

**On the possible mathematical connections between some equations of various topics concerning the D-Branes and some sectors of Number Theory (Rogers-Ramanujan continued fractions and mock theta functions).**

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**Abstract**

*In this research thesis, we have described some new mathematical connections between some equations of various topics concerning the D-Branes and some sectors of Number Theory (Rogers-Ramanujan continued fractions and mock theta functions).*

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<https://www.news.ucsb.edu/2012/013423/ucsb-physicist-receives-international-acclaim-his-work-theoretical-physics>



*Joseph Polchinski (1954-2018)*

From:

**Open String Tachyon in Supergravity Solution**  
*Shinpei Kobayashi, Tsuguhiko Asakawa and So Matsuura*  
arXiv:hep-th/0409044v4 13 Jun 2005

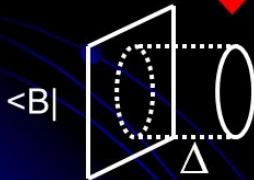
We have, from: “Gaussian Brane and Open String Tachyon Condensation”

e.g.) asymptotic behavior of  $\Phi$  of black p-brane

$$e^{-2\phi} = e^{-2\sqrt{2}\kappa\hat{\phi}} = f_p(r)^{-\sqrt{2}\kappa\frac{p-3}{2}},$$

$$\Rightarrow \hat{\phi}(r) = \frac{3-p}{2\sqrt{2}} \frac{T_p}{(7-p)\Omega_{8-p} r^{7-p}} - \frac{3-p}{2\sqrt{2}} \kappa \left( \frac{T_p}{(7-p)\Omega_{8-p} r^{7-p}} \right)^2 + \dots$$

leading term at infinity  
↔  
coincident



$$|\varphi\rangle = \langle \text{massless field} | \text{propagator} | \text{source} \rangle$$

$$= \epsilon_{MN}^{\langle\phi\rangle} \langle 0; k | \alpha_1^M \alpha_1^N D | B \rangle = \frac{3-p}{2\sqrt{2}} T_p V_{p+1} \frac{1}{k_i^2}$$

We can reproduce the leading term of a black p-brane solution (asymptotic behavior) via the boundary state.

From:

J. Polchinski, *String Theory Vol. I: An Introduction to the Bosonic String*

J. Polchinski, *String Theory Vol. II: Superstring Theory and Beyond*

1. The gravitational scale  $m_{\text{grav}} = \kappa^{-1} = 2.4 \times 10^{18}$  GeV, at which quantum gravitational effects become important; this is somewhat more useful than the Planck mass, which is a factor of  $(8\pi)^{1/2}$  greater.

$$\kappa^{-1} = 2.4 \times 10^{18}; \quad \kappa = 4.1666\dots \times 10^{-19}$$

From:

### Modular equations and approximations to $\pi$

Srinivasa Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

$$e^{\wedge}(-\text{Pi}^*\text{sqrt}(22)) = (((2508928 - e^{\wedge}(\text{Pi}(\text{sqrt}(22)) + 24))/4372$$

$$3.985727908284188 \times 10^{-7}$$

$$64(((1+\text{sqrt}(2))^{\wedge}12+(1-\text{sqrt}(2))^{\wedge}12))) = 2508928$$

For  $p = 6$  and  $\hat{\phi} = 1.5$ , from

$$e^{-2\phi} = e^{-2\sqrt{2}\kappa\hat{\phi}} = f_p(r)^{-\sqrt{2}\kappa\frac{p-3}{2}},$$

$$e^{-2\phi} = e^{-2\sqrt{2}\kappa\hat{\phi}} = f_p(r)^{-\sqrt{2}\kappa\frac{p-3}{2}}$$

we obtain:

$$\exp((-2*\text{sqrt}(2)*1.5* (((3.64993*(3.98573 \times 10^{-7})^{\wedge}(2.96))))$$

$$\text{where } (((3.64993*(3.98573 \times 10^{-7})^{\wedge}(2.96)))) = 4.16666 \times 10^{-19} = \kappa$$

### Input interpretation:

$$\exp\left(-2\sqrt{2} \times 1.5 \left(3.64993 (3.98573 \times 10^{-7})^{2.96}\right)\right)$$

### Result:

$$0.9999999999999998232236756350282896640258606643401914613\dots$$

Or:

$$\exp((-2*\text{sqrt}(2)*1.5*4.16666 \times 10^{-19}))$$

**Input interpretation:**

$$\exp\left(-2\sqrt{2} \times 1.5 \times 4.16666 \times 10^{-19}\right)$$

**Result:**

$$0.9999999999999998232235875460755936750481698119325820263\dots$$

$$0.999999999999999823223587546\dots$$

From:

$$\hat{\phi}(r) = \frac{3-p}{2\sqrt{2}} \frac{T_p}{(7-p)\Omega_{8-p} r^{7-p}} - \frac{3-p}{2\sqrt{2}} \kappa \left( \frac{T_p}{(7-p)\Omega_{8-p} r^{7-p}} \right)^2 + \dots$$

$$3 - 6 / 2\sqrt{2} = -3/(2 * \text{sqrt}(2))$$

**Input:**

$$-\frac{3}{2\sqrt{2}}$$

**Decimal approximation:**

$$-1.06066017177982128660126654315727355892725390653271105488\dots$$

$$-1.060660171779\dots = \frac{3-p}{2\sqrt{2}}$$

$$3 - 6 / 2\sqrt{2} * \kappa = (((-3/(2 * \text{sqrt}(2)))) * (((4.16666 \times 10^{-19})))$$

**Input interpretation:**

$$-\frac{3}{2\sqrt{2}} \times 4.16666 \times 10^{-19}$$

**Result:**

$$-4.41941\dots \times 10^{-19}$$

$$-4.41941\dots \times 10^{-19} = \frac{3-p}{2\sqrt{2}} \kappa$$

We have indeed:

