The horizon of two interacting bodies

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Abstract

Two concomitant Whitehead's models are used to describe the gravitational interaction of two point-like particles. If the two masses are different, and they are at rest, Newton's laws are slightly modified and for some particular values of the two masses and the distance separating them the force is a repulsion. This suggests a nice interpretation of the so-called horizon of the single particle model as the surface where gravitation becomes a repulsive force.

1 Two particles Whitehead-like model of Schwarzschild solution

I consider two point-like particles.One has mass m, the other has mass $\hat{m}$. The first is located at some point of Minkowsky's space x_i, the second is at at the same time located at some point $\hat{x}_i$.

The Whitehead-like model that I am using describes the gravitational field having as source these two point-like particles by two metric potentials. The first ones whose source is $hatm$ are:

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2\hat{m}}{(l_\rho u^\rho)^3} l_\mu l_\nu \tag{1}$$

with u^ρ being the future pointing arrow of time (from now on: $u^4 = 1,\ u^i = 0,\ i = 1, 2, 3$), and where $\eta_{\mu\nu}$ is Minkowsky's metric with $\eta_{44} = -1$ and where:

$$l_i = x_i - \hat{x}_i, \quad i, j = 1, 2, 3, \quad r \equiv l_4 = \sqrt{l_1^2 + l_2^2 + l_3^2}, \tag{2}$$

$\hat{x}^i$ the space location of the point source, and $\hat{m}$ is its passive mass. Therefore:

$$\eta_{\mu\nu} l^\mu l^\nu = 0 \tag{3}$$

where x^i is the space point where the field is evaluated. And,with $G = c = 1$,:

$$\begin{aligned}
g_{44} &= -1 + \tfrac{2\hat{m}}{r} & g_{31} &= \frac{2\hat{m} l_3 l_1}{r^3} && (4)\\
g_{11} &= 1 + \tfrac{2\hat{m} l_1^2}{r^3} & g_{14} &= \frac{2\hat{m} l_1}{r^2} && (5)\\
g_{22} &= 1 + \tfrac{2\hat{m} l_2^2}{r^3} & g_{24} &= \frac{2\hat{m} l_2}{r^2} && (6)\\
g_{33} &= 1 + \tfrac{2\hat{m} l_3^2}{r^3} & g_{34} &= \frac{2\hat{m} l_3}{r^2} && (7)\\
g_{12} &= \tfrac{2\hat{m} l_1 l_2}{r^3} & g_{23} &= \frac{2\hat{m} l_2 l_3}{r^3} && (8)\\
& & & && (9)
\end{aligned}$$

The second concomitant Whitehead-like model that I use, $\hat{g}_{\mu\nu}$, is:

$$\hat{g}_{\mu\nu} = \eta_{\mu\nu} + \frac{2m}{(l_\sigma u^\sigma)^3} \hat{l}_\mu \hat{l}_\nu \tag{10}$$

that follows from the preceding $g_{\mu\nu}$ by the following substitutions:

$$m, x^i \rightleftharpoons \hat{m},\ \hat{x}^i \tag{11}$$

2 Equations of motion

Let us start from the simplest case, the one particle problem, where a particle with neglectful mass is orbiting an object of mass $\hat{m}$ that is located at a point with coordinates $\hat{x}^i = 0$. Using t as parameter. The geodesic principle tells us that the orbit of the test particle will be a solution of the system of differential equations:

$$\frac{d^2 x^i}{dt^2} + \Gamma^i_{44} + 2\Gamma^i_{4j} v^j + \Gamma^i_{jk} v^j v^k = b v^i \tag{12}$$

$$b = \Gamma^4_{44} + 2\Gamma^4_{4j} v^j + \Gamma^4_{jk} v^j v^k \tag{13}$$

where $v^i = dx^i/dt$ and where the Christoffel symbols corresponding to Whitehead's model are:

$$\Gamma^i_{ii} = \frac{2\hat{m}^2 l_i^3 - 2\hat{m} r^3 l_i + 3\hat{m} r l_i^3}{r^6} \tag{14}$$

$$\Gamma^i_{jj} = \frac{-2\hat{m}^2 l_i l_j^2 + 2\hat{m} r^3 l_i - 3\hat{m} r l_i l_j^2}{r^6} \tag{15}$$

