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1. ABSTRACT

The hypothesis of this work is that D.M is generated by the own gravitational field, according an Universal mechanism based on a quantum gravitation phenomenon. In order to study DM as a Universal law is studied rotation curve of M31. The theory developed for galaxies is extended successfully to Coma cluster . In this paper is published by first time the called The equivalence theorem, which is a crucial tool to extend the theory to cluster scale.

Another crucial finding made in this paper is that a characteristic parameter of the theory has been calculated by data coming from Coma cluster and by data from M31. Both results match with a high precision.

Briefly will be explained method followed to develop this paper. Rotation curve data come from [5] Sofue,Y.2015. Thanks this remarkable rotation curve, the regression curve of velocity depending on radius has a correlation coefficient bigger than 0.96 and data range from 40 kpc up to 300 kpc.

In fourth chapter it is got the function of DM density depending on radius, called direct DM density.In fifth chapter it is demonstrated that function direct DM density is mathematically equivalent to the function DM density depending on E. Namely a power of E whose exponent is B= 1.6682.
In sixth chapter it is got that for radius bigger than 40 kpc the ratio baryonic density versus DM density is under 1% . So it is reasonable to consider negligible baryonic matter density in order to simplify calculus.

In seventh chapter it is got a Bernoulli differential equation for field and is solved.
In eighth chapter it is made dimensional analysis for magnitudes Density, field E and universal constants G, h and c. It is demonstrated that it is needed a formula with two Pi monomials. It is found that B = 5/3 is the value coherent with Buckingham theorem and differs only two thousandth regarding B=1.6682 which was got by regression analysis.In the chapter 9, considering B= 5/3 are recalculated the others parameters a&b and A and formulas of theory are rewriting.

In the tenth chapter are introduced the called reduced formulas for field, density and total mass. The variable of these functions is dimensionless, this way the formulas gain simplicity.
In the eleventh chapter, is calculated masses in M31 at different radius by the reduced formula of mass.

In the chapter 12 is introduced The equivalence theorem which is crucial in order to extend the dark gravitation theory to cluster scale. As mass and density are additive magnitudes it is found a new parameter D associated to the cluster, considering as start point the parameter D associated to one single galaxy.
In the thirteenth chapter are developed two methods to calculate parameter D in clusters and it is demonstrated that both are mathematically equivalents. Also it is calculated three different parameter D for Coma cluster using recent measures of Coma cluster mass.
In the fourteenth chapter is used the formula of total mass in Coma cluster by the three different parameter D calculated in previous chapter. Also it is discussed its result and compared with data measured of Coma cluster mass.

In the fifteenth chapter is introduced the average density formula, it is applied to Coma cluster and it is shown that at virial radius Coma cluster has an average density equal to 100 times the critical density of universe, which match very well with measures. Also it is calculated the radius of the sphere where the average density is equal to the Universal average D.M. density, $\Omega_{DM} = 0.27$
In the sixteenth chapter it is proposed a method to calculate parameters C and D, rightly from mass formula. This method is totally independent to the equivalence theorem, so it is highly interesting in order to compare both methods.

2. INTRODUCTION

For last years I have been publishing papers focused mainly in galactic rotation curves. Especially in M31 and Milky Way, because they have wide rotation curves far away from galactic disc. As the hypothesis of my theory is that DM follows a Universal law, the only way to look for a Universal law is searching in the halo region far away the baryonic matter.

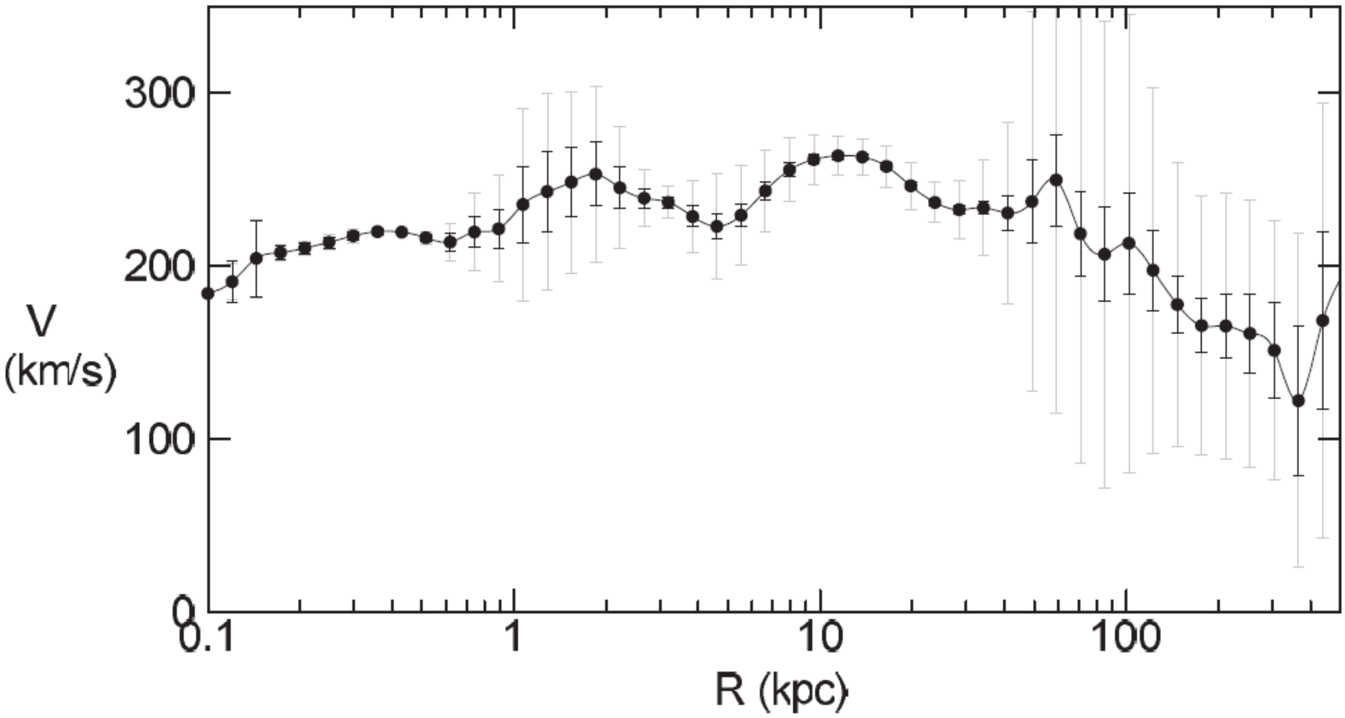
For some years I have try to connect the Dark gravitation theory developed for galaxies with galactic clusters. Finally thanks a new expression of functions for field, density and mass with a dimensionless variable it has been possible to develop a method to extend the theory to galaxy clusters. The equivalence theorem introduced in chapter 12 is the crucial tool to relate parameters at galactic scale with parameters at cluster scale.

As reader knows M31 is the twin galaxy of Milky Way in Local Group of galaxies. Its disk radius is approximately 35 kpc and according [5] Sofue, Y. 2015. Its baryonic mass is $M_{BARYONIC} = 1{,}61 \cdot 10^{11}\ M_{SUN}$
The DM theory introduced in [1] Abarca, M.2014. *Dark matter model by quantum vacuum* considers that DM is generated by the own gravitational field. Therefore, in order to study purely the phenomenon it is needed to consider a radius dominion where it is supposed that baryonic matter is negligible.

In this paper DM density profile has been got directly from a power regression function on rotation curve in halo region. This new DM profile has been called direct DM density because this profile is fitted directly from data measures inside halo region. In this work radius dominion begin at 40 kpc because at this distance baryonic density is negligible as it will be shown in chapter six. Therefore the only type of matter in halo region it is supposed to be non baryonic dark matter and it is quite simple to state the differential equation for field in these conditions.

It is known that there is baryonic dark matter such us giant planets, cold gas clouds, brown dwarfs but this kind of DM is more probable to be placed inside galactic disk. Reader can consult: [11] Nieuwenhuizen,T.M. 2010. [12] Nieuwenhuizen,T.M. 2012. [13] Nieuwenhuizen,T.M. 2010 [14] Wyrzykowski,L.2010. [15] M.R.S. Hawkins 2015.In fact there are an important amount of researchers in this way because baryonic DM and non baryonic DM are open problems still.

3. OBSERVATIONAL DATA FROM SOFUE. 2015 PAPER



Graphic come from [5] Sofue,Y. 2015. The axis for radius has logarithmic scale. In previous version V2 of this work, dominion extended up to 252 kpc, whereas in this version dominion reach up to 303 kpc. In the following epigraph it will explained the reason for this extension.

kpc	km/s
40,5	229,9
49,1	237,4
58,4	250,5
70,1	219,2
84,2	206,9
101,1	213,5
121,4	197,8
145,7	178,8
175	165,6
210,1	165,6
252,3	160,7
302,9	150,8

As in previous version, in chapter six will be shown reason why dominion data begin at 40 kpc in this work, although it is accepted that disk radius of M31 is approximately 35 kpc.

3.1 POWER REGRESSION TO ROTATION CURVE

The measures of rotation curve have a very good fitted curve by power regression.

In particular coefficients of $v = a{\cdot}r^b$ are in table below. Units are into I.S.

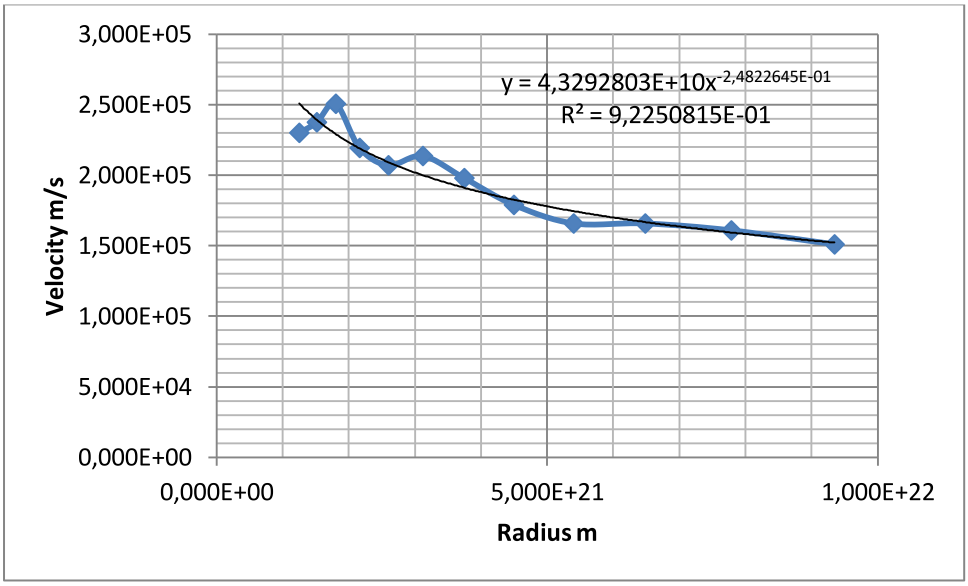
Power regression for M31 rot. curve	
V=a*r^b	
a	$4{,}32928{*}10^{10}$
b	-0.24822645
Correlation coeff.	0,96

Data fitted are in grey columns below.

