

# **On various Ramanujan formulas applied to some sectors of String Theory (open strings) and Particle Physics: Further new possible mathematical connections IV.**

**Michele Nardelli<sup>1</sup>, Antonio Nardelli**

## **Abstract**

*In this research thesis, we have analyzed and deepened various Ramanujan expressions applied to some sectors of String Theory (open strings) and Particle Physics. We have therefore described further new possible mathematical connections.*

---

<sup>1</sup> M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni “R. Caccioppoli” - Università degli Studi di Napoli “Federico II” – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy



<https://www.sciencephoto.com/media/228058/view/indian-mathematician-srinivasa-ramanujan>

From:

**Open Strings On The Rindler Horizon**

*Edward Witten* - arXiv:1810.11912v4 [hep-th] 26 Nov 201

We have that:

$$Z_{k,N}^F = (2 \sin(\pi k/N))^4 q^{1/3} \prod_{n=1}^{\infty} (1 - q^n \exp(2\pi i k/N))^4 (1 - q^n \exp(-2\pi i k/n))^4. \tag{3.9}$$

We have the following mock theta function:

([https://en.wikipedia.org/wiki/Mock\\_modular\\_form#Order\\_6](https://en.wikipedia.org/wiki/Mock_modular_form#Order_6))

$$\sigma(q) = \sum_{n \geq 0} \frac{q^{(n+1)(n+2)/2} (-q; q)_n}{(q; q^2)_{n+1}}$$

That is:

(A053271 sequence OEIS)

$$\text{Sum}_{\{n \geq 0\}} q^{((n+1)(n+2)/2)} (1+q)(1+q^2)\dots(1+q^n)/((1-q)(1-q^3)\dots(1-q^{2n+1}))$$

We have that:

$$\text{sum } q^{((n+1)(n+2)/2)} (1+q)(1+q^2)(1+q^n)/((1-q)(1-q^3)(1-q^{2n+1})), n = 0 \text{ to } k$$

$$\sum_{n=0}^k \frac{q^{1/2 (n+1)(n+2)} (1+q)(1+q^2)(1+q^n)}{(1-q)(1-q^3)(1-q^{2n+1})}$$

$$\sum_{n=0}^k \frac{q^{1/2(n+1)(n+2)} (1+q)(1+q^2)(1+q^n)}{(1-q)(1-q^3)(1-q^{2n+1})}$$

For  $q = 0.5$  and  $n = 2$ , we develop the above formula in the following way:

$$\frac{(((0.5^{(2+1)(2+2)/2} (1+0.5)(1+0.5^2)(1+0.5^2))))}{(((1-0.5)(1-0.5^3)(1-0.5^{2*2+1}))}$$

$$\frac{0.5^{(2+1) \times (2+2)/2} (1+0.5)(1+0.5^2)(1+0.5^2)}{(1-0.5)(1-0.5^3)(1-0.5^{2 \times 2+1})}$$

0.086405529953917050691244239631336405529953917050691244239...  
0.0864055...

From (3.9), for  $k = 2$ ,  $N = 5$ ,  $q = e^{2\pi} = 535.49165\dots$  and  $n$  from 1 to 0.0864055, we obtain:

$$(2\sin((4\pi)/5))^4 * 535.49165^{(1/3)} * \text{product } (1-535.49165^n \exp((8\pi*i)/5))^4 (1-535.49165^n \exp((-8\pi*i)/5))^4, n=1 \text{ to } 0.0864055$$

**Input interpretation:**

$$\left(2 \sin\left(\frac{4\pi}{5}\right)\right)^4 \sqrt[3]{535.49165} \prod_{n=1}^{0.0864055} \left(1 - 535.49165^n \exp\left(\frac{1}{5} (8\pi i)\right)\right)^4 \left(1 - 535.49165^n \exp\left(\frac{1}{5} (-8\pi i)\right)\right)^4$$

$i$  is the imaginary unit

**Result:**

15.5088  
15.5088

$$8 * ((((((2\sin((4\pi)/5))^4 * 535.49165^{(1/3)} * \text{product } (1-535.49165^n \exp((8\pi*i)/5))^4 (1-535.49165^n \exp((-8\pi*i)/5))^4, n=1 \text{ to } 0.0864055)))))) + \text{golden ratio}$$

where 8 is a Fibonacci number

**Input interpretation:**

$$8 \left( \left( 2 \sin\left(\frac{4\pi}{5}\right) \right)^4 \sqrt[3]{535.49165} \prod_{n=1}^{0.0864055} \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (8\pi i)\right) \right)^4 \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (-8\pi i)\right) \right)^4 \right) + \phi$$

*i* is the imaginary unit

$\phi$  is the golden ratio

**Result:**

125.689

125.689 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

8\*((((((2sin((4Pi)/5))^4\*535.49165^(1/3) \* product (1-535.49165^n exp((8Pi\*i)/5))^4 (1-535.49165^n exp((-8Pi\*i)/5))^4, n=1 to 0.0864055)))))))+13+golden ratio^2

where 13 is a Fibonacci number

**Input interpretation:**

$$8 \left( \left( 2 \sin\left(\frac{4\pi}{5}\right) \right)^4 \sqrt[3]{535.49165} \prod_{n=1}^{0.0864055} \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (8\pi i)\right) \right)^4 \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (-8\pi i)\right) \right)^4 \right) + 13 + \phi^2$$

*i* is the imaginary unit

$\phi$  is the golden ratio

**Result:**

139.689

139.689 result practically equal to the rest mass of Pion meson 139.57 MeV

64\*((((((2sin((4Pi)/5))^4\*535.49165^(1/3) \* product (1-535.49165^n exp((8Pi\*i)/5))^4 (1-535.49165^n exp((-8Pi\*i)/5))^4, n=1 to 0.0864055)))))))-55+1/golden ratio

where 55 is a Fibonacci number

**Input interpretation:**

$$64 \left( \left( 2 \sin\left(\frac{4\pi}{5}\right) \right)^4 \sqrt[3]{535.49165} \prod_{n=1}^{0.0864055} \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (8\pi i)\right) \right)^4 \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (-8\pi i)\right) \right)^4 \right) - 55 + \frac{1}{\phi}$$

*i* is the imaginary unit

$\phi$  is the golden ratio

**Result:**

938.183

938.183 result practically equal to the proton mass in MeV

76\*((((((2sin((4Pi)/5))^4\*535.49165^(1/3) \* product (1-535.49165^n exp((8Pi\*i)/5))^4 (1-535.49165^n exp((-8Pi\*i)/5))^4, n=1 to 0.0864055)))))))+11

where 76 and 11 are Lucas numbers

**Input interpretation:**

$$76 \left( \left( 2 \sin\left(\frac{4\pi}{5}\right) \right)^4 \sqrt[3]{535.49165} \prod_{n=1}^{0.0864055} \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (8\pi i)\right) \right)^4 \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (-8\pi i)\right) \right)^4 \right) + 11$$

*i* is the imaginary unit

**Result:**

1189.67

1189.67 result practically equal to the rest mass of Sigma baryon 1189.37

$$89 * \left( \left( \left( \left( \left( 2 \sin\left(\frac{4\pi}{5}\right) \right)^4 * 535.49165^{(1/3)} * \prod_{n=1}^{0.0864055} (1 - 535.49165^n \exp((8\pi*i)/5))^4 (1 - 535.49165^n \exp((-8\pi*i)/5))^4, n=1 \text{ to } 0.0864055) \right) \right) \right) \right) + \pi$$

where 89 is a Fibonacci number

**Input interpretation:**

$$89 \left( \left( 2 \sin\left(\frac{4\pi}{5}\right) \right)^4 \sqrt[3]{535.49165} \prod_{n=1}^{0.0864055} \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (8\pi i)\right) \right)^4 \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (-8\pi i)\right) \right)^4 \right) + \pi$$

*i* is the imaginary unit

**Result:**

1383.43

1383.43 result practically qual to the rest mass of Sigma baryon 1383.7

We have also:

$$76 * \left( \left( \left( \left( \left( 2 \sin\left(\frac{4\pi}{5}\right) \right)^4 * 535.49165^{(1/3)} * \prod_{n=1}^{0.0864055} (1 - 535.49165^n \exp((8\pi*i)/5))^4 (1 - 535.49165^n \exp((-8\pi*i)/5))^4, n=1 \text{ to } 0.0864055) \right) \right) \right) \right) - 11 - \pi$$

Where 76 and 11 are Lucas numbers

**Input interpretation:**

$$76 \left( \left( 2 \sin\left(\frac{4\pi}{5}\right) \right)^4 \sqrt[3]{535.49165} \prod_{n=1}^{0.0864055} \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (8\pi i)\right) \right)^4 \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (-8\pi i)\right) \right)^4 \right) - 11 - \pi$$

*i* is the imaginary unit

**Result:**

1164.53

1164.53 result very near to the following Ramanujan’s class invariant  $Q =$

$$(G_{505}/G_{101/5})^3 = 1164,2696$$

$[1/((((((2\sin((4\pi)/5))^4*535.49165^{(1/3)} * \text{product}(1-535.49165^n \exp((8\pi*i)/5))^4 (1-535.49165^n \exp((-8\pi*i)/5))^4, n=1 \text{ to } 0.0864055))))))]^{1/4096}$

**Input interpretation:**

$$\left(1 / \left( \left( 2 \sin\left(\frac{4\pi}{5}\right) \right)^4 \sqrt[3]{535.49165} \prod_{n=1}^{0.0864055} \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (8\pi i)\right) \right)^4 \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (-8\pi i)\right) \right)^4 \right) \right)^{(1/4096)}$$

*i* is the imaginary unit

**Result:**

0.999331

0.999331 result very near to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

Now, we have that:

Consider the function

$$K(z, N) = \sum_{k=1}^{N-1} \frac{\pi \sin \pi z}{\sin(\pi k/N) \sin \pi(z - k/N)}. \tag{2.5}$$

for  $N = 5, z = 1/2 - 0.0000864055i = 0.5 - 0.0000864055i, \pi = 180$ , we obtain:

sum (180\*sin (180\*(0.5-0.0000864055i)))/(((sin((180\*k)/5) sin180((0.5-0.0000864055i)-k/5))), k=1 to 4

**Sum:**

$$\sum_{k=1}^4 \frac{180 \sin(180 (0.5 - 0.0000864055 i))}{\sin\left(\frac{180k}{5}\right) \sin(180) \left( (0.5 - 0.0000864055 i) - \frac{k}{5} \right)} \approx -6435.352563006193048098502636338628964007 - 58.285299328127436938227745518659750643 i$$

*i* is the imaginary unit

**Decimal approximation:**

-6435.3525630061930480985026363386289640073804084536955773... - 58.285299328127436938227745518659750642622240480585974388... *i*

**Input interpretation:**

-6435.352563006193 + *i* × (-58.28529932812743)

*i* is the imaginary unit

**Result:**

-6435.352563006193... - 58.28529932812743... *i*

**Polar coordinates:**

*r* = 6435.616503980652 (radius), *θ* = -179.481083543176318° (angle)  
6435.616503980652

(((sum (180\*sin (180\*(0.5-0.0000864055i)))/(((sin((180\*k)/5) sin180((0.5-0.0000864055i)-k/5))), k=1 to 4))))+123+29+7

where 123, 29 and 7 are Lucas numbers

**Input interpretation:**

$$\sum_{k=1}^4 \frac{180 \sin(180 (0.5 + i \times (-0.0000864055)))}{\sin\left(\frac{180k}{5}\right) \sin(180) \left( (0.5 + i \times (-0.0000864055)) - \frac{k}{5} \right)} + 123 + 29 + 7$$

*i* is the imaginary unit

**Result:**

-6276.35 - 58.2853 *i*

**Input interpretation:**

$$-6276.35 + i \times (-58.2853)$$

*i* is the imaginary unit

**Result:**

$$-6276.35... - 58.2853... i$$

**Polar coordinates:**

$$r = 6276.62 \text{ (radius), } \theta = -179.468^\circ \text{ (angle)}$$

6276.62 result practically equal to the rest mass of charmed B meson 6276

$$(-6276.35 - 58.2853 i) + \text{golden ratio}$$

**Input interpretation:**

$$(-6276.35 + i \times (-58.2853)) + \phi$$

*i* is the imaginary unit

$\phi$  is the golden ratio

**Result:**

$$-6274.73... - 58.2853... i$$

**Polar coordinates:**

$$r = 6275. \text{ (radius), } \theta = -179.468^\circ \text{ (angle)}$$

6275 as above

for  $N = 5, k = 3, z = 1/2 - 0.0000864055i = 0.5 - 0.0000864055i, \pi = 180$ , we obtain also:

$$(180 * \sin(180 * (0.5 - 0.0000864055i))) / (((\sin((180 * 3) / 5) \sin 180 * ((0.5 - 0.0000864055i) - 3/5))))$$

**Input interpretation:**

$$\frac{180 \sin(180 (0.5 + i \times (-0.0000864055)))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left((0.5 + i \times (-0.0000864055)) - \frac{3}{5}\right)}$$

*i* is the imaginary unit

**Result:**

$$2167.47\dots + \\ 15.0216\dots i$$

**Polar coordinates:**

$$r = 2167.52 \text{ (radius)}, \quad \theta = 0.39708^\circ \text{ (angle)}$$

$$2167.52$$

**Alternative representations:**

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} = \\ \frac{180}{\csc(180 (0.5 - 0.0000864055 i)) \left( 0.5 - 0.0000864055 i - \frac{3}{5} \right)} = \\ \frac{180}{\csc(180) \csc\left(\frac{540}{5}\right)}$$

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} = \\ \frac{180 \cos\left(-180 (0.5 - 0.0000864055 i) + \frac{\pi}{2}\right)}{\cos\left(-180 + \frac{\pi}{2}\right) \cos\left(\frac{\pi}{2} - \frac{540}{5}\right) \left( 0.5 - 0.0000864055 i - \frac{3}{5} \right)}$$

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} = \\ \frac{180 \cos\left(180 (0.5 - 0.0000864055 i) + \frac{\pi}{2}\right)}{\cos\left(180 + \frac{\pi}{2}\right) \cos\left(\frac{\pi}{2} + \frac{540}{5}\right) \left( 0.5 - 0.0000864055 i - \frac{3}{5} \right)}$$

**Series representations:**

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} = \\ \frac{4.1664 \times 10^6 \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(180) T_{1+2k}(0.5 - 0.0000864055 i)}{(1157.33 + i) \sin(108) \sin(180)}$$

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} = \\ \frac{1.0416 \times 10^6 \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(90 - 0.015553 i)}{(1157.33 + i) \left( \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(108) \right) \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(180)}$$

### Integral representations:

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} = \frac{1.66667 \left( -5786.67 \int_0^1 \cos((90 - 0.015553 i) t) dt + i \int_0^1 \cos((90 - 0.015553 i) t) dt \right)}{(1157.33 + i) \left( \int_0^1 \cos(108 t) dt \right) \int_0^1 \cos(180 t) dt}$$

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} = \frac{\left( 6.66667 \left( -5786.67 \pi \mathcal{A} \int_{-\mathcal{A}_{\infty+\gamma}}^{\mathcal{A}_{\infty+\gamma}} \frac{e^{-(0.0000604739(5786.67-i)^2)/s+s}}{s^{3/2}} ds + i \pi \mathcal{A} \int_{-\mathcal{A}_{\infty+\gamma}}^{\mathcal{A}_{\infty+\gamma}} \frac{e^{-(0.0000604739(5786.67-i)^2)/s+s}}{s^{3/2}} ds \right) \right)}{\left( (1157.33 + i) \left( \int_{-\mathcal{A}_{\infty+\gamma}}^{\mathcal{A}_{\infty+\gamma}} \frac{e^{-8100/s+s}}{s^{3/2}} ds \right) \left( \int_{-\mathcal{A}_{\infty+\gamma}}^{\mathcal{A}_{\infty+\gamma}} \frac{e^{-2916/s+s}}{s^{3/2}} ds \right) \sqrt{\pi} \right)} \text{ for } \gamma > 0$$

### Multiple-argument formulas:

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} = \frac{1.59625 \times 10^{60} \prod_{k=0}^{179} \sin(0.5 - 0.0000864055 i + 0.00555556 k \pi)}{(1157.33 + i) \sin(108) \sin(180)}$$

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} = \frac{2.0832 \times 10^6 U_{179}(\sin(0.5 - 0.0000864055 i)) \cos(0.5 - 0.0000864055 i)}{(1157.33 + i) \sin(108) \sin(180)}$$

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} = \frac{2.0832 \times 10^6 U_{179}(\cos(0.5 - 0.0000864055 i)) \sin(0.5 - 0.0000864055 i)}{(1157.33 + i) \sin(108) \sin(180)}$$

And we obtain also:

$$(180 \cdot \sin(180 \cdot (0.5 - 0.0000864055i))) / (((\sin((180 \cdot 3)/5) \sin 180 \cdot ((0.5 - 0.0000864055i) - 3/5)))) - 55$$

where 55 is a Fibonacci number

**Input interpretation:**

$$\frac{180 \sin(180 (0.5 + i \times (-0.0000864055)))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 + i \times (-0.0000864055)) - \frac{3}{5} \right)} - 55$$

*i* is the imaginary unit

**Result:**

$$2112.47... + 15.0216... i$$

**Polar coordinates:**

$$r = 2112.53 \text{ (radius), } \theta = 0.407418^\circ \text{ (angle)}$$

2112.53 result practically equal to the rest mass of strange D meson 2112.3

**Alternative representations:**

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} - 55 = -55 + \frac{180}{\frac{\csc(180 (0.5 - 0.0000864055 i)) \left( (0.5 - 0.0000864055 i) - \frac{3}{5} \right)}{\csc(180) \csc\left(\frac{540}{5}\right)}}$$

