Theorem 1 Let G be a group and ϕ an automorphism of G. If $a \in G$ is of order $o(a) > 0$, then $o(\phi(a)) = o(a)$.
Proof.
This is Lemma 2.8.3 in [1].

Theorem 2 $V = \{e, (1\ \ 2)(3\ \ 4), (1\ \ 3)(2\ \ 4), (1\ \ 4)(2\ \ 3)\}$ is a characteristic subgroup of A_4.
Proof.
The elements of A_4 are

$$e, (1\ \ 2\ \ 3), (1\ \ 3\ \ 2), (1\ \ 2\ \ 4), (1\ \ 4\ \ 2), (1\ \ 3\ \ 4), (1\ \ 4\ \ 3), (2\ \ 3\ \ 4), (2\ \ 4\ \ 3), (1\ \ 2)(3\ \ 4), (1\ \ 3)(2\ \ 4), (1\ \ 4)(2\ \ 3).$$

Let ϕ be an automorphism of A_4. Clearly $\phi(e) = e$. If $a \neq e \in V$, then $o(a) = 2$ and hence $o(\phi(a)) = 2$ by Theorem 1. Since V contains all elements of A_4 of order 2, $\phi(a) \in V$.

References

[1] I. N. Herstein, *Topics in Algebra*, John Wiley & Sons, New York, 1975.