apply the force *long enough*. Newton shrugs his shoulders and sits down again. Now *Lorentz* gets up and points to the mass factor in the formula: m should go up with speed, he says. And it should go up progressively—as per God's design, he says. Lorentz is always well prepared, so he has a print-out with some formulas and graphs and sticks it on the blackboard. Here is an easy formula that does the trick, he says.

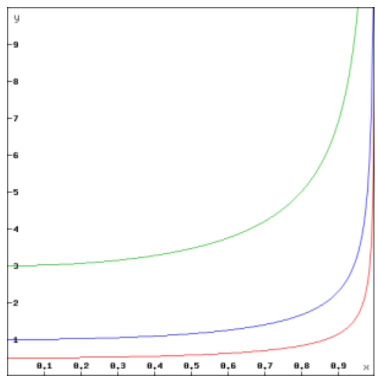


Figure 2: How to put a speed cap on bicycles, spaceships and galaxies

Look here, he says. The red graph is for m = 1/2, the blue one for m = 1, and the green one for m = 3. In the beginning, nothing much happens: the thing picks up speed but its mass doesn't increase all that much. Why not? Because you do want to allow everyone to move their stuff around, right? But when it gets a bit crazy, then the friction kicks in, and *very* progressively so as the speed gets closer to the speed of light.

Now you stare at this for a few seconds, and you think this is looking good. However, you tell Lorentz you don't want to discriminate: it looks like we're putting more aggressive brakes on the green thing than on the blue or the red thing, right? However, Lorentz says that is *not* the case. There is no discrimination here: his factor is the same per *unit* of mass. The graphs show the *product* of the mass and his Lorentz factor, which is actually represented by the blue line—because that's the one for m = 1. So, yes, the green thing will actually have better brakes, but that's just commensurate with its mass. You want the lorry to come with better brakes, right? And bicycle brakes won't do for a car, right?

You look again, and you think that makes sense. But then you hesitate, of course. You don't want to change the Laws of the Universe, as that would be messy. It would surely upset Newton, because he is pretty fussy about you tampering with stuff. So you look at both and you say: what's the implications for the force law? Newton nods in agreement: yes, what about it? We don't want to change it, he says, because there are a zillion devices that work on it now. We can't do a total recall, can we?

Lorentz is still at the blackboard but he tells Newton: it's not a problem. We're going to use the same force law. We're just going to distinguish *two* mass concepts: the mass at rest, and the mass at some velocity *v*. Just put a subscript – m_v – and then you use this. He jots this on the blackboard:

$$\boldsymbol{F} = \mathrm{m}_v \cdot \boldsymbol{a} = \frac{\mathrm{d}(\mathrm{m}_v \cdot \boldsymbol{v})}{\mathrm{d}t} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}$$

$$\mathrm{m}_v = \gamma \cdot \mathrm{m}_0 = \frac{1}{\sqrt{1 - v^2/c^2}} \cdot \mathrm{m}_0$$

Now Newton stares at that, and he takes a few minutes. You think he is going to turn it down, because his formula is… Well… Newton's formula, right? But… No. Something weird happens: Newton nods and agrees! He gets up, shakes hands with Lorentz and says: excellent job! Perfect fix! So you're delighted and you tell Lorentz he can pick and choose his men and build it.

Newton walks out, and Lorentz stays behind. Suddenly you see some worry on this face, and so you ask: what's up? You're not happy with your own thing? He sighs and says: my formula is the *only* thing that can do the trick because, yes, you want it to be progressive. It needs to be something based on the idea of the mass *unit*. But this mechanical thing has some weird implications. You ask: what implications? Now Lorentz starts a discussion on a guy you've never heard about – Albert Einstein – and he starts mumbling about time dilation and length contraction. He says Newton's formula came with Galilean relativity, and that we'll need a new concept of relativity. But you want to move on by now, and so you tell Lorentz to hire that Einstein and just get on with it. So… Well… That's what we'll do also. We'll just get on with it. However, before we do so, we'd like to add one or two other philosophical remarks:

1. We wrote that the Lorentz formula is the only one that can do the trick but, to be honest, we have no *proof* that other formulas would not work. While our Universe is what it is and, hence, we should just accept the Lorentz factor for what it is, it is an interesting exercise to try some other formulas. The $\sqrt{1-(v/c)^2}$ factor makes us think of the formula for a circle: $y = \sqrt{1 - \mathrm{x}^2}$, and you may, therefore, think some similar formula might also do the trick. Try it. It doesn't.[41]