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} - 55 = -55 + \frac{180 \cos\left(-180 (0.5 - 0.0000864055 i) + \frac{\pi}{2}\right)}{\cos\left(-180 + \frac{\pi}{2}\right) \cos\left(\frac{\pi}{2} - \frac{540}{5}\right) \left( (0.5 - 0.0000864055 i) - \frac{3}{5} \right)}$$

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} - 55 = -55 - \frac{180 \cos\left(180 (0.5 - 0.0000864055 i) + \frac{\pi}{2}\right)}{\cos\left(180 + \frac{\pi}{2}\right) \cos\left(\frac{\pi}{2} + \frac{540}{5}\right) \left( (0.5 - 0.0000864055 i) - \frac{3}{5} \right)}$$

### Series representations:

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left(0.5 - i 0.0000864055 - \frac{3}{5}\right)} - 55 =$$

$$-55 - \frac{4.1664 \times 10^6 \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(180) T_{1+2k}(0.5 - 0.0000864055 i)}{(1157.33 + i) \sin(108) \sin(180)}$$

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left(0.5 - i 0.0000864055 - \frac{3}{5}\right)} - 55 =$$

$$- \left( \left( 55 \left( 18938.2 \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(90 - 0.015553 i) + \right. \right. \right.$$

$$1157.33 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} (-1)^{k_1+k_2} J_{1+2k_1}(108) J_{1+2k_2}(180) +$$

$$\left. \left. i \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} (-1)^{k_1+k_2} J_{1+2k_1}(108) J_{1+2k_2}(180) \right) \right) /$$

$$\left( (1157.33 + i) \left( \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(108) \right) \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(180) \right)$$

### Integral representations:

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left(0.5 - i 0.0000864055 - \frac{3}{5}\right)} - 55 =$$

$$- \left( \left( 55 \left( 175.354 \int_0^1 \cos((90 - 0.015553 i) t) dt - \right. \right. \right.$$

$$0.030303 i \int_0^1 \cos((90 - 0.015553 i) t) dt +$$

$$\left. \left. 2 \int_0^1 \int_0^1 \cos(108 t_1) \cos(180 t_2) dt_2 dt_1 \right) \right) /$$

$$\left( (1157.33 + i) \left( \int_0^1 \cos(108 t) dt \right) \int_0^1 \cos(180 t) dt \right)$$

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left(0.5 - i 0.0000864055 - \frac{3}{5}\right)} - 55 =$$

$$\left( 6.66667 \left( -5786.67 \pi \mathcal{A} \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{-(0.0000604739(5786.67-i)^2)/s+s}}{s^{3/2}} ds + \right. \right.$$

$$i \pi \mathcal{A} \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{-(0.0000604739(5786.67-i)^2)/s+s}}{s^{3/2}} ds -$$

$$9548. \left( \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{-8100/s+s}}{s^{3/2}} ds \right) \left( \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{-2916/s+s}}{s^{3/2}} ds \right) \sqrt{\pi} -$$

$$8.25 i \left( \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{-8100/s+s}}{s^{3/2}} ds \right) \left( \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{-2916/s+s}}{s^{3/2}} ds \right) \sqrt{\pi} \right) /$$

$$\left( (1157.33 + i) \left( \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{-8100/s+s}}{s^{3/2}} ds \right) \left( \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{-2916/s+s}}{s^{3/2}} ds \right) \sqrt{\pi} \right) \text{ for } \gamma > 0$$

**Multiple-argument formulas:**

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} - 55 =$$

$$-55 + \frac{2.0832 \times 10^6 U_{179}(\sin(0.5 - 0.0000864055 i)) \cos(0.5 - 0.0000864055 i)}{(1157.33 + i) \sin(108) \sin(180)}$$

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} - 55 =$$

$$-55 - \frac{2.0832 \times 10^6 U_{179}(\cos(0.5 - 0.0000864055 i)) \sin(0.5 - 0.0000864055 i)}{(1157.33 + i) \sin(108) \sin(180)}$$

$$\frac{180 \sin(180 (0.5 - i 0.0000864055))}{\sin\left(\frac{180 \times 3}{5}\right) \sin(180) \left( (0.5 - i 0.0000864055) - \frac{3}{5} \right)} - 55 =$$

$$-55 + \left( -4.1664 \times 10^6 \cos(0.5 - 0.0000864055 i) \sin(89.5 - 0.0154666 i) + \right.$$

$$\left. 2.0832 \times 10^6 \sin(89 - 0.0153802 i) \right) / ((1157.33 + i) \sin(108) \sin(180))$$

$U_n(x)$  is the Chebyshev polynomial of the second kind

Now, we have that:

$$K_2(z, N) = \pi N \cot(\pi(N(z - 1/2) + 1/2)) - \pi \cot \pi z. \tag{2.10}$$

For  $z = 0.5 - 0.0000864055i$ ,  $N = 5$ ,  $\pi = 180$ , we obtain:

$$5 * 180 \cot(180(5(0.5 - 0.0000864055i - 0.5) + 0.5)) - ((180 \cot 180 * (0.5 - 0.0000864055i)))$$

**Input interpretation:**

$$5 \times 180 \cot(180 (5 (0.5 + i \times (-0.0000864055) - 0.5) + 0.5)) -$$

$$180 \cot(180) (0.5 + i \times (-0.0000864055))$$

$\cot(x)$  is the cotangent function

$i$  is the imaginary unit

**Result:**

$$-514.918... +$$

$$87.2733... i$$

**Polar coordinates:**

$$r = 522.262 \text{ (radius), } \theta = 170.38^\circ \text{ (angle)}$$

## 522.262 result very near to the Lucas number 521

### Alternative representations:

$$5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - \\ 180 (\cot(180) (0.5 - i 0.0000864055)) = 180 (0.5 - 0.0000864055 i) i \coth(-180 i) - \\ 900 i \coth(-180 i (0.5 + 5 (0 - 0.0000864055 i)))$$

$$5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - \\ 180 (\cot(180) (0.5 - i 0.0000864055)) = -180 (0.5 - 0.0000864055 i) i \coth(180 i) + \\ 900 i \coth(180 (0.5 + 5 (0 - 0.0000864055 i)))$$

$$5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - \\ 180 (\cot(180) (0.5 - i 0.0000864055)) = \\ -\frac{180 (0.5 - 0.0000864055 i)}{\tan(180)} + \frac{900}{\tan(180 (0.5 + 5 (0 - 0.0000864055 i)))}$$

### Series representations:

$$5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - \\ 180 (\cot(180) (0.5 - i 0.0000864055)) = \\ \sum_{k=-\infty}^{\infty} e^{-0.15553(-1157.33+i)k \mathcal{A}} \left( -900 + e^{(180.+0.15553 i)k \mathcal{A}} (90 - 0.015553 i) \right) \mathcal{A} \operatorname{sgn}(k)$$

$$5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - \\ 180 (\cot(180) (0.5 - i 0.0000864055)) = \\ \sum_{k=1}^{\infty} e^{-0.15553(-1157.33+i)k \mathcal{A}} \left( -900 + e^{(180.+0.15553 i)k \mathcal{A}} (90 - 0.015553 i) \right) \mathcal{A} + \\ \sum_{k=-\infty}^{-1} e^{-0.15553(-1157.33+i)k \mathcal{A}} \left( 900 + e^{(180.+0.15553 i)k \mathcal{A}} (-90 + 0.015553 i) \right) \mathcal{A}$$

$$5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - \\ 180 (\cot(180) (0.5 - i 0.0000864055)) = \\ \sum_{k=-\infty}^{\infty} \left( 2.49318 \times 10^9 - 137.155 i^2 + 0.0169299 i^3 - \right. \\ \left. 64800 k^2 \pi^2 + i (-2.01819 \times 10^6 + 67.1889 k^2 \pi^2) \right) / \\ ((-32400 + k^2 \pi^2) (-8100 + 13.9977 i - 0.00604739 i^2 + k^2 \pi^2))$$

**Integral representation:**

$$5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - 180 (\cot(180) (0.5 - i 0.0000864055)) = \int_{\frac{\pi}{2}}^{90-0.077765 i} \left( (162000 - 139.977 i - 900 \pi) \csc^2(t) + \frac{(-32400. + i(5.59908 - 0.015553 \pi) + 90 \pi) \csc^2\left(\frac{-2314.67 t + \pi(578.667 + 0.5 i + 6.42963 t)}{-1157.33 + i + 6.42963 \pi}\right)}{(-180 + 0.15553 i + \pi)} \right) dt$$

from which:

$$5 * 180 \cot(180(5(0.5-0.0000864055i-0.5)+0.5))-((180 \cot 180*(0.5-0.0000864055i))) - 24 - \text{golden ratio}$$

**Input interpretation:**

$$5 \times 180 \cot(180 (5 (0.5 + i \times (-0.0000864055) - 0.5) + 0.5)) - 180 \cot(180) (0.5 + i \times (-0.0000864055)) - 24 - \phi$$

$\cot(x)$  is the cotangent function

$i$  is the imaginary unit

$\phi$  is the golden ratio

**Result:**

$$-540.536... + 87.2733... i$$

**Polar coordinates:**

$$r = 547.536 \text{ (radius), } \theta = 170.828^\circ \text{ (angle)}$$

547.536 result practically equal to the rest mass of Eta meson 547.853

**Alternative representations:**

$$5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - 180 (\cot(180) (0.5 - i 0.0000864055)) - 24 - \phi = -24 - \phi + 180 (0.5 - 0.0000864055 i) i \coth(-180 i) - 900 i \coth(-180 i (0.5 + 5 (0 - 0.0000864055 i)))$$

$$\begin{aligned}
& 5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - \\
& 180 (\cot(180) (0.5 - i 0.0000864055)) - 24 - \phi = \\
& -24 - \phi - 180 (0.5 - 0.0000864055 i) i \coth(180 i) + \\
& 900 i \coth(180 (0.5 + 5 (0 - 0.0000864055 i)) i)
\end{aligned}$$

$$\begin{aligned}
& 5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - \\
& 180 (\cot(180) (0.5 - i 0.0000864055)) - 24 - \phi = \\
& -24 - \phi - \frac{180 (0.5 - 0.0000864055 i)}{\tan(180)} + \frac{900}{\tan(180 (0.5 + 5 (0 - 0.0000864055 i)))}
\end{aligned}$$

### Series representations:

$$\begin{aligned}
& 5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - \\
& 180 (\cot(180) (0.5 - i 0.0000864055)) - 24 - \phi = -24 - \phi + \\
& \sum_{k=-\infty}^{\infty} e^{-0.15553 (-1157.33+i)k \mathcal{A}} \left( -900 + e^{(180.+0.15553 i)k \mathcal{A}} (90 - 0.015553 i) \right) \mathcal{A} \operatorname{sgn}(k)
\end{aligned}$$

$$\begin{aligned}
& 5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - \\
& 180 (\cot(180) (0.5 - i 0.0000864055)) - 24 - \phi = \\
& -24 - \phi + \sum_{k=1}^{\infty} e^{-0.15553 (-1157.33+i)k \mathcal{A}} \left( -900 + e^{(180.+0.15553 i)k \mathcal{A}} (90 - 0.015553 i) \right) \mathcal{A} + \\
& \sum_{k=-\infty}^{-1} e^{-0.15553 (-1157.33+i)k \mathcal{A}} \left( 900 + e^{(180.+0.15553 i)k \mathcal{A}} (-90 + 0.015553 i) \right) \mathcal{A}
\end{aligned}$$

$$\begin{aligned}
& 5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - \\
& 180 (\cot(180) (0.5 - i 0.0000864055)) - 24 - \phi = \\
& -24 - \phi + \sum_{k=-\infty}^{\infty} \left( 2.49318 \times 10^9 - 137.155 i^2 + 0.0169299 i^3 - \right. \\
& \quad \left. 64800 k^2 \pi^2 + i (-2.01819 \times 10^6 + 67.1889 k^2 \pi^2) \right) / \\
& \quad \left( (-32400 + k^2 \pi^2) (-8100 + 13.9977 i - 0.00604739 i^2 + k^2 \pi^2) \right)
\end{aligned}$$

### Integral representation:

$$\begin{aligned}
& 5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - \\
& 180 (\cot(180) (0.5 - i 0.0000864055)) - 24 - \phi = \\
& -24 - \phi + \int_{\frac{\pi}{2}}^{90-0.077765i} \left( (162000 - 139.977 i - 900 \pi) \csc^2(t) + \right. \\
& \quad \left. \frac{(-32400. + i (5.59908 - 0.015553 \pi) + 90 \pi)}{\csc^2\left(\frac{-2314.67 t + \pi (578.667 + 0.5 i + 6.42963 t)}{-1157.33 + i + 6.42963 \pi}\right)} \right) / \\
& \quad (-180 + 0.15553 i + \pi) dt
\end{aligned}$$

From the formula of coefficients of the '5th order' mock theta function  $\psi_1(q)$ :  
 (A053261 OEIS Sequence)

$$\sqrt{\phi} * \exp(\text{Pi} * \sqrt{n/15}) / (2 * 5^{(1/4)} * \sqrt{n})$$

$$\sqrt{\text{golden ratio}} * \exp(\text{Pi} * \sqrt{140/15}) / (2 * 5^{(1/4)} * \sqrt{140}) - 7$$

where 7 is a Lucas number

**Input:**

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{140}{15}}\right)}{2^4 \sqrt{5} \sqrt{140}} - 7$$

$\phi$  is the golden ratio

**Exact result:**

$$\frac{e^{2\sqrt{7/3} \pi} \sqrt{\frac{\phi}{7}}}{4 \times 5^{3/4}} - 7$$

**Decimal approximation:**

522.5365205444131848886041148576074384081260329366703540246...

522.53652054...

**Property:**

$$-7 + \frac{e^{2\sqrt{7/3} \pi} \sqrt{\frac{\phi}{7}}}{4 \times 5^{3/4}} \text{ is a transcendental number}$$

**Alternate forms:**

$$\frac{1}{20} \sqrt{\frac{1}{14} (5 + \sqrt{5})} e^{2\sqrt{7/3} \pi} - 7$$

$$\frac{\sqrt{\frac{1}{14} (1 + \sqrt{5})} e^{2\sqrt{7/3} \pi}}{4 \times 5^{3/4}} - 7$$

$$\frac{1}{280} \left( \sqrt[4]{5} \sqrt{14(1+\sqrt{5})} e^{2\sqrt{7/3}\pi} - 1960 \right)$$

**Series representations:**

$$\begin{aligned} \frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{140}{15}}\right)}{2\sqrt[4]{5} \sqrt{140}} - 7 = & \left( -70 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (140 - z_0)^k z_0^{-k}}{k!} + \right. \\ & \left. 5^{3/4} \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{28}{3} - z_0\right)^k z_0^{-k}}{k!}\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!} \right) / \\ & \left( 10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (140 - z_0)^k z_0^{-k}}{k!} \right) \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)) \end{aligned}$$

$$\begin{aligned} \frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{140}{15}}\right)}{2\sqrt[4]{5} \sqrt{140}} - 7 = & \left( -70 \exp\left(i\pi \left\lfloor \frac{\arg(140-x)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (140-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \\ & \left. 5^{3/4} \exp\left(i\pi \left\lfloor \frac{\arg(\phi-x)}{2\pi} \right\rfloor\right) \exp\left(\pi \exp\left(i\pi \left\lfloor \frac{\arg\left(\frac{28}{3}-x\right)}{2\pi} \right\rfloor\right) \sqrt{x}\right) \right. \\ & \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{28}{3}-x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \sum_{k=0}^{\infty} \frac{(-1)^k (\phi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \\ & \left( 10 \exp\left(i\pi \left\lfloor \frac{\arg(140-x)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (140-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \end{aligned}$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{140}{15}}\right)}{2 \sqrt[4]{5} \sqrt{140}} - 7 = \left( \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(140-z_0)/(2\pi) \rfloor} z_0^{-1/2 \lfloor \arg(140-z_0)/(2\pi) \rfloor} \right. \\ \left. \left( -70 \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(140-z_0)/(2\pi) \rfloor} z_0^{1/2 \lfloor \arg(140-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (140-z_0)^k z_0^{-k}}{k!} + \right. \right. \\ \left. \left. 5^{3/4} \exp\left(\pi \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(\frac{28}{3}-z_0)/(2\pi) \rfloor} z_0^{1/2 (1+\lfloor \arg(\frac{28}{3}-z_0)/(2\pi) \rfloor)} \right. \right. \right. \\ \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{28}{3}-z_0\right)^k z_0^{-k}}{k!} \right) \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(\phi-z_0)/(2\pi) \rfloor} \right. \\ \left. \left. z_0^{1/2 \lfloor \arg(\phi-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi-z_0)^k z_0^{-k}}{k!} \right) \right) / \\ \left( 10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (140-z_0)^k z_0^{-k}}{k!} \right)$$

$n!$  is the factorial function

$(\alpha)_n$  is the Pochhammer symbol (rising factorial)

$\mathbb{R}$  is the set of real numbers

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

We have also:

$$\text{Pi}^*\left(\left(\left(5 \cdot 180 \cot(180(5(0.5 - 0.0000864055i) - 0.5) + 0.5)\right) - \left(180 \cot 180 \cdot (0.5 - 0.0000864055i)\right)\right)\right) - 89 - 1/\text{golden ratio}$$

where 89 is a Fibonacci number

**Input interpretation:**

$$\pi \left( 5 \times 180 \cot(180 (5 (0.5 + i \times (-0.0000864055) - 0.5) + 0.5)) - \right. \\ \left. 180 \cot(180) (0.5 + i \times (-0.0000864055)) \right) - 89 - \frac{1}{\phi}$$

