**On some new mathematical connections between Ramanujan's sum of two cubes, ζ(2), $\pi$, $\phi$, Ramanujan's mock theta functions and various sectors of Theoretical Physics**

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## Abstract

*In this research thesis, we have described some new possible mathematical connections between various equations concerning the Ramanujan's sum of two cubes, ζ(2), π, ϕ, Ramanujan's mock theta functions and some sectors of Theoretical Physics*

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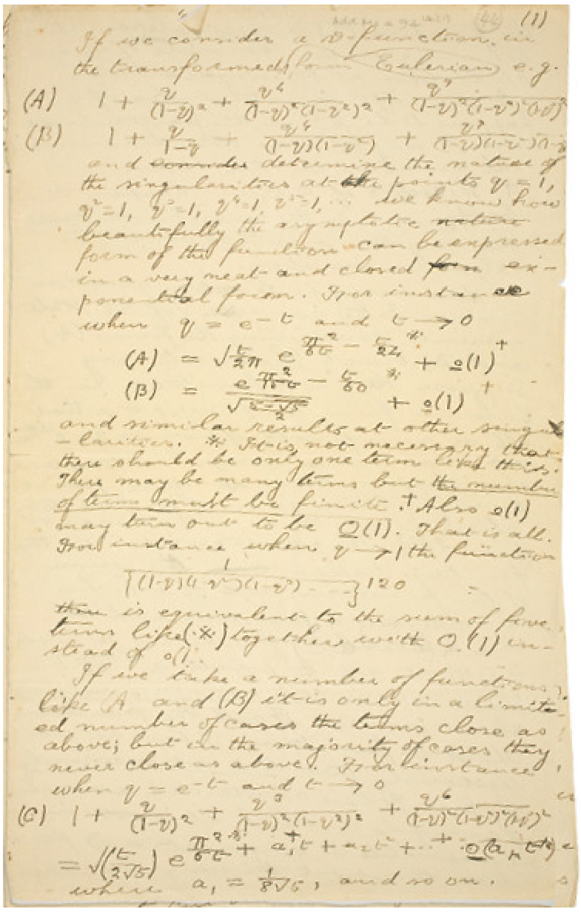
https://www.pinterest.it/pin/444237950734694507/?lp=true



(1)

If we consider a $\vartheta$-function in the transformed Eulerian form e.g.

(A) $1 + \frac{q}{(1-q)^2} + \frac{q^4}{(1-q)^2(1-q^2)^2} + \frac{q^9}{(1-q)^2(1-q^2)^2(1-q^3)^2} + \cdots$

(B) $1 + \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^2)} + \frac{q^9}{(1-q)(1-q^2)(1-q^3)} + \cdots$

and determine the nature of the singularities at the points $q = 1$, $q^2 = 1$, $q^3 = 1$, $q^4 = 1$, $q^5 = 1$, … we know how beautifully the asymptotic form of the function can be expressed in a very neat and closed exponential form. For instance when $q = e^{-t}$ and $t \to 0$

$$(A) = \sqrt{\frac{t}{2\pi}}\, e^{\frac{\pi^2}{6t} - \frac{t}{24}} + o(1)$$

$$(B) = \frac{e^{\frac{\pi^2}{15t} - \frac{t}{60}}}{\sqrt{\frac{5-\sqrt{5}}{2}}} + o(1)$$

and similar results at other singularities. * It is not necessary that there should be only one term like this. There may be many terms but the number of terms must be finite. † Also $o(1)$ may turn out to be $O(1)$. That is all. For instance when $q \to 1$ the function

$$\left\{\frac{1}{(1-q)(1-q^2)(1-q^3)\cdots}\right\}^{120}$$

is equivalent to the sum of five terms like (*) together with $O(1)$ instead of $o(1)$.

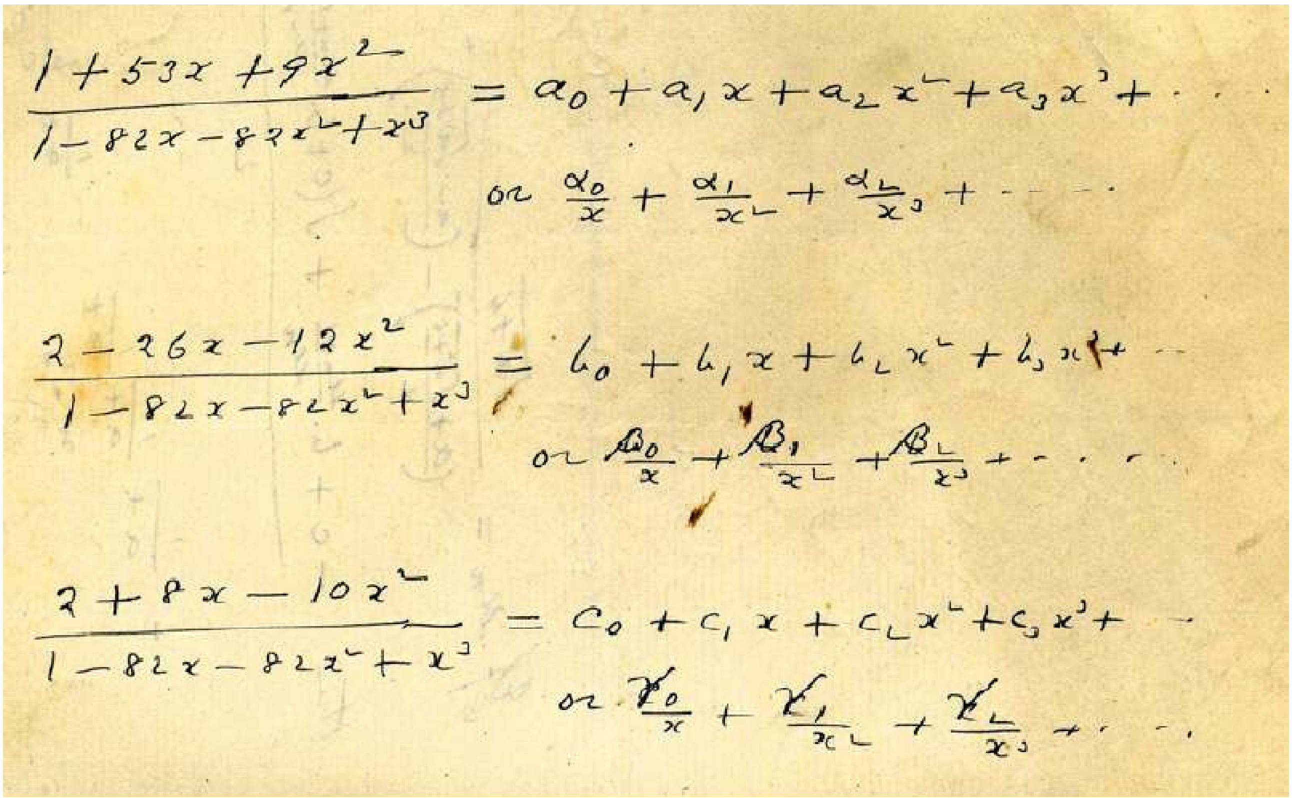
If we take a number of functions like (A) and (B) it is only in a limited number of cases the terms close as above; but in the majority of cases they never close as above. For instance, when $q = e^{-t}$ and $t \to 0$

(C) $1 + \frac{q}{(1-q)^2} + \frac{q^3}{(1-q)^2(1-q^2)^2} + \frac{q^6}{(1-q)^2(1-q^2)^2(1-q^3)^2} + \cdots$

$$= \sqrt{\left(\frac{t}{2\sqrt{5}}\right)}\, e^{\frac{\pi^2}{5t} + a_1 t + a_2 t^2 + \cdots + O(a_k t^k)}$$

where $a_1 = \frac{1}{8\sqrt{5}}$, and so on.

https://www.billtoole.net/wordpress/all/ramanujans-mock-theta-functions-letter-to-hardy-1920/



$$\frac{1+53x+9x^2}{1-82x-82x^2+x^3} = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$
$$\text{or } \frac{\alpha_0}{x} + \frac{\alpha_1}{x^2} + \frac{\alpha_2}{x^3} + \cdots$$

$$\frac{2-26x-12x^2}{1-82x-82x^2+x^3} = b_0 + b_1x + b_2x^2 + b_3x^3 + \cdots$$
$$\text{or } \frac{\beta_0}{x} + \frac{\beta_1}{x^2} + \frac{\beta_2}{x^3} + \cdots$$

$$\frac{2+8x-10x^2}{1-82x-82x^2+x^3} = c_0 + c_1x + c_2x^2 + c_3x^3 + \cdots$$
$$\text{or } \frac{\gamma_0}{x} + \frac{\gamma_1}{x^2} + \frac{\gamma_2}{x^3} + \cdots$$

For x = 2,

(1+53*2+9*4) / (1-82*2-82*4+8)

**Input:**

$$\frac{1+53\times2+9\times4}{1-82\times2-82\times4+8}$$

**Exact result:**

$$-\frac{143}{483}$$

**Decimal approximation:**

−0.29606625258799171842650103519668737060041407867494824016...

-0.29606625…

(2-26*2-12*4) / (1-82*2-82*4+8)

**Input:**

$$\frac{2-26\times2-12\times4}{1-82\times2-82\times4+8}$$

**Exact result:**

$$\frac{14}{69}$$

**Decimal approximation:**

0.202898550724637681159420289855072463768115942028985507246...

0.20289855…

(2+8*2-10*4) / (1-82*2-82*4+8)

**Input:**

$$\frac{2+8\times 2-10\times 4}{1-82\times 2-82\times 4+8}$$

**Exact result:**

$$\frac{22}{483}$$

**Decimal approximation:**

0.045548654244306418219461697722567287784679089026915113871...

0.04554865…

-2/(((((1+53*2+9*4) / (1-82*2-82*4+8))) * (((2-26*2-12*4) / (1-82*2-82*4+8))) * (((2+8*2-10*4) / (1-82*2-82*4+8))))

**Input:**

$$-\frac{2}{\frac{1+53\times 2+9\times 4}{1-82\times 2-82\times 4+8}\times\frac{2-26\times 2-12\times 4}{1-82\times 2-82\times 4+8}\times\frac{2+8\times 2-10\times 4}{1-82\times 2-82\times 4+8}}$$

**Exact result:**

- Step-by-step solution

$$\frac{2299563}{3146}$$

**Decimal approximation:**

- More digits

730.948188175460902733630006357279084551811824539097266369 9...

730.94818817…

-1/(((((1+53*2+9*4) / (1-82*2-82*4+8))) + (((2-26*2-12*4) / (1-82*2-82*4+8))) + (((2+8*2-10*4) / (1-82*2-82*4+8))))

**Input:**

$$-\frac{1}{\frac{1+53\times 2+9\times 4}{1-82\times 2-82\times 4+8}+\frac{2-26\times 2-12\times 4}{1-82\times 2-82\times 4+8}+\frac{2+8\times 2-10\times 4}{1-82\times 2-82\times 4+8}}$$

**Result:**

21

21 that is a Fibonacci number

-36-(21*4)/((((1+53*2+9*4) / (1-82*2-82*4+8))) + (((2-26*2-12*4) / (1-82*2-82*4+8))) + (((2+8*2-10*4) / (1-82*2-82*4+8))))

**Input:**

$$-36-\frac{21\times 4}{\frac{1+53\times 2+9\times 4}{1-82\times 2-82\times 4+8}+\frac{2-26\times 2-12\times 4}{1-82\times 2-82\times 4+8}+\frac{2+8\times 2-10\times 4}{1-82\times 2-82\times 4+8}}$$

**Result:**

1728

1728

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

-(21*4)/((((1+53*2+9*4) / (1-82*2-82*4+8))) + (((2-26*2-12*4) / (1-82*2-82*4+8))) + (((2+8*2-10*4) / (1-82*2-82*4+8

**Input:**

$$-\frac{21\times 4}{\frac{1+53\times 2+9\times 4}{1-82\times 2-82\times 4+8}+\frac{2-26\times 2-12\times 4}{1-82\times 2-82\times 4+8}+\frac{2+8\times 2-10\times 4}{1-82\times 2-82\times 4+8}}$$

**Result:**

1764

1764 result in the range of the mass of candidate "glueball" $f_0(1710)$ ("glueball" =1760 ± 15 MeV).

(((((-(21*4)/((((1+53*2+9*4) / (1-82*2-82*4+8)))+(((2-26*2-12*4) / (1-82*2-82*4+8)))+(((2+8*2-10*4) / (1-82*2-82*4+8))))))))^1/15

**Input:**

$$\sqrt[15]{-\frac{21\times 4}{\frac{1+53\times 2+9\times 4}{1-82\times 2-82\times 4+8}+\frac{2-26\times 2-12\times 4}{1-82\times 2-82\times 4+8}+\frac{2+8\times 2-10\times 4}{1-82\times 2-82\times 4+8}}}$$

**Result:**

$42^{2/15}$

**Decimal approximation:**

1.646012915965068680054939762967836675768640113228558037045...

$1.64601291.... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$

-89/((((1+53*2+9*4) / (1-82*2-82*4+8))) + (((2-26*2-12*4) / (1-82*2-82*4+8))) + (((2+8*2-10*4) / (1-82*2-82*4+8))))

**Input:**

$$-\frac{89}{\frac{1+53\times 2+9\times 4}{1-82\times 2-82\times 4+8}+\frac{2-26\times 2-12\times 4}{1-82\times 2-82\times 4+8}+\frac{2+8\times 2-10\times 4}{1-82\times 2-82\times 4+8}}$$

**Result:**

1869

1869 result practically equal to the rest mass of D meson 1869.62

-(55+8)/((((1+53*2+9*4) / (1-82*2-82*4+8))) + (((2-26*2-12*4) / (1-82*2-82*4+8))) + (((2+8*2-10*4) / (1-82*2-82*4+8))))

**Input:**

$$-\frac{55+8}{\frac{1+53\times 2+9\times 4}{1-82\times 2-82\times 4+8}+\frac{2-26\times 2-12\times 4}{1-82\times 2-82\times 4+8}+\frac{2+8\times 2-10\times 4}{1-82\times 2-82\times 4+8}}$$

**Result:**

1323

1323 result very near to the rest mass of Xi baryon 1321.71

1/((((((-40/((((1+53*2+9*4) / (1-82*2-82*4+8))) + (((2-26*2-12*4) / (1-82*2-82*4+8))) + (((2+8*2-10*4) / (1-82*2-82*4+8))))))))))^1/14

**Input:**

$$\frac{1}{\sqrt[14]{-\frac{40}{\frac{1+53\times2+9\times4}{1-82\times2-82\times4+8}+\frac{2-26\times2-12\times4}{1-82\times2-82\times4+8}+\frac{2+8\times2-10\times4}{1-82\times2-82\times4+8}}}}$$

**Result:**

$$\frac{1}{2^{3/14}\sqrt[14]{105}}$$

**Decimal approximation:**

0.6181913275158574233400158187012309128849089702945775502 43...

0.618191327…

1+1/((((((-40/((((1+53*2+9*4) / (1-82*2-82*4+8))) + (((2-26*2-12*4) / (1-82*2-82*4+8))) + (((2+8*2-10*4) / (1-82*2-82*4+8))))))))))^1/14

**Input:**

$$1+\frac{1}{\sqrt[14]{-\frac{40}{\frac{1+53\times2+9\times4}{1-82\times2-82\times4+8}+\frac{2-26\times2-12\times4}{1-82\times2-82\times4+8}+\frac{2+8\times2-10\times4}{1-82\times2-82\times4+8}}}}$$

**Result:**

$$1+\frac{1}{2^{3/14}\sqrt[14]{105}}$$

**Decimal approximation:**

1.6181913275158574233400158187012309128849089702945775502 43...

1.618191327…

This result is a very good approximation to the value of the golden ratio 1,618033988749...

**Alternate forms:**

$$\frac{1}{210}\left(210+2^{11/14}\times105^{13/14}\right)$$

- root of $840x^{14}-1$ near $x=0.618191$ $+1$
- $$\frac{1+2^{3/14}\sqrt[14]{105}}{2^{3/14}\sqrt[14]{105}}$$

**Minimal polynomial:**

$$840x^{14}-11760x^{13}+76440x^{12}-305760x^{11}+840840x^{10}-1681680x^9+2522520x^8-2882880x^7+2522520x^6-1681680x^5+840840x^4-305760x^3+76440x^2-11760x+839$$

((((((-64/((((1+53*2+9*4) / (1-82*2-82*4+8))) + (((2-26*2-12*4) / (1-82*2-82*4+8))) + (((2+8*2-10*4) / (1-82*2-82*4+8))))))))))^1/14

**Input:**

$$\sqrt[14]{-\frac{64}{\frac{1+53\times2+9\times4}{1-82\times2-82\times4+8}+\frac{2-26\times2-12\times4}{1-82\times2-82\times4+8}+\frac{2+8\times2-10\times4}{1-82\times2-82\times4+8}}}$$

**Result:**

$$2^{3/7}\sqrt[14]{21}$$

**Decimal approximation:**

- More digits

1.672850346449408368645044618012555038771633721803112068787...

1.67285034…. result practically equal to the proton mass without exponent

34+5+((((-1/((((1+53*2+9*4) / (1-82*2-82*4+8))) + (((2-26*2-12*4) / (1-82*2-82*4+8))) + (((2+8*2-10*4) / (1-82*2-82*4+8))))))))^3

**Input:**

$$34+5+\left(-\frac{1}{\frac{1+53\times2+9\times4}{1-82\times2-82\times4+8}+\frac{2-26\times2-12\times4}{1-82\times2-82\times4+8}+\frac{2+8\times2-10\times4}{1-82\times2-82\times4+8}}\right)^3$$

**Result:**

9300

9300 result equal to the rest mass of Bottom eta meson

1/5((((55+5+((((-1/((((1+53*2+9*4) / (1-82*2-82*4+8))) + (((2-26*2-12*4) / (1-82*2-82*4+8))) + (((2+8*2-10*4) / (1-82*2-82*4+8))))))))^3))))

**Input:**

$$\frac{1}{5}\left(55+5+\left(-\frac{1}{\frac{1+53\times2+9\times4}{1-82\times2-82\times4+8}+\frac{2-26\times2-12\times4}{1-82\times2-82\times4+8}+\frac{2+8\times2-10\times4}{1-82\times2-82\times4+8}}\right)^3\right)$$

**Exact result:**

$$\frac{9321}{5}$$

**Decimal form:**

1864.2

1864.2 result practically equal to the rest mass of D meson 1864.84

1/8((((55+((((-1/((((1+53*2+9*4) / (1-82*2-82*4+8))) + (((2-26*2-12*4) / (1-82*2-82*4+8))) + (((2+8*2-10*4) / (1-82*2-82*4+8))))))))^3))))

**Input:**

$$\frac{1}{8}\left(55+\left(-\frac{1}{\frac{1+53\times2+9\times4}{1-82\times2-82\times4+8}+\frac{2-26\times2-12\times4}{1-82\times2-82\times4+8}+\frac{2+8\times2-10\times4}{1-82\times2-82\times4+8}}\right)^3\right)$$

**Exact result:**

$$\frac{2329}{2}$$

**Decimal form:**

1164.5

1164.5 that is practically equal to the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164,2696

[1/8((((55+((((-1/((((1+53*2+9*4) / (1-82*2-82*4+8))) + (((2-26*2-12*4) / (1-82*2-82*4+8))) + (((2+8*2-10*4) / (1-82*2-82*4+8))))))))^3))))]^1/14

**Input:**

$$\sqrt[14]{\frac{1}{8}\left(55+\left(-\frac{1}{\frac{1+53\times2+9\times4}{1-82\times2-82\times4+8}+\frac{2-26\times2-12\times4}{1-82\times2-82\times4+8}+\frac{2+8\times2-10\times4}{1-82\times2-82\times4+8}}\right)^3\right)}$$

**Result:**

$$\sqrt[14]{\frac{2329}{2}}$$

**Decimal approximation:**

- More digits

1.655807951313615057272502519064082641119858385959953293525...

1.655807951… is very near to the 14th root of the above Ramanujan's class invariant i.e. 1,65578...

