Some Problems about the Neutrino Oscillation and New Explanation for the Neutrino Observations

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Abstract – We review the neutrino oscillation and find some problems about it. The original theory predicts the mass differences existing on three kinds of neutrinos. However, if no external energy or mass participates in the transformation process, it will experience the non-conservation of mass when one neutrino transforms to others and then transforms back to itself again. It also violates one of the conservation of laws of energy and momentum. Furthermore, the speeds of neutrinos before and after transformation must be different because the mass is non-conserved according to the conservation of momentum in the special relativity. It results in the special physical phenomena of self-acceleration and self-deceleration. Even the violation of the Lorentz invariance is proposed in the standard model extension to discuss the neutrino oscillation without the existence of the mass difference, the all other original elementary particles predicting by the standard model will lose their criteria because they seriously obey the Lorentz invariance. After reviewing the results of Super-Kamiokande Collaboration and Sudbury Neutrino Observatory, both results strongly imply the ratio of number between three kinds of neutrinos is approximately ν_e:ν_μ:ν_τ=1:1:1. According to this, we propose a new explanation for the observation data. The detection of neutrinos in the supernova SN 1987A event earlier than light may tell us the truth that the mass of neutrino is very possibly zero. Otherwise, the non-zero mass neutrino must be dragged by gravity to slow down its average velocity whole the traveling period.

Keywords: neutrino, neutrino oscillation, lepton, weak interaction, standard model

PACS: 14.69 Lm, 14.60 Pq, 97.60.Bw

I. Introduction

The neutrino was first proposed by W. Pauli in 1930 for explanation of missing energy and momentum in β-decays [1,2]. The investigation of the neutrino is based on the conservations of momentum and energy which must be satisfied for each detecting case. Since neutrinos interact very weakly with other known particles, they are much difficult to detect. The experimental group led by Cowan and Reines detected the electron neutrinos ν_e firstly produced by nuclear reactors in 1956 [1-4]. Next, the conclusion that the chiral characteristics of neutrinos produced by weak-interaction decay are all left-handed was obtained in 1958 [1,2,5]. The ν_μ neutrinos associated with the μ charged lepton were confirmed by experiments in 1962 [6]. Until 2000, the neutrino of the third-generation lepton, ν_τ, was detected in Fermilab [7]. Nowadays in the standard model, all the neutrinos only appear in the left-handed form and it is zero mass in this model.

Neutrino is thought to have a special characteristic, the neutrino oscillation, referring to the phenomenon that different types of neutrinos can transform from one to others which was first proposed in 1957 [8]. The mixing and oscillation of the different generation neutrinos was first discussed in 1962 [9]. The neutrino oscillation further indicates neutrinos having non-zero mass and it exists the mixing between the different flavors of neutrinos. However, the difficulty to detect neutrinos is that they really interact with matter very weakly. Averagely speaking, one neutrino has to pass through water more than several hundred light-years to take place one interaction. Therefore, the detection of neutrinos is a challenge that requires a lot of detectors and the cutting-edge technology.

Recently, neutrino detections in Super-Kamiokande (SK) Collaboration [10] and Sudbury Neutrino Observatory (SNO) [11,12] revealed the observation data which could prove the neutrino oscillation. But an immediate problem is that the existence of the mass difference between neutrinos causes neutrinos before and after transformation exhibit different speeds. If there were no other mass or energy involving this transformation, these neutrinos with different mass will result in the violation of some physical conservations. This problem has been tried to solve by the standard model extension which discusses the neutrino oscillation without the existence of the mass difference. Therefore, we propose some serious problems about the neutrino oscillation and offer new explanation for the neutrino observations in SK and SNO as well as the supernova SN 1987A event.

II. The Problems About The Neutrino Oscillation

In 1932, the electron neutrino ν_e was first investigated by Sir James Chadwick in the neutron beta-decay [2]

$$n^0 \rightarrow p^+ + e^- + \bar{\nu}_e. \qquad (1)$$

The Feynman diagram describing this reaction is shown in Fig. 1(a) which involves the weak interaction with the charged boson W^-. In 1956, Cowan and Reins measured this following reaction near the nuclear reactor [2]

$$\bar{\nu}_e + p^+ \rightarrow n^0 + e^+. \qquad (2)$$

The electron antineutrino was first found in the experiments. In 1962, the muon antineutrino was also investigated from the similar reaction at Brookhaven by replacing $(\bar{\nu}_e, e^+)$ with $(\bar{\nu}_\mu, \mu^+)$ [2]

$$\bar{\nu}_\mu + p^+ \rightarrow n^0 + \mu^+. \quad (3)$$

In 1956, the neutrino flux was recorded as high as $5 \times 10^{13}\, particles/cm^2 \cdot sec$ [2]. However, total 10^{14} antineutrinos from π^- decays were used in the experiments but only 29 instances were identified in 1962 [2]. Seriously speaking, the neutrino physics is like a field to study the highly invisible particles.

