GIACOMO LEOPARDI'S "L'INFINITO" AND DECISION LIMIT PROBLEMS

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When an observer stares out at a horizon that is miles away, the sight of an object caught in the middle allows her mind to imagine the occurrence of the infinity beyond the edge. According to this psychological account of infinity, put forward by Giacomo Leopardi in his masterpiece "*L'infinito*", a restricted view is required for our imagination to work properly: in absence of a bounded object with edges, our mind is unable to think about infinity. Here we show how Leopardi's account is correlated with recent topological approaches to closed curves, and with long-lasting medieval debates on decision limit problems: when a boat disappears over the horizon, what is the instant (o the minimum spatial range) of change between visibility and non visibility?

KEYWORDS: poetry; infinity; boundary; Jordan curve theorem; Walter Burley; Nicholas of Autrecourt.

The Jordan Curve Theorem (JCT) says that any continuous simple closed curve separates the plane in two disjoint regions, i.e., the "inside" and the "outside" (Jordan, 1893). In other words, the plane is divided into an "interior" region bounded by the curve and an "exterior" region containing all the nearby and far away unbounded points. This means that the curve stands for the common element of the bounded and the unbounded components. Nevertheless, this mathematical designation gives rises to practical difficulties, in particular in decision limit problems concerning physical/biological systems (Koczkodaj et al., 2017; Tozzi and Peters, 2019). To provide an example, when does the internal finish and the external start, in the case of a porous membrane of a living cell? Are membrane pores part of the interior of the cell, or of the surrounding environment? Where is the boundary between the inner cytoplasm and the environment surrounding the living cell? These seemingly scanty differences may be important in metrology: see, for example, Tozzi and Papo (2019).

Diverse types of limit decision problems, such as the problem of intension and remissions, were tackled by Medieval scholars through analytic arguments. The middle-age philosophers, in particular the Oxonians in the 15th century, focused on establishing the intrinsic limits of a power, such as time or distance (Offredus, 1478). At first, they considered the problem of *primo et ultimo istanti* concerning time passing, then they translated this temporal account in terms of *maximum and minimum* spatial distances: when a boat disappears over the horizon, what is the instant (o the minimum spatial range) of change between the whole state of visibility and that of non visibility?
Concerning the temporal analysis, Walter Burley's treatise "*De primo et ultimo instanti*" (Shapiro and Shapiro, 1965) was the most popular medieval work on the problem of assigning first and last instants of being to objects and events. This issue is related to the Aristotelian dilemma concerning the ceasing to be (i.e., to exist) of the present instant (Trifogli 2017): does the present instant cease to be when it is (dismissed by Aristotle), or when it is not (favoured by Aristotle)? In Burley's interpretation, the sentence "this instant ceases to be" may have two different meanings:
1) "this instant now is, and immediately afterwards will not be" (position of the present and negation of the future), or
2) "this instant now is not, and immediately beforehand was" (negation of the present and position of the past).
Burley believes that the first sentence is true, while the second is not, so that an instant ceases to be when it is, and not when it is not (Trifogli 2017).
Concerning the analysis of the spatial range, Nicholas of Autrecourt, in his question "*Utrum Visio*" dated about 1340, states that there is no maximum distance from which anything visible can be seen (O'Donnell, 1939). An object cannot be seen from every distance, because there can be a distance so great, that the object cannot be seen from it. Also, there is a minimum distance brought closed to the vision from which a visible thing cannot be seen (Walker, 2018). What about the medium instant between the whole time in which the thing can be seen, and the whole time in which it cannot be seen? In this medium instant, can the thing be seen, or cannot? Autrecourt concludes that there exist a minimum distance from which the object cannot be seen, instead of a maximum distance from which the object can be seen. We could ask: at the very point of intersection (the border), does the last point of the internal occur, or the first point of the external? If you assume that the border is not the last point of the internal, rather it is the first point of the external, you deny intrinsic limits (indeed, Autrecourt denies a "*minimum quod sic*" and "*a maximum quod sic*", but affirms "*a minimum and a maximum quod non*"). A visual account of the problem of the boundary is provided in **Figures A-C**.