$$-1.06066017177 * x - 4.16666e-19 * (x)^2 = 1.5$$

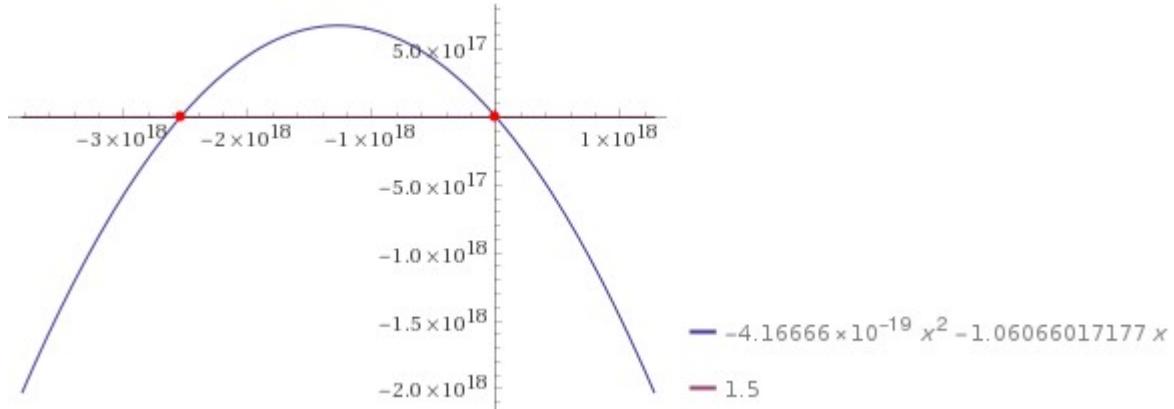
**Input interpretation:**

$$-1.06066017177 x - 4.16666 \times 10^{-19} x^2 = 1.5$$

**Result:**

$$-4.16666 \times 10^{-19} x^2 - 1.06066017177 x = 1.5$$

**Plot:**



- **Alternate forms:**

$$-4.16666 \times 10^{-19} x (x + 2.54559 \times 10^{18}) = 1.5$$

$$(-4.16666 \times 10^{-19} x - 1.06066) x = 1.5$$

$$-4.16666 \times 10^{-19} x^2 - 1.06066017177 x - 1.5 = 0$$

- **Alternate form assuming x is real:**

$$-4.16666 \times 10^{-19} x^2 - 1.06066017177 x + 0 = 1.5$$

- **Solutions:**

$$x = -2545588485189576192$$

$$x \approx -1.41421$$

- **Integer solution:**

$$x = -2545588485189576192$$

**Result:**

$$-2.545588485189576192 \times 10^{18}$$

$$-2.545588485189e+18 = \frac{T_p}{(7-p)\Omega_{8-p} r^{7-p}}$$

Thence, in conclusion:

$$((( -1.06066017177 * (-2.545588485189e+18))) - ((( -1.06066017177 * 4.16666e-19 * (-2.545588485189e+18)^2)))$$

**Input interpretation:**

$$-1.06066017177(-2.545588485189 \times 10^{18}) - \\ -1.06066017177 \times 4.16666 \times 10^{-19} (-2.545588485189 \times 10^{18})^2$$

**Result:**

$$5.56379136574024046717505346747334465756922 \times 10^{18}$$

$$\hat{\phi}(r) = 5.56379136574... * 10^{18}$$

Indeed, from:

$$\hat{\phi}(r) = \frac{3-p}{2\sqrt{2}} \frac{T_p}{(7-p)\Omega_{8-p} r^{7-p}} - \frac{3-p}{2\sqrt{2}} \kappa \left( \frac{T_p}{(7-p)\Omega_{8-p} r^{7-p}} \right)^2 + \dots$$

We obtain:

$$5.56379136574... * 10^{18} = (((-1.06066017177 * (-2.545588485189e+18)))) - (((-1.06066017177 * 4.16666e-19 * (-2.545588485189e+18)^2)))$$

**Result:**

True

Now, we have that:

$$\begin{aligned}
A(r) &= \frac{(7-p)(3-p)c_1}{64} h(r) - \frac{7-p}{16} \ln [\cosh(kh(r)) - c_2 \sinh(kh(r))], \\
B(r) &= \frac{1}{7-p} \ln(f_+ f_-) \\
&\quad + \frac{(p+1)(3-p)c_1}{64} h(r) + \frac{p+1}{16} \ln [\cosh(kh(r)) - c_2 \sinh(kh(r))], \\
\phi(r) &= \frac{(7-p)(p+1)c_1}{16} h(r) + \frac{3-p}{4} \ln [\cosh(kh(r)) - c_2 \sinh(kh(r))], \\
e^{\Lambda(r)} &= \pm (c_2^2 - 1)^{1/2} \frac{\sinh(kh(r))}{\cosh(kh(r)) - c_2 \sinh(kh(r))}, \\
h(r) &= \ln \left[ \frac{f_-(r)}{f_+(r)} \right], \quad f_\pm(r) = 1 \pm \left( \frac{r_0}{r} \right)^{7-p}, \\
k &= \sqrt{\frac{2(8-p)}{7-p} - \frac{(p+1)(7-p)}{16} c_1^2}.
\end{aligned}$$

tachyon vev ?

charge ?

mass ?

Now, we calculate  $h(r)$ ,  $f_+(r)$ ,  $f_-(r)$  and  $k$

$$r_0 = 2 \quad r = 4096 \quad (\text{for } r \rightarrow \text{infinity})$$

$$(1-2/4096) = 0.99951171875 = f_-(r)$$

$$(1-2/4096)$$

**Input:**

$$1 - \frac{2}{4096}$$

**Exact result:**

$$\frac{2047}{2048}$$

**Decimal form:**

$$0.99951171875$$

0.99951171875 result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{\frac{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt{5^3} - 1}} - \varphi + 1}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$(1+2/4096) = 1.00048828125 = f_+(r)$$

**Input:**

$$1 + \frac{2}{4096}$$

**Exact result:**

$$\frac{2049}{2048}$$

**Decimal form:**

$$1.00048828125$$

$$1.00048828125$$

$$h(r) = \ln (((((1-2/4096) / (1+2/4096)))))$$

**Input:**

$$\log\left(\frac{1 - \frac{2}{4096}}{1 + \frac{2}{4096}}\right)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$-\log\left(\frac{2049}{2047}\right)$$

**Decimal approximation:**

$$-0.00097656257761022565352201325254232083377845541641386422\dots$$

$$-0.00097656257761\dots$$

**Property:**

$-\log\left(\frac{2049}{2047}\right)$  is a transcendental number

•

**Alternate form:**

$$-\log(3) + \log(23) + \log(89) - \log(683)$$

### Alternative representations:

$$\log\left(\frac{1 - \frac{2}{4096}}{1 + \frac{2}{4096}}\right) = \log_e\left(\frac{1 - \frac{2}{4096}}{1 + \frac{2}{4096}}\right)$$

$$\log\left(\frac{1 - \frac{2}{4096}}{1 + \frac{2}{4096}}\right) = \log(a) \log_a\left(\frac{1 - \frac{2}{4096}}{1 + \frac{2}{4096}}\right)$$

$$\log\left(\frac{1 - \frac{2}{4096}}{1 + \frac{2}{4096}}\right) = -\text{Li}_1\left(1 - \frac{1 - \frac{2}{4096}}{1 + \frac{2}{4096}}\right)$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$\log\left(\frac{1 - \frac{2}{4096}}{1 + \frac{2}{4096}}\right) = \sum_{k=1}^{\infty} \frac{\left(-\frac{2}{2047}\right)^k}{k}$$

$$\log\left(\frac{1 - \frac{2}{4096}}{1 + \frac{2}{4096}}\right) = -2i\pi \left\lfloor \frac{\arg\left(\frac{2049}{2047} - x\right)}{2\pi} \right\rfloor - \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{2049}{2047} - x\right)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\begin{aligned} \log\left(\frac{1 - \frac{2}{4096}}{1 + \frac{2}{4096}}\right) &= -\left\lfloor \frac{\arg\left(\frac{2049}{2047} - z_0\right)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) - \\ &\log(z_0) - \left\lfloor \frac{\arg\left(\frac{2049}{2047} - z_0\right)}{2\pi} \right\rfloor \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{2049}{2047} - z_0\right)^k z_0^{-k}}{k} \end{aligned}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

### Integral representations:

$$\log\left(\frac{1 - \frac{2}{4096}}{1 + \frac{2}{4096}}\right) = - \int_1^{\frac{2049}{2047}} \frac{1}{t} dt$$

- $$\log\left(\frac{1 - \frac{2}{4096}}{1 + \frac{2}{4096}}\right) = \frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(\frac{2047}{2}\right)^s \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

We note that:

$$-1/h(r) = -1 / (((((\ln (((((1-2/4096) / (1+2/4096))))))))))$$

**Input:**

$$-\frac{1}{\log\left(\frac{1 - \frac{2}{4096}}{1 + \frac{2}{4096}}\right)}$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\frac{1}{\log\left(\frac{2049}{2047}\right)}$$

**Decimal approximation:**

1023.999918619786492651717085692952630502050233933264102464...