In quantum field theory (QFT), neutrinos are produced by the weak interactions. Each neutrino and its corresponding charged lepton are generated by the W^- or W^+ decays, where W^- and W^+ are the charged, spin-1, and massive bosons [1,2,13,14]. In the pion decay for the first generation in the lepton section, the reaction is [1,2, 13,14]

$$\pi^- \rightarrow e^- + \bar{\nu}_e, \quad (4)$$

and it takes place through the weak charged current [1,2,13,14] as shown in Fig. 1(b). The interaction is the Φ^3 structure and its Lagrangian is [1,2,13,14]

$$L_{W\pi} = -g_{W\pi}\left(J_\mu^{W^-} W^{+\mu} + J_\mu^{W^+} W^{-\mu}\right), \quad (5)$$

where $J_\mu^{W^-}$ and $J_\mu^{W^+}$ are weak charged currents and $g_{W\pi}$ is the coupling constant. It is similar to the one that can describe the pair-production process by a photon [13,14]

$$L_{EM} = eJ_{EM}^{\mu} A_\mu, \quad (6)$$

where e is the unit charge, J_{EM}^{μ} is the electromagnetic current, and A_μ is the photon field. The weak charged current has two parts which are shown as

$$J_\mu^{W^-} = J_{e\mu}^{W^-} + J_{H\mu}^{W^-}, \quad (7)$$

where

$$J_{e\mu}^{W^-} = \bar{\Psi}_e \gamma^\mu (1 - \gamma^5) \Psi_{\nu_e}, \quad (8)$$

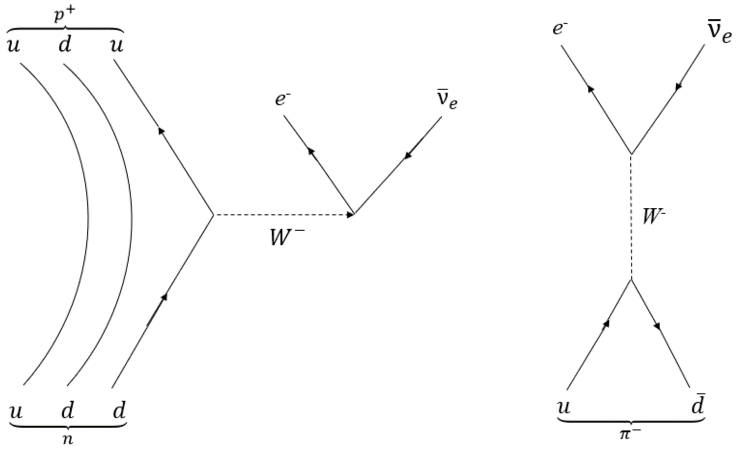
and

$$J_{H\mu}^{W^-} = f_\pi \partial_\mu \Phi_\pi, \quad (9)$$

where γ^μ $(\mu = 0,1,2,3)$ are the Dirac's 4 matrices, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, Φ_π is the positively charged pion field, and f_π is the pion decay constant. The general Lagrangian of the charged current for three generations of leptons is [13,14]

$$L = -\frac{g}{\sqrt{2}} \bar{\Psi}_L \gamma^\mu (1 - \gamma^5) \Psi_{L_\nu} W^- + H.C., \quad (10)$$

where all the first elements of the left-handed doublets for the three-generation leptons are $\Psi_L = \left(\Psi_e, \Psi_\mu, \Psi_\tau\right)^T$, and all the second elements of the left-handed doublets for the three-generation leptons are $\Psi_{L_\nu} = \left(\Psi_{e_\nu}, \Psi_{\mu_\nu}, \Psi_{\tau_\nu}\right)^T$. If the neutrinos have mass, their quantity would be very small. Direct measurements, such as the electron energy spectrum of the Tritium's beta decay, give the electron neutrino mass less than 1 eV [15]. But, in the standard model, the mass of each neutrino is supposed to be zero [13,14].



(a) (b)

Figure 1. (a) The neutron decay through the weak interaction [1,2]. (b) The negative pion decay in the weak interaction [1,2].

However, SK only tells us that the oscillation from ν_e to ν_μ or ν_τ can explain the flow problem of the solar neutrinos, but it does not prove that the missing part of ν_e just transforms to ν_μ and ν_τ. Fortunately, the experiments in SNO can give us more information about it. The SNO's experiments use heavy water D_2O as a target to detect neutrinos. They mainly measure three reaction processes [11]:

$$\nu_e + d^+ \rightarrow p^+ + p^+ + e^- \tag{11}$$

$$\nu_x + d^+ \rightarrow p^+ + n^0 + \nu_x \tag{12}$$

$$\nu_x + e^- \rightarrow \nu_x + e^- \tag{13}$$

The first reaction is the charged-current (CC) process only for ν_e, the second one is the neutral-current (NC) process for all three kinds of neutrinos, and the third one is the elastic process (ES) also for all three kinds of neutrinos. The first and third statistical data are respectively [11]

$$\Phi^{cc} = 1.75 \pm 0.07(stat.)^{+0.12}_{-0.11}(sys.) \pm 0.05(theor.) \times 10^6\ cm^{-2}s^{-1} \tag{14}$$

and

$$\Phi^{ES} = 2.39 \pm 0.34(stat.)^{+0.16}_{-0.14} \times 10^6\ cm^{-2}s^{-1}. \tag{15}$$