2. We also wrote – rather jokingly – that infinity is a nice mathematical concept but that it is weird to think of what it could possibly mean *in reality*. This is actually a rather deep philosophical statement. You should think through Zeno's paradoxes. Differential calculus shows that the idea that we can keep splitting some interval in time or in space in smaller and smaller bits – going on forever (so that's, funnily enough[42], the idea of a *limit* in math) – is *not* incompatible with Achilles overtaking the tortoise, or the idea of an arrow being somewhere while flying through space, but it is good to think through those paradoxes. We need math to describe reality – whatever *idea* we have about it – but Planck's quantum of action, and the finite speed of light, seems to tell us our mathematical ideas are what they are: idealized notions to describe something finite.[43]

[41] Having said that, the graph of the *inverse* Lorentz factor, as a function of the β = *v*/*c ratio*, is, in fact, just a simple circular arc—which is as it should be in light of the functional shape of the two formulas.

[42] Think about what I'd call that funny: the mathematical definition of a *limit* involves the idea of infinity. So that's a pretty clear example of a *contradictio in terminis*, no? 😊

[43] The rather philosophical discussion on the mathematical consistency of Dirac's delta function is a nice example of a paradox in quantum mechanics. We will not entertain such discussions in this book, however. Not because we don't like them – on the contrary – but because they have little *practical* value in trying to move towards some understanding of it all. However, we do encourage the reader to look into this. It's fun. For starters, the reader may want to think of how a *link* function can map the *in*finite [0, +∞] set of real numbers to the *finite* [0, *c*] interval.

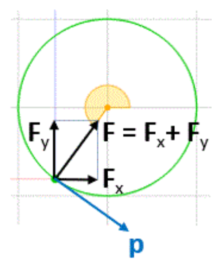
Annex 2: The effective mass of the *Zitterbewegung* charge

The electron and its charge

The *Zitterbewegung* model thinks of an electron as a hybrid thing: it consists of a pointlike charge which – in itself – has zero rest mass. However, its *Zitterbewegung* motion gives the electron as a whole the energy and, therefore, rest mass as measured in countless experiments. Hence, the 'elementary charge' that is whizzing around the center is a naked charge: it has no properties but its charge. Its *rest* mass is, therefore, zero, and it acquires all of its mass from its velocity. As such, some refer to it as some kind of toroidal photon, or an electron photon – but I don't like these terms because they are not only imprecise but also misleading: photons are not supposed to carry any charge.

Of course, the question is: how does a naked charge acquire mass? Just from whizzing around? The answer is positive. To keep an object with some momentum in a circular orbit, a centripetal force is needed, as shown in Figure 3. What is the nature of this force? Because the force can only grab onto the charge, it must be electromagnetic. We will come back to the force in a moment but, at this point, we will want to think about the nature of the momentum of the charge (**p**).

Figure 3: The *Zitterbewegung* model of an electron



The momentum **p** is relativistic momentum, of course, so its magnitude $|\mathbf{p}| = p$ is equal to:

$$p = mc = \gamma m_0 c$$

How should we calculate this? The Lorentz factor (γ) goes to infinity as the velocity goes to *c* and, as mentioned above, we assume the pointlike charge has zero rest mass, so $m_0 = 0$. So we are multiplying zero by infinity. What do we get? The behavior of the $p = \gamma m_0 v$ function is quite weird. The graph in Figure 4 shows what happens with the $p = m_v v = \gamma m_0 v$ for $m_0 = 0.001$ and v/c ranging between 0 and 1.[44]

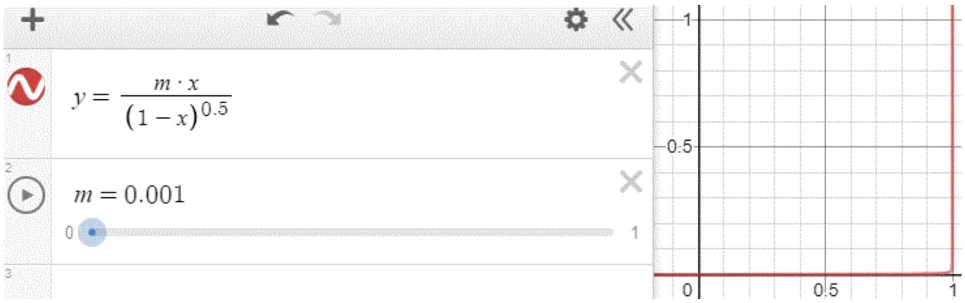
It is quite enlightening: p is (very close to) zero for v/c going from 0 to (very close to) 1 but then becomes infinity near or at $v/c = 1$ itself. What can we say about this? Perhaps we should say that the momentum of an object with zero rest mass is a nonsensical concept? Perhaps we should associate a tiny but non-zero rest mass with the pointlike charge? If it is *something*, then it should have some mass, shouldn't it?