In fifth column is shown results of fitted velocity and sixth column shows relative difference between measures and fitted results.

Radius kpc	Vel. km/s	Radius m	Vel. m/s	Vel. fitted	Relative Diff.
40,5	229,9	1,250E+21	2,299E+05	2,510E+05	8,397E-02
49,1	237,4	1,515E+21	2,374E+05	2,393E+05	7,777E-03
58,4	250,5	1,802E+21	2,505E+05	2,292E+05	-9,304E-02
70,1	219,2	2,163E+21	2,192E+05	2,190E+05	-8,154E-04
84,2	206,9	2,598E+21	2,069E+05	2,093E+05	1,138E-02
101,1	213,5	3,120E+21	2,135E+05	2,000E+05	-6,755E-02
121,4	197,8	3,746E+21	1,978E+05	1,911E+05	-3,500E-02
145,7	178,8	4,496E+21	1,788E+05	1,826E+05	2,107E-02
175	165,6	5,400E+21	1,656E+05	1,745E+05	5,115E-02
210,1	165,6	6,483E+21	1,656E+05	1,668E+05	7,100E-03
252,3	160,7	7,785E+21	1,607E+05	1,594E+05	-8,307E-03
302,9	150,8	9,347E+21	1,508E+05	1,523E+05	9,891E-03

Below is shown a graphic with measures data and power regression function.



Correlation coefficient equal to 0,96 is a hundredth bigger than data published in previous version V2 of this work, because this time it has been considerate a wide dominion up to 303 kpc. According theory of DM generated by field, halo extend up to a half of distance to Milky Way, 375 kpc, consequently the data for radius 303 kpc is trustworthy.

Furthermore, it has been calculated regression curve with another data placed at 363 kpc, but power regression is -0.28 and correlation coefficient is 0.954. This result shows that such data is not trustworthy because according dimensional analysis power has to be -0.25. As 363 kpc is placed in the border of M31 halo it is possible that such data might be influenced by a different field. Therefore it is better to study data only up to 303 kpc.

4. DIRECT FORMULA FOR DM DENSITY ON M31 HALO GOT FROM ROTATION CURVE

4.1 THEORETICAL DEVELOPPMENT FOR GALACTIC HALOS

Outside disk region, rotation curve it is fitted by power regression with a high correlation coefficient according formula $v = a \cdot r^b$. As $M(<r) = \frac{v^2 \cdot R}{G}$ represents total mass enclosed by a sphere with radius r, by substitution of velocity results $M = \frac{v^2 \cdot R}{G} = \frac{a^2 \cdot r^{2b+1}}{G}$

If it is considered outside region of disk where baryonic matter is negligible regarding dark matter it is possible to calculate DM density by a simple derivative. In next chapter will be show that for r > 40 kpc baryonic matter is negligible.

As density of D.M. is $D_{DM} = \frac{dm}{dV}$ where $d\,m = \frac{a^2 \cdot (2b+1) \cdot r^{2b} dr}{G}$ and $dV = 4\pi r^2 dr$ results

$$D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$$

Writing $L = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot$ results $D_{DM}(r) = L \cdot r^{2b-2}$. In case b = -1/2 DM density is cero which is Keplerian rotation.

4.2 DIRECT DM DENSITY FOR M31 HALO

Parameters a & b from power regression of M31 rotation curve allow calculate easily direct DM density

Direct DM density for M31 halo 40 < r < 300 kpc
$D_{DM}(r) = L \cdot r^{2b-2}$ kg/m^3

5. DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

As independent variable for this function is E, gravitational field, previously will be studied formula for E in the following paragraph.

5.1 GRAVITATIONAL FIELD E BY VIRIAL THEOREM

As it is known total gravitational field may be calculated through Virial theorem, formula $E = v^2/R$ whose I.S. unit is m/s^2 is well known. Hereafter, virial gravitational field, E, got through this formula will be called E.

By substitution of $v = a \cdot r^b$ in formula $E = \frac{v^2}{r}$ it is right to get $E = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ briefly $E = a^2 \cdot r^{2b-1}$

5.2 DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

According hypothesis dark matter by quantum vacuum $D_{DM} = A \cdot E^B$. Where A & B are parameters to be calculated. This hypothesis has been widely studied by author in previous papers. [1] Abarca,M. [2] Abarca,M. [8] Abarca,M. [9] Abarca,M. [10] Abarca,M.

As it is known direct DM density $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ depend on a & b parameters which come from power regression formula for velocity. In previous paragraph has been shown formula for gravitational field $E = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ which depend on a & b as well. Through a simple mathematical treatment it is possible to get A & B to find function of DM density depending on E. Specifically formulas are $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$.

According parameters a & b got in previous chapter, A& B parameters are:

M31 galaxy	$D_{DM} = A \cdot E^B$
A	$3{,}6559956 \cdot 10^{-6}$
B	1,6682469

As conclusion, in this chapter has been demonstrated that a power law for velocity $v = a \cdot r^b$ is mathematically equivalent to a power law for DM density depending on E. $D_{DM} = A \cdot E^B$

5.3 THE IMPORTANCE OF $D_{DM} = A \cdot E^B$

This formula is vital for theory of dark matter generated by gravitational field because it is supposed that DM is generated locally according an unknown quantum gravity mechanism. In other words, the propagation of gravitational field has this additional effect on the space as the gravitational wave goes by.

The formulas $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ and $E = a^2 \cdot r^{2b-1}$ have been got rightly from rotation curve. Therefore it can be considered more specific for each galaxy. However the formula $D_{DM} = A \cdot E^B$ is much more essential.

The basis of this theory is that such formula is right for different gravitational systems. Therefore A & B parameters have to be the same for different galaxies. This is the initial hypothesis of this theory. However, there is an important fact to highlight. It is clear that A depend on a and b, both parameters are global parameters.

As the gravitational interaction time between masses is proportional to distance, it is right to think that DM generated by a gravitational field has a bigger proportion as the system increase its size. For example inside the Solar system it is clear that Newton and General Relativity Theory is able to explain with total accuracy every gravitational phenomenon without DM hypothesis. Therefore it is right to conclude that DM arises when gravitational interaction takes a longer time to link the matter. Namely, for galaxy scale or bigger systems.

Furthermore, there are clear observational evidences that inside cluster of galaxies the proportion of DM is bigger than inside galaxies. In other words, it is right to think that A&B might be different a bigger scale. Namely galaxy cluster scale.

However, there are observational evidences of DM inside dwarf and medium size galaxies that show a bigger proportion of DM than inside giant galaxies.

In my opinion this fact could be explained by other reasons. For example dwarf galaxies are always orbiting near giant galaxies, so it is possible that the proportion of baryonic matter cold, which is unobservable, could be bigger. Anyway this is an open problem for current cosmology.

To sum up, regarding theory of DM generated by gravitational field, parameters A&B has to be the same for different gravitational system on condition they have the same size. i.e. two similar giant galaxies should have the same parameters A&B. However, a bigger gravitational system. i.e. galaxy cluster should have bigger parameter in order to produce a bigger fraction of D.M. Nonetheless, in chapter 9, it will be shown that total DM increase with the square root of distance. For example, the proportion of DM inside galactic disk of M31 is lower than the proportion when it is considered the whole halo whose radius is 350 kpc, so the maximum proportion goes up to 90% of DM versus baryonic matter.

6. RATIO BARYONIC MASS VERSUS DARK MATTER MASS DEPENDING ON RADIUS FOR M31

In this paragraph will be estimated radius which is needed to consider negligible baryonic density regarding DM density in M31 galaxy.

[5] According Sofue, Y. data for M31 disk are

M31 Galaxy	Baryonic Mass at disk	a_d	Σ_0
	$M_d = 2\pi\cdot\Sigma_0\cdot a^2{}_d$		
	$M_d = 1{,}26\cdot10^{11}$ Msun	5,28 kpc	1,5 kg/m^2

Where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ represents superficial density at disk. Total mass disk is given by integration of superficial density from cero to infinite. $M_d = \int_0^\infty 2\pi\cdot r\Sigma(r)\cdot dr = 2\pi\cdot\Sigma_0\cdot a^2{}_d$

In order to compare baryonic density and DM density it is considered differential baryonic mass and differential DM masses depending on radius.

$dM_{DISK} = 2\pi r\Sigma(r)dr$ where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ and

$dM_{DM} = 4\pi r^2 D_{DM}(r)dr$ where $D_{DM}(r) = \dfrac{a^2\cdot(2b+1)}{4\pi G}\cdot r^{2b-2}$

It is defined ratio function as quotient of both differential quantities $Ratio = \dfrac{dM_{DISK}}{dM_{DM}} = \dfrac{\Sigma(r)}{2\cdot r\cdot D_{DM}(r)}$

Radius	Radius	Ratio (r)	$\Sigma(r)$	Direct DM
Kpc	m	Ratio	kg/m^2	kg/m^3
36	1,110852E+21	2,310614E-02	1,64056151250E-03	3,1957946476E-23

38	1,172566E+21	1,715255E-02	1,12327743139E-03	2,7924857817E-23
40	1,234280E+21	1,268028E-02	7,69097762116E-04	2,4570213865E-23
42	1,295994E+21	9,339073E-03	5,26594188719E-04	2,1754010061E-23
44	1,357708E+21	6,854954E-03	3,60554214629E-04	1,9370002366E-23

For a radius 40 kpc ratio baryonic matter versus DM is only 1,2 % therefore is a good approximation to consider negligible baryonic mass density regarding DM density when radius is bigger than 40 kpc.
This is the reason why in this work dominion for radius begin at 40 kpc.

7. A DIFFERENTIAL EQUATION FOR A GRAVITATIONAL FIELD

7.1 INTRODUCTION

This formula $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ is a local formula because it has been got by differentiation. However E, which represents a local magnitude $E = \frac{G \cdot M(<r)}{r^2} = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ has been got through $v = a \cdot r^b$ whose parameters a & b were got by a regression process on the whole dominion of rotation speed curve. Therefore, D_{DM} formula has a character more local than E formula because the former was got by a differentiation process whereas the later involves M(<r) which is the mass enclosed by the sphere of radius r.

In other words, the process of getting D_{DM} involves a derivative whereas the process to get E(r) involves M(r) which is a global magnitude. This is a not suitable situation because the formula $D_{DM} = A \cdot E^B$ involves two local magnitudes. Therefore it is needed to develop a new process with a more local nature or character.

It is clear that a differential equation for E is the best method to study locally such magnitude.