In general, the neutrinos of the weakly acting eigenstates are not the eigenstates of mass ν_m. But quantum mechanics tells us that due to the completeness of the eigenstates, the eigenstates of the weak interactions and mass eigenstates can represent each other in this formula $N_L = V_{PMNS} N_L^m$, that is, ν_e is a linear combination of different ν_m

eigenstates. For three generations of neutrinos, V_{PMNS} is a 3 × 3 positive matrix, which describes the mixing characteristics of three generations of neutrinos. The matrix V_{PMNS} is often written as [13,14]

$$V_{PMNS} = \begin{pmatrix} V_{e1} \; V_{e2} \; V_{e3} \\ V_{\mu 1} \; V_{\mu 2} \; V_{\mu 3} \\ V_{\tau 1} \; V_{\tau 2} \; V_{\tau 3} \end{pmatrix}. \tag{16}$$

V_{PMNS} can be theoretically described by four independent parameters, three mixing angles θ_{12}, θ_{23}, θ_{13}, and one phase term δ. The commonly standard form for use is [16]

$$V_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times diag\left(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}\right), \tag{17}$$

where $c_{ij}=\cos\theta_{ij}$ and $s_{ij}=\sin\theta_{ij}$. The oscillation probability from ν_α to ν_β is given by the following formula [16]

$$P\left(\nu_\alpha \to \nu_\beta\right) = \delta_{\alpha\beta} - 4\sum_{i>j} Re\left(V_{\alpha i}^* V_{\alpha j} V_{\beta i} V_{\beta j}^*\right)\sin^2\left(\Delta m_{ij}^2 x/4E\right) - 2\sum_{i>j} Im\left(V_{\alpha i}^* V_{\alpha j} V_{\beta i} V_{\beta j}^*\right)\sin^2\left(\Delta m_{ij}^2 x/2E\right), \tag{18}$$

where x is the propagation distance from the origin at time t=0 and E is the total energy of neutrino. E is assumed to be a constant before and after the transformation.

Next, consider the case of two-kind neutrinos for the easily understanding purpose, i.e. ν_e and ν_μ, and θ is the mixing angle. Since neutrinos have very low activity interacting with other substances, the energy of the propagation process can be treated as a conserved quantity. The individual energy of the ν_m eigenstate is $E_i = (m_i^2 c^4 + c^2 p_i^2)^{1/2}$ where m_i is the mass and p_i is the momentum for the ith eigenstate with i=1,2. In the quantum-mechanical language, ν_e and ν_μ states can be related to the mass eigenstates of ν_{m1} and ν_{m2} by using the mixing matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix}. \tag{19}$$

This equation gives two ν_e- and ν_μ- neutrino states in terms of the mass eigenstates which are respectively

$$|\nu_e\rangle = \cos\theta|\nu_{m1}\rangle + \sin\theta\,|\nu_{m2}\rangle, \tag{20}$$

and

$$|\nu_\mu\rangle = -\sin\theta\,|\nu_{m1}\rangle + \cos\theta\,|\nu_{m2}\rangle. \tag{21}$$

However, both momenta of the mass eigenstates are p_1 and p_2, respectively. It means that two parts of ν_e move inconsistently in space if $v_1 \neq v_2$. That will cause ν_e separated as shown in Fig. 2, so its two parts must have the same speed at t=0. It means

$$\frac{p_1 c^2}{E_1} = v_1 = v_2 = \frac{p_2 c^2}{E_2}. \tag{22}$$

In reality, the experiments tell us that the detection of each coming particle shows the characteristic definitely belong to one of the three neutrinos, not the one part of one neutrino and the other part of the other neutrinos. Based on this, the neutrino doesn't move in separation. Otherwise, different parts of the original neutrino will move in different speed and we have to explain where the second part goes if only the early arrival part is detected?

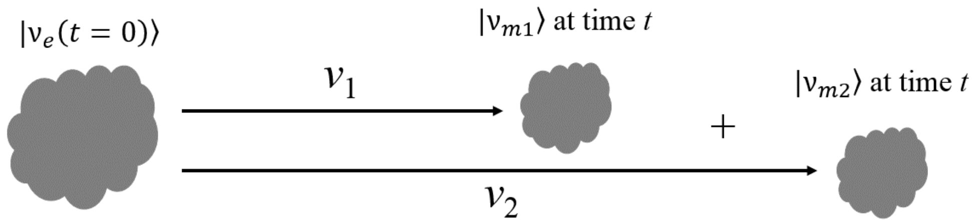


Figure 2. According to the *PMNS* matrix, if two parts of the neutrino ν_e moving in different speeds in space, they will separate from each other in some period.