$\cot(x)$  is the cotangent function

$i$  is the imaginary unit

$\phi$  is the golden ratio

**Result:**

$$-1707.28... + 274.177... i$$

**Polar coordinates:**

$$r = 1729.16 \text{ (radius), } \theta = 170.877^\circ \text{ (angle)}$$

1729.16

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternative representations:**

$$\begin{aligned} &\pi (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - \\ &\quad 180 (\cot(180) (0.5 - i 0.0000864055))) - 89 - \frac{1}{\phi} = \\ &-89 + \pi (180 (0.5 - 0.0000864055 i) i \coth(-180 i) - \\ &\quad 900 i \coth(-180 i (0.5 + 5 (0 - 0.0000864055 i)))) - \frac{1}{\phi} \end{aligned}$$

$$\begin{aligned} &\pi (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - \\ &\quad 180 (\cot(180) (0.5 - i 0.0000864055))) - 89 - \frac{1}{\phi} = \\ &-89 + \pi (-180 (0.5 - 0.0000864055 i) i \coth(180 i) + \\ &\quad 900 i \coth(180 (0.5 + 5 (0 - 0.0000864055 i) i))) - \frac{1}{\phi} \end{aligned}$$

$$\begin{aligned} &\pi (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - \\ &\quad 180 (\cot(180) (0.5 - i 0.0000864055))) - 89 - \frac{1}{\phi} = \\ &-89 - \frac{1}{\phi} + \pi \left( -\frac{180 (0.5 - 0.0000864055 i)}{\tan(180)} + \frac{900}{\tan(180 (0.5 + 5 (0 - 0.0000864055 i)))} \right) \end{aligned}$$

### Series representations:

$$\pi (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - 180 (\cot(180) (0.5 - i 0.0000864055))) - 89 - \frac{1}{\phi} = -89 - \frac{1}{\phi} + \sum_{k=-\infty}^{\infty} e^{-0.15553 (-1157.33+i)k \mathcal{A}} \left( -900 + e^{(180.+0.15553 i)k \mathcal{A}} (90 - 0.015553 i) \right) \pi \mathcal{A} \operatorname{sgn}(k)$$

$$\pi (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - 180 (\cot(180) (0.5 - i 0.0000864055))) - 89 - \frac{1}{\phi} = -89 - \frac{1}{\phi} + \sum_{k=-\infty}^{\infty} \left( \pi \left( 2.49318 \times 10^9 - 137.155 i^2 + 0.0169299 i^3 - 64800 k^2 \pi^2 + i \left( -2.01819 \times 10^6 + 67.1889 k^2 \pi^2 \right) \right) \right) / \left( (-32400 + k^2 \pi^2) (-8100 + 13.9977 i - 0.00604739 i^2 + k^2 \pi^2) \right)$$

$$\pi (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - 180 (\cot(180) (0.5 - i 0.0000864055))) - 89 - \frac{1}{\phi} = \frac{1}{\phi} \left( -1 - 89 \phi + \phi \sum_{k=1}^{\infty} e^{-0.15553 (-1157.33+i)k \mathcal{A}} \left( -900 + e^{(180.+0.15553 i)k \mathcal{A}} (90 - 0.015553 i) \right) \pi \mathcal{A} + \phi \sum_{k=-\infty}^{-1} e^{-0.15553 (-1157.33+i)k \mathcal{A}} \left( 900 + e^{(180.+0.15553 i)k \mathcal{A}} (-90 + 0.015553 i) \right) \pi \mathcal{A} \right)$$

### Integral representation:

$$\pi (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - 180 (\cot(180) (0.5 - i 0.0000864055))) - 89 - \frac{1}{\phi} = -89 - \frac{1}{\phi} + \int_{\frac{\pi}{2}}^{90-0.077765 i} \left( \left( \pi \left( (162000 - 139.977 i - 900 \pi) \csc^2(t) + \frac{(-32400. + i (5.59908 - 0.015553 \pi) + 90 \pi)}{\csc^2 \left( \frac{-2314.67 t + \pi (578.667 + 0.5 i + 6.42963 t)}{-1157.33 + i + 6.42963 \pi} \right)} \right) \right) / (-180 + 0.15553 i + \pi) dt$$

$$1/\pi * (((5 * 180 \cot(180(5(0.5 - 0.0000864055i - 0.5) + 0.5)) - ((180 \cot 180 * (0.5 - 0.0000864055i)))))) + 29 - \text{golden ratio}$$

where 29 is a Lucas number

**Input interpretation:**

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 + i \times (-0.0000864055) - 0.5) + 0.5)) - 180 \cot(180) (0.5 + i \times (-0.0000864055))) + 29 - \phi$$

$\cot(x)$  is the cotangent function

$i$  is the imaginary unit

$\phi$  is the golden ratio

**Result:**

$$-136.522... + 27.7800... i$$

**Polar coordinates:**

$$r = 139.319 \text{ (radius), } \theta = 168.498^\circ \text{ (angle)}$$

139.319 result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - 180 \cot(180) (0.5 - i 0.0000864055)) + 29 - \phi = 29 - \phi + \frac{1}{\pi} (180 (0.5 - 0.0000864055 i) i \coth(-180 i) - 900 i \coth(-180 i (0.5 + 5 (0 - 0.0000864055 i))))$$

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - 180 \cot(180) (0.5 - i 0.0000864055)) + 29 - \phi = 29 - \phi + \frac{1}{\pi} (-180 (0.5 - 0.0000864055 i) i \coth(180 i) + 900 i \coth(180 (0.5 + 5 (0 - 0.0000864055 i) i))$$

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - 180 \cot(180) (0.5 - i 0.0000864055)) + 29 - \phi = 29 - \phi + \frac{-\frac{180 (0.5 - 0.0000864055 i)}{\tan(180)} + \frac{900}{\tan(180 (0.5 + 5 (0 - 0.0000864055 i) i))}}{\pi}$$

### Series representations:

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5))) -$$

$$\sum_{k=-\infty}^{\infty} \frac{180 \cot(180) (0.5 - i 0.0000864055) + 29 - \phi = 29 - \phi + e^{-0.15553 (-1157.33+i)k \mathcal{A}} (-900 + e^{(180. + 0.15553 i)k \mathcal{A}} (90 - 0.015553 i)) \mathcal{A} \operatorname{sgn}(k)}{\pi}$$

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5))) -$$

$$180 \cot(180) (0.5 - i 0.0000864055) + 29 - \phi =$$

$$29 - \phi + \sum_{k=-\infty}^{\infty} \left( 2.49318 \times 10^9 - 137.155 i^2 + 0.0169299 i^3 - \right.$$

$$\left. \frac{64800 k^2 \pi^2 + i (-2.01819 \times 10^6 + 67.1889 k^2 \pi^2)}{\pi (-32400 + k^2 \pi^2) (-8100 + 13.9977 i - 0.00604739 i^2 + k^2 \pi^2)} \right)$$

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5))) -$$

$$180 \cot(180) (0.5 - i 0.0000864055) + 29 - \phi =$$

$$\left( -29 + \phi - \sum_{k=1}^{\infty} \frac{e^{-0.15553 (-1157.33+i)k \mathcal{A}} (-900 + e^{(180. + 0.15553 i)k \mathcal{A}} (90 - 0.015553 i)) \mathcal{A}}{\pi} - \right.$$

$$\left. \sum_{k=-\infty}^{-1} \frac{e^{-0.15553 (-1157.33+i)k \mathcal{A}} (900 + e^{(180. + 0.15553 i)k \mathcal{A}} (-90 + 0.015553 i)) \mathcal{A}}{\pi} \right)$$

### Integral representation:

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5))) -$$

$$180 \cot(180) (0.5 - i 0.0000864055) + 29 - \phi =$$

$$29 - \phi + \int_{\frac{\pi}{2}}^{90 - 0.077765 i} \left( (162000 - 139.977 i - 900 \pi) \csc^2(t) + \right.$$

$$\left. \frac{(-32400. + i (5.59908 - 0.015553 \pi) + 90 \pi)}{\csc^2\left(\frac{-2314.67 t + \pi (578.667 + 0.5 i + 6.42963 t)}{-1157.33 + i + 6.42963 \pi}\right)} \right) /$$

$$(\pi (-180 + 0.15553 i + \pi)) dt$$

$$1/\pi * (((5 * 180 \cot(180(5(0.5 - 0.0000864055i - 0.5) + 0.5))) - ((180 \cot 180 * (0.5 - 0.0000864055i)))))) + 47 - 4 - \text{golden ratio}$$

where 47 and 4 are Lucas numbers

**Input interpretation:**

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 + i \times (-0.0000864055) - 0.5) + 0.5)) - 180 \cot(180) (0.5 + i \times (-0.0000864055))) + 47 - 4 - \phi$$

$\cot(x)$  is the cotangent function

$i$  is the imaginary unit

$\phi$  is the golden ratio

**Result:**

$$-122.522... + 27.7800... i$$

**Polar coordinates:**

$$r = 125.631 \text{ (radius), } \theta = 167.225^\circ \text{ (angle)}$$

125.631 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - 180 \cot(180) (0.5 - i 0.0000864055)) + 47 - 4 - \phi = 43 - \phi + \frac{1}{\pi} (180 (0.5 - 0.0000864055 i) i \coth(-180 i) - 900 i \coth(-180 i (0.5 + 5 (0 - 0.0000864055 i))))$$

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5)) - 180 \cot(180) (0.5 - i 0.0000864055)) + 47 - 4 - \phi = 43 - \phi + \frac{1}{\pi} (-180 (0.5 - 0.0000864055 i) i \coth(180 i) + 900 i \coth(180 (0.5 + 5 (0 - 0.0000864055 i) i))$$

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5))) -$$

$$\frac{180 \cot(180) (0.5 - i 0.0000864055)) + 47 - 4 - \phi =$$

$$43 - \phi + \frac{-\frac{180 (0.5 - 0.0000864055 i)}{\tan(180)} + \frac{900}{\tan(180 (0.5 + 5 (0 - 0.0000864055 i)))}}{\pi}$$

### Series representations:

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5))) -$$

$$\frac{180 \cot(180) (0.5 - i 0.0000864055)) + 47 - 4 - \phi = 43 - \phi +$$

$$\sum_{k=-\infty}^{\infty} \frac{e^{-0.15553 (-1157.33+i)k \mathcal{A}} (-900 + e^{(180. + 0.15553 i)k \mathcal{A}} (90 - 0.015553 i)) \mathcal{A} \operatorname{sgn}(k)}{\pi}$$

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5))) -$$

$$\frac{180 \cot(180) (0.5 - i 0.0000864055)) + 47 - 4 - \phi =$$

$$43 - \phi + \sum_{k=-\infty}^{\infty} \left( 2.49318 \times 10^9 - 137.155 i^2 + 0.0169299 i^3 - \right.$$

$$\left. \frac{64800 k^2 \pi^2 + i (-2.01819 \times 10^6 + 67.1889 k^2 \pi^2)}{\pi (-32400 + k^2 \pi^2) (-8100 + 13.9977 i - 0.00604739 i^2 + k^2 \pi^2)} \right)$$

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5))) -$$

$$\frac{180 \cot(180) (0.5 - i 0.0000864055)) + 47 - 4 - \phi =$$

$$- \left( -43 + \phi - \sum_{k=1}^{\infty} \frac{e^{-0.15553 (-1157.33+i)k \mathcal{A}} (-900 + e^{(180. + 0.15553 i)k \mathcal{A}} (90 - 0.015553 i)) \mathcal{A}}{\pi} - \right.$$

$$\left. \sum_{k=-\infty}^{-1} \frac{e^{-0.15553 (-1157.33+i)k \mathcal{A}} (900 + e^{(180. + 0.15553 i)k \mathcal{A}} (-90 + 0.015553 i)) \mathcal{A}}{\pi} \right)$$

### Integral representation:

$$\frac{1}{\pi} (5 \times 180 \cot(180 (5 (0.5 - i 0.0000864055 - 0.5) + 0.5))) -$$

$$\frac{180 \cot(180) (0.5 - i 0.0000864055)) + 47 - 4 - \phi =$$

$$43 - \phi + \int_{\frac{\pi}{2}}^{90 - 0.077765 i} \left( \left( (162000 - 139.977 i - 900 \pi) \csc^2(t) + \right. \right.$$

$$\left. \frac{(-32400. + i (5.59908 - 0.015553 \pi) + 90 \pi)}{\csc^2\left(\frac{-2314.67 t + \pi (578.667 + 0.5 i + 6.42963 t)}{-1157.33 + i + 6.42963 \pi}\right)} \right) /$$

$$\left( \pi (-180 + 0.15553 i + \pi) \right) dt$$

Now, we have that:

$$\frac{16\eta^8(4i\tilde{T})}{\eta^{16}(2i\tilde{T})} = \frac{16 \prod_{n=1}^{\infty} (1 + \exp(-4\pi n\tilde{T}))^8}{1 - \exp(-4\pi n\tilde{T})^8} \quad (4.10)$$

16 product (1+exp(-4\*Pi\*n\*1729))^8, n=1 to infinity

**Input interpretation:**

$$16 \prod_{n=1}^{\infty} (1 + \exp(-4\pi n \times 1729))^8$$

**Result:**

$$\frac{1}{16} \left( (-1; e^{-6916\pi})_{\infty} \right)^8 \approx 16$$

16

1-exp(-4Pi\*n\*1729)^8

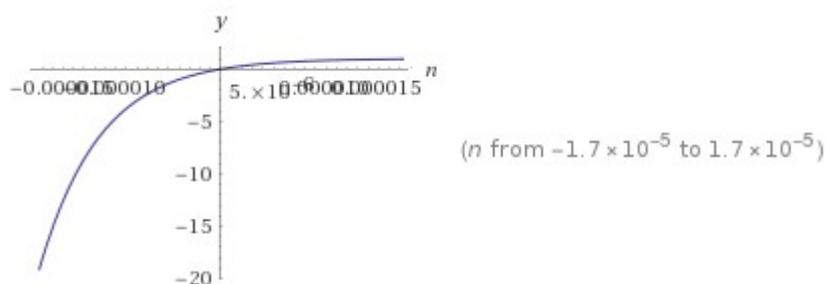
**Input:**

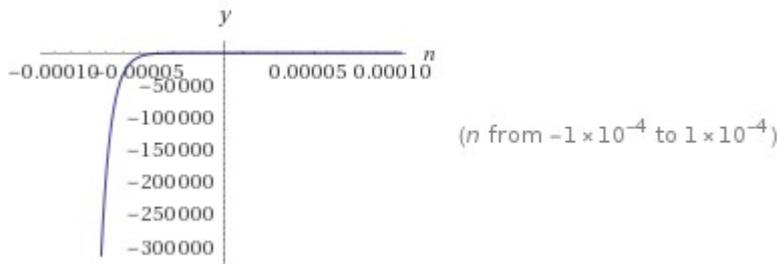
$$1 - \exp^8(-4\pi n \times 1729)$$

**Exact result:**

$$1 - e^{-55328\pi n}$$

**Plots:**





### Roots:

$$n = -\frac{im}{27664}, \quad m \in \mathbb{Z}$$

$\mathbb{Z}$  is the set of integers

### Periodicity:

periodic in  $n$  with period  $\frac{i}{27664}$

### Series expansion at $n = 0$ :

$$55\,328\pi n - 1\,530\,593\,792\pi^2 n^2 + \frac{84\,684\,693\,323\,776\pi^3 n^3}{3} - \frac{1\,171\,358\,678\,054\,469\,632\pi^4 n^4}{15} + \frac{64\,808\,932\,939\,397\,695\,799\,296\pi^5 n^5}{15} + O(n^6)$$

(Taylor series)

### Derivative:

$$\frac{d}{dn}(1 - \exp^8(-4\pi n \times 1729)) = 55\,328\pi e^{-55\,328\pi n}$$

### Indefinite integral:

$$\int (1 - e^{-55\,328\pi n}) dn = n + \frac{e^{-55\,328\pi n}}{55\,328\pi} + \text{constant}$$

### Limit:

$$\lim_{n \rightarrow \infty} (1 - e^{-55\,328\pi n}) = 1$$

### Series representations:

$$1 - \exp^8(-4\pi n \times 1729) = 1 - \sum_{k=0}^{\infty} \frac{(-n)^k (55\,328\pi)^k}{k!}$$

$$1 - \exp^8(-4\pi n \times 1729) = 1 - \sum_{k=-\infty}^{\infty} I_k(-55\,328\pi n)$$

$$1 - \exp^8(-4\pi n \times 1729) = 1 - e^{z_0} \sum_{k=0}^{\infty} \frac{(-55\,328\pi n - z_0)^k}{k!}$$

**Definite integral over a half-period:**

$$\int_0^{-\frac{i}{55328}} (1 - e^{-55328 n \pi}) dn = -\frac{2 + i \pi}{55328 \pi} \approx -0.0000115063 - 0.000018074 i$$

**Definite integral over a period:**

$$\int_0^{-\frac{i}{27664}} (1 - e^{-55328 n \pi}) dn = -\frac{i}{27664} \approx -0.0000361481 i$$

**Definite integral mean square:**

$$\int_0^{-\frac{i}{27664}} 27664 i (1 - e^{-55328 n \pi})^2 dn = 1$$

In conclusion, we obtain:

$$1/(((1 - e^{(-55328 * \pi)}))) \cdot 16 \cdot \prod_{n=1}^{\infty} (1 + \exp(-4 * \pi * n * 1729))^8, n=1 \text{ to infinity}$$

