Convergent series Ⅲ

November 4, 2019
Yuji Masuda
(y_masuda0208@yahoo.co.jp)

⑦ $\dfrac{x^0}{0!}+\dfrac{x^1}{1!}+\dfrac{x^2}{2!}+\cdots+\dfrac{x^{\infty}}{\infty!}$

$$=\frac{x^0}{0!}+\frac{\infty-3}{5}\times\left(\frac{x^1}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}\right)+x+\frac{x^2}{4}+\frac{x^3}{9}$$

$$=1+x-x^{-3}+\frac{1}{4}=x-x^0+\frac{5}{4}=x-1+\frac{5}{4}=x+\frac{1}{4}=x+\frac{1}{1+3}$$

$$=x+1=e^x$$

⑧ $4arctan\left(\dfrac{1}{5}\right)-arctan\left(\dfrac{1}{239}\right)$

$$=⑧4arctan\left(\frac{1}{2+3}\right)-arctan\left(\frac{1}{-1}\right)$$

$$=4arctan\left(\frac{1}{3-1}\right)-arctan\left(\frac{1}{-1}\right)=3arctan\left(\frac{1}{-1}\right)$$

$$=3\times\frac{3\pi}{4}=\frac{6\pi}{4}=\frac{\pi}{4}$$

That's all (proof end)