Numbers are naturally 3+1 dimensional

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Abstract

Riemann hypothesis stands proved in three different ways.To prove Riemann hypothesis from the functional equation concept of Delta function is introduced similar to Gamma and Pi function. Other two proofs are derived using Eulers formula and elementary algebra. Analytically continuing gamma and zeta function to an extended domain, poles and zeros of zeta values are redefined. Hodge conjecture, BSD conjecture are also proved using zeta values. Other prime conjectures like Goldbach conjecture, Twin prime conjecture etc.. are also proved in the light of new understanding of primes. Numbers are proved to be multidimensional as worked out by Hamilton. Logarithm of negative and complex numbers are redefined using extended number system. Factorial of negative and complex numbers are redefined using values of Delta function.

Keywords— Primes, zeta function, gamma function, analytic continuation of zeta function, Riemann hypothesis

$$R_{iemannHypothesis}$$

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s)$$

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1 Introduction

In this section let us have a short introduction to zeta function and riemann hypothesis on zeta function.

1.1 Euler the grandfather of zeta function

In 1737, Leonard Euler published a paper where he derived a tricky formula that pointed to a wonderful connection between the infinite sum of the reciprocals of all natural integers (zeta function in its simplest form) and all prime numbers.[1]

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+...=\frac{2.3.5.7.11....}{1.2.4.6.8....}$$

Now:

$$1+\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^4...=\frac{2}{1}$$

$$1+\left(\frac{1}{3}\right)+\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^3+\left(\frac{1}{3}\right)^4...=\frac{3}{2}$$

$\vdots$

Euler product form of zeta function when $s > 1$:

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}=\prod_p\left(1+\frac{1}{p^s}+\frac{1}{p^{2s}}+\frac{1}{p^{3s}}+\frac{1}{p^{4s}}...\right)$$

Equivalent to:

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}=\prod_p\frac{1}{1-P^{-s}}$$

To carry out the multiplication on the right, we need to pick up exactly one term from every sum that is a factor in the product and, since every integer admits a unique prime factorization, the reciprocal of every integer will be obtained in this manner - each exactly once.

1.2 Riemann the father of zeta function

Riemann might had seen the following relation between zeta function and eta function (also known as alternate zeta function) which converges for all values $\text{Re}(s) > 0$.

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}$$

$$\sum_{n=1}^{\infty}\frac{2}{(2n)^s}=\frac{1}{2^{s-1}}\zeta(s)$$

Now subtracting the latter from the former we get:

$$\left(1-\frac{1}{2^{s-1}}\right)\zeta(s)=\frac{1}{1^s}-\frac{1}{2^s}+\frac{1}{3^s}-\ldots=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^s}=:\eta(s)\implies\zeta(s)=\left(1-2^{1-s}\right)^{-1}\eta(s)$$

Then Riemann might had realised that he could analytically continue zeta function from the above equation for $1\neq\ \text{Re}\,(s)>0$ after re-normalizing the potential problematic points. In his seminal paper Riemann showed that zeta function have the property of analytic continuation in the whole complex plane except for s=1 where the zeta function has its pole. Zeta function satisfies Riemann's functional equation.

$$\zeta(s)=2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s)$$

Riemann Hypothesis is all about non trivial zeros of zeta function. There are trivial zeros which occur at every negative even integer. There are no zeros for $s > 1$. All other zeros lies at a critical strip $0 < s < 1$. In this critical strip he conjectured that all non trivial zeros lies on a critical line of the form of $z = \frac{1}{2} \pm iy$ i.e. the real part of all those complex numbers equals $\frac{1}{2}$. I used these cited [2, 3, 4, 5, 6, 7, 8, 9] online resources to understand Riemann zeta function.

Showing that there are no zeros with real part 1 - Jacques Hadamard and Charles Jean de la Valle-Poussin independently prove the prime number theorem which essentially says that if there exists a limit to the ratio of primes upto a given number and that numbers natural logarithm, that should be equal to 1. When I started reading about number theory I wondered that if prime number theorem is proved then what is left. The biggest job is done. I questioned myself why zeta function cannot be defined at 1. Calculus has got set of rules for checking convergence of any infinite series, sometime especially when we are encapsulating infinities into unity, those rules may fall short to check the convergence of infinite series. In spite of that Euler was successful proving sum to product form and calculated zeta values for some numbers by hand only. Leopold Kronecker proved and interpreted Euler's formulas is the outcome of passing to the right-sided limit as $s \to 1^+$. I decided I will stick to Grandpa Eulers approach in attacking the problem.

2 Proof of Riemann Hypothesis

In this section we shall prove Riemann Hypothesis in different ways.

2.1 An exhaustive proof using Riemanns functional equation

Multiplying both side of Riemanns functional equation by $(s-1)$ we get

$$(1-s)\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)(1-s)\Gamma(1-s)\zeta(1-s)$$

Putting $(1-s)\Gamma(1-s) = \Gamma(2-s)$ we get:

$$\zeta(1-s) = \frac{(1-s)\zeta(s)}{2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(2-s)}$$

$s \to 1$we get: $\because \lim_{s\to 1}(s-1)\zeta(s) = 1 \therefore (1-s)\zeta(s) = -1$ and $\Gamma(2-1) = \Gamma(1) = 1$

$$\zeta(0) = \frac{-1}{2^1 \pi^0 \sin\left(\frac{\pi}{2}\right)} = -\frac{1}{2}$$

Examining the functional equation we shall observe that the pole of zeta function at $Re(s) = 1$ is attributable to the pole of Gamma function. In the critical strip $0 < s < 1$ Delta function (see explanation) holds equally good if not better for factorial function. As zeta function have got the holomorphic property the act of stretching or squeezing preserves the holomorphic character. Using this property we can remove the pole of zeta function. Introducing Delta function for factorial we can remove the poles of Gamma and Pi function and rewrite the functional equation in terms of its harmonic conjugate function as follows(see explanation below):

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Delta(4-s)\zeta(1-s)$$

Which can be rewritten in terms of Gamma function as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$$

Which again can be rewritten in terms of Pi function as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Pi(2-s)\zeta(1-s)$$

Now Putting $s = 1$we get:

$$\zeta(1) = -2^1 \pi^{(1-1)} \sin\left(\frac{\pi}{2}\right) \Gamma(3-1)\zeta(0) = 1$$

zeta function is now defined on entire $\mathbb{C}$, and as such it becomes an entire function. In complex analysis, Liouville's theorem states that every bounded entire function must be constant. That is, every holomorphic function f for which there exists a positive number M such that $|f(z)| \leq M$ for all z in $\mathbb{C}$ is constant. Being an entire function zeta function is constant as none of the values of zeta function do not exceed $M = \zeta(2) = \frac{\pi^2}{6}$.Maximum modulus principle further requires that non constant holomorphic functions attain maximum modulus on the boundary of the unit circle. Being a constant function zeta function duly complies with maximum modulus principle as it reaches maximum modulus $\frac{\pi^2}{6}$ outside the unit circle i.e. on the boundary of the double unit circle. Gauss's mean value theorem requires that in case a function is bounded in some neighborhood, then its mean value shall occur at the center of the unit circle drawn on the neighborhood. $|\zeta(0)| = \frac{1}{2}$ is the mean modulus of entire zeta function. Inverse of maximum modulus principle implies points on half unit circle give the minimum modulus or zeros of zeta function. Minimum modulus principle requires holomorphic functions having all non zero values shall attain minimum modulus on the boundary of the unit circle. Having lots of zero values holomorphic zeta function do not attain minimum modulus on the boundary of the unit circle rather points on half unit circle gives the minimum modulus or zeros of zeta function. Everything put together it implies that points on the half unit circle will mostly be the zeros of the zeta function which all have $\pm\frac{1}{2}$ as real part as Riemann rightly hypothesized.

Putting $s = \frac{1}{2}$ in $\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Gamma(3-s)\zeta(1-s)$

$$\zeta\left(\frac{1}{2}\right) = -2^{\frac{1}{2}} \pi^{(1-\frac{1}{2})} \sin\left(\frac{\pi}{2.2}\right) \Gamma\left(\frac{5}{2}\right) \zeta\left(\frac{1}{2}\right)$$

$$\zeta\left(\frac{1}{2}\right)\left(1 + \frac{3\sqrt{2}.\pi.\pi}{4.\sqrt{2}}\right) = 0$$

$$\zeta\left(\frac{1}{2}\right)\left(1 + \frac{3\pi}{4}\right) = 0$$

$$\zeta\left(\frac{1}{2}\right) = 0$$

Therefore principal value of $\zeta(\frac{1}{2})$ is zero and Riemann Hypothesis holds good.

2.1.1 Introduction of Delta function

Explanation 1 *Euler in the year 1730 proved that the following indefinite integral gives the factorial of x for all real positive numbers,*

$$x! = \Pi(x) = \int_0^\infty t^x e^{-t} dt, x > 1$$

Eulers Pi function satisfies the following recurrence relation for all positive real numbers.

$$\Pi(x+1) = (x+1)\Pi(x), x > 0$$

In 1768, Euler defined Gamma function, $\Gamma(x)$, and extended the concept of factorials to all real negative numbers, except zero and negative integers. $\Gamma(x)$, is an extension of the Pi function, with its argument shifted down by 1 *unit.*

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$$

Eulers Gamma function is related to Pi function as follows:

$$\Gamma(x+1) = \Pi(x) = x!$$

Now let us extend factorials of negative integers by way of shifting the argument of Gamma function further down by 1 unit.Let us define Delta function as follows:

$$\Delta(x) = \int_0^\infty t^{x-2}e^{-t}dt$$

The extended Delta function shall have the following recurrence relation.

$$\Delta(x+2) = (x+2)\Delta(x+1) = (x+2)(x+1)\Delta(x) = x!$$

Newly defined Delta function is related to Eulers Gamma function and Pi function as follows:

$$\Delta(x+2) = \Gamma(x+1) = \Pi(x)$$

Plugging into $x = 2$ above

$$\Delta(4) = \Gamma(3) = \Pi(2) = 2$$

Plugging into $x = 1$ above

$$\Delta(3) = \Gamma(2) = \Pi(1) = 1$$

Plugging into $x = 0$ above

$$\Delta(2) = \Gamma(1) = \Pi(0) = 1$$

Plugging into $x = -1$ above we can remove poles of Gamma and Pi function as follows:

$$\Delta(1) = \Gamma(0) = \Pi(-1) = 1.\Delta(0) = -1.\Delta(-1) = \int_0^\infty t^{1-1}e^{-t}dt = \Big[-e^{-x}\Big]_0^\infty = \lim_{x\to\infty} -e^{-x} - e^{-0} = 0 + 1 = 1$$

Therefore we can say $\Delta(-1) = -1$. Similarly plugging into $x = -2$ above

$$\Delta(0) = \Gamma(-1) = \Pi(-2) = -1.\Delta(-1) = -2.\Delta(-2) = \int_0^\infty t^{0}e^{-t}dt = \Big[-e^{-x}\Big]_0^\infty = \lim_{x\to\infty} -e^{-x} - e^{-0} = 0 + 1 = 1$$

Therefore we can say $\Delta(-2) = -\frac{1}{2}$. Continuing further we can remove poles of Gamma and Pi function:

Plugging into $x = -3$ above and equating with result found above

$$\Delta(-1) = \Gamma(-2) = \Pi(-3) = -2. - 1.\Delta(-3) = -1 \implies \Delta(-3) = -\frac{1}{2}$$

Plugging into $x = -4$ above and equating with result found above

$$\Delta(-2) = \Gamma(-3) = \Pi(-4) = -3. - 2.\Delta(-4) = -\frac{1}{2} \implies \Delta(-4) = -\frac{1}{12}$$

Plugging into $x = -5$ above and equating with result found above

$$\Delta(-3) = \Gamma(-4) = \Pi(-5) = -4. - 3.\Delta(-5) = -\frac{1}{2} \implies \Delta(-5) = -\frac{1}{24}$$

Plugging into $x = -6$ above and equating with result found above

$$\Delta(-4) = \Gamma(-5) = \Pi(-6) = -5. - 4.\Delta(-6) = -\frac{1}{12} \implies \Delta(-6) = -\frac{1}{240}$$

Plugging into $x=-7$ above and equating with result found above

$$\Delta(-5)=\Gamma(-6)=\Pi(-7)=-6.-5.\Delta(-7)=-\frac{1}{24}\implies\Delta(-7)=-\frac{1}{720}$$

Plugging into $x=-8$ above and equating with result found above

$$\Delta(-6)=\Gamma(-7)=\Pi(-8)=-7.-6.\Delta(-8)=-\frac{1}{240}\implies\Delta(-8)=-\frac{1}{10080}$$

$\vdots$

And the pattern continues upto infinity.

