

# The Riemann metrics

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## Abstract

We define Riemann metrics for riemannian manifolds.

## 1 The Einstein metrics

Let  $(M, g)$  be a riemannian manifold. The riemannian curvature is:

$$R \in \Lambda^2(TM) \otimes \text{End}(TM)$$

$R$  is antisymmetric. The Ricci curvature is:

$$\text{Ric} \in \text{End}(TM)$$

Then the Einstein equations are:

$$\text{Ric} = \lambda \text{Id}$$

with  $\lambda$  a scalar.

## 2 The Riemann metrics

As  $R$  is antisymmetric, we have:

$$\text{tr}((R(X, Y))^2) \leq 0$$

So that the Riemann metrics are defined by the following equation:

$$-\text{tr}((R(X, Y))^2) = g(X, X)g(Y, Y) - g(X, Y)^2 \geq 0$$

## 3 Bibliography

A.Besse, "Einstein manifolds", Springer, Berlin, 1987.

J.Jost, "Riemannian Geometry and Geometric Analysis", Springer, Berlin, 2008.