Next, using Dirac's notation, the ν_e-neutrino state generated at x=0 and t = 0 is

$$|\nu_e(0)\rangle = |\nu_e\rangle = \cos\theta |\nu_{m1}\rangle + \sin\theta\, |\nu_{m2}\rangle, \tag{23}$$

where the ν_μ-neutrino state is

$$|\nu_\mu(0)\rangle = 0 \neq |\nu_\mu\rangle. \tag{24}$$

After the generation of neutrino, it will propagate in terms of the mass eigenstates in the free space. At time t and the propagation distance x, the ν_e-neutrino state becomes

$$|\nu_e(t)\rangle = \cos\theta\, e^{i(p_1 x - E_1 t)/\hbar} |\nu_{m1}\rangle + \sin\theta\, e^{i(p_2 x - E_2 t)/\hbar} |\nu_{m2}\rangle. \tag{25}$$

Hence, the probability amplitudes of ν_e and ν_μ measured at time t are

$$\langle \nu_e | \nu_e(t)\rangle = \cos^2\theta e^{i(p_1 x - E_1 t)/\hbar} + \sin^2\theta e^{i(p_2 x - E_2 t)/\hbar} \tag{26}$$

and

$$\langle \nu_\mu | \nu_e(t)\rangle = -\sin\theta\cos\theta\, e^{i(p_1 x - E_1 t)/\hbar} + \sin\theta\cos\theta\, e^{i(p_2 x - E_2 t)/\hbar}. \tag{27}$$

Therefore, the probability of the transformation from ν_e to ν_μ is

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \nu_e(t)\rangle|^2 = \sin^2(2\theta) \cdot \sin^2\left(\frac{\Delta p x - \Delta E t}{2\hbar}\right), \tag{28}$$

where $x/t=v=v_1=v_2$, $\Delta p = (p_2 - p_1)$, and $\Delta E = (E_2 - E_1)$. Similarly, the probability to hold on the v_e-neutrino state is

$$P(\nu_e \to \nu_e) = |\langle \nu_e | \nu_e(t) \rangle|^2 = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta px - \Delta Et}{2\hbar}\right). \qquad (29)$$

The above two equations show oscillation in time dependent on the initial condition $(v\Delta p - \Delta E)$ so the period T between two maxima that the part of v_e transforms mostly to v_μ is

$$T = \frac{\pi\hbar}{v\Delta p - \Delta E} \, . \qquad (30)$$

Considering the average momentum in the x direction at time t, it is

$$\overline{p(t)} = \sum_{i=e,\mu} \langle \nu_e(t) | \nu_i \rangle \langle \nu_i | \hat{p} | \nu_i \rangle \langle \nu_i | \nu_e(t) \rangle$$

$$= (p_1 \cos^2\theta + p_2 \sin^2\theta) + (p_2 - p_1) \cos 2\theta \sin^2(2\theta)\sin^2\left(\frac{\Delta px - \Delta Et}{2\hbar}\right), \quad (31)$$

where $\hat{p}$ is the momentum operator. This result tells us the variation of momentum in time for the initially free neutrino v_e. Furthermore, the average total mass M at time t is

$$\bar{M} = \sum_{i=e,\mu} \langle \nu_e(t) | \nu_i \rangle \langle \nu_i | \widehat{m} | \nu_i \rangle \langle \nu_i | \nu_e(t) \rangle$$

$$= (m_1 \cos^2\theta + m_2 \sin^2\theta) + (m_2 - m_1) \cos 2\theta \sin^2(2\theta)\sin^2\left(\frac{\Delta px - \Delta Et}{2\hbar}\right), (32)$$

where $\widehat{m}$ is the mass operator and m_1 and m_2 are the aforementioned eigenvalues of the two-mass eigenstates. It obviously that the average total mass M is not a constant and dependent on time. If the neutrino oscillation happens, mass will exist some difference in time after one neutrino transforms to another because the average total mass is non-conserved. According to the transition period T, a part of v_e transforms to v_μ and then transforms back to itself totally after another period time T as shown in Fig.3. The mass will change from m_{ve} to $m_{v\mu}$ and back to m_{ve} again. If there were no additional mass or energy participating in this transformation and $m_{\nu_\mu} \neq m_{\nu_e}$, then the conservation of mass is directly broken. If this transformation exists, the mass difference also causes one serious problem. Supposing the initial speed of the neutrino v_e is u_e and the final speed of the neutrino is u_μ. According to the conservation of energy in the special relativity, it gives the equivalence

$$\gamma_e m_{\nu_e} c^2 = \gamma_\mu m_{\nu_\mu} c^2 \qquad (33)$$

where γ is the Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}. \tag{34}$$

Substituting Eq. (34) into Eq. (33) and using the condition $m_{\nu\mu} \neq m_{\nu e}$, then we have