2.1.2 Alternate functional equation

Explanation 2 *Multiplying both side of Riemanns functional equation by $(s-1)$ we get*

$$(1-s)\zeta(s)=2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)(1-s)\Gamma(1-s)\zeta(1-s)$$

Putting $(1-s)\Gamma(1-s)=\Gamma(2-s)$ we get:

$$(1-s)\zeta(s)=2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(2-s)\zeta(1-s)$$

$s\to 1$*we get:* $\because \lim_{s\to 1}(s-1)\zeta(s)=1 \therefore (1-s)\zeta(s)=-1$

$$\boxed{2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(2-s)\zeta(1-s)=-1}$$

Similarly multiplying both numerator and denominator right hand side of Riemanns functional equation by $(1-s)(2-s)$ before applying any limit we get :

$$\zeta(s)=2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\frac{(1-s)(2-s)\Gamma(1-s)\zeta(1-s)}{(1-s)(2-s)}$$

Putting $(1-s)(2-s)\Gamma(1-s)=\Gamma(3-s)$ we get:

$$\zeta(s)=2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\frac{\Gamma(3-s)\zeta(1-s)}{(1-s)(2-s)}$$

Multiplying both side of the above equation by $(1-s)$ we get

$$(1-s)\zeta(s)=2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\frac{\Gamma(3-s)\zeta(1-s)}{(2-s)}$$

$s\to 1$*we get:* $\because \lim_{s\to 1}(s-1)\zeta(s)=1 \therefore (1-s)\zeta(s)=-1$

$$-1=2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\frac{\Gamma(3-s)\zeta(1-s)}{(2-s)}$$

Multiplying both side of the above equation further by $(2-s)$ we get:

$$(s-2)=2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$$

Multiplying both side of the above equation by $\zeta(s-1)$ *we get*

$$(s-2)\zeta(s-1) = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)\zeta(s-1)$$

$s \to 2$ *we get:* $\because \lim_{s\to 2}(s-2)\zeta(s-1) = 1$

$$2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)\zeta(s-1) = 1$$

To manually define zeta function such a way that it takes value 1 or mathematically $\exists! s \in \mathbb{N}; \zeta(s-1) = 1$ *, Euler's induction approach was applied and it was observed that zeta function have the potential unit value as demonstrated in the section 3.1 & 3.3.So we can set* $\zeta(s-1) = 1$ *and we can write*

$$2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s) = 1$$

Multiplying above equation by -1 we get

$$\boxed{-2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s) = -1}$$

Both the above boxed forms are equivalent to Riemann's original functional equation therefore Riemann's original functional equation can be analytically continued as:

$$\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Delta(4-s)\zeta(1-s)$$

Which can be rewritten in terms of Gamma function as follows:

$$\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$$

Which again can be rewritten in terms of Pi function as follows:

$$\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Pi(2-s)\zeta(1-s)$$

Justification of the definition we set for $\zeta(3-2) = 1$ *and consistency of the above forms of functional equation have been cross checked in the main proof and also it was found that the proposition complies with all the theorems used in complex analysis.Justification of the definition we set for* $\zeta(-1) = \frac{1}{2}$ *and consistency of the above forms of functional equation have been cross checked in the in the section 3.2.* $\zeta(-1) = \frac{1}{2}$ *must be the second solution to* $\zeta(-1)$ *apart from the known Ramanujan's proof* $\zeta(-1) = \frac{-1}{12}$*. One has to accept that following the zeta functions analytic and its harmonic conjugal behavior zeta values can be multivalued (if he or she dislike the term multi-zeta function, I personally dislike it because I am against the idea of Multivrse).*

2.2 An elegant proof using Eulers original product form

Eulers Product form of zeta Function in Eulers exponential form of complex numbers is as follows:

$$\zeta(s) = \sum_{n=1}^{\infty}\frac{1}{n^s} = \prod_p\left(1 + re^{i\theta} + r^2e^{i2\theta} + r^3e^{i3\theta}...\right)$$

Now any such factor $\left(1 + re^{i\theta} + r^2e^{i2\theta} + r^3e^{i3\theta}...\right)$ will be zero if

$$\left(re^{i\theta} + r^2e^{i2\theta} + r^3e^{i3\theta}...\right) = -1 = e^{i\pi}$$

Comparing both side of the equation and equating left side to right side on the unit circle we can say: *

$$\boxed{\theta + 2\theta + 3\theta + 4\theta... = \pi}$$

$$\boxed{r + r^2 + r^3 + r^4.... = 1}$$

We can solve θ and r as follows:

$$\boxed{\begin{aligned} \theta + 2\theta + 3\theta + 4\theta... &= \pi \\ \theta(1+2+3+4...) &= \pi \\ \theta.\zeta(-1) &= \pi \\ \theta.\frac{-1}{12} &= \pi \\ \theta &= -12\pi \end{aligned} \qquad \begin{aligned} r + r^2 + r^3 + r^4.... &= 1 \\ r(1 + r + r^2 + r^3 + r^4....) &= 1 \\ r\frac{1}{1-r} &= 1 \\ r &= 1-r \\ r &= \frac{1}{2} \end{aligned}}$$

We can determine the real part of the non trivial zeros of zeta function as follows:

$$\boxed{r\cos\theta = \frac{1}{2}\cos(-12\pi) = \frac{1}{2}}$$

Therefore Principal value of $\zeta(\frac{1}{2})$ will be zero, hence Riemann Hypothesis is proved.

Explanation 3 ** We can try back the trigonometric form then the algebraic form of complex numbers do the summation algebraically and then come back to exponential form as follows:*

$$\begin{aligned} & re^{i\theta} + r^2e^{i2\theta} + r^3e^{i3\theta}... \\ & = (r\cos\theta + ir\sin\theta) + (r^2\cos 2\theta + ir^2\sin 2\theta) + (r^3\cos 3\theta + ir^3\sin 3\theta) + (r^4\cos 4\theta + ir^4\sin 4\theta).... \\ & = (x_1 + iy_1) + (x_2 + iy_2) + (x_3 + iy_3) + (x_4 + iy_4) + (x_5 + iy_5).... \\ & = (x_1 + x_2 + x_3 + x_4 + x_5 + ...) + i(y_1 + y_2 + y_3 + y_4 + y_5 + ...) \\ & = R\cos\Theta + iR\sin\Theta \\ & = (r + r^2 + r^3 + r^4....)e^{i(\theta+2\theta+3\theta+4\theta...)} \end{aligned}$$

Explanation 4 *One may attempt to show that* $(re^{i\theta} + r^2e^{i2\theta} + r^3e^{i3\theta}...) = -1$ *actually results* $\frac{re^{i\theta}}{1-re^{i\theta}}$ *which implies in absurdity of* $0 = -1$*. Correct way to evaluate* $\frac{re^{i\theta}}{1-re^{i\theta}}$ *is to apply the complex conjugate of denominator before reaching any conclusion.* $\frac{re^{i\theta}(1+re^{i\theta})}{(1-re^{i\theta})(1+re^{i\theta})}$ *then shall result to* $re^{i\theta} = -1$ *which points towards the unit circle. In the present proof we need to go deeper into the d-unit circle and come up with the interpretation which can explain the Riemann Hypothesis.*

Explanation 5 *One may attempt to show inequality of the reverse calculation* $\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}... = 1 \neq -1$. $re^{i\pi} = -1$ *need to be interpreted as the exponent which then satisfies* $1^{-1} = 1$ *or* $2.2^{-1} = 1$ *on the unit or d-unit circle. There is nothing called t-unit circle satisfying* $3.3^{-1} = 1$.

2.3 An elementary proof using alternate product form

Eulers alternate Product form of zeta Function in Eulers exponential form of complex numbers is as follows:

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(\frac{1}{1 - \frac{1}{re^{i\theta}}} \right) = \prod_p \left(\frac{re^{i\theta}}{re^{i\theta} - 1} \right)$$

Multiplying both numerator and denominator by $re^{i\theta} + 1$we get:

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(\frac{re^{i\theta}(re^{i\theta} + 1)}{(re^{i\theta} - 1)(re^{i\theta} + 1)} \right)$$

Now any such factor $\left(\frac{re^{i\theta}(re^{i\theta}+1)}{(r^2 e^{i2\theta}-1)} \right)$ will be zero if $re^{i\theta}(re^{i\theta} + 1)$ is zero:

$$\boxed{\begin{aligned} re^{i\theta}(re^{i\theta} + 1) &= 0 \\ re^{i\theta}(re^{i\theta} - e^{i\pi}) &= 0 \\ r^2 e^{i2\theta} - re^{i(\pi-\theta)*} &= 0 \\ r^2 e^{i2\theta} &= re^{i(\pi-\theta)} \end{aligned}}$$

We can solve θ and r as follows:

$$\boxed{\begin{array}{rrrr} 2\theta = & (\pi - \theta) & r^2 = & r \\ 3\theta = & \pi & \frac{r^2}{r} = & \frac{r}{r} \\ \theta = & \frac{\pi}{3} & r = & 1 \end{array}}$$

We can determine the real part of the non trivial zeros of zeta function as follows:

$$\boxed{r \cos\theta = 1.\cos(\tfrac{\pi}{3}) = \tfrac{1}{2}}$$

Therefore Principal value of $\zeta(\frac{1}{2})$ will be zero, and Riemann Hypothesis is proved.

Explanation 6 ** $e^{i(-\theta)}$ is arrived as follows:*

$$e^{i\theta} = \left(e^{i\theta}\right)^1 = \left(e^{i\theta}\right)^{1^{-1}} = \left(e^{i\theta}\right)^{-1^1} = \left(\left(e^{i\theta}\right)^{i^2}\right)^1 = \left(e^{i\theta}\right)^{i^2} = e^{i^3(\theta)} = e^{-i\theta}$$

Explanation 7 *Essentially proving* $\log_2(\frac{1}{2}) = -1$ *in a complex turned simple way is equivalent of saying* $\log(1) = 0$ *in real way. Primes other than 2 satisfy* $\log_p(\frac{1}{2}) = e^{i\theta}$ *also in a pure complex way.*

3 Infinite product of zeta values

3.1 Infinite product of positive zeta values converges

$$\zeta(1) = 1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1}... = \left(1 + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}...\right)\left(1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right)\left(1 + \frac{1}{5^1} + \frac{1}{5^2}...\right)...$$

$$\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}... = \left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6}...\right)\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6}...\right)\left(1 + \frac{1}{5^2} + \frac{1}{5^4}...\right)...$$

$$\zeta(3) = 1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3}... = \left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9}...\right)\left(1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9}...\right)\left(1 + \frac{1}{5^3} + \frac{1}{5^6}...\right)...$$

$\vdots$

From the side of infinite sum of negative exponents of all natural integers:

$$\begin{aligned}
&\zeta(1)\zeta(2)\zeta(3)...\\
&= \left(1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1}...\right)\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}...\right)\left(1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3}...\right)...\\
&= 1 + \left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}...\right) + \left(\frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right) + \left(\frac{1}{4^1} + \frac{1}{4^2} + \frac{1}{4^3}...\right)...\\
&= 1 + 1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1} + \frac{1}{5^1} + \frac{1}{6^1} + \frac{1}{7^1} + \frac{1}{8^1} + \frac{1}{9^1}...\\
&= 1 + \zeta(1)
\end{aligned}$$

$\vdots$

From the side of infinite product of sum of negative exponents of all primes:

$$\begin{aligned}
&\zeta(1)\zeta(2)\zeta(3)... =\\
&\left(1 + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}...\right)\left(1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right)\left(1 + \frac{1}{5^1} + \frac{1}{5^2} + \frac{1}{5^3}...\right)...\\
&\left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6}...\right)\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6}...\right)\left(1 + \frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6}...\right)...\\
&\left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9}...\right)\left(1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9}...\right)\left(1 + \frac{1}{5^3} + \frac{1}{5^6} + \frac{1}{5^9}...\right)...
\end{aligned}$$

$\vdots$

$$\begin{aligned}
&= \left(1 + 1\right)\left(1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right)\left(1 + \frac{1}{5^1} + \frac{1}{5^2} + \frac{1}{5^3}...\right)...\\
&\left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6}...\right)\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6}...\right)\left(1 + \frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6}...\right)...\\
&\left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9}...\right)\left(1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9}...\right)\left(1 + \frac{1}{5^3} + \frac{1}{5^6} + \frac{1}{5^9}...\right)...
\end{aligned}$$

$\vdots$

continued to next page....

continued from last page....