7.2 A DIFFERENTIAL BERNOULLI EQUATION FOR GRAVITATIONAL FIELD IN A GALACTIC HALO

As it is known in this formula $\vec{E} = -G \frac{M(r)}{r^2} \hat{r}$, M(r) represents mass enclosed by a sphere with radius r. If it is considered a region where does not exit any baryonic matter, such as any galactic halo, then the derivative of M(r) depend on dark matter density essentially and therefore $M'(r) = 4\pi r^2 \varphi_{DM}(r)$.

If $E = G \frac{M(r)}{r^2}$, vector modulus, is differentiated then it is got $E'(r) = G \frac{M'(r) \cdot r^2 - 2rM(r)}{r^4}$

If $M'(r) = 4\pi r^2 \varphi_{DM}(r)$ is replaced above then it is got $E'(r) = 4\pi G \varphi_{DM}(r) - 2G \frac{M(r)}{r^3}$ As $\varphi_{DM}(r) = A \cdot E^B(r)$ it is right to get $E'(r) = 4\pi \cdot G \cdot A \cdot E^B(r) - 2\frac{E(r)}{r}$ which is a Bernoulli differential equation.

$$E'(r) = K \cdot E^B(r) - 2\frac{E(r)}{r} \quad \text{being } K = 4\pi \cdot G \cdot A$$

Calling y to E, the differential equation is written in this simple way $y` = K \cdot y^B - \frac{2 \cdot y}{r}$

Bernoulli family equations $y` = K \cdot y^B - \frac{2 \cdot y}{r}$ may be converted into a differential linear equation with this variable change $u = y^{1-B}$. Which is $\frac{u`}{1-B} + \frac{2u}{r} = K$

The homogenous equation is $\frac{u`}{1-B} + \frac{2u}{r} = 0$ Whose general solution is $u = C \cdot r^{2B-2}$ being C the integration constant.

If it is searched a particular solution for the complete differential equation with a simple linear function u=z*r then it is got that $z = \frac{K \cdot (1-B)}{3-2B}$. Therefore the general solution for u- equation is $u = C \cdot r^{2B-2} + z \cdot r$

When it is inverted the variable change it is got the general solution for field E.

General solution is $E(r) = \left(Cr^{2B-2} + \frac{Kr(1-B)}{3-2B} \right)^{\frac{1}{1-B}}$ $with\ B \neq 1\ and\ B \neq 3/2$ where C is the parameter of initial condition of gravitational field at a specific radius.

Calling $\alpha = 2B - 2$ $\beta = \frac{1}{1-B}$ and $D = \left(\frac{K(1-B)}{3-2B} \right)$ formula may be written as

$$E(r) = \left(Cr^{\alpha} + Dr \right)^{\beta}$$

Calculus of parameter C through initial conditions $R_0\ and\ E_0$

Suppose $R_0\ and\ E_0$ are the specific initial conditions for radius and gravitational field, then $C = \frac{E_0^{1/\beta} - D \cdot R_0}{R_0^{\alpha}}$

Final comment

It is clear that the Bernoulli solution contains implicitly the fact that it is supposed there is not any baryonic matter inside the radius dominion and the only DM matter is added by $\varphi_{DM}(r) = A \cdot E^B(r)$. Therefore this solution for field works only in the halo region and $R_0\ and\ E_0$ could be the border radius of galactic disk where it is supposed begins the halo region and the baryonic density is negligible.

8. DIMENSIONAL ANALYSIS FOR D.M. DENSITY AS POWER OF E FORMULA

8.1 POWER OF E BY BUCKINGHAM THEOREM

As it is supposed that DM density as power of E come from a quantum gravity theory, it is right to think that constant Plank h should be considered and universal constant of gravitation G as well.

So the elements for dimensional analysis are D, density of DM whose units are Kg/m^3, E gravitational field whose units are m/s^2, G and finally h.

In table below are developed dimensional expression for these four elements D, E, G and h.

	G	h	E	D
M	-1	1	0	1
L	3	2	1	-3
T	-2	-1	-2	0

According Buckingham theorem it is got the following formula for Density

$D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ being K a dimensionless number which may be understood as a coupling constant between field E and DM density.

As it is shown in previous epigraph, parameters for M31 is B = 1,6682469

In this case relative difference between B = 1,6682469 and 10/7 is 16,7 %. A 17% of error in cosmology could be acceptable. However by the end of the chapter it will be found a better solution.

8.2 POWER E FORMULA FOR DM DENSITY WITH TWO PI MONOMIALS

As this formula come from a quantum gravitation theory it is right to consider that Universal constants involved are G, h and c. So elements to make dimensional analysis are D, E, G,h and c =2.99792458·10^8 m/s.

	G	h	E	D	c
M	-1	1	0	1	0
L	3	2	1	-3	1
T	-2	-1	-2	0	-1

According Buckingham theorem, as matrix rank is three, there are two pi monomials. The first one was calculated in previous paragraph and the second one involves G, h, E and c.

These are both pi monomials $\pi_1 = D \cdot \sqrt[7]{G^9 \cdot h^2} \cdot E^{-\frac{10}{7}}$ and $\pi_2 = \frac{c}{\sqrt[7]{G \cdot h}} E^{-\frac{2}{7}}$. So formula for DM density through two pi monomials will be $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ being J a dimensionless number and $f(\pi_2)$ an unknown function, which can not be calculated by dimensional analysis theory.

8.3 MATHEMATICAL ANALYSIS TO DISCARD FORMULA WITH ONLY ONE PI MONOMIAL

As it was shown in paragraph **5.2** $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ and $B = \frac{2b-2}{2b-1}$. Being a, b parameters got to fit rotation curve of velocities $v = a \cdot r^b$

Conversely, it is right to clear up parameters a and b from above formulas.

Therefore $b = \dfrac{B-2}{2B-2}$ and $a = \left[\dfrac{4\pi GA(B-1)}{2B-3}\right]^{\frac{2b-1}{2}}$ being $B \neq 1$ *and* $B \neq 3/2$.

As A is a positive quantity then $2b+1>0$. As $2b+1 = \dfrac{2B-3}{B-1} > 0$ Therefore $B \in (-\infty,1) \cup (3/2,\infty)$.

If B=3/2 then 2b+1=0 and A=0 so dark matter density is cero which is Keplerian rotation curve.

In every galactic rotation curve studied, B parameter has been bigger than 3/2. See Abarca papers quoted in Bibliographic references. Therefore experimental data got in several galaxies fit perfectly with mathematical findings made in this paragraph especially for $B \in (3/2,\infty)$.

The main consequence this mathematical analysis is that formula $D = \dfrac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ got with only a pi monomial is wrong because B=10/7 = 1.428. Therefore formula $D = \dfrac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ got thorough dimensional analysis by two pi monomials it is more suitable formula.

This formula is physically more acceptable because it is got considering G, h and c as universal constant involved in formula of density. As according my theory, DM is generated through a quantum gravitation mechanism it is right to consider not only **G** and **h** but also **c** as well.

8.4 LOOKING FOR A D.M. DENSITY FUNCTION COHERENT WITH DIMENSIONAL ANALYSIS

It is right to think that $f(\pi_2)$ should be a power of π_2, because it is supposed that density of D.M. is a power of E.

M31 galaxy	$D_{DM} = A \cdot E^B$
A	$3,6559956 \cdot 10^{-6}$
B	1,6682469

Taking in consideration A &B parameters on the left, power for π_2 must be -5/6. This way, power of E in formula $D_{DM} = A \cdot E^B$ will be 5/3 = 1.666666 , which is the best approximation to B= 1.6682469.

Finally $D = \dfrac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ becomes $D = \dfrac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}} E^{\frac{5}{3}}$ being M a dimensionless number.

CALCULUS OF DIMENSIONLESS NUMBER INCLUDED IN FORMULA OF DARK MATTER DENSITY

By equation of $D = \dfrac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}} E^{\frac{5}{3}}$ and $D = A * E^B$

It is right that $A = \dfrac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}}$ and then $M = A \cdot \sqrt[6]{G^7 \cdot c^5 \cdot h}$

9. RECALCULATING FORMULAS IN M31 HALO WITH B = 5/3

Findings got through Buckingham theorem are crucial. It is clear that a physic formula has to be dimensionally coherent .Therefore it is a magnificent support to the theory of DM generated by gravitational field that statistical value got by regression analysis in M31, differs less than 2 thousandth regarding value got by Buckingham theorem.

Now it is needed to rewrite all the formulas considering B=5/3. Furthermore, with B= 5/3, a lot of parameters of the theory become simple fraction numbers. In other words, theory gains simplicity and credibility.

In chapter 5 was shown the relation between a&b parameters and A&B parameters. Now considering B= 5/3

as $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$. It is right to get $b = \frac{B-2}{2B-2} = -\frac{1}{4}$ and $A = \frac{a^{\frac{-4}{3}}}{8\pi G}$

Therefore, the central formula of theory becomes $D_{DM} = A \cdot E^{\frac{5}{3}} = \frac{a^{\frac{-4}{3}}}{8 \cdot \pi \cdot G} \cdot E^{\frac{5}{3}}$

9.1 RECALCULATING THE PARAMETER a IN M31 HALO

Table below comes from chapter 3 and represents regression curve of velocity depending on radius.

Regression for M31 dominion 40-303 kpc	
V=a*r^{b}	
a	4,32928*10^{10}
b	-0.24822645
Correlation coeff.	0,96

Due to Buckingham theorem it is needed that b= -1/4 . Therefore it is needed to recalculate parameter **a** in order to find a new couple of values a&b that fit perfectly to experimental measures of rotation curve in M31 halo.

RECALCULATING a WITH MINUMUN SQUARE METHOD

When it is searched the parameter a, a method widely used is called the minimum squared method. So it is searched a new parameter **a** for the formula V= a*r $^{-0.25}$ on condition that $\sum_e (v - v_e)^2$ has a minimum value. Where v represents the value fitted for velocity formula and v_e represents each measure of velocity. It is right to calculate the formula for a.

$a = \frac{\sum_e Ve \cdot r_e^{-0.25}}{\sum_e r_e^{-0.5}} = 4.727513 \cdot 10^{10}$ Where r_e represents each radius measure and v_e represents its velocity associated.

9.2 RECALCULATING PARAMETER A IN M31 HALO

At the beginning of this chapter was got that $A = \frac{a^{\frac{-4}{3}}}{8\pi G}$.

In previous epigraph has been recalculated the parameter a. Therefore A has to change according this new value.

The beside table shows the value of new parameters.

In addition it is defined a new parameter called D, because

This parameter is widely used in the Bernoulli solution for field.

