Representing basic physical fields by quaternionic fields

by J.A.J. van Leunen

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Summary

Basic physical fields are dynamic fields like our universe and the fields that are raised by electric charges. These fields are dynamic continuums. Most physical theories treat these fields by applying gravitational theories or by Maxwell equations. Mathematically these fields can be represented by quaternionic fields. Dedicated normal operators in quaternionic non-separable Hilbert spaces can represent these quaternionic fields in their continuum eigenspaces. Quaternionic functions can describe these fields. Quaternionic differential and integral calculus can describe the behavior of these fields and the interaction of these fields with countable sets of quaternions. All quaternionic fields obey the same quaternionic function theory. The basic fields differ in their start and boundary conditions.

1 Introduction

The fact that physical objects can be represented and modeled by mathematical constructs is applied in many physical theories.

Quite often function theory is applied and more seldomly the representation is embedded in a topological space, such as a Hilbert space. The Hilbert space has the advantage that it can act as a repository for dynamic geometrical data and for dynamic fields. If a system of Hilbert paces is applied, then a very powerful and flexible modeling platform results that can cope with the diversity and the dynamics of objects that are encountered in the universe. Mathematics severely restricts the possibilities of this platform. This appears an advantage rather than a discredit because it limits the extension of the model in arbitrary directions.

The base model is subject of a PowerPoint presentation

http://www.e-physics.eu/Base%20model.pptx.

The base model is part of the Hilbert Book Model. The Hilbert Book Model is subject of the Hilbert Book Model Project.

The Hilbert Book Model is treated in greater detail in "A Self-creating Model of Physical Reality".

2 Hilbert spaces

Hilbert spaces emerge from orthomodular lattices because the set of closed subspaces of a separable Hilbert space is a Hilbert lattice, which is isomorphic with an orthomodular lattice. Only a subtle difference exist between a Hilbert space and its underlying vector space. A separable Hilbert space is a complete vector space that features an inner product. The value of the inner product must be a member of an associative division ring. Only three suitable number systems exist that are associative division rings. Depending on their dimension these number systems exist in many versions that distinguish in the Cartesian and polar coordinate systems that sequence the members of the version. Each Hilbert space manages the selected version of the number system in the eigenspace of a dedicated normal operator that we call reference operator. This eigenspace acts as the private parameter space. A category of normal operators exists of which the members share the eigenvectors of the reference operators and apply a selected function and the parameter value that belongs to the eigenvector to generate a new eigenvalue by taking the target value of the function as the new eigenvalue. In this way, the eigenspace of the new operator becomes a sampled field.

3 A system of separable Hilbert spaces

Due to the subtle difference between a Hilbert space and its underlying vector space, and because number systems exist in many versions, a huge number of separable Hilbert spaces can share the same underlying vector space. Sharing the same underlying vector space appears to restrict the choice of the versions of the number system that can be selected. Only versions that have the axes of the Cartesian coordinates parallel to a background separable Hilbert space that is picked from the tolerated collection will be allowed. Only the sequencing of the elements along these axes can be selected freely. This limits the symmetries of the private parameter spaces to a short list. The difference between the symmetries reduces to the short list that also characterizes the list of electric charges and color charges that mark the elementary particles in the Standard Model. This is a remarkable result.

4 Adding a non-separable Hilbert space

Let us assume that the background separable Hilbert space has infinite dimensions. This separable Hilbert space owns a unique non-separable companion Hilbert space that embeds its separable partner. It also owns a reference operator that manages the private parameter space of the non-separable Hilbert space. The category of defined operators that apply a function to determine the eigenspace of the operator supports continuum eigenspaces. These eigenspaces represent continuum fields. In case of quaternionic Hilbert spaces these eigenspaces are dynamic fields. The resulting separable Hilbert spaces float with the geometrical center of their private parameter space over the background parameter space.

5 Modeling platform

Despite the strong restrictions this collection of Hilbert spaces represent a flexible and powerful modeling platform. It acts as a repository for the dynamic geometric data of point-like objects and for dynamic fields. Quaternions can act as storage bins of a scalar timestamp and a three-dimensional location. If the timestamps are sequenced, then the archive tells the life story of the point-like object as an ongoing hopping path. The Hilbert Book Model Project applies this model. In a creation episode the repository is filled with data. In a subsequent running mode, the archived hop landing locations are embedded in a selected field that we call the universe. The free-floating separable Hilbert spaces harbor elementary particles.

6 The universe field

The universe is a dynamic field that is represented by a dedicated normal operator in the non-separable Hilbert space, which is part of the background platform. This field exists always and everywhere in the parameter space of the platform. The field can vibrate, deform and expand as function of the real part of the parameter space. This real part represents proper time.

The universe is a mixed field. It can contain enclosed spatial regions that encapsulate a discontinuum. A discontinuum is a dense discrete set. A discontinuum is countable. The enclosure is a continuum with a lower dimension than the enclosed region. No field excitations exist inside the discontinuum. Thus, no field excitations can pass the enclosure.

7 Symmetry related fields

Each floating platform features its own private parameter space and with that parameter space it owns a symmetry. The difference in symmetry between the floating platform and the background platform defines a symmetry-related charge that is represented by a source or sink that locates at the geometric center of the private parameter space. The sources and sinks mark the locations of the geometrical centers of the corresponding floating platforms in the universe field. In the background platform these sources and sinks raise a symmetry related field that corresponds to the symmetry related charge of the floating platform. The symmetry related fields can superpose.

8 Field equations

Field equations are quaternionic functions or quaternionic differential and integral equations that describe the behavior of the continuum part of fields.

The differential change can be expressed in terms of a linear combination of partial differentials. Now the total differential change df of field f equals

$$df = \frac{\partial f}{\partial \tau} d\tau + \frac{\partial f}{\partial x} \vec{i} dx + \frac{\partial f}{\partial y} \vec{j} dy + \frac{\partial f}{\partial z} \vec{k} dz \qquad (8.1.1)$$

In this equation, the partial differentials $\frac{\partial f}{\partial \tau}, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial y}$ behave as quaternionic differential operators.