(24*3)+3+1/8((((((((-1/((((1+53*2+9*4) / (1-82*2-82*4+8))) + (((2-26*2-12*4) / (1-82*2-82*4+8))) + (((2+8*2-10*4) / (1-82*2-82*4+8))))))))^3))))

**Input:**

$$24\times3+3+\frac{1}{8}\left(-\frac{1}{\frac{1+53\times2+9\times4}{1-82\times2-82\times4+8}+\frac{2-26\times2-12\times4}{1-82\times2-82\times4+8}+\frac{2+8\times2-10\times4}{1-82\times2-82\times4+8}}\right)^3$$

**Exact result:**

$$\frac{9861}{8}$$

**Decimal form:**

1232.625

1232.625 result practically equal to the rest mass of Delta baryon 1232

**On the Ramanujan's Mock Theta Function definition: a simplified and efficient version**

**Ramanujan's Definition.** A mock theta function is a function $f$ of the complex variable $q$, defined by a $q$-series of a particular type (Ramanujan calls this the Eulerian form), which converges for $|q| < 1$ and satisfies the following conditions:

(1) infinitely many roots of unity are exponential singularities;

(2) for every root of unity $\xi$, there is a theta function $\vartheta_\xi(q)$, such that the difference $f(q) - \vartheta_\xi(q)$ is bounded as $q \to \xi$ radially; and

(3) $f$ is not the sum of two functions, one of which is a theta function and the other a function that is bounded radially toward all roots of unity.

Ramanujan gave 17 examples of functions he believed satisfied these properties. The most famous are

If we consider a $\vartheta$-function in the transformed Eulerian form e.g.

$$(A) \quad 1+\frac{q}{(1-q)^2}+\frac{q^4}{(1-q)^2(1-q^2)^2}+\frac{q^9}{(1-q)^2(1-q^2)^2(1-q^3)^2}+\ldots$$

$$(B) \quad 1+\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}+\ldots$$

for $|q|<1$, we obtain the following expressions:

1+((0.5/(1-0.5)^2))+((0.5^4/((1-0.5)^2(1-0.5^2)^2)))+((0.5^9/((1-0.5)^2(1-0.5^2)^2(1-0.5^3)^2)))

**Input:**

$$1+\frac{0.5}{(1-0.5)^2}+\frac{0.5^4}{(1-0.5)^2\,(1-0.5^2)^2}+\frac{0.5^9}{(1-0.5)^2\,(1-0.5^2)^2\,(1-0.5^3)^2}$$

**Result:**

3.462585034013605442176870748299319727891156462585034013605...

3.462585…

1+((0.5/(1-0.5)))+(0.5^4/((1-0.5)(1-0.5^2)))+(0.5^9/((1-0.5)(1-0.5^2)(1-0.5^3)))

**Input:**

$$1+\frac{0.5}{1-0.5}+\frac{0.5^4}{(1-0.5)\,(1-0.5^2)}+\frac{0.5^9}{(1-0.5)\,(1-0.5^2)\,(1-0.5^3)}$$

**Result:**

2.172619047619047619047619047619047619047619047619047619047...

2.17261904…

(3.462585034013605442176870748299319727891156462585034013605 - 2.172619047619047619047619047619047619047619047619047619047)^2

**Input interpretation:**

$(3.462585034013605442176870748299319727891156462585034013605 - 2.172619047619047619047619047619047619047619047619047619047)^2$

**Result:**

1.664012246054884538849553426812902031560923689203572585497...

1.664012246…. is very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164,2696 i.e. 1,65578...

1+ 1/(3.462585034013605442176870748299319727891156462585034013605 + 2.172619047619047619047619047619047619047619047619047619047)^1/4

**Input interpretation:**

1 + 1/((3.462585034013605442176870748299319727891156462585034013605 + 2.172619047619047619047619047619047619047619047619047619047) ^ (1/4))

**Result:**

1.649041681007932479373429578924270876187027361935728611356...

1.6490416…. ≈ ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

(3.462585034013605442176870748299319727891156462585034013605 * 2.172619047619047619047619047619047619047619047619047619047)^1/4

**Input interpretation:**

(3.462585034013605442176870748299319727891156462585034013605 × 2.172619047619047619047619047619047619047619047619047619047) ^ (1/4)

**Result:**

1.656136037748216945948207859006308663355637091932716757572...

1.656136037…. is very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164,2696 i.e. 1,65578...

1+(2.1726190476190 / 3.4625850340136)

**Input interpretation:**

$$1 + \frac{2.1726190476190}{3.4625850340136}$$

**Result:**

1.627455795677786840794963737189533665954793225200260832680...

1.62745579…

2Pi*exp(2.1726190476190 + 3.4625850340136)

**Input interpretation:**

$2\pi \exp(2.1726190476190 + 3.4625850340136)$

**Result:**

1760.021225745...

1760.021225745…. result in the range of the mass of candidate "glueball" $f_0(1710)$ ("glueball" =1760 ± 15 MeV).

-(34-3)+2Pi*exp(2.1726190476190 + 3.4625850340136)

**Input interpretation:**

$-(34-3) + 2\pi \exp(2.1726190476190 + 3.4625850340136)$

**Result:**

1729.021225745...

1729.021225745…

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

(2.1726190476190)^8

**Input interpretation:**

$2.1726190476190^8$

**Result:**

496.44216605801424174707009671896433402664295787279494824 01...

496.44216…. result very near to the dimensions of Lie group $E_8$ x $E_8$

(3.4625850340136)^6

**Input interpretation:**

$3.4625850340136^6$

**Result:**

1723.465862411363899675122982730129454502096118164527805896...

1723.46586….

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

(2.1726190476190)^(Pi)^2 – 377

**Input interpretation:**

$2.1726190476190^{\pi^2} - 377$

**Result:**

1740.847337076...

1740.8473…

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

**Alternative representations:**

- $2.17261904761900000^{\pi^2} - 377 = -377 + 2.17261904761900000^{(180\,°)^2}$
- $2.17261904761900000^{\pi^2} - 377 = -377 + 2.17261904761900000^{(-i\log(-1))^2}$
- $2.17261904761900000^{\pi^2} - 377 = -377 + 2.17261904761900000^{6\zeta(2)}$

**Series representations:**

- $2.17261904761900000^{\pi^2} - 377 = -377 + 2.17261904761900000^{16\left(\sum_{k=0}^{\infty} (-1)^k/(1+2k)\right)^2}$
- $2.17261904761900000^{\pi^2} - 377 = -377 + 2.17261904761900000^{4\left(-1+\sum_{k=1}^{\infty} 2^k/\binom{2k}{k}\right)^2}$
- $2.17261904761900000^{\pi^2} - 377 = -377 + 2.17261904761900000^{\left(\sum_{k=0}^{\infty} \left(2^{-k}(-6+50k)\right)/\binom{3k}{k}\right)^2}$

$\binom{n}{m}$ is the binomial coefficient

**Integral representations:**

$$2.17261904761900000^{\pi^2} - 377 = -377 + e^{3.10373349671684264 \left(\int_0^\infty 1/(1+t^2)\, dt\right)^2}$$

- $$2.17261904761900000^{\pi^2} - 377 = -377 + e^{12.4149339868673706 \left(\int_0^1 \sqrt{1-t^2}\, dt\right)^2}$$

- $$2.17261904761900000^{\pi^2} - 377 = -377 + e^{3.10373349671684264 \left(\int_0^\infty \sin(t)/t\, dt\right)^2}$$

((((1/2(2.1726190476190)^10))))^(1/14)

**Input interpretation:**

$$\sqrt[14]{\frac{1}{2} \times 2.1726190476190^{10}}$$

**Result:**

1.6565342381527...

1.6565342…. is very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164,2696$ i.e. 1,65578...

((((1/2(2.1726190476190)^10))))^(1/15)

**Input interpretation:**

$$\sqrt[15]{\frac{1}{2} \times 2.1726190476190^{10}}$$

**Result:**

1.6017216853870...

1.6017216…. result very near to the elementary charge

7/3 (3.4625850340136)^5

**Input interpretation:**

$$\frac{7}{3} \times 3.4625850340136^5$$

**Result:**

1161.3925163204297577653835744994904031455374619307574731 87...

1161.392516…

((((7/3 (3.4625850340136)^5))))^1/14

**Input interpretation:**

$$\sqrt[14]{\frac{7}{3}\times 3.4625850340136^5}$$

**Result:**

1.6554919492254...

1.6554919…. is very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164,2696 i.e. 1,65578...

16/10^3+((((7/3 (3.4625850340136)^5))))^1/14

**Input interpretation:**

$$\frac{16}{10^3}+\sqrt[14]{\frac{7}{3}\times 3.4625850340136^5}$$

**Result:**

1.6714919492254...

1.6714919….

We note that 1.6714919... is a result practically equal to the value of the formula:

$$m_{p'} = 2\times\frac{\eta}{R}m_P = 1.6714213\times 10^{-24}\text{ gm}$$

that is the holographic proton mass (N. Haramein)

1/3(1.6554919492253+3.2722730006610)

**Input interpretation:**

$$\frac{1}{3}(1.6554919492253+3.2722730006610)$$

**Result:**

1.642588316628766666666666666666666666666666666666666...

$1.64258831\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

-24/10^3+1/3(1.6554919492253+3.2722730006610)

**Input interpretation:**

$$-\frac{24}{10^3} + \frac{1}{3}(1.6554919492253 + 3.2722730006610)$$

**Result:**

1.618588316628766666666666666666666666666666666666666...

1.61858831….

This result is a very good approximation to the value of the golden ratio 1,618033988749...

From:

**Quantum Black Holes, Wall Crossing, and Mock Modular Forms**

*Atish Dabholkar, Sameer Murthy, and Don Zagier*

arXiv:1208.4074v2 [hep-th] 3 Apr 2014

$$\mathcal{F}_{7,2}(\tau) = -q^{-25/168} \sum_{n=1}^{\infty} \frac{q^{n^2}}{(1-q^n)\cdots(1-q^{2n-1})} \;=\; -q^{143/168}\left(1 + q + q^2 + 2q^3 + \cdots\right) . \quad (7.7)$$

For $q = e^{2\pi}$

-(((e^(2Pi))))^(143/168)*(1+e^(2Pi)+((e^(2Pi)))^2+2((e^(2Pi)))^3)

**Input:**

$$-\left(e^{2\pi}\right)^{143/168}\left(1 + e^{2\pi} + \left(e^{2\pi}\right)^2 + 2\left(e^{2\pi}\right)^3\right)$$

**Exact result:**

$$-e^{(143\pi)/84}\left(1 + e^{2\pi} + e^{4\pi} + 2e^{6\pi}\right)$$

**Decimal approximation:**

$-6.4622315336181667342598552391177946267368684590840 18\ldots \times 10^{10}$

-6.46223153… * $10^{10}$

**Property:**

$-e^{(143\pi)/84}\left(1+e^{2\pi}+e^{4\pi}+2e^{6\pi}\right)$ is a transcendental number

•

**Alternate forms:**

$e^{(143\pi)/84}\left(-1-e^{2\pi}-e^{4\pi}-2e^{6\pi}\right)$

$-e^{(143\pi)/84}-e^{(311\pi)/84}-e^{(479\pi)/84}-2e^{(647\pi)/84}$

**Alternative representations:**

$$-\left(e^{2\pi}\right)^{143/168}\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)= -\exp^{2\pi}(z)^{143/168}\left(1+\exp^{2\pi}(z)+\exp^{2\pi}(z)^2+2\exp^{2\pi}(z)^3\right) \text{ for } z=1$$

•

$$-\left(e^{2\pi}\right)^{143/168}\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)= -\left(1+e^{360^\circ}+\left(e^{360^\circ}\right)^2+2\left(e^{360^\circ}\right)^3\right)\left(e^{360^\circ}\right)^{143/168}$$

•

$$-\left(e^{2\pi}\right)^{143/168}\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)= -\exp^{2\cos^{-1}(-1)}(z)^{143/168}\left(1+\exp^{2\cos^{-1}(-1)}(z)+\exp^{2\cos^{-1}(-1)}(z)^2+2\exp^{2\cos^{-1}(-1)}(z)^3\right) \text{ for } z=1$$

$\cos^{-1}(x)$ is the inverse cosine function

**Series representations:**

$$-\left(e^{2\pi}\right)^{143/168}\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)= -e^{143/21\sum_{k=0}^{\infty}(-1)^k/(1+2k)}-e^{311/21\sum_{k=0}^{\infty}(-1)^k/(1+2k)}-e^{479/21\sum_{k=0}^{\infty}(-1)^k/(1+2k)}-2e^{647/21\sum_{k=0}^{\infty}(-1)^k/(1+2k)}$$

•

$$-\left(e^{2\pi}\right)^{143/168}\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)= -\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{(143\pi)/84}-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{(311\pi)/84}-\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{(479\pi)/84}-2\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{(647\pi)/84}$$

•

$$-(e^{2\pi})^{143/168}\left(1+e^{2\pi}+(e^{2\pi})^2+2(e^{2\pi})^3\right)=-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{(143\pi)/84}-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{(311\pi)/84}-\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{(479\pi)/84}-2\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{(647\pi)/84}$$

$n!$ is the factorial function

**Integral representations:**

$$-(e^{2\pi})^{143/168}\left(1+e^{2\pi}+(e^{2\pi})^2+2(e^{2\pi})^3\right)=-e^{143/42\int_0^\infty 1/(1+t^2)\,dt}-e^{311/42\int_0^\infty 1/(1+t^2)\,dt}-e^{479/42\int_0^\infty 1/(1+t^2)\,dt}-2\,e^{647/42\int_0^\infty 1/(1+t^2)\,dt}$$

- $$-(e^{2\pi})^{143/168}\left(1+e^{2\pi}+(e^{2\pi})^2+2(e^{2\pi})^3\right)=-e^{143/21\int_0^1\sqrt{1-t^2}\,dt}-e^{311/21\int_0^1\sqrt{1-t^2}\,dt}-e^{479/21\int_0^1\sqrt{1-t^2}\,dt}-2\,e^{647/21\int_0^1\sqrt{1-t^2}\,dt}$$

- $$-(e^{2\pi})^{143/168}\left(1+e^{2\pi}+(e^{2\pi})^2+2(e^{2\pi})^3\right)=-e^{143/42\int_0^1 1/\sqrt{1-t^2}\,dt}-e^{311/42\int_0^1 1/\sqrt{1-t^2}\,dt}-e^{479/42\int_0^1 1/\sqrt{1-t^2}\,dt}-2\,e^{647/42\int_0^1 1/\sqrt{1-t^2}\,dt}$$

ln-(((((-(((e^(2Pi))))^(143/168)*(1+e^(2Pi)+((e^(2Pi)))^2+2((e^(2Pi)))^3))))))

**Input:**

$$\log\left(-\left(-(e^{2\pi})^{143/168}\left(1+e^{2\pi}+(e^{2\pi})^2+2(e^{2\pi})^3\right)\right)\right)$$

$\log(x)$ is the natural logarithm

**Exact result:**

$$\log\left(e^{(143\pi)/84}\left(1+e^{2\pi}+e^{4\pi}+2\,e^{6\pi}\right)\right)$$

**Decimal approximation:**

24.89182562672535979547600930585067202243978208595540962198...

24.891825…

**Alternate forms:**

$$\frac{143\pi}{84}+\log\left(1+e^{2\pi}+e^{4\pi}+2e^{6\pi}\right)$$

- $$\frac{1}{84}\left(143\pi+84\log\left(1+e^{2\pi}+e^{4\pi}+2e^{6\pi}\right)\right)$$

## Alternative representations:

$$\log\left(-(-1)\left(\left(e^{2\pi}\right)^{143/168}\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)\right)\right)=\log_e\left(\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)\left(e^{2\pi}\right)^{143/168}\right)$$

- $$\log\left(-(-1)\left(\left(e^{2\pi}\right)^{143/168}\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)\right)\right)=\log(a)\log_a\left(\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)\left(e^{2\pi}\right)^{143/168}\right)$$

- $$\log\left(-(-1)\left(\left(e^{2\pi}\right)^{143/168}\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)\right)\right)=-\text{Li}_1\left(1-\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)\left(e^{2\pi}\right)^{143/168}\right)$$

$\log_b(x)$ is the base- $b$ logarithm

$\text{Li}_n(x)$ is the polylogarithm function

## Series representations:

$$\log\left(-(-1)\left(\left(e^{2\pi}\right)^{143/168}\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)\right)\right)=\log\left(-1+e^{(143\pi)/84}\left(1+e^{2\pi}+e^{4\pi}+2e^{6\pi}\right)\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+e^{(143\pi)/84}\left(1+e^{2\pi}+e^{4\pi}+2e^{6\pi}\right)}\right)^k}{k}$$

- $$\log\left(-(-1)\left(\left(e^{2\pi}\right)^{143/168}\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)\right)\right)=2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(e^{(143\pi)/84}\left(1+e^{2\pi}+e^{4\pi}+2e^{6\pi}\right)-z_0\right)^k z_0^{-k}}{k}$$

-

$$\log\left(-(-1)\left(\left(e^{2\pi}\right)^{143/168}\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)\right)\right) = 2 i \pi \left\lfloor \frac{\arg\left(e^{(143\pi)/84}\left(1+e^{2\pi}+e^{4\pi}+2e^{6\pi}\right)-x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(e^{(143\pi)/84}\left(1+e^{2\pi}+e^{4\pi}+2e^{6\pi}\right)-x\right)^k x^{-k}}{k} \quad \text{for } x < 0$$

arg(z) is the complex argument

⌊x⌋ is the floor function

**Integral representations:**

$$\log\left(-(-1)\left(\left(e^{2\pi}\right)^{143/168}\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)\right)\right) = \int_1^{e^{(143\pi)/84}\left(1+e^{2\pi}+e^{4\pi}+2e^{6\pi}\right)} \frac{1}{t}\,dt$$

- 

$$\log\left(-(-1)\left(\left(e^{2\pi}\right)^{143/168}\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)\right)\right) = -\frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1+e^{(143\pi)/84}\left(1+e^{2\pi}+e^{4\pi}+2e^{6\pi}\right)\right)^{-s}\Gamma(-s)^2\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1 < \gamma < 0$$

Γ(x) is the gamma function

1/(2Pi)*ln^2(((((ln-(((((-(((e^(2Pi))))^(143/168)*(1+e^(2Pi)+((e^(2Pi)))^2+2((e^(2Pi)))^3)))))))))))

**Input:**

$$\frac{1}{2\pi}\log^2\left(\log\left(-\left(-\left(e^{2\pi}\right)^{143/168}\left(1+e^{2\pi}+\left(e^{2\pi}\right)^2+2\left(e^{2\pi}\right)^3\right)\right)\right)\right)$$

log(x) is the natural logarithm

**Exact result:**

$$\frac{\log^2\left(\log\left(e^{(143\pi)/84}\left(1+e^{2\pi}+e^{4\pi}+2e^{6\pi}\right)\right)\right)}{2\pi}$$

**Decimal approximation:**

1.644590035831444017323160484253921846401584845668438998331...

1.644590035…. ≈ ζ(2) = $\frac{\pi^2}{6} = 1.644934$ ...