New parameters a&b and A&B	
B	5/3
$b = \frac{B-2}{2B-2}$	b = -1/4
a new	4,727513*10^{10}
$A = \frac{a^{\frac{-4}{3}}}{8\pi G}$	New parameter A 3.488152*10^{-6}
It is defined $D = 8\pi GA = a^{\frac{-4}{3}}$	D = 5.85*10^{-15}

9.4 BERNOULLI SOLUTION FOR E IN M31 HALO

In chapter 7 was got the solution for field in the halo region, now thanks dimensional analysis it is possible to get formulas far simple because some parameters are simple fractions.

$$E(r) = \left(Cr^{\alpha} + Dr\right)^{\beta} \text{ being } \alpha = 2B - 2 = \frac{4}{3} \quad \text{being } \beta = \frac{1}{1-B} = \frac{-3}{2} \quad \text{and } D = \left(\frac{4\cdot\pi\cdot G\cdot A(1-B)}{3-2B}\right) = 8\cdot\pi\cdot G\cdot A$$

Therefore $E(r) = \left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-3}{2}}$ being $D = 8\pi GA = a^{\frac{-4}{3}}$ = 5,85*10^{-15} being $C = \frac{E_0^{\frac{-2}{3}} - D\cdot R_0}{R_0^{\frac{4}{3}}}$ the initial condition of differential equation solution for E.

CALCULUS OF PARAMETER C

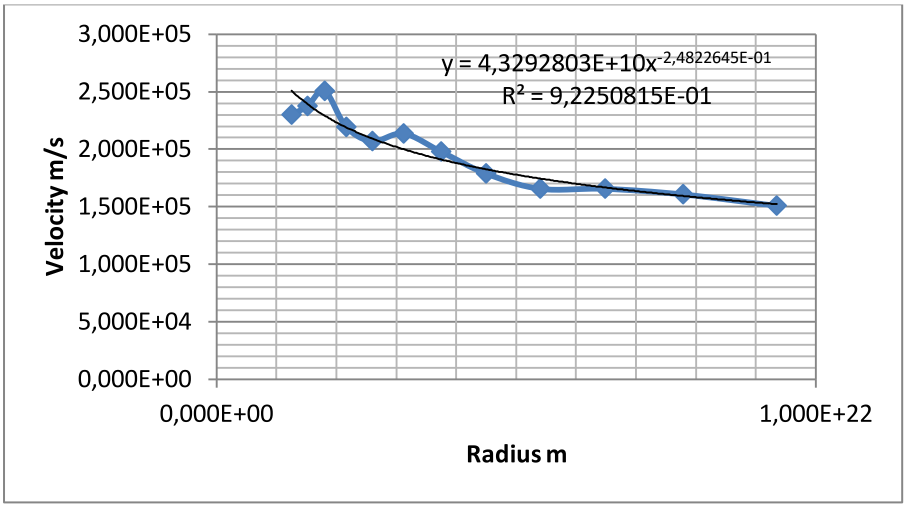
As it was pointed in the epigraph 7.2 C is calculated through the initial condition in the halo region. As it was shown in the chapter 6 at 40.5 kpc (below point P) radius the baryonic matter may be considerate negligible so it is reasonable to calculate C at this point with its formula $C = \frac{E_0^{\frac{-2}{3}} - D\cdot R_0}{R_0^{\frac{4}{3}}}$

Similarly it is possible to calculate C for different points inside the halo region. See in graph below points P, Q, R. They are the three first points to the left.

Points → Radius	Velocity m/s	E field.	Parameter C
P→ 40.5 kpc = 1.25*10^{21} m	2.29945*10^{5}	4,2293071*10^{-11}	6,88783573*10^{-23}
Q → 49.1 kpc	2.374 E5	3,719857E-11	6,36196464E-24
R → 58.4 kpc	2.505E5	3,482162E-11	-5,308924E-23

It is clear that there is a high difference between these three values for C. The reason is simple through the graph. There are two positives because such points (points P and Q) are below de regression curve, whereas the third (point R) is the above one.

The value of C associated to the second point, Q, placed at 49.1 kpc is far smaller than the other ones because it is very close to the regression curve. In the following epigraph it will be a bit clear the reason why Q is so small.



Studying case C = 0

Now It will be investigated the conditions to get C= 0. Then formula $C = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$ leds to

$E_0^{\frac{-2}{3}} = D \cdot R_0 = a^{\frac{-4}{3}} \cdot R_0$ and as $E = a^2 \cdot r^{2b-1}$ then $E_0^{\frac{-2}{3}} = a^{\frac{-4}{3}} \cdot R_0^{\frac{2-4b}{3}} = D * R_0 = a^{\frac{-4}{3}} \cdot R_0$ and by equation of power of R_0 $\frac{2-4b}{3} = 1$ it is got b= -1/4 .

At the beginning of chapter was shown that B= 5/3 leds rightly to b = -1/4. So b= -1/4 is rigorously the power of radius on the rotation curve of galaxy in the halo region, where there is not any baryonic matter. Namely formula is velocity, $V = a{*}r^{-0.25}$. Therefore C= 0 for every point belonging to regression curve whose power is -1 /4.

In the graph above, the point Q, at 49.1 kpc is very close to the regression curve so this is the reason why C is far smaller than the other two points. Unfortunately, measures of rotation curves might have considerable errors.

10. REDUCED BERNOULLI FORMULAS FOR DENSITY- MASS AND FIELD IN HALO REGION

Thanks to a simple mathematical treatment, it is possible to write a standard formula for field, for dark density and for dark gravitation mass in order to get a dimensionless variable X related to radius.

The method consist to rewrite the formulas using dimensionless variable X = R/ Rs where Rs is a parameter with length dimension defined by $Rs = \left(\frac{D}{C}\right)^3$ which can be called length scale.

In epigraph 9.4 has been shown that at radius 40.5 kpc (point P) the baryonic density is negligible versus dark gravitation density son this point is perfect to calculate parameter C. Made calculus it is got

$$Rs = \left(\frac{D}{C}\right)^3 = 19.85\,Mpc$$

10.1 REDUCED FORMULA FOR BERNOULLI FIELD WITH DIMENSIONLESS VARIABLE X

It is right to check that $E(r) = \left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-3}{2}}$ is mathematically equivalent to $E = \dfrac{Eo}{X^{3/2} \cdot \left(X^{1/3} + 1\right)^{3/2}}$

Where $Eo = D^{-3/2} \cdot Rs^{-3/2}$

Reduced formula for field E(r) =	$E = \dfrac{Eo}{X^{3/2} \cdot \left(X^{1/3} + 1\right)^{3/2}}$ Being X = R/ Rs	Being $Eo = D^{-3/2} \cdot Rs^{-3/2}$ and Rs = $(D/C)^3$

The following reduced formula for density it is a bit developed in order to help the reader to understand the process of variable changing.

10.2 REDUCED FORMULA FOR BERNOULLI DENSITY WITH DIMENSIONLESS VARIABLE X

From formula of density $D_{DM} = A*E^{5/3}$, by Bernoulli formula of field E, it is right to get

$$D_{DM}(r) = A\left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-5}{2}} = \frac{A}{r^{5/2} \cdot \left(C \cdot r^{1/3} + D\right)^{5/2}} = \frac{A}{r^{5/2} \cdot D^{5/2}\left(\frac{C}{D} \cdot r^{1/3} + 1\right)^{5/2}}$$

$$= \frac{A \cdot D^{-5/2}}{r^{5/2} \cdot \left(\frac{C}{D} \cdot r^{1/3} + 1\right)^{5/2}}$$ and as $A = \dfrac{D}{8\pi G}$ $$D_{DM}(r) = \frac{D^{-3/2}}{r^{5/2} \cdot \left(\frac{C}{D} \cdot r^{1/3} + 1\right)^{5/2} \cdot 8\pi G}$$

as $Rs = \left(\dfrac{D}{C}\right)^3$ then $$D_{DM}(R) = \frac{D^{-3/2}}{r^{5/2} \cdot \left(\left(\frac{r}{Rs}\right)^{1/3} + 1\right)^{5/2} \cdot 8\pi G}$$ in addition

$$D_{DM}(R) = \frac{\dfrac{D^{-3/2}}{Rs^{5/2}}}{\left(\frac{r}{Rs}\right)^{5/2} \cdot \left(\left(\frac{r}{Rs}\right)^{1/3} + 1\right)^{5/2} \cdot 8\pi G} = \frac{Do}{X^{5/2} \cdot \left(X^{1/3} + 1\right)^{5/2}}$$ being

$Do = D^{-3/2} \cdot Rs^{-5/2} / 8\pi G$ and $X = r / Rs$. Being Do a reference density and Rs a scale radius.

Bernoulli Den. DM = D_{DM} (R) =	$\dfrac{Do}{X^{5/2}\cdot\left(X^{1/3}+1\right)^{5/2}}$ Being X = r/ Rs	$Do = D^{-3/2}\cdot Rs^{-5/2}\,/\,8\pi G$ and Rs = (D/C) ^{3}

10.3 REDUCED FORMULA FOR TOTAL MASS IN THE HALO REGION

By a similar treatment is possible to get a formula for mass with the dimensionless variable X= R/ R_S

$$M_{DM} = \int_{R1}^{R2} 4\pi\cdot r^2\cdot\rho(r)dr = \int_{R1}^{R2} 4\pi\cdot \mathrm{r}^2 AE^B dr = 4\pi A\int_{R1}^{R2} \mathrm{r}^2\left[C\cdot r^{4/3}+D\cdot r\right]^{\frac{-5}{2}}\cdot dr$$

The indefinite integral $I = 4\pi A\cdot\int\dfrac{r^2\cdot dr}{\left(C\cdot r^{4/3}+D\cdot r\right)^{2.5}} = \dfrac{8\pi A\sqrt{r}}{D\cdot\left(C\cdot\sqrt[3]{r}+D\right)^{\frac{3}{2}}} = \dfrac{\sqrt{r}}{G\cdot\left(C\cdot\sqrt[3]{r}+D\right)^{\frac{3}{2}}}$

$$= \frac{\sqrt{r}}{G\cdot D^{3/2}\cdot\left(\frac{C}{D}\cdot\sqrt[3]{r}+1\right)^{\frac{3}{2}}} = \frac{D^{-3/2}\sqrt{r}}{G\cdot\left(\frac{C}{D}\cdot\sqrt[3]{r}+1\right)^{\frac{3}{2}}} = \frac{D^{-3/2}\sqrt{r}}{G\left(\sqrt[3]{\frac{r}{R_S}}+1\right)^{\frac{3}{2}}} = \frac{D^{-3/2}\sqrt{r}}{G\left(\sqrt[3]{X}+1\right)^{\frac{3}{2}}} = \frac{D^{-3/2}\sqrt{Rs}\sqrt{X}}{G\cdot\left(\sqrt[3]{X}+1\right)^{\frac{3}{2}}}$$

Indefinite integral for DM in spherical corona mass in halo. M_{DM} (< R) =	$=\dfrac{D^{-3/2}\sqrt{Rs}\sqrt{X}}{G\cdot\left(\sqrt[3]{X}+1\right)^{\frac{3}{2}}} = \dfrac{Mo\sqrt{X}}{\left(\sqrt[3]{X}+1\right)^{\frac{3}{2}}}$ Being X= r/Rs and	Being $Mo = D^{-3/2}\cdot\sqrt{Rs}\,/$ G and Rs = (D/C) ^{3}

PROPERTYES OF FORMULA FOR MASS

First- It is right to check that mass tend to Mo as X grow indefinitely. By the contrary formula NFW for mass is unbounded as radius grow. Therefore NFW has to be defined inside a bounded dominion.