The quaternionic nabla ∇ assumes the ***special condition*** that partial differentials direct along the axes of the Cartesian coordinate system. Thus

$$\nabla = \sum_{i=0}^{4} \vec{e}_i \frac{\partial}{\partial x_i} = \frac{\partial}{\partial \tau} + \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z} \qquad (8.1.2)$$

However, this way of notation is often considered as abusive. Still, we will apply that notation because the correct notation (8.1.1) leads to the same result. This will be shown in the next section by splitting both the quaternionic nabla and the function in a scalar part and a vector part.

8.1 Quaternionic differential calculus

The first order partial differential equations divide the first-order change of a field in five different parts that each represent a new field. We will represent the field change operator by a quaternionic nabla operator. This operator behaves as a quaternionic multiplier.

A quaternion can store a timestamp in its real part and a three-dimensional spatial location in its imaginary part. The quaternionic nabla ∇ acts as a quaternionic multiplying operator. Quaternionic multiplication obeys the equation

$$\begin{aligned} c = c_r + \vec{c} = ab = (a_r + \vec{a})(b_r + \vec{b}) \\ = a_r b_r - \langle \vec{a}, \vec{b} \rangle + a_r \vec{b} + \vec{a} b_r \pm \vec{a} \times \vec{b} \end{aligned} \quad (8.1.3)$$

The $\pm$ sign indicates the freedom of choice of the handedness of the product rule that exists when selecting a version of the quaternionic number system. The first order partial differential follows from

$$\nabla = \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} = \nabla_r + \vec{\nabla} \quad (8.1.4)$$

The spatial nabla $\vec{\nabla}$ is well-known as the del operator and is treated in detail in Wikipedia.

$$\begin{aligned} \phi = \nabla \psi = \left(\frac{\partial}{\partial \tau} + \vec{\nabla} \right)(\psi_r + \vec{\psi}) \\ = \nabla_r \psi_r - \langle \vec{\nabla}, \vec{\psi} \rangle + \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \end{aligned} \quad (8.1.5)$$

In a selected version of the quaternionic number system only the corresponding version of the quaternionic nabla is active.

The differential $\nabla \psi$ describes the change of field ψ. The five separate terms in the first-order partial differential have a separate physical meaning. All basic fields feature this decomposition. The terms may represent new fields.

$$\phi_r = \nabla_r \psi_r - \langle \vec{\nabla}, \vec{\psi} \rangle \tag{8.1.6}$$

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} = -\vec{E} \pm \vec{B} \tag{8.1.7}$$

$\vec{\nabla} f$ is the gradient of f.

$\langle \vec{\nabla}, \vec{f} \rangle$ is the divergence of $\vec{f}$.

$\vec{\nabla} \times \vec{f}$ is the curl of $\vec{f}$.

The conjugate of the quaternionic nabla operator defines another type of field change.

$$\nabla^* = \nabla_r - \vec{\nabla} \tag{8.1.8}$$

$$\begin{aligned} \zeta = \nabla^* \phi &= \left(\frac{\partial}{\partial \tau} - \vec{\nabla} \right) (\phi_r + \vec{\phi}) \\ &= \nabla_r \phi_r + \langle \vec{\nabla}, \vec{\phi} \rangle + \nabla_r \vec{\phi} - \vec{\nabla} \phi_r \mp \vec{\nabla} \times \vec{\phi} \end{aligned} \tag{8.1.9}$$

The quaternionic nabla is a normal operator.

$$\begin{aligned} \nabla^\dagger &= \nabla^* = \nabla_r - \vec{\nabla} = \nabla_r + \vec{\nabla}^\dagger = \nabla_r + \vec{\nabla}^* \\ \nabla^\dagger \nabla &= \nabla \nabla^\dagger = \nabla^* \nabla = \nabla \nabla^* = \nabla_r \nabla_r + \langle \vec{\nabla}, \vec{\nabla} \rangle \end{aligned} \tag{8.1.10}$$

The operators $\nabla_r \nabla_r$ and $\langle \vec{\nabla}, \vec{\nabla} \rangle$ are Hermitian operators. They can also be combined as $\square = \nabla_r \nabla_r - \langle \vec{\nabla}, \vec{\nabla} \rangle$. This is the d'Alembert operator.

8.2 Continuity equations

Continuity equations are partial quaternionic differential equations.

8.2.1 Field excitations

Field excitations are solutions of second-order partial differential equations.

One of the second-order partial differential equations results from combining the two first-order partial differential equations $\phi = \nabla \psi$ and $\zeta = \nabla^* \phi$.

$$\zeta = \nabla^* \varphi = \nabla^* \nabla \psi = \nabla \nabla^* \psi = \left(\nabla_r + \vec{\nabla}\right)\left(\nabla_r - \vec{\nabla}\right)\left(\psi_r + \vec{\psi}\right) \\ = \left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi \tag{8.2.1}$$

Integration over the time domain results in the Poisson equation

$$\rho = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi \tag{8.2.2}$$

Under isotropic conditions, a very special solution of the Poisson equation is the Green's function $\frac{1}{4\pi\left|\vec{q} - \vec{q'}\right|}$ of the affected field. This solution is the spatial Dirac $\delta(\vec{q})$ pulse response of the field under strict isotropic conditions.

$$\nabla \frac{1}{\left|\vec{q} - \vec{q'}\right|} = -\frac{\left(\vec{q} - \vec{q'}\right)}{\left|\vec{q} - \vec{q'}\right|^3} \tag{8.2.3}$$

$$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \frac{1}{\left|\vec{q} - \vec{q'}\right|} \equiv \left\langle \vec{\nabla}, \vec{\nabla} \frac{1}{\left|\vec{q} - \vec{q'}\right|} \right\rangle \\ = -\left\langle \vec{\nabla}, \frac{\left(\vec{q} - \vec{q'}\right)}{\left|\vec{q} - \vec{q'}\right|^3} \right\rangle = 4\pi\delta\left(\vec{q} - \vec{q'}\right) \tag{8.2.4}$$

Under isotropic conditions, the dynamic spherical pulse response of the field is a solution of a special form of the equation (8.2.1)

$$\left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi = 4\pi\delta\left(\vec{q} - \vec{q'}\right)\theta\left(\tau \pm \tau'\right) \tag{8.2.5}$$

Here $\theta(\tau)$ is a step function and $\delta(\vec{q})$ is a Dirac pulse response.