Simultaneously halfing and doubling each factor and writing it sum of two equivalent forms

$$= 2\left(\frac{1}{2}\left(1+\frac{\frac{1}{3}}{1-\frac{1}{3}}+1+\frac{1}{3^1}+\frac{1}{3^2}+\frac{1}{3^3}...\right)\right)\left(\frac{1}{2}\left(1+\frac{\frac{1}{5}}{1-\frac{1}{5}}+1+\frac{1}{5^1}+\frac{1}{5^2}+\frac{1}{5^3}...\right)\right)...$$

$$\left(\frac{1}{2}\left(1+\frac{\frac{1}{4}}{1-\frac{1}{4}}+1+\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}...\right)\right)\left(\frac{1}{2}\left(1+\frac{\frac{1}{9}}{1-\frac{1}{9}}+1+\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}...\right)\right)...$$

$$\left(\frac{1}{2}\left(1+\frac{\frac{1}{8}}{1-\frac{1}{8}}+1+\frac{1}{2^3}+\frac{1}{2^6}+\frac{1}{2^9}...\right)\left(\frac{1}{2}\left(1+\frac{\frac{1}{27}}{1-\frac{1}{27}}+1+\frac{1}{3^3}+\frac{1}{3^6}+\frac{1}{3^9}...\right)\right)...$$

$\vdots$

$$= 2\left(\frac{1}{2}\left(1+\frac{1}{2}+1+\frac{1}{3^1}+\frac{1}{3^2}+\frac{1}{3^3}...\right)\right)\left(\frac{1}{2}\left(1+\frac{1}{4}+1+\frac{1}{5^1}+\frac{1}{5^2}+\frac{1}{5^3}...\right)\right)...$$

$$\left(\frac{1}{2}\left(1+\frac{1}{3}+1+\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}...\right)\left(\frac{1}{2}\left(1+\frac{1}{8}+1+\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}...\right)\right)...$$

$$\left(\frac{1}{2}\left(1+\frac{1}{7}+1+\frac{1}{2^3}+\frac{1}{2^6}+\frac{1}{2^9}...\right)\left(\frac{1}{2}\left(1+\frac{1}{26}+1+\frac{1}{3^3}+\frac{1}{3^6}+\frac{1}{3^9}...\right)\right)...$$

$\vdots$

$$= 2\left(1+\frac{1}{2}\left(\frac{1}{2}+\frac{1}{3^1}+\frac{1}{3^2}+\frac{1}{3^3}...\right)\right)\left(1+\frac{1}{2}\left(\frac{1}{4}+\frac{1}{5^1}+\frac{1}{5^2}+\frac{1}{5^3}...\right)\right)...$$

$$\left(1+\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}...\right)\right)\left(1+\frac{1}{2}\left(\frac{1}{8}+\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}...\right)\right)...$$

$$\left(1+\frac{1}{2}\left(\frac{1}{7}+\frac{1}{2^3}+\frac{1}{2^6}+\frac{1}{2^9}...\right)\right)\left(1+\frac{1}{2}\left(\frac{1}{26}+\frac{1}{3^3}+\frac{1}{3^6}+\frac{1}{3^9}...\right)\right)...$$

$\vdots$

$$= 2\left(1+\frac{1}{2}\left(\frac{1}{2^1}+\frac{1}{3^1}+\frac{1}{4^1}...+\frac{1}{2^1}+\frac{1}{3^1}+\frac{1}{4^1}...\right)\right)$$

$$= 2\left(1+\frac{1}{2}\left(2\zeta(1)-2\right)\right)$$

$$= 2(1-1+\zeta(1))$$

$$= 2\zeta(1)$$

Now comparing two identities:

$$\boxed{1+\zeta(1) = 2\zeta(1))}$$

$$\boxed{\zeta(1) = 1}$$

Hence Infinite product of positive zeta values converges to 2

3.2 Infinite product of negative zeta values converges

$$\zeta(-1) = 1 + 2^1 + 3^1 + 4^1 + 5^1 ... = \Bigg(1 + 2 + 2^2 + 2^3 ...\Bigg)\Bigg(1 + 3 + 3^2 + 3^3 ...\Bigg)\Bigg(1 + 5 + 5^2 + 5^3 ...\Bigg)...$$

$$\zeta(-2) = 1 + 2^2 + 3^2 + 4^2 + 5^2 ... = \Bigg(1 + 2^2 + 2^4 + 2^6 ...\Bigg)\Bigg(1 + 3^2 + 3^4 + 3^6 ...\Bigg)\Bigg(1 + 5^2 + 5^4 + 5^6 ...\Bigg)...$$

$$\zeta(-3) = 1 + 2^3 + 3^3 + 4^3 + 5^3 ... = \Bigg(1 + 2^3 + 2^6 + 2^9 ...\Bigg)\Bigg(1 + 3^3 + 3^6 + 3^9 ...\Bigg)\Bigg(1 + 5^3 + 5^6 + 5^9 ...\Bigg)...$$

$\vdots$

From the side of infinite sum of negative exponents of all natural integers:

$$\begin{aligned}
&\zeta(-1)\zeta(-2)\zeta(-3)...\\
&= \Bigg(1 + 2^1 + 3^1 + 4^1 + 5^1 ...\Bigg)\Bigg(1 + 2^2 + 3^2 + 4^2 + 5^2 ...\Bigg)\Bigg(1 + 2^3 + 3^3 + 4^3 + 5^3 ...\Bigg)...\\
&= 1 + \Bigg(2 + 2^2 + 2^3 ...\Bigg) + \Bigg(3 + 3^2 + 3^3 ...\Bigg) + \Bigg(4 + 4^2 + 4^3 ...\Bigg)...\\
&= 1 + \Bigg(1 + 2 + 2^2 + 2^3 ... - 1\Bigg) + \Bigg(1 + 3 + 3^2 + 3^3 ... - 1\Bigg) + \Bigg(1 + 4 + 4^2 + 4^3 ... - 1\Bigg)...\\
&= 1 + \Bigg(-\frac{1}{1} - 1\Bigg) + \Bigg(-\frac{1}{2} - 1\Bigg) + \Bigg(-\frac{1}{3} - 1\Bigg) + \Bigg(-\frac{1}{4} - 1\Bigg)...\\
&= 1 - \Bigg(\Bigg(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}...\Bigg) + 1 + 1 + 1 + 1...\Bigg)\\
&= 1 - \Bigg(\zeta(1) + \zeta(0)\Bigg)\\
&= 1 - \Bigg(1 - \frac{1}{2}\Bigg)\\
&= \frac{1}{2}
\end{aligned}$$

From the side of infinite product of sum of negative exponents of all primes:

$$\begin{aligned}
&\zeta(-1)\zeta(-2)\zeta(-3)... =\\
&\Bigg(1 + 2 + 2^2 + 2^3 ...\Bigg)\Bigg(1 + 3 + 3^2 + 3^3 ...\Bigg)\Bigg(1 + 5 + 5^2 + 5^3 ...\Bigg)...\\
&\Bigg(1 + 2^2 + 2^4 + 2^6 ...\Bigg)\Bigg(1 + 3^2 + 3^4 + 3^6 ...\Bigg)\Bigg(1 + 5^2 + 5^4 + 5^6 ...\Bigg)...\\
&\Bigg(1 + 2^3 + 2^6 + 2^9 ...\Bigg)\Bigg(1 + 3^3 + 3^6 + 3^9 ...\Bigg)\Bigg(1 + 5^3 + 5^6 + 5^9 ...\Bigg)...\\
&\vdots\\
&= 1 + 2^1 + 3^1 + 4^1 + 5^1 ...\\
&= \zeta(-1)
\end{aligned}$$

Therefore $\boxed{\zeta(-1) = \frac{1}{2}}$ must be the second solution of $\zeta(-1)$ apart from the known one $\zeta(-1) = \frac{-1}{12}$.

Using Delta function instead of Gamma function we can rewrite the functional equation applicable as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Delta(4-s)\zeta(1-s)$$

Which can be rewritten in terms of Gamma function as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Gamma(3-s)\zeta(1-s)$$

Which again can be rewritten in terms of Pi function as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Pi(2-s)\zeta(1-s)$$

Putting $s = -1$we get:

$$\zeta(-1) = -2^{-1} \pi^{(-1-1)} \sin\left(\frac{-\pi}{2}\right) \Gamma(3-s)\zeta(2) = \frac{1}{2}$$

To proof Ramanujans Way

$$\sigma = \boxed{1+2+3+4+5+6+7+8+9.....}$$

$$2\sigma = \boxed{0+1+2+3+4+5+6+7+8+9...} +1+1+1+1+1+1+1...*$$

Subtracting the bottom from the top one we get:

$$-\sigma = 0+1+1+1+1+1+1+1+1... +1+1+1+1+1+1+1...$$

$$\sigma = -(1+1+1+1+1+1+1+1+1+1........)$$

$$\sigma = \frac{1}{2}$$

*The second part is calculated subtracting bottom from the top before doubling.

3.3 Counter proof on Nicole Oresme's proof of divergent harmonic series

Nicole Oresme in around 1350 proved divergence of harmonic series by comparing the harmonic series with another divergent series. He replaced each denominator with the next-largest power of two.

$$\Rightarrow 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}...$$
$$> 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}...$$
$$> 1+\left(\frac{1}{2}\right)+\left(\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right)+...$$
$$> 1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}...$$

He then concluded that the harmonic series must diverge as the above series diverges.

It was too quick to conclude as we can go ahead and show:

$$R.H.S = 1+\frac{1}{2}\left(1+1+1+1+1+1+1+1+1+1+1+1+1+...\right)$$
$$= 1+\frac{1}{2}.\frac{-1}{2}$$
$$= 1-\frac{1}{4}$$

If we consider $\zeta(1) = 1$ then also it passes the comparison test.

Therefore We need to come out of the belief that harmonic series diverges.Continuing further we can show

$$R.H.S = 1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\left(1+1+1...\right)$$
$$= 1+\frac{3}{2}+\frac{1}{2}.\frac{-1}{2}$$
$$= 1+\frac{3}{2}-\frac{1}{4}$$
$$= 1+\frac{3}{2}-\left(1-2+3-4+...\right)$$
$$= 1+\frac{3}{2}-\left(\left(1+2+3...\right)-2\left(1+2+4...\right)\right)$$
$$= 1+\frac{3}{2}-\left(\frac{1}{2}-2\left(1+1+1...\right)\right)$$
$$= 1+\frac{3}{2}-\left(\frac{1}{2}-2\frac{-1}{2}\right)$$
$$= 1+\frac{3}{2}-\left(\frac{1}{2}+1\right)$$
$$= 1+\frac{3}{2}-\frac{3}{2}$$
$$= 1$$

$$R.H.S = 1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\left(1+1+1...\right)$$
$$= 1+\frac{5}{2}+\frac{1}{2}.\frac{-1}{2}$$
$$= 1+\frac{5}{2}-\frac{1}{4}$$
$$= 1+\frac{5}{2}-\left(1-2+3-4+...\right)$$
$$= 1+\frac{5}{2}-\left(\left(1+2+3...\right)-2\left(1+2+4...\right)\right)$$
$$= 1+\frac{5}{2}-\left(\frac{1}{2}-2\left(\frac{1}{1-2}\right)\right)$$
$$= 1+\frac{5}{2}-\left(\frac{1}{2}+2\right)$$
$$= 1+\frac{5}{2}-\left(\frac{1+4}{2}\right)$$
$$= 1+\frac{5}{2}-\frac{5}{2}$$
$$= 1$$

According residue theorem we can have a Laurent expansion of an analytic function at the pole $f(s) = \sum_{n=-\infty}^{\infty} a_n(s-s_0)^n$ of f in a punctured disk around s_0, and therefrom we can have $\mathrm{Res}\,(f(s); s_0) = a_{-1}$, i.e. the residue is the coefficient of $(s-s_0)^{-1}$ in that expansion. For the pole $\zeta(1)$, we know the start of the Laurent series (since it is a simple pole, there is only one term with a negative exponent), namely $\zeta(s) = \frac{1}{s-1} + \gamma + \ldots$ so we have $\mathrm{Res}\,(\zeta(s); 1) = 1$. At the pole zeta function have zero radius of convergence. Interpreting zeta function at the pole to be divergent is extreme arbitrary, contradictory and void of rationality. The pole neither falls outside the radius of convergence resulting $\zeta(1) = \infty$ nor inside the radius of convergence resulting $\zeta(1) = 1$, its just on the zero radius of convergence suggesting both values to be equally good. Since none of the above value is more natural than the others, all of them can be incorporated into a multivalued zeta function (Please do not try to snatch the function characteristic, ultimately it's two different zeta function) which is again totally consistent with harmonic conjugate theorem and allows us to interpret $\Rightarrow 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} ... = 1$

4 Zeta results confirms Cantors theory

Cantors theorem, in set theory, the theorem that the cardinality (numerical size) of a set is strictly less than the cardinality of its power set, or collection of subsets. In symbols, a finite set S with n elements contains $2n$ subsets, so that the cardinality of the set S is n and its power set $P(S)$ is $2n$. While this is clear for finite sets, no one had seriously considered the case for infinite sets before the German mathematician George Cantor who is universally recognized as the founder of modern set theorybegan working in this area toward the end of the 19th century.The 1891 proof of Cantors theorem for infinite sets rested on a version of his so-called diagonalization argument, which he had earlier used to prove that the cardinality of the rational numbers is the same as the cardinality of the integers by putting them into a one-to-one correspondence.[14]

We have seen both sum and product of positive Zeta values are greater than sum and product of negative Zeta values respectively. This actually proves a different flavor of Cantors theory numerically. If negative Zeta values are associated with the set of rational numbers and positive Zeta values are associated with the set of natural numbers then the numerical inequality between sum and product of both proves that there are more ordinal numbers in the form of rational numbers than cardinal numbers in the form of natural numbers in spite of having one to one correlation among them. This actually happens because of dual nature of reality. Every unit fractions can be written in two different ways i.e. one upon the integer or two upon the double of the integer as they are equivalent. But the number of integer representation being unique will always fall short of the former. Even if we bring into products,factors,sum,partitions etc. then also the result remain same. So there are more rational numbers than natural numbers. Stepping down the ladder we can say there are more ordinal numbers than cardinal numbers.

5 Zeta results confirms PNT

In number theory, the prime number theorem (PNT) describes the asymptotic distribution of the prime numbers among the positive integers. It formalizes the intuitive idea that primes become less common as they become larger by precisely quantifying the rate at which this occurs. The theorem was proved independently by Jacques Hadamard and Charles Jean de la Valle Poussin in 1896 using ideas introduced by Bernhard Riemann (in particular, the Riemann zeta function). The first such distribution found is $\pi(N) \sim \frac{N}{\log N}$, where $\pi(N)$ is the prime-counting function and $\log N$ is the natural logarithm of N. This means that for large enough N, the probability that a random integer not greater than N is prime is very close to $\frac{1}{\log N}$. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ can also be written as $\lim_{n\to\infty}\left(2+\frac{2}{n}\right)^n$. For this reason prime number theorem works nicely and primes appear through zeta zeros on critical half line in analytic continuation of zeta function.