**Input interpretation:**

$$\frac{1}{1 - e^{-55328 \pi}} \times 16 \prod_{n=1}^{\infty} (1 + \exp(-4 \pi n \times 1729))^8$$

**Result:**

$$\frac{((-1; e^{-6916\pi})_{\infty})^8}{16 (1 - e^{-55328 \pi})} \approx 16$$

16

$(a; q)_n$  gives the  $q$ -Pochhammer symbol

**Alternate form:**

$$\frac{((-1; e^{-6916\pi})_{\infty})^8}{16 (e^{-55328 \pi} - 1)}$$

$$8 * (((1/(((1 - e^{(-55328 * \pi)}))) \cdot 16 \cdot \prod_{n=1}^{\infty} (1 + \exp(-4 * \pi * n * 1729))^8, n=1 \text{ to infinity})))) - 3 + 1/\text{golden ratio}$$

where 8 and 3 are Fibonacci numbers

**Input interpretation:**

$$8 \left( \frac{1}{1 - e^{-55328 \pi}} \times 16 \prod_{n=1}^{\infty} (1 + \exp(-4 \pi n \times 1729))^8 \right) - 3 + \frac{1}{\phi}$$

$\phi$  is the golden ratio

**Result:**

$$\frac{((-1; e^{-6916\pi})_{\infty})^8}{2(1 - e^{-55328\pi})} + \frac{1}{\phi} - 3 \approx 125.618$$

125.618 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

$(a; q)_n$  gives the  $q$ -Pochhammer symbol

**Alternate forms:**

$$-\frac{((-1; e^{-6916\pi})_{\infty})^8}{2(e^{-55328\pi} - 1)} + \frac{1}{\phi} - 3$$

$$\frac{((-1; e^{-6916\pi})_{\infty})^8}{2(1 - e^{-55328\pi})} + \frac{1}{2}(\sqrt{5} - 7)$$

$$\frac{-((( -1; e^{-6916\pi})_{\infty})^8 - 6)\phi + 2e^{-55328\pi}(1 - 3\phi) - 2}{2(e^{-55328\pi} - 1)\phi}$$

$8 * (((1/(((1 - e^{(-55328 * \pi)})))$  16 product  $(1 + \exp(-4 * \pi * n * 1729))^8$ ,  $n=1$  to infinity)))))+11+1/golden ratio

where 11 is a Lucas number

**Input interpretation:**

$$8 \left( \frac{1}{1 - e^{-55328\pi}} \times 16 \prod_{n=1}^{\infty} (1 + \exp(-4\pi n \times 1729))^8 \right) + 11 + \frac{1}{\phi}$$

$\phi$  is the golden ratio

**Result:**

$$\frac{((-1; e^{-6916\pi})_{\infty})^8}{2(1 - e^{-55328\pi})} + \frac{1}{\phi} + 11 \approx 139.618$$

139.618 result practically equal to the rest mass of Pion meson 139.57 MeV

$(a; q)_n$  gives the  $q$ -Pochhammer symbol

**Alternate forms:**

$$-\frac{((-1; e^{-6916\pi})_\infty)^8}{2(e^{-55328\pi} - 1)} + \frac{1}{\phi} + 11$$

$$\frac{((-1; e^{-6916\pi})_\infty)^8}{2(1 - e^{-55328\pi})} + 11 + \frac{2}{1 + \sqrt{5}}$$

$$\frac{-((( -1; e^{-6916\pi})_\infty)^8 + 22)\phi + 2e^{-55328\pi}(11\phi + 1) - 2}{2(e^{-55328\pi} - 1)\phi}$$

Now, we have that:

$$\frac{\eta^8(2i\tilde{T})}{\eta^8(i\tilde{T})\eta^8(4i\tilde{T})} = \exp(2\pi\tilde{T}) \frac{\prod_{n=1}^\infty (1 + \exp(-2\pi(2n-1)\tilde{T}))^8}{(1 - \exp(4\pi n\tilde{T}))^8} = \exp(2\pi\tilde{T}) + 8 + \mathcal{O}(\exp(-2\pi\tilde{T}))$$

$$\frac{\eta^8(i\tilde{T})}{\eta^{16}(2i\tilde{T})} = \exp(2\pi\tilde{T}) \frac{\prod_{n=1}^\infty (1 - \exp(-2\pi(2n-1)\tilde{T}))^8}{(1 - \exp(4\pi n\tilde{T}))^8} = \exp(2\pi\tilde{T}) - 8 + \mathcal{O}(\exp(-2\pi\tilde{T})) \quad (4.19)$$

$$\exp(2*\text{Pi}*(0.0864055))-8+(\exp(-2*\text{Pi}*(0.0864055)))$$

**Input interpretation:**

$$\exp(2\pi \times 0.0864055) - 8 + \exp(-2\pi \times 0.0864055)$$

**Result:**

$$-5.697947...$$

$$-5.697947...$$

$$\exp(2*\text{Pi}*(0.0864055))+8+(\exp(-2*\text{Pi}*(0.0864055)))$$

**Input interpretation:**

$$\exp(2\pi \times 0.0864055) + 8 + \exp(-2\pi \times 0.0864055)$$

**Result:**

10.30205...

10.30205...

From the difference between the two functions and squaring, we get:

$$\begin{aligned}
 & [((\exp(2*\text{Pi}*(0.0864055))-8+(\exp(-2*\text{Pi}*(0.0864055)))) - \\
 & ((\exp(2*\text{Pi}*(0.0864055))+8+(\exp(-2*\text{Pi}*(0.0864055)))))]^2
 \end{aligned}$$

**Input interpretation:**

$$\begin{aligned}
 & ((\exp(2 \pi \times 0.0864055) - 8 + \exp(-2 \pi \times 0.0864055)) - \\
 & (\exp(2 \pi \times 0.0864055) + 8 + \exp(-2 \pi \times 0.0864055)))^2
 \end{aligned}$$

**Result:**

256

$256 = 64 \times 4$

From the formula of coefficients of the '5th order' mock theta function  $\psi_1(q)$ :  
(A053261 OEIS Sequence)

$$\text{sqrt(golden ratio)} * \exp(\text{Pi}*\text{sqrt}(n/15)) / (2*5^{(1/4)}*\text{sqrt}(n))$$

for n = 117, we obtain:

$$\text{sqrt(golden ratio)} * \exp(\text{Pi}*\text{sqrt}(117/15)) / (2*5^{(1/4)}*\text{sqrt}(117)) + (2*0.9568666373)$$

where 0.9568666373 is the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}} \approx 0.9568666373$$

**Input interpretation:**

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{117}{15}}\right)}{2 \sqrt[4]{5} \sqrt{117}} + 2 \times 0.9568666373$$

$\phi$  is the golden ratio

**Result:**

256.083666904...

256.083666904...

**Series representations:**

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{117}{15}}\right)}{2 \sqrt[4]{5} \sqrt{117}} + 2 \times 0.956867 =$$

$$\left( 0.1 \left[ 19.1373 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (117 - z_0)^k z_0^{-k}}{k!} + 3.3437 \exp\left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{39}{5} - z_0\right)^k z_0^{-k}}{k!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!} \right] \right) /$$

$$\left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (117 - z_0)^k z_0^{-k}}{k!} \right) \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{117}{15}}\right)}{2 \sqrt[4]{5} \sqrt{117}} + 2 \times 0.956867 =$$

$$\left( 0.1 \left[ 19.1373 \exp\left(i \pi \left\lfloor \frac{\arg(117 - x)}{2 \pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (117 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \right.$$

$$\left. 3.3437 \exp\left(i \pi \left\lfloor \frac{\arg(\phi - x)}{2 \pi} \right\rfloor\right) \exp\left(\pi \exp\left(i \pi \left\lfloor \frac{\arg\left(\frac{39}{5} - x\right)}{2 \pi} \right\rfloor\right) \sqrt{x}\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{39}{5} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \sum_{k=0}^{\infty} \frac{(-1)^k (\phi - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right] \right) /$$

$$\left( \exp\left(i \pi \left\lfloor \frac{\arg(117 - x)}{2 \pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (117 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{117}{15}}\right)}{2 \sqrt[4]{5} \sqrt{117}} + 2 \times 0.956867 =$$

$$\left(0.1 \left(\frac{1}{z_0}\right)^{-1/2 [\arg(117-z_0)/(2\pi)]} z_0^{-1/2 [\arg(117-z_0)/(2\pi)]} \left(19.1373 \left(\frac{1}{z_0}\right)^{1/2 [\arg(117-z_0)/(2\pi)]} z_0^{1/2 [\arg(117-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (117-z_0)^k z_0^{-k}}{k!} + 3.3437 \exp\left(\pi \left(\frac{1}{z_0}\right)^{1/2 [\arg(\frac{39}{5}-z_0)/(2\pi)]} z_0^{1/2 (1+[\arg(\frac{39}{5}-z_0)/(2\pi)])} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{39}{5}-z_0\right)^k z_0^{-k}}{k!} \right) \left(\frac{1}{z_0}\right)^{1/2 [\arg(\phi-z_0)/(2\pi)]} z_0^{1/2 [\arg(\phi-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi-z_0)^k z_0^{-k}}{k!} \right) \Bigg/ \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (117-z_0)^k z_0^{-k}}{k!} \right)$$

$n!$  is the factorial function

$(\alpha)_n$  is the Pochhammer symbol (rising factorial)

$\mathbb{R}$  is the set of real numbers

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

Multiplying the two results, we obtain:

$$\left( \left( \left( \exp(2\pi i (0.0864055)) - 8 + \exp(-2\pi i (0.0864055)) \right) \right) \right) * \left( \left( \left( \exp(2\pi i (0.0864055)) + 8 + \exp(-2\pi i (0.0864055)) \right) \right) \right)$$

### Input interpretation:

$$\left( \exp(2\pi \times 0.0864055) - 8 + \exp(-2\pi \times 0.0864055) \right) \left( \exp(2\pi \times 0.0864055) + 8 + \exp(-2\pi \times 0.0864055) \right)$$

**Result:**

-58.7006...

-58.7006...

From which:

$$-2(((\exp(2*\text{Pi}*(0.0864055))-8+(\exp(-2*\text{Pi}*(0.0864055)))))) * \\ ((\exp(2*\text{Pi}*(0.0864055))+8+(\exp(-2*\text{Pi}*(0.0864055)))))+29-7$$

where 29 and 7 are Lucas numbers

**Input interpretation:**

$$-2 (\exp(2 \pi \times 0.0864055) - 8 + \exp(-2 \pi \times 0.0864055)) \\ (\exp(2 \pi \times 0.0864055) + 8 + \exp(-2 \pi \times 0.0864055)) + 29 - 7$$

**Result:**

139.4011...

139.4011... result practically equal to the rest mass of Pion meson 139.57 MeV

$$-2(((\exp(2*\text{Pi}*(0.0864055))-8+(\exp(-2*\text{Pi}*(0.0864055)))))) * \\ ((\exp(2*\text{Pi}*(0.0864055))+8+(\exp(-2*\text{Pi}*(0.0864055)))))+7+3\text{-golden ratio}$$

where 7 and 3 are Lucas numbers

**Input interpretation:**

$$-2 (\exp(2 \pi \times 0.0864055) - 8 + \exp(-2 \pi \times 0.0864055)) \\ (\exp(2 \pi \times 0.0864055) + 8 + \exp(-2 \pi \times 0.0864055)) + 7 + 3 - \phi$$

$\phi$  is the golden ratio

**Result:**

125.783...

125.783... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

$$-27 * (((\exp(2 * \pi * (0.0864055)) - 8 + (\exp(-2 * \pi * (0.0864055)))))) * \\
(((\exp(2 * \pi * (0.0864055)) + 8 + (\exp(-2 * \pi * (0.0864055)))))) + 123 + 18 + \text{golden ratio}^2$$

where 123 and 18 are Lucas numbers

**Input interpretation:**

$$-27 (\exp(2 \pi \times 0.0864055) - 8 + \exp(-2 \pi \times 0.0864055)) \\
(\exp(2 \pi \times 0.0864055) + 8 + \exp(-2 \pi \times 0.0864055)) + 123 + 18 + \phi^2$$

$\phi$  is the golden ratio

**Result:**

1728.533...

1728.533...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

We first consider the residue at  $z = \tau/2$  modulo the lattice. This corresponds to  $w = q^{1/2}$ , where  $w = \exp(2\pi iz)$ . From the product formula for  $G(z, \tau)$ , one finds that

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n).$$

$$G(z, \tau) \underset{z \rightarrow \tau/2}{\sim} \frac{1}{1 - qw^{-2}} \cdot \frac{q^{-1/6} \prod_{n=1}^{\infty} (1 - q^{n-1/2})^8}{\eta^8(\tau)}. \tag{3.32}$$

for  $q = e^{2\pi} = 535.49165\dots$ , we obtain:

535.49165^(1/24) product (1-535.49165^n), n=1 to 0.0864055

**Input interpretation:**

$$\sqrt[24]{535.49165} \prod_{n=1}^{0.0864055} (1 - 535.49165^n)$$

**Result:**

1.29927  
1.29927

And:

((535.49165^(1/24) product (1-535.49165^n), n=1 to 0.0864055))^8

**Input interpretation:**

$$\left( \sqrt[24]{535.49165} \prod_{n=1}^{0.0864055} (1 - 535.49165^n) \right)^8$$

**Result:**

8.12053  
8.12053

From:

$$G(z, \tau) \stackrel{z \rightarrow \tau/2}{\sim} \frac{1}{1 - qw^{-2}} \cdot \frac{q^{-1/6} \prod_{n=1}^{\infty} (1 - q^{n-1/2})^8}{\eta^8(\tau)}. \quad (3.32)$$

-(((1/(1-535.49165\*(535.49165^(1/2))^(-2)))) \* 1/8.12053 \* 535.49165^(-1/6) product ((1-535.49165^(n-0.5)))^8, n=1 to 0.0864055

**Input interpretation:**

$$-\frac{1}{1 + \frac{535.49165}{\sqrt{535.49165}^2}} \left( \frac{1}{8.12053} \times 535.49165^{-1/6} \right) \prod_{n=1}^{0.0864055} (1 - 535.49165^{n-0.5})^8$$

**Result:**

1.94618 × 10<sup>14</sup>  
1.94618\*10<sup>14</sup>

From which, we have:

$$\left( \left( \left( \left( \frac{1}{1 - 535.49165 \cdot (535.49165^{1/2})^{-2}} \right) \right) \cdot \frac{1}{8.12053} \cdot 535.49165^{-1/6} \right) \right) \cdot \left( \prod_{n=1}^{0.0864055} (1 - 535.49165^{n-0.5})^8 \right)^{1/3}$$

**Input interpretation:**

$$\sqrt[3]{ - \frac{1}{1 + \frac{535.49165}{\sqrt{535.49165^2}}} \left( \frac{1}{8.12053} \times 535.49165^{-1/6} \right) \prod_{n=1}^{0.0864055} (1 - 535.49165^{n-0.5})^8 }$$

**Result:**

57951.

**57951**

From the formula of coefficients of the '5th order' mock theta function  $\psi_1(q)$ :  
(A053261 OEIS Sequence)

$$\sqrt{\phi} \cdot \exp(\pi \cdot \sqrt{n/15}) / (2 \cdot 5^{1/4} \cdot \sqrt{n})$$

for  $n = 329$ , we obtain:

$$\sqrt{\phi} \cdot \exp(\pi \cdot \sqrt{329/15}) / (2 \cdot 5^{1/4} \cdot \sqrt{329}) + 377 + 34 + 8$$

where 377, 34 and 8 are Fibonacci numbers

**Input:**

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{329}{15}}\right)}{2 \sqrt[4]{5} \sqrt{329}} + 377 + 34 + 8$$

$\phi$  is the golden ratio

**Exact result:**

$$\frac{e^{\sqrt{329/15} \pi} \sqrt{\frac{\phi}{329}}}{2 \sqrt[4]{5}} + 419$$

**Decimal approximation:**

57951.35737436966704999608902807251901007379198071732333042...

57951.357...