Maybe. Maybe not. We are not in a position to say much about this right now, and so we won't. The discussion is, in any case, quite philosophical here and, therefore, not so relevant. What we want to do is to find some value for the *effective* mass and, preferable, a value that is expressed in terms of the

[44] We used the online desmos.com graphing tool to produce the graph.

actual rest mass of our *electron*: note that we distinguish the electron, as a whole, from the pointlike charge that (we think) is part of it !

Figure 4: $p = m_v v = \gamma m_0 v$ for $m_0 \to 0$



Calculating the effective mass of the charge

Let us distinguish the components of the momentum vector **p** in the *x*- and *y*-direction respectively. We write:

$$\mathbf{p} = \mathbf{p_x} + \mathbf{p_y}$$

The *magnitude* of these vectors can then be written as $|\mathbf{p}| = p$, $|\mathbf{p_x}| = p_x$, and $|\mathbf{p_s}| = p_y$ respectively. If we then write the effective mass as m_v or – even simpler – as m (as opposed to me), then we can write p_x and p_y as:

$$p_x = m v_x = \gamma m_0 v_x \text{ and } p_y = m v_y = \gamma m_0 v_y$$

The *origin* of both the force and momentum vectors is the position vector ***r***, which we can write using the elementary wavefunction, i.e. Euler's function:

$$\boldsymbol{r} = a \cdot e^{i\theta} = x + i \cdot y = a \cdot \cos(\theta) + i \cdot a \cdot \sin(\theta) = a \cdot \cos(\omega t) + i \cdot a \cdot \sin(\omega t) = (x, y)$$

We can also calculate the centripetal acceleration: it's equal to $a_c = v_t^2/a = a \cdot \omega^2$. This formula is relativistically correct. It might be useful to remind ourselves how we get this result. The position vector ***r*** has a horizontal and a vertical component: $x = a \cdot \cos(\omega t)$ and $y = a \cdot \sin(\omega t)$. We can now calculate the two components of the (tangential) velocity vector $\boldsymbol{v} = d\boldsymbol{r}/dt$ as $v_x = -a \cdot \omega \cdot \sin(\omega t)$ and $v_x\, y = -a \cdot \omega \cdot \cos(\omega t)$ and, in the next step, the components of the (centripetal) acceleration vector $\boldsymbol{a_c}$: $a_x = -a \cdot \omega^2 \cdot \cos(\omega t)$ and $a_y = -a \cdot \omega^2 \cdot \sin(\omega t)$. The magnitude of this vector is then calculated as follows:

$$a_c^2 = a_x^2 + a_y^2 = a^2 \cdot \omega^4 \cdot \cos^2(\omega t) + a^2 \cdot \omega^4 \cdot \sin^2(\omega t) = a^2 \cdot \omega^4 \Leftrightarrow a_c = a \cdot \omega^2 = v_t^2/a$$

Now, Newton's force law tells us that the magnitude of the centripetal force $|\mathbf{F}| = F$ will be equal to:

$$F = m \cdot a_c = m \cdot a \cdot \omega^2$$

However, we again have this problem of determining what the mass of our pointlike charge actually is: the m_0 in our $m = \gamma m_0$ is zero ! We should find another way !