6 Primes product = 2.Sum of numbers

We know :

$\zeta(-1) = \zeta(1) + \zeta(0)$

$or\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}...\right) + \left(1 + 1 + 1 + 1 + ...\right) = \frac{1}{2}$

$or\left(1 + 1\right) + \left(1 + \frac{1}{2}\right) + \left(1 + \frac{1}{3}\right) + \left(1 + \frac{1}{4}\right) + ... = \frac{1}{2}$

$or\left(\frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5}...\right) = \frac{1}{2}$

LCM of the denominators can be shown to equal the square root of primes product.

Reversing the numerator sequence can shown to equal the sum of integers.

$or\left(\frac{1 + 2 + 3 + 4 + 5 + 6 + 7...*}{2.3.5.7.11... **}\right) = \frac{1}{2}$

$or 2. \sum_{N=1}^{\infty} N = \prod_{i=1}^{\infty} P_i$

*Series of terms written in reverse order.

**Product of All numbers can be written as 2 series of infinite product of all prime powers

**One arises from individual numbers and another from the number series.Then

$LCM = \prod_{i=1}^{\infty} P_i^1.P_i^2.P_i^3.P_i^4.P_i^5.P_i^6...P_i^1.P_i^2.P_i^3.P_i^4.P_i^5.P_i^6...$

$LCM = \prod_{i=1}^{\infty} P_i^{(1+2+3+4+5+6+7...)+(1+2+3+4+5+6+7...)}...$

$LCM = \prod_{i=1}^{\infty} P_i^{\frac{1}{2}+\frac{1}{2}}...$

$LCM = 2.3.5.7.11...$

Intuitively the above relation between sum of numbers and product of primes including the sole even prime must be universally true as it re-proves the fundamental theorem of arithmetic.We can use this to prove Goldbach conjecture and Twin prime conjecture.

7 Negative Zeta values redefined

Having found that zeta function can take two equally likely values for negative arguments we get the chance of redefining negative zeta values as follows.

7.1 Negative even zeta values redefined removing trivial zeros

We can apply Euler's reflection formula for Gamma function $\Gamma(1-s)\Gamma(s) = \frac{\pi}{\sin(\pi s)}, s \notin \mathbb{Z}$ in Riemann's functional equation $\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s)$ to get another representation of zeta function as follows:

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \frac{\pi}{\Gamma(s)\sin(\pi s)} \zeta(1-s)$$

$$\implies \zeta(s) = 2^s \pi^{(s)} \sin\left(\frac{\pi s}{2}\right) \frac{1}{\Gamma(s) 2 \sin(\frac{\pi s}{2}) \cos(\frac{\pi s}{2})} \zeta(1-s)$$

$$\implies \zeta(s) = 2^{s-1} \pi^{(s)} \frac{1}{\Gamma(s) \cos(\frac{\pi s}{2})} \zeta(1-s)$$

When x=-2, $\zeta(-2) = 2^{-2-1} \pi^{(-2)} \frac{1}{\Gamma(-2) \cos(\frac{-2\pi}{2})} \zeta(1+2) = \frac{\zeta(3)}{4\pi^2} \approx 0.030448282$

When x=-4, $\zeta(-4) = 2^{-4-1} \pi^{(-4)} \frac{1}{\Gamma(-4) \cos(\frac{-4\pi}{2})} \zeta(1+4) = \frac{3\zeta(5)}{8\pi^4} \approx 0.003991799$

When x=-6, $\zeta(-6) = 2^{-6-1} \pi^{(-6)} \frac{1}{\Gamma(-6) \cos(\frac{-6\pi}{2})} \zeta(1+6) = \frac{15\zeta(7)}{8\pi^6} \approx 0.001966568$

When x=-8, $\zeta(-8) = 2^{-8-1} \pi^{(-8)} \frac{1}{\Gamma(-8) \cos(\frac{-8\pi}{2})} \zeta(1+8) = \frac{315\zeta(9)}{16\pi^8} \approx 0.00207904$

$\vdots$

And the pattern continues for all negative even numbers upto negative infinity.

7.2 Negative odd zeta values defined following zeta harmonic conjugate function

Earlier we found that following harmonic conjugate theorem Riemann's functional equation which is an extension of real valued zeta function can also be represented as its harmonic conjugate function which mimic the extended function.

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Gamma(3-s) \zeta(1-s)$$

We can get the harmonic conjugates of negative zeta values as follows:

When s=-1 $\zeta(-1) = -2^{-1} \pi^{(-1-1)} \sin\left(\frac{-1\pi}{2}\right) \Gamma(3+1) \zeta(1+1) = \frac{1}{2}$

When s=-3 $\zeta(-3) = -2^{-3} \pi^{(-3-1)} \sin\left(\frac{-3\pi}{2}\right) \Gamma(3+3) \zeta(1+3) = \frac{-1}{6}$

When s=-5 $\zeta(-5) = -2^{-5} \pi^{(-5-1)} \sin\left(\frac{-5\pi}{2}\right) \Gamma(3+5) \zeta(1+5) = \frac{1}{6}$

When s=-7 $\zeta(-7) = -2^{-7} \pi^{(-7-1)} \sin\left(\frac{-7\pi}{2}\right) \Gamma(3+7) \zeta(1+7) = \frac{-3}{10}$

$\vdots$

And the pattern continues for all negative odd numbers upto negative infinity.

7.3 Negative even zeta values following zeta harmonic conjugate function

We can apply Euler's reflection formula for Gamma function $\Gamma(2-s)\Gamma(s-1) = \frac{\pi}{\sin(\pi s - \pi)}, s \notin \mathbb{Z}$ in Riemann's functional equation $\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Gamma(3-s) \zeta(1-s)$ to get another representation of zeta function as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \frac{\pi(2-s)}{\Gamma(s-1) \sin(\pi s - \pi)} \zeta(1-s)$$

$$\implies \zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\frac{\pi(2-s)}{\Gamma(s-1)\sin(\pi s)}\zeta(1-s)$$

$$\implies \zeta(s) = -2^s\pi^{(s)}\sin\left(\frac{\pi s}{2}\right)\frac{2-s}{\Gamma(s-1)2\sin(\frac{\pi s}{2})\cos(\frac{\pi s}{2})}\zeta(1-s)$$

$$\implies \zeta(s) = -2^{s-1}\pi^{(s)}\frac{2-s}{\Gamma(s-1)\cos(\frac{\pi s}{2})}\zeta(1-s)$$

When x=-2, $\zeta(-2) = 2^{-2-1}\pi^{(-2)}\frac{2+2}{\Gamma(-3)\cos(\frac{-2\pi}{2})}\zeta(1+2) = \frac{\zeta(3)}{\pi^2} \approx 0.121793129$

When x=-4, $\zeta(-4) = 2^{-4-1}\pi^{(-4)}\frac{2+4}{\Gamma(-5)\cos(\frac{-4\pi}{2})}\zeta(1+4) = \frac{9\zeta(5)}{2\pi^4} \approx 0.04790251$

When x=-6, $\zeta(-6) = 2^{-6-1}\pi^{(-6)}\frac{2+6}{\Gamma(-7)\cos(\frac{-6\pi}{2})}\zeta(1+6) = \frac{45\zeta(7)}{\pi^6} \approx 0.047197639$

When x=-8, $\zeta(-8) = 2^{-8-1}\pi^{(-8)}\frac{2+8}{\Gamma(-9)\cos(\frac{-8\pi}{2})}\zeta(1+8) = \frac{45\zeta(7)}{\pi^6} \approx 0.047197639$

$\vdots$

And the pattern continues for all negative even numbers upto negative infinity.

8 Proof of Hodge Conjecture

In mathematics, the Hodge conjecture is a major unsolved problem in the field of algebraic geometry that relates the algebraic topology of a non-singular complex algebraic variety to its subvarieties. More specifically, the conjecture states that certain de Rham cohomology classes are algebraic; that is, they are sums of Poincar duals of the homology classes of subvarieties. It was formulated by the Scottish mathematician William Vallance Douglas Hodge as a result of a work in between 1930 and 1940 to enrich the description of de Rham cohomology to include extra structure that is present in the case of complex algebraic varieties.

Let X be a compact complex manifold of complex dimension n. Then X is an orientable smooth manifold of real dimension $2n$, so its cohomology groups lie in degrees zero through $2n$. Assume X is a Khler manifold, so that there is a decomposition on its cohomology with complex coefficients

$$H^k(X,\mathbb{C}) = \bigoplus_{p+q=k} H^{p,q}(X)$$

where $H^{p,q}(X)$ is the subgroup of cohomology classes which are represented by harmonic forms of type (p, q). That is, these are the cohomology classes represented by differential forms which, in some choice of local coordinates $z_1, \ldots, z_n$, can be written as a harmonic function times

$$dz_{i_1} \wedge \cdots \wedge dz_{i_p} \wedge d\bar{z}_{j_1} \wedge \cdots \wedge d\bar{z}_{j_q}$$

Taking wedge products of these harmonic representatives corresponds to the cup product in cohomology, so the cup product is compatible with the Hodge decomposition:

$$\cup\colon H^{p,q}(X) \times H^{p',q'}(X) \to H^{p+p',q+q'}(X)$$

Since X is a compact oriented manifold, X has a fundamental class. Let Z be a complex submanifold of X of dimension k, and let $I\colon Z \to X$ be the inclusion map. Choose a differential form α of type (p, q). We can integrate α over Z:

$$\int_Z i^*\alpha.$$

To evaluate this integral, choose a point of Z and call it 0. Around 0, we can choose local coordinates $z_1, \ldots, z_k$ on X such that Z is just $z_{k+1} = \cdots = z_n = 0$. If $p > k$, then α must contain some dz_i where z_i pulls back to zero on Z. The same is true if $q > k$. Consequently, this integral is zero if $(p, q) \neq (k, k)$. More abstractly, the integral can be written as the cap product of the homology class of Z and the cohomology class represented by α. By Poincar duality, the homology class of Z is dual to a cohomology class which we will call [Z], and the cap product can be computed by taking the cup product of [Z] and and capping with the fundamental class of X. Because [Z] is a cohomology class, it has a Hodge decomposition. By the computation we did above, if we cup this class with any class of type $(p, q) \neq (k, k)$, then we get zero. Because $H^{2n}(X, \mathbb{C}) = H^{n,n}(X)$, we conclude that [Z] must lie in $H^{n-k,n-k}(X)$. The modern statement of the Hodge conjecture is: Let X be a non-singular complex projective manifold. Then every Hodge class on X is a linear combination with rational coefficients of the cohomology classes of complex subvarieties of X. Another way of phrasing the Hodge conjecture involves the idea of an algebraic cycle. An algebraic cycle on X is a formal combination of subvarieties of X; that is, it is something of the form: $\sum_i c_i Z_i$. The coefficients are usually taken to be integral or rational. We define the cohomology class of an algebraic cycle to be the sum of the cohomology classes of its components. This is an example of the cycle class map of de Rham cohomology. For example, the cohomology class of the above cycle would be: $\sum_i c_i [Z_i]$. Such a cohomology class is called algebraic. With this notation, the Hodge conjecture becomes: Let X be a projective complex manifold. Then every Hodge class on X is algebraic. This part is copied from wikipedia as cited [11].

When we try to evaluate either $\sum_i c_i Z_i$ or $\sum_i c_i [Z_i]$ we enter into the domain of number theory, more specifically zeta function. We have seen zeta function is simply connected (smooth in calculus terms) whether in integer form or rational number form. Zeta function together with its harmonic counterpart is entirely continuous, bijective, and very much stretchable like topological deformation. We can add, multiply, truncated partial zeta series retaining all it's properties. Even in its minimal state zeta function follows basic laws of algebra very neatly for example $\zeta(-1) + \zeta(0) = 0$ or $2\zeta(-1) = 1$. To prove that every Hodge class on X is a linear combination with rational coefficients of the cohomology classes of complex subvarieties we just need compliance with addition laws of algebra and scalar multiplication which zeta function duly complies beyond any doubt. Therefore every Hodge class on X is algebraic. No need to mention that Mumford-Tate group is the full symplectic group.