$$u_\mu \neq u_e. \tag{35}$$

It clearly shows that the speed before and after transformation must be different because two neutrinos have different mass if no other particles participate in the transformation. Then neutrino will perform self-acceleration or self-deceleration without any external force after time $2T$! This directly violate the conservation of momentum especially in the elementary particle physics. It means that the neutrino oscillation cannot happen due to the violations of the conservations of mass and momentum. No matter what kind of neutrinos is detected, the equipment must detect one complete neutrino belong to one of the three neutrinos, ν_e, ν_ν, and ν_μ, in each detection like the reaction in Eq. (11). Therefore, we can check the mass before and after the transformation by the conservations of momentum and energy in the special relativity. The conservation of momentum is

$$\gamma_e m_{\nu_e} u_e = \gamma_\mu m_{\nu_\mu} u_\mu. \tag{36}$$

Combing Eq. (36) with Eq. (33), it gives

$$u_\mu = u_e, \tag{37}$$

and

$$m_{\nu_\mu} = m_{\nu_e}. \tag{38}$$

The laws of conservations of momentum and energy clearly tell us that even the transformation exists, the mass as well as velocity must be constant before and after the transformation!

Even some research points out the neutrino oscillation possibly existing at Lorentz and *CPT* violation [17], the direct violation of Lorentz invariance still makes some serious problem. The Lorentz violation of neutrino means the existence of this violation from the beginning since the neutrino's birth. Such unique spacetime for the neutrino makes it inconsistent with other elementary particles describing by the standard model based on the Lorentz invariance so as to result in neutrino oscillation more questionable and doubtful. Then without the Lorentz invariance, how to describe and calculate the following reaction because it obeys the Lorentz invariance [2]?

$$\nu + n \to p^+ + e^-. \tag{39}$$

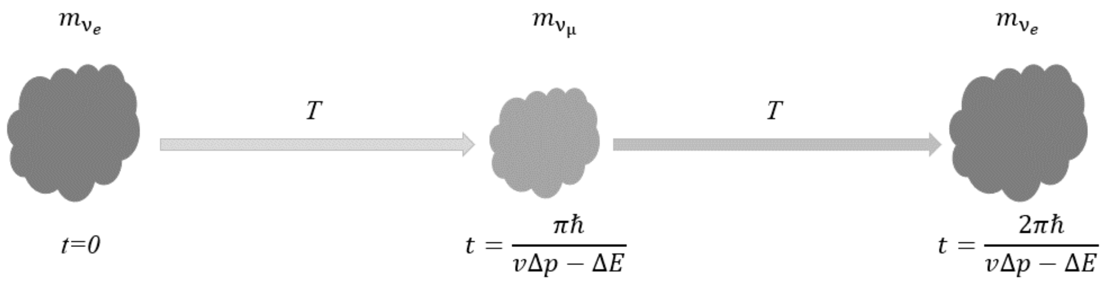


Figure 3. ν_e transforms to ν_μ maximally after time T and transforms itself totally after $2T$. If there were no additional mass or energy and the mass of both neutrinos $m_{\nu_\mu} \neq m_{\nu_e}$, then the conservations of mass as well as momentum are directly broken in this case which are shown in Eqs. (31) and (32).

III. Dose The Heisenberg Uncertainty Principle Allow The Violation Of The Conservations Of Energy And Momentum Significantly?

Since we deal with the quantum problem, then we have to face the Heisenberg uncertainty Principle. Dose the Heisenberg uncertainty principle allow the significant violation of the conservations of momentum and energy? Someone might say that the Heisenberg uncertainty principle allows for coherence of two superposition mass eigenstates, without violating the conservations of energy and momentum. We might ask whether it is true or not?

We can explain what problems the neutrino oscillation phenomenon will encounter when applying the Heisenberg uncertainty principle. The upper limit of three generations of lepton neutron mass is reported as 0.3 eV [18], so the average maximum mass of each neutrino is 0.1 eV. Considering that solar neutrinos have an initial energy of 1 MeV, it belongs to an extremely relativistic case, and the corresponding Lorentz factor γ is about 10^7. Using Eq. (28) for the two neutrino oscillation probabilities, we suppose that the maximum probability of an electron neutrino becoming a muon neutrino is 0.5, and the mass difference between the two neutrinos is about Δm~0.0086 eV [16]. Then the average energy measured at the maximum transforming probability on the Earth is

$$E \sim \gamma[0.5 \times (0.1) + 0.5 \times (0.1 + 0.086)] = 1.043\ MeV. \tag{40}$$

That is to say, when this neutrino travels from the sun to the Earth, the average energy at the maximal transforming probability of neutrons will increase by 43,000 eV. Where does so much energy come from? When calculating the energy uncertainty according to the uncertainty principle, in order to produce such an energy uncertainty effect, it is necessary to be able to measure the neutrino position to an accuracy of 1Å! For neutrinos, such accuracy is a fantasy. If we think that the Lorentz factor using in Eq. (40) is not a constant, then we must use the conservations of energy and momentum to calculate the Lorentz factor. Therefore, we cannot avoid to use the conservation of energy and momentum here. In the field of astronomy, neutrino energy can be as high as 1 GeV or even 1 TeV. For a 1 TeV neutrino, if the aforementioned two neutrino oscillations exist, the energy gap will reach 43 GeV! Such energy gap means that we have measure the position of neutrino at much higher accuracy than the solar neutrino. Thus, it even leads to such conclusion that energy comes from nothing, and it cannot satisfy the conservation of momentum!

