

## Addendum to vixra:2001.0061

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### *Abstract*

We have previously shown that 3-dimensional space equipped with minimal fractality provides a qualitative explanation for both rotation curves of disk galaxies and cosmological expansion. This brief Addendum brings up an additional argument in support of our findings.

Consider a non-relativistic, homogeneous and spherically symmetric distribution of matter surrounding a massive object placed at the center of the sphere. Let  $R$  denote the sphere radius and let's assume that the Newtonian potential falls off as [1]

$$\varphi'(r) \sim -\frac{1}{r^{1\mp\varepsilon}}, \quad \varepsilon \ll 1 \quad (1)$$

at large distances from the center. No matter is assumed to exist inside a small spherical neighborhood of radius  $\delta$  ( $0 < \delta \ll R$ ), whose function is to regularize the potential (1) at distances  $O(\delta)$  from the center. Following [1], we assume that (1) describes reasonably well gravitation on ultrashort time scales  $t \ll O(M_{EW}^{-1})$ , where  $M_{EW}$  is the Fermi scale. The energy of the gravitating system in nearly 3-dimensional space may be estimated as [2]

$$E(\varepsilon) = \int_{\delta}^R \frac{\rho G_N}{r^{1\mp\varepsilon}} d^3r = \frac{4\pi}{2 \pm \varepsilon} \rho G_N (R^{2\pm\varepsilon} - \delta^{2\pm\varepsilon}) \quad (2)$$

in which  $\rho$  stands for the mass density. Taking the ratio of (2) to the energy of the system in Newtonian mechanics at  $\varepsilon = 0$  leads to the approximation

$$\lambda(\varepsilon) = \frac{E(\varepsilon)}{E(0)} \sim R^{\pm\varepsilon} \quad (3)$$

A similar scaling behavior emerges by comparing the corresponding gravitational forces computed at  $\varepsilon \neq 0$  and  $\varepsilon = 0$ , respectively, i.e.

$$\lambda(\varepsilon) = \frac{f_N(\varepsilon)}{F_N(0)} \sim R^{\pm\varepsilon} \quad (4)$$

Both (3) and (4) are undefined in the combined limit  $R \rightarrow \infty, \varepsilon \rightarrow 0$ . Following again [1], we introduce the constraint

$$\left| \frac{dR}{d\tau} \right| \gg \left| \frac{d\varepsilon}{d\tau} \right| \quad (5)$$

where  $\tau$  plays the role of a time-like autonomous parameter. Under these conditions, it is seen that the scaling  $\lambda = R^\varepsilon$  mimics the *gravitational effect of Dark Matter*, whereas  $\lambda = R^{-\varepsilon}$  the *cosmological expansion* of the Universe. It is conceivable that (3) and (4) stem from the nonlocal structure of spacetime in the early Universe and carry over as residual effects to the observable cosmological scales [1].

## **References**

[1] Available at the following sites:

[viXra:2001.0061](https://arxiv.org/abs/2001.0061)

[https://www.researchgate.net/publication/338409772\\_Fractional\\_Spacetime\\_and\\_the\\_Emergence\\_of\\_the\\_Dark\\_Sector](https://www.researchgate.net/publication/338409772_Fractional_Spacetime_and_the_Emergence_of_the_Dark_Sector)

[https://www.academia.edu/41523295/Fractional Spacetime and the Emergence of the Dark Sector](https://www.academia.edu/41523295/Fractional_Spacetime_and_the_Emergence_of_the_Dark_Sector)

[2] <https://arxiv.org/pdf/0907.0323.pdf>