In topological words, given a closed curve (say a spherical object), are the points lying on its boundary encompassed inside or outside the sphere? If the simple closed curve is the boundary of each the two component, does this curve lie inside or outside? In operational terms, if I have to calculate the area of the interior, must I encompass the area of the boundary? The answer provided by current topology to the problem of boundary relies on the Lebesgue's density theorem: the "edge" of A, i.e., the set of points in A whose "neighborhood" is partially in A and partially outside of A, is negligible.; or, in other words, the "density" of A is 0 or 1 at almost every point in the plane. This means that every point can be sharply either internal, or external, without the possibility of fuzzy interpretations, and the border can be indifferently approached from the inside and the outside of an object. Despite this topological standardization, the answer to the problem of the boundary is less trivial and straightforward than it could appear, because, if the points were intersections pertinent of both internal and external, there would not be a distinction between inside and outside. If the limit can be both inside and outside, what does make me sure me that the interior does not extend throughout the exterior? Or, in turn, why couldn't the external fill the internal? As suggested above, in the interpretation of physical/biological features, the problem is correlated with the indeterminacy of the delimitation of an object (encompassed in a closed curve) and the difficulty to determine and quantify its boundaries.

Here Giacomo Leopardi comes into play, providing an unexpected answer to the problem of boundary in his 1819 masterpiece "*L'infinito"* (Rigoni, 1996). Leopardi here uses a psychological approach to solve decision limit problem, in particular through the relationships between imagination and infinity (**Figure D**). According to Leopardi, infinity stands for the mental concept that originates when one looks at bounded things. The bounded object, i.e., "this hedge, which, from so many parts/ of the far horizon, the sight excludes" (*questa siepe, che da tanta parte/ dell'ultimo orizzonte il guardo esclude*) allows the observer to see in her mind's eye the infinity beyond the object: "but sitting and gazing, endless/ spaces beyond it, …/ I fake myself in my thoughts" (*ma sedendo e mirando, interminati/ spazi di là da quella …./io nel pensier mi fingo*). According to Leopardi, and contrary to Nicholas of Autrecourt, infinity is an object that can be seen from every distance, independent of visual boundaries. Therefore, we can state that the idea of infinity in Leopardi is built on the concept of the edge: a narrow object equipped with borders allows us imagine the occurrence of an infinity beyond it (Damiani, 1996). Without the occurrence of a bounded object, our mind would not be able to imagine the infinity, because the human soul is able to imagine just what she cannot observe. The hypothetical sight of an infinite horizon devoid of real objects prohibits us to imagine the infinity, because our mind requires a limited sight, otherwise imagination could not work properly and be able to think of infinity.
Therefore, Leopardi solves the decision limit problems in a way that is different both from the medieval philosophers and the current topology: according to "*L'infinito"*, the border does not restrict our view of the world, rather it allows to use the powers of imagination to look over the edge. The border widens our sight, rather than shrink it.