Neutrino mixing is a quantum phenomena and can be described in Quantum Mechanics. Neutrino states are represented as a wave function describing probability of measurements as shown in Eqs. (18), (28), and (29). Actually, we do not know which state it will be before measurement. In a measurement, when one of actual eigenstates is discovered, this neutrino interacts with the experimental setup and we will know which kind of neutrinos is detected. All the detections can give a statistical distribution and, therefore, the probability of each neutrino at certain measuring place can be known. As mentioned in Introduction, the neutrino was first proposed by W. Pauli in 1930 for explanation of missing energy and momentum in β-decays [1,2]. Since neutrinos interact very weakly with other known particles, they are much difficult to detect. The investigation of the neutrino is based on the conservations of momentum and energy which must be satisfied for each detecting case. The other thing that we have to notice is the value in Eq. (40), where the detecting place is at the maximum transforming probability in Eq. (28). If we adopt to detect muon neutrino at its minimum transforming probability, then we will not detect anyone of them. It means that the total detecting neutrino is electron neutrino and thus, the conservations of energy and momentum are automatically preserved. This place is not like the one describing in Eq. (40) and on the contrary, the Heisenberg uncertainty principle is not necessary to be considered. Therefore, the detecting places at the maximum transforming probability and the minimum transforming probability give a total different results. The former needs to consider the energy uncertainty and the later does not need to think about it. These two places give the violation results for the same detecting thing.

Except for the viewpoint of the Heisenberg uncertainty principle, let us look at how to calculate some decay rates and scattering involving the weak interaction as well as neutrinos. The Golden rule gives the decay rate of the muon decay in the muon rest frame [2]

$$d\Gamma = \frac{\langle |M|^2 \rangle}{2\hbar m_\mu}\left[\frac{c d^3\boldsymbol{p}_2}{(2\pi^3)2E_2}\right]\left[\frac{c d^3\boldsymbol{p}_3}{(2\pi^3)2E_3}\right]\left[\frac{c d^3\boldsymbol{p}_4}{(2\pi^3)2E_4}\right](2\pi^4)\delta^4(p_1 - p_2 - p_3 - p_4), \quad (41)$$

where p_1, p_2, p_3, and p_4 are the four momenta of muon, electron anti-neutrino, muon neutrino, and electron, respectively, m_μ is the muon mass, and the average amplitude square with the weak coupling constant g_w and the mass of the W boson M_W is [2]

$$\langle |M|^2 \rangle = 2\left(\frac{g_W}{M_W c}\right)^4 (p_1 \cdot p_2)(p_3 \cdot p_4). \tag{42}$$

Similarly, in the neutron decay as the reaction in Eq. (1), the Golden rule gives the decay rate in the rest frame of the neutron [2]

$$d\Gamma = \frac{\langle |M|^2 \rangle}{2\hbar m_n}\left[\frac{c d^3 \boldsymbol{p}_2}{(2\pi^3)2E_2}\right]\left[\frac{c d^3 \boldsymbol{p}_3}{(2\pi^3)2E_3}\right]\left[\frac{c d^3 \boldsymbol{p}_4}{(2\pi^3)2E_4}\right](2\pi^4)\delta^4(p_1 - p_2 - p_3 - p_4), \tag{43}$$

where p_1, p_2, p_3, and p_4 are the four momenta of neutron, electron antineutrino, proton, and electron, respectively, m_n is the neutron mass, and the expression of the average amplitude square is the same as Eq. (42). Another case is about the elastic muon neutrino-electron scattering in the CM frame, in which the Golden rule gives the differential cross-section [2]

$$d\sigma = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{\langle |M|^2 \rangle c}{(E_1 + E_2)|\boldsymbol{p}_1|}\frac{d^3 \boldsymbol{p}_3 d^3 \boldsymbol{p}_4}{E_3 E_4}\delta^4(p_1 + p_2 - p_3 - p_4), \tag{44}$$

where p_1, p_2, p_3, and p_4 are the four momenta of the incoming muon neutrino, incoming electron, outgoing muon neutrino, and outgoing electron, respectively, and the average amplitude square with the weak coupling constant g_z, the mass of the Z boson M_Z, and the electron mass $m_e \rightarrow 0$ is [2]

$$\langle |M|^2 \rangle = 2\left(\frac{g_z E}{M_Z c^2}\right)^4 \left[(c_V + c_A)^2 + (c_V - c_A)^2 \cos^4\frac{\theta}{2}\right]. \tag{45}$$

In the above equation c_V and c_A are the neutral and axial vector couplings. The same significance in Eqs. (41), (44), and (45) is the conservations of momentum and energy. In the quantum field theory or high-energy physics, when we calculate the decay rate and scattering cross-section by the Feynman rules, it is necessary to consider the conservations of momentum energy at each vortex. At most parts of the high-energy physics, we have to consider the conservations of momentum and energy to explain a lot of phenomena. Even in the Noether's theorem, it shows a conserved quantity associated with every continuous symmetry [13]. For example, the conservation of energy is associated with invariance under time translation, and the conservation of momentum is associated with invariance under space translation [13]. Therefore, the conservations of momentum and energy are indeed the bases in the quantum field theory.