We may note the horizontal and vertical force component behave like the restoring force causing linear harmonic oscillation. This restoring force depends linearly on the (horizontal or vertical) displacement from the center, and the (linear) proportionality constant is usually written as k. In case of a mechanical spring, this constant will be the *stiffness* of the spring. We don't have a spring here so it is tempting to think it models some elasticity of space itself. However, we should probably not engage in such philosophical thought. Let us just write down the formulas:

$$F_x = dp_x/dt = -k\cdot x = -k\cdot a\cdot\cos(\omega t) = -F\cdot\cos(\omega t)$$
$$F_y = dp_y/dt = -k\cdot y = -k\cdot a\cdot\sin(\omega t) = -F\cdot\sin(\omega t)$$

Now, it is quite straightforward to show that the constant (k) can always be written as:

$$k = m\cdot\omega^2$$

We get that from the *solution* we find for ω when solving the differential equations $F_x = dp_x/dt = -k\cdot x$ and $F_y = dp_y/dt = F_y = dp_y/dt = -k\cdot y$ and assuming there is nothing particular about p and m. In other words, we assume there is nothing wrong with this $p = m\cdot v = \gamma m_0 v$ relation. So we just don't think about the weird behavior of that function. It's a bit like what Dirac did when he *defined* his rather (in)famous Dirac function: the function doesn't make sense mathematically but it works – i.e. we get the right answers – when we use it.

So now we have the $k = m\cdot\omega^2$ equation and we know m is *not* the rest mass of our electron here. We referred to it as the *effective* mass of our pointlike charge as it's whizzing around at the speed of light. We need to remember mass is a measure of inertia and, hence, we can measure that inertia along the horizontal and vertical axis respectively. Hence, we can write something like this: $m = m_\gamma = m_x = m_y$, in line with the distinction we made between p, p_x and p_y. Why m_γ? The notation is just a placeholder: we need to remind ourselves it is a relativistic mass concept and so I used γ (the symbol for the Lorentz factor) to remind ourselves of that. So let us write this:

$$k = m_\gamma\cdot\omega^2$$

From the equations for F_x and F_x, we know that $k\cdot a = F$, so $k = F/a$. Hence, the following equality must hold:

$$F/a = m_\gamma\cdot\omega^2 \Leftrightarrow F = m_\gamma\cdot a\cdot\omega^2 \Leftrightarrow F/a = m_\gamma\cdot a^2\cdot\omega^2 = \Leftrightarrow F/a\cdot m_\gamma = a^2\cdot\omega^2$$

We know the sum of the potential and kinetic energy in a linear oscillator adds up to $E = m\cdot a^2\cdot\omega^2/2$. We have *two* independent linear oscillations here so we can just add their energies and the ½ factor vanishes. Now I am going to ask you to accept Einstein's mass-energy equivalence relation should apply, so I am asking you to accept that **the total energy in this oscillation must be equal to $E = m\cdot c^2$**. The mass factor here is the *rest mass of our electron*, so it's *not* that weird relativistic m_γ concept. *However*, we did equate *c* to $a\cdot\omega^2$. Hence, we can now write the following:

$$E = m\cdot c^2 = m\cdot a^2\cdot\omega^2 = m\cdot F/a\cdot m_\gamma$$

The force is, therefore, equal to:

$$F = (m_\gamma/m)\cdot(E/a)$$

Now what can we say about the m_γ/m ratio? We know m_γ is sort of undefined—but it shouldn't be zero and it shouldn't be infinity. It is also quite sensible to think m_γ should be smaller than m. It cannot be larger because than the energy of the oscillation would be larger than $E = mc^2$. What could it be? 1/2, 1/2π? Rather than guessing, we may want to remind ourselves that we know the angular momentum: $L = \hbar/2$. We calculated it using the $L = I\cdot\omega$ formula and using an educated guess for the moment of inertia

(I = m·a^2/2), but we also have the **L** = ***r*** × **p** formula, of course! The lever arm is the radius here, so we can write:

1. L = ħ/2 ⇔ p = L/a = (ħ/2)/a = (ħ/2)·mc/ħ = mc/2
2. p = $m_\gamma c$

   ⇒ $m_\gamma c$ = mc/2 ⇔ m_γ = m/2

We found the grand result we expected to find: the *effective* mass of the pointlike charge – as it whizzes around the center of the two-dimensional oscillation that makes up our electron – is half of the (rest) mass of the electron.

