Fusion Reactor with Electrodynamic Stabilization

Octavian Balaci

https://orcid.org/0000-0002-0705-1906

Abstract

The magnetic confinement of thermonuclear plasma can be significantly improved by using the reaction of an electrical conductive wall in combination with the AC driving of the plasma current. In this way the magnetic fields can be confined in some well defined space and with it the plasma itself. Also many plasma instabilities are suppressed or reduced.

1 Introduction

The fusion reactor described here has the purpose to generate usable energy from the fusion reaction between the hydrogen isotopes, mainly between deuterium and tritium which require a lower temperature than other reactions. It is based on an improved version of tokamak magnetic confinement, using electrodynamic suspension and stabilization of plasma in a toroidal chamber that has a high electrical conductivity metallic shell around the plasma column. While the conductive shell stabilizing properties have been used before in various tokamak reactors, these effects are short in duration because of the penetration of magnetic field in the conductor due to its resistivity. This situation can be changed completely if the current is induced in the plasma alternatingly in both directions, so virtually by using an AC driving of plasma with an appropriate frequency. In this case the stabilization will become permanent, also will allow that the reactor to be operated continously. The alternating currents induced in this conductive shell will produce the electrodynamic suspension of plasma relative to the walls, the variable currents and magnetic fields will produce the reactive field expulsion from the conductive shell and plasma because of their good electrical conductivity. The alternating magnetic field created by the current flowing along the z axis of plasma, will be expelled from the high conductivity shell and from plasma, the field lines being forced to close in the space between the plasma surface and the inner surface of the shell. This will squeeze the field in that space and will keep the plasma away from the walls even against significant forces. The metallic confinement shell will react to changes in plasma position and will stabilize it inside its walls, as long as the alternating of current direction is maintained.

In addition to positional stabilization exerted by the combination af an AC current and the conductive shell, the changing of current direction will change the direction of rotation of the resultant magnetic field from a half of a period to the next. This will limit the evolution of some specific instabilities which have longer evolution time. A cyclic plasma compression and expansion is also produced with the variation of the current, however this effect is limited. A magnetic field along the z axis created by a toroidal coil will stabilize the plasma against some specific MHD instabilities, will also contribute to the confinement, in the same way like in the usual tokamak reactors.

2 The confinement system

The structure of principle of the system is shown in figure 2.1. It resemble an usual tokamak with some changes coresponding to the new working conditions. Multipole induction driving using multiple primary coils to ease the circulation of the necessary driving power at the required frequency (typical in the range 100 - 1000 Hz). Circular shape of the conductive shell and the shell wall being close to the plasma surface to increase the effectiveness of its stabilization. Relatively high tickness of the shell wall to decrease plasma inductance (reduce the amount of reactive energy circulated), and to decrease the amount of Joule power released in the conductive shell.

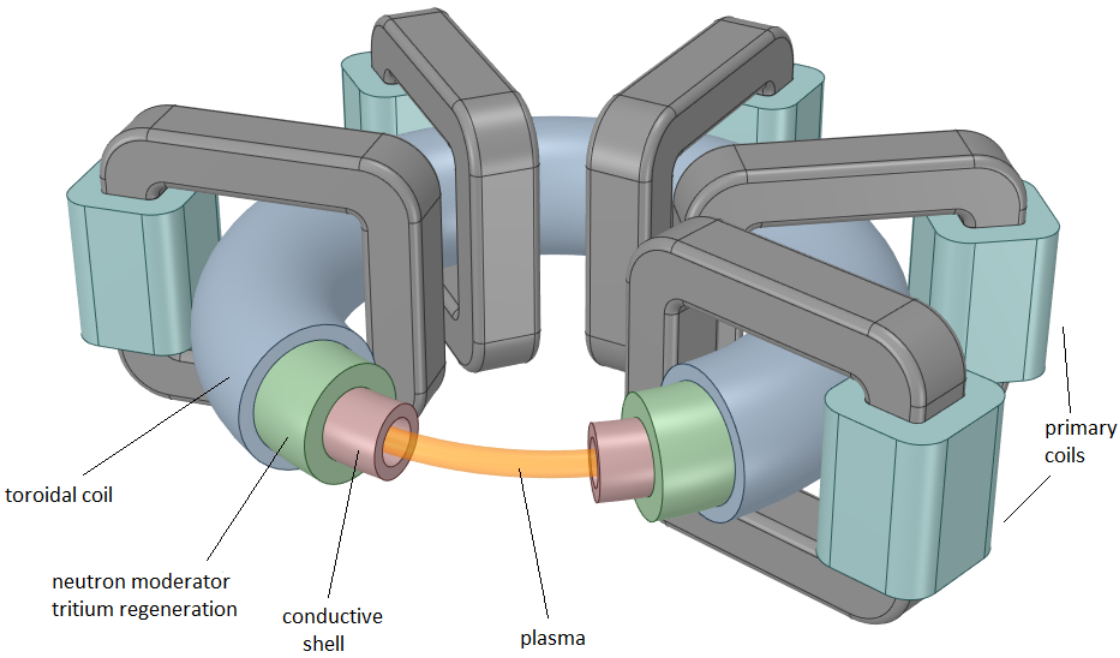

Figure 2.1: Electrodynamic stabilization the structure of principle

Because the toroidal plasma ring has a cylindrical symmetry, we will use a cylindrical coordinate system, with the z axis along the toroidal ring of plasma, the r radius transversal on the z axis, the azimuthal φ angle around the z axis. Now the z axis is following the toroidal curvature, however locally we approximate this with a straight z axis cylindrical coordinates. To prevent that the metallic confinement shell to be induced with toroidal currents by the primary coils variable magnetic field, the toroidal shell will be assembled from segments electrically insulated between them. In this way a close electrical path similar to the plasma column is avoided and the plasma column is the only secondary turn for the primary coils.

2.1 Electrodynamic suspension and stabilization

An alternating current in the axial direction will be induced in the plasma column by the primary coils, this current will create an alternating magnetic field around the plasma. The variations in this azimuthal magnetic field will induce currents in the metallic shell placed around the plasma column, currents that will create their own magnetic field that will oppose and expel the inductive magnetic field from the metal. For this the material of the metallic shell must have a high electrical conductivity and no ferromagnetic properties, like Cu for example. Because the magnetic field has no divergence, the azimuthal field lines are forced to close in the space between the plasma outer surface and the inner surface of the conductive shell. In this way it will create an electrodynamic suspension or levitation of plasma column inside the metallic shell. This will prevent the plasma to go close to the shell walls and also will limit the local bending of the plasma column. Virtually the azimuthal magnetic field around the plasma column is confined between the plasma surface and the walls, both being of high electrical conductivity. The azimuthal magnetic field created by the axial current through plasma, must change the direction periodically, with a high enough frequency, in this way the suspension effect of the plasma column inside the metallic shell will be permanent. This will produce a series of compression, during the current increase, followed by expansion during the current decrease, then the current change the direction and repeat the compression expansion cycle but with a reversed direction and azimuthal magnetic field this time. To change the current direction with the required frequency, several sources of alternating voltage with the average value of zero will be used. This can be implemented in various ways, but is simpler and easier to use resonant driving. In the case of resonant driving the plasma inductance will form a resonant LC circuit with an external capacitor connected at every primary coil and syncronized together. The external sources of power will feed these circuits only for the compensation of Joule loses, while the reactive energy circulate only inside the resonant circuits. A diferent approach is to use some AC voltage generators like multipolar rotary generators or inverters based on power electronic devices, but in this case these generators must handle the full active and reactive power and this can be unpractical. The equivalent electrical circuit is presented in figure 2.2.

