Chapter 1. Exercise 5.8 (a)

$\overline{[(A \cap X) \cup (B \cap \overline{X})]} = (\overline{A} \cap X) \cup (\overline{B} \cap \overline{X})$

Pr. $\overline{[(A \cap X) \cup (B \cap \overline{X})]}$

$= \overline{(A \cap X)} \cap \overline{(B \cap \overline{X})}$

$= (\overline{A} \cup \overline{X}) \cap (\overline{B} \cup X)$

$= [(\overline{A} \cup \overline{X}) \cap \overline{B}] \cup [(\overline{A} \cup \overline{X}) \cap X]$

$= [\overline{B} \cap (\overline{A} \cup \overline{X})] \cup [X \cap (\overline{A} \cup \overline{X})]$

$= [\overline{B} \cap (\overline{A} \cup \overline{X})] \cup [(X \cap \overline{A}) \cup (X \cap \overline{X})]$

$= [\overline{B} \cap (\overline{A} \cup \overline{X})] \cup (X \cap \overline{A})$

$= (X \cap \overline{A}) \cup [\overline{B} \cap (\overline{A} \cup \overline{X})]$

$= (\overline{A} \cap X) \cup [\overline{B} \cap (\overline{A} \cup \overline{X})]$

$= [(\overline{A} \cap X) \cup \overline{B}] \cap [(\overline{A} \cap X) \cup (\overline{A} \cup \overline{X})]$

$= [(\overline{A} \cap X) \cup \overline{B}] \cap (\overline{A} \cup \overline{X})$ since $\overline{A} \cap X \subseteq \overline{A} \subseteq \overline{A} \cup \overline{X}$.

$(\overline{A} \cap X) \cup (\overline{B} \cap \overline{X})$

$= [(\overline{A} \cap X) \cup \overline{B}] \cap [(\overline{A} \cap X) \cup \overline{X}]$

$= [(\overline{A} \cap X) \cup \overline{B}] \cap [\overline{X} \cup (\overline{A} \cap X)]$

$= [(\overline{A} \cap X) \cup \overline{B}] \cap [(\overline{X} \cup \overline{A}) \cap (\overline{X} \cup X)]$

$= [(\overline{A} \cap X) \cup \overline{B}] \cap (\overline{X} \cup \overline{A})$

$= [(\overline{A} \cap X) \cup \overline{B}] \cap (\overline{A} \cup \overline{X})$

REFERENCE

Stoll, R. R. 1979. Set theory and logic. New York: Dover Publications, Inc.

$\overline{[(A \cap X) \cup (B \cap \overline{X})]} = (\overline{A} \cap X) \cup (\overline{B} \cap \overline{X})$

Pr. $\overline{[(A \cap X) \cup (B \cap \overline{X})]}$

$= \overline{(A \cap X)} \cap \overline{(B \cap \overline{X})}$

$= (\overline{A} \cup \overline{X}) \cap (\overline{B} \cup X)$

$= [(\overline{A} \cup \overline{X}) \cap \overline{B}] \cup [(\overline{A} \cup \overline{X}) \cap X]$

$= [\overline{B} \cap (\overline{A} \cup \overline{X})] \cup [X \cap (\overline{A} \cup \overline{X})]$

$= [(\overline{B} \cap \overline{A}) \cup (\overline{B} \cap \overline{X})] \cup [(X \cap \overline{A}) \cup (X \cap \overline{X})]$

$= [(\overline{B} \cap \overline{A}) \cup (\overline{B} \cap \overline{X})] \cup (X \cap \overline{A})$

$= (\overline{B} \cap \overline{A}) \cup [(\overline{B} \cap \overline{X}) \cup (X \cap \overline{A})]$

$= (\overline{B} \cap \overline{X}) \cup (X \cap \overline{A})$ since $\overline{B} \cap \overline{A} \subseteq (\overline{B} \cap \overline{X}) \cup (X \cap \overline{A})$. For the proof see below.

$= (\overline{B} \cap \overline{X}) \cup (\overline{A} \cap X)$

$= (\overline{A} \cap X) \cup (\overline{B} \cap \overline{X})$

$\overline{B} \cap \overline{A}$

$= (\overline{B} \cap U) \cap \overline{A}$ U is the universal set.

$= [\overline{B} \cap (\overline{X} \cup X)] \cap \overline{A}$

$= [(\overline{B} \cap \overline{X}) \cup (\overline{B} \cap X)] \cap \overline{A}$

$= \overline{A} \cap [(\overline{B} \cap \overline{X}) \cup (\overline{B} \cap X)]$

$= [\overline{A} \cap (\overline{B} \cap \overline{X})] \cup [\overline{A} \cap (\overline{B} \cap X)]$

$= [\overline{A} \cap (\overline{B} \cap \overline{X})] \cup [\overline{A} \cap (X \cap \overline{B})]$

$= [\overline{A} \cap (\overline{B} \cap \overline{X})] \cup [(\overline{A} \cap X) \cap \overline{B}]$

$= [\overline{A} \cap (\overline{B} \cap \overline{X})] \cup [(X \cap \overline{A}) \cap \overline{B}]$

$\subseteq (\overline{B} \cap \overline{X}) \cup (X \cap \overline{A})$