**Alternate forms:**

$$\frac{\log^2\left(\frac{143\pi}{84} + \log\left(1 + e^{2\pi} + e^{4\pi} + 2 e^{6\pi}\right)\right)}{2\pi}$$

$$\frac{\left(\log\left(\frac{1}{21}\left(143\pi + 84\log\left(1 + e^{2\pi} + e^{4\pi} + 2 e^{6\pi}\right)\right)\right) - 2\log(2)\right)^2}{2\pi}$$

**Alternative representations:**

$$\frac{\log^2\left(\log\left(-\left(-\left(e^{2\pi}\right)^{143/168}\left(1 + e^{2\pi} + \left(e^{2\pi}\right)^2 + 2\left(e^{2\pi}\right)^3\right)\right)\right)\right)}{2\pi} = \frac{\log_e^2\left(\log\left(\left(1 + e^{2\pi} + \left(e^{2\pi}\right)^2 + 2\left(e^{2\pi}\right)^3\right)\left(e^{2\pi}\right)^{143/168}\right)\right)}{2\pi}$$

- 

$$\frac{\log^2\left(\log\left(-\left(-\left(e^{2\pi}\right)^{143/168}\left(1 + e^{2\pi} + \left(e^{2\pi}\right)^2 + 2\left(e^{2\pi}\right)^3\right)\right)\right)\right)}{2\pi} = \frac{\left(\log(a)\log_a\left(\log\left(\left(1 + e^{2\pi} + \left(e^{2\pi}\right)^2 + 2\left(e^{2\pi}\right)^3\right)\left(e^{2\pi}\right)^{143/168}\right)\right)\right)^2}{2\pi}$$

- 

$$\frac{\log^2\left(\log\left(-\left(-\left(e^{2\pi}\right)^{143/168}\left(1 + e^{2\pi} + \left(e^{2\pi}\right)^2 + 2\left(e^{2\pi}\right)^3\right)\right)\right)\right)}{2\pi} = \frac{\left(-\text{Li}_1\left(1 - \log\left(\left(1 + e^{2\pi} + \left(e^{2\pi}\right)^2 + 2\left(e^{2\pi}\right)^3\right)\left(e^{2\pi}\right)^{143/168}\right)\right)\right)^2}{2\pi}$$

$\log_b(x)$ is the base- $b$ logarithm

$\text{Li}_n(x)$ is the polylogarithm function

**Series representations:**

$$\frac{\log^2\left(\log\left(-\left(-\left(e^{2\pi}\right)^{143/168}\left(1 + e^{2\pi} + \left(e^{2\pi}\right)^2 + 2\left(e^{2\pi}\right)^3\right)\right)\right)\right)}{2\pi} = \frac{\left(\log\left(-1 + \frac{143\pi}{84} + \log\left(1 + e^{2\pi} + e^{4\pi} + 2e^{6\pi}\right)\right) - \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1 + \frac{143\pi}{84} + \log\left(1 + e^{2\pi} + e^{4\pi} + 2e^{6\pi}\right)}\right)^k}{k}\right)^2}{2\pi}$$

- $$\frac{\log^2(\log(-(-(e^{2\pi})^{143/168}(1+e^{2\pi}+(e^{2\pi})^2+2(e^{2\pi})^3))))}{2\pi} = \frac{\left(2 i \pi \left\lfloor \frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{143\pi}{84}+\log(1+e^{2\pi}+e^{4\pi}+2e^{6\pi})-z_0\right)^k z_0^{-k}}{k}\right)^2}{2\pi}$$

- $$\frac{\log^2(\log(-(-(e^{2\pi})^{143/168}(1+e^{2\pi}+(e^{2\pi})^2+2(e^{2\pi})^3))))}{2\pi} = \frac{1}{2\pi}\left(2 i \pi \left\lfloor \frac{\arg(-x+\log(e^{(143\pi)/84}(1+e^{2\pi}+e^{4\pi}+2e^{6\pi})))}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} \left(\frac{143\pi}{84}-x+\log(1+e^{2\pi}+e^{4\pi}+2e^{6\pi})\right)^k}{k}\right)^2 \quad \text{for } x < 0$$

arg(z) is the complex argument

$\lfloor x \rfloor$ is the floor function

From:

$$\chi_{12}(n) = \begin{cases} +1 \text{ if } n \equiv \pm 1 \pmod{12} \\ -1 \text{ if } n \equiv \pm 5 \pmod{12} \\ 0 \quad \text{if } (n, 12) > 1 . \end{cases} \qquad (3.13)$$

$$\sum_{n \equiv 2 \pmod 7} \chi_{12}(n)\, n\, q^{n^2/168} ,$$

2 (mod 7) * ((e^(2Pi)))^((((2 mod 7)^2)) / 168)

**Input:**

$2 \bmod 7\, (e^{2\pi})^{2 \bmod 7^2/168}$

**Exact result:**

$2\, e^{\pi/21}$

**Decimal approximation:**

2.32273839136955481745722137307531990802932187825168293993 2...

2.3227383…

**Property:**

$2\, e^{\pi/21}$ is a transcendental number

**Alternative representations:**

$2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168} = \left(e^{2\pi}\right)^{1/168\,(2-7\,\text{Quotient}[2,7])^2}\ (2-7\,\text{Quotient}[2,7])$

- $2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168} = 7\,\text{frac}\left(\frac{2}{7}\right)\left(e^{2\pi}\right)^{1/168\,(7\,\text{frac}(2/7))^2}$

- $2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168} = \left(2+7\left\lceil -\frac{2}{7}\right\rceil\right)\left(e^{2\pi}\right)^{1/168\,(2+7\lceil -2/7\rceil)^2}$

**Series representations:**

$2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168} = 2\, e^{4/21 \sum_{k=0}^{\infty} (-1)^k/(1+2k)}$

- $2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168} = 2\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{\pi/21}$

- $2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168} = 2\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{\pi/21}$

$n!$ is the factorial function

**Integral representations:**

$$2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168} = 2\, e^{4/21 \int_0^1 \sqrt{1-t^2}\, dt}$$

- $$2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168} = 2\, e^{2/21 \int_0^1 1/\sqrt{1-t^2}\, dt}$$

- $$2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168} = 2\, e^{2/21 \int_0^\infty 1/(1+t^2)\, dt}$$

1/ sqrt((((2 (mod 7) * ((e^(2Pi)))^((((2 mod 7)^2)) /168)))))

**Input:**

$$\frac{1}{\sqrt{2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168}}}$$

**Exact result:**

$$\frac{e^{-\pi/42}}{\sqrt{2}}$$

**Decimal approximation:**

0.656145041017085236998475257544419320965944349369747292111...

0.65614504…

**Property:**

$\frac{e^{-\pi/42}}{\sqrt{2}}$ is a transcendental number

**Alternative representations:**

$$\frac{1}{\sqrt{2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168}}} = \frac{1}{\sqrt{\left(e^{2\pi}\right)^{1/168\,(2-7\,\text{Quotient}[2,7])^2}\,(2-7\,\text{Quotient}[2,7])}}$$

- $$\frac{1}{\sqrt{2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168}}} = \frac{1}{\sqrt{7\,\text{frac}\left(\frac{2}{7}\right)\left(e^{2\pi}\right)^{1/168\,(7\,\text{frac}(2/7))^2}}}$$

- 

$$\frac{1}{\sqrt{2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168}}} = \frac{1}{\sqrt{\left(2+7\left\lceil -\frac{2}{7}\right\rceil\right)\left(e^{2\pi}\right)^{1/168\,(2+7\lceil -2/7\rceil)^2}}}$$

frac($x$) is the fractional part function

$\lceil x \rceil$ is the ceiling function

## Series representations:

$$\frac{1}{\sqrt{2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168}}} = \frac{1}{\sqrt{-1+\left(e^{2\pi}\right)^{2 \bmod 7^2/168}\, 2 \bmod 7}\, \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-1+\left(e^{2\pi}\right)^{2 \bmod 7^2/168}\, 2 \bmod 7\right)^{-k}}$$

- $$\frac{1}{\sqrt{2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168}}} = \frac{1}{\sqrt{-1+\left(e^{2\pi}\right)^{2 \bmod 7^2/168}\, 2 \bmod 7}\, \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\left(e^{2\pi}\right)^{2 \bmod 7^2/168}\, 2 \bmod 7\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

- $$\frac{1}{\sqrt{2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168}}} = \frac{1}{\sqrt{\frac{7\left(e^{2\pi}\right)^{\left(7\left(\pi-2\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}\right)^2\right)/(96\pi^2)} \left(\pi-2\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}\right)}{2\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

A067661 Number of partitions of n into distinct parts such that number of parts is even.

a(n) ~ exp(Pi*sqrt(n/3)) / (8*3^(1/4)*n^(3/4))

exp(Pi*sqrt(n/3)) / (8*3^(1/4)*n^(3/4))

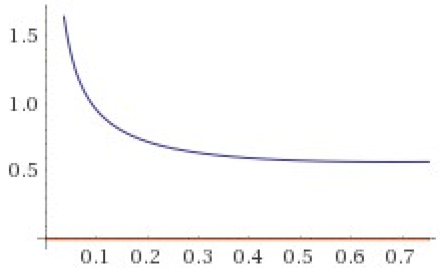
**Input:**

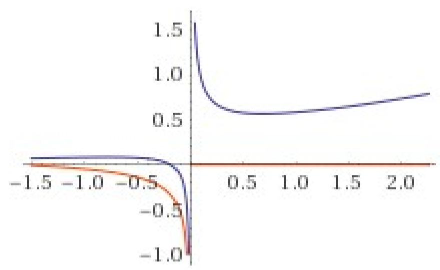
$$\frac{\exp\left(\pi\sqrt{\frac{n}{3}}\right)}{8\sqrt[4]{3}\,n^{3/4}}$$

**Exact result:**

$$\frac{e^{(\pi\sqrt{n})/\sqrt{3}}}{8\sqrt[4]{3}\,n^{3/4}}$$

**Plots:**




- 


- **Roots:**

(no roots exist)

- **Series expansion at n = 0:**

$$O\left(\frac{1}{n^7}\right)$$

(Taylor series)

**Derivative:**

$$\frac{d}{dn}\left(\frac{\exp\left(\pi\sqrt{\frac{n}{3}}\right)}{8\sqrt[4]{3}\,n^{3/4}}\right) = \frac{e^{(\pi\sqrt{n})/\sqrt{3}}\,(2\pi\sqrt{n} - 3\sqrt{3})}{32\times 3^{3/4}\,n^{7/4}}$$

- **Indefinite integral:**

$$\int \frac{\exp\left(\pi\sqrt{\frac{n}{3}}\right)}{8\sqrt[4]{3}\,n^{3/4}}\,dn = \frac{1}{4}\operatorname{erfi}\left(\frac{\sqrt{\pi}\,\sqrt[4]{n}}{\sqrt[4]{3}}\right) + \text{constant}$$

erfi$(x)$ is the imaginary error function

- **Global minimum:**

$$\min\left\{\frac{\exp\left(\pi\sqrt{\frac{n}{3}}\right)}{8\sqrt[4]{3}\,n^{3/4}}\right\} = \frac{(e\,\pi)^{3/2}}{18\sqrt{6}} \quad \text{at } n = \frac{27}{4\pi^2}$$

- **Limit:**

$$\lim_{n\to-\infty}\frac{e^{(\sqrt{n}\,\pi)/\sqrt{3}}}{8\sqrt[4]{3}\,n^{3/4}} = 0$$

- **Series representations:**

$$\frac{\exp\left(\pi\sqrt{\frac{n}{3}}\right)}{8\sqrt[4]{3}\,n^{3/4}} = \frac{\sum_{k=0}^{\infty}\frac{3^{-k/2}\,n^{k/2}\,\pi^k}{k!}}{8\sqrt[4]{3}\,n^{3/4}}$$

- 

$$\frac{\exp\left(\pi\sqrt{\frac{n}{3}}\right)}{8\sqrt[4]{3}\,n^{3/4}} = \frac{\sum_{k=0}^{\infty}\frac{3^{-k}\,n^k\,\pi^{2k}\left(1+2k+\frac{\sqrt{n}\,\pi}{\sqrt{3}}\right)}{(1+2k)!}}{8\sqrt[4]{3}\,n^{3/4}}$$

- 

$$\frac{\exp\left(\pi\sqrt{\frac{n}{3}}\right)}{8\sqrt[4]{3}\,n^{3/4}} = \frac{\sum_{k=0}^{\infty}\frac{3^{-k}\,n^{-1/2+k}\,\pi^{-1+2k}\left(2\sqrt{3}\,k+\sqrt{n}\,\pi\right)}{(2k)!}}{8\sqrt[4]{3}\,n^{3/4}}$$

For n = 21, we obtain:

exp(Pi*sqrt(21/3)) / (8*3^(1/4)*21^(3/4))

**Input:**

$$\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\times 21^{3/4}}$$

**Exact result:**

$$\frac{e^{\sqrt{7}\,\pi}}{24\times 7^{3/4}}$$

**Decimal approximation:**

39.424460160228368341295261116367602216142812049771067637 98...

39.42446016…

**Property:**

$\frac{e^{\sqrt{7}\,\pi}}{24\times 7^{3/4}}$ is a transcendental number

**Series representations:**

$$\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\ 21^{3/4}} = \frac{\exp\left(\pi\sqrt{6}\ \sum_{k=0}^{\infty} 6^{-k}\binom{\frac{1}{2}}{k}\right)}{24\times 7^{3/4}}$$

- 

$$\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\ 21^{3/4}} = \frac{\exp\left(\pi\sqrt{6}\ \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{6}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}{24\times 7^{3/4}}$$

- 

$$\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\ 21^{3/4}} = \frac{\exp\left(\frac{\pi\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}\,6^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)}{24\times 7^{3/4}}$$

We know that for n = 21, a(n) = 38. With a little adjustment, subtracting √2, we obtain:

(((exp(Pi*sqrt(21/3)) / (8*3^(1/4)*21^(3/4))-((sqrt(2))))))

**Input:**

$$\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\times 21^{3/4}}-\sqrt{2}$$

**Exact result:**

$$\frac{e^{\sqrt{7}\,\pi}}{24\times 7^{3/4}}-\sqrt{2}$$

**Decimal approximation:**

38.010246597855273292493572392157904137573140174394119564 80...

38.0102465… ≈ 38

**Property:**

$$-\sqrt{2}+\frac{e^{\sqrt{7}\,\pi}}{24\times 7^{3/4}} \text{ is a transcendental number}$$

- 

**Alternate forms:**

$$\frac{1}{168}\left(\sqrt[4]{7}\,e^{\sqrt{7}\,\pi}-168\sqrt{2}\right)$$

- 

$$\frac{e^{\sqrt{7}\,\pi}-24\sqrt{2}\,7^{3/4}}{24\times 7^{3/4}}$$

**Series representations:**

$$\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\;21^{3/4}}-\sqrt{2}=\frac{1}{168}\left(\sqrt[4]{7}\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k z_0^{-k}}{k!}\right)-168\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right) \text{ for not } ((z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$$

- 

$$\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\ 21^{3/4}} - \sqrt{2} =$$

$$\frac{1}{168}\left(\sqrt[4]{7}\ \exp\left(\pi \exp\left(i\pi\left\lfloor\frac{\arg(7-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (7-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) - 168\right.$$
$$\left.\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

- 

$$\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\ 21^{3/4}} - \sqrt{2} = \frac{1}{168}$$
$$\left(\sqrt[4]{7}\ \exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(7-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(7-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (7-z_0)^k z_0^{-k}}{k!}\right) - \right.$$
$$\left.168\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}\right)$$

Thence, with this expression, from the previous formula that is equal to 0.65614504..., we obtain:

1/[1/ sqrt((((2 (mod 7) * ((e^(2Pi)))^((((2 mod 7)^2)) /168))))) - (((((((1/10^3*(((exp(Pi*sqrt(21/3)) / (8*3^(1/4)*21^(3/4))-((sqrt(2)))))))))))))]

**Input:**

$$\frac{1}{\frac{1}{\sqrt{2 \bmod 7\left(e^{2\pi}\right)^{2 \bmod 7^2/168}}} - \frac{1}{10^3}\left(\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\times 21^{3/4}} - \sqrt{2}\right)}$$

**Exact result:**

$$\frac{1}{\frac{e^{-\pi/42}}{\sqrt{2}} + \frac{\sqrt{2} - \frac{e^{\sqrt{7}\pi}}{24\times 7^{3/4}}}{1000}}$$

**Decimal approximation:**

1.6177701191203003073089828927061533009631763822440980269 96...

1.6177701191…

This result is a very good approximation to the value of the golden ratio 1,618033988749...

**Alternate forms:**

$$-\frac{24000\sqrt{2}\;7^{3/4}}{-48\times 7^{3/4} - 24000\times 7^{3/4}\, e^{-\pi/42} + \sqrt{2}\; e^{\sqrt{7}\,\pi}}$$

$$-\frac{168000\, e^{\pi/42}}{-84000\sqrt{2} - 168\sqrt{2}\; e^{\pi/42} + \sqrt[4]{7}\; e^{\pi/42+\sqrt{7}\,\pi}}$$

$$\frac{168000\, e^{\pi/42}}{84000\sqrt{2} + 168\sqrt{2}\; e^{\pi/42} - \sqrt[4]{7}\; e^{\pi/42+\sqrt{7}\,\pi}}$$

**Alternative representations:**

$$\frac{1}{\frac{1}{\sqrt{2 \bmod 7\,(e^{2\pi})^{2 \bmod 7^2/168}}} - \frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\;21^{3/4}} - \sqrt{2}}{10^3}} = \frac{1}{\frac{1}{\sqrt{\left(2+7\left\lceil -\frac{2}{7}\right\rceil\right)(e^{2\pi})^{1/168\,(2+7\lceil -2/7\rceil)^2}}} - \frac{\frac{\exp\left(\pi\sqrt{7}\right)}{8\sqrt[4]{3}\;21^{3/4}} - \sqrt{2}}{10^3}}$$

$$\frac{1}{\frac{1}{\sqrt{2 \bmod 7\,(e^{2\pi})^{2 \bmod 7^2/168}}} - \frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\;21^{3/4}} - \sqrt{2}}{10^3}} = \frac{1}{\frac{1}{\sqrt{\left(9-7\left\lceil \frac{2}{7}\right\rceil\right)(e^{2\pi})^{1/168\,(9-7\lceil 2/7\rceil)^2}}} - \frac{\frac{\exp\left(\pi\sqrt{7}\right)}{8\sqrt[4]{3}\;21^{3/4}} - \sqrt{2}}{10^3}}$$

$$\frac{1}{\frac{1}{\sqrt{2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168}}} - \frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\ 21^{3/4}} - \sqrt{2}}{10^3}} =$$

$$\frac{1}{\frac{1}{\sqrt{\left(2-7\left\lfloor\frac{2}{7}\right\rfloor+7\right)\left(w^a\right)^{1/168\,(2-7\lfloor 2/7\rfloor+7)^2}}} - \frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\ 21^{3/4}} - \sqrt{2}}{10^3}} \quad \text{for } a = \frac{2\pi}{\log(w)}$$

**Series representations:**

$$\frac{1}{\frac{1}{\sqrt{2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168}}} - \frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\ 21^{3/4}} - \sqrt{2}}{10^3}} =$$

$$-\left(\left(168\,000\sqrt{\left(\sum_{k=0}^{\infty}\frac{2^k\pi^k}{k!}\right)^{1/168\left(7/2-\left(7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}\right)/\pi\right)^2}\left(\frac{7}{2}-\frac{7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}}{\pi}\right)}\right)\right/$$

$$\left(-168\,000+\sqrt[4]{7}\ \exp\left(\pi\sqrt{7}\right)\sqrt{\left(\sum_{k=0}^{\infty}\frac{2^k\pi^k}{k!}\right)^{1/168\left(7/2-\left(7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}\right)/\pi\right)^2}\left(\frac{7}{2}-\frac{7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}}{\pi}\right)} - 168\sqrt{2}\sqrt{\left(\sum_{k=0}^{\infty}\frac{2^k\pi^k}{k!}\right)^{1/168\left(7/2-\left(7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}\right)/\pi\right)^2}\left(\frac{7}{2}-\frac{7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}}{\pi}\right)}\right)\right)$$

$$\frac{1}{\frac{1}{\sqrt{2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168}}} - \frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\ 21^{3/4}} - \sqrt{2}}{10^3}} =$$

$$-\left(\left(168\,000 \sqrt{\left(\sum_{k=-\infty}^{\infty} I_k(2\pi)\right)^{1/168\left(7/2-\left(7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}\right)/\pi\right)^2} \left(\frac{7}{2} - \frac{7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}}{\pi}\right)}\right) \Big/ \left(-168\,000 + \sqrt[4]{7}\exp\left(\pi\sqrt{7}\right) \sqrt{\left(\sum_{k=-\infty}^{\infty} I_k(2\pi)\right)^{1/168\left(7/2-\left(7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}\right)/\pi\right)^2} \left(\frac{7}{2} - \frac{7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}}{\pi}\right)} - 168\sqrt{2}\sqrt{\left(\sum_{k=-\infty}^{\infty} I_k(2\pi)\right)^{1/168\left(7/2-\left(7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}\right)/\pi\right)^2} \left(\frac{7}{2} - \frac{7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}}{\pi}\right)}\right)\right)$$

$$\frac{1}{\frac{1}{\sqrt{2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168}}} - \frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\ 21^{3/4}} - \sqrt{2}}{10^3}} =$$

$$-\left(\left(168\,000 \sqrt{\left(-\frac{1}{2}\left(\sum_{k=0}^{\infty}\frac{2^k \pi^k}{k!}\right)^{1/672\left(-7+\sum_{k=1}^{6}\cot((k\pi)/7)\sin((4k\pi)/7)\right)^2} \left(-7 + \sum_{k=1}^{6}\cot\left(\frac{k\pi}{7}\right)\sin\left(\frac{4k\pi}{7}\right)\right)\right)}\right) \Big/ \left(-168\,000 + \sqrt[4]{7}\exp\left(\pi\sqrt{7}\right)\sqrt{\left(-\frac{1}{2}\left(\sum_{k=0}^{\infty}\frac{2^k \pi^k}{k!}\right)^{1/672\left(-7+\sum_{k=1}^{6}\cot((k\pi)/7)\sin((4k\pi)/7)\right)^2} \left(-7 + \sum_{k=1}^{6}\cot\left(\frac{k\pi}{7}\right)\sin\left(\frac{4k\pi}{7}\right)\right)\right)} - 168\sqrt{2}\sqrt{\left(-\frac{1}{2}\left(\sum_{k=0}^{\infty}\frac{2^k \pi^k}{k!}\right)^{1/672\left(-7+\sum_{k=1}^{6}\cot((k\pi)/7)\sin((4k\pi)/7)\right)^2} \left(-7 + \sum_{k=1}^{6}\cot\left(\frac{k\pi}{7}\right)\sin\left(\frac{4k\pi}{7}\right)\right)\right)}\right)\right)$$

Furthermore, we obtain also:

29/10^3+1/[1/ sqrt((((2 (mod 7) * ((e^(2Pi)))^((((2 mod 7)^2)) /168))))) - (((((((1/10^3*(((exp(Pi*sqrt(21/3)) / (8*3^(1/4)*21^(3/4))-((sqrt(2)))))))))))))]

Where 29 is a Lucas number

**Input:**

$$\frac{29}{10^3} + \frac{1}{\frac{1}{\sqrt{2 \bmod 7 \left(e^{2\pi}\right)^{2 \bmod 7^2/168}}} - \frac{1}{10^3}\left(\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\times 21^{3/4}} - \sqrt{2}\right)}$$

**Exact result:**

$$\frac{29}{1000} + \frac{1}{\frac{e^{-\pi/42}}{\sqrt{2}} + \frac{\sqrt{2} - \frac{e^{\sqrt{7}\pi}}{24\times 7^{3/4}}}{1000}}$$

**Decimal approximation:**

1.646770119120300307308982892706153300963176382244098026996...