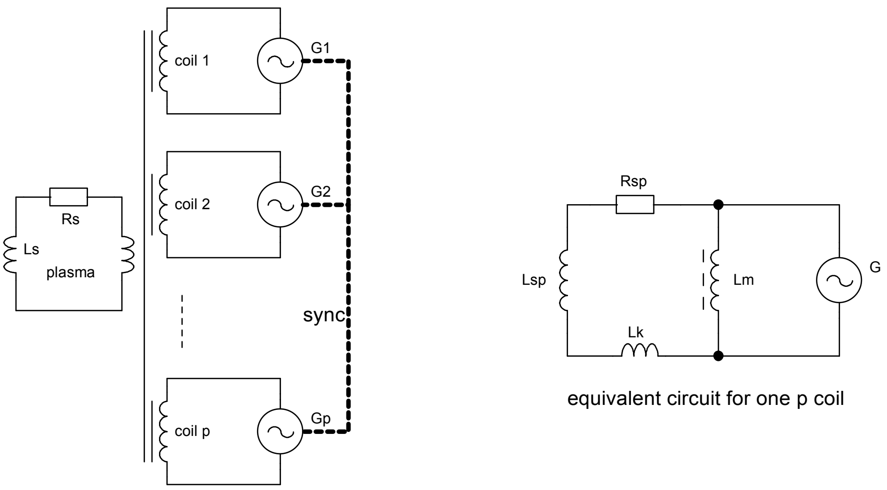

Figure 2.2: Equivalent electrical circuit

We will consider that we have a number of p primary coils, every one supplied with alternating voltage. All these primary coils will be driven synchronously. The voltage wave-form will be sinusoidal in the case of resonant driving. The plasma ring act like a short-circuit turn and is induced with a voltage and current by the oscillating magnetic field of all primary coils, but in the same time it has its own inductance L_s the effect of its own magnetic field which is not coupled with the magnetic cores (mainly the field bellow the inner surface of the conductive shell). We can represent an equivalent circuit with all the elements reflected in the primary side of every coil, with L_{sp} the reflected inductance of plasma, L_k the additional leakage inductance, L_m the magnetization inductance of one primary coil. We can write for the voltage induced in the plasma ring

$$u_s = -\frac{d\Phi_{sum}}{dt} = -p \cdot \frac{d\Phi_p}{dt} \tag{2.1}$$

in the same time the voltage induced in one of the primary coils is

$$u_p = -N_p \cdot \frac{d\Phi_p}{dt} \tag{2.2}$$

where N_p is the number of turns of the primary coil. So the voltage ratio will be

$$u_s = u_p \cdot \frac{p}{N_p} \tag{2.3}$$

For every magnetic field line passing through one of the magnetic cores, we have the Ampere equation

$$\oint Hdl = N_p i_{ptot} + i_z = N_p i_m \tag{2.4}$$

where i_m is the magnetization current reflected in the primary coil, i_z is the axial current through plasma, so for the primary current we can write

$$i_{ptot} = -\frac{i_z}{N_p} + i_m = i_p + i_m \tag{2.5}$$

only the magnetization current effectively create a magnetic field line that encircle both current loops (primary and secondary), the rest of the currents primary i_p and secondary(plasma) i_z are reflected currents and their magnetic fields cancel each other, the minus sign indicate exactly this cancellation. So for the reflected currents, neglecting the direction of flow we have

$$i_z = i_p N_p \tag{2.6}$$

with this we can also verify the conservation of reflected energy $u_s i_z = u_p i_p p$. A magnetic field line that do not encircle both current loops but only one is produced by the entire current that it encircle and do not contribute to the reflection of energy, but contribute to the uncoupled inductance $L_k + L_s$.

Any impedance from the secondary(plasma ring) is reflected in all the primary coils as

$$Z_{sp} = \frac{u_p}{i_p} = \frac{u_s}{i_z} \cdot \frac{N_p^2}{p} = Z_s \cdot \frac{N_p^2}{p} \tag{2.7}$$

because $Z = R + \mathrm{i}\omega L$ we can also write

$$L_{sp} = L_s \cdot \frac{N_p^2}{p} \tag{2.8}$$

$$R_{sp} = R_s \cdot \frac{N_p^2}{p} \tag{2.9}$$

The electrodynamic stabilization inside the conductive shell is illustrated in figure 2.3. In the normal position the azimuthal magnetic field is equal around the plasma and magnetic forces are canceling each other. When the plasma position is shifting toward one of the walls, the magnetic field on that side become stronger because it is repelled out from the conductive wall due to the induction of eddy currents. This will create a difference of forces that will push the plasma back into the normal position.

The penetration depth of a sinusoidal varying magnetic field in a conductive medium is limited in a layer close to the surface by the induction of eddy currents in the conductive medium

$$\delta = \frac{1}{\sqrt{\pi\mu\sigma f}} \tag{2.10}$$

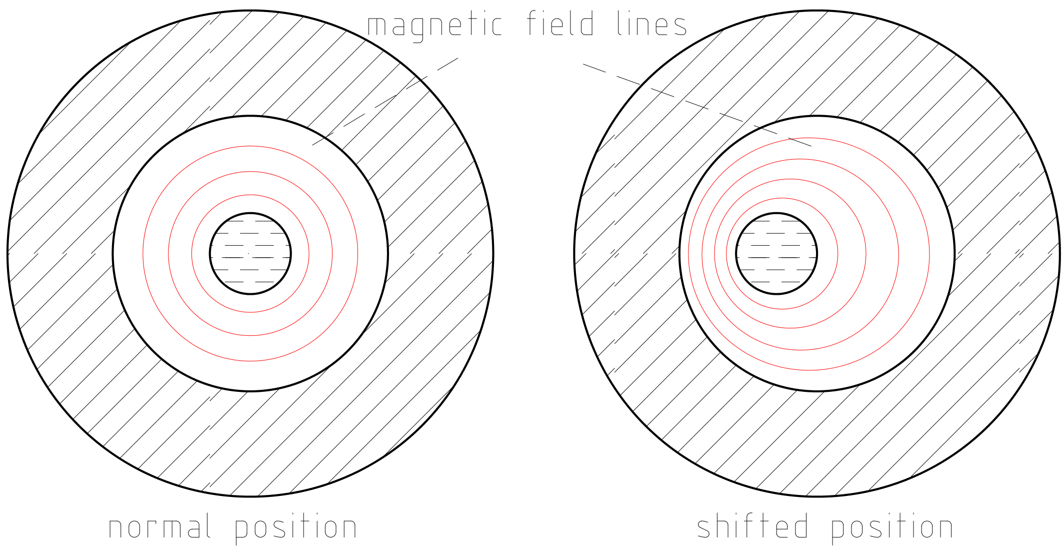

Figure 2.3: Plasma levitation inside the metalic shell

where δ is the penetration distance into conductive material where B decrease e times from the surface, σ is the material electrical conductivity, μ is the magnetic permeability, f is the frequency of oscillations of the magnetic field. The thickness of the metallic wall must be at least several times bigger than δ, the magnetic field at distance x inside the conductive medium metal or plasma will decrease exponentially

$$B_x = B_s \exp\left(-\frac{x}{\delta}\right) \tag{2.11}$$

where B_s is the field at the surface.

To avoid to short circuit the plasma ring in the presence of the induced axial electric field, the high conductivity metallic shell must be assembled from multiple tubular segments over the length of the torus.

These segments must be connected together with an insulation interface between them to prevent the circulation of a toroidal current. However the localized axial currents are able to circulate inside every segment and in this way to expel the azimuthal magnetic field from inside. Also they will allow the circulation of azimuthal induced currents by the axial magnetic field. Because these currents can be relatively high and also because of the plasma radiation, a significant amount of heat will be produced inside these segments and an active cooling will be necessary to keep them at a low enough temperature.