**Property:**

$419 + \frac{e^{\sqrt{329/15} \pi} \sqrt{\frac{\phi}{329}}}{2 \sqrt[4]{5}}$  is a transcendental number

**Alternate forms:**

$$419 + \frac{1}{2} \sqrt{\frac{5 + \sqrt{5}}{3290}} e^{\sqrt{329/15} \pi}$$

$$419 + \frac{\sqrt{\frac{1}{658} (1 + \sqrt{5})} e^{\sqrt{329/15} \pi}}{2 \sqrt[4]{5}}$$

$$\frac{2757020 + 5^{3/4} \sqrt{658 (1 + \sqrt{5})} e^{\sqrt{329/15} \pi}}{6580}$$

**Series representations:**

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{329}{15}}\right)}{2 \sqrt[4]{5} \sqrt{329}} + 377 + 34 + 8 = \left( 4190 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (329 - z_0)^k z_0^{-k}}{k!} + 5^{3/4} \right. \\ \left. \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{329}{15} - z_0\right)^k z_0^{-k}}{k!}\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!} \right) / \\ \left( 10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (329 - z_0)^k z_0^{-k}}{k!} \right) \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\begin{aligned}
& \frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{329}{15}}\right)}{2 \sqrt[4]{5} \sqrt{329}} + 377 + 34 + 8 = \\
& \left( 4190 \exp\left(i \pi \left\lfloor \frac{\arg(329-x)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (329-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& \quad 5^{3/4} \exp\left(i \pi \left\lfloor \frac{\arg(\phi-x)}{2\pi} \right\rfloor\right) \exp\left(\pi \exp\left(i \pi \left\lfloor \frac{\arg\left(\frac{329}{15}-x\right)}{2\pi} \right\rfloor\right)\right) \sqrt{x} \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{329}{15}-x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \sum_{k=0}^{\infty} \frac{(-1)^k (\phi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \\
& \left( 10 \exp\left(i \pi \left\lfloor \frac{\arg(329-x)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (329-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{329}{15}}\right)}{2 \sqrt[4]{5} \sqrt{329}} + 377 + 34 + 8 = \\
& \left( \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(329-z_0)/(2\pi) \rfloor} z_0^{-1/2 \lfloor \arg(329-z_0)/(2\pi) \rfloor} \left( 4190 \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(329-z_0)/(2\pi) \rfloor} \right. \right. \\
& \quad \left. \left. z_0^{1/2 \lfloor \arg(329-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (329-z_0)^k z_0^{-k}}{k!} + \right. \right. \\
& \quad 5^{3/4} \exp\left(\pi \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg\left(\frac{329}{15}-z_0\right)/(2\pi) \rfloor} z_0^{1/2 (1 + \lfloor \arg\left(\frac{329}{15}-z_0\right)/(2\pi) \rfloor)} \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{329}{15}-z_0\right)^k z_0^{-k}}{k!} \right) \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(\phi-z_0)/(2\pi) \rfloor} \\
& \quad \left. \left. z_0^{1/2 \lfloor \arg(\phi-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi-z_0)^k z_0^{-k}}{k!} \right) \right) / \\
& \left( 10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (329-z_0)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\mathbb{R}$  is the set of real numbers

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$$55 + \left( \left( \left( \left( \frac{1}{1 - 535.49165 \cdot (535.49165^{1/2})^{-2}} \right) \right) \cdot \frac{1}{8.12053} \cdot 535.49165^{-1/6} \right) \right) \cdot \prod_{n=1}^{0.0864055} (1 - 535.49165^{n-0.5})^8$$

where 55 is a Fibonacci number

**Input interpretation:**

$$55 + \sqrt[5]{-\frac{1}{1 + \frac{535.49165}{\sqrt{535.49165^2}}} \left( \frac{1}{8.12053} \times 535.49165^{-1/6} \right)^{0.0864055} \prod_{n=1} (1 - 535.49165^{n-0.5})^8}$$

**Result:**

775.836

775.836 result practically equal to the rest mass of Neutral rho meson 775.49

$$8 + 10^3 + \left( \left( \left( \left( \frac{1}{1 - 535.49165 \cdot (535.49165^{1/2})^{-2}} \right) \right) \cdot \frac{1}{8.12053} \cdot 535.49165^{-1/6} \right) \right) \cdot \prod_{n=1}^{0.0864055} (1 - 535.49165^{n-0.5})^8$$

where 8 is a Fibonacci number

**Input interpretation:**

$$8 + 10^3 + \sqrt[5]{-\frac{1}{1 + \frac{535.49165}{\sqrt{535.49165^2}}} \left( \frac{1}{8.12053} \times 535.49165^{-1/6} \right)^{0.0864055} \prod_{n=1} (1 - 535.49165^{n-0.5})^8}$$

**Result:**

1728.84

1728.84

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$55+8+10^3+\left(\left(\left(\left(\frac{1}{1-535.49165*(535.49165^{(1/2)})^{-2}}\right)\right)\right)\right)^{1/8} \cdot 1/8.12053 * 535.49165^{(-1/6)} \text{ product } \left(\left(1-535.49165^{(n-0.5)}\right)\right)^8, n=1 \text{ to } 0.0864055)^{1/5}$$

where 55 and 8 are Fibonacci numbers

**Input interpretation:**

$$55 + 8 + 10^3 + \sqrt[5]{-\frac{1}{1 + \frac{535.49165}{\sqrt{535.49165^2}}} \left(\frac{1}{8.12053} \times 535.49165^{-1/6}\right)^{0.0864055} \prod_{n=1} (1 - 535.49165^{n-0.5})^8}$$

**Result:**

1783.84

1783.84 result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

$$\left(\left(\left(\left(\frac{1}{1-535.49165*(535.49165^{(1/2)})^{-2}}\right)\right)\right)\right)^{1/8} * 1/8.12053 * 535.49165^{(-1/6)} \text{ product } \left(\left(1-535.49165^{(n-0.5)}\right)\right)^8, n=1 \text{ to } 0.0864055)^{1/4} - 123 + 11 - \text{golden ratio}$$

where 123 and 11 are Lucas numbers

**Input interpretation:**

$$\sqrt[4]{-\frac{1}{1 + \frac{535.49165}{\sqrt{535.49165^2}}} \left(\frac{1}{8.12053} \times 535.49165^{-1/6}\right)^{0.0864055} \prod_{n=1} (1 - 535.49165^{n-0.5})^8} - 123 + 11 - \phi$$

φ is the golden ratio

**Result:**

3621.43

3621.43 result practically equal to the rest mass of double charmed Xi baryon 3621.40

$1/2[(((1/(-535.49165*(535.49165^{(1/2))^{-2}}))) * 1/8.12053 * 535.49165^{(-1/6)}$   
 $\text{product } ((1-535.49165^{(n-0.5)))^8, n=1 \text{ to } 0.0864055))]^{1/4} + \text{golden ratio}$

**Input interpretation:**

$$\frac{1}{2} \sqrt[4]{-\frac{1}{1 + \frac{535.49165}{\sqrt{535.49165}^2}} \left( \frac{1}{8.12053} \times 535.49165^{-1/6} \right)^{0.0864055} \prod_{n=1}^{0.0864055} (1 - 535.49165^{n-0.5})^8} + \phi$$

$\phi$  is the golden ratio

**Result:**

1869.14

1869.14 result practically equal to the rest mass of D meson 1869.62

And:

$$G(z, \tau) \stackrel{z \rightarrow (1+\tau)/2}{\sim} \frac{1}{1 - qw^{-2}} \cdot \frac{q^{-1/6} \prod_{n=1}^{\infty} (1 + q^{n-1/2})^8}{\eta(\tau)^8}, \tag{3.34}$$

$(((1/(-535.49165*(-535.49165^{(1/2))^{-2}}))) * 1/8.12053 * 535.49165^{(-1/6)}$   
 $\text{product } ((1+535.49165^{(n-0.5)))^8, n=1 \text{ to } 0.0864055$

**Input interpretation:**

$$\frac{1}{1 + \frac{535.49165}{(-\sqrt{535.49165})^2}} \times \frac{1}{8.12053} \times 535.49165^{-1/6} \prod_{n=1}^{0.0864055} (1 + 535.49165^{n-0.5})^8$$

**Result:**

$-1.94618 \times 10^{14}$   
 $-1.94618 * 10^{14}$

We have that:

$$G(z, \tau) \stackrel{z \rightarrow 1/2}{\sim} \frac{1}{w - w^{-1}} \cdot \frac{16q^{1/3} \prod_{n=1}^{\infty} (1 + q^n)^8}{\eta^8(\tau)}, \tag{3.36}$$

$-1/(\sqrt{535.49165} - 1/(535.49165^{1/2})) * 1/8.12053 * (((16 * 535.49165^{1/3})$   
 $\text{product} ((1+535.49165^n)^8, n=1 \text{ to } 0.0864055))$

**Input interpretation:**

$$\frac{\frac{1}{8.12053} \left( 16 \sqrt[3]{535.49165} \prod_{n=1}^{0.0864055} (1 + 535.49165^n)^8 \right)}{\sqrt{535.49165} - \frac{1}{\sqrt{535.49165}}}$$

**Result:**

-0.692716  
 -0.692716

Multiplying the two results, we obtain:

$-0.692716 * ((((((1/(1-535.49165 * (-535.49165^{1/2}))^{-2})))) * 1/8.12053 * 535.49165^{-1/6}$   
 $\text{product} ((1+535.49165^{(n-0.5)}))^8, n=1 \text{ to } 0.0864055))))$

**Input interpretation:**

$$-0.692716 \left( \frac{1}{1 + \frac{535.49165}{(-\sqrt{535.49165})^2}} \times \frac{1}{8.12053} \times 535.49165^{-1/6} \prod_{n=1}^{0.0864055} (1 + 535.49165^{n-0.5})^8 \right)$$

**Result:**

$1.34815 \times 10^{14}$   
 $1.34815 * 10^{14}$

From which:

$4 \ln[-0.692716 * ((((((1/(1-535.49165 * (-535.49165^{1/2}))^{-2})))) * 1/8.12053 * 535.49165^{-1/6}$   
 $\text{product} ((1+535.49165^{(n-0.5)}))^8, n=1 \text{ to } 0.0864055)))))] - 5$

where 5 is a Fibonacci number

**Input interpretation:**

$$4 \log \left( -0.692716 \left( \frac{1}{1 + \frac{535.49165}{(-\sqrt{535.49165})^2}} \times \frac{1}{8.12053} \times 535.49165^{-1/6} \prod_{n=1}^{0.0864055} (1 + 535.49165^{n-0.5})^8 \right) - 5 \right)$$

log(x) is the natural logarithm

**Result:**

125.14

125.14 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

$4 \ln[-0.692716 * ((((((1/(1-535.49165 * (-535.49165^{(1/2)})^{-2})))) * 1/8.12053 * 535.49165^{-1/6} \text{ product } ((1+535.49165^{(n-0.5)})^8, n=1 \text{ to } 0.0864055)))))] + 11 - 2 + 1/\text{golden ratio}$

where 11 and 2 are Lucas numbers

**Input interpretation:**

$$4 \log \left( -0.692716 \left( \frac{1}{1 + \frac{535.49165}{(-\sqrt{535.49165})^2}} \times \frac{1}{8.12053} \times 535.49165^{-1/6} \prod_{n=1}^{0.0864055} (1 + 535.49165^{n-0.5})^8 \right) + 11 - 2 + \frac{1}{\phi} \right)$$

log(x) is the natural logarithm

φ is the golden ratio

**Result:**

139.758

139.758 result practically equal to the rest mass of Pion meson 139.57 MeV

We have also:

$$27 \times 2 \ln \left[ -0.692716 \times \left( \frac{1}{1 - 535.49165 \times (-535.49165)^{1/2}} \right)^{-2} \right] \times \frac{1}{8.12053} \times 535.49165^{-1/6} \prod_{n=1}^{0.0864055} \left( (1 + 535.49165^{n-0.5})^8 \right) - 29 + \frac{1}{\phi}$$

where 29 is a Lucas number

From Wikipedia:

*“The fundamental group of the complex form, compact real form, or any algebraic version of  $E_6$  is the cyclic group  $\mathbf{Z}/3\mathbf{Z}$ , and its outer automorphism group is the cyclic group  $\mathbf{Z}/2\mathbf{Z}$ . Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics,  $E_6$  plays a role in some grand unified theories”.*

**Input interpretation:**

$$27 \times 2 \log \left( -0.692716 \left( \frac{1}{1 + \frac{535.49165}{(-\sqrt{535.49165})^2}} \times \frac{1}{8.12053} \times 535.49165^{-1/6} \prod_{n=1}^{0.0864055} (1 + 535.49165^{n-0.5})^8 \right) - 29 + \frac{1}{\phi} \right)$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Result:**

1728.5

1728.5

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729



$((535.49165^{1/24} \prod_{n=1}^{0.0864055} (1 - 535.49165^n))^{12})^6$

**Input interpretation:**

$$\left( \sqrt[24]{535.49165} \prod_{n=1}^{0.0864055} (1 - 535.49165^n) \right)^6$$

**Result:**

4.81048

$$4.81048 = \eta^6(\tau)$$

$1 / (((535.49165^{1/12} \prod_{n=1}^{0.0864055} (1 - 535.49165^n \exp((2 * 4 \pi * i) / 5))) (1 - 535.49165^n \exp((-2 * 4 \pi * i) / 5)))) 4.81048$ ,  $n=1$  to  $0.0864055$ ))

**Input interpretation:**

$$\frac{1}{\left( \sqrt[12]{535.49165} \prod_{n=1}^{0.0864055} \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (2 \times 4 (\pi i))\right) \right) \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (-2 \times 4 (\pi i))\right) \right) \times 4.81048 \right)}$$

$i$  is the imaginary unit

**Result:**

0.592385

**0.592385**

$[1 / (((535.49165^{1/12} \prod_{n=1}^{0.0864055} (1 - 535.49165^n \exp((2 * 4 \pi * i) / 5))) (1 - 535.49165^n \exp((-2 * 4 \pi * i) / 5)))) 4.81048$ ,  $n=1$  to  $0.0864055$ )))]^{1/1024}

**Input interpretation:**

$$\left( \frac{1}{\left( \sqrt[12]{535.49165} \prod_{n=1}^{0.0864055} \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (2 \times 4 (\pi i))\right) \right) \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (-2 \times 4 (\pi i))\right) \right) \times 4.81048 \right)} \right)^{1/1024}$$

$i$  is the imaginary unit

**Result:**

0.999489

0.999489 result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\phi^5 4\sqrt{5^3} - 1}}}{\sqrt{5}} - \phi + 1$$

and to the dilaton value **0.989117352243 =  $\phi$**

1/8 log base 0.999489 [1/((((535.49165^1/12 product ((1-535.49165^n exp((2\*4Pi\*i)/5))) ((1-535.49165^n exp((-2\*4Pi\*i)/5))) 4.81048 , n=1 to 0.00864055)))]-e

**Input interpretation:**

$$\frac{1}{8} \log_{0.999489} \left( 1 / \left( \sqrt[12]{535.49165} \prod_{n=1}^{0.00864055} \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (2 \times 4 (\pi i))\right) \right) \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (-2 \times 4 (\pi i))\right) \right) \times 4.81048 \right) \right) - e$$

log<sub>b</sub>(x) is the base- b logarithm

i is the imaginary unit

**Result:**

125.331

125.331 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

$\frac{1}{8} \log_{0.999489} \left[ \frac{1}{\left( \left( \left( \left( 535.49165^{1/12} \prod_{n=1}^{\infty} \left( (1 - 535.49165^n \exp\left(\frac{2 \times 4 \pi i}{5}\right)) \right) \right) \right) \left( (1 - 535.49165^n \exp\left(\frac{-2 \times 4 \pi i}{5}\right)) \right) \right) \right) \right) \right] + 11 + \frac{1}{\text{golden ratio}}$

where 11 is a Lucas number

**Input interpretation:**

$$\frac{1}{8} \log_{0.999489} \left( \frac{1}{\left( \left( \left( \sqrt[12]{535.49165} \prod_{n=1}^{0.00864055} \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (2 \times 4 (\pi i))\right) \right) \right) \right) \left( 1 - 535.49165^n \exp\left(\frac{1}{5} (-2 \times 4 (\pi i))\right) \right) \right) \right) \times 4.81048 \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$i$  is the imaginary unit

$\phi$  is the golden ratio

**Result:**

139.667

139.667 result practically equal to the rest mass of Pion meson 139.57 MeV

Now, we have that:

$$Z_{k,N}^B = \frac{V}{(8\pi^2 \alpha' T)^{(p-1)/2}} \frac{1}{4 \sin^2(2\pi k/N) q^{1/12} \prod_{n=1}^{\infty} (1 - q^n \exp(4\pi i k/N))(1 - q^n \exp(-4\pi i k/N))} \frac{1}{\eta^6(\tau)} \quad (3.22)$$

$$\frac{1}{q^{1/12} \prod_{n=1}^{\infty} ((1 - q^n \exp(4\pi i k/N))(1 - q^n \exp(-4\pi i k/N)) \eta^6(\tau))} \quad (3.14)$$

= 0.592385

$$1/(4 \sin^2(4\pi/5))$$

**Input:**

$$\frac{1}{4 \sin^2\left(4 \times \frac{\pi}{5}\right)}$$

**Exact result:**

$$\frac{1}{4\left(\frac{5}{8} - \frac{\sqrt{5}}{8}\right)}$$

**Decimal approximation:**

0.723606797749978969640917366873127623544061835961152572427...

0.723606797....

**Alternate forms:**

$$\frac{1}{10} (5 + \sqrt{5})$$

$$-\frac{2}{\sqrt{5} - 5}$$

$$\frac{\sqrt{5}}{10} + \frac{1}{2}$$

**Minimal polynomial:**

$$5x^2 - 5x + 1$$

**Alternative representations:**

$$\frac{1}{4 \sin^2\left(\frac{4\pi}{5}\right)} = \frac{1}{4 \left(\frac{1}{\csc\left(\frac{4\pi}{5}\right)}\right)^2}$$

$$\frac{1}{4 \sin^2\left(\frac{4\pi}{5}\right)} = \frac{1}{4 \cos^2\left(\frac{\pi}{2} - \frac{4\pi}{5}\right)}$$

$$\frac{1}{4 \sin^2\left(\frac{4\pi}{5}\right)} = \frac{1}{4 \left(-\cos\left(\frac{\pi}{2} + \frac{4\pi}{5}\right)\right)^2}$$

**Series representations:**

$$\frac{1}{4 \sin^2\left(\frac{4\pi}{5}\right)} = \frac{1}{4 \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{9}{100}\right)^k \pi^{2k}}{(2k)!}\right)^2}$$

$$\frac{1}{4 \sin^2\left(\frac{4\pi}{5}\right)} = \frac{1}{16 \left(\sum_{k=0}^{\infty} (-1)^k J_{1+2k}\left(\frac{4\pi}{5}\right)\right)^2}$$

$$\frac{1}{4 \sin^2\left(\frac{4\pi}{5}\right)} \propto \frac{\theta\left(\frac{4\pi}{5}\right)^2}{4 \left(\sum_{k=0}^{\infty} (-1)^k \frac{\partial^{2k}}{\partial\left(\frac{4\pi}{5}\right)^{2k}} \delta\left(\frac{4\pi}{5}\right)\right)^2}$$

$$1/(8\pi^2 * 0.9568666373 * 0.0864055)^3$$

**Input interpretation:**

$$\frac{1}{(8 \pi^2 \times 0.9568666373 \times 0.0864055)^3}$$

**Result:**

0.00359462...

