Twin Prime Conjeture

Xuan Zhong Ni, Campbell, CA, USA

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Abstract

In this article, we use method of a modified sieve of Eratosthenes to prove the twin prime conjecture.

We use p_i for all the primes, 2,3,5,7,11,13,....., i=1,2,3,.....,

If a prime pair (p_m, p_{m+1}) is a twin prime, then it can be written as (6k-1, 6k+1) for some k.

Theorem;

If (p_j, p_{j+1}) is a twin prime, then there are approximately $(\frac{p_j^2-1}{6})\Pi_{(3\leq i\leq j)}(1-\frac{2}{p_i})$ numbers of twin primes in the range of (p_{j+1}, p_{j+1}^2).

let set M $=\{m; m=1,...,\frac{p_{j+1}^2-1}{6}\}$,

set N $=\{n=6m+1; m\in M\}$,

P $=\Pi_{3\leq i\leq j}p_i$,

for $d\mid P$ set $M_d=\{$ $m\in M$; 6m+1 =0, mod d$\}$,

obviously, $|M_d|=[\frac{|M|}{d}]$,

if $d'\mid P$, set $M_{d,d'}=\{m\in M_d$, 6m+1 = 2, mod d'$\}$, If $(p_j,\, p_{j+1})$ is a twin prime,

By seiving of the Eratosthenes for all the primes (p_i, i = 3,4,...,j),

The total of remaining numbers n of N which are those numbers in the following set,

$\{n \in N;\, n \neq 0$ mod p $\forall p \mid P$, and $n \neq 2$ mod p $\forall p \mid P\}$,

and it equals to,

$$B(N) = \Sigma_{n\in N}(\Sigma_{d|(n,P)}\mu(d))(\Sigma_{d'|(n-2,P)}\mu(d')) \tag{1}$$

we have,

$$B(N) = \Sigma_{(d|P,d'|P)}\mu(d)\mu(d')(\Sigma_{(n\in N,d|(n,P),d'|(n-2,P))}1), \tag{2}$$

The summation in the last blank is zero for those $d \mid$ d' or $d' \mid d$ when d or d' is not equal 1.

It is easy to prove that,

$$||M_{d,d'}| - \frac{|M|}{LCM(d,d')}| \leq 1, \tag{3}$$

we have,

$$B(N) \approx \Sigma_{(d|P,d'|P)}\mu(d)\mu(d')\frac{|M|}{LCM(d,d')} = |M|\Pi_{(3\leq i\leq j)}(1 - \frac{2}{p_i}), \tag{4}$$

So there are approximately $(\frac{p_j^2-1}{6})\Pi_{(3\leq i\leq j)}(1 - \frac{2}{p_i})$ twin primes in the range $(p_{j+1},\, p_{j+1}^2)$.

This also proves the twin prime conjecture.