1024

**Property:**

$\frac{1}{\log\left(\frac{2049}{2047}\right)}$  is a transcendental number

**Alternate form:**

$$\frac{1}{\log(3) - \log(23) - \log(89) + \log(683)}$$

**Alternative representations:**

$$-\frac{1}{\log\left(\frac{1 - \frac{2}{4096}}{1 + \frac{2}{4096}}\right)} = -\frac{1}{\log_e\left(\frac{1 - \frac{2}{4096}}{1 + \frac{2}{4096}}\right)}$$

$$-\frac{1}{\log\left(\frac{1-\frac{2}{4096}}{1+\frac{2}{4096}}\right)} = -\frac{1}{\log(a) \log_a\left(\frac{1-\frac{2}{4096}}{1+\frac{2}{4096}}\right)}$$

$$-\frac{1}{\log\left(\frac{1-\frac{2}{4096}}{1+\frac{2}{4096}}\right)} = \frac{-1}{-\text{Li}_1\left(1 - \frac{1-\frac{2}{4096}}{1+\frac{2}{4096}}\right)}$$

$\log_b(x)$  is the base-  $b$  logarithm  
 $\text{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$-\frac{1}{\log\left(\frac{1-\frac{2}{4096}}{1+\frac{2}{4096}}\right)} = -\frac{1}{\sum_{k=1}^{\infty} \frac{(-\frac{2}{2047})^k}{k}}$$

$$-\frac{1}{\log\left(\frac{1-\frac{2}{4096}}{1+\frac{2}{4096}}\right)} = \frac{1}{2i\pi \left\lfloor \frac{\arg\left(\frac{2049}{2047}-x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{2049}{2047}-x\right)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$-\frac{1}{\log\left(\frac{1-\frac{2}{4096}}{1+\frac{2}{4096}}\right)} = \frac{1}{\log(z_0) + \left\lfloor \frac{\arg\left(\frac{2049}{2047}-z_0\right)}{2\pi} \right\rfloor \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{2049}{2047}-z_0\right)^k z_0^{-k}}{k}}$$

$\arg(z)$  is the complex argument  
 $\lfloor x \rfloor$  is the floor function

### Integral representations:

$$-\frac{1}{\log\left(\frac{1-\frac{2}{4096}}{1+\frac{2}{4096}}\right)} = \frac{1}{\int_1^{2047} \frac{1}{t} dt}$$

$$-\frac{1}{\log\left(\frac{1-\frac{2}{4096}}{1+\frac{2}{4096}}\right)} = \frac{2i\pi}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(\frac{2047}{2}\right)^s \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

From the following data:

$$c_1 \geq 0 \text{ and } k \geq 0$$

$$(r_0^{7-p} \geq 0, c_2 \geq 0) \text{ for } p = 3, \dots, 6$$

$$p = 6; c_1 = 1/4 = 0.25$$

$$\sqrt{2 \times 2 - \frac{7}{16} \times 0.25^2} = \mathbf{k = 1.9931523}$$

**Input:**

$$\sqrt{2 \times 2 - \frac{7}{16} \times 0.25^2}$$

**Result:**

$$1.9931523\dots$$

$$1.9931523\dots$$

The general asymptotically flat solution of the equations of motion of the p-brane is given by (see above slide):

$$\begin{aligned} A(r) &= \frac{(7-p)(3-p)c_1}{64} h(r) - \frac{7-p}{16} \ln [\cosh(kh(r)) - c_2 \sinh(kh(r))], \\ B(r) &= \frac{1}{7-p} \ln [f_-(r) f_+(r)] \\ &\quad + \frac{(p+1)(3-p)c_1}{64} h(r) + \frac{p+1}{16} \ln [\cosh(kh(r)) - c_2 \sinh(kh(r))], \\ \phi(r) &= \frac{(p+1)(7-p)c_1}{16} h(r) + \frac{3-p}{4} \ln [\cosh(kh(r)) - c_2 \sinh(kh(r))], \\ e^{\Lambda(r)} &= -\eta(c_2^2 - 1)^{1/2} \frac{\sinh(kh(r))}{\cosh(kh(r)) - c_2 \sinh(kh(r))}, \end{aligned}$$

Now, we calculate  $A(r)$

$$p = 6, c_1 = 1/4 = 0.25 \quad k = 1.9931523 \quad c_2 = 1/3 = 0.333333\dots$$

$$h(r) = -0.00097656257761\dots$$

$$(-3*1/4)/64 * (-0.00097656257761) - 1/16 \ln (((\cosh(1.9931523 * -0.00097656257761) - 1/3 \sinh(1.9931523 * -0.00097656257761))))$$

### Input interpretation:

$$\left( \frac{1}{64} \left( -3 \times \frac{1}{4} \right) \right) \times (-0.00097656257761) - \frac{1}{16} \log \left( \cosh(1.9931523 \times (-0.00097656257761)) - \frac{1}{3} \sinh(1.9931523 \times (-0.00097656257761)) \right)$$

$\cosh(x)$  is the hyperbolic cosine function

$\sinh(x)$  is the hyperbolic sine function

$\log(x)$  is the natural logarithm

### Result:

$$-0.000029211892\dots$$

$$-0.000029211892\dots = A(r)$$

### Alternative representations:

$$-\frac{0.000976562577610000 (-3)}{4 \times 64} - \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) = \\ -\frac{1}{16} \log \left( \frac{1}{6} \left( -\frac{1}{e^{0.00194644}} + e^{0.00194644} \right) + \frac{1}{2} \left( \frac{1}{e^{0.00194644}} + e^{0.00194644} \right) \right) + \frac{0.00292968773283000}{4 \times 64}$$

$$-\frac{0.000976562577610000 (-3)}{4 \times 64} - \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) = \\ -\frac{1}{16} \log \left( \cos(-0.00194644 i) + \frac{1}{6} \left( -\frac{1}{e^{0.00194644}} + e^{0.00194644} \right) \right) + \frac{0.00292968773283000}{4 \times 64}$$

$$\begin{aligned}
& -\frac{0.000976562577610000 (-3)}{4 \times 64} - \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\
& \quad \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) = \\
& -\frac{1}{16} \log \left( \cos(0.00194644 i) + \frac{1}{6} \left( -\frac{1}{e^{0.00194644}} + e^{0.00194644} \right) \right) + \\
& \frac{0.00292968773283000}{4 \times 64}
\end{aligned}$$