Second - The indefinite integral calculated at radius where is calculated C, gives rightly the dynamical mass at such radius. Therefore this formula calculate total mass enclosed by the sphere with radius r.

Third – When the definite integral is calculated inside a region without baryonic matter then the result is dark matter purely.

Foruth – If parameter C is calculated at a radius $R_{C,}$ where baryonic density is negligible versus dark gravitation density is negligible then indefinite integral gives the total mass for radius bigger than $R_{C.}$ This property is a consequence of second and third properties.

It is possible to demonstrated it mathematically, and in the following chapter it will be shown that such formula of mass for M31 gives the dynamical mass at 40.5, because at this radius was calculated the parameter C.

10.4 NEWTON'S THEOREM

The name for this theorem has been chosen why the relation between field E and total mass M(<r) is the same that in Newton's theory.

From Bernoulli field $E(r) = \left(Cr^{\frac{4}{3}} + Dr \right)^{\frac{-3}{2}} = \dfrac{1}{r^{3/2} \cdot \left(C \cdot r^{1/3} + D \right)}$

From Total mass formula $M_{TOTAL}(<r) = \dfrac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D \right)^{\frac{3}{2}}}$ so $G \cdot M_{TOTAL}(<r) = \dfrac{\sqrt{r}}{\left(C \cdot \sqrt[3]{r} + D \right)^{\frac{3}{2}}}$ and

$$\frac{G \cdot M(<r)}{r^2} = \frac{\sqrt{r}}{r^2 \left(C \cdot \sqrt[3]{r} + D \right)^{\frac{3}{2}}} = \frac{1}{r^{3/2} \cdot \left(C \cdot r^{1/3} + D \right)^{3/2}} = E(r)$$

Therefore $$E(r) = \frac{G \cdot M(<r)}{r^2}$$

10.5 REDUCED FORMULAS FOR M31 HALO

In the epigraph 9.3 was shown that the best value for parameter D = $5.85 \cdot 10^{-15}$. In the epigraph 9.4 was shown that parameter C = $6,88783573 \cdot 10^{-23}$ calculated at radius 40.5 kpc because at this radius baryonic density is negligible versus DM density. Consequently $Rs = (D/C)^3$ = 19.85 Mpc. Variable X = radius/ Rs

Reduced formula for field E(r) =	$E = \dfrac{Eo}{X^{3/2} \cdot \left(X^{1/3} + 1 \right)^{3/2}}$	Being $Eo = D^{-3/2} \cdot Rs^{-3/2}$ = $4.66 \cdot 10^{-15}$ m/s^2

Bernoulli Den. DM = D_{DM} (R) =	$\dfrac{Do}{X^{5/2} \cdot \left(X^{1/3} + 1 \right)^{5/2}}$	$Do = D^{-3/2} \cdot Rs^{-5/2} / 8\pi G$ = $4.54 \cdot 10^{-30}$ Kg/m^3

Indefinite integral for DM in spherical corona mass in halo. M_{DM} (< R) =	$= \dfrac{D^{-3/2} \sqrt{Rs} \sqrt{X}}{G \cdot \left(\sqrt[3]{X} + 1 \right)^{\frac{3}{2}}} = \dfrac{Mo \sqrt{X}}{\left(\sqrt[3]{X} + 1 \right)^{\frac{3}{2}}}$	Being $Mo = D^{-3/2} \cdot \sqrt{Rs} /$ G = $1.32 \cdot 10^{13}$ Msun

11. MASSES IN M31

It will calculated in this chapter some different types of masses related to M31 and will be compared with got by [5] Sofue.

11.1 DYNAMICAL MASSES

As it is known, this type of mass represents the mass enclosed by a sphere with a radius r in order to produce a balanced rotation with a specific velocity at such radius.

The formula of dynamical mass is $M_{DYN}(<r) = \frac{V^2 \cdot r}{G}$. Namely, taking rightly data from rotation curve, it is got dynamical mass at 40.5 kpc = $4.973 \cdot 10^{11}$ M sun.

11.2 TOTAL MASS IN M31

In the epigraph 10.3 was shown as a property of reduced mass formula that such formula at 40.5 kpc gives rightly the dynamical mass. In other words, thanks the election of Rs at 40.5 kpc, the value of indefinite integral at such radius gives the dynamical mass enclosed by the sphere with radius 40.5 kpc. Dynamical mass is considered the total mass, namely baryonic and dark matter. Now it is possible to check such property.

$$M(<R) = \frac{Mo\sqrt{X}}{\left(\sqrt[3]{X}+1\right)^{\frac{3}{2}}}$$ Being Mo = $1.32 \cdot 10^{13}$ Msun.

As at radius = 40.5 kpc the variable X = $2.04 \cdot 10^{-3}$ so, it is right to check that M = $4.9846 \cdot 10^{11}$ Msun which match perfectly with dynamical mass.

The main consequence of this property is that total mass enclosed by a sphere of radius R is given by the above formula of mass. M(<R).

Below is tabulated some specific total masses at different radius.

Radius kpc	X dimensionless	Mtotal <R Msun
40,50	2,04E-03	4,98E+11
100,00	5,04E-03	7,37E+11
200,00	1,01E-02	9,86E+11
385,00	1,94E-02	1,28E+12
770,00	3,88E-02	1,68E+12
1314,00	6,62E-02	2,04E+12
2230,00	1,12E-01	2,45E+12

In the following epigraph will be compared data highlight in grey with results got by Sofue.

According Sofue Baryonic matter in M31 = $1.6 \cdot 10^{11}$ Msun so the proportion Baryonic mass versus Total mass = 8.8% at 385 kpc.

11.2.1 COMPARING TOTAL MASS FORMULA AND TOTAL MASS WITH A NFW METHOD

In [5] Sofue, the author has published the following results.

Total Mass up to 200 kpc by Sofue	13.9 ± 2.6 x10 ^{11} Msun
Total mass up to 385 kpc by Sofue	19.9 ± 3.9 x10 ^{11} Msun
Total mas up to 200 kpc by reduced formula	9.86 *10 ^{11} Msun
Total mass up to 385 kpc by reduced formula	12.8 *10 ^{11} Msun

Total masses by reduced formula, above data in grey are lightly below the Sofue results when it is considered the error in calculus.

It is important to highlight that reduced formula gives results a bit lower than NFW by reasons I have already explained in several previous papers. i.e. [10] Abarca.

12.THE EQUIVALENCE THEOREM FOR ADDITION OF FIELD - DENSITY AND TOTAL MASS

This theorem is an important tool in order to get a method to estimate the magnitudes field, density or total mass when are involved several galaxies. In other words, this theorem is the crucial tool to extend the dark gravitation theory to galaxy clusters.

12.1 EQUIVALENT FORMULAS FOR A COUPLE OF TWIN GALAXIES

When are considered two close twin galaxies it is right to estimate the total magnitudes for field, density and total mass in the global halo region as the sum of values generated by both galaxies of each magnitudes.

Physically it is right to considerate that the total value of each magnitude will be two times the value generated by one single galaxy.

It is only a method to estimate such magnitudes because the total field have to be calculated rigorously by vector addition, whereas in this paper field is added without vector theory. Therefore these results might be considerate as an upper bound of field and consequently also for density and total mass.

Below are the formulas of the three magnitudes: modulus of field, dark gravitation density and total mass.

$$E(r) = \frac{D^{-3/2} \cdot Rs^{-3/2}}{X^{3/2} \cdot \left(X^{1/3}+1\right)^{3/2}} \quad D(r) = \frac{D^{-3/2} \cdot R_s^{-5/2} / 8\pi G}{X^{5/2} \cdot \left(X^{1/3}+1\right)^{5/2}} \quad \text{and} \quad M(<r) = \frac{D^{-3/2}\sqrt{Rs}\sqrt{X}}{G \cdot \left(\sqrt[3]{X}+1\right)^{\frac{3}{2}}}$$

To begin it is important to highlight that denominators depend on dimensionless variable X = radius / Rs and numerator depend on parameter $D^{-3/2}$ and Rs.

Watching denominators of formulas of the three magnitudes, it is right to see that at a specific radius R, the variable X will be the same for both galaxies on condition that both galaxies have the same Rs.

This condition is quite plausible, because $Rs = (D/C)^3$ being D universally the same for all galaxies with the same mass as it is related to dark gravitation and C is right to be the same as this parameter is involved to dynamical mass at the border of halo at a radius where baryonic mass is negligible versus dark gravitational mass.

Watching numerators it is deduced that at a specific radius, both numerators are the same by the same reasons explained for denominators.

A crucial property found by these formulas is that they depend on D according with the same power = -3/2. The same power for D is the only way to get that these three magnitudes follow the same rule when is added the field, the dark gravitation density or the total mass produced by two or more galaxies.

In fact this happy found have suggested me that these formulas have to be considered as serious candidates in order to extend the theory of gravitation to galaxy clusters.

For example, for field E_{TOTAL} symbolically the sum is represented in this way $E_{TOTAL} = Nu/De + Nu/De = 2Nu/De$

Now it is possible to ask for a new parameter D associated to total field E. This new parameter $D_{TOTAL} = 2^{-2/3}*D$ because $(D_{TOTAL})^{-3/2} = 2*D$. The other factor of numerator, Rs and X, are the same for both galaxies by reasons explained above.

Similarly, it is right to consider that the dark gravitation density or total mass (<R) is two times when is considered two twin galaxies.

Therefore, thanks these simple reasons it is possible to state an interesting theorem.

12.1.1 THE TWIN GALAXIES THEOREM

The field, density and total mass associated to twin galaxies is given by the same parameter Rs associated to a single galaxies and by $D_{TWIN} = 2^{-2/3} *D$.

As a consequence of this theorem, if Milky Way is considerate as twin galaxy of M31 then it is possible to accept that the Local Group of galaxies is defined by Rs = 19.85 Mpc and $D_{LOCAL\ GROUP} = 2^{-2/3} D_{M31} = 3.685*10^{-15}$.

By these parameters it is right to calculate the total mass at a specific radius. For example, the total mass inside a sphere with radius lower than 3 Mpc is $M(< 3\ Mpc) = 5{,}4*10^{12}$ Msun which is a very good estimation of total mass of Local Group at such radius. To do this calculus is enough to considerate X =3/ 19.85 and $D_{LOCAL\ GROUP}$ for total mass formula.

12.2 THE EQUIVALENCE THEOREM FOR GALAXY CLUSTERS

It is a generalization of previous theorem when $M_{CLUSTER} = 2^N *M_{GALAXY}$ being $N = \log(M_{CLUSTER} / M_{GALAXY}) / \log 2$.