$1.6467701191.... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$

**Alternate forms:**

$$\frac{29}{1000} - \frac{24000\sqrt{2}\,7^{3/4}}{-48\times 7^{3/4} - 24000\times 7^{3/4}\,e^{-\pi/42} + \sqrt{2}\,e^{\sqrt{7}\pi}}$$

$$\frac{29}{1000} - \frac{168000\,e^{\pi/42}}{-84000\sqrt{2} - 168\sqrt{2}\,e^{\pi/42} + \sqrt[4]{7}\,e^{\pi/42+\sqrt{7}\pi}}$$

- $$\frac{-48\times 7^{3/4}\left(29 + 500000\sqrt{2}\right) - 696000\times 7^{3/4}\,e^{-\pi/42} + 29\sqrt{2}\,e^{\sqrt{7}\pi}}{1000\left(-48\times 7^{3/4} - 24000\times 7^{3/4}\,e^{-\pi/42} + \sqrt{2}\,e^{\sqrt{7}\pi}\right)}$$

**Alternative representations:**

$$\frac{29}{10^3}+\frac{1}{\frac{1}{\sqrt{2 \bmod 7\left(e^{2\pi}\right)^{2 \bmod 7^2/168}}}-\frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\,21^{3/4}}-\sqrt{2}}{10^3}}=$$

$$\frac{29}{10^3}+\frac{1}{\frac{1}{\sqrt{\left(2+7\left\lceil-\frac{2}{7}\right\rceil\right)\left(e^{2\pi}\right)^{1/168\,(2+7\lceil-2/7\rceil)^2}}}-\frac{\frac{\exp\left(\pi\sqrt{7}\right)}{8\sqrt[4]{3}\,21^{3/4}}-\sqrt{2}}{10^3}}$$

- $$\frac{29}{10^3}+\frac{1}{\frac{1}{\sqrt{2 \bmod 7\left(e^{2\pi}\right)^{2 \bmod 7^2/168}}}-\frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\,21^{3/4}}-\sqrt{2}}{10^3}}=$$

$$\frac{29}{10^3}+\frac{1}{\frac{1}{\sqrt{\left(9-7\left\lceil\frac{2}{7}\right\rceil\right)\left(e^{2\pi}\right)^{1/168\,(9-7\lceil 2/7\rceil)^2}}}-\frac{\frac{\exp\left(\pi\sqrt{7}\right)}{8\sqrt[4]{3}\,21^{3/4}}-\sqrt{2}}{10^3}}$$

- $$\frac{29}{10^3}+\frac{1}{\frac{1}{\sqrt{2 \bmod 7\left(e^{2\pi}\right)^{2 \bmod 7^2/168}}}-\frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\,21^{3/4}}-\sqrt{2}}{10^3}}=$$

$$\frac{29}{10^3}+\frac{1}{\frac{1}{\sqrt{\left(2-7\left\lceil\frac{2}{7}\right\rceil+7\right)\left(w^{a}\right)^{1/168\,(2-7\lceil 2/7\rceil+7)^2}}}-\frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\,21^{3/4}}-\sqrt{2}}{10^3}} \quad \text{for } a=\frac{2\pi}{\log(w)}$$

**Series representations:**

$$\frac{29}{10^3}+\frac{1}{\frac{1}{\sqrt{2 \bmod 7\,(e^{2\pi})^{2 \bmod 7^2/168}}}-\frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\ 21^{3/4}}-\sqrt{2}}{10^3}}=\frac{29}{1000}+\frac{1}{\frac{-\frac{\exp(\pi\sqrt{7})}{24\times 7^{3/4}}+\sqrt{2}}{1000}+\frac{1}{\sqrt{\left(\sum_{k=0}^{\infty}\frac{2^k\pi^k}{k!}\right)^{1/168\left(7/2-\left(7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}\right)/\pi\right)^2}\left(\frac{7}{2}-\frac{7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}}{\pi}\right)}}}$$

- $$\frac{29}{10^3}+\frac{1}{\frac{1}{\sqrt{2 \bmod 7\,(e^{2\pi})^{2 \bmod 7^2/168}}}-\frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\ 21^{3/4}}-\sqrt{2}}{10^3}}=\frac{29}{1000}+1\Bigg/\left(\frac{-\frac{\exp(\pi\sqrt{7})}{24\times 7^{3/4}}+\sqrt{2}}{1000}+1\Bigg/\left(\sqrt{\left(-\frac{1}{2}\left(\sum_{k=0}^{\infty}\frac{2^k\pi^k}{k!}\right)^{1/672\left(-7+\sum_{k=1}^{6}\cot((k\pi)/7)\sin((4k\pi)/7)\right)^2}\left(-7+\sum_{k=1}^{6}\cot\left(\frac{k\pi}{7}\right)\sin\left(\frac{4k\pi}{7}\right)\right)\right)}\right)\right)$$

- $$\frac{29}{10^3}+\frac{1}{\frac{1}{\sqrt{2 \bmod 7\,(e^{2\pi})^{2 \bmod 7^2/168}}}-\frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\ 21^{3/4}}-\sqrt{2}}{10^3}}=\frac{29}{1000}+\frac{1}{\frac{-\frac{\exp(\pi\sqrt{7})}{24\times 7^{3/4}}+\sqrt{2}}{1000}+\frac{1}{\sqrt{\left(\sum_{k=-\infty}^{\infty}I_k(2\pi)\right)^{1/168\left(7/2-\left(7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}\right)/\pi\right)^2}\left(\frac{7}{2}-\frac{7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}}{\pi}\right)}}}$$

(47+7)/10^3+1/[1/ sqrt((((2 (mod 7) * ((e^(2Pi)))^((((2 mod 7)^2)) /168))))) - (((((((1/10^3*(((exp(Pi*sqrt(21/3)) / (8*3^(1/4)*21^(3/4))-((sqrt(2))))))))))))]

Where 47 and 7 are Lucas numbers

**Input:**

$$\frac{47+7}{10^3}+\frac{1}{\frac{1}{\sqrt{2 \bmod 7\left(e^{2\pi}\right)^{2 \bmod 7^2/168}}}-\frac{1}{10^3}\left(\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\times 21^{3/4}}-\sqrt{2}\right)}$$

**Exact result:**

$$\frac{27}{500}+\frac{1}{\frac{e^{-\pi/42}}{\sqrt{2}}+\frac{\sqrt{2}-\frac{e^{\sqrt{7}\pi}}{24\times 7^{3/4}}}{1000}}$$

**Decimal approximation:**

1.671770119120300307308982892706153300963176382244098026996...

1.6717701191…

We note that 1.6717701191... is a result practically equal to the value of the formula:

$$m_{p'} = 2\times\frac{\eta}{R}m_P = 1.6714213\times 10^{-24}\text{ gm}$$

that is the holographic proton mass (N. Haramein)

**Alternate forms:**

$$\frac{27}{500}-\frac{24000\sqrt{2}\,7^{3/4}}{-48\times 7^{3/4}-24000\times 7^{3/4}\,e^{-\pi/42}+\sqrt{2}\,e^{\sqrt{7}\pi}}$$

$$\frac{27}{500}-\frac{168000\,e^{\pi/42}}{-84000\sqrt{2}-168\sqrt{2}\,e^{\pi/42}+\sqrt[4]{7}\,e^{\pi/42+\sqrt{7}\pi}}$$

•

$$\frac{3\left(-16\times 7^{3/4}\left(27+250\,000\sqrt{2}\right)-216\,000\times 7^{3/4}\,e^{-\pi/42}+9\sqrt{2}\,e^{\sqrt{7}\,\pi}\right)}{500\left(-48\times 7^{3/4}-24\,000\times 7^{3/4}\,e^{-\pi/42}+\sqrt{2}\,e^{\sqrt{7}\,\pi}\right)}$$

## Alternative representations:

$$\frac{47+7}{10^3}+\frac{1}{\frac{1}{\sqrt{2 \bmod 7\left(e^{2\pi}\right)^{2 \bmod 7^2/168}}}-\frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\;21^{3/4}}-\sqrt{2}}{10^3}}=\frac{54}{10^3}+\frac{1}{\frac{1}{\sqrt{\left(2+7\left\lceil-\frac{2}{7}\right\rceil\right)\left(e^{2\pi}\right)^{1/168\,(2+7\lceil-2/7\rceil)^2}}}-\frac{\frac{\exp\left(\pi\sqrt{7}\right)}{8\sqrt[4]{3}\;21^{3/4}}-\sqrt{2}}{10^3}}$$

- $$\frac{47+7}{10^3}+\frac{1}{\frac{1}{\sqrt{2 \bmod 7\left(e^{2\pi}\right)^{2 \bmod 7^2/168}}}-\frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\;21^{3/4}}-\sqrt{2}}{10^3}}=\frac{54}{10^3}+\frac{1}{\frac{1}{\sqrt{\left(9-7\left\lceil\frac{2}{7}\right\rceil\right)\left(e^{2\pi}\right)^{1/168\,(9-7\lceil 2/7\rceil)^2}}}-\frac{\frac{\exp\left(\pi\sqrt{7}\right)}{8\sqrt[4]{3}\;21^{3/4}}-\sqrt{2}}{10^3}}$$

- $$\frac{47+7}{10^3}+\frac{1}{\frac{1}{\sqrt{2 \bmod 7\left(e^{2\pi}\right)^{2 \bmod 7^2/168}}}-\frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\;21^{3/4}}-\sqrt{2}}{10^3}}=\frac{47+7}{10^3}+\frac{1}{\frac{1}{\sqrt{\left(2-7\left\lceil\frac{2}{7}\right\rceil+7\right)\left(w^{a}\right)^{1/168\,(2-7\lceil 2/7\rceil+7)^2}}}-\frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\;21^{3/4}}-\sqrt{2}}{10^3}} \quad \text{for } a=\frac{2\pi}{\log(w)}$$

## Series representations:

$$\frac{47+7}{10^3}+\frac{1}{\frac{1}{\sqrt{2 \bmod 7\,(e^{2\pi})^{2 \bmod 7^2/168}}}-\frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\,21^{3/4}}-\sqrt{2}}{10^3}}=$$

$$\frac{27}{500}+\frac{1}{\frac{-\frac{\exp(\pi\sqrt{7})}{24\times 7^{3/4}}+\sqrt{2}}{1000}+\frac{1}{\sqrt{\left(\sum_{k=0}^{\infty}\frac{2^k\pi^k}{k!}\right)^{1/168\left(7/2-\left(7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}\right)/\pi\right)^2\left(\frac{7}{2}-\frac{7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}}{\pi}\right)}}}}$$

- $$\frac{47+7}{10^3}+\frac{1}{\frac{1}{\sqrt{2 \bmod 7\,(e^{2\pi})^{2 \bmod 7^2/168}}}-\frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\,21^{3/4}}-\sqrt{2}}{10^3}}=\frac{27}{500}+$$

$$\frac{1}{\frac{-\frac{\exp(\pi\sqrt{7})}{24\times 7^{3/4}}+\sqrt{2}}{1000}+\frac{1}{\sqrt{\left(\sum_{k=-\infty}^{\infty}I_k(2\pi)\right)^{1/168\left(7/2-\left(7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}\right)/\pi\right)^2\left(\frac{7}{2}-\frac{7\sum_{k=1}^{\infty}\frac{\sin\left(\frac{4k\pi}{7}\right)}{k}}{\pi}\right)}}}}$$

- $$\frac{47+7}{10^3}+\frac{1}{\frac{1}{\sqrt{2 \bmod 7\,(e^{2\pi})^{2 \bmod 7^2/168}}}-\frac{\frac{\exp\left(\pi\sqrt{\frac{21}{3}}\right)}{8\sqrt[4]{3}\,21^{3/4}}-\sqrt{2}}{10^3}}=\frac{27}{500}+$$

$$1\Bigg/\left(\frac{-\frac{\exp(\pi\sqrt{7})}{24\times 7^{3/4}}+\sqrt{2}}{1000}+1\Bigg/\left(\sqrt{\left(\left(\sum_{k=0}^{\infty}\frac{2^k\pi^k}{k!}\right)^{1/168\,(7/2-1/2\sum_{k=1}^{6}\cot((k\pi)/7)\sin((4k\pi)/7))^2}\left(\frac{7}{2}-\frac{1}{2}\sum_{k=1}^{6}\cot\left(\frac{k\pi}{7}\right)\sin\left(\frac{4k\pi}{7}\right)\right)\right)}\right)\right)$$

We have that:

$$E_4(\tau) \;=\; 1 + 240\sum_{n=1}^{\infty}\frac{n^3q^n}{1-q^n} \;=\; 1 + 240q + 2160q^2 + \cdots\,, \qquad (3.4)$$

$$E_6(\tau) \;-\; 1 \;-\; 504\sum_{n=1}^{\infty}\frac{n^5q^n}{1-q^n} \;-\; 1 - 504q - 16632q^2 - \cdots\,. \qquad (3.5)$$

$$\Delta \;=\; \left(E_4^3 - E_6^2\right)/1728.$$

[((((1+240*e^(2Pi)+2160*(((e^(2Pi)))^2)))))))^3 - ((((1-504*e^(2Pi)-16632*(((e^(2Pi)))^2)))))))^2] * 1/1728

**Input:**

$$\left(\left(1+240\,e^{2\pi}+2160\left(e^{2\pi}\right)^2\right)^3-\left(1-504\,e^{2\pi}-16\,632\left(e^{2\pi}\right)^2\right)^2\right)\times\frac{1}{1728}$$

**Exact result:**

$$\frac{\left(1+240\,e^{2\pi}+2160\,e^{4\pi}\right)^3-\left(1-504\,e^{2\pi}-16\,632\,e^{4\pi}\right)^2}{1728}$$

**Decimal approximation:**

$1.375954376559217296160708198823526929838952845263565\,4\ldots \times 10^{23}$

1.375954376… * $10^{23}$
1.375954376559217296e+23

**Property:**

$$\frac{-\left(1-504\,e^{2\pi}-16\,632\,e^{4\pi}\right)^2+\left(1+240\,e^{2\pi}+2160\,e^{4\pi}\right)^3}{1728}$$ is a transcendental number

**Alternate forms:**

$$e^{2\pi}-24\,e^{4\pi}+98\,e^{6\pi}+64\,017\,e^{8\pi}+1\,944\,000\,e^{10\pi}+5\,832\,000\,e^{12\pi}$$

- 

$$e^{2\pi}\left(1 - 24\,e^{2\pi} + 98\,e^{4\pi} + 64\,017\,e^{6\pi} + 1\,944\,000\,e^{8\pi} + 5\,832\,000\,e^{10\pi}\right)$$

- $$\frac{(1 + 240\,e^{2\pi} + 2160\,e^{4\pi})^3 - (-1 + 504\,e^{2\pi} + 16\,632\,e^{4\pi})^2}{1728}$$

**Alternative representations:**

$$\frac{(1 + 240\,e^{2\pi} + 2160\,(e^{2\pi})^2)^3 - (1 - 504\,e^{2\pi} - 16\,632\,(e^{2\pi})^2)^2}{1728} = \frac{-(1 - 504\,e^{360^\circ} - 16\,632\,(e^{360^\circ})^2)^2 + (1 + 240\,e^{360^\circ} + 2160\,(e^{360^\circ})^2)^3}{1728}$$

- $$\frac{(1 + 240\,e^{2\pi} + 2160\,(e^{2\pi})^2)^3 - (1 - 504\,e^{2\pi} - 16\,632\,(e^{2\pi})^2)^2}{1728} = \frac{(1 + 240\,\exp^{2\pi}(z) + 2160\,\exp^{2\pi}(z)^2)^3 - (1 - 504\,\exp^{2\pi}(z) - 16\,632\,\exp^{2\pi}(z)^2)^2}{1728} \text{ for } z = 1$$

- $$\frac{(1 + 240\,e^{2\pi} + 2160\,(e^{2\pi})^2)^3 - (1 - 504\,e^{2\pi} - 16\,632\,(e^{2\pi})^2)^2}{1728} = \frac{1}{1728}\left(-\left(1 - 504\,e^{-2\,i\log(-1)} - 16\,632\left(e^{-2\,i\log(-1)}\right)^2\right)^2 + \left(1 + 240\,e^{-2\,i\log(-1)} + 2160\left(e^{-2\,i\log(-1)}\right)^2\right)^3\right)$$

**Comparison:**

≈ (0.1 to 1000) × the number of grains of sand on the earth (≈ $10^{20}$ to $10^{24}$)

**Series representations:**

$$\frac{(1 + 240\,e^{2\pi} + 2160\,(e^{2\pi})^2)^3 - (1 - 504\,e^{2\pi} - 16\,632\,(e^{2\pi})^2)^2}{1728} = e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\left(1 - 24\,e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)} + 98\,e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)} + 64\,017\,e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)} + 1\,944\,000\,e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)} + 5\,832\,000\,e^{40\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)$$

- 

$$\frac{(1+240\,e^{2\pi}+2160\,(e^{2\pi})^2)^3-(1-504\,e^{2\pi}-16\,632\,(e^{2\pi})^2)^2}{1728}=$$
$$\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\left(1-24\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}+98\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}+64\,017\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi}+1\,944\,000\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}+5\,832\,000\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{10\pi}\right)$$

- $$\frac{(1+240\,e^{2\pi}+2160\,(e^{2\pi})^2)^3-(1-504\,e^{2\pi}-16\,632\,(e^{2\pi})^2)^2}{1728}=$$
$$\left(1-24\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}+98\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{4\pi}+64\,017\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{6\pi}+1\,944\,000\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{8\pi}+5\,832\,000\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{10\pi}\right)\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{2\pi}$$

$n!$ is the factorial function

## Integral representations:

$$\frac{(1+240\,e^{2\pi}+2160\,(e^{2\pi})^2)^3-(1-504\,e^{2\pi}-16\,632\,(e^{2\pi})^2)^2}{1728}=$$
$$e^{4\int_0^{\infty}1/(1+t^2)\,dt}\left(1-24\,e^{4\int_0^{\infty}1/(1+t^2)\,dt}+98\,e^{8\int_0^{\infty}1/(1+t^2)\,dt}+64\,017\,e^{12\int_0^{\infty}1/(1+t^2)\,dt}+1\,944\,000\,e^{16\int_0^{\infty}1/(1+t^2)\,dt}+5\,832\,000\,e^{20\int_0^{\infty}1/(1+t^2)\,dt}\right)$$

- $$\frac{(1+240\,e^{2\pi}+2160\,(e^{2\pi})^2)^3-(1-504\,e^{2\pi}-16\,632\,(e^{2\pi})^2)^2}{1728}=$$
$$e^{4\int_0^{\infty}\sin(t)/t\,dt}\left(1-24\,e^{4\int_0^{\infty}\sin(t)/t\,dt}+98\,e^{8\int_0^{\infty}\sin(t)/t\,dt}+64\,017\,e^{12\int_0^{\infty}\sin(t)/t\,dt}+1\,944\,000\,e^{16\int_0^{\infty}\sin(t)/t\,dt}+5\,832\,000\,e^{20\int_0^{\infty}\sin(t)/t\,dt}\right)$$

- $$\frac{(1+240\,e^{2\pi}+2160\,(e^{2\pi})^2)^3-(1-504\,e^{2\pi}-16\,632\,(e^{2\pi})^2)^2}{1728}=$$
$$e^{8\int_0^{1}\sqrt{1-t^2}\,dt}\left(1-24\,e^{8\int_0^{1}\sqrt{1-t^2}\,dt}+98\,e^{16\int_0^{1}\sqrt{1-t^2}\,dt}+64\,017\,e^{24\int_0^{1}\sqrt{1-t^2}\,dt}+1\,944\,000\,e^{32\int_0^{1}\sqrt{1-t^2}\,dt}+5\,832\,000\,e^{40\int_0^{1}\sqrt{1-t^2}\,dt}\right)$$

Logically:

[((((1+240*e^(2Pi)+2160*(((e^(2Pi)))^2)))))))^3 - ((((1-504*e^(2Pi)-16632*(((e^(2Pi)))^2)))))))^2]*1/1.37595437655921729616 × 10^23

**Input interpretation:**

$$((1+240\,e^{2\pi}+2160\,(e^{2\pi})^2)^3-(1-504\,e^{2\pi}-16632\,(e^{2\pi})^2)^2)\times\frac{1}{1.37595437655921729616\times10^{23}}$$

**Result:**

1728.0000000000000000...