$i$  is the imaginary unit

### Series representation:

$$\begin{aligned}
& -\frac{0.000976562577610000 (-3)}{4 \times 64} - \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\
& \quad \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) = 0.0000114440927063672 + \\
& 0.0625000000000000 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3} \right)^k}{k}
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& -\frac{0.000976562577610000 (-3)}{4 \times 64} - \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\
& \quad \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) = 0.0000114440927063672 - \\
& 0.0625000000000000 \int_1^{\cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3}} \frac{1}{t} dt
\end{aligned}$$

$$\begin{aligned}
& -\frac{0.000976562577610000 (-3)}{4 \times 64} - \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\
& \quad \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) = \\
& 0.0000114440927063672 - 0.0625000000000000 \\
& \log \left( 1 + \int_0^1 (0.000648813 \cosh(-0.00194644 t) - 0.00194644 \right. \\
& \quad \left. \sinh(-0.00194644 t)) dt \right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{0.000976562577610000 (-3)}{4 \times 64} - \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\
& \quad \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) = \\
& 0.0000114440927063672 - 0.0625000000000000 \\
& \log \left( \frac{\sqrt{\pi}}{i \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{0.5 e^{0.47155 \times 10^{-7}/s+s} (0.000324406 + s)}{s^{3/2}} ds \right) \text{ for } \gamma > 0
\end{aligned}$$

The reciprocal is:

$$1 / ((((((-3*1/4)/64 * (-0.00097656257761) - 1/16 \ln (((\cosh(1.9931523 * -0.00097656257761) - 1/3 \sinh(1.9931523 * -0.00097656257761)))))))$$

### Input interpretation:

$$1 / \left( \left( \frac{1}{64} \left( -3 \times \frac{1}{4} \right) \right) \times (-0.00097656257761) - \frac{1}{16} \log \left( \cosh(1.9931523 \times (-0.00097656257761)) - \frac{1}{3} \sinh(1.9931523 \times (-0.00097656257761)) \right) \right)$$

$\cosh(x)$  is the hyperbolic cosine function

$\sinh(x)$  is the hyperbolic sine function

$\log(x)$  is the natural logarithm

### Result:

-34232.634...

-34232.634.... =  $1/A(r)$

### Alternative representations:

$$1 / \left( -\frac{0.000976562577610000 (-3)}{4 \times 64} - \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) \right) = \frac{1}{-\frac{1}{16} \log_e \left( \cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3} \right) + \frac{0.00292968773283000}{4 \times 64}}$$

$$1 / \left( -\frac{0.000976562577610000 (-3)}{4 \times 64} - \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) \right) = \frac{1}{-\frac{1}{16} \log \left( \cos(-0.00194644 i) + \frac{1}{6} \left( -\frac{1}{e^{0.00194644}} + e^{0.00194644} \right) \right) + \frac{0.00292968773283000}{4 \times 64}}$$

$$1 / \left( -\frac{0.000976562577610000 (-3)}{4 \times 64} - \frac{\frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right)}{1} = \right.$$

$$\left. -\frac{\frac{1}{16} \log(a) \log_a \left( \cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3} \right) + \frac{0.00292968773283000}{4 \times 64}}{\log_b(x) \text{ is the base- } b \text{ logarithm}} \right)$$

$i$  is the imaginary unit

### Series representation:

$$1 / \left( -\frac{0.000976562577610000 (-3)}{4 \times 64} - \frac{\frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right)}{16.000000000000000} = \right.$$

$$\left. -\frac{-0.000183105483301875 - \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + \cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3})^k}{k}}{-0.000183105483301875 - \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + \cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3})^k}{k}} \right)$$

### Integral representations:

$$1 / \left( -\frac{0.000976562577610000 (-3)}{4 \times 64} - \frac{\frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right)}{16.000000000000000} = \right.$$

$$\left. -\frac{-0.000183105483301875 + \int_1^{\cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3}} \frac{1}{t} dt}{-0.000183105483301875 + \int_1^{\cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3}} \frac{1}{t} dt} \right)$$

$$1 / \left( -\frac{0.000976562577610000 (-3)}{4 \times 64} - \frac{\frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right)}{16.000000000000000} = \right.$$

$$\left. -\left( 16.00000000000000 / \left( -0.000183105483301875 + \log \left( 1 + \int_0^1 (0.000648813 \cosh(-0.00194644 t) - 0.00194644 \sinh(-0.00194644 t)) dt \right) \right) \right)$$

$$\begin{aligned}
& 1 / \left( -\frac{0.000976562577610000 (-3)}{4 \times 64} - \right. \\
& \quad \left. \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \right. \\
& \quad \left. \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) \right) = \\
& - \left( 16.00000000000000 / \left( -0.000183105483301875 + \right. \right. \\
& \quad \left. \left. \log \left( \int_0^1 \left( 0.000648813 \cosh(-0.00194644 t) + (-0.00194644 - 0.5 i \pi) \right. \right. \right. \right. \\
& \quad \left. \left. \left. \sinh \left( 0 - \frac{1}{2} i \pi (-1 + t) - 0.00194644 t \right) \right] dt \right) \right)
\end{aligned}$$

We note that, from the computations of the following Ramanujan mock theta functions, 4267.24; 2498.27; 2122.1867 and 33021.1 we obtain:

$$-(((4267.24 - 2498.27 + 33021.1 - (2122.1867 / 4) - \sqrt{729})))$$

### **Input interpretation:**

$$-\left( 4267.24 - 2498.27 + 33021.1 - \frac{2122.1867}{4} - \sqrt{729} \right)$$

### **Result:**

$$-34232.523325$$

**-34232.523325**

This result is practically equal to the value of  $1/A(r) = \textcolor{red}{-34232.634\dots}$

$$\begin{aligned}
& 1 / \left( \left( \frac{1}{64} \left( -3 \times \frac{1}{4} \right) \right) \times (-0.00097656257761) - \right. \\
& \quad \left. \frac{1}{16} \log \left( \cosh(1.9931523 \times (-0.00097656257761)) - \right. \right. \\
& \quad \left. \left. \frac{1}{3} \sinh(1.9931523 \times (-0.00097656257761)) \right) \right) \Rightarrow \\
& \Rightarrow - \left( 4267.24 - 2498.27 + 33021.1 - \frac{2122.1867}{4} - \sqrt{729} \right) \\
& = \textcolor{red}{-34232.523325} \approx -34232.634
\end{aligned}$$

Now:

$$\ln(1.00048828125 \times 0.99951171875) + (((6+1)*(3-6)*0.25)) / 64 * (-0.00097656257761) + 7/16 * \ln(((\cosh(1.9931523 \times -0.00097656257761) - 1/3 \sinh(1.9931523 \times -0.00097656257761))))$$

### Input interpretation:

$$\log(1.00048828125 \times 0.99951171875) + \left( \frac{1}{64} ((6+1)(3-6) \times 0.25) \right) \times (-0.00097656257761) + \frac{7}{16} \log \left( \cosh(1.9931523 \times (-0.00097656257761)) - \frac{1}{3} \sinh(1.9931523 \times (-0.00097656257761)) \right)$$