By the same mathematical reason used to explain the Twin Galaxies theorem, it is right to get that in this case

$D_{CLUSTER} = 2^{-2N/3} * D_{GALAXY}$ where $D_{CLUSTER}$ represents parameter D for the galaxy cluster because the dark gravitation matter generated by all the galaxies belonging to the cluster is equivalent to the dark matter generated by a set of identical galaxies with the same mass, the same D and the same C. Obviously the galaxy of reference will be M31.

13. TWO METHODS MATHEMATICALLY EQUIVALENTS TO GET PARAMETER D IN CLUSTERS

It is clear that The equivalence theorem is one of them. In this chapter it will be shown a second way to calculate it and it will be checked that both methods give the same value with total accuracy.

In order to be able to explain with calculus it will used this data from Coma cluster.

[14] Joo Heon Yoon provided data for Virial mass and radius in Coma cluster. As it is known at the Virial radius the whole galaxies enclosed inside a sphere with such radius are in dynamical equilibrium.

[16] Jubee Sohn, Margaret J. Geller provided M_{200} and R_{200} . As it is known R_{200} is the radius of the sphere inside a galactic cluster whose mean density is 200 times the universal critic density. $\rho_{CRITIC} = \frac{3H^2}{8\pi G} = 9.2 \cdot 10^{-27}\ kg/m^3$ being H the Hubble constant. M_{200} is the mass of such sphere. Similarly are defined R_{500} and M_{500} . [18] De Martino.2016.

Rvirial 2.9 Mpc	M_{VIRIAL} 1.4*10^{15} Msun	Data Joo Heon Yoon- 2017
R_{200} 2.23 Mpc	M_{200} 1.26*10^{15} Msun	Jubee Sohn- M. Geller- 2016
R_{500} 1.314 Mpc	M_{500} 6.45*10^{14} Msun	De Martino.2016.

13.1 METHOD BY FORMULA OF INITIAL CONDITION IN BERNOULLI DIFFERENTIAL EQUATION

The field E calculated by the theory at galactic scale is $E(r) = \left(Cr^{\frac{4}{3}} + Dr \right)^{\frac{-3}{2}}$ as solution the Bernoulli differential equation of field by the rotation curve. A crucial condition for the rightness of this solution is that in the rotation curve the stars are in dynamical equilibrium between field and its rotation velocity.

In galaxy cluster might happen something similar, the galaxies are in dynamical equilibrium this way the cluster is bound gravitationally. In fact the cluster virial radius is the radius of a sphere where the galaxies are mainly in dynamical equilibrium. For bigger radius the galaxies are falling toward the cluster and they are not in dynamical equilibrium properly. However, Newton formula for field works perfectly because of dynamical equilibrium $E_V = \frac{G \cdot M_V}{R_V{}^2}$ for the galaxies inside the virial radius.

Furthermore D and C are galactic parameter well known in previous chapter whose proportion D/C remain constant even at cluster scale. Being $R_s = (D/C)^3$ a constant with length dimension. Its value for M31 is Rs = 19.85 Mpc and even such value has been adopted for Coma cluster. From such formula it is got $C = \frac{D}{\sqrt[3]{R_s}}$

From the Bernoulli field formula it is possible to clear up $C = \frac{E_o{}^{\frac{-2}{3}} - D \cdot R_o}{R_o{}^{\frac{4}{3}}}$ In fact this formula was used to calculate C as initial condition parameter.

By equation of two formulas for C it is got

$$D\left(\frac{R_o^{\ 4/3}}{\sqrt[3]{R_S}}+R_o\right)=E_o^{\ -2/3}$$ and by substitution of E_o is got $$D\left(\frac{R_o^{\ 4/3}}{\sqrt[3]{R_S}}+R_o\right)=\frac{R_o^{\ 4/3}}{(GM_o)^{2/3}}.$$

In table below are the three different results of parameter D calculated at three different radii.

PARAMETER D BY CONDITION INITIAL FORMULA AND R_S AS CONSTANT		
Rvirial 2.9 Mpc	M_{VIRIAL} $1.4*10^{15}$ Msun	D_{VIRIAL}= 8.994314506E-17
R_{200} 2.23 Mpc	M_{200} $1.26*10^{15}$ Msun	D_{200} =9.103099788E-17
R_{500} 1.314 Mpc	M_{500} $6.45*10^{14}$ Msun	D_{500} = 1.258822191E-16

It is remarkable that Dvirial and D_{200} are similar whereas D_{500} is a 25 % bigger. Forward will be explained this fact.

13.2 METHOD BY FORMULA THE EQUIVALENCE THEOREM

Firstly it is got the total mass of M31 at different radii using its parameters D_{M31} = 5.85E-15 , Rs = 19.85 Mpc and X= R/ Rs by formula $$M(<r)=\frac{D^{-3/2}\sqrt{Rs}\sqrt{X}}{G\cdot\left(\sqrt[3]{X}+1\right)^{\frac{3}{2}}}$$

PARAMETER D BY EQUIVALENCE THEOREM			
Rvirial = 2.9 Mpc	M_{VIRIAL} = $1.4*10^{15}$ Msun	M_{M31} (< 2.9 Mpc) =	$2.668986278*10^{12}$ Msun
R_{200} = 2.23 Mpc	M_{200} $1.26*10^{15}$ Msun	M_{M31} (< 2.23 Mpc) =	$2.445798652*10^{12}$ Msun
R_{500}= 1.314 Mpc	M_{500} $6.45*10^{14}$ Msun	M_{M31} (< 1.314 Mpc) =	$2.035975282*10^{12}$ Msun

Secondly is calculated N from the equation $M_{COMA} = 2^N *M_{M31}$ being N = log(M_{COMA} / M_{M31}) / log 2

and finally by the equivalence theorem it is calculated $D_{VIRIAL\text{-}COMA} = 2^{-2N/3} *D_{M31}$

PARAMETER D BY EQUIVALENCE THEOREM				
Rvirial	M_{VIRIAL}	M_{M31} (< 2.9 Mpc) = $2.668986278*10^{12}$	N_{VIRIAL} 9.034919224	D_{VIRIAL} 8.994314507E-17
R_{200}	M_{200}	M_{M31} (< 2.23 Mpc) = $2.445798652*10^{12}$	N_{200} 9.008902378	D_{200} = 9.103099789E-17
R_{500}	M_{500}	M_{M31} (< 1.314 Mpc) = $2.035975282*10^{12}$	N_{500} 8.307435304	D_{500} = 1.258822191E-16

Comparing results highlighted in grey with the ones in cian, it is clear that both methods are mathematically equivalents.

This fact back fully the rightness of all mathematical process.

13.3 DISCUSSION OF DIFFERENCES BETWEEN PARAMETER D AT DIFFERENT RADII

It is clear that differences between Dvirial and D_{200} are far lower than versus D_{500}.

According dark gravitation theory the only reason for a different D, even tiny like Dvirial versus D_{200} is the baryonic matter there is between both radii. In other words the difference of masses M_{VIRIAL} (< 2.9 Mpc) - M_{200} (< 2.23 Mpc) = $1.4*10^{14}$ Msun is mainly dark gravitational mass generated by total mass inside the radius R_{200} . However, M_{200} (< 2.23 Mpc) - M_{500} (< 1.314 Mpc) = $6.15*10^{14}$ Msun contains a fraction non negligible of baryonic matter and dark gravitational generated by this matter apart form dark gravitational generated for total matter included inside of radius R_{500}

14. MASSES IN COMA CLUSTER BY THE EQUIVALENCE THEOREM

The remarkable measures from Coma cluster are tabulated and quoted below.

[14] Joo Heon Yoon provided data for Virial mass and radius in Coma cluster. As it is known at the Virial radius the whole galaxies enclosed inside a sphere with such radius are in dynamical equilibrium.

[16] Jubee Sohn, Margaret J. Geller provided M_{200} and R_{200} . As it is known R_{200} is the radius of the sphere inside a galactic cluster whose mean density is 200 times the universal critic density. $\rho_{CRITIC} = \frac{3H^2}{8\pi G} = 9.2 \cdot 10^{-27}\, kg/m^3$ being H the Hubble constant. M_{200} is the mass of such sphere. Similarly are defined R_{500} and M_{500} . [18] De Martino.2016.

Rvirial 2.9 Mpc	M_{VIRIAL} 1.4*10^{15} Msun	Data Joo Heon Yoon- 2017
R_{200} 2.23 Mpc	M_{200} 1.26*10^{15} Msun	Jubee Sohn- M. Geller- 2016
R_{500} 1.314 Mpc	M_{500} 6.45*10^{14} Msun	De Martino.2016.

It is important to highlight that the mass in cluster are measured at different radius, therefore it is possible to associate a different parameter D at each radius. For example in Coma cluster R_{500} = 1.3 Mpc which is quite small because in such sphere there are a high concentration of galaxies.

When it is considered a bigger radius the mean density decrease but total mass increase because of a bigger radius, for example R_{200} the number of galaxies inside the sphere is far bigger than galaxies inside R_{500}. In fact, there are some galaxies outside the sphere defined by virial radius.

Below are the parameters D calculated in the previous chapter at different radii.

Rvirial 2.9 Mpc	M_{VIRIAL}	D_{VIRIAL} 8.994314507*10^{-17}
R_{200} 2.23 Mpc	M_{200}	D_{200} = 9.103099789*10^{-17}
R_{500} 1.314 Mpc	M_{500}	D_{500} = 1.258822191*10^{-16}

In this chapter will be calculated the masses of Coma cluster at these three different radii with this three different parameter D. It will be show that some calculus match perfectly with measures whereas other differs. It will be analyzed the reason to explain these results.

The formula of total mass is $M(<r) = \frac{D^{-3/2}\sqrt{Rs}\sqrt{X}}{G \cdot \left(\sqrt[3]{X}+1\right)^{\frac{3}{2}}}$

14.1 ESTIMATING COMA MASSES BY D_{VIRIAL}

COMA MASSES ESTIMATED BY D_{VIRIAL} 8.994314507*10^{-17}					Data measured of mass
Mpc	X = R/Rs	X		By Formula M(<r) *10^{15} Msun	*10^{15} Msun
Rvirial 2.9	X_{VIRIAL}	0.1468354	M_{VIRIAL}	1.39878	1.4
R_{200} 2.23	X_{200}	0.1129114	M_{200}	1.28187	1.26
R_{500} 1.314	X_{500}	0.06653164	M_{500}	1.06718	0.645

Data highlighted in grey match very well. Below is explained the reason.

As it was expected M_{VIRIAL} has been calculated with total agreement versus data measures. M_{200} calculus is very good, and it is a bit bigger because D_{VIRIAL} belong to R_{VIRIAL} and it is associated with a sphere with a bit more baryonic matter than sphere with R_{200} radius.

Clearly the calculus fail for R_{500} and it was expected the mass calculated is far bigger than measure of M_{500}.

