Goldbach Conjeture

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Abstract

In this article, we use method of a modified sieve of Eratosthenes to prove the Goldbach conjecture.

We use p_i for all the primes, 2,3,5,7,11,13,....., i=1,2,3,.....,

Let $p_j < \sqrt{N}$, and $p_{j+1} > \sqrt{N}$,

N is a large even integer.

Let set M = $\{n \in$(1, N); and n odd number$\}$

P $=\Pi_{2\leq i\leq j} p_i$,

for $d \mid P$ set $M_d = \{$ m ; $md \in M\}$,

obviously, $|M_d| = [\frac{|M|}{d}]$,

if $d' \mid P$, set $M_{d,d'} = \{m \in M_d$, N-md = 0, mod d'$\}$,

By seiving of the Eratosthenes for all the primes (p_i, i = 2,3,...,j),

The total of remaining numbers n of M which are those numbers in the following set,

$\{n \in M; n \neq 0 \bmod \text{p} \ \forall p \mid P, \text{ and } N - n \neq 0 \bmod \text{p} \ \forall p \mid P\}$,

and it equals to,

$$B(M) = \Sigma_{n \in M}(\Sigma_{d|(n,P)}\mu(d))(\Sigma_{d'|(N-n,P)}\mu(d')) \quad (1)$$

we have,

$$B(M) = \Sigma_{(d|P,d'|P)}\mu(d)\mu(d')(\Sigma_{(n \in M, d|(n,P), d'|(N-n,P))}1), \quad (2)$$

The summation in the last blank is zero for those $d \mid$ d' or $d' \mid d$ when d or d' is not equal 1.

It is easy to prove that,

$$||M_{d,d'}| - \frac{|M|}{LCM(d,d')}| \leq 1, \quad (3)$$

we have,

$$B(M) \approx \Sigma_{(d|P,d'|P)}\mu(d)\mu(d')\frac{|M|}{LCM(d,d')} = |M|\Pi_{(2 \leq i \leq j)}(1 - \frac{2}{p_i}), \quad (4)$$

So there are approximately $(\frac{N-1}{2})\Pi_{(2 \leq i \leq j)}(1 - \frac{2}{p_i})$ such primes in the range (1, N).

This number is larger than 3. There is at least one n which is not 1 or N-1, and obviously it is a prime number.

Also N - n is a prime number too.

This proves the Goldbach conjecture.