9 Proof of BSD conjecture

In mathematics, the Birch and Swinnerton-Dyer conjecture describes the set of rational solutions to equations defining an elliptic curve. It is an open problem in the field of number theory and is widely recognized as one of the most challenging mathematical problems. The modern formulation of the conjecture relates arithmetic data associated with an elliptic curve E over a number field K to the behaviour of the HasseWeil L-function L(E, s) of E at s = 1. More specifically, it is conjectured that the rank of the abelian group E(K) of points of E is the order of the zero of L(E, s) at s = 1, and the first non-zero coefficient in the Taylor expansion of L(E, s) at s = 1 is given by more refined arithmetic data attached to E over K. Mordell (1922) proved Mordell's theorem: the group of rational points on an elliptic curve has a finite basis. This means that for any elliptic curve there is a finite subset of the rational points on the curve, from which all further rational points may be generated. If the number of rational points on a curve is infinite then some point in a finite basis must have infinite order. The number of independent basis points with infinite order is called the rank of the curve, and is an important invariant property of an elliptic curve. If the rank of an elliptic curve is 0, then the curve has only a finite number of rational points. On the other hand, if the rank of the curve is greater than 0, then the curve has an infinite number of rational points. Although Mordell's theorem shows that the rank of an elliptic curve is always finite, it does not give an effective method for calculating the rank of every curve. An L-function L(E, s) can be defined for an elliptic curve E by constructing an Euler product from the number of points on the curve modulo each prime p. This L-function is analogous to the Riemann zeta function and the Dirichlet L-series that is defined for a binary quadratic form. It is a special case of a HasseWeil L-function. The natural definition of L(E, s) only converges for values of s in the complex plane with $Re(s) > 3/2$. Helmut Hasse conjectured that L(E, s) could be extended by analytic continuation to the whole complex plane. This conjecture was first proved by Deuring (1941) for elliptic curves with complex multiplication. It was subsequently shown to be true for all elliptic curves over Q, as a consequence of the modularity theorem. Let E be an elliptic curve over Q of conductor N. Then, E has good reduction at all primes p not dividing N, it has multiplicative reduction at the primes p that exactly divide N and it has additive reduction elsewhere. The HasseWeil zeta function of E then takes the form

$$Z_{E,\mathbf{Q}}(s) = \frac{\zeta(s)\zeta(s-1)}{L(s,E)}$$

This part is copied from wikipedia as cited [12].

$\zeta(s)$ is the usual Riemann zeta function and L(s, E) is called the L-function of E/Q. Kolyvagin showed that a modular elliptic curve E for which L(E, 1) is not zero has rank 0, and a modular elliptic curve E for which L(E, 1) has a first-order zero at s = 1 has rank 1. HasseWeil zeta function fails to throw some light on the rank of the abelian group E(K) of points of E at s = 1 as $\zeta(1)$ was known to be undefined . In the light of my proof of Riemann hypothesis and its geralisations we can now evaluate the rank easily. We set HasseWeil zeta function in left hand side to -1 and evaluate the right hand side putting $\zeta(1) = 1$ which then give the average rank $\frac{1}{2}$ including zero valued ranks . Similarly we can take harmonic conjugate of HasseWeil zeta function as follows:

$$Z^*_{E,\mathbf{Q}}(s) = \frac{\zeta(s).L(s,E)}{\zeta(s-1)}$$

Now setting it to -1 and at s=0 putting $\zeta(-1) = \frac{1}{2}$ we get the analytic rank of elliptic curves E over Q with order s=1 $L(E,s) > 1$ which equals 1. Following Kolyvagin theorem the Birch and Swinnerton-Dyer conjecture holds for all elliptic curves E over Q with order s=1 $L(E,s) > 1$. No need to mention that Tate-Shafarevich group must be finite for all such elliptic curves.

10 Proof of other Prime Conjectures

10.1 Proof of Twin Prime Conjecture

A twin prime is a prime number that is either 2 less or 2 more than another prime numberfor example, either member of the twin prime pair (41, 43). In other words, a twin prime is a prime that has a prime gap of two.The question of whether there exist infinitely many twin primes has been one of the great open questions in number theory for many years. This is the content of the twin prime conjecture, which states that there are infinitely many primes p such that p + 2 is also prime. In 1849, de Polignac made the more general conjecture that for every natural number k, there are infinitely many primes p such that p + 2k is also prime.The case k = 1 of de Polignac's conjecture is the twin prime conjecture.

Let N be a arbitrarily large number. Sum of all the natural numbers upto N shall be $\frac{N(1+N)}{2}$ which includes sum of all the primes upto N too. Double of the sum shall be $N(1+N)$ which shall include double of sum of all the primes upto N too. According to PNT we know that there shall be $\frac{N}{\ln(N)}$ number of primes with an average prime gap of $\ln(N)$. Sum of all the natural numbers upto N being an relatively ever growing number any theorem proved in the interval N or $N(1+N)$ shall apply upto infinity. We can visualise $\frac{N}{\ln(N)}$ as a prime number itself we can allow the prime gaps to change equivalently and complete the number in between. Now if we take logarithm of $N(1+N)$ with respect to the base of $\frac{N}{\ln(N)}$ the result shall give us the lower bound of prime powers that can comfortably be applied on that prime less than N to reach double of the sum of all the natural numbers upto N i.e. $N(1+N)$. In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of $P < \frac{N}{\ln(N)}$ then that will lead us also to lower limit of prime gaps which will satisfy the equation $P + R = P^{\log_{\frac{N}{\ln(N)}} N(1+N)} = N(1+N)$ where $R \geq$ lowest bound of prime gap. As we are comparing double of the sum of all the natural numbers we can always half it and do the same test again and again to descend along the even number line from any arbitrarily large height. If our resultant exponent is greater than 2 (ideally it should be greater than or equal to 2 as we have ensured all primes are summed up 2 times) then that would imply that there shall be a lower bound of prime gaps and that bound will lie near to very initial gaps along the number line whereas due to continuity there shall not be any upper bound on the prime gaps, it may grow as the number sequence grows. Clearly the result $\log_{\frac{N}{\ln(N)}} N(1+N) = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N)$ shall be greater than 2 meaning that the lower bound of prime gaps would be the gap between sole even prime 2 and its immediate successor even number i.e. 4. Thus the lower bound of prime gaps equals 2. As a prime gap of 2 is lesser than the above highest possible exponent, there shall be infinitely many twin primes satisfying the equation $p_1 + 2 = N(1+N) - 1 = p_2$.Hence Twin prime conjecture stands proved and it can be called as Twin prime theorem.

10.2 Proof of Goldbach's Conjecture

Goldbach's conjecture is one of the oldest unsolved problems in number theory and all of mathematics. It states:

Every even integer greater than 2 can be expressed as the sum of two primes.

The conjecture has been shown to hold for all integers less than 4×10^{18} but remains unproven to date.

Similarly we can proof Goldbach conjecture too. Before we proceed to proof Goldbach conjecture let us have an understanding how it works. We take the identity $(p+q)^2 = p^2 + q^2 + 2pq$. Now let us set p equals an odd prime p_1 and q equals the sole even prime 2.As a result $(p_1+2)^2$ gives a confirmed odd number as follows:$(p_1+2)^2 = p_1^2 + 4 + 4p_1$.This can be rewritten as sum of one even and one one odd prime as $(p_1+2)^2 = (2)+(p_1^2+4p_1+2)$ as $p_1^2+4p_1+2$ cannot be factorized in a real way.We know that there are infinite number of primes out of which 2 is the sole even prime which essentially means there are infinite number of odd primes.For all this odd primes there will be infinite number of odd numbers which differs an odd prime by 2.Ensuring that atleast one odd prime is there in the right hand side by way of adding such an odd number r to both side of $(p_1+2)^2 = 2+p_1^2+4p_1+2$ we will turn both side into an even number capable of being expressed as sum of two odd primes as follows:$(p_1+2)^2+r = (2+r)+(p_1^2+4p_1+2) = p_2+p_3$.$\mathbf{(p_1+2)^2+r = (2+r)+(p_1^2+4p_1+2) = p_2+p_3}$ can be regarded as standard prime sum form. Standard prime sum form can also be written in vertex form $y = \frac{1}{2}(p_1+2)^2 + (\frac{r}{2} - 1)$.On which, due to infinitude of prime, there shall be infinite number of points satisfying the equation. Now to prove that above equation goes through all the even numbers we go back to our earlier approach of using arithmetic sum.

Let N be a arbitrarily large number. Sum of all the natural numbers upto N shall be $\frac{N(1+N)}{2}$ which includes sum of all the primes upto N too. Double of the sum shall be $N(1+N)$ which shall include double of sum of all the primes upto N too. According to PNT we know that there shall be $\frac{N}{\ln(N)}$ number of primes with an average prime gap of $\ln(N)$. Sum of all the natural numbers upto N being an ever growing number any theorem proved in the interval N or $N(1+N)$ shall apply upto infinity. We can visualise $\frac{N}{\ln(N)}$ as a prime number itself we can allow the prime gaps to change equivalently and complete the number in between. Now if we take logarithm of $N(1+N)$ with respect to the base of $\frac{N}{\ln(N)}$ the result shall give us the lower bound of prime powers that can comfortably be applied on that prime less than N to reach double of the sum of all the natural numbers upto N i.e. $N(1+N)$. In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of $P < \frac{N}{\ln(N)}$ then that will lead us also to lower limit of number of primes sum of which will satisfy the equation $\sum p_i = P^{\log_{\frac{N}{\ln(N)}} N(1+N)} = N(1+N)$ where $i =$ integer sequence less than N. As we are comparing double of the sum of all the natural numbers we can always half it and do the same test again and again to descend along the even number line from any arbitrarily large height. If our resultant exponent is greater than 2 then that would imply that there shall be a lower bound of number of primes, sum of which can express all the even numbers less than or equal to $N(1+N)$ and that bound will lie near to very initial primes along the number line whereas due to continuity there shall not be any upper bound on the same, it may grow as the number sequence grows. Clearly the result $\log_{\frac{N}{\ln(N)}} N(1+N) = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N)$ shall be greater than 2 meaning that the lower bound of Goldbach partitions would be the same of number 4 the very first non-prime even number. 4 can be written 4=2+2 i.e 4 has got 2 Goldbach partitions. As 2 Goldbach partition is always lesser than the general value of the exponent as calculated above, all the even numbers greater than 2 can be expressed as sum of two primes $p_1 + p_2 = N(1+N)$.Hence Goldbach conjecture stands proved and it can be called as Goldbach theorem. The weaker version of Goldbach conjecture (ternary Goldbach conjecture) immediately follows from the stronger version (binary Goldbach conjecture) proved above.

10.3 Legendre's prime conjecture

Conjecture. (Adrien-Marie Legendre) There is always a prime number between n^2 and $(n+1)^2$ provided that $n \neq -1$ or 0. In terms of the prime counting function, this would mean that $\pi((n+1)^2) - \pi(n^2) > 0$ for all $n > 0$. Jing Run Chen proved in 1975 that there is always a prime or a semiprime between n^2 and $(n+1)^2$ for large enough n. A natural question to ask is: Why doesn't Bertrand's postulate prove Legendre's conjecture? The reason is that actually $(n+1)^2 < 2n^2$ when $n > 2$. For example, for $n = 3$, Bertrand's postulate guarantees that there is at least one prime between 9 and 18, but for Legendre's conjecture to be true we need a prime between 9 and 16. Suppose, just for the sake of argument, that 17 is prime but 11 and 13 are composite. Bertrand's postulate would still be true but Legendre's conjecture would be false. Of course the gap between $(n+1)^2$ and $2n^2$ gets larger as n gets larger, Legendre's conjecture holds true for $n = 3$, and indeed it has been checked up to $n = 10^{10}$.

Let N be a arbitrarily large number. Sum of squares of all the natural numbers upto N shall be $\frac{N(N+1)(2N+1)}{6}$. Double of the sum shall be $\frac{N(N+1)(2N+1)}{3}$. According to PNT we know that there shall be $\frac{N}{\ln(N)}$ number of primes with an average prime gap of $\ln(N)$. Sum of squares of all the natural numbers upto N being an ever growing number any theorem proved in the interval N or $\frac{N(N+1)(2N+1)}{3}$ shall apply upto infinity. We can visualise $\frac{N}{\ln(N)}$ as a prime number itself we can allow the prime gaps to change equivalently and complete the number in between.Now if we take logarithm of $\frac{N(N+1)(2N+1)}{3}$ with respect to the base of $\frac{N}{\ln(N)}$ the result shall give us the lower bound of prime powers that can comfortably be applied on that prime less than N to reach double of the sum of squares of all the natural numbers upto N i.e. $\frac{N(N+1)(2N+1)}{3}$. In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of $P < \frac{N(N+1)(2N+1)}{3}$ then that will lead us also to lower bound of primes which will satisfy the equation $P + R = P^{\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3}}$ where $R \geq$ lowest bound of prime gap. Similarly replacing sum of N^2 by sum of $(N+1)^2$ we get $P + R = P^{\log_{\frac{N}{\ln(N)}} \frac{(N+1)(N+2)(2N+3)}{3}} = P^{\log_{\frac{N}{\ln(N)}} \frac{(N+1)(N+2)(2N+3)}{3}}$. As we are comparing double of the sum of squares of all the natural numbers or its successors we can always half it and do the same test again and again to descend along the even number line from any arbitrarily large height. If our resultant exponent is greater than 2 then that would imply that there shall be a lower bound of prime gaps in the interval and that bound will lie near to very initial gaps along the number line whereas due to continuity there shall not be any upper bound on the prime gaps, it may grow as the number sequence grows. Clearly the result $\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3} = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (N+1) + \log_{\frac{N}{\ln(N)}} (2N+1)$ shall be significantly lower than $\log_{\frac{N}{\ln(N)}} \frac{(N+1)(N+2)(2N+3)}{3} = \log_{\frac{N}{\ln(N)}} (N+1)((N+1)+1)(\frac{2N}{3}+1)$ (due to complete pattern of extra little quantity of +1) such that another prime can occur in the interval meaning that the lower bound of number of primes in the interval between $\frac{N(N+1)(2N+1)}{3}$) and $N(1+N)$ would be greater than 1. Thus there shall be atleast one prime between n^2 and $(n+1)^2$ as Legendre conjectured.Hence Legendre's prime conjecture stands proved and it can be called as Legendre's theorem.