14.2 ESTIMATING COMA MASSES BY D_{200}

Coma masses estimated by $D_{200} = 9.103099789 \cdot 10^{-17}$					Data measured of mass
Mpc	X = R/Rs	X		By Formula M(<r) $\cdot 10^{15}$ Msun	$\cdot 10^{15}$ Msun
Rvirial 2.9	X_{VIRIAL}	0.1468354	M_{VIRIAL}	1.3738	1.4
R_{200} 2.23	X_{200}	0.1129114	M_{200}	1.2589	1.26
R_{500} 1.314	X_{500}	0.06653164	M_{500}	1.0481	0.645

Data highlighted in grey match very well. Below is explained the reason.

The estimation of M_{500} by D_{200} is bad because there a non negligible amount of baryonic matter between both radii.

The estimation of M_{200} is perfect as it was expected by the second property of mass, (epigraph 10.3). The estimation of M_{VIRIAL} is very good because there are very few of baryonic matter between both radii. In fact the data is lightly bigger, because the difference of masses $(1.3738 - 1.2589) \cdot 10^{15}$ Msun is purely dark gravitational mass, by the third property of mass, (see epigraph 10.3), whereas the total mass measured at $R_{VIRIAL} = 1.4 \cdot 10^{15}$ Msun include a little of baryonic mass.

14.3 ESTIMATING COMA MASSES BY D_{500}

Coma masses estimated by $D_{500} = 1.258822191 \cdot 10^{-16}$					Data measured of mass
Mpc	X = R/Rs	X		By Formula M(<r) $\cdot 10^{15}$ Msun	$\cdot 10^{15}$ Msun
Rvirial 2.9	X_{VIRIAL}	0.1468354	M_{VIRIAL}	0.8448	1.4
R_{200} 2.23	X_{200}	0.1129114	M_{200}	0.7742	1.26
R_{500} 1.314	X_{500}	0.06653164	M_{500}	0.6445	0.645

DISCUSSION

Data highlighted in grey match mathematically with data measured , whereas the other data does not agree. Calculus at R_{200} or R_{VIRIAL} are far lower than measures. For example inside the spherical corona defined by R_{200} and R_{500} has been calculated $(0.77442-0.6445) \cdot 10^{15}$ Msun of dark gravitation mass, which has been generated by total mass inside R_{500}. Therefore $(1.26-0.7742) \cdot 10^{15}$ Msun is composed by an important fraction of baryonic matter and the dark gravitation mass generated by this baryonic mass.

Something similar happens with calculus at R_{VIRIAL}.

15 AVERAGE DENSITY FORMULA BY DARK GRAVITATION THEORY

Formula of average density is got rightly from D = M/V where $M(<r) = \dfrac{D^{-3/2}\sqrt{Rs}\sqrt{X}}{G\cdot\left(\sqrt[3]{X}+1\right)^{\frac{3}{2}}}$

As formula of total mass depend on dimension less X = R/Rs it is needed to undo the change of variable in order to write its formula depending on radius so the formula remain $M(<r) = \dfrac{D^{-3/2}\sqrt{Rs}\sqrt{r}}{G\cdot\left(\sqrt[3]{r}+\sqrt[3]{R_S}\right)^{\frac{3}{2}}}$ So average density formula is got by $\overline{D}(r) = \dfrac{D^{-3/2}\sqrt{Rs}\sqrt{r}}{G\cdot\left(\sqrt[3]{r}+\sqrt[3]{R_S}\right)^{\frac{3}{2}}} \div \dfrac{4\pi R^3}{3}$ and finally $\overline{D} = \dfrac{3D^{-3/2}\sqrt{Rs}\cdot r^{-5/2}}{4\pi G\cdot\left(\sqrt[3]{r}+\sqrt[3]{R_S}\right)^{\frac{3}{2}}}$

COMPARING THE DM DENSITY FORMULA WITH THE AVERAGE DENSITY FORMULA

From epigraph 10.2 it is right to get this formula for DM density $D_{DM}(R) = \dfrac{\sqrt[6]{Rs^5}\,D^{-3/2}\cdot r^{-5/2}}{\left(\sqrt[3]{r}+\sqrt[3]{Rs}\right)^{5/2}\cdot 8\pi G}$.

It is clear that this density decrease faster as radius grows. This fact, it is totally right because DM density D(r) is a local magnitude whereas $\overline{D}$ (r) is a global magnitude.

15.1 AVERAGE DENSITY FORMULA FOR COMA CLUSTER PARAMETERS

As in previous chapter has been got different parameter D. It is interesting to do calculus of average density using these different parameters. This way it is possible to understand better the condition when formulas match perfectly with measures and by the contrary when does not match with measures.

15.1.1 AVERAGE DENSITY AT DIFFERENT RADII BY D_{VIRIAL}

$$\overline{D}_{COMA} = \frac{3D^{-3/2}{}_{VIRIAL}\sqrt{Rs}\cdot r^{-5/2}}{4\pi G\cdot\left(\sqrt[3]{r}+\sqrt[3]{R_S}\right)^{\frac{3}{2}}}$$

Where D_{VIRIAL} = 8.9943145*10^{-17} R_{VIRIAL} = 2.9 Mpc and Rs = 19.85 Mpc

AVERAGE DENSITY AT R_{VIRIAL} = 2.9 MPC

This formula is totally exact for R_{VIRIAL} = 2.9 Mpc . Also it is quite right for bigger or lower radius where baryonic matter is negligible. It is clear that for bigger radius there is a wide dominion where this formula is acceptable.

Namely for R_{VIRIAL} = 2.9 Mpc $\overline{D}_{COMA} = 9.281911174 * 10^{-25}$ which is exactly $\overline{D} = 100.89\,\varphi_C$ where $\varphi_C = \dfrac{3H^2}{8\pi G} = 9.2 * 10^{-27}$ when it is considered h = 0.7

The average density got rightly from data is $\overline{D}_{DATA} = 9.28*10^{-25}$ which match perfectly with average density calculated by formula.

AVERAGE DENSITY AT R_{200} = 2.23 MPC

According [16] Jubee Sohn at 2.23 Mpc is R_{200}. Thanks the formula got for average density it is possible to calculate rightly $\overline{D}_{COMA}(2.23Mpc) = 1.87064412*10^{-24}$ so $\overline{D} = 203.33\varphi_C$ This result match very well with measures, which produce $\overline{D}_{MEASURES} = 200\varphi_C$

AVERAGE DENSITY AT R_{500} = 1.314 MPC

It worth to calculate density at this radius because there is measure as well. Data source is Plank collaboration project-2013. According its measures R_{500} = 1.314 Mpc and of course $\overline{D}_{MEASURES} = 500\varphi_C$

From formula of average density $\overline{D}_{COMA}(1.314Mpc) = 7.6115213*10^{-24}$ so $\overline{D} = 827.34\varphi_C$

It is clear that at this radius formula does not agree with measures at all. The reason is that D_{VIRIAL} is totally wrong at this radius, because there is a non negligible of baryonic matter between R_{500} and R_{VIRIAL}.

In the following epigraph it will be made the same calculus for these three different radii but using D_{500}. It is clear that by this parameter D, calculus at R_{500} will be quite accurate whereas calculus for R_{200} and R_{VIRIAL} will be inaccurate.

15.1.2 AVERAGE DENSITY AT DIFFERENT RADII BY D_{500}

$$\overline{D}_{COMA} = \frac{3D^{-3/2}{}_{500}\sqrt{Rs}\cdot r^{-5/2}}{4\pi G\cdot\left(\sqrt[3]{r}+\sqrt[3]{R_S}\right)^{\frac{3}{2}}}$$

Where D_{500} = 1.258822191 $*10^{-16}$ R_{500} = 1.314 Mpc and Rs = 19.85 Mpc

AVERAGE DENSITY AT R_{500} = 1.314 MPC

As it is right at R_{500} = 1.314 Mpc this formula is totally accurate. In order to check it, it is easy to do the calculus.

$\overline{D}_{COMA}(1.314Mpc) = 4.597026261*10^{-24}$ so $\overline{D} = 499.68\varphi_C$ as it was expected the result is perfect.

AVERAGE DENSITY AT R_{200} = 2.23 MPC

Although it is expected a bad agreement it worth to calculate it to compare with measures.

According [16] Jubee Sohn at 2.23 Mpc is R_{200}. It is right to calculate $\overline{D}_{COMA}(2.23Mpc) = 1.12979*10^{-24}$ so $\overline{D} = 122.8\varphi_C$ This result is far lower than D_{200} because parameter D_{500} works perfectly at R_{500} but between R_{500} and R_{200} there is a lot of baryonic matter. Therefore D_{500} is wrong to calculate masses and density at R_{200}.

AVERAGE DENSITY AT R_{VIRIAL} = 2.9 MPC

Similarly to previous calculus, it is expected a bad agreement with measures.

Namely for R_{VIRIAL} = 2.9 Mpc $\overline{D}_{COMA} = 5.605867*10^{-25}$ which is exactly $\overline{D} = 60.9\varphi_C$ which is far lower than $\overline{D}_{DATA} = 9.28*10^{-25}$ by the same reason explained above.

15.1.3 AVERAGE DENSITY AT DIFFERENT RADII BY D_{200}

$\overline{D}_{COMA} = \dfrac{3D^{-3/2}{}_{200}\sqrt{Rs}\cdot r^{-5/2}}{4\pi G\cdot\left(\sqrt[3]{r}+\sqrt[3]{R_S}\right)^{\frac{3}{2}}}$ Where D_{200} = 9.1031 $*10^{-17}$ R_{200} = 2.23 Mpc and Rs = 19.85 Mpc

AVERAGE DENSITY AT R_{500} = 1.314 MPC

At R_{500} = 1.314 Mpc this formula is quite wrong because there are a lot of baryonic matter between R_{500} and R_{200}

$\overline{D}_{COMA}(1.314Mpc) = 7.4754888*10^{-24}$ so $\overline{D} = 812.5\varphi_C$ as it was expected the result is quite bad , because it should be $\overline{D}_{MEASURES} = 500\varphi_C$

AVERAGE DENSITY AT R_{200} = 2.23 MPC

According [16] Jubee Sohn at 2.23 Mpc is R_{200}. It is right to calculate $\overline{D}_{COMA}(2.23Mpc) = 1.837212*10^{-24}$ so $\overline{D} = 199.7\varphi_C$ This result is perfect, as it was expected.

AVERAGE DENSITY AT R_{VIRIAL} = 2.9 MPC

Similarly to previous calculus, it is expected a good agreement with measures.

Namely for R_{VIRIAL} = 2.9 Mpc $\overline{D}_{COMA} = 9.1160257 *10^{-25}$ which is very close to $\overline{D}_{DATA} = 9.28*10^{-25}$ because there are few baryonic matter between R_{200} and R_{VIRIAL}

SUMMARASING

Parameters D_{VIRIAL} and D_{200} works perfectly at radii R_{VIRIAL} and R_{200} whereas fail at R_{500} . Conversely D_{500} works mathematically right at R_{500} and fail at the other two radii. It happen exactly the same as happened to the calculus of Coma cluster mass in previous chapter.