$\log(x)$  is the natural logarithm

$\cosh(x)$  is the hyperbolic cosine function

$\sinh(x)$  is the hyperbolic sine function

### Result:

$$0.0003644621\dots$$

$$0.0003644621 = B(r)$$

### Alternative representations:

$$\begin{aligned} & \log(1.000488281250000 \times 0.999511718750000) - \\ & \frac{1}{64} \times 0.000976562577610000 ((6+1)(3-6) 0.25) + \\ & \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) \\ & 7 = \log(0.99999976158142) + \\ & \frac{7}{16} \log \left( \frac{1}{6} \left( -\frac{1}{e^{0.00194644}} + e^{0.00194644} \right) + \frac{1}{2} \left( \frac{1}{e^{0.00194644}} + e^{0.00194644} \right) \right) + \\ & \frac{0.00512695}{64} \end{aligned}$$

$$\begin{aligned} & \log(1.000488281250000 \times 0.999511718750000) - \\ & \frac{1}{64} \times 0.000976562577610000 ((6+1)(3-6) 0.25) + \\ & \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) \\ & 7 = \log(0.99999976158142) + \\ & \frac{7}{16} \log \left( \cos(-0.00194644 i) + \frac{1}{6} \left( -\frac{1}{e^{0.00194644}} + e^{0.00194644} \right) \right) + \frac{0.00512695}{64} \end{aligned}$$

$$\begin{aligned}
& \log(1.000488281250000 \times 0.999511718750000) - \\
& \frac{1}{64} \times 0.000976562577610000 ((6 + 1)(3 - 6) 0.25) + \\
& \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\
& \quad \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 7 = \log(0.99999976158142) + \\
& \frac{7}{16} \log \left( \cos(0.00194644 i) + \frac{1}{6} \left( -\frac{1}{e^{0.00194644}} + e^{0.00194644} \right) \right) + \frac{0.00512695}{64}
\end{aligned}$$

$i$  is the imaginary unit

### Series representation:

$$\begin{aligned}
& \log(1.000488281250000 \times 0.999511718750000) - \\
& \frac{1}{64} \times 0.000976562577610000 ((6 + 1)(3 - 6) 0.25) + \\
& \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\
& \quad \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 7 = \\
& 0.0000801086 + \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \left( -(-2.3841858 \times 10^{-7})^k - \right. \\
& \quad \left. 0.4375 \left( -1 + \cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3} \right)^k \right)
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& \log(1.000488281250000 \times 0.999511718750000) - \\
& \frac{1}{64} \times 0.000976562577610000 ((6 + 1)(3 - 6) 0.25) + \\
& \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\
& \quad \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 7 = 0.0000801086 + \\
& \log(0.99999976158142) + 0.4375 \log \left( 1 + \int_0^1 (0.000648813 \cosh(-0.00194644 t) - \right. \\
& \quad \left. 0.00194644 \sinh(-0.00194644 t)) dt \right)
\end{aligned}$$

•

$$\begin{aligned}
& \log(1.000488281250000 \times 0.999511718750000) - \\
& \frac{1}{64} \times 0.000976562577610000 ((6+1)(3-6)0.25) + \\
& \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\
& \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right)^7 = \\
& 0.0000801086 + \log(0.99999976158142) + \\
& 0.4375 \log \left( \frac{\sqrt{\pi}}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{0.5 e^{0.47155 \times 10^{-7}/s+s} (0.000324406 + s)}{s^{3/2}} ds \right) \text{ for } \gamma > 0
\end{aligned}$$

$$\begin{aligned}
& \log(1.000488281250000 \times 0.999511718750000) - \\
& \frac{1}{64} \times 0.000976562577610000 ((6+1)(3-6)0.25) + \\
& \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\
& \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right)^7 = \\
& 0.0000801086 + \int_1^{0.99999976158142} \left( \frac{1}{t} + \right. \\
& \left. (-0.4375 + 0.4375 \cosh(-0.00194644) - 0.145833 \sinh(-0.00194644))/ \right. \\
& \left. (1. - \cosh(-0.00194644) + t (-1 + \cosh(-0.00194644) - 0.333333 \right. \\
& \left. \sinh(-0.00194644)) + 0.333333 \sinh(-0.00194644)) \right) dt
\end{aligned}$$

$$\begin{aligned}
& 1 / (((((\ln(1.00048828125 * 0.99951171875) + (((6+1)*(3-6)*0.25))) / 64 * (- \\
& 0.00097656257761) + 7/16 * \ln(((\cosh(1.9931523 * -0.00097656257761) - 1/3 \\
& \sinh(1.9931523 * -0.00097656257761)))))))
\end{aligned}$$

### Input interpretation:

$$1 / \left( \log(1.00048828125 \times 0.99951171875) + \right. \\
\left( \frac{1}{64} ((6+1)(3-6) \times 0.25) \right) \times (-0.00097656257761) + \\
\frac{7}{16} \log \left( \cosh(1.9931523 \times (-0.00097656257761)) - \right. \\
\left. \left. \frac{1}{3} \sinh(1.9931523 \times (-0.00097656257761)) \right) \right)$$

$\log(x)$  is the natural logarithm

$\cosh(x)$  is the hyperbolic cosine function

$\sinh(x)$  is the hyperbolic sine function

### Result:

2743.769...

$2743.769\dots = 1/B(r)$

### Alternative representations:

$$1 / \left( \log(1.000488281250000 \times 0.999511718750000) - \frac{1}{64} \times 0.000976562577610000 ((6+1)(3-6)0.25) + \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 7 \right) = \\ 1 / \left( \log(0.99999976158142) + \frac{7}{16} \log \left( \cos(-0.00194644 i) + \frac{1}{6} \left( -\frac{1}{e^{0.00194644}} + e^{0.00194644} \right) \right) + \frac{0.00512695}{64} \right)$$

$$1 / \left( \log(1.000488281250000 \times 0.999511718750000) - \frac{1}{64} \times 0.000976562577610000 ((6+1)(3-6)0.25) + \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 7 \right) = \\ 1 / \left( \log(0.99999976158142) + \frac{7}{16} \log \left( \cos(0.00194644 i) + \frac{1}{6} \left( -\frac{1}{e^{0.00194644}} + e^{0.00194644} \right) \right) + \frac{0.00512695}{64} \right)$$

$$1 / \left( \log(1.000488281250000 \times 0.999511718750000) - \frac{1}{64} \times 0.000976562577610000 ((6+1)(3-6)0.25) + \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 7 \right) = \\ 1 / \left( \log_e(0.99999976158142) + \frac{7}{16} \log_e \left( \cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3} \right) + \frac{0.00512695}{64} \right)$$

$i$  is the imaginary unit  
 $\log_b(x)$  is the base- $b$  logarithm

### Series representation:

$$\begin{aligned}
& 1 / \left( \log(1.000488281250000 \times 0.999511718750000) - \right. \\
& \quad \frac{1}{64} \times 0.000976562577610000 ((6 + 1)(3 - 6) 0.25) + \\
& \quad \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \right. \\
& \quad \left. \sinh(1.99315 (-0.000976562577610000)) \right) 7 = \\
& 1 / \left( 0.0000801086 + \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \left( -(-2.3841858 \times 10^{-7})^k - \right. \right. \\
& \quad \left. \left. 0.4375 \left( -1 + \cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3} \right)^k \right) \right)
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& 1 / \left( \log(1.000488281250000 \times 0.999511718750000) - \right. \\
& \quad \frac{1}{64} \times 0.000976562577610000 ((6 + 1)(3 - 6) 0.25) + \\
& \quad \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \right. \\
& \quad \left. \sinh(1.99315 (-0.000976562577610000)) \right) 7 = \\
& 1 / \left( 0.0000801086 + \log(0.99999976158142) + \right. \\
& \quad \left. 0.4375 \log \left( 1 + \int_0^1 (0.000648813 \cosh(-0.00194644 t) - \right. \right. \\
& \quad \left. \left. 0.00194644 \sinh(-0.00194644 t)) dt \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 1 / \left( \log(1.000488281250000 \times 0.999511718750000) - \right. \\
& \quad \frac{1}{64} \times 0.000976562577610000 ((6 + 1)(3 - 6) 0.25) + \\
& \quad \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \right. \\
& \quad \left. \sinh(1.99315 (-0.000976562577610000)) \right) 7 = \\
& 1 / \left( 0.0000801086 + \int_1^{0.99999976158142} \left( \frac{1}{t} + (-0.4375 + 0.4375 \cosh(-0.00194644) - \right. \right. \\
& \quad \left. \left. 0.145833 \sinh(-0.00194644)) / (1. - \cosh(-0.00194644) + t (-1 + \cosh(-0.00194644) - 0.333333 \sinh(-0.00194644)) + \right. \right. \\
& \quad \left. \left. 0.333333 \sinh(-0.00194644)) \right) dt \right)
\end{aligned}$$

