# Decomposition of the Riemann and Ricci curvatures

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**Abstract**

We propose to decompose the Riemann and Ricci curvatures according to symmetric parallel endomorphisms.

## 1 The curvatures

The Riemann curvature $R$ is a tensor obtained by compositions of the Levi-Civita connection. The Ricci curvature $Ric$ is then defined with help of the Riemann curvature by a contraction with respect to the metric.

## 2 The decomposition of the Riemann curvature

The Riemann curvature posses symmetries like the Bianchi identities. The first Bianchi identity seems to imply that:

$$g(R(x,y)z,t) = \sum_i \pm[g(S_i(x),z)g(S_i(y),t) - g(S_i(x),t)g(S_i(y),z)]$$

with $g$ the riemannian metric and $S_i$ symmetric endomorphisms of the tangent vector bundle $S_i^* = S_i$. Now, the second Bianchi identity gives $\nabla S_i = 0$, the endomorphisms $S_i$ are parallel with respect of the Levi-Civita connection $\nabla$.

## 3 Decomposition of the Ricci curvature

According to the decomposition of the Riemann curvature, the Ricci curvature is:

$$Ric = \sum_i \pm[tr(S_i)S_i - S_i^2]$$

with $\nabla S_i = 0$.