15.2 RADIUS FOR COMA CLUSTER WHERE $\overline{D}$ IS EQUAL TO UNIVERSAL D.M. DENSITY

It is well known that $\Omega_{DM} = 0.27$ which means that the average of DM density in the Universe is $0.27\varphi_C = 2.485*10^{-27}$ kg/m^3. Therefore by equation formula of average density to $0.27\varphi_C$

$\overline{D} = \dfrac{3D^{-3/2}\sqrt{Rs}\cdot r^{-5/2}}{4\pi G\cdot\left(\sqrt[3]{r}+\sqrt[3]{R_S}\right)^{\frac{3}{2}}} = 2.485*10^{-27}$ it is got an equation where the only unknown is the radius.

It is not hard to get a more simple equation equivalent. $\left[\sqrt[3]{r}+\sqrt[3]{Rs}\right]^3 * r^5 = 1.7415\cdot 10^{144}$ in meters.

The solution of this equation has been got by wolfram alpha software and it is radius = 25.7 Mpc

This value is bigger than the average distance between galaxy clusters. The reason is that Coma cluster is one of the most massive cluster in the nearby universe, so it is right that it needs a bigger sphere to get the universal average of dark matter density.

16. PARAMETERS C AND D GOT RIGHTLY FROM MASS FORMULA AND TWO CLUSTER DATA

R1 = Rvirial 2.9 Mpc	M_{VIRIAL} $1.4*10^{15}$ Msun	Data Joo Heon Yoon- 2017
R2 = R_{200} 2.23 Mpc	M_{200} $1.29*10^{15}$ Msun	Jubee Sohn- M. Geller- 2016

In previous chapter has been analysed widely these data of Coma cluster and it has been concluded that the amount of baryonic matter enclosed by the spherical corona R_{200} and R_{VIRIAL} is negligible versus dark gravitation matter.

According theory of dark gravitation, if there were not baryonic matter in the spherical corona defined by R1 and R2 then parameters C and D would be mathematically the same for R1 and R2.

Therefore it is worth to set a couple of equation with these data in order to calculate parameter C and D by the formula of mass. Their values may be compared with parameters got by the equivalence theorem.

Hereafter these data will be named:

R_2 = Rvirial = 2.9 Mpc	M_2 = M_{VIRIAL} $1.4*10^{15}$ Msun
R_1 = R_{200} = 2.23 Mpc	M_1 = M_{200} = $1.29*10^{15}$ Msun

In epigraph 10.3 has been shown that indefinite integral of mass gives total mass on condition that parameter C has been calculated at a radius where baryonic density is negligible versus dark gravitation density.

$M_{TOTAL}(<r) = \dfrac{\sqrt{r}}{G\cdot\left(C\cdot\sqrt[3]{r}+D\right)^{\frac{3}{2}}}$ From formula of mass it is right to get $\left[C\cdot\sqrt[3]{r}+D\right]^3 = \dfrac{r}{G^2\cdot M^2}$ or

$C\sqrt[3]{r}+D=\sqrt[3]{\dfrac{r}{G^2M^2}}$ By substitution of above data in this formula it is got a pair of equations with the two unknowns C and D.

$C\sqrt[3]{r_1}+D=\sqrt[3]{\dfrac{r_1}{G^2{M_1}^2}}$ and $C\sqrt[3]{r_2}+D=\sqrt[3]{\dfrac{r_2}{G^2{M_2}^2}}$ By simple algebraic calculus it is got

$C(\sqrt[3]{r_2}-\sqrt[3]{r_1})=\sqrt[3]{\dfrac{r_2}{G^2{M_2}^2}}-\sqrt[3]{\dfrac{r_1}{G^2{M_1}^2}}$ and $D=\sqrt[3]{\dfrac{r_2}{G^2{M_2}^2}}-C\sqrt[3]{r_2}$ it is right to get

$D_{MASS} = 8.41257*10^{-17}$ and $C_{MASS} = 1.1891436*10^{-24}$. These parameters are sub indexed as **MASS** because they have been got rightly from total mass formula. Furthermore $R_S = \left(\frac{D}{C}\right)^3 = 11.47\,Mpc$ which is similar to Rs of M31 = 19.85 Mpc

16.1 COMPARISON WITH PARAMETERS D AND C GOT BY THE EQUIVALENCE THEOREM

In the chapter 13 were got these parameters by the equivalence theorem and a second method, its results are below.

PARAMETER D BY THE EQUIVALENCE THEOREM				
Rvirial	M_{VIRIAL}	D_{VIRIAL} $8.9943*10^{-17}$	$C = D/\sqrt[3]{R_S}$	$C_{VIRIAL} = 1.0591*10^{-24}$

The main advantage to calculate D by the equivalence theorem is that it is needed a single data for radius and mass. Obviously this calculus is possible because the theorem considers the galaxy M31 as a reference, for mass, for parameter D and for Rs = 19.85 Mpc.

Therefore it is highly interesting to calculate the same parameter with a different method. Below are the relative differences of parameters C and D when they are calculated by those methods.

Relative difference D_{VIRIAL} vs D_{MASS} %	$(D_{VIRIAL} - D_{MASS}) / D_{VIRIAL}$	6.5 %
Relative difference Cvirial vs C_{MASS} %	$(C_{MASS}$ - Cvirial) / Cvirial	12.3 %

The relative difference is due to two factors, by one side measures about a galaxy cluster 80 Mpc have inevitably errors. The second factor is that there is a little amount of baryonic matter enclosed by R_{200} and R_{VIRIAL}. This fact originates that according dark gravitation theory each radius has a different parameter C and D. In other words there is a C_{200} and a D_{200} for R_{200} and a C_{VIRIAL} and a D_{VIRIAL} for R_{VIRIAL}. However this simple method associates a single parameter D and C for both radius. In other words, baryonic matter enclosed by the spherical corona has been considered dark gravitation mass. Thanks that the amount of baryonic matter enclosed between both radius is negligible versus the amount of dark gravitational matter enclosed between both radius it is possible to consider a single parameter C and a single parameter C, and relative errors are so small.

In fact total mass calculated by formula $M(<r) = \frac{D^{-3/2}\sqrt{Rs}\sqrt{X}}{G\cdot\left(\sqrt[3]{X}+1\right)^{\frac{3}{2}}}$ at R_{200} = 2.23 Mpc using D_{VIRIAL} is M(<2.23 Mpc) = $1{,}2829163*10^{15}$ Msun whereas the data of $M_{200} = 1.29*10^{15}$ Msun. The difference of masses is even lower that error in measures. Therefore, it is concluded that baryonic mass enclosed in the spherical corona is negligible versus dark gravitational mass.

Furthermore, considering that parameters D and C have 21 and 28 digital ciphers, and that measures come from Coma clusters 80 Mpc away such relative differences are a total success, because the two methods used to calculate these parameters are absolutely independents.

17. CONCLUSION

This work is focused in halo region of M31 where baryonic density is negligible regarding DM non baryonic. The reason is that according the main hypothesis of this theory, the non baryonic DM is generated locally by the gravitational field. Therefore it is needed to study DM on the radius dominion where it is possible to study gravitational field propagation without interference of baryonic mass density or at least where this density is negligible.

In order to defend properly the conclusion of this paper, it is important to emphasize that correlation coefficient of power regression over velocity measures in rotation curve in halo region is bigger than 0,96. See chapter 3 where was got coefficients a& b for $v = a \cdot r^b$ law.

In chapter four was mathematically demonstrated that the power law $v = a \cdot r^b$ in halo region is equivalent a DM density called direct DM , whose formula is $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$.

In chapter five was demonstrated mathematically that the power law for velocity $v = a \cdot r^b$ on the rotation curve is mathematically equivalent to a power law for DM density depending on E. $D_{DM} = A \cdot E^B$.

Where $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$.

Therefore joining chapters 3,4 and 5 it is concluded that the high correlation coefficient bigger than 0.96 at power regression law for rotation curve $v = a \cdot r^b$ in halo region support strongly that DM density inside halo region is a power of gravitational field $D_{DM} = A \cdot E^B$ whose parameters A & B are written above.

As it was pointed at introduction, it is known that there is baryonic dark matter such as giant planets, cold gas clouds, brown dwarfs but this type of DM is more probable to be placed inside galactic disk and bulge.

Reader can consult these papers about this open problem: [11] Nieuwenhuizen,T.M. 2010. [12] Nieuwenhuizen,T.M. 2012. [13] Nieuwenhuizen,T.M. 2010 [14] Wyrzykowski,L.2010. [15] M.R.S. Hawkins 2015

In chapter seven was got a Bernoulli differential equation for M31 halo in order to look for a local method to calculate the local field E.

In the chapter eight is made dimensional analysis for magnitudes Density and field E and for universal constants G, h and c. It is demonstrated that formula for DM density need two Pi monomials. Furthermore it is found that coefficient B = 5/3, power of E, is coherent with Buckingham theorem and it differs only two thousandth regarding B value got by statistical regression of rotation curve in M31 halo.

In the chapter 9, considering B= 5/3 are recalculated the others parameters a&b and A and formulas of theory are rewriting. This way, these formulas achieve dimensional coherence and the theory gains simplicity and credibility.

In the tenth chapter are introduced the called reduced formulas for field, density and total mass. The variable of these functions is dimensionless, this way the formulas gain simplicity.

In the eleventh chapter, is calculated masses in M31 at different radius by the reduced formula of mass.

In the chapter 12 is introduced The equivalence theorem which is crucial in order to extend the dark gravitation theory to cluster scale. As mass and density are additive magnitudes it is found a new parameter D associated to the cluster, considering as start point the parameter D associated to one single galaxy.

In the thirteenth chapter are developed two methods to calculate parameter D in clusters and it is demonstrated that both are mathematically equivalents. Also it is calculated three different parameter D for Coma cluster using recent measures of Coma cluster mass.

In the fourteenth chapter is used the formula of total mass in Coma cluster by the three different parameter D calculated in previous chapter. Also it is discussed its result and compared with data measured of Coma cluster mass.

In the fifteenth chapter is introduced the average density formula, it is applied to Coma cluster and it is shown that at virial radius Coma cluster has an average density equal to 100 times the critical density of universe, which match very well with measures. Also it is calculated the radius of sphere where the average density is equal to the Universal average D.M. density, $\Omega_{DM} = 0.27$, and its value is 25.7 Mpc. It is a big value but Coma cluster is the most massive cluster in the nearby Universe so it is right that such radius has to be so big.
In the sixteenth chapter it is proposed a method to calculate parameters C and D, rightly from mass formula. This method is totally independent to the equivalence theorem, so it is highly interesting in order to compare both methods. Data needed are masses at two radii on condition that the amount of baryonic matter between both radii should be negligible versus dark gravitation matter.

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A f(r)-gravity model of the sunyaev-zeldovich profile of the coma cluster compatible with planck data