

The coupled Einstein equations

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November 22, 2019

Abstract

The coupled Einstein equations are defined for a manifold with two riemannian metrics.

1 The Einstein equations

Let (M, g) be a riemannian manifold with riemannian curvature R .

$$r_g(x, y, z, t) = g(R(x, y)z, t)$$

$$Ric(g)(x, y) = \sum_i r_g(x, e_i, y, e_i)$$

The Einstein equations are then [Be]:

$$Ric(g) = \lambda g$$

2 The coupled Einstein equations

Let (g, g') be two metrics over the manifold M , then the coupled Einstein equations are:

$$Ric(g) = \lambda g'$$

$$Ric(g') = \lambda' g$$

If $g = g'$, these equations are obviously the Einstein equations.

References

[Be] A.Besse, "Einstein Manifolds", Springer Verlag, Berlin, 1987.