10.4 Sophie Germain prime conjecture

In number theory, a prime number p is a Sophie Germain prime if 2p + 1 is also prime. The number 2p + 1 associated with a Sophie Germain prime is called a safe prime. For example, 11 is a Sophie Germain prime and 2 11 + 1 = 23 is its associated safe prime. Sophie Germain primes are named after French mathematician Sophie Germain, who used them in her investigations of Fermat's Last Theorem.

The conjecture states that there are infinitely many prime numbers of the form $2P + 1$.

Sum of all the natural numbers upto N shall be $\frac{N(1+N)}{2}$ which includes sum of all the primes upto N too. Double of the sum shall be $N(1+N)$ which shall include double of sum of all the primes upto N too. According to

PNT we know that there shall be $\frac{N}{\ln(N)}$ number of primes with an average prime gap of $\ln(N)$. Sum of all the natural numbers upto N being an ever growing number any theorem proved in the interval N or $N(1+N)$ shall apply upto infinity. We can visualise $\frac{N}{\ln(N)}$ as a prime number itself we can allow the prime gaps to change equivalently and complete the number in between.Now if we take logarithm of $N(1+N)$ with respect to the base of $\frac{N}{\ln(N)}$ the result shall give us the lower bound of prime powers that can comfortably be applied on that prime less than N to reach double of the sum of all the natural numbers upto N i.e. $N(1+N)$. In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of $P < \frac{N}{\ln(N)}$ then that will lead us also to lower limit of prime gaps which will satisfy the equation $P + R = P^{\log_{\frac{N}{\ln(N)}} N(1+N)} = N(1+N)$ where $R \geq$ lowest bound of prime gap. As we are comparing double of the sum of all the natural numbers we can always half it and do the same test again and again to descend along the even number line from any arbitrarily large height. If our resultant exponent is greater than 2 which is the lower bound of prime gaps then due to continuity infinitude of prime of the underlying pattern is guaranteed otherwise not. Clearly the result $\log_{\frac{N}{\ln(N)}} N(1+N) = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N)$ shall be greater than 2 meaning that there shall be infinitely many primes of the form $2P+1$.Hence Sophie Germain conjecture stands proved and it can be called as Sophie Germain's prime theorem.

10.5 Landau's prime conjecture

The conjecture states that there are infinitely many prime numbers of the form N^2+1.

Let N be a arbitrarily large number. Sum of square of all the natural numbers upto N shall be $\frac{N(N+1)(2N+1)}{6}$. Double of the sum shall be $\frac{N(N+1)(2N+1)}{3}$. According to PNT we know that there shall be $\frac{N}{\ln(N)}$ number of primes with an average prime gap of $\ln(N)$. Sum of squares of all the natural numbers upto N being an ever growing number any theorem proved in the interval N or $\frac{N(N+1)(2N+1)}{3}$ shall apply upto infinity. We can visualise $\frac{N}{\ln(N)}$ as a prime number itself we can allow the prime gaps to change equivalently and complete the number in between. Now if we take logarithm of $\frac{N(N+1)(2N+1)}{3}$ with respect to the base of $\frac{N}{\ln(N)}$ the result shall give us the lower bound of prime powers that can comfortably be applied on that prime less than N to reach double of the sum of squares of all the natural numbers upto N i.e. $\frac{N(N+1)(2N+1)}{3}$. In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of $P < \frac{N(N+1)(2N+1)}{3}$ then that will lead us also to lower bound of primes which will satisfy the equation $P + R = P^{\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3}}$ where $R \geq$ lowest bound of prime gap. As we are comparing double of the sum of square of all the natural numbers we can always half it and do the same test again and again to descend along the even number line from any arbitrarily large height. If our resultant exponent is greater than 2 which is the lower bound of prime gaps then due to continuity infinitude of prime of the underlying pattern is guaranteed otherwise not. Clearly the result $\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3} = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (N+1) + \log_{\frac{N}{\ln(N)}} (2N+1)$ shall be significantly lower than $\log_{\frac{N}{\ln(N)}} \frac{(N+1)(N+2)(2N+3)}{3} = \log_{\frac{N}{\ln(N)}} (N+1)((N+1)+1)(\frac{2N}{3}+1)$ (due to complete pattern of extra little quantity of +1) such that another prime can occur in the interval meaning that there shall be infinitely many primes of the form N^2+1.Hence Landau's prime conjecture stands proved and it can be called as Landau's prime theorem.

10.6 Brocard's prime conjecture

Brocard's conjecture pertains to the squares of prime numbers. Here we denote the nth prime as p_n. With the exception of 4, there are always at least four primes between the square of a prime and the square of the next prime. In terms of the prime counting function, this would mean that $\pi({p_{n+1}}^2) - \pi({p_n}^2) > 3$ for all $n > 1$.

Let N be a arbitrarily large number. Sum of squares of all the natural numbers upto N shall be $\frac{N(N+1)(2N+1)}{6}$. Double of the sum shall be $\frac{N(N+1)(2N+1)}{3}$. Sum of all the natural numbers upto N shall be $\frac{N(1+N)}{2}$ which includes sum of all the primes upto N too. Double of the sum shall be $N(1+N)$ which shall include double of sum of all the primes upto N too. According to PNT we know that there shall be $\frac{N}{\ln(N)}$ number of primes with an average prime gap of $\ln(N)$. Sum of squares of all the natural numbers upto N being an ever growing number any theorem proved in the interval N or $\frac{N(N+1)(2N+1)}{3}$ shall apply upto infinity. We can visualise $\frac{N}{\ln(N)}$ as a prime number itself we can allow the prime gaps to change equivalently and complete the number in between. Now if we take logarithm of $N(1+N)$ or $\frac{N(N+1)(2N+1)}{3}$ with respect to the base of $\frac{N}{\ln(N)}$ the result shall give us the lower bound of prime powers that can comfortably be applied on that prime less than N to reach double of the sum of all the natural numbers upto N i.e. $N(1+N)$ or double of the sum of squares of all the natural numbers upto N i.e. $\frac{N(N+1)(2N+1)}{3}$ respectively. Clearly both the result $\log_{\frac{N}{\ln(N)}} N(1+N) = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N)$ or $\log_{\frac{N}{\ln(N)}} N(1+N) = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N)$ shall be greater than 2. In case of interval between two consecutive primes the above limit get raised to the power of its own value meaning that there shall be at least 4 primes the square of a prime and the square of the next prime. Hence Brocard's prime conjecture stands proved and it can be called as Landau's prime theorem.

10.7 Opperman's prime conjecture

Oppermann's conjecture is an unsolved problem in mathematics on the distribution of prime numbers. It is closely related to but stronger than Legendre's conjecture, Andrica's conjecture, and Brocard's conjecture. It is named after Danish mathematician Ludvig Oppermann, who announced it in an unpublished lecture in March 1877.The conjecture states that, for every integer $x > 1$, there is at least one prime number between $x(x-1)$ and x^2,and at least another prime between x^2 and $x(x+1)$.It can also be phrased equivalently as stating that the prime-counting function must take unequal values at the endpoints of each range.That is: $\pi(x^2-x) < \pi(x^2) < \pi(x^2+x)$ for $x > 1$ with $\pi(x)$ being the number of prime numbers less than or equal to x. The end points of these two ranges are a square between two pronic numbers, with each of the pronic numbers being twice a pair triangular number. The sum of the pair of triangular numbers is the square.

Let N be a arbitrarily large number. Sum of square of all the natural numbers upto N shall be $\frac{N(N+1)(2N+1)}{6}$. Double of the sum shall be $\frac{N(N+1)(2N+1)}{3}$. Sum of all the natural numbers upto N shall be $\frac{N(1+N)}{2}$ which includes sum of all the primes upto N too. According to PNT we know that there shall be $\frac{N}{\ln(N)}$ number of primes with an average prime gap of $\ln(N)$. N being relatively an ever growing number any theorem proved in the interval N or $N(1+N)$ or $\frac{N(N+1)(2N+1)}{3}$ shall apply upto infinity. We can visualise $\frac{N}{\ln(N)}$ as a prime number itself we can allow the prime gaps to change equivalently and complete the numbers in between. Now if we take logarithm of $N(N+1).\frac{4N-1}{3}$ with respect to the base of $\frac{N}{\ln(N)}$ the result shall give us the lower bound of prime powers that can comfortably be applied on that prime less than N to reach double of the sum of squares of all the natural numbers upto N less the double of the sum of all the natural numbers upto N i.e. $N(N+1).\frac{4N-1}{3}$. In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of $P < N(N+1).\frac{4N-1}{3}$ then that will lead us also to lower bound of primes which will satisfy the equation $P + R = P^{\log_{\frac{N}{\ln(N)}} N(N+1).\frac{4N-1}{3}}$ where $R \geq$ lowest bound of prime gap. Clearly the result $\log_{\frac{N}{\ln(N)}} N(N+1).\frac{4N-1}{3} = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N) + \log_{\frac{N}{\ln(N)}} \frac{4N-1}{3}$ shall be greater than 2 meaning that there shall be atleast one prime between $x(x-1)$ and x^2. Again adding $\frac{N(1+N)}{2}$ with $\frac{N(N+1)(2N+1)}{3}$ we get $\frac{N(N+1)(2N+1)}{3} + \frac{N(1+N)}{2} = N(N+1).\frac{2(2N+1)+3}{6} = N(N+1).\frac{4N+5}{6}$. Double of such difference shall be $N(N+1).\frac{4N+5}{3}$. Clearly the result $\log_{\frac{N}{\ln(N)}} N(N+1).\frac{4N+5}{3} = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N) + \log_{\frac{N}{\ln(N)}} \frac{4N+5}{3}$ shall be greater than 2 meaning that there shall be atleast one prime between x^2 and $x(x+1)$. Altogether Opperman's conjecture stands proved and it can be called as Opperman's theorem.

10.8 Collatz conjecture

The Collatz conjecture is a conjecture in mathematics that concerns a sequence defined as follows: start with any positive integer n. Then each term is obtained from the previous term as follows: if the previous term is even, the next term is one half the previous term. If the previous term is odd, the next term is 3 times the previous term plus 1. The conjecture is that no matter what value of n, the sequence will always reach 1.

Collatz conjectured operations on any number (i.e. halving the even numbers or simultaneously tripling and adding 1 to odd numbers) may either blow up to infinity or come down to singularity. Tripling and adding 1 to odd numbers will always land on an even number. Now to end the game we just need to step upon an even number which is of the form 2^n. That will happen when odd primes are tripled and added to 1. We have seen that among the odd numbers odd primes are descendants of sole even prime 2. This small bias turns the game of equal probability into one sided game i.e Collatz conjecture cannot blow upto infinity, it ends with 2 and one last step before the final whistle bring it down to singularity 1 as Collatz conjectured. Hence Collatz conjecture is proved to be trivial.

11 Complex logarithm simplified

11.1 Fallacies in present concept of Complex logarithm and way out

The complex exponential function is not injective, because $ew + 2\pi i = ew$ for any w, since adding $i\theta$ to w has the effect of rotating ew counterclockwise θ radians. So the points equally spaced along a vertical line, are all mapped to the same number by the exponential function. That is why the exponential function does not have an inverse (Complex logarithm) function in true sense.One is to restrict the domain of the exponential function to a region that does not contain any two numbers differing by an integer multiple of $2\pi i$: this leads naturally to the definition of branches of log z, which are certain functions that single out one logarithm of each number in their domains. Another way to resolve the indeterminacy is to view the logarithm as a function whose domain is not a region in the complex plane, but a Riemann surface that covers the punctured complex plane in an infinite-to-1 way. Branches have the advantage that they can be evaluated at complex numbers. On the other hand, the function on the Riemann surface is elegant in that it packages together all branches of the logarithm and does not require an arbitrary choice as part of its definition. The function Log z is discontinuous at each negative real number, but continuous everywhere else in $\mathbb{C}^{\times}$. To explain the discontinuity, consider what happens to Arg z as z approaches a negative real number a. If z approaches a from above, then Arg z approaches π, which is also the value of Arg a itself. But if z approaches a from below, then Arg z approaches $-\pi$. So Arg z "jumps" by 2 as z crosses the negative real axis, and similarly Log z jumps by $2\pi i$. All logarithmic identities are satisfied by complex numbers. It is true that $e^{\ln z} = z$ for all $z \neq 0$ (this is what it means for Log z to be a logarithm of z), but the identity Log $e^z = z$ fails for z outside the strip S. For this reason, one cannot always apply Log to both sides of an identity $e^z = e^w$ to deduce z = w. Also, the identity $\ln z_1 z_2 = \ln z_1 + \ln z_2$ can fail: the two sides can differ by an integer multiple of $2\pi i$: for instance,

$$\text{Log}((-1)i) = \text{Log}(-i) = \ln(1) - \frac{\pi i}{2} = -\frac{\pi i}{2}$$

but

$$\text{Log}(-1) + \text{Log}(i) = (\ln(1) + \pi i) + \left(\ln(1) + \frac{\pi i}{2}\right) = \frac{3\pi i}{2} \neq -\frac{\pi i}{2}$$

This part is copied from wikipedia as cited [12].

Bringing two more complex number analogous to imaginary number i we can fix the problem in defining the principal logarithm as follows: $\ln 1 = 0 = \ln -1. -1 = \ln i^2.j^2.k^2.i.j.k = 3(\ln i + \ln j + \ln k) = 3.0 = 0$.