Even in the quantum mechanics, the Heisenberg uncertainty principle is the characteristic of the particle-wave duality. The Golden rule to describe the transition from the initial state i to the final state f is [19]

$$R_{i\to f} = \frac{2\pi}{\hbar}\int\prod_{k}\frac{Vd^3\boldsymbol{p}_k}{(2\pi\hbar)^3}\left|M_{fi}\right|^2\delta\left(E_i^0 - E_f^0 - \sum_{K} E_k\right)\delta\left(\boldsymbol{p}_i - \boldsymbol{p}_f - \sum_{k}\boldsymbol{p}_k\right), \quad (46)$$

where E_i^0, E_f^0, and E_k are the energy of the initial state, final state, and the joining particles due to decay, respectively, $\boldsymbol{p}_i$, $\boldsymbol{p}_f$, and $\boldsymbol{p}_k$ are the momenta of the initial state, final state, and the joining particles due to decay, V is the occupied space, and M_{fi} is the matrix element of the perturbation between the initial and final states. Two delta functions express the conservations of energy and momentum, and the energy and momentum carried off by the free particle are equal to the change in the system [19].

IV. The Estimation Of The Upper Rest Mass Limit For Neutrino From The Supernova SN 1987A Event

If neutrino has mass, then it must be affected by gravity so the Lorentz violation would be questionable and unbelievable based on the instantaneously inertial coordinate frame of General Relativity. According to the supernova SN1987A event [20-25], neutrinos were detected three hours earlier than photons that causes a problem: if neutrinos have mass, why they came to the Earth faster than photons? The role of this supernova, SN 1987A, is 168,000 light years far away from the Earth [26]. It means that the early arrival neutrinos move averagely faster than photons even their speeds should be very slightly slower than the speed of light in the free space. The time difference of three hours means the average speed difference between neutrinos and photons from SN 1987A is $2\text{x}10^{-9}$ or 0.6 m/sec. Maybe someone might say that photons were detected later than neutrinos just because the core collapse process could emit neutrino way earlier than the photon. However, this traveling time is 168,000 years, the core collapse process is only a very tiny period comparing to this traveling time. Therefore, we still can identify that the neutrinos arrived Earth 3 hours than the photons. Even the core collapse last for three hours, it still means that the average speed of the detected neutrinos is almost the same as that of the detected photons. If the rest mass of one neutrino m_ν were non-zero, then the relativistic effect and gravitational affection must be considered. Supposing the speed of neutrino very close to the speed of light, the difference is as small as 10^{-8} in speed. According to the Lorentz factor, this difference in speed gives $\gamma\approx 7{,}000$. If the supernova event exhausted one solar mass to transform energy to all neutrinos and produce approximate 10^{60} neutrinos, then the upper rest mass limit of each neutrino would be

$$m_\nu c^2 \approx (1.99\times 10^{30})\times(3\times 10^8)^2/10^{60}/\gamma \approx 150\ eV. \quad (47)$$

However, the energy of one neutrino occupies the nuclear reaction very small, the upper mass limit should be at least 10^3 times smaller than the above value. Roughly speaking, the upper mass limit of one neutrino is

$$m_\nu c^2 \le 0.15\ eV. \tag{48}$$

This rough estimation is from the SN 1987A event. Theoretically speaking, the neutrinos must be affected by the gravitation if they have non-zero mass. However, the early arrival neutrinos revealed that their average speeds were larger than the photons based on the three-hour difference. As we know, the escaping velocity to get rid of the solar gravitation, the third cosmic velocity, is $1.67 \times 10^4\ m/\sec \approx\ 5.57 \times 10^{-5}\ c$, so each neutrino escaping the gravitation of the supernova would also lose its speed at the same order in the whole time. If the photons interact with the interstellar materials very few, their average speeds should be still very close to c. Only when photons pass through the interstellar materials and interact with them, photons will slow down. On the other hand, in the empty space far away from gravitation, they will propagate in the velocity of light. But it is not the same thing for the neutrinos with non-zero mass. Once the neutrinos leave the initial gravitation, their velocities are affected and decrease until they arrive the detectors on the Earth, about 168,000 light years [26]. Their average speeds must be slower than the speed of light in the free space during the whole traveling time. Hence, an easy way to explain the phenomenon of the arrival neutrinos three hours earlier than photons is the zero-mass neutrino. Photons take part in the electromagnetic interaction so they slow down when passing through the interstellar materials but it is not the truth for the zero-mass neutrinos. They are very hard to interact with matters so not easily decelerated by matters. According to the supernova SN 1987A event, the average speed of neutrinos is 2×10^{-9} faster than the average speeds of photons, therefore it is a reasonable factor that neutrinos have zero mass resulting in them not slowing down by gravitation!