1728

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

From which:

(((((((((([((((1+240*e^(2Pi)+2160*(((e^(2Pi)))^2)))))))^3 - ((((1-504*e^(2Pi)-16632*(((e^(2Pi)))^2)))))))^2]*1/1.37595437655921729616 × 10^23)))))^1/15

**Input interpretation:**

$$\left(((1+240\,e^{2\pi}+2160\,(e^{2\pi})^2)^3-(1-504\,e^{2\pi}-16632\,(e^{2\pi})^2)^2)\times\frac{1}{1.37595437655921729616\times10^{23}}\right)\hat{}\,(1/15)$$

**Result:**

1.64375182951722576231...

1.643751829…. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

And:

29/10^3+[[((1+240*e^(2Pi)+2160*(((e^(2Pi)))^2)))^3 - ((((1-504*e^(2Pi)-16632*(((e^(2Pi)))^2)))))^2]*1/1.3759543765e+23)]^1/15

Where 29 is a Lucas number

**Input interpretation:**

$$\frac{29}{10^3}+\left(\left(\left(1+240\,e^{2\pi}+2160\left(\left(e^{2\pi}\right)^2\right)\right)^3-\left(1-504\,e^{2\pi}-16\,632\left(\left(e^{2\pi}\right)^2\right)\right)^2\right)\times\frac{1}{1.3759543765\times10^{23}}\right)^{\wedge}(1/15)$$

**Result:**

1.6727518295...

1.6727518295…. result practically equal to the proton mass without exponent

-(29-4)/10^3+[[((1+240*e^(2Pi)+2160*(((e^(2Pi)))^2)))^3 - ((((1-504*e^(2Pi)-16632*(((e^(2Pi)))^2)))))^2]*1/1.3759543765e+23)]^1/15

**Input interpretation:**

$$-\frac{29-4}{10^3}+\left(\left(\left(1+240\,e^{2\pi}+2160\left(\left(e^{2\pi}\right)^2\right)\right)^3-\left(1-504\,e^{2\pi}-16\,632\left(\left(e^{2\pi}\right)^2\right)\right)^2\right)\times\frac{1}{1.3759543765\times10^{23}}\right)^{\wedge}(1/15)$$

**Result:**

1.6187518295...

1.6187518295…

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Now, we have that:

$$2+q^{1/120}\,h_1(\tau)\;=\;\chi_1(q)\;=\;\sum_{n=0}^{\infty}\frac{q^n}{(1-q^{n+1})\cdots(1-q^{2n})}\,,$$

$$q^{-71/120}\,h_2(\tau)\;=\;\chi_2(q)\;=\;\sum_{n=0}^{\infty}\frac{q^n}{(1-q^{n+1})\cdots(1-q^{2n+1})}\,.$$

for n = 2 and q = 0.8:

((0.8)^2) / [(((((1-(((0.8))^3))))) ((((1-(((0.8)))^4))))]

**Input:**

$$\frac{0.8^2}{(1-0.8^3)(1-0.8^4)}$$

**Result:**

2.221333688746723532809098582789106579590386067795104180550...

2.22133688…

**Repeating decimal:**

2.221333688746723532809098582789106579590386067795104180550...

(period 60)

((0.8)^2) / [((1-((((0.8))^3))) (((1-(((0.8)))^5)))]

**Input:**

$$\frac{0.8^2}{(1-0.8^3)(1-0.8^5)}$$

**Result:**

1.950671421103143702062249826390243521820210516459765451268...

1.950671421…

(2.2213336887*1.9506714211)^1/3

**Input interpretation:**

$$\sqrt[3]{2.2213336887\times 1.9506714211}$$

**Result:**

1.6302941674...

1.6302941674…

(11/10^3+3/10^3)+(2.2213336887*1.9506714211)^1/3

where 11 and 3 are Lucas numbers

**Input interpretation:**

$$\left(\frac{11}{10^3}+\frac{3}{10^3}\right)+\sqrt[3]{2.2213336887\times 1.9506714211}$$

**Result:**

1.6442941674...

1.6442941674…. ≈ ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

(-11/10^3-3/10^3+2/10^3)+(2.2213336887*1.9506714211)^1/3

where 2, 3 and 11 are Lucas numbers

**Input interpretation:**

$$\left(-\frac{11}{10^3}-\frac{3}{10^3}+\frac{2}{10^3}\right)+\sqrt[3]{2.2213336887\times 1.9506714211}$$

**Result:**

1.6182941674...

1.6182941674…

This result is a very good approximation to the value of the golden ratio 1,618033988749...

We have that:

Since the degeneracies of immortal black holes are independent of the moduli by definition, they are actually invariant under $S$-duality:

$$d^*(n+mb^2+b, \ell+2mb, m) = d^*(n,\ell,m)\,. \tag{11.5}$$

From the mathematical point of view, this is precisely the action of the elliptic part of the Jacobi transformations (4.2) on the Fourier coefficients of a Jacobi (or mock Jacobi) form.

The immortal degeneracies $d^*(n,\ell,m)$ are computed using the attractor contour (6.13), for which the imaginary parts of the potentials are proportional to the charges and scale as

$$\mathrm{Im}(\sigma) = 2n/\varepsilon\,, \qquad \mathrm{Im}(\tau) = 2m/\varepsilon\,, \qquad \mathrm{Im}(z) = -\ell/\varepsilon\,, \tag{11.6}$$

with $\varepsilon$ very small and positive. In other words,

$$|p| = \lambda^{2n}\,, \quad |q| = \lambda^{2m}\,, \quad |y| = \lambda^{-\ell}\,, \qquad \text{with } \lambda = \exp(-2\pi/\epsilon) \to 0 \tag{11.7}$$

on the attractor contour. We assume that $n > m$ without loss of generality, otherwise one can simply exchange $\sigma$ and $\tau$. Moreover, using the spectral flow symmetry (11.5) we can always bring $\ell$ to the window $0 \le \ell < 2m$ for the immortal degeneracies.

For m equal to a positive integer, n > m; m = 3, n = 8 ε very small and positive, ε = 0.0833333... and $r > 0$; $r = 2$ $s > 0$; $s = 5$

$$\lambda^{2(\sqrt{nr}-\sqrt{ms})^2}$$

(((exp(-2Pi/0.0833))))^((((2(((sqrt(16)-sqrt(15)))^2)))))

**Input:**

$$\exp^{2\left(\sqrt{16}-\sqrt{15}\right)^2}\left(-2\times\frac{\pi}{0.0833}\right)$$

**Result:**

0.0877023...

0.0877023…

- **Series representations:**

$$\exp^{2\left(\sqrt{16}-\sqrt{15}\right)^2}\left(-\frac{2\pi}{0.0833}\right)=\exp^{2\left(\sum_{k=0}^{\infty}210^{-k}\binom{\frac{1}{2}}{k}\left(15^k\sqrt{14}-14^k\sqrt{15}\right)\right)^2}(-24.0096\,\pi)$$

- $$\exp^{2\left(\sqrt{16}-\sqrt{15}\right)^2}\left(-\frac{2\pi}{0.0833}\right)=\exp^{2\left(\sum_{k=0}^{\infty}\frac{15^{-k}\left(-\frac{1}{2}\right)_k\left(\left(-\frac{15}{14}\right)^k\sqrt{14}+(-1)^{1+k}\sqrt{15}\right)}{k!}\right)^2}(-24.0096\,\pi)$$

- $$\exp^{2\left(\sqrt{16}-\sqrt{15}\right)^2}\left(-\frac{2\pi}{0.0833}\right)=\exp^{2\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left((15-z_0)^k-(16-z_0)^k\right)z_0^{-k}}{k!}\right)^2}(-24.0096\,\pi)$$

for not $((z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0))$

**Integral representation:**

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\,\pi\,i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

(23/28) 1/ (((exp(-2Pi/0.0833))))^((((2(((sqrt(16)-sqrt(15)))^2)))))

**Input:**

$$\frac{23}{28} \times \frac{1}{\exp^{2\left(\sqrt{16}-\sqrt{15}\right)^2}\left(-2 \times \frac{\pi}{0.0833}\right)}$$

**Result:**

9.36610…

9.36610… result practically equal to the black hole entropy 9.3664

**Series representations:**

$$\frac{23}{\exp^{2\left(\sqrt{16}-\sqrt{15}\right)^2}\left(-\frac{2\pi}{0.0833}\right)28} = \frac{23}{28}\exp^{-2\left(\sum_{k=0}^{\infty} 210^{-k}\binom{\frac{1}{2}}{k}\left(15^k\sqrt{14}-14^k\sqrt{15}\right)\right)^2}(-24.0096\,\pi)$$

- $$\frac{23}{\exp^{2\left(\sqrt{16}-\sqrt{15}\right)^2}\left(-\frac{2\pi}{0.0833}\right)28} = \frac{23}{28}\exp^{-2\left(\sum_{k=0}^{\infty}\frac{15^{-k}\left(-\frac{1}{2}\right)_k\left(\left(-\frac{15}{14}\right)^k\sqrt{14}+(-1)^{1+k}\sqrt{15}\right)}{k!}\right)^2}(-24.0096\,\pi)$$

- $$\frac{23}{\exp^{2\left(\sqrt{16}-\sqrt{15}\right)^2}\left(-\frac{2\pi}{0.0833}\right)28} = \frac{23}{28}\exp^{-2\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left((15-z_0)^k-(16-z_0)^k\right)z_0^{-k}}{k!}\right)^2}(-24.0096\,\pi)$$

  for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

We have that:

$$p_{24}(m+1) \cdot \frac{1}{\eta^{24}(\tau)} \cdot \sum_{\ell>0} \ell\, y^{\ell},$$

From Wikipedia:

$$\eta(\tau) = e^{\frac{\pi i \tau}{12}} \prod_{n=1}^{\infty}(1-q^n).$$

$$\Delta = (2\pi)^{12}\eta^{24}(\tau)$$

1/(2Pi)^12 * ((((([(((((1+240*e^(2Pi)+2160*(((e^(2Pi)))^2)))))))^3 - ((((1-504*e^(2Pi)-16632*(((e^(2Pi)))^2)))))))^2] * 1/1728))))

**Input:**

$$\frac{1}{(2\pi)^{12}}\left(\left((1+240\,e^{2\pi}+2160\,(e^{2\pi})^2)^3-(1-504\,e^{2\pi}-16632\,(e^{2\pi})^2)^2\right)\times\frac{1}{1728}\right)$$

**Exact result:**

$$\frac{(1+240\,e^{2\pi}+2160\,e^{4\pi})^3-(1-504\,e^{2\pi}-16632\,e^{4\pi})^2}{7077888\,\pi^{12}}$$

**Decimal approximation:**

$3.6345078704342506936264153454173598583489249628752803\ldots\times 10^{13}$

$3.63450787\ldots * 10^{13}$

**Alternate forms:**

$$\frac{(1+240\,e^{2\pi}+2160\,e^{4\pi})^3-(-1+504\,e^{2\pi}+16632\,e^{4\pi})^2}{7077888\,\pi^{12}}$$

- $$\frac{e^{2\pi}\,(1-24\,e^{2\pi}+98\,e^{4\pi}+64017\,e^{6\pi}+1944000\,e^{8\pi}+5832000\,e^{10\pi})}{4096\,\pi^{12}}$$

- $$\frac{(1+240\,e^{2\pi}\,(1+9\,e^{2\pi}))^3-(1-504\,e^{2\pi}\,(1+33\,e^{2\pi}))^2}{7077888\,\pi^{12}}$$

- **Alternative representations:**

$$\frac{(1+240\,e^{2\pi}+2160\,(e^{2\pi})^2)^3-(1-504\,e^{2\pi}-16\,632\,(e^{2\pi})^2)^2}{1728\,(2\,\pi)^{12}} = \frac{-(1-504\,e^{360\,^\circ}-16\,632\,(e^{360\,^\circ})^2)^2+(1+240\,e^{360\,^\circ}+2160\,(e^{360\,^\circ})^2)^3}{1728\,(360\,^\circ)^{12}}$$

- 

$$\frac{(1+240\,e^{2\pi}+2160\,(e^{2\pi})^2)^3-(1-504\,e^{2\pi}-16\,632\,(e^{2\pi})^2)^2}{1728\,(2\,\pi)^{12}} = \frac{(1+240\exp^{2\pi}(z)+2160\exp^{2\pi}(z)^2)^3-(1-504\exp^{2\pi}(z)-16\,632\exp^{2\pi}(z)^2)^2}{1728\,(2\,\pi)^{12}} \text{ for } z=1$$

- 

$$\frac{(1+240\,e^{2\pi}+2160\,(e^{2\pi})^2)^3-(1-504\,e^{2\pi}-16\,632\,(e^{2\pi})^2)^2}{1728\,(2\,\pi)^{12}} = \frac{1}{1728\,(-2\,i\log(-1))^{12}}\left(-\left(1-504\,e^{-2\,i\log(-1)}-16\,632\left(e^{-2\,i\log(-1)}\right)^2\right)^2+\left(1+240\,e^{-2\,i\log(-1)}+2160\left(e^{-2\,i\log(-1)}\right)^2\right)^3\right)$$

**Series representations:**

$$\frac{(1+240\,e^{2\pi}+2160\,(e^{2\pi})^2)^3-(1-504\,e^{2\pi}-16\,632\,(e^{2\pi})^2)^2}{1728\,(2\,\pi)^{12}} = \left(e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\left(1-24\,e^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+98\,e^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+64\,017\,e^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+1\,944\,000\,e^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+5\,832\,000\,e^{40\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\right)\right)\Bigg/\left(68\,719\,476\,736\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^{12}\right)$$

- 

$$\frac{(1+240\,e^{2\pi}+2160\,(e^{2\pi})^2)^3-(1-504\,e^{2\pi}-16\,632\,(e^{2\pi})^2)^2}{1728\,(2\,\pi)^{12}} = \frac{1}{4096\,\pi^{12}}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}\left(1-24\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{2\pi}+98\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4\pi}+64\,017\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{6\pi}+1\,944\,000\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\pi}+5\,832\,000\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{10\pi}\right)$$

- $$\frac{(1+240 e^{2\pi}+2160 (e^{2\pi})^2)^3-(1-504 e^{2\pi}-16632 (e^{2\pi})^2)^2}{1728 (2\pi)^{12}} = \frac{1}{4096 \pi^{12}} \left(1-24\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{2\pi}+98\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{4\pi}+64017\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{6\pi}+ 1944000\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{8\pi}+5832000\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{10\pi}\right)\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{2\pi}$$

$n!$ is the factorial function

## Integral representations:

$$\frac{(1+240 e^{2\pi}+2160 (e^{2\pi})^2)^3-(1-504 e^{2\pi}-16632 (e^{2\pi})^2)^2}{1728 (2\pi)^{12}} = \left(e^{4\int_0^\infty \sin(t)/t\,dt}\left(1-24 e^{4\int_0^\infty \sin(t)/t\,dt}+98 e^{8\int_0^\infty \sin(t)/t\,dt}+64017 e^{12\int_0^\infty \sin(t)/t\,dt}+ 1944000 e^{16\int_0^\infty \sin(t)/t\,dt}+5832000 e^{20\int_0^\infty \sin(t)/t\,dt}\right)\right)\Big/ \left(16777216\left(\int_0^\infty \frac{\sin(t)}{t}\,dt\right)^{12}\right)$$

- $$\frac{(1+240 e^{2\pi}+2160 (e^{2\pi})^2)^3-(1-504 e^{2\pi}-16632 (e^{2\pi})^2)^2}{1728 (2\pi)^{12}} = \left(e^{4\int_0^\infty 1/(1+t^2)\,dt}\left(1-24 e^{4\int_0^\infty 1/(1+t^2)\,dt}+98 e^{8\int_0^\infty 1/(1+t^2)\,dt}+64017 e^{12\int_0^\infty 1/(1+t^2)\,dt}+ 1944000 e^{16\int_0^\infty 1/(1+t^2)\,dt}+5832000 e^{20\int_0^\infty 1/(1+t^2)\,dt}\right)\right)\Big/ \left(16777216\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^{12}\right)$$

$$\frac{(1+240 e^{2\pi}+2160 (e^{2\pi})^2)^3-(1-504 e^{2\pi}-16632 (e^{2\pi})^2)^2}{1728 (2\pi)^{12}} = \left(e^{4\int_0^\infty \sin^2(t)/t^2\,dt}\left(1-24 e^{4\int_0^\infty \sin^2(t)/t^2\,dt}+98 e^{8\int_0^\infty \sin^2(t)/t^2\,dt}+64017 e^{12\int_0^\infty \sin^2(t)/t^2\,dt}+ 1944000 e^{16\int_0^\infty \sin^2(t)/t^2\,dt}+5832000 e^{20\int_0^\infty \sin^2(t)/t^2\,dt}\right)\right)\Big/ \left(16777216\left(\int_0^\infty \frac{\sin^2(t)}{t^2}\,dt\right)^{12}\right)$$

$\eta^{24}(\tau)$ = 3.63450787043425069362641534541735985834892496287528O3 × 10^13

$$p_{24}(m+1) \cdot \frac{1}{\eta^{24}(\tau)} \cdot \sum_{\ell>0} \ell\, y^{\ell},$$

p(4) = 5

5 * 1/(3.6345078704342506936264153454173598 × 10^13) * 4 * 0.86^4

**Input interpretation:**

$$5 \times \frac{1}{3.6345078704342506936264153454173598 \times 10^{13}} \times 4 \times 0.86^4$$

**Result:**

$$3.0100810316013623666071171675333115139424117734748922... \times 10^{-13}$$

3.010081031601…*$10^{-13}$

We have also that:

(1.375954376559217296e+23)/ (3.6345078704342506936e+13)

**Input interpretation:**

$$\frac{1.375954376559217296 \times 10^{23}}{3.634507870434250693 \times 10^{13}}$$

**Result:**

$$3.7858065675197406669780030835879081327166899123945O567... \times 10^{9}$$

3.7858065675… * $10^9$

And:

13/10^3 + (29+7) / ln((((1.375954376559217296e+23)/ (3.634507870434250693e+13))))

with 29 and 7 that are Lucas numbers

**Input interpretation:**

$$\frac{13}{10^3} + \frac{29+7}{\log\left(\frac{1.375954376559217296 \times 10^{23}}{3.634507870434250693 \times 10^{13}}\right)}$$

log(x) is the natural logarithm

**Result:**

1.6453180994151531342...