After the instant τ', this solution is described by

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q}'\right| \pm c(\tau-\tau')\vec{n}\right)}{\left|\vec{q}-\vec{q}'\right|} \tag{8.2.6}$$

The normalized vector $\vec{n}$ can be interpreted as the spin of the solution. The spherical pulse response acts either as an expanding or as a contracting spherical shock front. Over time this pulse response integrates into the Green's function. This means that the expanding pulse injects the volume of the Green's function into the field. Subsequently, the front spreads this volume over the field. The contracting shock front collects the volume of the Green's function and sucks it out of the field. The $\pm$ sign in equation (8.2.5) selects between injection and subtraction.

Apart from the spherical pulse response equation (8.2.5) supports a one-dimensional pulse response that acts as a one-dimensional shock front. This solution is described by

$$\psi = f\left(\left|\vec{q}-\vec{q}'\right| \pm c(\tau-\tau')\vec{n}\right) \tag{8.2.7}$$

Here, the normalized vector $\vec{n}$ can be interpreted as the polarization of the solution. Shock fronts only occur in one and three dimensions. A pulse response can also occur in two dimensions, but in that case, the pulse response is a complicated vibration that looks like the result of a throw of a stone in the middle of a pond.

Equations (8.2.1) and (8.2.2) show that the operators $\frac{\partial^2}{\partial\tau^2}$ and $\langle\vec{\nabla},\vec{\nabla}\rangle$ are valid second-order partial differential operators. These operators combine in the quaternionic equivalent of the wave equation.

$$\varphi = \left(\frac{\partial^2}{\partial\tau^2} - \langle\vec{\nabla},\vec{\nabla}\rangle\right)\psi \tag{8.2.8}$$

This equation also offers one-dimensional and three-dimensional shock fronts as its solutions.

$$\psi = \frac{f\left(\left|\vec{q}-\vec{q'}\right| \pm c\left(\tau-\tau'\right)\right)}{\left|\vec{q}-\vec{q'}\right|} \tag{8.2.9}$$

$$\psi = f\left(\left|\vec{q}-\vec{q'}\right| \pm c\left(\tau-\tau'\right)\right) \tag{8.2.10}$$

These pulse responses do not contain the normed vector $\vec{n}$. Apart from pulse responses, the wave equation offers waves as its solutions.

By splitting the field into the time-dependent part $T(\tau)$ and a location-dependent part, $A(\vec{q})$, the homogeneous version of the wave equation can be transformed into the Helmholtz equation.

$$\frac{\partial^2\psi}{\partial\tau^2} = \left\langle\vec{\nabla},\vec{\nabla}\right\rangle\psi = -\omega^2\psi \tag{8.2.11}$$

$$\psi(\vec{q},\tau) = A(\vec{q})T(\tau) \tag{8.2.12}$$

$$\frac{1}{T}\frac{\partial^2 T}{\partial\tau^2} = \frac{1}{A}\left\langle\vec{\nabla},\vec{\nabla}\right\rangle A = -\omega^2 \tag{8.2.13}$$

$$\left\langle\vec{\nabla},\vec{\nabla}\right\rangle A + \omega^2 A \tag{8.2.14}$$

The time-dependent part $T(\tau)$ depends on initial conditions, or it indicates the switch of the oscillation mode. The switch of the oscillation mode means that temporarily the oscillation is stopped and instead an object is emitted or absorbed that compensates the difference in potential energy. The location-dependent part of the field $A(\vec{q})$ describes the possible oscillation modes of the field and depends on boundary conditions. The oscillations have a binding effect. They keep the moving objects within a bounded region.

For three-dimensional isotropic spherical conditions, the solutions have the form

$$A(r,\theta,\varphi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left\{\left(a_{lm}j_l(kr)\right)+b_{lm}Y_l^m(\theta,\varphi)\right\} \qquad (8.2.15)$$

Here j_l and y_l are the spherical Bessel functions, and Y_l^m are the spherical harmonics . These solutions play a role in the spectra of atomic modules.

Planar and spherical waves are the simpler wave solutions of the equation (8.2.11)

$$\psi(\vec{q},\tau)=\exp\left\{\vec{n}\left(\left\langle\vec{k},\vec{q}-\vec{q}_0\right\rangle-\omega\tau+\varphi\right)\right\} \qquad (8.2.16)$$

$$\psi(\vec{q},\tau)=\frac{\exp\left\{\vec{n}\left(\left\langle\vec{k},\vec{q}-\vec{q}_0\right\rangle-\omega\tau+\varphi\right)\right\}}{\left|\vec{q}-\vec{q}_0\right|} \qquad (8.2.17)$$

A more general solution is a superposition of these basic types.

Two quite similar homogeneous second-order partial differential equations exist. They are the homogeneous versions of equation (8.2.5) and equation (8.2.8). The first equation has spherical shock front solutions with a spin vector that behaves like the spin of elementary fermionic particles. The second equation has spherical shock front solutions that behave more like elementary bosons.

The inhomogeneous pulse activated equations are

$$\left(\nabla_r\nabla_r\pm\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\right)\psi=4\pi\delta\left(\vec{q}-\vec{q}'\right)\theta(\tau\pm\tau') \qquad (8.2.18)$$

The paper treats quaternionic differential equations more extensively in chapter 14.

8.3 Enclosure balance equations

Enclosure balance equations are quaternionic integral equations that describe the balance between the inside and the outside of an enclosure.