0.00359462...

**Alternative representations:**

$$\frac{1}{(8 \pi^2 \times 0.956867 \times 0.0864055)^3} = \frac{1}{(0.661428 (180^\circ)^2)^3}$$

$$\frac{1}{(8 \pi^2 \times 0.956867 \times 0.0864055)^3} = \frac{1}{(3.96857 \zeta(2))^3}$$

$$\frac{1}{(8 \pi^2 \times 0.956867 \times 0.0864055)^3} = \frac{1}{(0.661428 \cos^{-1}(-1)^2)^3}$$

**Series representations:**

$$\frac{1}{(8 \pi^2 \times 0.956867 \times 0.0864055)^3} = \frac{0.000843707}{\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^6}$$

$$\frac{1}{(8 \pi^2 \times 0.956867 \times 0.0864055)^3} = \frac{0.0539973}{\left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}\right)^6}$$

$$\frac{1}{(8 \pi^2 0.956867 \times 0.0864055)^3} = \frac{3.45582}{\left( \sum_{k=0}^{\infty} \frac{2^{-k} (-6+50k)}{\binom{3k}{k}} \right)^6}$$

**Integral representations:**

$$\frac{1}{(8 \pi^2 0.956867 \times 0.0864055)^3} = \frac{0.0539973}{\left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^6}$$

$$\frac{1}{(8 \pi^2 0.956867 \times 0.0864055)^3} = \frac{0.000843707}{\left( \int_0^1 \sqrt{1-t^2} dt \right)^6}$$

$$\frac{1}{(8 \pi^2 0.956867 \times 0.0864055)^3} = \frac{0.0539973}{\left( \int_0^{\infty} \frac{\sin(t)}{t} dt \right)^6}$$

$$(0.592385 * 0.723606797 * 0.00359462)$$

**Input interpretation:**

$$0.592385 \times 0.723606797 \times 0.00359462$$

**Result:**

$$0.0015408475672761102539$$

**Repeating decimal:**

$$0.0015408475672761102539000$$

$$0.0015408475672....$$

$$\text{golden ratio}/(0.592385 * 0.723606797 * 0.00359462) + 64 + \text{golden ratio}$$

**Input interpretation:**

$$\frac{\phi}{0.592385 \times 0.723606797 \times 0.00359462} + 64 + \phi$$

$\phi$  is the golden ratio

**Result:**

1115.71...

1115.71... result practically equal to the rest mass of Lambda baryon 1115.683

**Alternative representations:**

$$\frac{\phi}{0.592385 \times 0.723607 \times 0.00359462} + 64 + \phi = 64 + 2 \sin(54^\circ) + \frac{2 \sin(54^\circ)}{0.00154085}$$

$$\frac{\phi}{0.592385 \times 0.723607 \times 0.00359462} + 64 + \phi = 64 - 2 \cos(216^\circ) - \frac{2 \cos(216^\circ)}{0.00154085}$$

$$\frac{\phi}{0.592385 \times 0.723607 \times 0.00359462} + 64 + \phi = 64 - 2 \sin(666^\circ) - \frac{2 \sin(666^\circ)}{0.00154085}$$

$$e/(0.592385 * 0.723606797 * 0.00359462) - 47 + 11$$

where 47 and 11 are Lucas numbers

**Input interpretation:**

$$\frac{e}{0.592385 \times 0.723606797 \times 0.00359462} - 47 + 11$$

**Result:**

1728.15...

1728.15...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternative representation:**

$$\frac{e}{0.592385 \times 0.723607 \times 0.00359462} - 47 + 11 =$$

$$\frac{\exp(z)}{0.592385 \times 0.723607 \times 0.00359462} - 47 + 11 \text{ for } z = 1$$

**Series representations:**

$$\frac{e}{0.592385 \times 0.723607 \times 0.00359462} - 47 + 11 = -36 + 648.993 \sum_{k=0}^{\infty} \frac{1}{k!}$$

$$\frac{e}{0.592385 \times 0.723607 \times 0.00359462} - 47 + 11 = -36 + 324.497 \sum_{k=0}^{\infty} \frac{1+k}{k!}$$

$$\frac{e}{0.592385 \times 0.723607 \times 0.00359462} - 47 + 11 = -36 + \frac{648.993 \sum_{k=0}^{\infty} \frac{-1+k+z}{k!}}{z}$$

Pi/(0.592385 \* 0.723606797 \* 0.00359462)-256-55+1/golden ratio

where 55 is a Fibonacci number

**Input interpretation:**

$$\frac{\pi}{0.592385 \times 0.723606797 \times 0.00359462} - 256 - 55 + \frac{1}{\phi}$$

ϕ is the golden ratio

**Result:**

1728.49...

1728.49...

This result is very near to the mass of candidate glueball f<sub>0</sub>(1710) meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternative representations:**

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00359462} - 256 - 55 + \frac{1}{\phi} = -311 + \frac{\pi}{0.00154085} + \frac{1}{2 \cos(216^\circ)}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00359462} - 256 - 55 + \frac{1}{\phi} = -311 + \frac{180^\circ}{0.00154085} + \frac{1}{2 \cos(216^\circ)}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00359462} - 256 - 55 + \frac{1}{\phi} = -311 + \frac{\pi}{0.00154085} + \frac{1}{2 \cos\left(\frac{\pi}{5}\right)}$$

### Series representations:

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00359462} - 256 - 55 + \frac{1}{\phi} = -311 + \frac{1}{\phi} + 2595.97 \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00359462} - 256 - 55 + \frac{1}{\phi} = -1608.99 + \frac{1}{\phi} + 1297.99 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00359462} - 256 - 55 + \frac{1}{\phi} = -311 + \frac{1}{\phi} + 648.993 \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}}$$

### Integral representations:

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00359462} - 256 - 55 + \frac{1}{\phi} = -311 + \frac{1}{\phi} + 1297.99 \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00359462} - 256 - 55 + \frac{1}{\phi} = -311 + \frac{1}{\phi} + 2595.97 \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00359462} - 256 - 55 + \frac{1}{\phi} = -311 + \frac{1}{\phi} + 1297.99 \int_0^{\infty} \frac{\sin(t)}{t} dt$$

$1/5 * 1/(0.592385 * 0.723606797 * 0.00359462) + 11$  - golden ratio

where 11 is a Lucas number

**Input interpretation:**

$$\frac{1}{5} \times \frac{1}{0.592385 \times 0.723606797 \times 0.00359462} + 11 - \phi$$

$\phi$  is the golden ratio

**Result:**

139.181...

139.181... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$\frac{1}{(0.592385 \times 0.723607 \times 0.00359462) 5} + 11 - \phi = 11 + \frac{1}{0.00154085 \times 5} - 2 \sin(54^\circ)$$

$$\frac{1}{(0.592385 \times 0.723607 \times 0.00359462) 5} + 11 - \phi = 11 - 2 \cos\left(\frac{\pi}{5}\right) + \frac{1}{0.00154085 \times 5}$$

$$\frac{1}{(0.592385 \times 0.723607 \times 0.00359462) 5} + 11 - \phi = 11 + 2 \cos(216^\circ) + \frac{1}{0.00154085 \times 5}$$

$1/5 * 1/(0.592385 * 0.723606797 * 0.00359462) - \pi$  - golden ratio

**Input interpretation:**

$$\frac{1}{5} \times \frac{1}{0.592385 \times 0.723606797 \times 0.00359462} - \pi - \phi$$

$\phi$  is the golden ratio

**Result:**

125.039...

125.039... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$\frac{1}{(0.592385 \times 0.723607 \times 0.00359462) 5} - \pi - \phi = -\pi - 2 \cos\left(\frac{\pi}{5}\right) + \frac{1}{0.00154085 \times 5}$$

$$\frac{1}{(0.592385 \times 0.723607 \times 0.00359462) 5} - \pi - \phi = -\pi + 2 \cos(216^\circ) + \frac{1}{0.00154085 \times 5}$$

$$\frac{1}{(0.592385 \times 0.723607 \times 0.00359462) 5} - \pi - \phi = -180^\circ - 2 \cos\left(\frac{\pi}{5}\right) + \frac{1}{0.00154085 \times 5}$$

**Series representations:**

$$\frac{1}{(0.592385 \times 0.723607 \times 0.00359462) 5} - \pi - \phi = 129.799 - \phi - 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}$$

$$\frac{1}{(0.592385 \times 0.723607 \times 0.00359462) 5} - \pi - \phi = 131.799 - \phi - 2 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$\frac{1}{(0.592385 \times 0.723607 \times 0.00359462) 5} - \pi - \phi = 129.799 - \phi - \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}}$$

**Integral representations:**

$$\frac{1}{(0.592385 \times 0.723607 \times 0.00359462) 5} - \pi - \phi = 129.799 - \phi - 2 \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$\frac{1}{(0.592385 \times 0.723607 \times 0.00359462) 5} - \pi - \phi = 129.799 - \phi - 4 \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{1}{(0.592385 \times 0.723607 \times 0.00359462) 5} - \pi - \phi = 129.799 - \phi - 2 \int_0^{\infty} \frac{\sin(t)}{t} dt$$

1/4 \* 1/(0.592385 \* 0.723606797 \* 0.00359462)-29+golden ratio

where 29 is a Lucas number

**Input interpretation:**

$$\frac{1}{4} \times \frac{1}{0.592385 \times 0.723606797 \times 0.00359462} - 29 + \phi$$

**Result:**

134.866...

134.866... result practically equal to the rest mass of Pion meson 134.9766

**Alternative representations:**

$$\frac{1}{(0.592385 \times 0.723607 \times 0.00359462) 4} - 29 + \phi = -29 + \frac{1}{0.00154085 \times 4} + 2 \sin(54^\circ)$$

$$\frac{1}{(0.592385 \times 0.723607 \times 0.00359462) 4} - 29 + \phi = -29 - 2 \cos(216^\circ) + \frac{1}{0.00154085 \times 4}$$

$$\frac{1}{(0.592385 \times 0.723607 \times 0.00359462) 4} - 29 + \phi = -29 + \frac{1}{0.00154085 \times 4} - 2 \sin(666^\circ)$$

$$3/2 * 1 / (0.592385 * 0.723606797 * 0.00359462) - 34$$

where 34 is a Fibonacci number

**Input interpretation:**

$$\frac{3}{2} \times \frac{1}{0.592385 \times 0.723606797 \times 0.00359462} - 34$$

**Result:**

939.4901958223420908074370253488557488974940913785005582941...

939.49019... result practically equal to the neutron mass in MeV

Now, from the previous equation

$$Z_{k,N}^B = \frac{V}{(8\pi^2 \alpha' T)^{(p-1)/2} 4 \sin^2(2\pi k/N) q^{1/12} \prod_{n=1}^{\infty} (1 - q^n \exp(4\pi i k/N) (1 - q^n \exp(-4\pi i k/N)) \eta^6(\tau)} \frac{1}{\eta^6(\tau)}. \quad (3.22)$$

we have also, for  $V = 1.9559391549$

$$1.9559391549/(8\pi^2 \times 0.9568666373 \times 0.0864055)^3$$

**Input interpretation:**

$$\frac{1.9559391549}{(8\pi^2 \times 0.9568666373 \times 0.0864055)^3}$$

**Result:**

0.00703085...

0.00703085...

**Alternative representations:**

$$\frac{1.95593915490000}{(8\pi^2 \times 0.956867 \times 0.0864055)^3} = \frac{1.95593915490000}{(0.661428 (180^\circ)^2)^3}$$

$$\frac{1.95593915490000}{(8\pi^2 \times 0.956867 \times 0.0864055)^3} = \frac{1.95593915490000}{(3.96857 \zeta(2))^3}$$

$$\frac{1.95593915490000}{(8\pi^2 \times 0.956867 \times 0.0864055)^3} = \frac{1.95593915490000}{(0.661428 \cos^{-1}(-1)^2)^3}$$

**Series representations:**

$$\frac{1.95593915490000}{(8\pi^2 \times 0.956867 \times 0.0864055)^3} = \frac{0.00165024}{\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^6}$$

$$\frac{1.95593915490000}{(8\pi^2 \times 0.956867 \times 0.0864055)^3} = \frac{0.105615}{\left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}\right)^6}$$

$$\frac{1.95593915490000}{(8\pi^2 \times 0.956867 \times 0.0864055)^3} = \frac{6.75938}{\left(\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}\right)^6}$$

**Integral representations:**

$$\frac{1.95593915490000}{(8 \pi^2 0.956867 \times 0.0864055)^3} = \frac{0.105615}{\left(\int_0^\infty \frac{1}{1+t^2} dt\right)^6}$$

$$\frac{1.95593915490000}{(8 \pi^2 0.956867 \times 0.0864055)^3} = \frac{0.00165024}{\left(\int_0^1 \sqrt{1-t^2} dt\right)^6}$$

$$\frac{1.95593915490000}{(8 \pi^2 0.956867 \times 0.0864055)^3} = \frac{0.105615}{\left(\int_0^\infty \frac{\sin(t)}{t} dt\right)^6}$$

$$1 / \left( \left( \left( \frac{1.9559391549}{(8\pi^2 \times 0.9568666373 \times 0.0864055)^3} \right) \right)^3 \right) - 3$$

where 3 is a Fibonacci number

**Input interpretation:**

$$\frac{\frac{1}{\frac{1.9559391549}{(8 \pi^2 \times 0.9568666373 \times 0.0864055)^3}}}{-3}$$

**Result:**

139.230...

139.230... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$\frac{1}{\frac{1.95593915490000}{(8 \pi^2 0.956867 \times 0.0864055)^3}} - 3 = -3 + \frac{1}{\frac{1.95593915490000}{(0.661428 (180^\circ)^2)^3}}$$

$$\frac{1}{\frac{1.95593915490000}{(8 \pi^2 0.956867 \times 0.0864055)^3}} - 3 = -3 + \frac{1}{\frac{1.95593915490000}{(3.96857 \zeta(2))^3}}$$

$$\frac{1}{\frac{1.95593915490000}{(8 \pi^2 0.956867 \times 0.0864055)^3}} - 3 = -3 + \frac{1}{\frac{1.95593915490000}{(0.661428 \cos^{-1}(-1)^2)^3}}$$

**Series representations:**

$$\frac{1}{\frac{1.95593915490000}{(8\pi^2 \cdot 0.956867 \times 0.0864055)^3}} - 3 = -3 + 605.973 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^6$$

$$\frac{1}{\frac{1.95593915490000}{(8\pi^2 \cdot 0.956867 \times 0.0864055)^3}} - 3 = -3 + 9.46832 \left( -1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}} \right)^6$$

$$\frac{1}{\frac{1.95593915490000}{(8\pi^2 \cdot 0.956867 \times 0.0864055)^3}} - 3 = -3 + 0.147943 \left( \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}} \right)^6$$

**Integral representations:**

$$\frac{1}{\frac{1.95593915490000}{(8\pi^2 \cdot 0.956867 \times 0.0864055)^3}} - 3 = -3 + 9.46832 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^6$$

$$\frac{1}{\frac{1.95593915490000}{(8\pi^2 \cdot 0.956867 \times 0.0864055)^3}} - 3 = -3 + 605.973 \left( \int_0^1 \sqrt{1-t^2} dt \right)^6$$

$$\frac{1}{\frac{1.95593915490000}{(8\pi^2 \cdot 0.956867 \times 0.0864055)^3}} - 3 = -3 + 9.46832 \left( \int_0^{\infty} \frac{\sin(t)}{t} dt \right)^6$$

Thence, we obtain:

$$(0.592385 * 0.723606797 * 0.00703085)$$

**Input interpretation:**

$$0.592385 \times 0.723606797 \times 0.00703085$$

**Result:**

$$0.00301380065719971506825$$

$$0.00301380065719971506825$$

From which:

$$1/(0.592385 * 0.723606797 * 0.00703085)$$

**Input interpretation:**

$$\frac{1}{0.592385 \times 0.723606797 \times 0.00703085}$$

**Result:**

331.8069486815873212540190048201382812511441443306746388516...

331.80694868...

From the formula of coefficients of the '5th order' mock theta function  $\psi_1(q)$ :  
(A053261 OEIS Sequence)

$$\sqrt{\phi} * \exp(\pi * \sqrt{n/15}) / (2 * 5^{1/4} * \sqrt{n})$$

for  $n = 125$ , we obtain:

$$\sqrt{\phi} * \exp(\pi * \sqrt{125/15}) / (2 * 5^{1/4} * \sqrt{125}) + \phi$$

**Input:**

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{125}{15}}\right)}{2 \sqrt[4]{5} \sqrt{125}} + \phi$$

$\phi$  is the golden ratio

**Exact result:**

$$\frac{e^{(5\pi)/\sqrt{3}} \sqrt{\phi}}{10 \times 5^{3/4}} + \phi$$

**Decimal approximation:**

331.8975144032454894461212136088952958184224685185000611495...

331.8975144...