Basic calculations

We can now calculate the force using the F = (m_γ/m)·(E/a) formula:

$$\mathrm{F} = \frac{1}{2}\frac{\mathrm{E}}{a} \approx \frac{8.187 \times 10^{-14}\ \mathrm{J}}{\frac{2}{2\pi} \cdot 2.246\ \times 10^{-12}\ \mathrm{m}} \approx 0.115\ \mathrm{N}$$

This force is equivalent to a force that gives a mass of about 115 *gram* (1 g = 10^{-3} kg) an acceleration of 1 m/s per second. This is *huge* at the sub-atomic scale. Does it make sense? We think it does. We can also calculate the current and the other properties of the electron, such as the angular momentum and the magnetic moment[45]:

$$\mathrm{I} = \mathrm{q_e}f = \mathrm{q_e}\frac{\mathrm{E}}{h} \approx (1.6 \times 10^{-19}\ \mathrm{C})\frac{8.187 \times 10^{-14}\ \mathrm{J}}{6.626 \times 10^{-34}\ \mathrm{Js}} \approx 19.8\ \mathrm{A}\ (ampere)$$

The magnetic moment is equal to the current times the area of the loop and is, therefore, equal to:

$$\mu = \mathrm{I} \cdot \pi a^2 = \mathrm{q_e}\frac{\mathrm{m}c^2}{h} \cdot \pi a^2 = \mathrm{q_e}c\frac{\pi a^2}{2\pi a} = \frac{\mathrm{q_e}c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

This calculation is consistent with the presumed angular momentum of an electron, which is that of a spin-1/2 particle. Here we must make some assumption as to how the effective mass of the electron will be spread over the disk. Our oscillator model assumes it is spread uniformly over the whole disk[46], then we can use the 1/2 form factor for the moment of inertia (I). We write:

$$\mathrm{L} = I \cdot \omega = \frac{ma^2}{2}\frac{c}{a} = \frac{mc}{2}\frac{\hbar}{mc} = \frac{\hbar}{2}$$

We now get the correct g-factor for the pure spin moment of an electron:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g} = 2$$

[45] See: Jean Louis Van Belle, *The Electron as a Harmonic Electromagnetic Oscillator*, 1 June 2019 (http://vixra.org/abs/1905.0521).

[46] See Annex 3 for our oscillator model, which effectively assumes the energy is all over the 'disk' of this two-dimensional oscillation.

Annex 3: Oscillator math

In this Annex, we will walk you through the math of our 'two-dimensional oscillator' model which, initially, I used to refer to as the 'flywheel model' of matter-particles.[47] It is all pretty straightforward but let us just make sure you're somewhat comfortable with it.[48]

If the magnitude of the oscillation is equal to a, then the motion of the piston (or the mass on a spring) will be described by $x = a\cdot\cos(\omega\cdot t + \Delta)$.[49] Needless to say, Δ is just a phase factor which defines our $t = 0$ point, and ω is the *natural* angular frequency of our oscillator. Because of the 90° angle between the two cylinders, Δ would be 0 for one oscillator, and –π/2 for the other. Hence, the motion of one piston is given by $x = a\cdot\cos(\omega\cdot t)$, while the motion of the other is given by $x = a\cdot\cos(\omega\cdot t - \pi/2) = a\cdot\sin(\omega\cdot t)$. The kinetic and potential energy of *one* oscillator (think of one piston or one spring only) can then be calculated as:

1. $\text{K.E.} = T = m\cdot v^2/2 = (1/2)\cdot m\cdot\omega^2\cdot a^2\cdot\sin^2(\omega\cdot t + \Delta)$
2. $\text{P.E.} = U = k\cdot x^2/2 = (1/2)\cdot k\cdot a^2\cdot\cos^2(\omega\cdot t + \Delta)$

The coefficient k in the potential energy formula characterizes the restoring force: $F = -k\cdot x$. From the dynamics involved, it is obvious that k must be equal to $m\cdot\omega^2$. Hence, the total energy is equal to:

$$E = T + U = (1/2)\cdot m\cdot\omega^2\cdot a^2\cdot[\sin^2(\omega\cdot t + \Delta) + \cos^2(\omega\cdot t + \Delta)] = m\cdot a^2\cdot\omega^2/2$$

The formulas above are illustrated below.

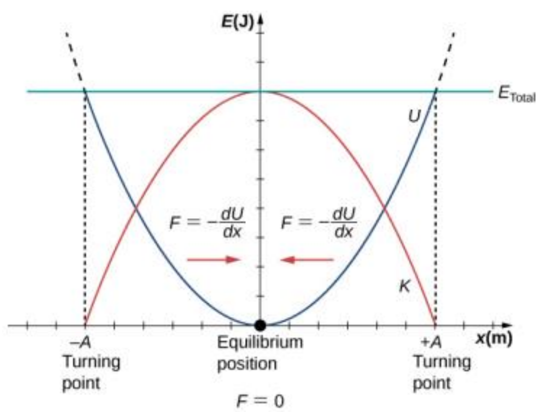


Figure 5: Kinetic (K) and potential energy (U) of an oscillator[50]

Now, if the amplitude of the oscillation is equal to a, then we know that the sum of the kinetic and potential energy of the oscillator will be equal to $(1/2)\cdot m\cdot a^2\cdot\omega^2$. Now, if we have two oscillators –

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[47] See: https://readingfeynman.org/2017/11/19/the-flywheel-model-of-an-electron/.