These integral balance equations base on replacing the del operator $\vec{\nabla}$ by a normed vector $\vec{n}$.

$$\begin{aligned} \vec{\nabla}\psi_r &\Leftrightarrow \vec{n}\psi_r \\ \vec{\nabla}\times\vec{\psi} &\Leftrightarrow \vec{n}\times\vec{\psi} \\ \left\langle\vec{\nabla},\vec{\psi}\right\rangle &\Leftrightarrow \left\langle\vec{n},\vec{\psi}\right\rangle \end{aligned} \tag{8.3.1}$$

With respect to a local part of a closed boundary that is oriented perpendicular to vector $\vec{n}$ the partial differentials relate as

$$\begin{aligned} \vec{\nabla}\psi &= -\left\langle\vec{\nabla},\vec{\psi}\right\rangle + \vec{\nabla}\psi_r \pm \vec{\nabla}\times\vec{\psi} \Leftrightarrow \vec{n}\psi \\ &= -\left\langle\vec{n},\vec{\psi}\right\rangle + \vec{n}\psi_r \pm \vec{n}\times\vec{\psi} \end{aligned} \tag{8.3.2}$$

This is exploited in the surface-volume integral equations that are known as Stokes and Gauss theorems.

$$\iiint \vec{\nabla}\psi\, dV = \oiint \vec{n}\psi\, dS \tag{8.3.3}$$

$$\iiint \left\langle\vec{\nabla},\vec{\psi}\right\rangle dV = \oiint \left\langle\vec{n},\vec{\psi}\right\rangle dS \tag{8.3.4}$$

$$\iiint \vec{\nabla}\times\vec{\psi}\, dV = \oiint \vec{n}\times\vec{\psi}\, dS \tag{8.3.5}$$

$$\iiint \vec{\nabla}\psi_r\, dV = \oiint \vec{n}\psi_r\, dS \tag{8.3.6}$$

This result turns terms in the differential continuity equation into a set of corresponding integral balance equations.

The method also applies to other partial differential equations. For example

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{\psi}\right)=\vec{\nabla}\left\langle\vec{\nabla},\vec{\psi}\right\rangle-\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\vec{\psi}\Leftrightarrow\vec{\nabla}\times\left(\vec{\nabla}\times\vec{\psi}\right) \\ =\vec{n}\langle\vec{n},\vec{\psi}\rangle-\langle\vec{n},\vec{n}\rangle\vec{\psi} \tag{8.3.7}$$

$$\iiint_V\left\{\vec{\nabla}\times\left(\vec{\nabla}\times\vec{\psi}\right)\right\}dV=\oiint_S\left\{\vec{\nabla}\left\langle\vec{\nabla},\vec{\psi}\right\rangle\right\}dS-\oiint_S\left\{\left\langle\vec{\nabla},\vec{\nabla}\right\rangle\vec{\psi}\right\}dS \tag{8.3.8}$$

One dimension less, a similar relation exists.

$$\iint_S\left(\left\langle\vec{\nabla}\times\vec{a},\vec{n}\right\rangle\right)dS=\oint_C\left\langle\vec{a},d\vec{l}\right\rangle \tag{8.3.9}$$

The curl can be presented as a line integral

$$\left\langle\vec{\nabla}\times\vec{\psi},\vec{n}\right\rangle\equiv\lim_{A\to 0}\left(\frac{1}{A}\oint_C\langle\vec{\psi},d\vec{r}\rangle\right) \tag{8.3.10}$$

8.4 Derivation of physical laws

The quaternionic equivalents of Ampère's law are

$$\vec{J}\equiv\vec{\nabla}\times\vec{B}=\nabla_r\vec{E}\Leftrightarrow\vec{J}\equiv\vec{n}\times\vec{B}=\nabla_r\vec{E} \tag{8.4.1}$$

$$\iint_S\left\langle\vec{\nabla}\times\vec{B},\vec{n}\right\rangle dS=\oint_C\left\langle\vec{B},d\vec{l}\right\rangle=\iint_S\left\langle\vec{J}+\nabla_r\vec{E},\vec{n}\right\rangle dS \tag{8.4.2}$$

The quaternionic equivalents of Faraday's law are:

$$\nabla_r\vec{B}=\vec{\nabla}\times(\nabla_r\vec{\psi})=-\vec{\nabla}\times\vec{E}\Leftrightarrow\nabla_r\vec{B}=\vec{n}\times\left(\nabla_r\vec{\psi}\right)=-\vec{\nabla}\times\vec{E} \tag{8.4.3}$$

$$\oint_c\left\langle\vec{E},d\vec{l}\right\rangle=\iint_S\left\langle\vec{\nabla}\times\vec{E},\vec{n}\right\rangle dS=-\iint_S\left\langle\nabla_r\vec{B},\vec{n}\right\rangle dS \tag{8.4.4}$$

$$\vec{J}=\vec{\nabla}\times\left(\vec{B}-\vec{E}\right)=\vec{\nabla}\times\vec{\varphi}-\nabla_r\vec{\varphi}=\vec{v}\rho \tag{8.4.5}$$

$$\iint_S\left\langle\vec{\nabla}\times\vec{\varphi},\vec{n}\right\rangle dS=\oint_C\left(\left\langle\vec{\varphi},d\vec{l}\right\rangle\right)=\iint_S\langle\vec{v}\rho+\nabla_r\vec{\varphi},\vec{n}\rangle dS \tag{8.4.6}$$

The equations (8.4.4) and (8.4.6) enable the derivation of the ***Lorentz force***.

$$\vec{\nabla}\times\vec{E}=-\nabla_r\vec{B} \tag{8.4.7}$$

$$\frac{d}{d\tau}\iint_{S}\left\langle \vec{B},\vec{n}\right\rangle dS=\iint_{S(\tau_0)}\left\langle \dot{\vec{B}}(\tau_0),\vec{n}\right\rangle ds+\frac{d}{d\tau}\iint_{S(\tau)}\left\langle \vec{B}(\tau_0),\vec{n}\right\rangle ds \qquad (8.4.8)$$