1.645318…. ≈ ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

(21/10^3+5/10^3+13/10^3) + (29+7) / ln((((1.375954376559217296e+23)/(3.634507870434250693e+13))))

**Input interpretation:**

$$\left(\frac{21}{10^3}+\frac{5}{10^3}+\frac{13}{10^3}\right)+\frac{29+7}{\log\left(\frac{1.375954376559217296\times 10^{23}}{3.634507870434250693\times 10^{13}}\right)}$$

log(x) is the natural logarithm

**Result:**

1.6713180994151531342...

1.671318….

We note that 1.671318... is a result practically equal to the value of the formula:

$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24}$ gm

that is the holographic proton mass (N. Haramein)

(13/10^3-29/10^3+2/10^3) + (29+7) / ln((((1.375954376559217296e+23)/(3.634507870434250693e+13))))

With 2 and 29 that are Lucas numbers and 13 that is a Fibonacci number

**Input interpretation:**

$$\left(\frac{13}{10^3}-\frac{29}{10^3}+\frac{2}{10^3}\right)+\frac{29+7}{\log\left(\frac{1.375954376559217296\times 10^{23}}{3.634507870434250693\times 10^{13}}\right)}$$

log(x) is the natural logarithm

**Result:**

1.6183180994151531342...

1.618318....

This result is a very good approximation to the value of the golden ratio 1,618033988749...

We have also:

(3.634507870434 × 10^13 * 3.010081031601*10^-13)^1/5

**Input interpretation:**

$\sqrt[5]{3.634507870434\times 10^{13}\times 3.010081031601\times 10^{-13}}$

**Result:**

1.613632974681...

1.613632974...

(2/10^3+29/10^3)+(3.634507870434 × 10^13 * 3.010081031601*10^-13)^1/5

Where 2 and 29 are Lucas numbers

**Input interpretation:**

$\left(\frac{2}{10^3}+\frac{29}{10^3}\right)+\sqrt[5]{3.634507870434\times 10^{13}\times 3.010081031601\times 10^{-13}}$

**Result:**

1.644632974681...

1.64463297.... ≈ $\zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$

Now, we have that, for n = 27 and n = 49, where 49 = 47 + 2 and 27 = 29 – 2, where 2, 29 and 47 are Lucas numbers:

a(n) ~ exp(Pi*sqrt(n/3)) / (8*3^(1/4)*n^(3/4)) (((exp(Pi*sqrt(27/3)) / (8*3^(1/4)*27^(3/4))))) + (((exp(Pi*sqrt(49/3)) / (8*3^(1/4)*49^(3/4)))))+11

**Input:**

$$\frac{\exp\left(\pi\sqrt{\frac{27}{3}}\right)}{8\sqrt[4]{3}\times 27^{3/4}}+\frac{\exp\left(\pi\sqrt{\frac{49}{3}}\right)}{8\sqrt[4]{3}\times 49^{3/4}}+11$$

**Exact result:**

$$11+\frac{e^{3\pi}}{72\sqrt{3}}+\frac{e^{(7\pi)/\sqrt{3}}}{56\sqrt[4]{3}\sqrt{7}}$$

**Decimal approximation:**

1785.478249795823306587715128189583509213998637921956234650...

1785.4782 result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

- **Alternate forms:**

$$\frac{116424+49\sqrt{3}\,e^{3\pi}+9\times 3^{3/4}\sqrt{7}\,e^{(7\pi)/\sqrt{3}}}{10584}$$

- $$\frac{e^{(7\pi)/\sqrt{3}}}{56\sqrt[4]{3}\sqrt{7}}+\frac{1}{216}\left(2376+\sqrt{3}\,e^{3\pi}\right)$$

- $$\frac{1}{216}e^{3\pi}\sqrt{3}+\frac{e^{(7\pi)/\sqrt{3}}\left(3^{3/4}\sqrt{7}\right)}{1176}+11$$

**Series representations:**

$$\frac{\exp\left(\pi\sqrt{\frac{27}{3}}\right)}{8\sqrt[4]{3}\ 27^{3/4}} + \frac{\exp\left(\pi\sqrt{\frac{49}{3}}\right)}{8\sqrt[4]{3}\ 49^{3/4}} + 11 = \frac{1}{10584}\left(116\,424 + 49\sqrt{3}\ \exp\left(\pi\sqrt{8}\sum_{k=0}^{\infty} 8^{-k}\binom{\frac{1}{2}}{k}\right) + 9\times 3^{3/4}\sqrt{7}\ \exp\left(\pi\sqrt{\frac{46}{3}}\sum_{k=0}^{\infty}\left(\frac{46}{3}\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

- 

$$\frac{\exp\left(\pi\sqrt{\frac{27}{3}}\right)}{8\sqrt[4]{3}\ 27^{3/4}} + \frac{\exp\left(\pi\sqrt{\frac{49}{3}}\right)}{8\sqrt[4]{3}\ 49^{3/4}} + 11 = \frac{1}{10584}\left(116\,424 + 49\sqrt{3}\ \exp\left(\pi\sqrt{8}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{8}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right) + 9\times 3^{3/4}\sqrt{7}\ \exp\left(\pi\sqrt{\frac{46}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{46}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

- 

$$\frac{\exp\left(\pi\sqrt{\frac{27}{3}}\right)}{8\sqrt[4]{3}\ 27^{3/4}} + \frac{\exp\left(\pi\sqrt{\frac{49}{3}}\right)}{8\sqrt[4]{3}\ 49^{3/4}} + 11 = \frac{1}{10584}\left(116\,424 + 49\sqrt{3}\ \exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (9-z_0)^k z_0^{-k}}{k!}\right) + 9\times 3^{3/4}\sqrt{7}\ \exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{49}{3}-z_0\right)^k z_0^{-k}}{k!}\right)\right)$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

And:

((((((((exp(Pi*sqrt(27/3)) / (8*3^(1/4)*27^(3/4))))) + (((exp(Pi*sqrt(49/3)) / (8*3^(1/4)*49^(3/4)))))+11)))))^1/15

**Input:**

$$\sqrt[15]{\frac{\exp\left(\pi\sqrt{\frac{27}{3}}\right)}{8\sqrt[4]{3}\times 27^{3/4}}+\frac{\exp\left(\pi\sqrt{\frac{49}{3}}\right)}{8\sqrt[4]{3}\times 49^{3/4}}+11}$$

**Exact result:**

$$\sqrt[15]{11+\frac{e^{3\pi}}{72\sqrt{3}}+\frac{e^{(7\pi)/\sqrt{3}}}{56\sqrt[4]{3}\sqrt{7}}}$$

**Decimal approximation:**

1.647341493354365565831362202983137980267972133278168044047...

1.64734149…. ≈ ζ(2) = $\frac{\pi^2}{6} = 1.644934$ ...

**Alternate forms:**

$$\frac{\sqrt[15]{116424+49\sqrt{3}\,e^{3\pi}+9\times 3^{3/4}\sqrt{7}\,e^{(7\pi)/\sqrt{3}}}}{\sqrt[5]{6}\,7^{2/15}}$$

- $$\frac{\sqrt[15]{5544\sqrt{21}+7\sqrt{7}\,e^{3\pi}+9\sqrt[4]{3}\,e^{(7\pi)/\sqrt{3}}}}{\sqrt[5]{2}\sqrt[6]{3}\sqrt[10]{7}}$$

-29/10^3+((((((((exp(Pi*sqrt(27/3)) / (8*3^(1/4)*27^(3/4))))) + (((exp(Pi*sqrt(49/3)) / (8*3^(1/4)*49^(3/4)))))+11)))))^1/15

**Input:**

$$-\frac{29}{10^3}+\sqrt[15]{\frac{\exp\left(\pi\sqrt{\frac{27}{3}}\right)}{8\sqrt[4]{3}\times 27^{3/4}}+\frac{\exp\left(\pi\sqrt{\frac{49}{3}}\right)}{8\sqrt[4]{3}\times 49^{3/4}}+11}$$

**Exact result:**

$$\sqrt[15]{11+\frac{e^{3\pi}}{72\sqrt{3}}+\frac{e^{(7\pi)/\sqrt{3}}}{56\sqrt[4]{3}\sqrt{7}}}-\frac{29}{1000}$$

**Decimal approximation:**

1.618341493354365565831362202983137980267972133278168044047...

1.61834149335…

This result is a very good approximation to the value of the golden ratio 1,618033988749...

- **Alternate forms:**

$$\frac{500\times 6^{4/5}\times 7^{13/15}\sqrt[15]{116424+49\sqrt{3}\,e^{3\pi}+9\times 3^{3/4}\sqrt{7}\,e^{(7\pi)/\sqrt{3}}}-609}{21000}$$

- 

$$\frac{\sqrt[15]{5544\sqrt{21}+7\sqrt{7}\,e^{3\pi}+9\sqrt[4]{3}\,e^{(7\pi)/\sqrt{3}}}}{\sqrt[5]{2}\sqrt[6]{3}\sqrt[10]{7}}-\frac{29}{1000}$$

$$\frac{\frac{1000\sqrt[15]{5544\sqrt{21}+7\sqrt{7}\,e^{3\pi}+9\sqrt[4]{3}\,e^{(7\pi)/\sqrt{3}}}}{\sqrt[30]{21}}-29\sqrt[5]{2}\,3^{2/15}\sqrt[15]{7}}{1000\sqrt[5]{2}\,3^{2/15}\sqrt[15]{7}}$$

- **Series representations:**

$$-\frac{29}{10^3}+\sqrt[15]{\frac{\exp\left(\pi\sqrt{\frac{27}{3}}\right)}{8\sqrt[4]{3}\,27^{3/4}}+\frac{\exp\left(\pi\sqrt{\frac{49}{3}}\right)}{8\sqrt[4]{3}\,49^{3/4}}+11}=\frac{1}{21000}\left(-609+500\times 6^{4/5}\times 7^{13/15}\left(116424+49\sqrt{3}\exp\left(\pi\sqrt{8}\sum_{k=0}^{\infty}8^{-k}\binom{\frac{1}{2}}{k}\right)+9\times 3^{3/4}\sqrt{7}\exp\left(\pi\sqrt{\frac{46}{3}}\sum_{k=0}^{\infty}\left(\frac{46}{3}\right)^{-k}\binom{\frac{1}{2}}{k}\right)\right)\hat{}\,(1/15)\right)$$

- 

$$-\frac{29}{10^3}+\sqrt[15]{\frac{\exp\left(\pi\sqrt{\frac{27}{3}}\right)}{8\sqrt[4]{3}\,27^{3/4}}+\frac{\exp\left(\pi\sqrt{\frac{49}{3}}\right)}{8\sqrt[4]{3}\,49^{3/4}}+11}=\frac{1}{21000}\left(-609+500\times 6^{4/5}\times 7^{13/15}\left(116424+49\sqrt{3}\exp\left(\pi\sqrt{8}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{8}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+9\times 3^{3/4}\sqrt{7}\exp\left(\pi\sqrt{\frac{46}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{46}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\hat{}\,(1/15)\right)$$

•

$$-\frac{29}{10^3}+\sqrt[15]{\frac{\exp\left(\pi\sqrt{\frac{27}{3}}\right)}{8\sqrt[4]{3}\;27^{3/4}}+\frac{\exp\left(\pi\sqrt{\frac{49}{3}}\right)}{8\sqrt[4]{3}\;49^{3/4}}+11} = \frac{1}{21000}\left(-609+500\times 6^{4/5}\times 7^{13/15}\left(116424+49\sqrt{3}\,\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(9-z_0)^k z_0^{-k}}{k!}\right)+9\times 3^{3/4}\sqrt{7}\,\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{49}{3}-z_0\right)^k z_0^{-k}}{k!}\right)\right)^\wedge(1/15)\right)$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

From:

CERN-PH-TH/2009-108 - UMN–TH–2803/09 - FTPI–MINN–09/23
**Nucleosynthesis Constraints on a Massive Gravitino in Neutralino Dark Matter Scenarios**
*Richard H. Cyburt, John Ellis, Brian D. Fields,*
*Feng Luo, Keith A. Olive, and Vassilis C. Spanos*

Massive Gravitino with a mass $m_{3/2}$ = 250-500-750-1000-5000 GeV

$m_{3/2}$= 250 GeV

$m_{3/2}$= 500 GeV

$m_{3/2}$= 750 GeV

$m_{3/2}$=1000 GeV

$m_{3/2}$=5000 GeV

Note that, from the following Lucas numbers:

5778-3571; 1364-843; 843-521; 521-322; 322-199

Calculating the square root of the sum of the two ratios for each corresponding mass value (example: 1/ √(5778/5000 + 5000/3571) = 0.62551724...), we obtain:

5000 = 0.62551724; 1000 = 0.62619487; 750 = 0.624568398; 500 = 0.6207953;

250 = 0.62692765

Note that the ratios of the above Lucas numbers are always an excellent approximations to the golden ratio. For example: 5778/3571 = 1,618034164...

Now, we have the following mean:

**Input interpretation:**

$$\frac{1}{5}(0.62551724 + 0.62619487 + 0.624568398 + 0.6207953 + 0.62692765)$$

**Result:**

0.6248006916

0.6248006916...

And:

1+(18+3)/10^3+1/5(0.62551724 + 0.62619487 + 0.624568398 + 0.6207953 + 0.62692765)

**Input interpretation:**

$$1 + \frac{18+3}{10^3} + \frac{1}{5}(0.62551724 + 0.62619487 + 0.624568398 + 0.6207953 + 0.62692765)$$

**Result:**

1.6458006916

$1.6458006916 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

1+18/10^3+29/10^3+1/5(0.62551724 + 0.62619487 + 0.624568398 + 0.6207953 + 0.62692765)

**Input interpretation:**

$$1 + \frac{18}{10^3} + \frac{29}{10^3} + \frac{1}{5}(0.62551724 + 0.62619487 + 0.624568398 + 0.6207953 + 0.62692765)$$

**Result:**

1.6718006916

1.6718006916

We note that 1.6718006916... is a result practically equal to the value of the formula:

$m_{p\prime} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24}$ gm

that is the holographic proton mass (N. Haramein)

And:

1-18/10^3+11/10^3+1/5(0.62551724 + 0.62619487 + 0.624568398 + 0.6207953 + 0.62692765)

**Input interpretation:**

$$1 - \frac{18}{10^3} + \frac{11}{10^3} + \frac{1}{5}(0.62551724 + 0.62619487 + 0.624568398 + 0.6207953 + 0.62692765)$$

**Result:**

1.6178006916

1.6178006916

This result is a very good approximation to the value of the golden ratio 1,618033988749...

And again:

1/1.6178006916

**Input interpretation:**

$$\frac{1}{1.6178006916}$$

**Result:**

0.6181231131821330963263385960589856382776193393487853296 94...

0.61812311…

Furthermore, we obtain the following approximation to $\pi$

sqrt[6(((((1+(18+3)/10^3+1/5(0.62551724 + 0.62619487 + 0.624568398 + 0.6207953 + 0.62692765))))))]

**Input interpretation:**

$$\sqrt{\left(6\left(1+\frac{18+3}{10^3}+\frac{1}{5}(0.62551724+0.62619487+0.624568398+0.6207953+0.62692765)\right)\right)}$$

**Result:**

3.142420110297157578321330398698059883194129495068481633758...

3.14242011.... ≈ $\pi$

Note that:

750 -21 = 729; where 729 = $9^3$ is a Ramanujan number (Ramanujan cubes)

1/sqrt((((987/729)+ (729/610))))

Where 987 and 610 are Fibonacci numbers

**Input:**

$$\frac{1}{\sqrt{\frac{987}{729}+\frac{729}{610}}}$$

**Result:**

$$9\sqrt{\frac{1830}{377837}}$$

**Decimal approximation:**

0.626348168937984866181218607547686136740494311480382051248...

0.626348168…

From:

**ON RECURSIONS FOR COEFFICIENTS OF MOCK THETA FUNCTIONS**
*SONG HENG CHAN, RENRONG MAO, AND ROBERT OSBURN*

arXiv:1502.01429v2 [math.NT] 12 Nov 2015

For q = 0.5 and n = 2, l = 3, j = 5

**Corollary 2.2.** *We have*

$$\frac{(q)_\infty^3}{(-q)_\infty^2} f(q) = -4\sum_{n=1}^{\infty} \frac{q^n}{(1-q^n)^2} - 16\sum_{n=1}^{\infty} \frac{q^{2n}}{(1-q^{2n})^2} + 1 + 4\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{q^{n(3n+1)/2}}{(1-q^n)^2} + 2\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{(6n-1)q^{n(3n+1)/2}}{1-q^n}, \tag{2.8}$$

$$q\frac{(q^2;q^2)_\infty^3}{(-q;q^2)_\infty^2}\omega(-q) = \sum_{n=1}^{\infty} \frac{q^{2n-1}}{(1+q^{2n-1})^2} - 2\sum_{n=1}^{\infty} \frac{q^{2n}}{(1-q^{2n})^2} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{q^{3n^2}}{(1-q^{2n})^2} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{(3n-1)q^{3n^2}}{1-q^{2n}}. \tag{2.9}$$

-4*(((0.5^2/(1-0.5^2)^2)))-16*(((0.5^4/(1-0.5^4)^2)))+1+4*(((0.5^7/(1-0.5^2)^2)))+2*(((11*0.5^7)/(1-0.5^2)))

**Input:**

$$-4\times\frac{0.5^2}{(1-0.5^2)^2} - 16\times\frac{0.5^4}{(1-0.5^4)^2} + 1 + 4\times\frac{0.5^7}{(1-0.5^2)^2} + 2\times\frac{11\times 0.5^7}{1-0.5^2}$$

**Result:**

-1.63083333333333333333333333333333333333333333333333333333...

-1.630833333….

**Repeating decimal:**

$-1.6308\overline{3}$ (period 1)

(((0.5^3/(1+0.5^3)^2)))-2*(((0.5^4/(1-0.5^4)^2)))+(((0.5^12/(1-0.5^4)^2)))+(((5*0.5^12)/(1-0.5^4)))

**Input:**

$$\frac{0.5^3}{(1+0.5^3)^2} - 2\times\frac{0.5^4}{(1-0.5^4)^2} + \frac{0.5^{12}}{(1-0.5^4)^2} + \frac{5\times 0.5^{12}}{1-0.5^4}$$

**Result:**

−0.04187692901234567901234567901234567901234567901234567901234567901...

-0.041876929…

**Repeating decimal:**

$-0.0418769\overline{290123456}7$ (period 9)

1+(-((((((0.5^3/(1+0.5^3)^2)))-2*(((0.5^4/(1-0.5^4)^2)))+(((0.5^12/(1-0.5^4)^2)))+(((5*0.5^12)/(1-0.5^4)))))))^1/8

**Input:**

$$1+\sqrt[8]{-\left(\frac{0.5^3}{(1+0.5^3)^2}-2\times\frac{0.5^4}{(1-0.5^4)^2}+\frac{0.5^{12}}{(1-0.5^4)^2}+\frac{5\times 0.5^{12}}{1-0.5^4}\right)}$$

**Result:**

1.67258...