The confinement of plasma is done by both the azimuthal and the axial magnetic fields. Because of the additional stabilization effect that the metallic shell has it over the kink instability, the azimuthal magnetic pressure can be higher here than in a typical tokamak system. The axial magnetic field created by the external toroidal coil have the additional role to keep the plasma shape and stabilize its surface against some types of instabilities. This axial field while is created by a stationary current, is also “frozen" inside plasma and its value will change with the plasma radius.

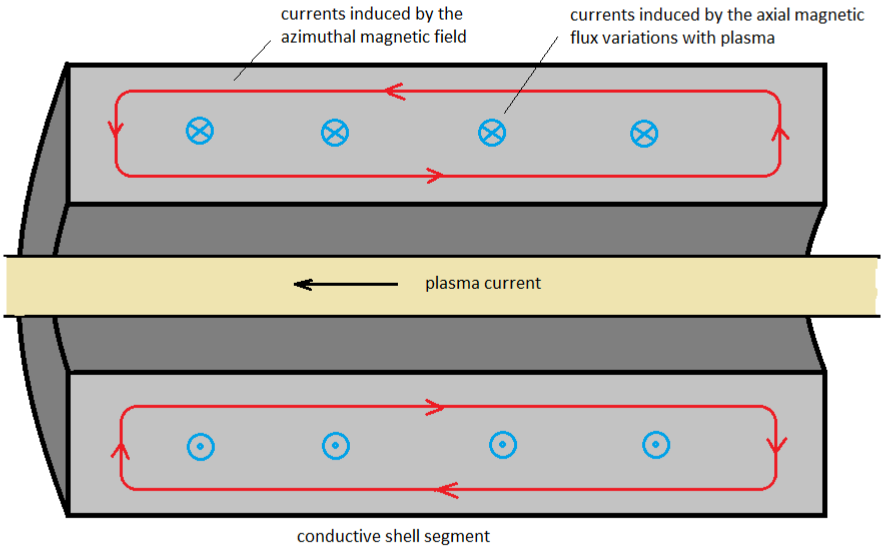

Figure 2.4: Section view through one segment of the conductive shell

2.2 The balance of plasma surface

The plasma is subjected to the action of two magnetic fields, the azimuthal field created by the current induced in the plasma by the primary coils, the axial field created by an external coil placed along the z axis of the torus. As a result we will have a magnetic pressure exerted on the plasma surface by these fields and the particles pressure from inside the plasma. In the case of a thermonuclear plasma with the usual particles density in the range $10^{20} \div 10^{22} m^{-3}$, at the driving frequencies no higher than 1000 Hz, the plasma mass will exert a negligible inertial pressure in comparison with the particles and magnetic pressures. In these conditions the influence of plasma mass, internal viscosity and tensions, over the movement and balance of the plasma surface can be neglected, in consequence the balance of the surface of plasma will be reduced at the balance of magnetic and particles pressures. The equation of motion is reduced at

$$\vec{j} \times \vec{B} - \nabla p = 0 \tag{2.12}$$

we also have $\mu_0 \vec{j} = \nabla \times \vec{B}$ and after some transformations result

$$\frac{1}{\mu_0} \left(\vec{B} \cdot \nabla \right) \vec{B} - \nabla \left(\frac{B^2}{2\mu_0} + p \right) = 0 \tag{2.13}$$

where p is the plasma particle pressure. The first term from left is the magnetic tension force density that reflect the change in the direction of magnetic field lines, in a symetric and stable configuration this can be neglected in this balance equation. The equation become

$$\frac{B^2}{2\mu_0} + p = const \tag{2.14}$$

Because at high temperature the plasma has a very good electrical conductivity and also because the axial current from the plasma and its azimuthal magnetic field change with a relatively high frequency, most of the axial current will flow in a relatively thin layer at the surface of plasma. In consequence the azimuthal magnetic field will be weak or null inside the plasma and will be present only in the exterior of plasma and in the transition layer close to the surface where most of the current is flowing. The thickness of this transition layer is comparable with the penetration distance of the magnetic field. Another effect of high conductivity of plasma is the behavior of the axial magnetic field, it will be frozen inside plasma during the relatively short time of the cycle. Here the cycle time is the time interval coresponding to a complete compression - expansion cycle, that is equal with the half of period of the driving current $t_c = 1/(2f_{drv})$. The thickness of the transition layer from the internal to the external axial magnetic field will be again comparable with the penetration distance. In consequence both magnetic pressures are built up inside the transition layer located at the plasma surface, being balanced by the particles pressure from the inside of plasma.

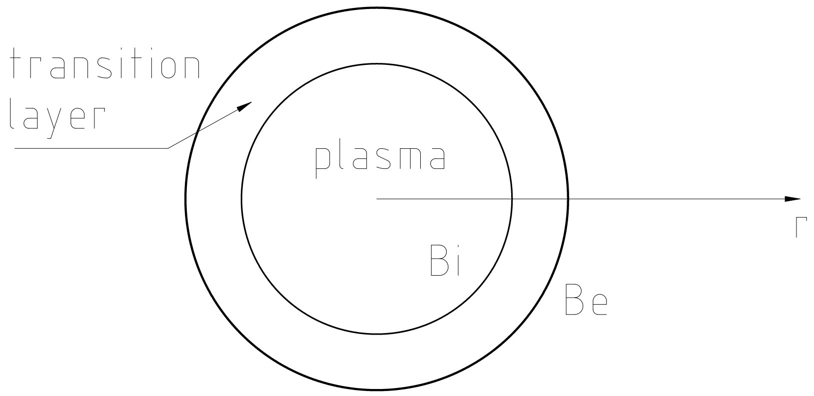

Figure 2.5: Magnetic pressure in the transition layer of plasma

In the vacuum surrounding the plasma the particles pressure is zero and the external magnetic field B_e is present and we have

$$\frac{B_e^2}{2\mu_0} = const$$

Inside the plasma the particles thermal pressure is no longer zero and the magnetic field B_i is lower, so the equation (2.14) become

$$p = \frac{1}{2\mu_0}\left(B_e^2 - B_i^2\right) \tag{2.15}$$

where the right hand term represent the magnetic pressure. Because the axial and azimuthal components of the magnetic field are perpendicular to each other, their individual magnetic pressures are additive and can be considered separately. The azimuthal magnetic pressure (produced by the azimuthal component of the magnetic field) and the axial magnetic pressure (produced by the axial component of the magnetic field).