The Leibniz integral equation states

$$\begin{aligned}&\frac{d}{dt}\iint_{S(\tau)}\left\langle \vec{X}(\tau_0),\vec{n}\right\rangle dS\\&=\iint_{S(\tau_0)}\left\langle \dot{\vec{X}}(\tau_0)+\left\langle \vec{\nabla},\vec{X}(\tau_0)\right\rangle \vec{v}(\tau_0),\vec{n}\right\rangle dS-\oint_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{X}(\tau_0),d\vec{l}\right\rangle\end{aligned} \qquad (8.4.9)$$

With $\vec{X}=\vec{B}$ and $\left\langle \vec{\nabla},\vec{B}\right\rangle=0$ follows

$$\begin{aligned}&\frac{d\Phi_B}{d\tau}=\\&\frac{d}{d\tau}\iint_{S(\tau)}\left\langle \dot{\vec{B}}(\tau),\vec{n}\right\rangle dS=\iint_{S(\tau_0)}\left\langle \vec{B}(\tau_0),\vec{n}\right\rangle dS-\oint_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{B}(\tau_0),d\vec{l}\right\rangle\\&=-\oint_{C(\tau_0)}\left\langle E(\tau_0),d\vec{l}\right\rangle-\oint_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{B}(\tau_0),d\vec{l}\right\rangle\end{aligned} \qquad (8.4.10)$$

The electromotive force (EMF) ε equals

$$\begin{aligned}\varepsilon&=\oint_{C(\tau_0)}\left\langle \frac{\vec{F}(\tau_0)}{q},d\vec{l}\right\rangle=-\frac{d\Phi_B}{d\tau}\bigg|_{\tau=\tau_0}\\&=\oint_{C(\tau_0)}\left\langle \vec{E}(\tau_0),d\vec{l}\right\rangle+\oint_{C(\tau_0)}\left\langle \vec{v}(\tau_0)\times\vec{B}(\tau_0),d\vec{l}\right\rangle\end{aligned} \qquad (8.4.11)$$

$$\vec{F}=q\vec{E}+q\vec{v}\times\vec{B} \qquad (8.4.12)$$

9 Stochastic control

Stochastic processes that own a characteristic function control the coherence and part of the dynamics of most of the discrete objects in the model. A displacement generator that can be considered as part of the characteristic function determines the location of geometric center of the object.

9.1 Elementary modules

Elementary modules are controlled by the first type of stochastic processes. These processes are inhomogeneous spatial Poisson point processes. They can be considered as a combination of a genuine Poisson process and a binomial process that is implemented by a spatial point spread function. The process generates an ongoing hopping path that recurrently regenerates a coherent hop landing location swarm. A location density distribution describes this swarm and equals the Fourier transform of the characteristic function of the process. Further it equals the square of the modulus of what physicists would call the wavefunction of the elementary module.

Each elementary particle behaves as an elementary module. Together, the elementary modules constitute all modules that exist in the universe. Some modules constitute modular systems.

9.2 Composite modules

Composite modules are controlled by the second type of stochastic processes. The characteristic function of these stochastic processes are dynamic superpositions of the characteristic functions of their components. The superposition coefficients act as displacement generators. This means that the composition of composite modules is defined in Fourier space. In that environment location in configuration space has no significance. Thus, components of a composite can locate far from each other in configuration space. This is the reason that entanglement exists. Entanglement becomes noticeable when components obey exclusion principles.

9.3 Atoms

Compound modules are composed modules for which the geometric centers of the platforms of the components coincide. The charges of the platforms of the elementary modules establish the binding of the corresponding platforms. Physicists and chemists call these compound modules atoms or atomic ions.

In free compound modules, the symmetry-related charges do not take part in the oscillations. The targets of the private stochastic processes of the elementary modules oscillate. This means that the hopping path of the elementary module folds around the oscillation path and the hop landing location swarm gets smeared along the oscillation path. The oscillation path is a solution of the Helmholtz equation. Each fermion must use a different oscillation mode. A change of the oscillation mode goes together with the emission or the absorption of a photon. The center of emission coincides with the geometrical center of the compound module. During the emission or absorption, the oscillation mode and the hopping path halt, such that the emitted photon does not lose its integrity. Since all photons share the same emission duration, that duration must coincide with the regeneration cycle of the hop landing location swarm. Absorption cannot be interpreted so easily. In fact, it can only be comprehended as a time-reversed emission act. Otherwise, the absorption would require an incredible aiming precision for the photon.

The type of stochastic process that controls the binding of components appears to be responsible for the absorption and emission of photons and the change of oscillation modes. If photons arrive with too low energy, then the energy is spent on the kinetic energy of the common platform. If photons arrive with too high energy, then the energy is distributed over the available oscillation modes, and the rest is spent on the kinetic energy of the common platform, or it escapes into free space. The process must somehow archive the modes of the components. It can apply the private platform of the components for that purpose. Most probably, the current value of the dynamic superposition coefficient is stored in the eigenspace of a special superposition operator.

9.4 Molecules

Molecules are conglomerates of compound modules that each keep their private geometrical center. However, electron oscillations are shared among the compound modules. Together with the symmetry-related charges, this binds the compound modules into the molecule.

10 Gravity

Mainstream physics considers the origin of the deformation of our living space as an unsolved problem. It presents the Higgs mechanism as the explanation of why some elementary particles get their mass. The Hilbert Book Model relates mass to deformation of the field that represents our universe. This deformation causes the mutual attraction of massive objects.

10.1 Difference between the Higgs field and the universe field

The Higgs field corresponds with a Higgs boson. The dynamic field that represents our universe does not own a field generating particle like the Higgs boson that is supposed to generate the Higgs field. The universe field exists always and everywhere. In fact, a private stochastic process generates each elementary particle. The stochastic process produces quaternions that break the symmetry of the background parameter space. Consequently, the embedded quaternion breaks the symmetry of the functions that apply this parameter pace. Thus, the quaternion breaks the symmetry of the field that represents the universe. However, only isotropic symmetry breaks can produce the spherical pulse responses that temporarily deform the universe field. These spherical pulse responses act as spherical shock fronts. The pulse injects volume into the field, and the shock front distributes this volume over the whole field. The volume expands the field persistently, but the initial deformation fades away. The front wipes the deformation away from the location of the pulse.