1.67258…. result practically equal to the proton mass without exponent

$$\frac{[-q^{l+j},q^{2l};q^{6l}]_\infty(q^{6l};q^{6l})_\infty^2}{[-q^j,q^l,-q^{2l+j};q^{6l}]_\infty}=\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{3ln(n+1)+ln+l}}{1+q^{6ln+j+2l}}+\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{3ln(n+1)-in}}{1+q^{6ln+j}} \qquad (2.17)$$

(((0.5^63 / (1+0.5^47))) + (((0.5^48 / (1+0.5^41)))

**Input:**

$$\frac{0.5^{63}}{1+0.5^{47}}+\frac{0.5^{48}}{1+0.5^{41}}$$

**Result:**

$3.5528220990161338918947582079223091702647754153694110...\times 10^{-15}$

$3.55282209…*10^{-15}$

1+((((((0.5^63 / (1+0.5^47))) + (((0.5^48 / (1+0.5^41))))))))^1/76

**Input:**

$$1+\sqrt[76]{\frac{0.5^{63}}{1+0.5^{47}}+\frac{0.5^{48}}{1+0.5^{41}}}$$

**Result:**

1.645470...

1.645470…. ≈ ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

For n = 2, m = 3

$$\frac{(q)_\infty^3}{(-q)_\infty^2} f(q) = 1 - 4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} nq^{mn} - 16\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} nq^{2mn} + 4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}(6n+2m-1)q^{n(3n-1+2m)/2}. \tag{3.1}$$

1-4(2*0.5^6)-16(2*0.5^6)+4(12+6-1)*0.5^11

**Input:**

$1 - 4\left(2\times 0.5^6\right) - 16\left(2\times 0.5^6\right) + 4\,(12+6-1)\times 0.5^{11}$

**Result:**

0.408203125

0.408203125

4((((1-4(2*0.5^6)-16(2*0.5^6)+4(12+6-1)*0.5^11))))

**Input:**

$4\left(1 - 4\left(2\times 0.5^6\right) - 16\left(2\times 0.5^6\right) + 4\,(12+6-1)\times 0.5^{11}\right)$

**Result:**

1.6328125

1.6328125

For l = 3,

$$\sum_{l=-\infty}^{\infty} \frac{(6l+1)q^{3l^2+2l}}{1-q^{6l+1}}, \tag{3.8}$$

(((19*0.5^33))) / (((1-0.5^19)))

**Input:**

$$\frac{19 \times 0.5^{33}}{1 - 0.5^{19}}$$

**Result:**

$2.2118953335673018785512515091924842691121465914661244... \times 10^{-9}$

1/(((((((19*0.5^33))) / (((1-0.5^19)))))))^1/40

**Input:**

$$\frac{1}{\sqrt[40]{\frac{19 \times 0.5^{33}}{1-0.5^{19}}}}$$

**Result:**

1.6458145055025318615105453716175844979289263855822378345 41...

$1.6458145.... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$

For n = 2

$$\sum_{n=0}^{\infty} S_2(n) q^n = \sum_{n=-\infty}^{\infty} \frac{(2n-1)q^{3n^2+2n-2}}{1-q^{6n-3}}. \tag{3.9}$$

(((3*0.5^14))) / (((1-0.5^9)))

**Input:**

$$\frac{3 \times 0.5^{14}}{1 - 0.5^{9}}$$

**Result:**

- More digits

0.00018346379647749510763209393346379647749510763209393346 3...

0.00018346379….

**Repeating decimal:**

$0.0001\overline{834637964774951076320939 3}$ (period 24)

(((1/(((3*0.5^14))) / (((1-0.5^9))))))^1/17

**Input:**

$$\sqrt[17]{\frac{\frac{1}{3\times 0.5^{14}}}{1-0.5^9}}$$

**Result:**

1.659170601151739390407742508918299759042606867524294079769...

1.6591706…. is very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164,2696 i.e. 1,65578...

For n = 2

$$2\frac{(q^2;q^2)_\infty^2}{(-q;q^2)_\infty^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{n(3n+1)}}{1+q^{2n}} = \sum_{n=-\infty}^{\infty}\frac{(6n+1)q^{3n^2+2n}}{1-q^{6n+1}} - 3\sum_{n=-\infty}^{\infty}\frac{(2n-1)q^{3n^2+2n-2}}{1-q^{6n-3}}. \quad (3.11)$$

(((13*0.5^16) / (1-0.5^13))) - (((3*3*0.5^14) / (1-0.5^9)))

**Input:**

$$\frac{13\times 0.5^{16}}{1-0.5^{13}} - \frac{3\times 3\times 0.5^{14}}{1-0.5^9}$$

**Result:**

−0.000352002914276826673158765013674260876753422029476770 48...

-0.00035200291427….

1/(-((((((13*0.5^16) / (1-0.5^13))) - (((3*3*0.5^14) / (1-0.5^9)))))))^1/16

**Input:**

$$\frac{1}{\sqrt[16]{-\left(\frac{13\times 0.5^{16}}{1-0.5^{13}} - \frac{3\times 3\times 0.5^{14}}{1-0.5^9}\right)}}$$

**Result:**

1.643769276379747269473388236869983308933390108205898271234...

1.643769276…. ≈ ζ(2) = $\frac{\pi^2}{6} = 1.644934$ ...

For n = 2

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(3n+1)}}{1+q^{2n}} = \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(3n+1)}(1-q^{2n}+q^{4n})}{1+q^{6n}}.$$

((0.5^14 * (1-0.5^4+0.5^8))) / ((1+0.5^12))

**Input:**

$$\frac{0.5^{14}\,(1-0.5^4+0.5^8)}{1+0.5^{12}}$$

**Result:**

0.00005744485294117647058823529411764705882352941176470588 2...

0.00005744485294…

-1/6ln((0.5^14 * (1-0.5^4+0.5^8))) / ((1+0.5^12))

**Input:**

$$-\frac{1}{6}\times\frac{\log(0.5^{14}\,(1-0.5^4+0.5^8))}{1+0.5^{12}}$$

$\log(x)$ is the natural logarithm

**Result:**

1.627009620659021595507900096306228364323031979255584180140...

1.62700962…

**Alternative representations:**

$$\frac{\log(0.5^{14}\,(1-0.5^4+0.5^8))\,(-1)}{(1+0.5^{12})\,6} = -\frac{\log_e((1-0.5^4+0.5^8)\,0.5^{14})}{6\,(1+0.5^{12})}$$

- $$\frac{\log(0.5^{14}\,(1-0.5^4+0.5^8))\,(-1)}{(1+0.5^{12})\,6} = -\frac{\log(a)\log_a((1-0.5^4+0.5^8)\,0.5^{14})}{6\,(1+0.5^{12})}$$

-

$$\frac{\log(0.5^{14}(1-0.5^4+0.5^8))(-1)}{(1+0.5^{12})6} = \frac{\text{Li}_1(1-(1-0.5^4+0.5^8)0.5^{14})}{6(1+0.5^{12})}$$

- **Series representations:**

$$\frac{\log(0.5^{14}(1-0.5^4+0.5^8))(-1)}{(1+0.5^{12})6} = 0.166626\sum_{k=1}^{\infty}\frac{(-1)^k(-0.999943)^k}{k}$$

- $$\frac{\log(0.5^{14}(1-0.5^4+0.5^8))(-1)}{(1+0.5^{12})6} = -0.333252\, i\pi\left\lfloor\frac{\arg(0.0000574589-x)}{2\pi}\right\rfloor - 0.166626\log(x) + 0.166626\sum_{k=1}^{\infty}\frac{(-1)^k(0.0000574589-x)^k x^{-k}}{k} \quad \text{for } x<0$$

- $$\frac{\log(0.5^{14}(1-0.5^4+0.5^8))(-1)}{(1+0.5^{12})6} = -0.166626\left\lfloor\frac{\arg(0.0000574589-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right) - 0.166626\log(z_0) - 0.166626\left\lfloor\frac{\arg(0.0000574589-z_0)}{2\pi}\right\rfloor\log(z_0) + 0.166626\sum_{k=1}^{\infty}\frac{(-1)^k(0.0000574589-z_0)^k z_0^{-k}}{k}$$

- **Integral representation:**

$$\frac{\log(0.5^{14}(1-0.5^4+0.5^8))(-1)}{(1+0.5^{12})6} = -0.166626\int_1^{0.0000574589}\frac{1}{t}\,dt$$

-11/10^3+2/10^3-1/6ln((0.5^14 * (1-0.5^4+0.5^8))) / ((1+0.5^12))

Where 11 and 2 are Lucas numbers

**Input:**

$$-\frac{11}{10^3}+\frac{2}{10^3}-\frac{1}{6}\times\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{1+0.5^{12}}$$

log(x) is the natural logarithm

**Result:**

1.618009620659021595507900096306228364323031979255584180140...

1.61800962….

This result is a very good approximation to the value of the golden ratio 1,618033988749...

**Alternative representations:**

$$-\frac{11}{10^3}+\frac{2}{10^3}-\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{(1+0.5^{12})6}=-\frac{\log_e((1-0.5^4+0.5^8)0.5^{14})}{6(1+0.5^{12})}-\frac{9}{10^3}$$

- 

$$-\frac{11}{10^3}+\frac{2}{10^3}-\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{(1+0.5^{12})6}=-\frac{\log(a)\log_a((1-0.5^4+0.5^8)0.5^{14})}{6(1+0.5^{12})}-\frac{9}{10^3}$$

- 

$$-\frac{11}{10^3}+\frac{2}{10^3}-\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{(1+0.5^{12})6}=\frac{\text{Li}_1(1-(1-0.5^4+0.5^8)0.5^{14})}{6(1+0.5^{12})}-\frac{9}{10^3}$$

- 

**Series representations:**

$$-\frac{11}{10^3}+\frac{2}{10^3}-\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{(1+0.5^{12})6}=-0.009+0.166626\sum_{k=1}^{\infty}\frac{(-1)^k(-0.999943)^k}{k}$$

- 

$$-\frac{11}{10^3}+\frac{2}{10^3}-\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{(1+0.5^{12})6}=$$
$$-0.009-0.333252\,i\,\pi\left\lfloor\frac{\arg(0.0000574589-x)}{2\pi}\right\rfloor-0.166626\log(x)+$$
$$0.166626\sum_{k=1}^{\infty}\frac{(-1)^k(0.0000574589-x)^k x^{-k}}{k}\quad\text{for } x<0$$

- 

$$-\frac{11}{10^3}+\frac{2}{10^3}-\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{(1+0.5^{12})6} = -\frac{9}{1000}-0.166626\left\lfloor\frac{\arg(0.0000574589-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)- 0.166626\log(z_0)-0.166626\left\lfloor\frac{\arg(0.0000574589-z_0)}{2\pi}\right\rfloor\log(z_0)+ 0.166626\sum_{k=1}^{\infty}\frac{(-1)^k(0.0000574589-z_0)^k z_0^{-k}}{k}$$

- **Integral representation:**

$$-\frac{11}{10^3}+\frac{2}{10^3}-\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{(1+0.5^{12})6} = -0.009-0.166626\int_1^{0.0000574589}\frac{1}{t}\,dt$$

(47/10^3-3/10^3)-1/6ln((0.5^14 * (1-0.5^4+0.5^8))) / ((1+0.5^12))

**Input:**

$$\left(\frac{47}{10^3}-\frac{3}{10^3}\right)-\frac{1}{6}\times\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{1+0.5^{12}}$$

$\log(x)$ is the natural logarithm

**Result:**

1.67101...

1.67101…

We note that 1.67101... is a result practically equal to the value of the formula:

$m_{p\prime} = 2\times\frac{\eta}{R}m_P = 1.6714213\times10^{-24}$ gm

that is the holographic proton mass (N. Haramein)

**Alternative representations:**

$$\left(\frac{47}{10^3}-\frac{3}{10^3}\right)-\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{(1+0.5^{12})6} = -\frac{\log_e((1-0.5^4+0.5^8)\,0.5^{14})}{6(1+0.5^{12})}+\frac{44}{10^3}$$

- 

$$\left(\frac{47}{10^3}-\frac{3}{10^3}\right)-\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{(1+0.5^{12})6}=-\frac{\log(a)\log_a((1-0.5^4+0.5^8)0.5^{14})}{6(1+0.5^{12})}+\frac{44}{10^3}$$

- $$\left(\frac{47}{10^3}-\frac{3}{10^3}\right)-\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{(1+0.5^{12})6}=\frac{\text{Li}_1(1-(1-0.5^4+0.5^8)0.5^{14})}{6(1+0.5^{12})}+\frac{44}{10^3}$$

- **Series representations:**

$$\left(\frac{47}{10^3}-\frac{3}{10^3}\right)-\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{(1+0.5^{12})6}=0.044+0.166626\sum_{k=1}^{\infty}\frac{(-1)^k(-0.999943)^k}{k}$$

- $$\left(\frac{47}{10^3}-\frac{3}{10^3}\right)-\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{(1+0.5^{12})6}= 0.044-0.333252\,i\,\pi\left\lfloor\frac{\arg(0.0000574589-x)}{2\pi}\right\rfloor-0.166626\log(x)+ 0.166626\sum_{k=1}^{\infty}\frac{(-1)^k(0.0000574589-x)^k x^{-k}}{k}\quad\text{for } x<0$$

- $$\left(\frac{47}{10^3}-\frac{3}{10^3}\right)-\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{(1+0.5^{12})6}= \frac{11}{250}-0.166626\left\lfloor\frac{\arg(0.0000574589-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)- 0.166626\log(z_0)-0.166626\left\lfloor\frac{\arg(0.0000574589-z_0)}{2\pi}\right\rfloor\log(z_0)+ 0.166626\sum_{k=1}^{\infty}\frac{(-1)^k(0.0000574589-z_0)^k z_0^{-k}}{k}$$

- **Integral representation:**

$$\left(\frac{47}{10^3}-\frac{3}{10^3}\right)-\frac{\log(0.5^{14}(1-0.5^4+0.5^8))}{(1+0.5^{12})6}=0.044-0.166626\int_1^{0.0000574589}\frac{1}{t}\,dt$$

For n = 2

$$\frac{q(q^2;q^2)_\infty^3}{(-q;q^2)_\infty^2}\omega(q) = -3\sum_{\substack{n=-\infty\\n\neq 0}}^{\infty}\frac{nq^{3n^2+2n}}{1+q^{6n}} - \sum_{n=-\infty}^{\infty}\frac{(3n+2)q^{3n^2+8n+4}}{1+q^{6n+4}}$$
$$= -3\sum_{n=-\infty}^{\infty}\frac{nq^{3n^2+2n}}{1+q^{6n}} + \sum_{n=-\infty}^{\infty}\frac{(3n+1)q^{3n^2+4n+1}}{1+q^{6n+2}}. \qquad (3.25)$$

-3*(2*0.5^16)/(1+0.5^12) + (7*0.5^21)/(1+0.5^14)

**Input:**

$$-3\times\frac{2\times 0.5^{16}}{1+0.5^{12}} + \frac{7\times 0.5^{21}}{1+0.5^{14}}$$

**Result:**

−0.000088192731695803079860231021172633220240715889600889 14...

-0.00008819273169…

-1/6* colog((((-3*(2*0.5^16)/(1+0.5^12) + (7*0.5^21)/(1+0.5^14)))))

**Input:**

$$-\frac{1}{6}\left(-\log\left(-3\times\frac{2\times 0.5^{16}}{1+0.5^{12}} + \frac{7\times 0.5^{21}}{1+0.5^{14}}\right)\right)$$

log(x) is the natural logarithm

**Result:**

$-1.55600... + 0.523599...\, i$

**Polar coordinates:**

$r = 1.64173$ (radius), $\theta = 161.402°$ (angle)

$1.64173 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$

**Alternative representations:**

$$\frac{1}{6}\left(-\log\left(-\frac{3(2\times 0.5^{16})}{1+0.5^{12}} + \frac{7\times 0.5^{21}}{1+0.5^{14}}\right)\right)(-1) = \frac{1}{6}\log_e\left(-\frac{6\times 0.5^{16}}{1+0.5^{12}} + \frac{7\times 0.5^{21}}{1+0.5^{14}}\right)$$

•

$$\frac{1}{6}\left(-\log\left(-\frac{3(2\times 0.5^{16})}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)\right)(-1)=\frac{1}{6}\log(a)\log_a\left(-\frac{6\times 0.5^{16}}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)$$

•

$$\frac{1}{6}\left(-\log\left(-\frac{3(2\times 0.5^{16})}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)\right)(-1)=-\frac{1}{6}\operatorname{Li}_1\left(1+\frac{6\times 0.5^{16}}{1+0.5^{12}}-\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)$$

•

**Series representations:**

$$\frac{1}{6}\left(-\log\left(-\frac{3(2\times 0.5^{16})}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)\right)(-1)=\frac{\log(-1.00009)}{6}-\frac{1}{6}\sum_{k=1}^{\infty}\frac{e^{-0.0000881888\,k}}{k}$$

•

$$\frac{1}{6}\left(-\log\left(-\frac{3(2\times 0.5^{16})}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)\right)(-1)=\frac{1}{3}\,i\,\pi\left\lfloor\frac{\arg(-0.0000881927-x)}{2\,\pi}\right\rfloor+\frac{\log(x)}{6}-\frac{1}{6}\sum_{k=1}^{\infty}\frac{(-1)^k\,(-0.0000881927-x)^k\,x^{-k}}{k}\quad\text{for } x<0$$

•

$$\frac{1}{6}\left(-\log\left(-\frac{3(2\times 0.5^{16})}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)\right)(-1)=\frac{1}{6}\left\lfloor\frac{\arg(-0.0000881927-z_0)}{2\,\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\frac{\log(z_0)}{6}+\frac{1}{6}\left\lfloor\frac{\arg(-0.0000881927-z_0)}{2\,\pi}\right\rfloor\log(z_0)-\frac{1}{6}\sum_{k=1}^{\infty}\frac{(-1)^k\,(-0.0000881927-z_0)^k\,z_0^{-k}}{k}$$

•

**Integral representation:**

$$\frac{1}{6}\left(-\log\left(-\frac{3(2\times 0.5^{16})}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)\right)(-1)=\frac{1}{6}\int_1^{-0.0000881927}\frac{1}{t}\,dt$$

(29/10^3+2/10^3)+1/6* colog((((-3*(2*0.5^16)/(1+0.5^12) + (7*0.5^21)/(1+0.5^14)))))

**Input:**

$$\left(\frac{29}{10^3}+\frac{2}{10^3}\right)+\frac{1}{6}\left(-\log\left(-3\times\frac{2\times 0.5^{16}}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)\right)$$

log(x) is the natural logarithm

**Result:**

1.58700... −
0.523599... $i$

**Polar coordinates:**

$r = 1.67114$ (radius), $\theta = -18.2593°$ (angle)

1.67114

We note that 1.67114... is a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass (N. Haramein)

**Alternative representations:**

$$\left(\frac{29}{10^3} + \frac{2}{10^3}\right) - \frac{1}{6} \log\left(-\frac{3(2 \times 0.5^{16})}{1 + 0.5^{12}} + \frac{7 \times 0.5^{21}}{1 + 0.5^{14}}\right) = -\frac{1}{6} \log_e\left(-\frac{6 \times 0.5^{16}}{1 + 0.5^{12}} + \frac{7 \times 0.5^{21}}{1 + 0.5^{14}}\right) + \frac{31}{10^3}$$

- $$\left(\frac{29}{10^3} + \frac{2}{10^3}\right) - \frac{1}{6} \log\left(-\frac{3(2 \times 0.5^{16})}{1 + 0.5^{12}} + \frac{7 \times 0.5^{21}}{1 + 0.5^{14}}\right) = -\frac{1}{6} \log(a) \log_a\left(-\frac{6 \times 0.5^{16}}{1 + 0.5^{12}} + \frac{7 \times 0.5^{21}}{1 + 0.5^{14}}\right) + \frac{31}{10^3}$$

- $$\left(\frac{29}{10^3} + \frac{2}{10^3}\right) - \frac{1}{6} \log\left(-\frac{3(2 \times 0.5^{16})}{1 + 0.5^{12}} + \frac{7 \times 0.5^{21}}{1 + 0.5^{14}}\right) = \frac{1}{6} \text{Li}_1\left(1 + \frac{6 \times 0.5^{16}}{1 + 0.5^{12}} - \frac{7 \times 0.5^{21}}{1 + 0.5^{14}}\right) + \frac{31}{10^3}$$

-

**Series representations:**

$$\left(\frac{29}{10^3} + \frac{2}{10^3}\right) - \frac{1}{6} \log\left(-\frac{3(2 \times 0.5^{16})}{1 + 0.5^{12}} + \frac{7 \times 0.5^{21}}{1 + 0.5^{14}}\right) = \frac{31}{1000} - \frac{\log(-1.00009)}{6} + \frac{1}{6} \sum_{k=1}^{\infty} \frac{e^{-0.0000881888 k}}{k}$$

- $$\left(\frac{29}{10^3}+\frac{2}{10^3}\right)-\frac{1}{6}\log\left(-\frac{3\left(2\times 0.5^{16}\right)}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)=$$
$$\frac{31}{1000}-\frac{1}{3}i\pi\left\lfloor\frac{\arg(-0.0000881927-x)}{2\pi}\right\rfloor-\frac{\log(x)}{6}+\frac{1}{6}\sum_{k=1}^{\infty}\frac{(-1)^k(-0.0000881927-x)^k x^{-k}}{k} \quad \text{for } x<0$$

- $$\left(\frac{29}{10^3}+\frac{2}{10^3}\right)-\frac{1}{6}\log\left(-\frac{3\left(2\times 0.5^{16}\right)}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)=$$
$$\frac{31}{1000}-\frac{1}{6}\left\lfloor\frac{\arg(-0.0000881927-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)-\frac{\log(z_0)}{6}-\frac{1}{6}\left\lfloor\frac{\arg(-0.0000881927-z_0)}{2\pi}\right\rfloor\log(z_0)+\frac{1}{6}\sum_{k=1}^{\infty}\frac{(-1)^k(-0.0000881927-z_0)^k z_0^{-k}}{k}$$

- **Integral representation:**

$$\left(\frac{29}{10^3}+\frac{2}{10^3}\right)-\frac{1}{6}\log\left(-\frac{3\left(2\times 0.5^{16}\right)}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)=\frac{31}{1000}-\frac{1}{6}\int_1^{-0.0000881927}\frac{1}{t}\,dt$$

(21/10^3+4/10^3)-1/6* colog((((-3*(2*0.5^16)/(1+0.5^12) + (7*0.5^21)/(1+0.5^14)))))

**Input:**

$$\left(\frac{21}{10^3}+\frac{4}{10^3}\right)-\frac{1}{6}\left(-\log\left(-3\times\frac{2\times 0.5^{16}}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)\right)$$

log(x) is the natural logarithm

**Result:**

$-1.53100\ldots +$
$0.523599\ldots i$

**Polar coordinates:**

$r = 1.61806$ (radius), $\theta = 161.119°$ (angle)

1.61806

This result is a very good approximation to the value of the golden ratio 1,618033988749...