The azimuthal magnetic field from the inside of plasma under the transition layer is weak and its contribution to the magnetic pressure will be neglected, the external field just on the external surface of plasma will be at its maximum value

$$B_{\varphi a} = \frac{\mu_0 i_z}{2\pi a} \tag{2.16}$$

where a is the radius of the plasma column. The azimuthal magnetic pressure become

$$p_{m\varphi} = \frac{B_{\varphi a}^2}{2\mu_0} = \frac{\mu_0 i_z^2}{8\pi^2 a^2} = \frac{\mu_0 i_z^2}{8\pi S_i} \tag{2.17}$$

this is the pressure below the inner limit of the transition layer, $S_i = \pi a^2$ is the radial section area of the plasma. In the transition layer the azimuthal pressure decrease progressively from its maximum value (on the inner limit of the layer) to zero (on the outer limit). In consequence some of the particles close to the outer surface of the plasma, being exposed to a low pressure and receiving thermal energy from the inner particles, will have the tendency to escape from plasma. Because of the magnetic fields the particles will move on a circular path, eventually returning back into plasma. Additionally the variable magnetic fields created by the primary coils and the variable azimuthal magnetic field will induce an electric field along the z axis. Inside plasma and at the plasma surface, the electric field induced by the primary coils will be balanced by the electric field induced by the variation of the azimuthal magnetic field of the plasma, the remaining field will compensate the voltage drop on the plasma resistivity. This axial electric field at the surface of plasma will always have the direction of the z current, as such the escaping particles will be accelerated by it, they will interact with the azimuthal magnetic field and the resultant Lorentz force will prevent the particles to leave the plasma surface. This is true at the plasma surface and very close to it, further away from the surface in the radial direction, the electric field induced by the primary coils is no longer fully balanced by the electric field induced by the azimuthal magnetic field variation. As a result far away from the plasma surface the electric field will be more intense and will have the same direction as the z current only during the plasma compression, during the expansion will have an opposite direction to the z current, so any charged particles here during the expansion stage will be pushed toward the tube walls and toward the plasma surface during the compression stage. This effect of the induced electric field can be used to inject charged particles into plasma.

The axial magnetic field is created from exterior by the toroidal coil using a constant current, so that in a stationary plasma it will be constant and weaker inside the plasma because of the plasma diamagnetism. The variations of the axial current will modulate corespondigly the azimuthal magnetic pressure and with it the plasma volume and the axial magnetic field. When the radial plasma section crossed by the axial magnetic field, change with the radius change, the inner magnetic flux will change and because of the plasma high conductivity, an azimuthal current will be induced that will try to keep the inner flux constant close to the average value for the cycles. This current will circulate in the transition layer close to the surface, and will produce the frozen effect of the axial magnetic field inside the plasma of high conductivity. A similar effect appear inside the confinement shell that is also highly conductive and its segments allow for an azimuthal current to close around it. So while the magnetic field created by the toroidal coil is constant outside the conductive shell, inside it and inside plasma this field will be changed by the change in the plasma radial section, but only if these changes evolve fast compared with penetration times. For the magnetic field inside the plasma, in a time frame much smaller than the penetration time, we have

$$B_{zi} S_i = B_1 S_1 = const \tag{2.18}$$

where S_i is the radial section of the plasma, B_1 is the average axial field inside the plasmal, S_1 is the plasma radial section coresponding to B_1 so that (2.18) hold. The average inner field $B_1 = B_0 - B_{dia}$ is smaller than the field produced by the toroidal coil because of plasma diamagnetism. The axial magnetic pressure will be

$$p_{mz} = \frac{1}{2\mu_0}\left(B_{ze}^2 - B_{zi}^2\right) \tag{2.19}$$

where B_{zi} is the axial field inside the plasma, B_{ze} is the field outside the plasma up to the confinement shell, B_0 is the axial field created by the toroidal coil. Because the magnetic flux inside the plasma column is constant, with the change of the plasma radius the area of the circular crown between the plasma outer radius and the radius of the shell inner wall will also change. This will produce a change in the magnetic flux passing through this circular crown and as so the entire flux passing through the inner section of the confinement shell will change. This will induce azimuthal currents around the conductive shell that will keep the flux constant.

$$B_{ze}S_e = B_0 S_2 = const \tag{2.20}$$

where $S_e = S_0 - S_i$ is the circular crown area, $S_2 = S_0 - S_1$, S_0 is the radial section delimited by the inner wall of the stabilization shell. During compression when $S_i < S_1$ the axial magnetic field inside the plasma will be higher than the average and the axial magnetic pressure will decrease and may be even reversed. During expansion when $S_i > S_1$ the axial magnetic field inside the plasma will be lower than the average and the axial magnetic pressure will increase and oppose further expansion. Replacing in (2.19) we have

$$p_{mz} = \frac{B_0^2}{2\mu_0} \cdot \frac{(S_0 - S_1)^2}{(S_0 - S_i)^2} - \frac{B_1^2}{2\mu_0} \cdot \frac{S_1^2}{S_i^2} \tag{2.21}$$

The equilibrium of pressures on the plasma surface is

$$p = n_i k T_i + n_e k T_e = p_{m\varphi} + p_{mz} \tag{2.22}$$

where the particles pressure is the sum of ions and electrons pressures.

If during the cyclic changes of the axial current we consider that the plasma will behave like an ideal gas composed from two particles ions and electrons, which will suffer a series of adiabatic transformations. This will determine a corresponding change of the plasma volume and its temperature. Then we have the equations

$$pV^{\gamma} = const \tag{2.23}$$

and

$$TV^{\gamma-1} = const \tag{2.24}$$

between plasma perticles pressure, volume and temperature.

$$\gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f} \tag{2.25}$$

is the adiabatic index, with f the number of degrees of freedom. Both ions and electrons have 3 degrees of freedom so they will have $\gamma = 5/3$. Assuming that p_1 is the plasma thermal pressure when $S_i = S_1$, also if the length of the plasma column remain approximatively constant during the cycle, the plasma thermal pressure will also be

$$p = p_1 \left(\frac{S_1}{S_i}\right)^{\gamma} \tag{2.26}$$

The pressure balance equation of plasma surface will be

$$\frac{\mu_0 i_z^2}{8\pi S_i} + \frac{B_0^2}{2\mu_0}\left(\frac{S_0 - S_1}{S_0 - S_i}\right)^2 - \frac{B_1^2}{2\mu_0}\left(\frac{S_1}{S_i}\right)^2 - p_1\left(\frac{S_1}{S_i}\right)^{\gamma} = 0 \tag{2.27}$$

here S_1 is the plasma inner section area were the average of B_{zi} is equal with B_1. From (2.18), if we average during the cycle time, we have

$$\frac{1}{t_c}\int_0^{t_c} B_{zi}dt = B_1 S_1 \cdot \frac{1}{t_c}\int_0^{t_c} \frac{1}{S_i}dt = B_1$$

$$\Rightarrow S_1 = \frac{1}{\frac{1}{t_c}\int_0^{t_c}\frac{1}{S_i}dt}$$

Once we have S_1 then p_1 can be calculated as the plasma particles thermal pressure at the coresponding section area.

2.3 Plasma polarization

One aspect that must be taken into consideration is the possibility that the plasma column can become electrically charged relative to the metallic shell, through exchange of charged particles. This will produce a radial electric field between the plasma surface and the tube walls, field that will produce attraction forces between the two and also will push the positive or negative particles (dependent of the field direction) toward the wall. Once the plasma column is out of its central position inside the tube and closer to a wall than to the other side wall, on the closer distance the radial electric field will become stronger and its forces will become unbalanced with the tendency to push the plasma into the tube wall if the electrical charge built in plasma become high enough.

To eliminate this problem we can use injectors of ions and electrons into plasma. Some injectors will accelerates positive ions of deuterium and tritium into plasma, the others will accelerates electrons into plasma. The amount of ions and electrons injected will be controlled by a system that measure the intensity and direction of the radial electric field and take action to keep it to a minimum. The problem with this approach is that the magnetic field will prevent the injection of charged particles into plasma, the injected particles will rather rotate around field lines somewere outside plasma. However in the case of this system the situation is a little different because of the presence of the induced axial electric field. This induced electric field is particularly strong outside the plasma. If the injection take place only during the compression time intervals, the induced electric field will accelerate the injected particles in such a direction that the azimuthal magnetic field from around the plasma will push them into plasma. In addition to this the $E \times B$ drift in this case will also push the particles into plasma. The injection should stop during the expansion time intervals when these phenomena are not favorable and any particles that are far enough from the plasma surface, will be pushed toward the wall.