$$\begin{aligned}
& 1 / \left( \log(1.000488281250000 \times 0.999511718750000) - \right. \\
& \quad \frac{1}{64} \times 0.000976562577610000 ((6+1)(3-6)0.25) + \\
& \quad \frac{1}{16} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \right. \\
& \quad \left. \sinh(1.99315 (-0.000976562577610000)) \right) 7 \Big) = \\
& 1 / \left( 0.0000801086 + \log(0.99999976158142) + \right. \\
& \quad 0.4375 \log \left( \int_0^1 \left( 0.000648813 \cosh(-0.00194644 t) + (-0.00194644 - 0.5 i \pi) \right. \right. \\
& \quad \left. \left. \sinh \left( 0 - \frac{1}{2} i \pi (-1+t) - 0.00194644 t \right) \right) dt \right) \Big)
\end{aligned}$$

We note that  $33021.10 / 12 = 2.751,758333333$  is very near to the value of  $1/B(r) = 2743.769$  where 33021.10 is a Ramanujan mock theta function

Now, we have that:

$$(((6+1)*0.25)) / 16 * (-0.00097656257761) - 3/4 * \ln(((\cosh(1.9931523 * -0.00097656257761) - 1/3 \sinh(1.9931523 * -0.00097656257761))))$$

### Input interpretation:

$$\begin{aligned}
& \left( \frac{1}{16} ((6+1) \times 0.25) \right) \times (-0.00097656257761) - \\
& \quad \frac{3}{4} \log \left( \cosh(1.9931523 \times (-0.00097656257761)) - \right. \\
& \quad \left. \frac{1}{3} \sinh(1.9931523 \times (-0.00097656257761)) \right)
\end{aligned}$$

$\cosh(x)$  is the hyperbolic cosine function

$\sinh(x)$  is the hyperbolic sine function

$\log(x)$  is the natural logarithm

### Result:

-0.0005946833...

-0.0005946833... =  $\phi(r)$

### Alternative representations:

$$\begin{aligned} & \frac{1}{16} (-0.000976562577610000) ((6 + 1) 0.25) - \\ & \frac{1}{4} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\ & \quad \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 3 = \\ & -\frac{3}{4} \log \left( \frac{1}{6} \left( -\frac{1}{e^{0.00194644}} + e^{0.00194644} \right) + \frac{1}{2} \left( \frac{1}{e^{0.00194644}} + e^{0.00194644} \right) \right) - \frac{0.00170898}{16} \end{aligned}$$

$$\begin{aligned} & \frac{1}{16} (-0.000976562577610000) ((6 + 1) 0.25) - \\ & \frac{1}{4} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\ & \quad \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 3 = \\ & -\frac{3}{4} \log \left( \cos(-0.00194644 i) + \frac{1}{6} \left( -\frac{1}{e^{0.00194644}} + e^{0.00194644} \right) \right) - \frac{0.00170898}{16} \end{aligned}$$

$$\begin{aligned} & \frac{1}{16} (-0.000976562577610000) ((6 + 1) 0.25) - \\ & \frac{1}{4} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\ & \quad \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 3 = \\ & -\frac{3}{4} \log \left( \cos(0.00194644 i) + \frac{1}{6} \left( -\frac{1}{e^{0.00194644}} + e^{0.00194644} \right) \right) - \frac{0.00170898}{16} \end{aligned}$$

$i$  is the imaginary unit

### Series representation:

$$\begin{aligned} & \frac{1}{16} (-0.000976562577610000) ((6 + 1) 0.25) - \\ & \frac{1}{4} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\ & \quad \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 3 = \\ & -0.000106812 + 0.75 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3} \right)^k}{k} \end{aligned}$$

### Integral representations:

$$\begin{aligned} & \frac{1}{16} (-0.000976562577610000) ((6 + 1) 0.25) - \\ & \frac{1}{4} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\ & \quad \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 3 = \\ & -0.000106812 - 0.75 \int_1^{\cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3}} \frac{1}{t} dt \end{aligned}$$

$$\begin{aligned} & \frac{1}{16} (-0.000976562577610000) ((6 + 1) 0.25) - \\ & \frac{1}{4} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\ & \quad \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 3 = \\ & -0.000106812 - 0.75 \log \left( 1 + \int_0^1 (0.000648813 \cosh(-0.00194644 t) - \right. \\ & \quad \left. 0.00194644 \sinh(-0.00194644 t)) dt \right) \end{aligned}$$

$$\begin{aligned} & \frac{1}{16} (-0.000976562577610000) ((6 + 1) 0.25) - \\ & \frac{1}{4} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\ & \quad \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 3 = -0.000106812 - \\ & 0.75 \log \left( \frac{\sqrt{\pi}}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{0.5 e^{9.47155 \times 10^{-7}/s+s} (0.000324406 + s)}{s^{3/2}} ds \right) \text{ for } \gamma > 0 \end{aligned}$$

$$1 / (((((6+1)*0.25))) / 16 * (-0.00097656257761) - 3/4 * \ln(((\cosh(1.9931523 * -0.00097656257761) - 1/3 \sinh(1.9931523 * -0.00097656257761))))))$$

**Input interpretation:**

$$1 / \left( \left( \frac{1}{16} ((6 + 1) \times 0.25) \right) \times (-0.00097656257761) - \right. \\ \left. \frac{3}{4} \log \left( \cosh(1.9931523 \times (-0.00097656257761)) - \right. \right. \\ \left. \left. \frac{1}{3} \sinh(1.9931523 \times (-0.00097656257761)) \right) \right)$$

$\cosh(x)$  is the hyperbolic cosine function

$\sinh(x)$  is the hyperbolic sine function

$\log(x)$  is the natural logarithm

**Result:**

-1681.567...

$$-1681.567\dots = 1/\phi(r)$$

### Alternative representations:

$$\frac{1}{\left(\frac{1}{16} (-0.000976562577610000) ((6 + 1) 0.25) - \frac{1}{4} \log\left(\cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000))\right) 3\right)} =$$
$$\frac{1}{-\frac{3}{4} \log_e\left(\cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3}\right) - \frac{0.00170898}{16}}$$

$$\frac{1}{\left(\frac{1}{16} (-0.000976562577610000) ((6 + 1) 0.25) - \frac{1}{4} \log\left(\cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000))\right) 3\right)} =$$
$$\frac{1}{-\frac{3}{4} \log\left(\cos(-0.00194644 i) + \frac{1}{6} \left(-\frac{1}{e^{0.00194644}} + e^{0.00194644}\right)\right) - \frac{0.00170898}{16}}$$

$$\frac{1}{\left(\frac{1}{16} (-0.000976562577610000) ((6 + 1) 0.25) - \frac{1}{4} \log\left(\cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000))\right) 3\right)} =$$
$$\frac{1}{-\frac{3}{4} \log(a) \log_a\left(\cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3}\right) - \frac{0.00170898}{16}}$$