10.2 Center of mass

In a system of massive objects $p_i, i = 1,2,3,...n$, each with static mass m_i at locations r_i, the center of mass $\vec{R}$ follows from

$$\sum_{i=1}^{n} m_i \left(\vec{r}_i - \vec{R}\right) = \vec{0} \tag{10.2.1}$$

Thus

$$\vec{R} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{r}_i \tag{10.2.2}$$

Where

$$M = \sum_{i=1}^{n} m_i \tag{10.2.3}$$

In the following, we will consider an ensemble of massive objects that own a center of mass $\vec{R}$ and a fixed combined mass M as a single massive object that locates at $\vec{R}$. $\vec{R}$ can be a dynamic location. In that case, the ensemble must move as one unit. In physical reality, this construct has no point-like equivalent that owns a fixed mass. The problem with the treatise in this paragraph is that in physical reality, point-like objects that possess a static mass do not exist. Only pulse responses that temporarily deform the field exist. Except for black holes, these pulse responses constitute all massive objects that exist in universe.

10.3 Newton

Newton's laws are nearly correct in nearly flat field conditions. The main formula for Newton's laws is

$$\vec{F} = m\vec{a} \tag{10.3.1}$$

Another law of Newton treats the mutual attraction between massive objects.

$$\vec{F}(\vec{r}_1 - \vec{r}_2) = M_1 \vec{a} = \frac{G M_1 M_2 (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3} \tag{10.3.2}$$

Newton deduced this universal law of gravitation from results of experiments, but this gravitational attraction can also be derived theoretically from the gravitational potential that is produced by spherical pulse responses.

Massive objects deform the field that embeds these objects. At large distances, a simplified form of the gravitational potential describes properly what occurs.

The following relies heavily on the chapters on quaternionic differential and integral calculus.

10.4 Gauss law

The Gauss law for gravitation is

$$\oiint_{\partial V} \langle \vec{g}, dA \rangle = \iiint_V \langle \vec{\nabla}, \vec{g} \rangle dV = -4\pi G \iiint_V \rho dV = -4\pi G M \tag{10.4.1}$$

Here $\vec{g}$ is the gravitational field. G is the gravitational constant. M is the encapsulated mass. ρ is the mass density distribution. The differential form of Gauss law is

$$\langle \vec{\nabla}, \vec{g} \rangle = \langle \vec{\nabla}, \vec{\nabla} \rangle \phi = -4\pi G \rho \tag{10.4.2}$$

$$\vec{g} = -\vec{\nabla} \phi \tag{10.4.3}$$

ϕ is the gravitational field. Far from the center of mass this gravitation potential equals

$$\phi(r) = \frac{MG}{r} \tag{10.4.4}$$

10.5 A deforming field excitation

A spherical pulse response is a solution of a homogeneous second-order partial differential equation that was triggered by an isotropic pulse. The corresponding field equation and the corresponding solution are repeated here.

$$\left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi = 4\pi\delta\left(\vec{q} - \vec{q'}\right)\theta\left(\tau \pm \tau'\right) \tag{10.5.1}$$

Here the $\pm$ sign represents time inversion.

$$\psi = \frac{f\left(\left|\vec{q} - \vec{q'}\right| \pm c\left(\tau - \tau'\right)\vec{n}\right)}{\left|\vec{q} - \vec{q'}\right|} \tag{10.5.2}$$

The spherical pulse response integrates over time into the Green's function of the field. The Green's function is a solution of the Poisson equation.

$$\rho = \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \psi \tag{10.5.3}$$

The Green's function occupies some volume.

$$g(\vec{q}) = \frac{1}{4\pi\left|\vec{q} - \vec{q'}\right|} \tag{10.5.4}$$

This means that locally the pulse pumps some volume into the field, or it subtracts volume out of the field. The selection between injection and subtraction depends on the sign in the step function in the equation (10.5.1). The dynamics of the spherical pulse response shows that the injected volume quickly spreads over the field. In the case of volume subtraction, the front first collects the volume and finally subtracts it at the trigger location. Gravitation considers the case in which the pulse response injects volume into the field.

Thus, locally and temporarily, the pulse deforms the field, and the injected volume persistently expands the field.

This paper postulates that the spherical pulse response is the only field excitation that temporarily deforms the field, while the injected volume persistently expands the field.

The effect of the spherical pulse response is so tiny and so temporarily that no instrument can ever measure the effect of a single spherical pulse response in isolation. However, when recurrently regenerated in huge numbers in dense and coherent swarms, the pulse responses can cause a significant and persistent deformation that instruments can detect. This is achieved by the stochastic processes that generate the footprint of elementary modules.

The spherical pulse responses are straightforward candidates for what physicists call dark matter objects. A halo of these objects can cause gravitational lensing.

10.6 Gravitational potential

A massive object at a large distance acts as a point-like mass. Far from the center of mass, the gravitational potential of a group of massive particles with combined mass M is

$$\phi(r) \approx \frac{GM}{r} \qquad (10.6.1)$$

At this distance the gravitation potential shows the shape of the Green's function of the field; however, the amplitude differs. The formula does not indicate that the gravitational potential can cause acceleration for a uniformly moving massive object. However, the gravitational potential is the gravitational potential energy per unit mass. The relation to Newton's law is shown by the following.