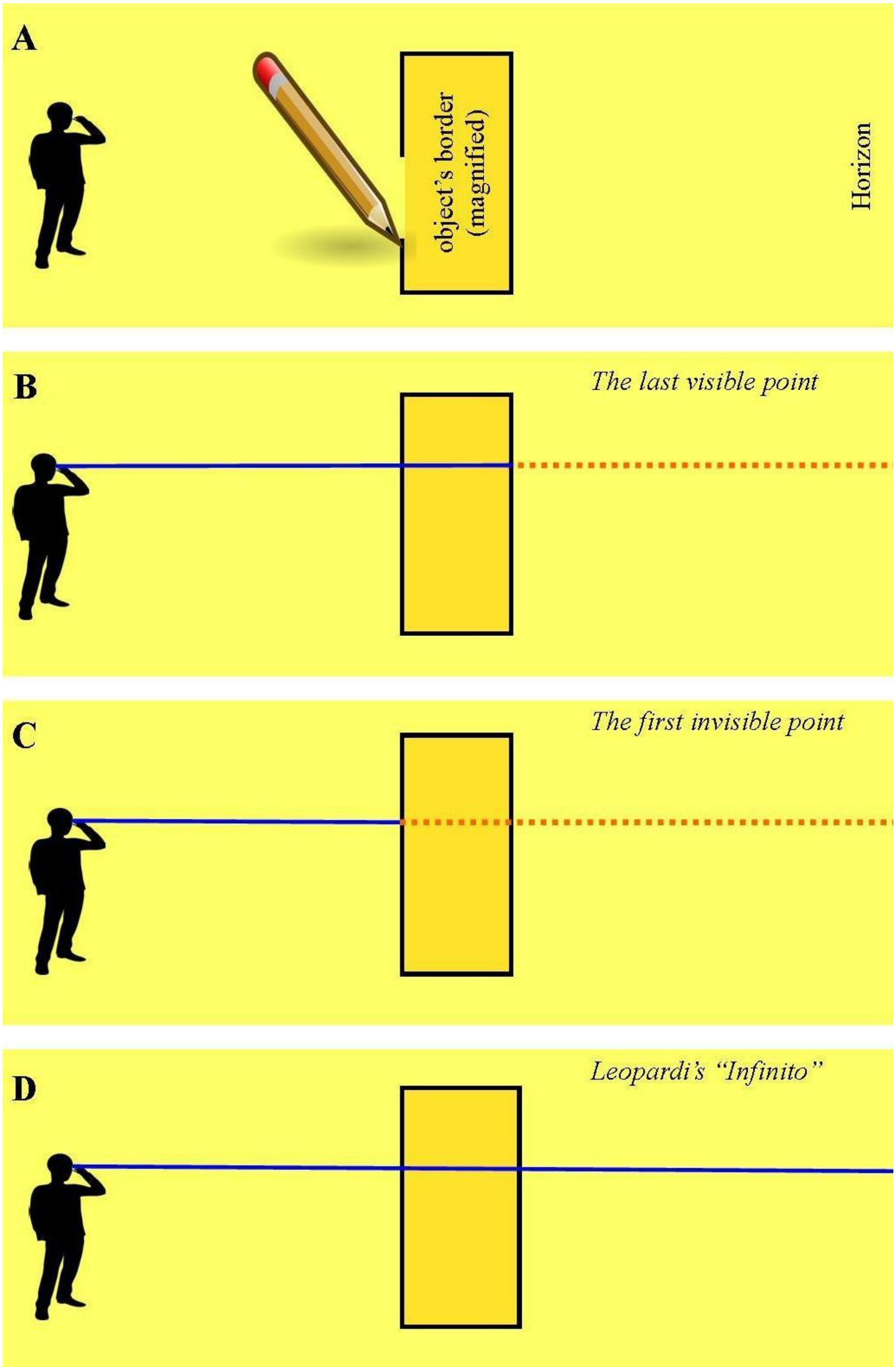

Figure. Different standpoints of an observer looking at her infinite horizon. **Figure A**. The rectangle stands for the magnified boundary of an object that hinders the observer's sight of the infinite horizon. **Figure B**. According to a medieval account, the observer watches the interior of the boundary (blue solid curve), so that the interpretation of a last visible point is preferable to the interpretation a first invisible point. **Figure C**. According to an opposite medieval account, the observer cannot watch the interior of the boundary (red dotted curve), so that the interpretation of a first invisible point is preferable to the interpretation a last visible point. **Figure D**. According to Leopardi, the occurrence of a boundary does not preclude, rather makes it possible to look for the infinite horizon beyond the object.

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