$\log_b(x)$  is the base- $b$  logarithm

$i$  is the imaginary unit

### Series representation:

$$\frac{1}{\left(\frac{1}{16} (-0.000976562577610000) ((6 + 1) 0.25) - \frac{1}{4} \log\left(\cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000))\right) 3\right)} =$$
$$\frac{1.33333}{0.000142415 - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3}\right)^k}{k}}$$

## Integral representations:

$$1 / \left( \frac{1}{16} (-0.000976562577610000) ((6 + 1) 0.25) - \frac{1}{4} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 3 \right) = \\ - \frac{1.33333}{0.000142415 + \int_1^{\cosh(-0.00194644) - \frac{\sinh(-0.00194644)}{3}} \frac{1}{t} dt}$$

$$1 / \left( \frac{1}{16} (-0.000976562577610000) ((6 + 1) 0.25) - \frac{1}{4} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 3 \right) = \\ - \left( 1.33333 / \left( 0.000142415 + \log \left( 1 + \int_0^1 (0.000648813 \cosh(-0.00194644 t) - 0.00194644 \sinh(-0.00194644 t)) dt \right) \right) \right)$$

$$1 / \left( \frac{1}{16} (-0.000976562577610000) ((6 + 1) 0.25) - \frac{1}{4} \log \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) 3 \right) = - \left( 1.33333 / \left( 0.000142415 + \log \left( \int_0^1 (0.000648813 \cosh(-0.00194644 t) + (-0.00194644 - 0.5 i \pi) \sinh(0 - \frac{1}{2} i \pi (-1 + t) - 0.00194644 t)) dt \right) \right) \right)$$

From the calculations of the following Ramanujan mock theta functions, we obtain:  
 $-(-33021.10 / 10 + 2122.18 + 2498.27 + 4267.24 / 12)$

## Input interpretation:

$$-\left( -\frac{33021.10}{10} + 2122.18 + 2498.27 + \frac{4267.24}{12} \right)$$

## Result:



$$\frac{\left( \sqrt{\left(\frac{1}{3}\right)^2 - 1} \sinh(1.99315 (-0.000976562577610000)) \right) / \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) = \left( \frac{1}{e^{0.00194644}} - e^{0.00194644} \right) \sqrt{-1 + \left(\frac{1}{3}\right)^2}}{2 \left( \cos(0.00194644 i) + \frac{1}{6} \left( -\frac{1}{e^{0.00194644}} + e^{0.00194644} \right) \right)}$$

$$\frac{\left( \sqrt{\left(\frac{1}{3}\right)^2 - 1} \sinh(1.99315 (-0.000976562577610000)) \right) / \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) = \left( \frac{1}{e^{0.00194644}} - e^{0.00194644} \right) \sqrt{-1 + \left(\frac{1}{3}\right)^2}}{2 \left( \frac{1}{6} \left( -\frac{1}{e^{0.00194644}} + e^{0.00194644} \right) + \frac{1}{2} \left( \frac{1}{e^{0.00194644}} + e^{0.00194644} \right) \right)}$$

*i* is the imaginary unit

### Series representations:

$$\frac{\left( \sqrt{\left(\frac{1}{3}\right)^2 - 1} \sinh(1.99315 (-0.000976562577610000)) \right) / \left( \cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) = \left( \frac{1}{e^{0.00194644}} - e^{0.00194644} \right) \sqrt{-1 + \left(\frac{1}{3}\right)^2}}{\sum_{k=0}^{\infty} \frac{(-0.00194644)^{1+2k}}{(1+2k)!} \left( \frac{3}{(2k)!} + \frac{0.00194644}{(1+2k)!} \right)}$$

$$\begin{aligned} & \left( \sqrt{\left(\frac{1}{3}\right)^2 - 1} \sinh(1.99315 (-0.000976562577610000)) \right) / \\ & \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\ & \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) = \\ & - \frac{4i\sqrt{2} \sum_{k=0}^{\infty} I_{1+2k}(-0.00194644)}{\sum_{k=0}^{\infty} \left( 2I_{1+2k}(-0.00194644) - \frac{3(-0.00194644)^{2k}}{(2k)!} \right)} \end{aligned}$$

$$\begin{aligned} & \left( \sqrt{\left(\frac{1}{3}\right)^2 - 1} \sinh(1.99315 (-0.000976562577610000)) \right) / \\ & \left( \cosh(1.99315 (-0.000976562577610000)) - \right. \\ & \left. \frac{1}{3} \sinh(1.99315 (-0.000976562577610000)) \right) = \\ & - \frac{2i\sqrt{2} i \sum_{k=0}^{\infty} \frac{(-0.00194644 - \frac{i\pi}{2})^{2k}}{(2k)!}}{\sum_{k=0}^{\infty} \frac{-3(-0.00194644)^{2k} + i(-0.00194644 - \frac{i\pi}{2})^{2k}}{(2k)!}} \end{aligned}$$

$$1/(((((((((1/3)^2-1))^1/2))) * ((\sinh(1.9931523*-0.00097656257761))) / (((((\cosh(1.9931523*-0.00097656257761)-1/3 \sinh(1.9931523*-0.00097656257761)))))))))))$$

**Input interpretation:**

$$\frac{1}{\sqrt{\left(\frac{1}{3}\right)^2 - 1} \times \frac{\sinh(1.9931523 \times (-0.00097656257761))}{\cosh(1.9931523 \times (-0.00097656257761)) - \frac{1}{3} \sinh(1.9931523 \times (-0.00097656257761))}}$$

$\sinh(x)$  is the hyperbolic sine function  
 $\cosh(x)$  is the hyperbolic cosine function

**Result:**

$$545.27794... i$$

**Polar coordinates:**

$$r = 545.278 \text{ (radius)}, \theta = 90^\circ \text{ (angle)}$$

$$545.278 = 1/e^{\Lambda(r)}$$

## Alternative representations:

$$\frac{1}{\sqrt{\left(\frac{1}{3}\right)^2 - 1} \sinh(1.99315 (-0.000976562577610000))} =$$

$$\frac{1}{\cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000))} =$$

$$\frac{1}{\left(\frac{1}{e^{0.00194644}} - e^{0.00194644}\right) \sqrt{-1 + \left(\frac{1}{3}\right)^2}}$$

$$\frac{1}{2 \left(\cos(0.00194644 i) + \frac{1}{6} \left(-\frac{1}{e^{0.00194644}} + e^{0.00194644}\right)\right)}$$

$$\frac{1}{\sqrt{\left(\frac{1}{3}\right)^2 - 1} \sinh(1.99315 (-0.000976562577610000))} =$$

$$\frac{1}{\cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000))} =$$

$$\frac{1}{\left(\frac{1}{e^{0.00194644}} - e^{0.00194644}\right) \sqrt{-1 + \left(\frac{1}{3}\right)^2}}$$

$$\frac{1}{2 \left(\cos(-0.00194644 i) + \frac{1}{6} \left(-\frac{1}{e^{0.00194644}} + e^{0.00194644}\right)\right)}$$

$$\frac{1}{\sqrt{\left(\frac{1}{3}\right)^2 - 1} \sinh(1.99315 (-0.000976562577610000))} =$$