**Property:**

$$\frac{e^{(5\pi)/\sqrt{3}} \sqrt{\phi}}{10 \times 5^{3/4}} + \phi \text{ is a transcendental number}$$

**Alternate forms:**

$$\frac{\left(10 \times 5^{3/4} \sqrt{\phi} + e^{(5\pi)/\sqrt{3}}\right) \sqrt{\phi}}{10 \times 5^{3/4}}$$

$$\frac{1}{2} (1 + \sqrt{5}) + \frac{1}{50} \sqrt{\frac{1}{2} (5 + \sqrt{5})} e^{(5\pi)/\sqrt{3}}$$

$$\frac{1}{100} \left(50 + 50 \sqrt{5} + \sqrt[4]{5} \sqrt{2(1 + \sqrt{5})} e^{(5\pi)/\sqrt{3}}\right)$$

**Series representations:**

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{125}{15}}\right)}{2 \sqrt[4]{5} \sqrt{125}} + \phi = \left(10 \phi \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (125 - z_0)^k z_0^{-k}}{k!} + \right. \\ \left. 5^{3/4} \exp\left[\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{25}{3} - z_0\right)^k z_0^{-k}}{k!}\right] \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!}\right) / \\ \left(10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (125 - z_0)^k z_0^{-k}}{k!}\right) \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{125}{15}}\right)}{2 \sqrt[4]{5} \sqrt{125}} + \phi = \left(10 \phi \exp\left(i \pi \left[\frac{\arg(125 - x)}{2 \pi}\right]\right) \sum_{k=0}^{\infty} \frac{(-1)^k (125 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \\ \left. 5^{3/4} \exp\left(i \pi \left[\frac{\arg(\phi - x)}{2 \pi}\right]\right) \exp\left[\pi \exp\left(i \pi \left[\frac{\arg\left(\frac{25}{3} - x\right)}{2 \pi}\right]\right) \sqrt{x}\right] \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{25}{3} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right) / \\ \left(10 \exp\left(i \pi \left[\frac{\arg(125 - x)}{2 \pi}\right]\right) \sum_{k=0}^{\infty} \frac{(-1)^k (125 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{125}{15}}\right)}{2 \sqrt[4]{5} \sqrt{125}} + \phi =$$

$$\left( \left(\frac{1}{z_0}\right)^{-1/2 [\arg(125-z_0)/(2\pi)]} z_0^{-1/2 [\arg(125-z_0)/(2\pi)]} \left( 10 \phi \left(\frac{1}{z_0}\right)^{1/2 [\arg(125-z_0)/(2\pi)]} \right. \right.$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (125-z_0)^k z_0^{-k}}{k!} + \right.$$

$$\left. 5^{3/4} \exp\left(\pi \left(\frac{1}{z_0}\right)^{1/2 [\arg(\frac{25}{3}-z_0)/(2\pi)]} z_0^{1/2 (1+[\arg(\frac{25}{3}-z_0)/(2\pi)])} \right. \right.$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{25}{3}-z_0\right)^k z_0^{-k}}{k!} \right) \left(\frac{1}{z_0}\right)^{1/2 [\arg(\phi-z_0)/(2\pi)]} z_0^{1/2 [\arg(\phi-z_0)/(2\pi)]}$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi-z_0)^k z_0^{-k}}{k!} \right) \left/ \left( 10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (125-z_0)^k z_0^{-k}}{k!} \right) \right.$$

$n!$  is the factorial function

$(\alpha)_n$  is the Pochhammer symbol (rising factorial)

$\mathbb{R}$  is the set of real numbers

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

golden ratio/ (0.592385 \* 0.723606797 \* 0.00703085) + 11

where 11 is a Lucas number

**Input interpretation:**

$$\frac{\phi}{0.592385 \times 0.723606797 \times 0.00703085} + 11$$

$\phi$  is the golden ratio

**Result:**

547.875...

547.875... result practically equal to the rest mass of Eta meson 547.853

**Alternative representations:**

$$\frac{\phi}{0.592385 \times 0.723607 \times 0.00703085} + 11 = 11 + \frac{2 \sin(54^\circ)}{0.0030138}$$

$$\frac{\phi}{0.592385 \times 0.723607 \times 0.00703085} + 11 = 11 - \frac{2 \cos(216^\circ)}{0.0030138}$$

$$\frac{\phi}{0.592385 \times 0.723607 \times 0.00703085} + 11 = 11 - \frac{2 \sin(666^\circ)}{0.0030138}$$

Pi/ (0.592385 \* 0.723606797 \* 0.00703085) - 21 - golden ratio

where 21 is a Fibonacci number

**Input interpretation:**

$$\frac{\pi}{0.592385 \times 0.723606797 \times 0.00703085} - 21 - \phi$$

$\phi$  is the golden ratio

**Result:**

1019.78...

1019.78... result practically equal to the rest mass of Phi meson 1019.445

**Alternative representations:**

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} - 21 - \phi = -21 + 2 \cos(216^\circ) + \frac{\pi}{0.0030138}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} - 21 - \phi = -21 - 2 \cos\left(\frac{\pi}{5}\right) + \frac{\pi}{0.0030138}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} - 21 - \phi = -21 + 2 \cos(216^\circ) + \frac{180^\circ}{0.0030138}$$

**Series representations:**

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} - 21 - \phi = -21 - \phi + 1327.23 \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2^k}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} - 21 - \phi = -684.614 - \phi + 663.614 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} - 21 - \phi = -21 - \phi + 331.807 \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}}$$

**Integral representations:**

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} - 21 - \phi = -21 - \phi + 663.614 \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} - 21 - \phi = -21 - \phi + 1327.23 \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} - 21 - \phi = -21 - \phi + 663.614 \int_0^{\infty} \frac{\sin(t)}{t} dt$$

$$5 / (0.592385 * 0.723606797 * 0.00703085) + 76 - 7$$

where 76 and 7 are Lucas numbers

**Input interpretation:**

$$\frac{5}{0.592385 \times 0.723606797 \times 0.00703085} + 76 - 7$$

**Result:**

1728.034743407936606270095024100691406255720721653373194258...

1728.0347434....

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic

curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$4 / (0.592385 * 0.723606797 * 0.00703085) + 55$$

where 55 is a Fibonacci number

**Input interpretation:**

$$\frac{4}{0.592385 \times 0.723606797 \times 0.00703085} + 55$$

**Result:**

1382.227794726349285016076019280553125004576577322698555406...

1382.227794... result practically equal to the rest mass of Sigma baryon 1382.8

$$\pi / (0.592385 * 0.723606797 * 0.00703085) + 199 - 11 + \text{golden ratio}$$

where 199 and 11 are Lucas numbers

**Input interpretation:**

$$\frac{\pi}{0.592385 \times 0.723606797 \times 0.00703085} + 199 - 11 + \phi$$

$\phi$  is the golden ratio

**Result:**

1232.02...

1232.02... result practically equal to the rest mass of Delta baryon 1232

**Alternative representations:**

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 199 - 11 + \phi = 188 - 2 \cos(216^\circ) + \frac{\pi}{0.0030138}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 199 - 11 + \phi = 188 + 2 \cos\left(\frac{\pi}{5}\right) + \frac{\pi}{0.0030138}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 199 - 11 + \phi = 188 - 2 \cos(216^\circ) + \frac{180^\circ}{0.0030138}$$

### Series representations:

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 199 - 11 + \phi = 188 + \phi + 1327.23 \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 199 - 11 + \phi = -475.614 + \phi + 663.614 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 199 - 11 + \phi = 188 + \phi + 331.807 \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}}$$

### Integral representations:

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 199 - 11 + \phi = 188 + \phi + 663.614 \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 199 - 11 + \phi = 188 + \phi + 1327.23 \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 199 - 11 + \phi = 188 + \phi + 663.614 \int_0^{\infty} \frac{\sin(t)}{t} dt$$

(((Pi / (0.592385 \* 0.723606797 \* 0.00703085) + 123)))

where 123 is a Lucas number

### Input interpretation:

$$\frac{\pi}{0.592385 \times 0.723606797 \times 0.00703085} + 123$$

**Result:**

1165.40...

1165.40... result very near to the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$

**Alternative representations:**

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 123 = 123 + \frac{180^\circ}{0.0030138}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 123 = 123 - \frac{i \log(-1)}{0.0030138}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 123 = 123 + \frac{\cos^{-1}(-1)}{0.0030138}$$

**Series representations:**

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 123 = 123 + 1327.23 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 123 = -540.614 + 663.614 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 123 = 123 + 331.807 \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}}$$

**Integral representations:**

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 123 = 123 + 663.614 \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 123 = 123 + 1327.23 \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 123 = 123 + 663.614 \int_0^{\infty} \frac{\sin(t)}{t} dt$$

$$(((\pi / (0.592385 * 0.723606797 * 0.00703085) + 123)))^{1/14}$$

**Input interpretation:**

$$\sqrt[14]{\frac{\pi}{0.592385 \times 0.723606797 \times 0.00703085} + 123}$$

**Result:**

1.6558995573137767205200140987540148666363054874833473603063...

1.655899557313.... result very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

$$(((\pi / (0.592385 * 0.723606797 * 0.00703085) + 123)))^{1/14} - (29+7+2)/10^3$$

where 29, 7 and 2 are Lucas numbers

**Input interpretation:**

$$\sqrt[14]{\frac{\pi}{0.592385 \times 0.723606797 \times 0.00703085} + 123} - \frac{29 + 7 + 2}{10^3}$$

**Result:**

1.6178995573137767205200140987540148666363054874833473603063...

1.6178995573.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

**Alternative representations:**

$$\sqrt[14]{\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085} + 123} - \frac{29 + 7 + 2}{10^3} = \sqrt[14]{123 + \frac{180^\circ}{0.0030138} - \frac{38}{10^3}}$$

$$\sqrt[14]{\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085}} + 123 - \frac{29 + 7 + 2}{10^3} =$$

$$\sqrt[14]{123 - \frac{i \log(-1)}{0.0030138} - \frac{38}{10^3}}$$

$$\sqrt[14]{\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085}} + 123 - \frac{29 + 7 + 2}{10^3} =$$

$$\sqrt[14]{123 + \frac{\cos^{-1}(-1)}{0.0030138} - \frac{38}{10^3}}$$

### Series representations:

$$\sqrt[14]{\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085}} + 123 - \frac{29 + 7 + 2}{10^3} =$$

$$-\frac{19}{500} + \sqrt[14]{123 + 1327.23 \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}}$$

$$\sqrt[14]{\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085}} + 123 - \frac{29 + 7 + 2}{10^3} =$$

$$-\frac{19}{500} + \sqrt[14]{-540.614 + 663.614 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}$$

$$\sqrt[14]{\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085}} + 123 - \frac{29 + 7 + 2}{10^3} =$$

$$-\frac{19}{500} + \sqrt[14]{123 + 331.807x + 663.614 \sum_{k=1}^{\infty} \frac{\sin(kx)}{k}} \text{ for } (x \in \mathbb{R} \text{ and } x > 0)$$

### Integral representations:

$$\sqrt[14]{\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085}} + 123 - \frac{29 + 7 + 2}{10^3} =$$

$$-\frac{19}{500} + \sqrt[14]{123 + 663.614 \int_0^{\infty} \frac{1}{1+t^2} dt}$$

$$\begin{aligned}
& \sqrt[14]{\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085}} + 123 - \frac{29 + 7 + 2}{10^3} = \\
& -\frac{19}{500} + \sqrt[14]{123 + 1327.23 \int_0^1 \sqrt{1-t^2} dt} \\
& \sqrt[14]{\frac{\pi}{0.592385 \times 0.723607 \times 0.00703085}} + 123 - \frac{29 + 7 + 2}{10^3} = \\
& -\frac{19}{500} + \sqrt[14]{123 + 663.614 \int_0^\infty \frac{\sin(t)}{t} dt}
\end{aligned}$$

Now, we have that:

$$\frac{1}{4} \exp(2\pi\tilde{T}) \sum_{s \in \mathbb{Z}} (-1)^s \left( \tanh \pi N s / 4\tilde{T} - \frac{1}{N} \tanh \pi s / 4\tilde{T} \right) \frac{1}{\sinh \pi s / 2\tilde{T}}. \quad (4.20)$$

For  $s = 2, N = 5, \tilde{T} = 0.0864055$

$1/4 \exp(2 * \text{Pi} * 0.0864055) \text{ sum } ((-1)^s (\tanh ((5\text{Pi} * s) / (4 * 0.0864055))) - 1/5 \tanh((\text{Pi} * s) / (4 * 0.0864055))) * 1 / ((\sinh(\text{Pi} * s) / (2 * 0.0864055))), s = 1 \text{ to } 233$

**Input interpretation:**

$$\frac{1}{4} \exp(2\pi \times 0.0864055) \sum_{s=1}^{233} \left( (-1)^s \tanh\left(\frac{5\pi s}{4 \times 0.0864055}\right) - \frac{1}{5} \left( \tanh\left(\frac{\pi s}{4 \times 0.0864055}\right) \times \frac{1}{\frac{\sinh(\pi s)}{2 \times 0.0864055}} \right) \right)$$

$\tanh(x)$  is the hyperbolic tangent function

$\sinh(x)$  is the hyperbolic sine function

**Result:**

-0.431594

-0.431594

From which:

$$\frac{1}{10^{27}} \left[ \left( \left( -\frac{2}{\left( \frac{1}{4} \exp(2\pi \cdot 0.0864055) \sum_{s=1}^{233} (-1)^s \tanh\left(\frac{5\pi s}{4 \cdot 0.0864055}\right) - \frac{1}{5} \tanh\left(\frac{\pi s}{4 \cdot 0.0864055}\right) \right) \right) \cdot \frac{1}{\left( \frac{\sinh(\pi s)}{2 \cdot 0.0864055} \right)} \right) \right]^{1/3 + \frac{5}{10^3}}$$

where 5 is a Fibonacci number

**Input interpretation:**

$$\frac{1}{10^{27}} \left( \left( -\frac{2}{\left( \frac{1}{4} \exp(2\pi \times 0.0864055) \sum_{s=1}^{233} (-1)^s \tanh\left(\frac{5\pi s}{4 \times 0.0864055}\right) - \frac{1}{5} \left( \tanh\left(\frac{\pi s}{4 \times 0.0864055}\right) \times \frac{1}{\frac{\sinh(\pi s)}{2 \times 0.0864055}} \right) \right) \right) \right)^{\left( \frac{1}{3} + \frac{5}{10^3} \right)}$$

tanh(x) is the hyperbolic tangent function

sinh(x) is the hyperbolic sine function

**Result:**

$$1.67219 \times 10^{-27}$$

1.67219 \* 10<sup>-27</sup> result practically equal to the proton mass in kg

We have also:

$$V(x) = \left( \tanh \pi N x / 4 - \frac{1}{N} \tanh \pi x / 4 \right) \frac{1}{\sinh \pi x / 2}. \tag{4.23}$$

$$N = 5, \quad x = 1/5$$

$$\left( \tanh \left( \frac{5\pi}{20} \right) - \frac{1}{5} \tanh \left( \frac{\pi}{20} \right) \right) \cdot \frac{1}{\sinh \left( \frac{\pi}{10} \right)}$$

**Input:**

$$\left( \tanh \left( 5 \times \frac{\pi}{20} \right) - \frac{1}{5} \tanh \left( \frac{\pi}{20} \right) \right) \times \frac{1}{\sinh \left( \frac{\pi}{10} \right)}$$

tanh(x) is the hyperbolic tangent function

sinh(x) is the hyperbolic sine function

**Exact result:**

$$\left(\tanh\left(\frac{\pi}{4}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)\right) \operatorname{csch}\left(\frac{\pi}{10}\right)$$

$\operatorname{csch}(x)$  is the hyperbolic cosecant function

**Decimal approximation:**

1.955939154900132951224555504284020433882363208631026457577...

1.9559391549....