The potential ϕ of a unit mass m at a distance r from a point-mass of mass M can be defined as the work W that needs to be done by an external agent to bring the unit mass in from infinity to that point.

$$\phi(\vec{r}) \approx \frac{W}{m} = \frac{1}{m}\int_{\infty}^{\vec{r}} \left\langle \vec{F}, d\vec{r} \right\rangle = \frac{1}{m}\int_{\infty}^{\vec{r}} \left\langle \frac{GmM\,\vec{r}}{|\vec{r}|^3}, d\vec{r} \right\rangle = \frac{GM}{|\vec{r}|} \qquad (10.6.2)$$

10.7 Pulse location density distribution

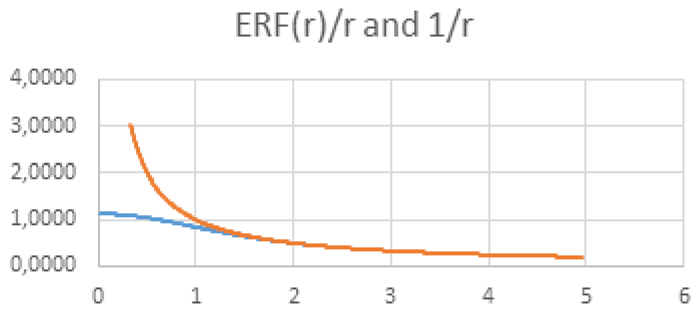
It is false to treat a pulse location density distribution as a set of point-like masses as is done in formulas (10.2.1) and (10.2.2). Instead, the gravitational potential follows from the convolution of the location density distribution and the Green's function. This calculation is still not correct, because the exact result depends on the fact that the deformation that is due to a pulse response quickly fades away and the result also depends on the density of the distribution. If these effects can be ignored, then the resulting gravitational potential of a Gaussian density distribution would be given by

$$g(r) \approx GM\,\frac{ERF(r)}{r} \qquad (10.7.1)$$

Where $ERF(r)$ is the well-known error function. Here the gravitational potential is a perfectly smooth function that at some distance from the center equals the approximated gravitational potential that was described above in equation (10.6.1). As indicated above, the convolution only offers an approximation because this computation does not account for the influence of the density of the swarm and it does not compensate for the fact that the deformation by the individual pulse responses quickly fades away. Thus, the exact result depends on the duration of the recurrence cycle of the swarm.

In the example, we apply a normalized location density distribution, but the actual location density distribution might have a higher amplitude.

This might explain why some elementary module types exist in three generations.



Due to the convolution, and the coherence of the location density distribution, the blue curve does not show any sign of the singularity that is contained in the red curve, which shows the Green's function.

In physical reality, no point-like static mass object exists. The most important lesson of this investigation is that far from the gravitational center of the distribution the deformation of the field is characterized by the here shown simplified form of the gravitation potential

$$\phi(r) \approx \frac{GM}{r} \tag{10.7.2}$$

Warning: This simplified form shares its shape with the Green's function of the deformed field. This does not mean that the Green's function owns a mass that equals $M_G = \frac{1}{G}$. The functions only share the form of their tail.

10.8 Inertia

The relation between inertia and mass is complicated. We apply a field that resists its changing. The condition that for each type of massive object, the gravitational potential is a static function and the condition that in free space, the massive object moves uniformly, establish that inertia rules the dynamics of the situation. These conditions define an artificial quaternionic field that does not change. The real part of the artificial field is represented by the gravitational potential, and the uniform speed of the massive object represents the imaginary (vector) part of the field.

The change of the quaternionic field can be divided into five separate changes that partly can compensate each other.

The first-order change of a field contains five terms. Mathematically, the statement that in first approximation nothing in the field ξ changes, indicates that locally, the first-order partial differential $\nabla\xi$ will be equal to zero.

$$\zeta = \nabla\xi = \nabla_r\xi_r - \langle\vec{\nabla},\vec{\xi}\rangle + \vec{\nabla}\xi_r + \nabla_r\vec{\xi} \pm \vec{\nabla}\times\vec{\xi} = 0 \qquad (10.8.1)$$

Thus

$$\zeta_r = \nabla_r\xi_r - \langle\vec{\nabla},\vec{\xi}\rangle = 0 \qquad (10.8.2)$$

$$\vec{\zeta} = \vec{\nabla}\xi_r + \nabla_r\vec{\xi} \pm \vec{\nabla}\times\vec{\xi} = 0 \qquad (10.8.3)$$

These formulas can be interpreted independently. For example, according to equation (10.8.2) the variation in time of ξ_r must equal the divergence of $\vec{\xi}$. The terms that are still eligible for change must together be equal to zero. For our purpose, the curl $\vec{\nabla}\times\vec{\xi}$ of the vector field $\vec{\xi}$ is expected to be zero. The resulting terms of equation (10.8.3) are

$$\nabla_r\vec{\xi} + \vec{\nabla}\xi_r = 0 \qquad (10.8.4)$$

In the following text plays $\vec{\xi}$ the role of the vector field and ξ_r plays the role of the scalar gravitational potential of the considered object. For elementary modules, this special field supports the hop landing location swarm that resides on the floating platform. It reflects the activity of the stochastic process, and the uniform movement in free space of the floating platform over the background platform. It is characterized by a mass value and by the uniform velocity of the platform with respect to the background platform. The real part conforms to the deformation that the stochastic process causes. The imaginary part conforms to the speed of movement of the floating platform. The main characteristic of this field is that it tries to keep its overall change zero. We call ξ the ***conservation field***.

At a large distance r, we approximate this potential by using formula

$$\phi(r) \approx \frac{GM}{r} \tag{10.8.5}$$

The new artificial field $\xi = \left\{ \frac{GM}{r}, \vec{v} \right\}$ considers a uniformly moving mass as a normal situation. It is a combination of the scalar potential $\frac{GM}{r}$ and the uniform speed $\vec{v}$.

If this object accelerates, then the new field $\left\{ \frac{GM}{r}, \vec{v} \right\}$ tries to counteract the change of the field $\dot{\vec{v}}$ by compensating this with an equivalent change of the real part $\frac{GM}{r}$ of the new field. According to the equation (10.8.4), this equivalent change is the gradient of the real part of the field.

$$\vec{a} = \dot{\vec{v}} = -\vec{\nabla}\left(\frac{GM}{r}\right) = \frac{GM\,\vec{r}}{|\vec{r}|^3} \tag{10.8.6}$$

This generated vector field acts on masses that appear in its realm.

