

We take

$\mathbb{N} = \{0, 1, 2, 3, \dots\}$ the set of natural numbers obtained by Peano's axioms

$\mathbb{Z} = \{p - q | \forall p, q \in \mathbb{N}\} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ the set of integer numbers

$\mathbb{Q} = \{\frac{r}{s} | r \in \mathbb{Z}, s \in \mathbb{Z} - \{0\}\}$ the set of rational numbers

\mathbb{R} = the set of real numbers constructed by rational numbers through Dedekind's sections.

A function, f , is a relation between two sets where to each member of first set is linked at most one member of second set.

A real succession $(a_n)_{n \in \mathbb{N}}$ is a function from \mathbb{N} to \mathbb{R} defined by $n \mapsto a_n$. The most simply succession is $(n)_{n \in \mathbb{N}}$ this succession is divergent.

Let $(a_n)_{n \in \mathbb{N}}$ a real succession is say a serie the sum $\sum_{n \in \mathbb{N}} a_n = a_0 + a_1 + a_2 + \dots$

The natural set can be sayed like the successions of partial sum of the sum $0+1+1+1+1+\dots$ in fact $s_0 = 0, s_1 = 0+1 = 1, s_2 = 0+1+1 = 2, \dots$ so a serie $\sum_{n \in \mathbb{N}} a_n \geq 0+1+1+1+1+\dots$ is a divergent serie because not exist a real number grater than a real succession $(n)_{n \in \mathbb{N}}$ so not exist a real grater than the succession of the partial sum of the serie. Conversely, if $\sum_{n \in \mathbb{N}} a_n \leq 0+1+1+1+1+1+\dots$ the real exists for the construction of real numbers, in general this real depends by infinity.

For example the armonic serie $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$ have for domain the set of natural numbers for codomain the real set that contained the natural set obtained from the sum $0+1+1+1+1+\dots$ so the harmonic serie is minor than $0+1+1+1+\dots$ so a real number in codomain grater than succession of the partial sum of harmonic serie exists. In general the serie $\sum_{n \in \mathbb{N}} \frac{1}{n^s}$ converges for $s > 0$.

The same discuss is true for the functions in fact if get a function from reals to reals, for example if we get the esponential function we note that it diverges more fast of the function that a real x maps the real x so exist a real a such that the limit of $\exp(x)$ for x tends to a is infinity and for the inverse of $\exp(x)$ is true the opposite that is it changes the domain with codomain.