In the same time this injection system ca be used to supply the plasma with new nuclei of deuterium and tritium to replace the ones that had been converted to helium, also can play a role in the initial formation and heating of the plasma.

2.4 Sausage instability

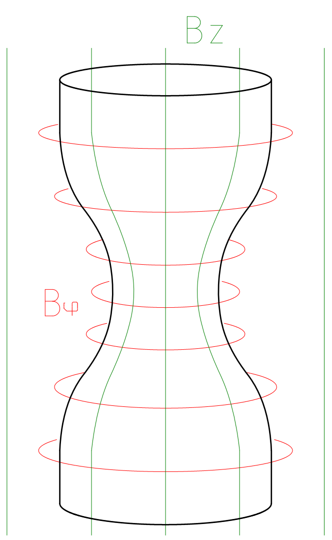

Figure 2.6: Local sausage deformation

The sausage instability (figure 2.6) appear as a local reduction in the radius of the plasma column, that in turn lead to the local increase of the azimuthal magnetic field and its pressure which trigger a further propagation of the deformation. It is stabilized by the axial magnetic field. The inner axial field is pinched with the plasma and this lead to its increase and the increase of the inner axial pressure. From equations (2.17) and (2.21) we can observe that while the axial current remain constant, the azimuthal magnetic pressure will change slower with the inner section area of the plasma column than the axial magnetic pressure, so if the axial magnetic field from inside the plasma is strong enough the deformation will be stable. For very small deformations and considering the worst case when only the inner axial field is modified by the deformation, the condition of stability is

$$B_{zi}^2 > \frac{B_{\varphi a}^2}{2} \tag{2.28}$$

When the plasma is at its maximum radius the axial current is zero and the particle pressure is balanced only by the axial pressure which now is at its maximum. Here the plasma is stable because the azimuthal field is zero. When the plasma is at its minimum radius the azimuthal magnetic field will be maxim and this is the point of maximum instability, so if the condition (2.28) is satisfied for $a = a_{min}$ then the plasma will be stable for the entire cycle.

2.5 Kink instability

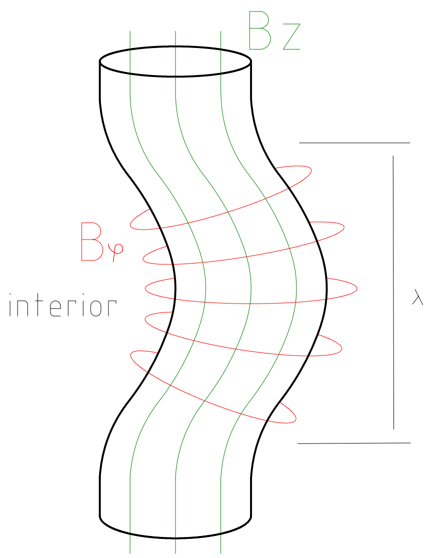

Figure 2.7: Local kink deformation

The kink instability (figure 2.7) appear as a local bending of the plasma column, that in turn lead to the increase of the azimuthal field on the interior of bending compared to the field on the exterior of it, the difference in azimuthal magnetic pressures will push the deformation even further. It is stabilized by the conductive shell in combination with the axial magnetic field from inside the plasma column. The conductive shell will change the value of azimuthal magnetic field around the deformation (see figure 2.3) until the difference in azimuthal magnetic field will be eliminated and the deformation will no longer progress. The inner axial magnetic field lines will follow the bend of the plasma column by the induction of additional currents in plasma at the area of deformation. The tension forces of the curved field lines will act as a restoring force and push the deformation back. In this case, both the conductive shell and the axial magnetic field have an active role in stopping the deformation, the conductive shell is more effective for deformations with large λ, while the axial field for the ones with a small λ. The inner axial field will stabilize a kink deformation if

$$\frac{B_{zi}^2}{B_{\varphi a}^2} > \ln\left(\frac{\lambda}{a}\right) \tag{2.29}$$

for worst case when $a = a_{min}$. The conductive shell can effectively stabilize a kink deformation if λ is much bigger than the shell inner radius r_0, that is for $\lambda > 10 \cdot r_0$ the kink deformation is stabilized by the shell alone. In consequence the axial field must provide stability just up to this, which is much smaller than in a classic tokamak, as a consequence in this case the control of kink instability is more easily achieved. Also considerable higher induced current in plasma can be used.

2.6 Interchange instability

In areas with “bad" magnetic field lines curvature, that is with the field lines concave toward the plasma, this is on the exterior of the toroidal curvature, the interchange instability can develop. The plasma surface will develop flutes along the combined magnetic field lines. However in this case if the cycle time is too small for a significant evolution of the flutes and also because of the skin effect of the plasma current, the deformation will be suppressed. This happens because the direction of the azimuthal field and the direction of rotation of the helicoidal combined field will change from one cycle to the next due to the change in the direction of axial current. The surface of plasma will be wiped by the combined magnetic field lines when they change direction of rotation from left to right and vice versa with the sinusoidal variations of the axial current. This will prevent or reduce the dvelopment of any instabilities that depend on the direction of rotation of the combined field and are slower than the cycle time. In addition to this the plasma axial current will be pushed to the plasma surface during the cycle time. The azimuthal magnetic field lines has the tendency to follow the shortest path that is around the top of the flutes with much lower values between the flutes. In consequence the magnetic force will push the top of the flutes back into plasma. This stabilizing effect will become progresively less effective while the current is allowed to penetrate into plasma, this penetration will not happen in this configuration, but happen in the case of classic tokamak.

2.7 Resistive instabilities

In classic tokamaks the penetration of the current into the plasma with a strong current density in the center of plasma will have an increased tendency to trigger tearing instabilities. In our case the current will be expelled to the plasma surface and just a weak current density will exist inside plasma, this will reduce the posibility to create disruptive tearing instabilities in plasma.

3 Parameters of fully ionized plasma

These parameters are valid in the condition that all charged particles have a Maxwell-Boltzmann distribution.

3.1 Debye length and plasma frequency

We consider the presence in plasma of electrons, ions of hydrogen and ions with Z_a, the Debye length is

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k}{q_e^2 \left(\frac{n_e}{T_e} + \frac{n_i}{T_i} + \frac{Z_a^2 n_a}{T_a}\right)}} \tag{3.1}$$

where k is the Boltzmann constant and n,T is the coresponding particle density and temperature. If we take $n_a \simeq 0$, electrons and ions temperatures equal $T_e = T_i = T$ the Debye length become

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k T}{q_e^2 (n_e + n_i)}} \tag{3.2}$$

Because the mass of the ions is much higher than the mass of electrons, the plasma frequency can be approximate as

$$\omega_p = \sqrt{\frac{q_e^2 n_e}{\varepsilon_0 m_e}} \tag{3.3}$$

Plasma absorption of electromagnetic radiation is influenced by the plasma frequency, the plasma is opaque for radiation with frequency $\omega \ll \omega_p$ and is transparent for $\omega \gg \omega_p$.