Thus, if two uniformly moving masses M_1 and M_2 exist in each other's neighborhood, then any disturbance of the situation will cause the gravitational force

$$\vec{F}(\vec{r}_1 - \vec{r}_2) = M_1\vec{a} = \frac{GM_1M_2(\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3} \tag{10.8.7}$$

The disturbance by the ongoing expansion of the embedding field suffices to put the gravitational force into action. The description also holds when the field ξ describes a conglomerate of platforms and M represents the mass of the conglomerate.

The artificial field ξ represents the habits of the underlying model that ensures the constancy of the gravitational potential and the uniform floating of the considered massive objects in free space.

Inertia ensures that the third-order differential (the third-order change) of the deformed field is minimized. It does that by varying the speed of the platforms on which the massive objects reside.

Inertia bases mainly on the definition of mass that applies to the region outside the sphere where the gravitational potential behaves as the Green's function of the field. There the formula $\xi_r = \frac{m}{r}$ applies. Further, it bases in the intention of modules to keep the gravitational potential inside the mentioned sphere constant. At least that holds when this potential is averaged over the regeneration period. In that case, the overall change ζ of the conservation field ξ equals zero. Next, the definition of the conservation field supposes that the swarm which causes the deformation moves as one unit. Further, the fact is used that the solutions of the homogeneous second-order partial differential equation can superpose in new solutions of that same equation.

The popular sketch in which the deformation of our living space is presented by smooth dips is obviously false. The story that is represented in this paper shows the deformations as local extensions of the field, which represents the universe. In both sketches, the deformations elongate the information path, but none of the sketches explain why two masses attract each other. The above explanation founds on the habit of the stochastic process to recurrently regenerate the same time average of the gravitational potential, even when that averaged potential moves uniformly. Without the described habit of the stochastic processes, inertia would not exist.

The applied artificial field also explains the gravitational attraction by black holes.

The artificial field that implements mass inertia also plays a role in other fields. Similar tricks can be used to explain the electrical force from the fact that the electrical field is produced by sources and pits that can be described with the Green's function.

10.9 Elementary particles

For elementary particles, a private stochastic process generates the hop landing locations of the ongoing hopping path that recurrently forms the same hop landing location density distribution. The characteristic function of the stochastic process ensures that the same location density distribution is generated. This does not mean that the same hop landing location swarm is generated! The squared modulus of the wavefunction of the elementary particle equals the generated location density distribution. This explanation means that all elementary particles and all conglomerates of elementary particles are recurrently regenerated.

10.10 Mass

Mass is a property of objects, which has its own significance. Since at large distance, the gravitational potential always has the shape $\phi(r) \approx \frac{GM}{r}$, it does not matter what the massive object is. The formula can be used to determine the mass, even if only is known that the object in question deforms the embedding field. In that case, the formula can still be applied. This is used in the chapter about mixed fields.

In physical reality, no static point-like mass object exists.

10.11 Hop landing generation

The generation of the hopping path is an ongoing process. The generated hop landing location swarm contains a huge number of elements. Each elementary module type is controlled by a corresponding type of stochastic process. For the stochastic process, only the Fourier transform of the location density distribution of the swarm is important. Consequently, for a selected type of elementary module, it does not matter at what instant of the regeneration of the hop landing location swarm the location density distribution is determined. Thus, even when different types are bonded into composed modules, there is no need to synchronize the regeneration cycles of different types. This freedom also means that the number of elements in a hop landing location swarm may differ between elementary module types. This means that the strength of the deformation of the embedding field can differ between elementary module types. The strength of deformation relates to the mass of the elementary modules according to formula (10.6.1).

The requirement for regeneration represents a great mystery. All mass that by elementary modules generate appears to dilute away and must be recurrently regenerated. This fact conflicts with the conservation laws of mainstream physics. The deformation work done by the stochastic processes vanishes completely. What results is the ongoing expansion of the field. Thus, these processes must keep generating the particle to which they belong. The stochastic process accurately regenerates the hop landing location swarm, such that its rest mass stays the same.

Only the ongoing embedding of the content that is archived in the floating platform into the embedding field can explain the activity of the stochastic process. This supposes that at the instant of creation, the creator already archived the dynamic geometric data of his creatures into the eigenspaces of the footprint operators. These data consist of a scalar timestamp and a three-dimensional spatial location. The quaternionic eigenvalues act as storage bins.

After the instant of creation, the creator left his creation alone. The set of floating separable Hilbert spaces, together with the background Hilbert space, act as a read-only repository. After sequencing the timestamps, the stochastic processes read the storage bins and trigger the embedding of the location into the embedding field in the predetermined sequence.

10.11.1 Open question

If the instant of archival proceeds the passage of the window that scans the Hilbert Book Base Model as a function of progression, then the behavior of the model does not change. This indicates a freedom of the described model.

10.12 Symmetry-related charges

Symmetry-related charges only appear at the geometric center of the private parameter space of the separable Hilbert space that acts as the floating platform for an elementary particle. These charges represent sources or sinks for the corresponding symmetry-related field. Since these phenomena disturb the corresponding symmetry-related field in a static way that can be described by the Green's function of the field, the same trick that was used to explain inertia can be used here to explain the attraction or the repel of two symmetry-related charges Q_1 and Q_2.

$$\vec{a} = \dot{\vec{v}} = -\vec{\nabla}\left(\frac{Q}{r}\right) = \frac{Q\vec{r}}{|\vec{r}|^3} \tag{10.12.1}$$

$$\vec{F}\left(\vec{r}_1 - \vec{r}_2\right) = Q_1\vec{a} = \frac{Q_1 Q_2\left(\vec{r}_1 - \vec{r}_2\right)}{\left|\vec{r}_1 - \vec{r}_2\right|^3} \tag{10.12.2}$$

10.13 Color confinement

Some elementary particle types do not possess an isotropic symmetry. Mainstream physics indicates this fact with a corresponding color charge. Spherical pulse responses require an isotropic pulse. Thus, colored elementary particles cannot generate a gravitational potential. They must first cling together into colorless conglomerates before they can manifest as massive objects. Mesons and baryons are the colorless conglomerates that become noticeable as particles that attract other massive particles.