V. New Explanation For The Neutrino Observations

According to the SNO's observations in 2001, the occupation of ν_e from the sun is about 0.32, close to 1/3. If no neutrino oscillation takes place, ν_μ and ν_τ will occupy about 2/3 neutrino flux from the sun. One thing is possible that the solar model needs to be corrected, and the other thing is to boldly predict that only one kind of neutrino exists which can be a linear combination of three different neutrino states. Because neutrino is hard to detect and the present recorded data cannot completely avoid such possibility. For example, the decays of Z^0 bosons can produce three kinds of neutrino. The atmospheric observations from SK in 1998 also revealed the close 1:1 ratio between ν_μ and ν_e, and the missing part of ν_μ was very possibly to be ν_τ which is roughly equal to ν_μ [10]. Both results of SK and SNO imply three equal neutrinos in number and lead to a unified neutrino state as a linear combination of three neutrino states

$$\left|\nu_{unified}\right\rangle = \frac{1}{\sqrt{3}}\left|\nu_e\right\rangle + \frac{1}{\sqrt{3}}\left|\nu_\mu\right\rangle + \frac{1}{\sqrt{3}}\left|\nu_\tau\right\rangle. \tag{49}$$

This situation is similar to the neutral K mesons which are linear combinations of neutral K^0 and its antiparticle $\overline{K}_0$ [1,2]. The laws of conservations of energy and momentum in Eqs. (33) and (36) clearly tell us the result in Eq. (37) that the mass of neutrino is fixed no matter which kind of neutrino is detected. Therefore, all three kinds of neutrinos have the same rest mass zero or non-zero and the Eq. (49) is a reasonable expression. Besides, we also have to consider the possibility of the cross terms at the same time that all neutrinos may interact with all leptons so the correction of Lagrangian is multiplied by a matrix U

$$U = \begin{pmatrix} 1-\delta_{e2}-\delta_{e3} & \delta_{e2} & \delta_{e3} \\ \delta_{\mu1} & 1-\delta_{\mu2}-\delta_{\mu2} & \delta_{\mu3} \\ \delta_{\tau1} & \delta_{\tau2} & 1-\delta_{\tau3}-\delta_{\tau3} \end{pmatrix}, \tag{50}$$

where δ_{e2}, δ_{e3}, $\delta_{\mu1}$, $\delta_{\mu3}$, $\delta_{\tau1}$, and $\delta_{\tau2}$ are possibly non-zero values, and each element should be not negative. Therefore, the matrix will appear in the Lagrangian which becomes

$$L = -\frac{g}{\sqrt{2}}\bar{\psi}_L U\gamma^\mu(1-\gamma^5)\left(\sqrt{3}\Psi_{L_\nu}\right)W^- + H.C. \tag{51}$$

In fact, the inverse muon decay [27] reveals the possibility of the form in Eq. (50). The reaction is

$$\nu_\mu + e^- \to \mu^- + \nu_e. \tag{52}$$

The role of the muon neutrino is changed to the electron neutrino after this reaction so it makes the unified neutrino model become much reasonable.

VI. Conclusions

In conclusion, the neutrino oscillation violates several conservations if the mass difference exists. The original theory predicts the mass differences existing on three kinds of neutrino. However, one neutrino transforms to another and then transforms back to itself again that causes the mass non-conservation if no external energy or additional mass participates in the transforming process. It also violates one of the conservations of energy and momentum and the speeds of neutrinos before and after transformation must be different that results in self-acceleration and deceleration. Even the Lorentz violation is proposed in the standard model extension, the all other originally elementary particles predicting by the standard model will lose their criteria because all of them obey the Lorentz invariance. Based on the assumption, once neutrino is generated, its spacetime will violate the Lorentz symmetry everywhere.

Neutrino shall not have so special spacetime different from all other elementary particles. Thus, the Lorentz violation is still not reasonable to explain the neutrino oscillation even the mass differences between three kinds of neutrinos are not existing. The non-zero mass of neutrinos might cause a problem because they must be affected by gravity. If so, the fact that the neutrinos arrived Earth three hours than photons in the supernova SN 1987A event would be not easy to reasonably explain because the traveling distance is 16,800 light years! An easy way to explain the observations from the supernova SN 1987A is the zero mass of neutrinos.

After reviewing the results of SK and SNO, both results strongly imply the ratio of number between three kinds of neutrinos is approximate ν_e:ν_μ:ν_τ=1:1:1. According to this, we propose a new explanation for the observation data. Only one unified neutrino exists in nature which is a linear combination of three neutrino states. This situation is similar to the neutral K mesons which are linear combinations of K^0 and its antiparticle $\overline{K}_0$. This is a reasonable expression because the laws of conservations of energy and momentum tell us that the three kinds of neutrinos have the same mass zero or non-zero. Each lepton not only interacts with the corresponding neutrino state, but also interacts with other lepton neutrino states and the correction of the Largrangian described by a new matrix is revealed here.

Acknowledgement

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