**Alternative representations:**

$$\left(\frac{21}{10^3}+\frac{4}{10^3}\right)--\frac{1}{6}\log\left(-\frac{3\,(2\times 0.5^{16})}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)=\frac{1}{6}\log_e\left(-\frac{6\times 0.5^{16}}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)+\frac{25}{10^3}$$

- 

$$\left(\frac{21}{10^3}+\frac{4}{10^3}\right)--\frac{1}{6}\log\left(-\frac{3\,(2\times 0.5^{16})}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)=\frac{1}{6}\log(a)\log_a\left(-\frac{6\times 0.5^{16}}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)+\frac{25}{10^3}$$

- 

$$\left(\frac{21}{10^3}+\frac{4}{10^3}\right)--\frac{1}{6}\log\left(-\frac{3\,(2\times 0.5^{16})}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)=-\frac{1}{6}\operatorname{Li}_1\left(1+\frac{6\times 0.5^{16}}{1+0.5^{12}}-\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)+\frac{25}{10^3}$$

- 

**Series representations:**

$$\left(\frac{21}{10^3}+\frac{4}{10^3}\right)--\frac{1}{6}\log\left(-\frac{3\,(2\times 0.5^{16})}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)=\frac{1}{40}+\frac{\log(-1.00009)}{6}-\frac{1}{6}\sum_{k=1}^{\infty}\frac{e^{-0.0000881888\,k}}{k}$$

- 

$$\left(\frac{21}{10^3}+\frac{4}{10^3}\right)--\frac{1}{6}\log\left(-\frac{3\,(2\times 0.5^{16})}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)=\frac{1}{40}+\frac{1}{3}\,i\,\pi\left\lfloor\frac{\arg(-0.0000881927-x)}{2\,\pi}\right\rfloor+\frac{\log(x)}{6}-\frac{1}{6}\sum_{k=1}^{\infty}\frac{(-1)^k\,(-0.0000881927-x)^k\,x^{-k}}{k}\quad\text{for } x<0$$

- 

$$\left(\frac{21}{10^3}+\frac{4}{10^3}\right)--\frac{1}{6}\log\left(-\frac{3\,(2\times 0.5^{16})}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)=\frac{1}{40}+\frac{1}{6}\left\lfloor\frac{\arg(-0.0000881927-z_0)}{2\,\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\frac{\log(z_0)}{6}+\frac{1}{6}\left\lfloor\frac{\arg(-0.0000881927-z_0)}{2\,\pi}\right\rfloor\log(z_0)-\frac{1}{6}\sum_{k=1}^{\infty}\frac{(-1)^k\,(-0.0000881927-z_0)^k\,z_0^{-k}}{k}$$

- 

**Integral representation:**

$$\left(\frac{21}{10^3}+\frac{4}{10^3}\right)--\frac{1}{6}\log\left(-\frac{3(2\times 0.5^{16})}{1+0.5^{12}}+\frac{7\times 0.5^{21}}{1+0.5^{14}}\right)=\frac{1}{40}+\frac{1}{6}\int_1^{-0.0000881927}\frac{1}{t}\,dt$$

For n = 2

$$2\frac{(q^2;q^2)_\infty^3}{(-q;q^2)_\infty^2}\omega(q)=\sum_{n=-\infty}^{\infty}\frac{(6n+2)q^{3n^2+4n}}{1+q^{6n+2}}-\sum_{n=-\infty}^{\infty}\frac{6nq^{3n^2+2n-1}}{1+q^{6n}}$$

(14*0.5^20) / (1+0.5^14) – (12*0.5^15) / (1+0.5^12)

**Input:**

$$\frac{14\times 0.5^{20}}{1+0.5^{14}}-\frac{12\times 0.5^{15}}{1+0.5^{12}}$$

**Result:**

−0.00035277092678321231944092408469053288096286355840355659...

-0.0003527709267….

1/6(((((8+ln((((((14*0.5^20) / (1+0.5^14) – (12*0.5^15) / (1+0.5^12))))))))))^2

**Input:**

$$\frac{1}{6}\left(8+\log\left(\frac{14\times 0.5^{20}}{1+0.5^{14}}-\frac{12\times 0.5^{15}}{1+0.5^{12}}\right)\right)^2$$

log(x) is the natural logarithm

**Result:**

− 1.64451... +
0.0526828... $i$

**Polar coordinates:**

$r = 1.64536$ (radius), $\theta = 178.165°$ (angle)

$1.64536 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

29/10^3-2/10^3-1/6(((((8+ln((((((14*0.5^20) / (1+0.5^14) – (12*0.5^15) / (1+0.5^12)))))))))))^2

**Input:**

$$\frac{29}{10^3} - \frac{2}{10^3} - \frac{1}{6}\left(8 + \log\left(\frac{14\times 0.5^{20}}{1+0.5^{14}} - \frac{12\times 0.5^{15}}{1+0.5^{12}}\right)\right)^2$$

log(x) is the natural logarithm

**Result:**

1.67151... –
0.0526828... $i$

**Polar coordinates:**

$r = 1.67234$ (radius), $\theta = -1.80525°$ (angle)

1.67234 result very near to the proton mass without exponent

29/10^3-2/10^3+1/6(((((8+ln((((((14*0.5^20) / (1+0.5^14) – (12*0.5^15) / (1+0.5^12)))))))))))^2

**Input:**

$$\frac{29}{10^3} - \frac{2}{10^3} + \frac{1}{6}\left(8 + \log\left(\frac{14\times 0.5^{20}}{1+0.5^{14}} - \frac{12\times 0.5^{15}}{1+0.5^{12}}\right)\right)^2$$

log(x) is the natural logarithm

**Result:**

– 1.61751... +
0.0526828... $i$

**Polar coordinates:**

$r = 1.61837$ (radius), $\theta = 178.135°$ (angle)

1.61837

This result is a very good approximation to the value of the golden ratio 1,618033988749...

From the Last Letter to Hardy from Srinivasa Ramanujan:

Mock $\vartheta$-functions (of 7th order)

$$\text{(i)} \quad 1+\frac{q}{1-q^2}+\frac{q^4}{(1-q^3)(1-q^4)}+\frac{q^9}{(1-q^4)(1-q^5)(1-q^6)}+\ldots$$

$$\text{(ii)} \quad \frac{q}{1-q}+\frac{q^4}{(1-q^2)(1-q^3)}+\frac{q^9}{(1-q^3)(1-q^4)(1-q^5)}+\ldots$$

$$\text{(iii)} \quad \frac{1}{1-q}+\frac{q^2}{(1-q^2)(1-q^3)}+\frac{q^6}{(1-q^3)(1-q^4)(1-q^5)}+\ldots$$

Now, as for the Rogers-Ramanujan equation, we put q = 0.5 $|q| < 1$; $q = 0.5$ that is < 1. We obtain the following further results:

(i) 1+0.5/(1-0.5^2)+0.5^4/(((1-0.5^3)(1-0.5^4)))+0.5^9/(((1-0.5^4)(1-0.5^5)(1-0.5^6)))

**Input:**

$$1+\frac{0.5}{1-0.5^2}+\frac{0.5^4}{(1-0.5^3)(1-0.5^4)}+\frac{0.5^9}{(1-0.5^4)(1-0.5^5)(1-0.5^6)}$$

**Result:**

1.745041816009557945041816009557945041816009557945041816009...

1.745041816…

(ii) 0.5/(1-0.5)+0.5^4/(((1-0.5^2)(1-0.5^3)))+0.5^9/(((1-0.5^3)(1-0.5^4)(1-0.5^5)))

**Input:**

$$\frac{0.5}{1-0.5}+\frac{0.5^4}{(1-0.5^2)(1-0.5^3)}+\frac{0.5^9}{(1-0.5^3)(1-0.5^4)(1-0.5^5)}$$

**Result:**

1.097695852534562211981566820276497695852534562211981566820...

1.0976958525…

(iii) 1/(1-0.5)+0.5^2/(((1-0.5^2)(1-0.5^3)))+0.5^6/(((1-0.5^3)(1-0.5^4)(1-0.5^5)))

**Input:**

$$\frac{1}{1-0.5}+\frac{0.5^2}{(1-0.5^2)(1-0.5^3)}+\frac{0.5^6}{(1-0.5^3)(1-0.5^4)(1-0.5^5)}$$

**Result:**

2.400614439324116743471582181259600614439324116743471582181...

2.400614439…

The sum of (i), (ii) and (iii) is equal to:

5.24335210786823690049496501109404335210786823690049496501.

From this result, we obtain the following expressions:

1+ 1 / (5.24335210786)^1/4

**Input interpretation:**

$$1+\frac{1}{\sqrt[4]{5.24335210786}}$$

**Result:**

1.660842163368...

1.6608421.... is very near to the 14th root of the following Ramanujan's class invariant $Q=\left(G_{505}/G_{101/5}\right)^3$ = 1164,2696 i.e. 1,65578...

(8/10^3+3/10^3 +1) + 1 / (5.24335210786)^1/4

**Input interpretation:**

$$\left(\frac{8}{10^3}+\frac{3}{10^3}+1\right)+\frac{1}{\sqrt[4]{5.24335210786}}$$

**Result:**

1.671842163368...

1.67184216...

We note that 1.67184216... is a result practically equal to the value of the formula:

$m_{p\prime}=2\times\frac{\eta}{R}m_P=1.6714213\times 10^{-24}$ gm that is the holographic proton mass (N. Haramein)

(-2*8/10^3 +1) + 1 / (5.24335210786)^1/4

**Input interpretation:**

$$\left(-2\times\frac{8}{10^3}+1\right)+\frac{1}{\sqrt[4]{5.24335210786}}$$

**Result:**

1.644842163368...

1.6448421.... ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

((-21*2)/10^3-(55+21+5)/10^5)+1+ 1 / (5.24335210786)^1/4

**Input interpretation:**

$$\left(\frac{-21\times 2}{10^3}-\frac{55+21+5}{10^5}\right)+1+\frac{1}{\sqrt[4]{5.24335210786}}$$

**Result:**

1.618032163368...

1.61803216...

This result is a very good approximation to the value of the golden ratio 1,618033988749...

And:
sqrt(((((((((6*((((((-2*8/10^3 +1) + 1/(5.24335210786)^1/4)))))))))))))

**Input interpretation:**

$$\sqrt{6\left(\left(-2\times\frac{8}{10^3}+1\right)+\frac{1}{\sqrt[4]{5.24335210786}}\right)}$$

**Result:**

3.141504891005...

3.1415048.... ≈ $\pi$

The difference of (i), (ii) and (iii) is equal to:

-0.44212322922000341355180064857484212322922000341355180064

From this result, we obtain the following expressions:

-(5+2)/(21+3)*1/ -0.44212322922

**Input interpretation:**

$$-\frac{5+2}{21+3}\left(-\frac{1}{0.44212322922}\right)$$

**Result:**

0.659695413835706143417339682721922616890694270557573275897...

0.6596954…

(1+ 12/10^3)-7/24*(1/ -0.44212322922000341)

**Input interpretation:**

$$\left(1+\frac{12}{10^3}\right)-\frac{7}{24}\left(-\frac{1}{0.44212322922000341}\right)$$

**Result:**

1.671695413835701055330551869178867727543539375236348598741...

1.671695413…

We note that 1.671695413... is a result practically equal to the value of the formula:

$$m_{p\prime} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass (N. Haramein)

3/10^3+(1+ 12/10^3)-8/30*(1/ -0.44212322922000341)

**Input interpretation:**

$$\frac{3}{10^3}+\left(1+\frac{12}{10^3}\right)-\frac{8}{30}\left(-\frac{1}{0.44212322922000341}\right)$$

**Result:**

1.618150092649783822016504566106393350896950285930375861706...

1.61815009…

This result is a very good approximation to the value of the golden ratio 1,618033988749...

The product of (i), (ii) and (iii) is equal to:

4.598437367426429319522631500841602646078590395426256688283

From this result, we obtain the following expression:

1/(((0.021+0.08+0.03)*4.59843736742))))

**Input interpretation:**

$$\frac{1}{(0.021+0.08+0.03)\times 4.59843736742}$$

**Result:**

1.660039525675315212779583355824250595058320361616371100407...

1.6600395256…. is very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164,2696 i.e. 1,65578...

And, as above, from this result we obtain ζ(2), $\phi$ and Haramein's proton mass

**Input interpretation:**

$$\left(\frac{8}{10^3}+\frac{3}{10^3}\right)+\frac{1}{(0.021+0.08+0.03)\times 4.59843736742}$$

**Result:**

1.671039525675315212779583355824250595058320361616371100407...

1.671039525…

We note that 1.671039525... is a result practically equal to the value of the formula:

$m_{p\prime} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24}$ gm

that is the holographic proton mass (N. Haramein)

**Input interpretation:**

$$-2\times\frac{8}{10^3}+\frac{1}{(0.021+0.08+0.03)\times 4.59843736742}$$

**Result:**

1.644039525675315212779583355824250595058320361616371100407...

1.644039525…. ≈ ζ(2) = $\frac{\pi^2}{6} = 1.644934$ ...

**Input interpretation:**

$$\left(\frac{-21\times 2+1}{10^3}-\frac{55+21+5}{10^5}\right)+\frac{1}{(0.021+0.08+0.03)\times 4.59843736742}$$

**Result:**

1.618229525675315212779583355824250595058320361616371100407...

1.618229525…

This result is a very good approximation to the value of the golden ratio 1,618033988749...

And:

**Input interpretation:**

$$\sqrt{6\left(-2\times\frac{8}{10^3}+\frac{1}{(0.021+0.08+0.03)\times 4.59843736742}\right)}$$

**Result:**

3.140738313526278374374154872101819782968668320527016503676...

3.14073831…. ≈ $\pi$

The division of (i), (ii) and (iii) is equal to:

1.253240878545823789333232159172708086218320272094108525 15

From this result, we obtain the following expression:

1+(1/2*1.253240878)

**Input interpretation:**

$$1 + \frac{1}{2} \times 1.253240878$$

**Result:**

1.626620439

1.626620439

(21/10^3-3/10^3)+1+(1/2*1.253240878)

**Input interpretation:**

$$\left(\frac{21}{10^3} - \frac{3}{10^3}\right) + 1 + \frac{1}{2} \times 1.253240878$$

**Result:**

1.644620439

1.644620439 ≈ $\zeta(2) = \frac{\pi^2}{6} = 1.644934$ ...

(21*2/10^3+3/10^3)+1+(1/2*1.253240878)

**Input interpretation:**

$$\left(21 \times \frac{2}{10^3} + \frac{3}{10^3}\right) + 1 + \frac{1}{2} \times 1.253240878$$

**Result:**

1.671620439

1.671620439

-8/10^3+1+(1/2*1.253240878)

**Input interpretation:**

$$-\frac{8}{10^3} + 1 + \frac{1}{2} \times 1.253240878$$

**Result:**

1.618620439

1.618620439

This result is a very good approximation to the value of the golden ratio 1,618033988749...

**Table 1**

**Pseudoscalar and vector mesons**

5412,8 yroot 18 = 1,6121755567625040236799532125479

5325,1 yroot 18 = 1,6107131685717719630155684260401

2112,3 yroot 15 = 1,6659055431460041004403700717161

2006,97 yroot 15 = 1,6602343383181435201804528280 31

2010,27 yroot 15 = 1,6604161903558361840489468678013

896 yroot 14 = 1,6250964572948424170159035095751

891,66 yroot 14 = 1,6245329345441220430679921671897

9460,3 yroot 18 = 1,6629669038782188107556740904 72

3096,916 yroot 16 = 1,6526583762364355038554903110161

1019,445 yroot 14 = 1,6401483394181421821467273712845

782,65 yroot 14 = 1,6094719021392010260511399296 48

775,49 yroot 14 = 1,6084156869226947936552702850866

6276 yroot 17 = 1,6726039336098606692726386510316

5366,3 yroot 17 = 1,6572676157466882922176668111163

5279,34 yroot 17 = 1,6556756912546367526430981992312

1968,49 yroot 15 = 1,6580929809993139004336803545987

1864,84 yroot 15 = 1,6521245006260682233073185757 67

1869,62 yroot 15 = 1,6524064810048469221056275242794

497,614 yroot 13 = 1,6123275548441915501272274613228

493,677 yroot 13 = 1,611342696472979949148755230894

9300 yroot 18 = 1,6613887866520912603544395524522

2980,3 yroot 16 = 1,6486985231996318451920295756186

957,66 yroot 14 = 1,6328401386516461371221715914625

547,853 yroot 13 = 1,6243008251603635725574534450629

134,9766 yroot 10 = 1,633149147099935626086407262524

139,57 yroot 10 = 1,6386236091622671930819981196728

Now, we summing:

1.6121755567625

1.6107131685717

1.6659055431460

1.6602343383181

1.6604161903558

1.6250964572948

1.6245329345441

1.6629669038782

1.6526583762364

1.6401483394181

1.6094719021392

1.6084156869226

1.6726039336098

1.6572676157466

1.6556756912546

1.65809298099931

1.65212450062606

1.65240648100484

1.61232755484419

1.61134269647297

1.66138878665209

1.64869852319963

1.63284013865164

1.62430082516036

1.63314914709993

1.63862360916226

The sum is: 42,64357788207178

The mean is:

42,64357788207178 ÷ 26 = 1,640137610848914615384615384 where
1.64013761... ≈ ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

Note that the index of roots are min = 10 – max = 18 (the mean is 15$^{th}$)

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