[48] In case you wonder, the remark is inspired by various 'crackpot theorist' remarks.

[49] The origin of our oscillator idea goes back to a discussion on the efficiency of the Ducati V-2 engines, which has the pistons in a 90-degree angle as opposed to, say, a Harley-Davidson. Of course, when thinking about engines, we should note that – because of the sideways motion of the connecting rods – the sinusoidal function will describe the linear motion only *approximately*. Springs connected to a crankshaft will give you the same issue. However, you can easily imagine the idealized limit situation.

[50] You will find this diagram in many texts, but we took this one from the https://phys.libretexts.org/ site—which is a great hub for open-access textbooks.

working in tandem at a 90-degree angle – then we can *add* their kinetic and potential energies. Why? Because of the 90-degree phase difference. Think of the V-2 metaphor—or of *two* springs working in tandem on the same crankshaft: it is a perpetuum mobile. Let us show you the associated math.

To facilitate the calculations, we will briefly assume that $k = m \cdot \omega^2$ and *a* are both equal to 1. Think of it as a normalization.[51] The motion of our first oscillator is given by the $\cos(\omega \cdot t) = \cos\theta$ function (so the phase varies with time only: $\theta = \omega \cdot t$). Its kinetic energy will be equal to $\sin^2\theta$. Hence, the (instantaneous) *change* in kinetic energy at some point in time – any point in time, really – will be equal to:

$$d(\sin^2\theta)/d\theta = 2\cdot\sin\theta\cdot d(\sin\theta)/d\theta = 2\cdot\sin\theta\cdot\cos\theta$$

Let us look at the second oscillator now. Just think of the second piston going up and down in the V-2 engine. Its motion is given by the $\sin\theta$ function, which is equal to $\cos(\theta-\pi/2)$. Hence, its kinetic energy is equal to $\sin^2(\theta-\pi/2)$, and how it *changes* – as a function of θ – will be equal to:

$$2\cdot\sin(\theta-\pi/2)\cdot\cos(\theta-\pi/2) = -2\cdot\cos\theta\cdot\sin\theta = -2\cdot\sin\theta\cdot\cos\theta$$

We have our *perpetuum mobile*! While transferring kinetic energy from one piston to the other, the crankshaft will rotate with a constant angular velocity: linear motion becomes circular motion, and vice versa, and the *total* energy that is stored in the system is $T + U = ma^2\omega^2$. We have a great *metaphor* here. Somehow, in this beautiful interplay between linear and circular motion, energy is borrowed from one place and then returns to the other, cycle after cycle.

We know the wavefunction consist of a sine and a cosine: the cosine is the real component, and the sine is the imaginary component. We believe they are equally real. And we believe each of the two oscillations carries *half* of the total energy of our particle.

The relativistic oscillator

You may wonder if the math holds for relativistic speeds. If the velocity of our mass on this spring – on the *two* springs, really – becomes a sizable fraction of the speed of light, then we can no longer treat the mass as a constant factor: it will vary with velocity, and its variation is given by the Lorentz factor (γ). While we will not work out each and every detail, we will show you the basics of why our reasoning above isn't faulty – even when relativistic speeds are involved.

The relativistically correct force equation for *one* oscillator is:

$$F = dp/dt = F = -kx \text{ with } p = m_v v = \gamma m_0 v$$

The $m_v = \gamma m_0$ varies with speed because γ varies with speed:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}} = \frac{dt}{d\tau}$$

What's the $dt/d\tau$ here? Don't worry about it. We actually don't need it for what follows, but we quickly wanted to insert it so as to remind you that we no longer have a unique concept of time: there is the

[51] There is no trick here. You can check for yourself by writing it all out. In fact, we advise that – as an exercise – you re-do the calculations for $a \neq 1$ and $k = m\cdot\omega^2 \neq 1$. It's easy enough: you can treat both as a constant factor in the derivations.

time in our reference frame (t) – aka as the *coordinate time* – and the time in the reference frame of the object itself (τ) – which is known as the *proper time*. We may want to use these two concepts of time in a later development, so it's good to introduce them here. But let's get on with that equation above. It actually *is* a differential equation (it involves a derivative), but you'll agree it's a very simple one. In fact, when you first learned about an equation like this, no one probably told you it's a proper differential equation. However, simple as it is, we're not going to solve it – because we don't have to. We'll just derive an energy conservation equation from it.