11.2 Eulers formula, the unit circle, the unit sphere

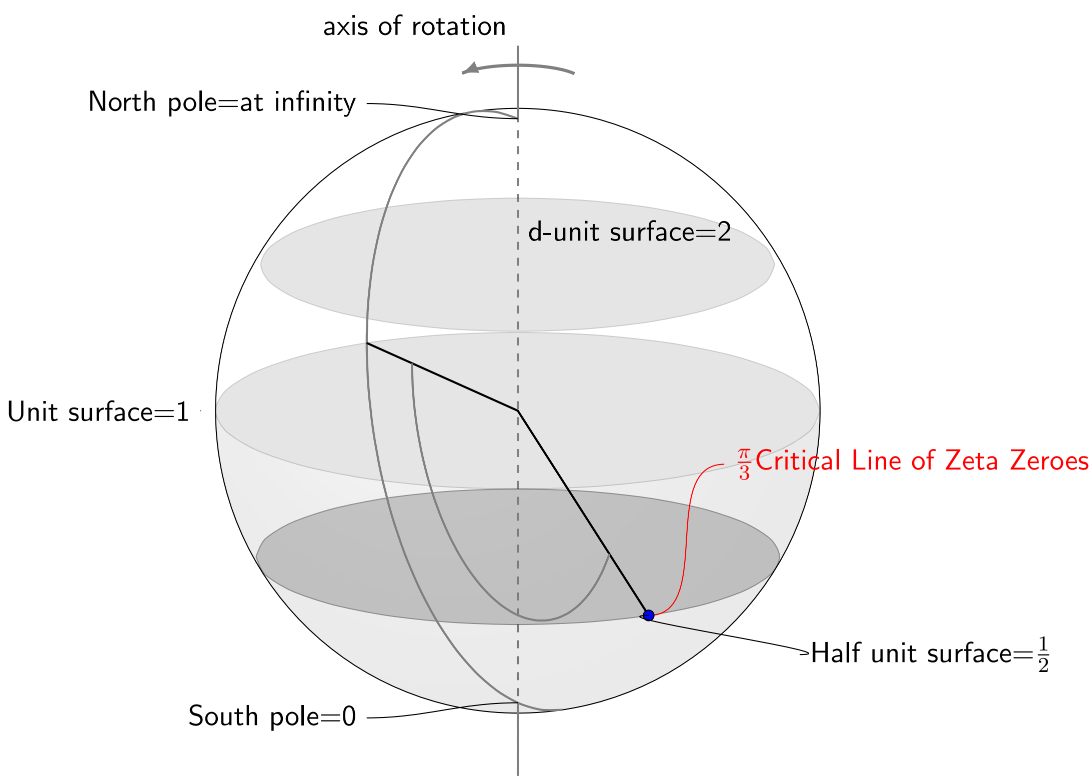
$z = r(\cos x + i \sin x)$ is the trigonometric form of complex numbers. Using Eulers formula $e^{ix} = \cos x + i \sin x$ we can write $z = re^{ix}$. Putting $x = \pi$ in Eulers formula we get , $e^{i\pi} = -1$.Putting $x = \frac{\pi}{2}$ we get $e^{\frac{i\pi}{2}} = i$. So the general equation of the points lying on unit circle $|z| = |e^{ix}| = 1$. But that's not all. If $x = \frac{\pi}{3}$ in trigonometric form then $z = cos(\frac{\pi}{3}) + i.sin(\frac{\pi}{3}) = \frac{1}{2}(\sqrt{3}+i)$.So $|z| = r = \sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2} = \frac{1}{2}.\sqrt{4} = \frac{1}{2}.2 = 1$.So another equation of the points lying on unit circle $|\mathbf{z}| = |\frac{\mathbf{1}}{\mathbf{2}}\mathbf{e}^{\mathbf{ix}}| = \mathbf{1}$. Although both the equation are of unit circle, usefulness of $|\mathbf{z}| = |\frac{\mathbf{1}}{\mathbf{2}}\mathbf{e}^{\mathbf{ix}}| = \mathbf{1}$ is greater than $|z| = |e^{ix}| = 1$ as $|\mathbf{z}| = |\frac{\mathbf{1}}{\mathbf{2}}\mathbf{e}^{\mathbf{ix}}| = \mathbf{1}$ bifurcates mathematical singularity and introduces unavoidable mathematical duality particularly in studies of primes and Zeta function. $|\mathbf{z}| = |\frac{\mathbf{1}}{\mathbf{2}}\mathbf{e}^{\mathbf{ix}}| = \mathbf{1}$ can be regarded as d-unit circle. When Unit circle in complex plane is stereo-graphically projected to unit sphere the points within the area of unit circle gets mapped to southern hemisphere, the points on the unit circle gets mapped to equatorial plane, the points outside the unit circle gets mapped to northern hemisphere. d-unit circle can also be easily projected to Riemann sphere. Projection of d-unit circle to d-unit sphere will have three parallel disc (like three dimensions hidden in one single dimension of numbers) for three (equivalent unit values in three different sense) magnitude of $\frac{1}{2}, 1, 2$ in the southern hemisphere, on the equator, in the northern hemisphere respectively as shown in the following diagram.

Explanation 8 *One may attempt to show that* $|\mathbf{z}| = |\mathbf{\frac{1}{2}e^{ix}}| = \mathbf{1}$ *will mean 1= 2. This may not be right interpretation. Correct way to interpret is given here under.*

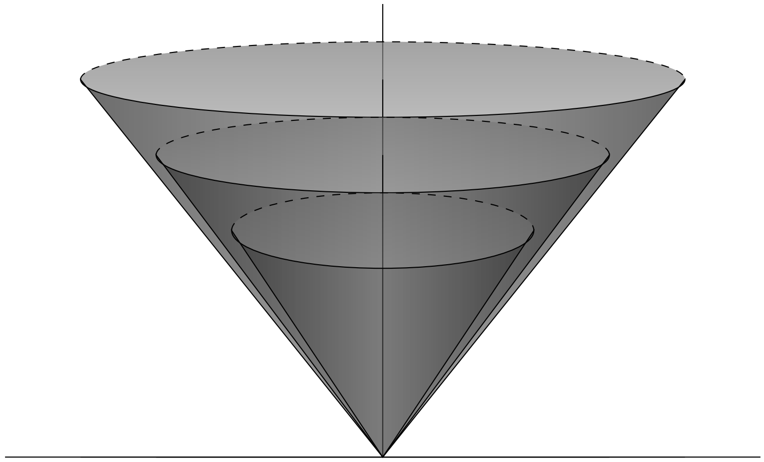
We know: $e^{ix} = r(\cos\theta + i\sin\theta)$. *Taking derivative both side we get*

$$ie^{ix} = (\cos\theta + i\sin\theta)\frac{dr}{dx} + r(-\sin\theta + i\cos\theta)\frac{d\theta}{dx}.$$

Now Substituting $r(\cos\theta + i\sin\theta)$ *for* e^{ix} *and equating real and imaginary parts in this formula gives* $\frac{dr}{dx} = 0$ *and* $\frac{d\theta}{dx} = 1$. *Thus, r is a constant, and θ is x + C for some constant C. Now if we assign* $r = \frac{1}{2}$ *and* $ix = \ln 2$ *then* $re^{ix} = \frac{1}{2}.e^{\ln 2} = 1$ *The initial value x=1 then gives* $i = \ln 2$. *This proves the formula* $|\mathbf{z}| = |\mathbf{\frac{1}{2}e^{ix}}| = \mathbf{1}$. *Thus we see* $ix = \ln(\cos\theta + i\sin\theta)$ *is a multivalued function not only because of infinite rotation around the unit circle but also due to different real solutions to i in higher dimensions. Square root of minus 1 is a general concept of complex numbers which can have different real values.*



If we wish to ascend along the number line then we need to keep open the d-unit sphere in the direction of both positive infinity and negative infinity, which will then look like a double cone. Three parallel surfaces in a single cone will look like (of course ignoring the complex part involving non commutative math altogether) as follows.



11.3 Introduction of Quaternions for closure of logarithmic operation

Hamilton knew that the complex numbers could be interpreted as points in a plane, and he was looking for a way to do the same for points in three-dimensional space. Points in space can be represented by their coordinates, which are triples of numbers, and for many years he had known how to add and subtract triples of numbers. However, Hamilton had been stuck on the problem of multiplication and division for a long time. He could not figure out how to calculate the quotient of the coordinates of two points in space. The great breakthrough in quaternions finally came on Monday 16 October 1843 in Dublin, when Hamilton was on his way to the Royal Irish Academy where he was going to preside at a council meeting. Hamilton could not resist the urge to carve the formula for the quaternions, $\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$ into the stone of Brougham Bridge as he paused on it. A quaternion is an expression of the form : $a + b\ \mathbf{i} + c\ \mathbf{j} + d\ \mathbf{k}$ where a, b, c, d, are real numbers, and i, j, k, are symbols that can be interpreted as 'imaginary operators' which define how the scalar values combine. The set of quaternions is made a 4 dimensional vector space over the real numbers, with $\{1, \mathbf{i}, \mathbf{j}, \mathbf{k}\}$ as a basis, by the componentwise addition

$$(a_1 + b_1\ \mathbf{i} + c_1\ \mathbf{j} + d_1\ \mathbf{k}) + (a_2 + b_2\ \mathbf{i} + c_2\ \mathbf{j} + d_2\ \mathbf{k}) = (a_1 + a_2) + (b_1 + b_2)\ \mathbf{i} + (c_1 + c_2)\ \mathbf{j} + (d_1 + d_2)\ \mathbf{k}$$

and the componentwise scalar multiplication

$$\lambda(a + b\ \mathbf{i} + c\ \mathbf{j} + d\ \mathbf{k}) = \lambda a + (\lambda b)\ \mathbf{i} + (\lambda c)\ \mathbf{j} + (\lambda d)\ \mathbf{k}\ .$$

A multiplicative group structure, called the Hamilton product, can be defined on the quaternions. The real quaternion 1 is the identity element.The real quaternions commute with all other quaternions, that is aq = qa for every quaternion q and every real quaternion a. In algebraic terminology this is to say that the field of real quaternions are the center of this quaternion algebra. The product is first given for the basis elements, and then extended to all quaternions by using the distributive property and the center property of the real quaternions. The Hamilton product is not commutative, but associative, thus the quaternions form an associative algebra over the reals.

For two elements $a_1 + b_1 i + c_1 j + d_1 k$ and $a_2 + b_2 i + c_2 j + d_2 k$, their product, called the Hamilton product $(a_1 + b_1 i + c_1 j + d_1 k)(a_2 + b_2 i + c_2 j + d_2 k)$, is determined by the products of the basis elements and the distributive law.The distributive law makes it possible to expand the product so that it is a sum of products of basis elements. This gives the following expression:

$$a_1a_2 + a_1b_2i + a_1c_2j + a_1d_2k + b_1a_2i + b_1b_2i^2 + b_1c_2ij + b_1d_2ik$$

$$+c_1a_2j + c_1b_2ji + c_1c_2j^2 + c_1d_2jk + d_1a_2k + d_1b_2ki + d_1c_2kj + d_1d_2k^2$$

Now the basis elements can be multiplied using the rules given above to get:

$$a_1a_2 - b_1b_2 - c_1c_2 - d_1d_2 + (a_1b_2 + b_1a_2 + c_1d_2 - d_1c_2)i$$

$$+(a_1c_2 - b_1d_2 + c_1a_2 + d_1b_2)j + (a_1d_2 + b_1c_2 - c_1b_2 + d_1a_2)k$$

This part is copied from wikipedia as cited [10].

If some ask what quaternion has to do with complex logarithm then I wont say "shut up and calculate" (quantum mechanics instructors famous instruction). First let us fix the problem we faced in complex logarithm defining the principal value by way of introducing quaternions in the picture. If we visualise natural logarithm of product of two pairs of -1 as natural logarithm of two pairs of quaternion then we can arive zero at part with the definition of logarithm and solve the issue of indeterminacy of the principal value i.e. $\ln 1 = 0 = \ln -1. -1 = \ln i^2.j^2.k^2.i.j.k = 3(\ln i + \ln j + \ln k)$.Any guess what angle can make vector-sum of three equal vactors equal to zero? As shown in my Riemann hypothesis proof it's 120 degree in 3D or 60 degree in 4D. This way numbers are very complexly 3 dimensional hidden in other hidden dimensions of quaternions although we do not feel it in our everyday use of numbers. Now let see how quaternion helps in simplifying the complex logarithm. For simplification let us use a single alphabet for expressing quaternion. Let us recall the power addition identity, which is,

$$e^{(a+b)} = e^a * e^b$$

However this only applies when 'a' and 'b' commute, so it applies when a or b is a scalar for instance. The more general case where 'a' and 'b' don't necessarily commute is given by:

$$e^c = e^a * e^b$$

where:

$$c = c = a + b + ab + 1/3(a(ab) + b(ba)) + \text{...series known as the Baker-Campbell-Hausdorff formula}$$

where:x = vector cross product. This shows that if when a_1 and a_2 become close to becoming parallel then ab approaches zero and c approaches a + b so the rotation algebra approaches vector algebra. As we have seen all the three unit discs appear parallel to each other our life gets easier and we can do complex exponentiation and logarithm as we do natural logarithm in real life. This becomes real and simple logarithm.