**Property:**

$\operatorname{csch}\left(\frac{\pi}{10}\right)\left(-\frac{1}{5} \tanh\left(\frac{\pi}{20}\right) + \tanh\left(\frac{\pi}{4}\right)\right)$  is a transcendental number

**Alternate forms:**

$$-\frac{1}{5} \left(\tanh\left(\frac{\pi}{20}\right) - 5 \tanh\left(\frac{\pi}{4}\right)\right) \operatorname{csch}\left(\frac{\pi}{10}\right)$$

$$\tanh\left(\frac{\pi}{4}\right) \operatorname{csch}\left(\frac{\pi}{10}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right) \operatorname{csch}\left(\frac{\pi}{10}\right)$$

$$\frac{4 \left(4 \cosh\left(\frac{\pi}{10}\right) - 1\right)}{5 \left(1 - 2 \cosh\left(\frac{\pi}{10}\right) + 2 \cosh\left(\frac{\pi}{5}\right)\right)}$$

$\cosh(x)$  is the hyperbolic cosine function

**Alternative representations:**

$$\frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} = \frac{-1 + \frac{2}{1+e^{-(10\pi)/20}} - \frac{1}{5} \left(-1 + \frac{2}{1+e^{-(2\pi)/20}}\right)}{i \cos\left(\frac{\pi}{2} + \frac{i\pi}{10}\right)}$$

$$\frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} = \frac{-1 + \frac{2}{1+e^{-(10\pi)/20}} - \frac{1}{5} \left(-1 + \frac{2}{1+e^{-(2\pi)/20}}\right)}{\frac{1}{2} \left(-e^{-\pi/10} + e^{\pi/10}\right)}$$

$$\frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} = -\frac{-1 + \frac{2}{1+e^{-(10\pi)/20}} - \frac{1}{5} \left(-1 + \frac{2}{1+e^{-(2\pi)/20}}\right)}{i \cos\left(\frac{\pi}{2} - \frac{i\pi}{10}\right)}$$

**Series representations:**

$$\frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} = \sum_{k=1}^{\infty} \frac{768 (1 - 2k)^2 \operatorname{csch}\left(\frac{\pi}{10}\right)}{(5 - 16k + 16k^2)(101 - 400k + 400k^2)\pi}$$

$$\frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} = \frac{4\left(-2 + \sum_{k=0}^{\infty} (-1)^{1+k} e^{-1/2(1+k)\pi} \left(-5 + e^{2/5(1+k)\pi}\right)\right)\left(1 + 2 \sum_{k=1}^{\infty} \frac{(-1)^k}{1+100k^2}\right)}{\pi}$$

$$\frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} = -4\left(-2 + \sum_{k=0}^{\infty} (-1)^{1+k} e^{-1/2(1+k)\pi} \left(-5 + e^{2/5(1+k)\pi}\right)\right) \sum_{k=-\infty}^{\infty} \frac{(-1)^k}{\pi + 100k^2 \pi}$$

**Integral representations:**

$$\frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} = -\frac{2\left(\int_0^{\frac{\pi}{20}} \operatorname{sech}^2(t) dt - 5 \int_0^{\frac{\pi}{4}} \operatorname{sech}^2(t) dt\right)}{\pi \int_0^1 \cosh\left(\frac{\pi t}{10}\right) dt}$$

$$\frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} = \int_0^{\frac{\pi}{20}} -\frac{8i(\operatorname{sech}^2(t) - 25 \operatorname{sech}^2(5t))}{\sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^2/(400s)+s}}{s^{3/2}} ds} dt \text{ for } \gamma > 0$$

From which:

$$\left(\left(\left(\tanh\left(5\frac{\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)\right) \times \frac{1}{\sinh\left(\frac{\pi}{10}\right)}\right)\right)^{11+123+\text{golden ratio}^2}$$

where 123 is a Lucas number

**Input:**

$$\left(\left(\tanh\left(5 \times \frac{\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)\right) \times \frac{1}{\sinh\left(\frac{\pi}{10}\right)}\right)^{11} + 123 + \phi^2$$

$\tanh(x)$  is the hyperbolic tangent function

$\sinh(x)$  is the hyperbolic sine function

$\phi$  is the golden ratio

**Exact result:**

$$\phi^2 + 123 + \left( \tanh\left(\frac{\pi}{4}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right) \right)^{11} \operatorname{csch}^{11}\left(\frac{\pi}{10}\right)$$

$\operatorname{csch}(x)$  is the hyperbolic cosecant function

**Decimal approximation:**

1728.526591678978524326466630150302026002712558017996618425...

1728.52659...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Property:**

$123 + \phi^2 + \operatorname{csch}^{11}\left(\frac{\pi}{10}\right) \left( -\frac{1}{5} \tanh\left(\frac{\pi}{20}\right) + \tanh\left(\frac{\pi}{4}\right) \right)^{11}$  is a transcendental number

**Alternate forms:**

$$\frac{1}{2} (249 + \sqrt{5}) + \left( \tanh\left(\frac{\pi}{4}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right) \right)^{11} \operatorname{csch}^{11}\left(\frac{\pi}{10}\right)$$

$$\frac{1}{2} (249 + \sqrt{5}) - \frac{\left( \tanh\left(\frac{\pi}{20}\right) - 5 \tanh\left(\frac{\pi}{4}\right) \right)^{11} \operatorname{csch}^{11}\left(\frac{\pi}{10}\right)}{48828125}$$

$$123 + \frac{1}{4} (1 + \sqrt{5})^2 + \left( \tanh\left(\frac{\pi}{4}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right) \right)^{11} \operatorname{csch}^{11}\left(\frac{\pi}{10}\right)$$

**Alternative representations:**

$$\left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^{11} + 123 + \phi^2 =$$

$$123 + \phi^2 + \left( \frac{-1 + \frac{2}{1+e^{-(10\pi)/20}} - \frac{1}{5} \left( -1 + \frac{2}{1+e^{-(2\pi)/20}} \right)}{i \cos\left(\frac{\pi}{2} + \frac{i\pi}{10}\right)} \right)^{11}$$

$$\left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^{11} + 123 + \phi^2 =$$

$$123 + \phi^2 + \left( \frac{-1 + \frac{2}{1+e^{-(10\pi)/20}} - \frac{1}{5} \left(-1 + \frac{2}{1+e^{-(2\pi)/20}}\right)}{\frac{1}{2}(-e^{-\pi/10} + e^{\pi/10})} \right)^{11}$$

$$\left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^{11} + 123 + \phi^2 =$$

$$123 + \phi^2 + \left( \frac{-1 + \frac{2}{1+e^{-(10\pi)/20}} - \frac{1}{5} \left(-1 + \frac{2}{1+e^{-(2\pi)/20}}\right)}{i \cos\left(\frac{\pi}{2} - \frac{i\pi}{10}\right)} \right)^{11}$$

### Series representations:

$$\left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^{11} + 123 + \phi^2 =$$

$$123 + \phi^2 + \operatorname{csch}^{11}\left(\frac{\pi}{10}\right) \left( \sum_{k=1}^{\infty} \frac{768(1-2k)^2}{(5-16k+16k^2)(101-400k+400k^2)\pi} \right)^{11}$$

$$\left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^{11} + 123 + \phi^2 =$$

$$\frac{1}{2} \left( 249 + \sqrt{5} - 4096 \left( \sum_{k=1}^{\infty} \frac{768(1-2k)^2}{(5-16k+16k^2)(101-400k+400k^2)\pi} \right)^{11} \right.$$

$$\left. \left( \sum_{k=1}^{\infty} q^{-1+2k} \right)^{11} \right) \text{ for } q = e^{\pi/10}$$

$$\left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^{11} + 123 + \phi^2 = 123 + \phi^2 +$$

$$\left( \frac{10}{\pi} + \frac{1}{5} \pi \sum_{k=1}^{\infty} \frac{100(-1)^k}{(1+100k^2)\pi^2} \right)^{11} \left( \sum_{k=1}^{\infty} \frac{768(1-2k)^2}{(5-16k+16k^2)(101-400k+400k^2)\pi} \right)^{11}$$

$(((((\tanh(5\pi/20) - 1/5 \tanh(\pi/20)) * 1/(\sinh(\pi/10))))))^{7+29+1/\text{golden ratio}}$

where 29 is a Lucas number

**Input:**

$$\left( \left( \tanh\left(5 \times \frac{\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right) \right) \times \frac{1}{\sinh\left(\frac{\pi}{10}\right)} \right)^7 + 29 + \frac{1}{\phi}$$

$\tanh(x)$  is the hyperbolic tangent function

$\sinh(x)$  is the hyperbolic sine function

$\phi$  is the golden ratio

**Exact result:**

$$\frac{1}{\phi} + 29 + \left( \tanh\left(\frac{\pi}{4}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right) \right)^7 \operatorname{csch}^7\left(\frac{\pi}{10}\right)$$

$\operatorname{csch}(x)$  is the hyperbolic cosecant function

**Decimal approximation:**

139.1365082334322762072623701285966484343160032636900658669...

139.136508... result practically equal to the rest mass of Pion meson 139.57 MeV

**Property:**

$29 + \frac{1}{\phi} + \operatorname{csch}^7\left(\frac{\pi}{10}\right) \left( -\frac{1}{5} \tanh\left(\frac{\pi}{20}\right) + \tanh\left(\frac{\pi}{4}\right) \right)^7$  is a transcendental number

**Alternate forms:**

$$\frac{1}{2} (57 + \sqrt{5}) + \left( \tanh\left(\frac{\pi}{4}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right) \right)^7 \operatorname{csch}^7\left(\frac{\pi}{10}\right)$$

$$29 + \frac{2}{1 + \sqrt{5}} + \left( \tanh\left(\frac{\pi}{4}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right) \right)^7 \operatorname{csch}^7\left(\frac{\pi}{10}\right)$$

$$29 + \frac{1}{\phi} + \frac{\left( -\frac{\sinh\left(\frac{\pi}{20}\right)}{5 \cosh\left(\frac{\pi}{20}\right)} + \frac{\sinh\left(\frac{\pi}{4}\right)}{\cosh\left(\frac{\pi}{4}\right)} \right)^7}{\sinh^7\left(\frac{\pi}{10}\right)}$$

$\cosh(x)$  is the hyperbolic cosine function

**Expanded form:**

$$\begin{aligned} & \frac{1}{\phi} + 29 - \frac{\tanh^7\left(\frac{\pi}{20}\right) \operatorname{csch}^7\left(\frac{\pi}{10}\right)}{78\,125} + \tanh^7\left(\frac{\pi}{4}\right) \operatorname{csch}^7\left(\frac{\pi}{10}\right) + \\ & \frac{7 \tanh^6\left(\frac{\pi}{20}\right) \tanh\left(\frac{\pi}{4}\right) \operatorname{csch}^7\left(\frac{\pi}{10}\right)}{15\,625} - \frac{7}{5} \tanh\left(\frac{\pi}{20}\right) \tanh^6\left(\frac{\pi}{4}\right) \operatorname{csch}^7\left(\frac{\pi}{10}\right) - \\ & \frac{21 \tanh^5\left(\frac{\pi}{20}\right) \tanh^2\left(\frac{\pi}{4}\right) \operatorname{csch}^7\left(\frac{\pi}{10}\right)}{31\,250} + \frac{21}{25} \tanh^2\left(\frac{\pi}{20}\right) \tanh^5\left(\frac{\pi}{4}\right) \operatorname{csch}^7\left(\frac{\pi}{10}\right) + \\ & \frac{7}{125} \tanh^4\left(\frac{\pi}{20}\right) \tanh^3\left(\frac{\pi}{4}\right) \operatorname{csch}^7\left(\frac{\pi}{10}\right) - \frac{7}{25} \tanh^3\left(\frac{\pi}{20}\right) \tanh^4\left(\frac{\pi}{4}\right) \operatorname{csch}^7\left(\frac{\pi}{10}\right) \end{aligned}$$

**Alternative representations:**

$$\begin{aligned} & \left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^7 + 29 + \frac{1}{\phi} = \\ & 29 + \frac{1}{\phi} + \left( \frac{-1 + \frac{2}{1+e^{-(10\pi)/20}} - \frac{1}{5} \left(-1 + \frac{2}{1+e^{-(2\pi)/20}}\right)}{i \cos\left(\frac{\pi}{2} + \frac{i\pi}{10}\right)} \right)^7 \end{aligned}$$

$$\begin{aligned} & \left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^7 + 29 + \frac{1}{\phi} = \\ & 29 + \frac{1}{\phi} + \left( \frac{-1 + \frac{2}{1+e^{-(10\pi)/20}} - \frac{1}{5} \left(-1 + \frac{2}{1+e^{-(2\pi)/20}}\right)}{\frac{1}{2} (-e^{-\pi/10} + e^{\pi/10})} \right)^7 \end{aligned}$$

$$\begin{aligned} & \left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^7 + 29 + \frac{1}{\phi} = \\ & 29 + \frac{1}{\phi} + \left( -\frac{-1 + \frac{2}{1+e^{-(10\pi)/20}} - \frac{1}{5} \left(-1 + \frac{2}{1+e^{-(2\pi)/20}}\right)}{i \cos\left(\frac{\pi}{2} - \frac{i\pi}{10}\right)} \right)^7 \end{aligned}$$

**Series representations:**

$$\begin{aligned} & \left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^7 + 29 + \frac{1}{\phi} = \\ & 29 + \frac{1}{\phi} + \operatorname{csch}^7\left(\frac{\pi}{10}\right) \left( \sum_{k=1}^{\infty} \frac{768 (1-2k)^2}{(5-16k+16k^2)(101-400k+400k^2)\pi} \right)^7 \end{aligned}$$

$$\left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^7 + 29 + \frac{1}{\phi} = 29 + \frac{1}{\phi} + \left( \frac{10}{\pi} + \frac{1}{5} \pi \sum_{k=1}^{\infty} \frac{100 (-1)^k}{(1 + 100 k^2) \pi^2} \right)^7 \left( \sum_{k=1}^{\infty} \frac{768 (1 - 2k)^2}{(5 - 16k + 16k^2)(101 - 400k + 400k^2) \pi} \right)^7$$

$$\left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^7 + 29 + \frac{1}{\phi} = 29 + \frac{1}{\phi} - 128 \left( 1 - 2 \sum_{k=0}^{\infty} (-1)^k e^{-1/2(1+k)\pi} + \frac{1}{5} \left( -1 + 2 \sum_{k=0}^{\infty} (-1)^k e^{-1/10(1+k)\pi} \right) \right)^7 \left( \sum_{k=1}^{\infty} q^{-1+2k} \right)^7 \text{ for } q = e^{\pi/10}$$

(((tanh (5Pi/20)-1/5 tanh (Pi/20)) \* 1/(sinh (Pi/10))))^7+11+Pi+golden ratio

where 11 is a Lucas number

**Input:**

$$\left( \left( \tanh\left(5 \times \frac{\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right) \right) \times \frac{1}{\sinh\left(\frac{\pi}{10}\right)} \right)^7 + 11 + \pi + \phi$$

tanh(x) is the hyperbolic tangent function

sinh(x) is the hyperbolic sine function

φ is the golden ratio

**Exact result:**

$$\phi + 11 + \pi + \left( \tanh\left(\frac{\pi}{4}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right) \right)^7 \operatorname{csch}^7\left(\frac{\pi}{10}\right)$$

csch(x) is the hyperbolic cosecant function

**Decimal approximation:**

125.2781008870220694457250135118761513185131726630651716879...

125.2781008... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternate forms:**

$$\frac{1}{2} (23 + \sqrt{5}) + \pi + \left( \tanh\left(\frac{\pi}{4}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right) \right)^7 \operatorname{csch}^7\left(\frac{\pi}{10}\right)$$

$$11 + \frac{1}{2} (1 + \sqrt{5}) + \pi + \left( \tanh\left(\frac{\pi}{4}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right) \right)^7 \operatorname{csch}^7\left(\frac{\pi}{10}\right)$$

$$11 + \phi + \pi + \frac{\left( -\frac{\sinh\left(\frac{\pi}{20}\right)}{5 \cosh\left(\frac{\pi}{20}\right)} + \frac{\sinh\left(\frac{\pi}{4}\right)}{\cosh\left(\frac{\pi}{4}\right)} \right)^7}{\sinh^7\left(\frac{\pi}{10}\right)}$$

$\cosh(x)$  is the hyperbolic cosine function

**Alternative representations:**

$$\left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^7 + 11 + \pi + \phi =$$

$$11 + \phi + \pi + \left( \frac{-1 + \frac{2}{1+e^{-(10\pi)/20}} - \frac{1}{5} \left( -1 + \frac{2}{1+e^{-(2\pi)/20}} \right)}{i \cos\left(\frac{\pi}{2} + \frac{i\pi}{10}\right)} \right)^7$$

$$\left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^7 + 11 + \pi + \phi =$$

$$11 + \phi + \pi + \left( \frac{-1 + \frac{2}{1+e^{-(10\pi)/20}} - \frac{1}{5} \left( -1 + \frac{2}{1+e^{-(2\pi)/20}} \right)}{\frac{1}{2} (-e^{-\pi/10} + e^{\pi/10})} \right)^7$$

$$\left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^7 + 11 + \pi + \phi =$$

$$11 + \phi + \pi + \left( -\frac{-1 + \frac{2}{1+e^{-(10\pi)/20}} - \frac{1}{5} \left( -1 + \frac{2}{1+e^{-(2\pi)/20}} \right)}{i \cos\left(\frac{\pi}{2} - \frac{i\pi}{10}\right)} \right)^7$$

**Series representations:**

$$\left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^7 + 11 + \pi + \phi =$$

$$11 + \phi + \pi + \operatorname{csch}^7\left(\frac{\pi}{10}\right) \left( \sum_{k=1}^{\infty} \frac{768 (1 - 2k)^2}{(5 - 16k + 16k^2)(101 - 400k + 400k^2)\pi} \right)^7$$

$$\left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^7 + 11 + \pi + \phi =$$

$$11 + \phi + \pi + 16384 \left( 2 + \sum_{k=0}^{\infty} (-1)^k e^{-1/2(1+k)\pi} (-5 + e^{2/5(1+k)\pi}) \right)^7 \left( \sum_{k=-\infty}^{\infty} \frac{(-1)^k}{\pi + 100k^2\pi} \right)^7$$

$$\left( \frac{\tanh\left(\frac{5\pi}{20}\right) - \frac{1}{5} \tanh\left(\frac{\pi}{20}\right)}{\sinh\left(\frac{\pi}{10}\right)} \right)^7 + 11 + \pi + \phi =$$

$$\frac{1}{2} \left( 23 + \sqrt{5} + 2\pi - 256 \left( \sum_{k=1}^{\infty} \frac{768 (1 - 2k)^2}{(5 - 16k + 16k^2)(101 - 400k + 400k^2)\pi} \right)^7 \right.$$

$$\left. \left( \sum_{k=1}^{\infty} q^{-1+2k} \right)^7 \right) \text{ for } q = e^{\pi/10}$$

## References

### **Open Strings On The Rindler Horizon**

*Edward Witten* - arXiv:1810.11912v4 [hep-th] 26 Nov 201

With regard the formula of coefficients of the '5th order' mock theta function  $\psi_1(q)$ , see:

- a) *Srinivasa Ramanujan, Collected Papers*, Chelsea, New York, 1962, pp. 354-355
  
- b) *Srinivasa Ramanujan, The Lost Notebook and Other Unpublished Papers*, Narosa Publishing House, New Delhi, 1988, pp. 19, 21, 22