3.2 Coulomb collisions

Because is fully ionized the particles from plasma will exchange momentum and energy between them through Coulomb collisions. The momentum transfer collision frequency, which is approximately equal with the inverse of maxwellization time, [4, 8, 9] for electron-ions collisions

$$\nu_{ei} = \frac{1}{\tau_{ei}} = \frac{\sqrt{2\pi} Z_i^2 q_e^4 n_i \ln \Lambda_{ei}}{12\pi^2 \varepsilon_0^2 \sqrt{m_e} (kT_e)^{3/2}} \tag{3.4}$$

for electron-electrons collisions

$$\nu_{ee} = \frac{1}{\tau_{ee}} = \frac{\sqrt{\pi} q_e^4 n_e \ln \Lambda_{ee}}{12\pi^2 \varepsilon_0^2 \sqrt{m_e} (kT_e)^{3/2}} \tag{3.5}$$

for ion-ions collisions

$$\nu_{ii} = \frac{1}{\tau_{ii}} = \frac{\sqrt{\pi} Z_i^4 q_e^4 n_i \ln \Lambda_{ii}}{12\pi^2 \varepsilon_0^2 \sqrt{m_i} (kT_i)^{3/2}} \tag{3.6}$$

for ion-electrons collisions

$$\nu_{ie} = \frac{1}{\tau_{ie}} = \frac{\sqrt{2\pi} Z_i^2 q_e^4 n_e \ln \Lambda_{ie}}{12\pi^2 \varepsilon_0^2 \sqrt{m_i} (kT_e)^{3/2}} \tag{3.7}$$

where $\ln \Lambda$ is Coulomb logarithm between b_{min} and $b_{max} = \lambda_D$ of impact parameter. For an electron temperature above 1 MK and ion velocity much smaller than electron thermal velocity

$$v_i \ll v_{Te} = \sqrt{\frac{3kT_e}{m_e}} \tag{3.8}$$

we have the condition $\hbar = m_e v_{Te} b_{min}$, Coulomb logaritm for both electron-ion and ion-electron collisions become

$$\ln \Lambda_{ei} = \ln \Lambda_{ie} = \ln \left(\sqrt{\frac{3}{2}} \cdot \frac{kT_e}{\hbar \omega_p} \right) \tag{3.9}$$

For other collisions the situation can be more complicated and some convenient formula can be found in literature [4, 5]. Total collision frequencies will be $\nu_e = \nu_{ei} + \nu_{ee}$ respectively $\nu_i = \nu_{ie} + \nu_{ii}$.

3.3 Electrical conductivity

If we neglect the contribution of ions to conductivity, the current density in plasma will be

$$j = n_e q_e v_d \tag{3.10}$$

where v_d is the electrons drift velocity. The classic Spitzer electrical conductivity of plasma is

$$\sigma_e = \gamma_E \cdot \frac{64\sqrt{2\pi}\varepsilon_0^2 (kT_e)^{3/2}}{Z_i q_e^2 \sqrt{m_e} \ln \Lambda_{ei}} \tag{3.11}$$

where $\gamma_E = 0.582$ for $Z_i = 1$ [4, 15]. This formula of conductivity remain valid as long as $v_d \ll v_{Te}$. When these velocities become comparable the drift movement start to contribute to the electron kinetic energy and the conductivity start to increase with the current density.

If the electrical field inside the plasma will exceed the value of Dreicer field, then the electrons become runaway electrons. For $v_d \ll v_{Te}$ the Dreicer field is

$$E_D = \frac{n_e q_e^3 \ln \Lambda_e}{12\pi\varepsilon_0^2 kT_e} \tag{3.12}$$

In the presence of a magnetic field inside plasma, the electrical conductivity along the magnetic field lines will be unmodified $\sigma_{\|} = \sigma_e$, but the conductivity perpendicular to the field lines will be reduced

$$\sigma_{\perp} = \frac{\sigma_{\|}}{1 + \frac{\omega_{ce}^2}{\nu_e^2}} \tag{3.13}$$

where ω_{ce} is the cyclotron frequency of electrons. The transverse conductivity remain reduced as long as the circular movement of particles around the field lines remain undisturbed. At the plasma surface the transverse conductivity will tend to increase toward the normal parallel conductivity.

3.4 Thermal bremsstrahlung

This radiation is produced by the coulomb collisions and braking of charged particles in plasma. The emitted spectra is continuous up to a photon energy comparable with the average thermal energy of electrons

$$\nu_{max} = \frac{kT_e}{h} \tag{3.14}$$

The plasma has a good transparency for the frequencies of this radiation that are higher than plasma frequency, so most of the power emitted will escape from the volume of plasma to the walls of the metallic shell. The total radiated power over all frequencies per unit volume will be

$$\frac{dP_{br}}{dV} = C_2 \left(n_i + Z_a^2 n_a\right) n_e \sqrt{T_e} \tag{3.15}$$

where $C_2 = 1.92 \cdot 10^{-40}$ [SI].

3.5 Cyclotron motion and radiation

In the presence of a magnetic field B in plasma, the charged particles ions and electrons will start to rotate around the magnetic field lines. If we neglect the collisions, we have

$$\frac{mv^2}{r_c} = qvB \tag{3.16}$$

Because of the rotation around the magnetic field lines the parallel velocity is not contributing, the average thermal velocity of rotation will be

$$v = \sqrt{\frac{2kT}{m}} \tag{3.17}$$

For electrons the gyroradius

$$r_{ce} = \frac{\sqrt{2m_e kT_e}}{q_e B} \tag{3.18}$$

from $v = \omega r$ the electrons cyclotron frequency

$$\omega_{ce} = \frac{q_e B}{m_e} \tag{3.19}$$

And similar for ions

$$r_{ci} = \frac{\sqrt{2m_i kT_i}}{Z_i q_e B} \tag{3.20}$$

and

$$\omega_{ci} = \frac{Z_i q_e B}{m_i} \tag{3.21}$$

This rotation being accelerated movement will produce the emission of cyclotronic radiation with the corresponding frequency. The power radiated by the electrons per unit volume is

$$\frac{dP_{cy}}{dV} = \frac{n_e \sigma_t B^2 2kT_e}{c\mu_0 m_e} \tag{3.22}$$

where $\sigma_t = 6.65 \cdot 10^{-29}\ \left[\mathrm{m}^2\right]$ is the Thomson cross section for electrons and c the speed of light. The power radiated by ions is much smaller. The metallic shell will reflect a good part of this radiation back into plasma, reducing the power loss through cyclotronic radiation.

3.6 Plasma diamagnetism

Here we are interested in the diamagnetism of plasma in relation with the axial magnetic field that is approximately uniform. The ions and electrons rotate around field lines and are forming small current loops, every loop will have a cyclotronic magnetic moment that is opposing to the external field.

$$m_{cy} = i_c \pi r_c^2 = \frac{q_e}{t_{cy}} \cdot \pi \cdot \frac{2mkT}{q_e^2 B_0^2} \tag{3.23}$$

where the cyclotronic time per revolution is

$$t_{cy} = \frac{2\pi}{\omega_c} = \frac{2\pi m}{q_e B_0} \tag{3.24}$$

so the magnetic moment of one particle is

$$m_{cy} = \frac{kT}{B_0} \tag{3.25}$$

which is independent of particle mass. The total magnetic moment due to all cyclotronic loops inside plasma volume is

$$M_{cy} = \frac{nkT}{B_0} \cdot V \tag{3.26}$$

In addition to this we must consider the drift current from the transition layer at the plasma surface. We can have two possible sources of drift here, $F \times B$ drift is one and the other is the drift caused by the radial gradient of magnetic field. In a plasma whose surface is in balance the forces are canceling each other so the force drift is zero. The diamagnetism itself will create a difference between inner B_1 and outer B_0 magnetic fields. This drift current will create a magnetic moment that will oppose to the cyclotronic loops magnetic moment, reducing the diamagnetic effect. The drift velocity is

$$v_{db} = \frac{v_\perp r_c}{2B_0} \cdot \frac{\partial B}{\partial r} \tag{3.27}$$

where $v_\perp$ is given by (3.17). If the transition layer δ is small then the radial gradient of magnetic field can be approximate

$$\frac{\partial B}{\partial r} \simeq \frac{B_0 - B_1}{\delta} \tag{3.28}$$

the drift velocity gets

$$v_{db} = \frac{\sqrt{\frac{2kT}{m}} \cdot \frac{\sqrt{2mkT}}{q_e B_0}}{2B_0} \cdot \frac{B_0 - B_1}{\delta} = \frac{kT}{q_e B_0^2} \cdot \frac{B_0 - B_1}{\delta} \tag{3.29}$$