11.4 Properties of Real and simple (RS) Logarithm

Thanks to Roger cots who first time used i in complex logarithm. Thanks to euler who extended it to exponential function and tied i, pi and exponential function to unity in his famous formula. Now taking lead from both of their work and applying results of Zeta function we can redefine complex logarithm as follows inspired from Thukral and Parkash's work [2]. If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ then RS logarithm has the following property.

Theorem 1

$$\ln(z_1.z_2) = \ln(\mathrm{Re}(z_1)) + \ln(\mathrm{Re}(z_2)) + i(\ln(\mathrm{Im}(z_1) + \ln(\mathrm{Im}(z_2))$$

Proof:

$$\begin{aligned}
&\ln(z_1.z_2.z_3.z_4.z_5.z_6.z_7....)\\
&= \ln\Bigg(p_1.p_2.p_3.p_4.p_5.p_6.p_7...\Bigg) + i\ln\Bigg(p_1.p_2.p_3.p_4.p_5.p_6.p_7...\Bigg)\\
&= \ln(1) + \ln(2) + \ln(3) + \ln(4) + \ln(5) + ... + i\ln\Bigg(\ln(1) + \ln(2) + \ln(3) + \ln(4) + \ln(5) + ...\Bigg)\\
&= \ln(\mathrm{Re}(z_1)) + \ln(\mathrm{Re}(z_2)) + \ln(\mathrm{Re}(z_3)) + ... + i\Bigg(\ln(\mathrm{Im}(z_1)) + \ln(\mathrm{Im}(z_2)) + \ln(\mathrm{Im}(z_3)) + ...\Bigg)
\end{aligned}$$

Following Zeta functions analytic continuation or bijective holomorphic property, we can write:

$$\ln(z_1.z_2) = \ln(\mathrm{Re}(z_1)) + \ln(\mathrm{Re}(z_2)) + i\Bigg(\ln(\mathrm{Im}(z_1)) + \ln(\mathrm{Im}(z_2))\Bigg)$$

Corrolary 1

$$\ln(\frac{z_1}{z_2}) = \ln(\mathrm{Re}(z_1)) - \ln(\mathrm{Re}(z_2)) + i\Bigg(\ln(\mathrm{Im}(z_1)) - \ln(\mathrm{Im}(z_2))\Bigg)$$

Corrolary 2

$$\exp(z_1+z_2)=\exp(\mathrm{Re}(z_1)).\exp(\mathrm{Re}(z_2))+i\Bigg(\exp(\mathrm{Im}(z_1)).\exp(\mathrm{Im}(z_2))\Bigg)$$

Corrolary 3

$$\exp(z_1-z_2)=\frac{\exp(\mathrm{Re}(z_1))}{\exp(\mathrm{Re}(z_2))}+i\Bigg(\frac{\exp(\mathrm{Im}(z_1))}{\exp(\mathrm{Im}(z_2))}\Bigg)$$

Corrolary 4

$$\ln(z_1+z_2)=\ln(\mathrm{Re}(z_1+z_2))+i\Bigg(\ln(\mathrm{Im}(z_1+z_2))\Bigg)$$

Corrolary 5

$$\ln(z_1-z_2)=\ln(\mathrm{Re}(z_1-z_2))+i\Bigg(\ln(\mathrm{Im}(z_1-z_2))\Bigg)$$

12 Pi based logarithm

One thing to notice is that pi is intricately associated with e. We view pi mostly associated to circles, what it has to do with logarithm? Can it also be a base to complex logarithm? Although base pi logarithm are not common but this can be handy in complex logarithm. We know:

$$\begin{aligned}
&ln(2).\frac{\pi}{4}\\
&=\left(\frac{1}{1}-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots\right)\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{11}-\frac{1}{13}+\cdots\right)\\
&=\left(1+\frac{1}{\lfloor 3}-\frac{1}{\lfloor 5}+\frac{1}{\lfloor 7}-\cdots\right)+\left(1+\frac{1}{\lfloor 2}+\frac{1}{\lfloor 4}+\frac{1}{\lfloor 6}+\cdots\right)-\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\cdots\right)\\
&=\left(1-\frac{i^3}{\lfloor 3}+\frac{i^5}{\lfloor 5}-\frac{i^7}{\lfloor 7}-\cdots\right)+\left(1-\frac{i^2}{\lfloor 2}+\frac{i^4}{\lfloor 4}-\frac{i^6}{\lfloor 6}+\cdots\right)-\frac{1}{1-\frac{1}{2}}\\
&=\sin(i)+\cos(i)-2
\end{aligned}$$

Lets set:$\pi=\sin(i)+\cos(i)$ and replacing $\pi-2=\ln(\pi)$ we can write

$$\frac{\ln\left(e^{\frac{ln(2)}{4}}\right)}{\ln(\pi)}=\frac{1}{\pi}=\pi^{-1}\text{Lets set:}e^{\frac{ln(2)}{4}}=\pi^{\pi^{je}}\text{ we can write }\boxed{\pi^{je}=-1}$$

13 Factorial functions revisited

The factorial function is defined by the product

$$n!=1\cdot 2\cdot 3\cdots(n-2)\cdot(n-1)\cdot n,$$

for integer $n\geq 1$ This may be written in the Pi product notation as

$$n!=\prod_{i=1}^{n} i.$$

$$n!=n\cdot(n-1)!.$$

Euler in the year 1730 proved that the following indefinite integral gives the factorial of x for all real positive numbers,

$$x!=\Pi(x)=\int_0^\infty t^x e^{-t}dt, x>1$$

Eulers Pi function satisfies the following recurrence relation for all positive real numbers.

$$\Pi(x+1) = (x+1)\Pi(x), x > 0$$

In 1768, Euler defined Gamma function, $\Gamma(x)$, and extended the concept of factorials to all real negative numbers, except zero and negative integers. $\Gamma(x)$, is an extension of the Pi function, with its argument shifted down by 1 unit.

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$$

Eulers Gamma function is related to Pi function and factorial function as follows:

$$\Gamma(x+1) = \Pi(x) = x!$$

Ibrahim [1] defined the factorial of negative integer n as the product of first n negative integers.

$$-n! = \prod_{k=1}^{n}(-1)^k, -n \leq -1$$

The relation $n! = n \cdot (n-1)!$ allows one to compute the factorial for an integer given the factorial for a smaller integer. The relation can be inverted so that one can compute the factorial for an integer given the factorial for a larger integer:

$$(n-1)! = \frac{n!}{n}$$

13.1 Factorial of rational numbers

For positive half-integers, factorials are given exactly by

$$\Gamma\left(\tfrac{n}{2}\right) = \left(\tfrac{n}{2}-1\right)! = \sqrt{\pi}\frac{(n-2)!!}{2^{\frac{n-1}{2}}}$$

or equivalently, for non-negative integer values of n:

$$\Gamma\left(\tfrac{1}{2}+n\right) = \left(n-\tfrac{1}{2}\right)! = \frac{(2n-1)!!}{2^n}\sqrt{\pi} = \frac{(2n)!}{4^n n!}\sqrt{\pi}$$
$$\Gamma\left(\tfrac{1}{2}-n\right) = \left(-n-\tfrac{1}{2}\right)! = \frac{(-2)^n}{(2n-1)!!}\sqrt{\pi} = \frac{(-4)^n n!}{(2n)!}\sqrt{\pi}$$

similarly based on gamma function factorials can be calculated for other rational numbers as follows,

$$\Gamma\left(n+\tfrac{1}{3}\right) = \left(n-\tfrac{2}{3}\right)! = \Gamma\left(\tfrac{1}{3}\right)\frac{(3n-2)!!!}{3^n}$$
$$\Gamma\left(n+\tfrac{1}{4}\right) = \left(n-\tfrac{3}{4}\right)! = \Gamma\left(\tfrac{1}{4}\right)\frac{(4n-3)!!!!}{4^n}$$
$$\Gamma\left(n+\tfrac{1}{p}\right) = \left(n-1+\tfrac{1}{p}\right)! = \Gamma\left(\tfrac{1}{p}\right)\frac{(pn-(p-1))!^{(p)}}{p^n}$$

13.2 Limitation of factorial functions

However, this recursion does not permit us to compute the factorial of a negative integer; use of the formula to compute $(-1)!$ would require a division by zero, and thus blocks us from computing a factorial value for every negative integer. Similarly, the gamma function is not defined for zero or negative integers, though it is defined for all other complex numbers.Representation through the gamma function also allows evaluation of factorial of complex argument.

$$z! = (x+iy)! = \Gamma(x+iy+1), z = \mathbb{C} \setminus \{0, -1, -2, \dots\}$$

For example the gamma function with real and complex unit arguments returns

$$\Gamma(1+i) = i! = i\Gamma(i) \approx 0.498 - 0.155i$$

$$\Gamma(1-i) = -i! = -i\Gamma(-i) \approx 0.498 + 0.155i$$

13.3 Extended factorials using Delta function

Now let us extend factorials of negative integers by way of shifting the argument of Gamma function further down by 1 unit.Let us define Delta function as follows:

$$\Delta(x) = \int_0^\infty t^{x-2} e^{-t} dt$$

The extended Delta function shall have the following recurrence relation.

$$\Delta(x+2) = (x+2)\Delta(x+1) = (x+2)(x+1)\Delta(x) = x!$$

Newly defined Delta function is related to Eulers Gamma function and Pi function as follows:

$$\Delta(x+2) = \Gamma(x+1) = \Pi(x)$$

Plugging into $x = 2$ above

$$\Delta(4) = \Gamma(3) = \Pi(2) = 2$$

Putting $x = 1$ above

$$\Delta(3) = \Gamma(2) = \Pi(1) = 1$$

Putting $x = 0$ above

$$\Delta(2) = \Gamma(1) = \Pi(0) = 1$$

Putting $x = -1$ above we can remove poles of Gamma and Pi function as follows:

$$\Delta(1) = \Gamma(0) = \Pi(-1) = 1.\Delta(0) = -1.\Delta(-1) = \int_0^\infty t^{1-1} e^{-t} dt = \Big[- e^{-x} \Big]_0^\infty = \lim_{x\to\infty} -e^{-x} - e^{-0} = 0 + 1 = 1$$

Therefore we can say $\Delta(-1) = -1$. Similarly Putting $x = -2$ above

$$\Delta(0) = \Gamma(-1) = \Pi(-2) = -1.\Delta(-1) = -2.\Delta(-2) = \int_0^\infty t^0 e^{-t} dt = \Big[- e^{-x} \Big]_0^\infty = \lim_{x\to\infty} -e^{-x} - e^{-0} = 0 + 1 = 1$$

Therefore we can say $\Delta(-2) = -\frac{1}{2}$. Continuing further we can remove poles of Gamma and Pi function:

Putting $x = -3$ above and equating with result found above

$$\Delta(-1) = \Gamma(-2) = \Pi(-3) = -2. - 1.\Delta(-3) = -1 \implies \Delta(-3) = -\frac{1}{2}$$

Putting $x = -4$ above and equating with result found above

$$\Delta(-2) = \Gamma(-3) = \Pi(-4) = -3. - 2.\Delta(-4) = -\frac{1}{2} \implies \Delta(-4) = -\frac{1}{12}$$

Putting $x = -5$ above and equating with result found above

$$\Delta(-3) = \Gamma(-4) = \Pi(-5) = -4. - 3.\Delta(-5) = -\frac{1}{2} \implies \Delta(-5) = -\frac{1}{24}$$

Putting $x = -6$ above and equating with result found above

$$\Delta(-4) = \Gamma(-5) = \Pi(-6) = -5. - 4.\Delta(-6) = -\frac{1}{12} \implies \Delta(-6) = -\frac{1}{240}$$

$\vdots$

And the pattern continues upto negative infinity.
We can extend concept of factorials as follows:

1. We can define $(-1)! = \Delta(-1) = \Gamma(-2) = \Pi(-3) = -1$.

2. We can use Delta function to formulate factorial of negative integer $-n < -1$ as follows:
For even negative integers factorial can be obtained using the following formula:

$$(-n-1)! = \frac{-1}{\Delta(-n-2)} = \frac{-1}{\Gamma(-n-3)} = \frac{-1}{\pi(-n-4)}$$

For odd negative integers factorial can be obtained using the following formula:

$$-n! = \frac{-1}{(-n+1)\Delta(-n-1)} = \frac{-1}{(-n+1)\Gamma(-n-2)} = \frac{-1}{(-n+1)\Pi(-n-3)}$$

3. Through the extended Delta, Gamma, Pi function trio we can evaluate factorial of all complex argument.

$$z! = (x+iy)! = \Delta(x+iy+2) = \Gamma(x+iy+1) = \Pi(x+iy)$$

4. Hence factorials satisfy the closure property and $\mathbb{C}$ is closed under the factorial operation.

14 Bibilography

Contents freely available on internet (generated by Google search) were referred for this piece of research work. Notable few are listed here. Cross-reference citations are given where substantial amount of text has been verbatim copied and used to save unproductive time spent on typing, formatting etc.

Wikipedia, educational websites, educational youtube channels. etc..

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Sources

I do not have much access to mathematical journals, archives etc. However my friend Google searched for me a few free published mathematical papers. I would like to mention two such papers which inspired me to extend the negative or complex factorials using the newly discovered delta function and extend the negative or complex logarithms using the newly discovered value for imaginary number i .

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Papers

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