We do so by multiplying both sides with v = dx/dt. I am skipping a few steps (we're not going to do *all* of the work for you) but you should be able to verify the following:

$$v\frac{\mathrm{d}(\gamma \mathrm{m}_0 v)}{\mathrm{dt}} = -\mathrm{kx}v \Leftrightarrow \frac{\mathrm{d}(\mathrm{m}c^2)}{\mathrm{dt}} = -\frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{1}{2}\mathrm{k}x^2\right] \Leftrightarrow \frac{\mathrm{dE}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{1}{2}\mathrm{k}x^2 + \mathrm{m}c^2\right] = 0$$

So what's the energy concept here? We recognize the potential energy: it is the same $\mathrm{k}x^2/2$ formula we got for the non-relativistic oscillator. No surprises: potential energy depends on position only, not on *velocity*, and there is nothing relative about position. However, the $(½)\mathrm{m}_0v^2$ term that we would get when using the non-relativistic formulation of Newton's Law is now replaced by the $\mathrm{m}c^2 = \gamma \mathrm{m}_0c^2$ term.

You should note this $\mathrm{m}c^2 = \gamma \mathrm{m}_0c^2$ is *not* a constant: it varies with time – just like $\mathrm{k}x^2/2$ – because of the use of the relativistic mass concept.

So how can we calculate the energy? The total energy is constant at any point, so we may equate *x* to 0 and calculate the energy there. At that point, the potential energy will be zero and, crossing the *x* = 0 point, our pointlike charge will also reach the speed of light. The m in the formula will, therefore, be equal to $\mathrm{m}_\gamma = \mathrm{m}_c = \mathrm{m}_e/2$. We write:

$$\mathrm{E} = \frac{1}{2}\mathrm{k}\cdot 0^2 + \mathrm{m}_{x\,=\,0}c^2 = \mathrm{m}_\gamma \ = \frac{\mathrm{m_e}c^2}{2}$$

We can now add the energies in both oscillators so as to arrive at the *total* energy of the electron:

$$\mathrm{E} = \mathrm{m_e}c^2$$

It is a wonderful result. In fact, we think it amounts to a rather elegant and intuitive common-sense explanation of Einstein's mass-energy equivalence relation.

Should we regret this? Did we take the magic away? We don't think so. While we've made the magic somewhat more comprehensible, perhaps, it is still there: we used the metaphor of two oscillators working in tandem, *somehow*. As such, it may feel like squaring the circle. However, how this 'machinery' actually works – *exactly*, that is – is a deep mystery.

Annex 4: Equivalent models?

We spoke about the 'machinery' of the *Zitterbewegung* in Annex 3. Most theorists – such as Hestenes, for example – think of the machinery as being electromagnetic. The approach is quite sensible but, at the same time, it raises its own questions. In fact, one of the people who wrote me accused me of using Hestenes' model and, worse, of using it *wrongly*, because Hestenes' model yields a radius that is *twice* as large as mine. The exchange hereunder may be enlightening in that regard:

> "Hello Jean Louis – Thanks for your latest article. I'm not sure why you don't consider the circulating internal momentum in your electron model as the source of the electron's inertial mass. David Hestenes' Zitter electron model has radius $\hbar/2mc$. This is the same amplitude (see the excerpt below) that Barut and Bracken found in 1981 ("*Zitterbewegung* and the internal geometry of the electron", Phys Rev D, Vol 23, No. 10) when they went through Schrodinger's analysis of the Dirac equation. Their full paper is attached below the excerpt from their paper which shows this result. Note in the excerpt that the zitterbewegung's characteristic frequency is $\omega = 2mc^2/\hbar$. Geometrical models of the electron with radius $\hbar/mc$ and corresponding circulating frequency $\omega = mc^2/\hbar$ therefore don't correspond to Schrodinger's zitterbewegung result from the Dirac equation. All the best – [X]."