The magnetic moment of this drift will be

$$M_{db} = -I_{db} S_i = -q_e n v_{db} \cdot l_p \delta \cdot S_i = -q_e n v_{db} \delta \cdot V \tag{3.30}$$

which like the cyclotronic moment is independent of the particles mass, $l_p S_i$ are plasma length and inner radial section area. The total diamagnetic moment of plasma is the sum of cyclotronic and drift magnetic moments

$$M_{dia} = \frac{nkT}{B_0} \cdot V - \frac{nkT\,(B_0 - B_1)}{B_0^2} \cdot V = \frac{1}{\mu_0} B_{dia} \cdot V \tag{3.31}$$

$$B_{dia} = \frac{\mu_0 nkT}{B_0} \left(1 - \frac{B_0 - B_1}{B_0}\right) = \frac{\mu_0 nkT B_1}{B_0^2} \tag{3.32}$$

This is the diamagnetic field of plasma if collisions are negligible. The collisions will perturb the cyclotronic rotation and will reduce the diamagnetic effect. The situation is similar with the case of transverse conductivity so we exepect something similar with $1 + \omega_c^2 \tau_{col}^2$ but decreasing while collision frequency increase. Considering the effect of both the ions and the electrons, $n_i = n_e$ and includind a collision factor, we have

$$B_{dia} = \frac{\mu_0 nk B_1}{B_0^2} \left(\frac{T_e}{1 + \frac{\nu_e^2}{\omega_{ce}^2}} + \frac{T_i}{1 + \frac{\nu_i^2}{\omega_{ci}^2}} \right) \tag{3.33}$$

From

$$B_0 = B_1 + B_{dia} = B_1 \left[1 + \frac{\mu_0 nk}{B_0^2} \left(\frac{T_e}{1 + \frac{\nu_e^2}{\omega_{ce}^2}} + \frac{T_i}{1 + \frac{\nu_i^2}{\omega_{ci}^2}} \right) \right] \tag{3.34}$$

result

$$B_1 = \frac{B_0^3}{B_0^2 + \mu_0 nk \left(\frac{T_e}{1+\frac{\nu_e^2}{\omega_{ce}^2}} + \frac{T_i}{1+\frac{\nu_i^2}{\omega_{ci}^2}} \right)} \tag{3.35}$$

or if $\nu_{col} \ll \omega_c$ it gets

$$B_1 = \frac{B_0^3}{B_0^2 + \mu_0 nk\,(T_e + T_i)} \tag{3.36}$$

3.7 Thermalization of fast charged particles

We have fast particles with Z_a mass m_a and energy ϵ released into plasma. These particles will transfer energy to both ions and electrons, so we have two thermalization processes [4]. On ions if

$$v_a = \sqrt{\frac{2\epsilon}{m_a}} \gg \sqrt{\frac{3kT_i}{m_i}} \tag{3.37}$$

then the thermalization time can be approximate with

$$\tau_{ai} = \frac{8\pi\varepsilon_0^2 m_i \epsilon^{3/2}\sqrt{2m_a}}{(m_i + m_a)\, Z_a^2 q_e^4 n_i \ln\Lambda_{ii}} \tag{3.38}$$

On electrons if

$$v_a \ll \sqrt{\frac{3kT_e}{m_e}} \tag{3.39}$$

in this case the thermalization time can be approximate with

$$\tau_{ae} = \frac{6\pi^2\varepsilon_0^2 m_a \sqrt{2\pi}\,(kT_e)^{3/2}}{\sqrt{m_e} Z_a^2 q_e^4 n_e \ln\Lambda_{ee}} \tag{3.40}$$

The temperature equilibration between ions and electrons, $n_i = n_e$ and maxwellian distribution

$$\tau_{eq} = \frac{3\pi\varepsilon_0^2\sqrt{2\pi} m_e m_i}{n q_e^4 \ln\Lambda_{qe}} \cdot \left(\frac{kT_e}{m_e} + \frac{kT_i}{m_i}\right)^{3/2} \tag{3.41}$$

4 Energy balance

In a thermonuclear plasma of high temperature, the amount of energy produced by fusions reactions is proportional with plasma density, temperature and the type of reaction involved. To have a positive energy output, this fusion energy must exceed the energy lost through radiation, energy disipated by AC driving and any additional energy loss in the system. Under the influence of thermal movement the nuclei from plasma will suffer repeated coulomb collisions between them, if in such a collision the two nuclei have enough thermal energy to overcome the electric repulsive force and are getting close enough a process of nuclear fusion will happen. This define a limit cross section for the approaching nuclei, inside which a nuclear fusion reaction will happen. If we consider one nucleus, the number of reactions per second is dependent by the number of other nuclei it meet inside the cross volume defined by the cross section moving with the thermal velocity $\sigma_r v$. Because the reaction cross section is dependent on the approaching velocity of the two nuclei, the cross volume per second must be integrated over velocity distribution to obtain the reactivity or average cross volume per second

$$\langle\sigma_r v\rangle = \int_0^\infty \sigma_r v f(v) dv \tag{4.1}$$

In the case of thermonuclear reactions a maxwellian distribution for velocity end energy is considered. In the temperature range of 15...390 MK (2...50 keV) the reactivity for deuterium-tritium D-T reaction is 40 to 100 times higher than for D-D or T-T reactions [5]. This indicates that the D-T reaction is the most favorable in this temperature interval. Also indicates that in a D-T plasma the D-D and T-T reactions are happening at a very small rate compared with the D-T reactions which will be dominant. In table 1 are presented the reactivity at various temperatures and corresponding thermal energy $3/2kT$.

keV (MK)	D-T	D-D (1+2)	T-T
1 (7.7)	$5.48 \cdot 10^{-27}$	$1.52 \cdot 10^{-28}$	$3.28 \cdot 10^{-28}$
2 (15)	$2.62 \cdot 10^{-25}$	$5.42 \cdot 10^{-27}$	$7.09 \cdot 10^{-27}$
3 (23)	$1.71 \cdot 10^{-24}$	$2.95 \cdot 10^{-26}$	$3.03 \cdot 10^{-26}$
4 (31)	$5.58 \cdot 10^{-24}$	$8.46 \cdot 10^{-26}$	$7.46 \cdot 10^{-26}$
5 (39)	$1.28 \cdot 10^{-23}$	$1.77 \cdot 10^{-25}$	$1.4 \cdot 10^{-25}$
6 (46)	$2.42 \cdot 10^{-23}$	$3.09 \cdot 10^{-25}$	$2.26 \cdot 10^{-25}$
7 (54)	$3.98 \cdot 10^{-23}$	$4.81 \cdot 10^{-25}$	$3.29 \cdot 10^{-25}$
8 (62)	$5.94 \cdot 10^{-23}$	$6.89 \cdot 10^{-25}$	$4.47 \cdot 10^{-25}$
9 (70)	$8.26 \cdot 10^{-23}$	$9.32 \cdot 10^{-25}$	$5.79 \cdot 10^{-25}$
10 (77)	$1.09 \cdot 10^{-22}$	$1.21 \cdot 10^{-24}$	$7.22 \cdot 10^{-25}$
15 (116)	$2.65 \cdot 10^{-22}$	$2.96 \cdot 10^{-24}$	$1.56 \cdot 10^{-24}$
20 (155)	$4.24 \cdot 10^{-22}$	$5.16 \cdot 10^{-24}$	$2.51 \cdot 10^{-24}$
25 (193)	$5.59 \cdot 10^{-22}$	$7.6 \cdot 10^{-24}$	$3.51 \cdot 10^{-24}$
30 (232)	$6.65 \cdot 10^{-22}$	$1.02 \cdot 10^{-23}$	$4.54 \cdot 10^{-24}$
35 (270)	$7.45 \cdot 10^{-22}$	$1.28 \cdot 10^{-23}$	$5.57 \cdot 10^{-24}$
40 (309)	$8.02 \cdot 10^{-22}$	$1.54 \cdot 10^{-23}$	$6.6 \cdot 10^{-24}$
45 (348)	$8.43 \cdot 10^{-22}$	$1.81 \cdot 10^{-23}$	$7.63 \cdot 10^{-24}$
50 (387)	$8.7 \cdot 10^{-22}$	$2.08 \cdot 10^{-23}$	$8.65 \cdot 10^{-24}$