$$\Gamma^i_{jk} = \frac{(2\hat{m}+3r)\hat{m} l_i l_j l_k}{r^6} \tag{16}$$

$$\Gamma^i_{ij} = \frac{(2\hat{m}+3r)\hat{m} l_i^2 l_j}{r^6}, \qquad \Gamma^i_{4i} = -\frac{2l_i^2 \hat{m}^2}{r^5} \tag{17}$$

$$\Gamma^i_{4j} = -\frac{2 l_i l_j \hat{m}^2}{r^5}, \qquad \Gamma^i_{44} = \frac{(-2\hat{m}+r)\hat{m} l_i}{r^4} \tag{18}$$

$$\Gamma^4_{ii} = \frac{2\hat{m}(\hat{m} l_i^2 - r^3 + 2 r l_i^2)}{r^5}, \qquad \Gamma^4_{ij} = \frac{2(\hat{m}+2r)\hat{m} l_i l_j}{r^5} \tag{19}$$

$$\Gamma^4_{i4} = \frac{(2\hat{m}+r)\hat{m} l_i}{r^4}, \qquad \Gamma^4_{4,4} = \frac{2\hat{m}^2}{r^3} \tag{20}$$

In these formulas i, j, k run from 1 to 3 but $i \neq j$, $i \neq k$ and $j \neq k$

3 Two bodies geodesic equations

In this paper I consider an important generalization by considering two point objects with non negligible masses m and $\hat{m}$ describing orbits $x_i(t)$ and $\hat{x}_i(t)$ as functions of a common time parameter t. This idea can be very simply implemented rewriting the Whitehead model as a two body coupled bi-metric model using both:

The resulting coupled system of differential equations to be considered is then the following:

$$\frac{d^2 x^i}{dt^2} + \Gamma^i_{44} + 2\Gamma^i_{4j} v^j + \Gamma^i_{jk} v^j v^k = b v^i \tag{21}$$

$$\frac{d^2 \hat{x}^i}{dt^2} + \hat{\Gamma}^i_{44} + 2\hat{\Gamma}^i_{4j} \hat{v}^j + \hat{\Gamma}^i_{jk} \hat{v}^j \hat{v}^k = \hat{b} \hat{v}^i \tag{22}$$

$$\tag{23}$$

with:

$$b = \Gamma^4_{44} + 2\Gamma^4_{4j} v^j + \Gamma^4_{jk} v^j v^k \tag{24}$$

$$\hat{b} = \hat{\Gamma}^4_{44} + 2\hat{\Gamma}^4_{4j} \hat{v}^j + \hat{\Gamma}^4_{jk} \hat{v}^j \hat{v}^k \tag{25}$$

4 Reconsideration of Newton's laws

Let us consider two point-like bodies with masses m and $\hat{m}$ at rest at a distance r apart. At this moment the forces acting on them derived from the geodesic equations above with $v^i = 0,\ \hat{w}^i = 0$ will be

$$f^i = \left(\frac{d^2\hat{x}^i}{dt^2}\right)_0 = -\frac{m\hat{m}(x^i - \hat{x}^i)}{r^3}\left(1 - \frac{2\hat{m}}{r}\right) \tag{26}$$

$$\hat{f}^i = \left(\frac{d^2 x^i}{dt^2}\right)_0 = -\frac{\hat{m}m(\hat{x}^i - x^i)}{r^3}\left(1 - \frac{2m}{r}\right) \tag{27}$$

therefore:

$$f^i + \hat{f}^i = 2\frac{m\hat{m}}{r^4}(x^i - \hat{x}^i)(\hat{m} - m) \tag{28}$$

therefore if m is not equal to $\hat{m}$ the third law of Newton is not satisfied. But still more spectacular is that the force becomes repulsive for those values of r, m and $\hat{m}$ for which:

$$g_{44} = -1 + \frac{2m}{r} < 0 \quad \text{or} \quad \hat{g}_{44} = -1 + \frac{2\hat{m}}{r} < 0 \tag{29}$$

These conclusion partially subsist if one of the bodies is not a point like object but, for example, a spherical object of radius R and constant density ρ, in which case the force acting on the point-like object of mass m located at a distance r from the center of the sphere would be:

$$\hat{f}^i = \frac{d^2 x^i}{dt^2} = -\frac{\hat{m}mx^i}{r^3}\left(1 - \frac{8\pi}{3r}\rho R^3\right) \tag{30}$$

In particular f^i would be zero for $r = R$ if:

$$\rho = \frac{3}{8\pi R^2} \tag{31}$$

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