I responded as follows:

> "Dear [X] – We talked about the ½ factor a couple of times before. The essence of 'my' *zbw* idea is an oscillation in two dimensions – so I am not thinking in terms of a current ring and the magnetic field it creates, although the 'electromagnetic' *zbw* model must be equivalent somehow. The 'ring current' idea is *secondary* only to this basic geometric interpretation of the motion of the pointlike charge.
>
> The idea is that the circulating pointlike charge has an *effective* mass (because of its velocity *c*) that is *half* the electron mass (m/2). However, just like a photon, it does have this effective mass and, hence, some centripetal force must keep the pointlike charge in place – even if its rest mass is zero. That centripetal force can be broken down in two perpendicular oscillations, each of which is associated with energy (so we add the energies of two linear harmonic oscillators to arrive at the energy of a two-dimensional harmonic oscillator). [Note the ½ factor also vanishes when using the relativistically correct formula for the energy of a harmonic oscillator – see p. 66 of http://vixra.org/pdf/1901.0105vG.pdf).]
>
> The summary formula in Consa's article sums up the 'Big Issue' with the 'electromagnetic' *zbw* model of an electron, as he writes: $L = m \cdot a \cdot c = \hbar$. This makes the "1/2 mistake" two times, which is why the gets the 'right' result:
>
> 1. The electron mass (m_e) is the electron's rest mass. It is non-zero and, therefore, such mass cannot travel around at the speed of light. One should use the effective mass of the electron in calculations involving angular momentum or – in my calculations – the energy of the two perpendicular oscillations that make up the circular current.
> 2. The angular momentum of an electron is not $\hbar$ but $\hbar/2$: that's why electrons are considered to be spin-1/2 particles.

One can ‘avoid’ one of the two mistakes above by accepting a *zbw* radius that is twice the Compton radius, but that just creates another problem then.

So my answer to your question is simple: my model is *not* a copy of Hestenes’ or any other model. It’s a totally different ‘philosophy’, so to speak, but it gives the right answers – to me, at least. I also feel it has an advantage over the ‘electromagnetic’ model because it gives us an explanation to the quintessential question: there is nothing ‘mechanical’ (no wire or so) to ‘constrain’ the *zbw* charge in its circular motion. Hence, the ‘electromagnetic’ model offers a delicate ‘equilibrium’ trajectory that can easily be disturbed. As such, I don’t like the comparison with superconducting currents or so – because superconducting currents do need a ‘mechanical’ constraint ensuring the charge doesn’t go ‘off-track’.

Having said that, I’ve started to realize it must, effectively, look like a ‘crackpot’ theory to most, especially because it also offers an easy geometric interpretation of the previously mysterious mass-energy equivalence relation which – I realize – sounds terribly arrogant.

Kindest regard – JL”

It is an interesting discussion. I hope someone will be able to show the two models (electromagnetic and two-dimensional) are equivalent. However, in order to do so, Hestenes’ followers will need to explain the missing ½ factor based on a thorough review of their calculations. To be precise, they will need to present more convincing calculations of the electric and magnetic energies in their models, and show how these add up to the total energy of the electron.[52]

Of course, one may think the question can be settled by a careful consideration of the *experimentally measured* charge radius of an electron, as is done in photon scattering experiments, so that is something to look into. In addition, the various models use different energy concepts (the ring electron model is associated with electric and magnetic energies, while the oscillator model uses kinetic versus potential energy – in a very ‘mechanical’ sense, that is). These energies must, of course, add up to the total electron energy.

I should look in more detail, but in *zbw* models I see good calculations of force based on spin and angular momentum – and calculations of the strength of the magnetic field (e.g. in Consa’s article[53]) – but I have not seen any *integration* of (field) energies over space: does the energy in the electromagnetic fields of the ring electron add up to 0.511 MeV/c^2?

They should, of course, and so all of the models of the electron that are currently around (Burinskii, Hestenes and mine) should then be equivalent, *somehow* – just showing different *aspects* of (how we think about) the *reality* of an electron.

[52] To be honest, they will also need to explain why they don’t seem to care about integrating the Planck-Einstein relation, or Einstein’s mass-energy equivalence relation, into their rather elaborate computations.
[53] See: Oliver Consa, *The Helical Solenoid Model of the Electron* (http://www.ptep-online.com/2018/PP-53-06.PDF).