Table 1: Reactivity [m^3/s] (source [6])

For energies up to 25 keV (190 MK) the D-T reactivity in [m^3/s] can be approximate with

$$\langle \sigma_{dt} v \rangle = \frac{3.68 \cdot 10^{-18}}{\epsilon_T^{2/3}} \cdot \exp\left(-\frac{19.94}{\epsilon_T^{1/3}}\right) \tag{4.2}$$

were ϵ_T is the ions thermal energy in [keV]. In an unit volume per second, for every deuterium with density n_d we will have a number of reactions with tritium nuclei with density n_t encountered inside the average cross volume. The fusion power released per unit volume will be

$$\frac{dP_{dt}}{dV} = \langle \sigma_{dt} v \rangle \, n_d n_t \epsilon_{dt} \tag{4.3}$$

where ϵ_{dt} is the amount of energy released per one D-T fusion process.

One deuterium and one tritium nucleus will undergo the fusion reaction

$$\mathrm{D}^2 + \mathrm{T}^3 \longrightarrow \mathrm{He}^4 + \mathrm{n} + 17.6\mathrm{MeV} \tag{4.4}$$

that produce a helium-4 nucleus and a neutron and will release 17.6 MeV of energy. This energy is distributed as kinetic energy, 3.5 MeV on the He nucleus and 14.1 MeV on the neutron. The helium nucleus will suffer Coulomb collisions with the plasma ions and electrons and some of its kinetic energy will be transferred to plasma. The neutron will interact very little with plasma, will pass through the metallic shell and will transfer most of its energy in the moderator placed around the metallic shell. This energy can be converted then into electricity. The slowed down neutron can be captured by a lithium-6 isotope to regenerate back the tritium used, through the reaction

$$\mathrm{Li}^6 + \mathrm{n} \longrightarrow \mathrm{He}^4 + \mathrm{T}^3 + 4.8\mathrm{MeV} \tag{4.5}$$

that also release additional 4.8 MeV of energy. In this way the tritium will be recirculated and the nuclear fuel actually used is deuterium and lithium-6, producing a total of 22.4 MeV of energy on every pair of reactions. Not all neutrons resulted from the reaction 4.4 will be captured by lithium-6, some of them will be lost. To keep producing enough tritium some neutrons multiplication must be employed. If some lithium-7 is introduced in the moderator, then it can react with fast neutrons as follows

$$\mathrm{Li}^7 + \mathrm{n} - 2.47\mathrm{MeV} \longrightarrow \mathrm{He}^4 + \mathrm{T}^3 + \mathrm{n} \tag{4.6}$$

reaction that consume some energy from the incoming neutron, produce one tritium nucleus and also a new neutron that can enter into a lithium-6 reaction producing an additional tritium. In addition to this a neutrons multiplier and reflector like beryllium can be used.

The alpha heat released in plasma by the helium nuclei, together with the Joule heat, count for the compensation of the plasma loses through radiation and other ways, while the rest of energy is converted into electricity with a limited efficiency of about 40%. The conversion of plasma radiation loses into electricity is limited by the temperature of the metallic shell used for confinement and stabilization. This layer electrical conductivity decrease with the increase of its temperature, also will increase its emission of impurities into plasma. As a consequence is preferable to keep this layer at a low temperature. In any case the energy released into plasma as alpha heat and as Joule heat from the axial current, cannot be higher than the total loses from plasma otherwise the plasma will overheat and may expand too much. So we can say that its maximum energy production is reached when alpha heat and Joule heat equal the plasma loses. Bellow this point the plasma need additional heating, above this point it need additional cooling. The level of energy can be controlled through the amplitude of the axial current that influence both the Joule heat and the compression, and also through the axial magnetic field that will influence the compression and the cyclotronic radiation.

5 Conclusion

The thermonuclear plasma can be effectively confined by using a magnetic expulsion wall in the form of a conductive shell in combination with an alternating axial current to create a permanent variable magnetic field so that the expulsion effect become permanent. This is contrary to the typical tokamak systems where the plasma current flow in the same direction and after the initial dynamic evolution ends and become stationary, the plasma lose the stability and the confinement. In addition an alternating current always keep changing the conditions from plasma, like the magnetic field direction, the radius and temperature, the behavior of the axial magnetic field, preventing in this way the development of instabilities. So rather than being just a stationary magnetic confinement system, in this case is an electrodynamic one, where the plasma column is kept inside a conductive tube in a way similar to the electrodynamic levitation. Also the system allow for a continuous run of the reactor.

References

[1] Samuel Glasstone, Ralph H. Lovberg, “Controlled Thermonuclear Reactions", Robert E. Krieger Publishing Company, reprint 1975

[2] L. Spitzer Jr., “Physics of Fully Ionized Gases", Interscience Publishers, New York, 1962

[3] Richard Fitzpatrick, “Introduction to Plasma Physics", 1998

[4] Andre Anders, “A Formulary for Plasma Physics", Berlin, Akademie-Verlag, 1990

[5] NRL Plasma Formulary, Naval Research Laboratory, Washington DC, 2018

[6] Miley G.H., Towner H., Ivich N., “Fusion Cross Sections and Reactivities", United States, 1974, Web. doi:10.2172/4014032

[7] Xing Z. Li, Qing M. Wei, Bin Liu, “A new simple formula for fusion cross-sections of light nuclei", Nucl. Fusion 48 (2008) 125003, doi:10.1088/0029-5515/48/12/125003

[8] J.D. Callen, “Fundamentals of Plasma Physics", draft 2006

[9] K. Miyamoto, “Plasma Physics for Nuclear Fusion", third edition

[10] D.J. Rose, M. Clark Jr., “Plasma Physics and Controlled Fusion"

[11] John Wesson, “Tokamaks", 4th edition, 2012, Contemporary Physics 53(5):450-451, doi:10.1080/00107514.2012.720285

[12] B.A. Trubnikov, “Particle Interactions in a Fully Ionized Plasma", Reviews of Plasma Physics vol.1, 1965

[13] K.A. Gunther, R. Radtke, “Electrical Properties of Weakly Nonideal Plasmas", 1984

[14] Cohen Robert S., Spitzer Jr. Lyman, McR. Routly Paul, 1950, "The Electrical Conductivity of an Ionized Gas", Physical Review. 80 (2): 230–238. doi:10.1103/PhysRev.80.230

[15] Spitzer Jr. Lyman, Harm Richard, 1953, "Transport Phenomena in a completely ionized gas", Physical Review. 89 (5): 977–981. doi:10.1103/PhysRev.89.977