$i$  is the imaginary unit

## Series representations:

$$\frac{1}{\sqrt{\left(\frac{1}{3}\right)^2 - 1} \sinh(1.99315 (-0.000976562577610000))} =$$

$$\frac{1}{\cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000))}$$

$$\frac{i \left(2 \sum_{k=0}^{\infty} I_{1+2k}(-0.00194644) - 3 \sum_{k=0}^{\infty} \frac{(-0.00194644)^{2k}}{(2k)!}\right)}{4 \sqrt{2} \sum_{k=0}^{\infty} I_{1+2k}(-0.00194644)}$$

$$\frac{1}{\sqrt{\left(\frac{1}{3}\right)^2 - 1} \sinh(1.99315 (-0.000976562577610000))} =$$

$$\frac{\cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000))}{i \left( 3 \sum_{k=0}^{\infty} \frac{(-0.00194644)^{2k}}{(2k)!} - \sum_{k=0}^{\infty} \frac{(-0.00194644)^{1+2k}}{(1+2k)!} \right)}$$

$$- \frac{2\sqrt{2} \sum_{k=0}^{\infty} \frac{(-0.00194644)^{1+2k}}{(1+2k)!}}$$

$$\frac{1}{\sqrt{\left(\frac{1}{3}\right)^2 - 1} \sinh(1.99315 (-0.000976562577610000))} =$$

$$\frac{\cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000))}{i \left( - \sum_{k=0}^{\infty} \frac{(-0.00194644)^{1+2k}}{(1+2k)!} + 3i \sum_{k=0}^{\infty} \frac{(-0.00194644 - \frac{i\pi}{2})^{1+2k}}{(1+2k)!} \right)}$$

$$- \frac{2\sqrt{2} \sum_{k=0}^{\infty} \frac{(-0.00194644)^{1+2k}}{(1+2k)!}}$$

$I_n(z)$  is the modified Bessel function of the first kind  
 $n!$  is the factorial function

## Integral representations:

$$\frac{1}{\sqrt{\left(\frac{1}{3}\right)^2 - 1} \sinh(1.99315 (-0.000976562577610000))} =$$

$$\frac{\cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000))}{\left( 0.353553i \left( (1541.28 + 0i) + 1 + 0i \int_0^1 \cosh(-0.00194644t) dt - \right. \right.}$$

$$\left. \left. 3 + 0i \int_0^1 \sinh(-0.00194644t) dt \right) \right/ \left( \int_0^1 \cosh(-0.00194644t) dt \right)}$$

$$\frac{1}{\sqrt{\left(\frac{1}{3}\right)^2 - 1} \sinh(1.99315 (-0.000976562577610000))} =$$

$$\frac{\cosh(1.99315 (-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315 (-0.000976562577610000))}{\left( 0.353553i \left( 1 + 0i \int_0^1 \cosh(-0.00194644t) dt + 1541.28 + 0i \int_{\frac{i\pi}{2}}^{-0.00194644} \sinh(t) dt \right) \right.}$$

$$\left. \int_0^1 \cosh(-0.00194644t) dt \right)$$

$$\frac{1}{\sqrt{\left(\frac{1}{3}\right)^2 - 1 \sinh(1.99315(-0.000976562577610000))}} =$$

$$\cosh(1.99315(-0.000976562577610000)) - \frac{1}{3} \sinh(1.99315(-0.000976562577610000))$$

$$\left(0.353553 i \left((1+0 i) i \pi \int_0^1 \cosh(-0.00194644 t) dt + (770.638 + 0 i) \sqrt{\pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{0.47155 \times 10^{-7}/s+s}}{\sqrt{s}} ds\right)\right)/$$

$$\left(i \pi \int_0^1 \cosh(-0.00194644 t) dt\right) \text{ for } \gamma > 0$$

From the following Ramanujan mock theta function, we obtain:

$$12 + (4267.24 / 8)$$

**Input interpretation:**

$$12 + \frac{4267.24}{8}$$

**Result:**

$$545.405$$

545.405 result practically equal to the solution of  $1/e^{\Lambda(r)} = 545.278$

Thence we have the following results:

$$-0.000029211892\dots = A(r) \quad 0.0003644621 = B(r) \quad -0.0005946833\dots = \phi(r)$$

$$0.00183393 = e^{\Lambda(r)}$$

We have the following ratio (A. Nardelli) concerning the general asymptotically flat solution of the equations of motion of the p-brane:

$$\begin{aligned}
A(r) &= \frac{(7-p)(3-p)c_1}{64} h(r) - \frac{7-p}{16} \ln [\cosh(kh(r)) - c_2 \sinh(kh(r))] , \\
B(r) &- \frac{1}{7-p} \ln [f_-(r)f_+(r)] \\
&- \frac{(p+1)(3-p)c_1}{64} h(r) + \frac{p+1}{16} \ln [\cosh(kh(r)) - c_2 \sinh(kh(r))] , \\
\phi(r) &= \frac{(p+1)(7-p)c_1}{16} h(r) + \frac{3-p}{4} \ln [\cosh(kh(r)) - c_2 \sinh(kh(r))] , \\
e^{\Lambda(r)} &= -\eta(c_2^2 - 1)^{1/2} \frac{\sinh(kh(r))}{\cosh(kh(r)) - c_2 \sinh(kh(r))} ,
\end{aligned}$$

$$A(r) \times \frac{1}{B(r)} \left( -\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}}$$

**Input interpretation:**

$$-0.000029211892 \times \frac{1}{0.0003644621} \left( -\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393}$$

**Result:**

$$73491.78832548118710549159572042220548025195726563413398700\dots$$

[73491.7883254...](#)

We have the following mathematical connection with the Karatsuba's equation concerning the Dirichlet series:

$$\begin{aligned}
I_{21} &\ll \int_{-\infty}^{+\infty} \exp \left( -\left( \frac{t}{H} \right)^2 \right) \left| \sum_{\lambda \leq P^{1-\varepsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(t+r)} \right|^2 dt \ll \\
&\ll H \left( \sum_{\lambda \leq P^{1-\varepsilon_2}} \frac{|a(\lambda)|^2}{\lambda} \cdot \frac{4^r}{(\log P \lambda^{-1})^{2r}} + |W_2| \right) \ll \\
&\ll H \left( \left( \frac{4}{\varepsilon_2 \log T} \right)^{2r} \sum_{\lambda \leq P} \frac{|a(\lambda)|^2}{\lambda} + |W_2| \right), \\
I_{21} &\ll H \left\{ \left( \frac{4}{\varepsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + \right. \\
&\quad \left. + (\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r}) T^{-\varepsilon_1} \right\} . \\
&= 793139765.05275
\end{aligned}$$

Indeed:

$$(793139765.05275/10^4) - 4096 - 2048 + 322$$

Where  $4096 = 64^2$ ,  $2048 = 64*32$  and 322 is a Lucas number

### Input interpretation:

$$\frac{7.9313976505275 \times 10^8}{10^4} - 4096 - 2048 + 322$$

### Result:

73491.976505275

**73491.976505275**

$$\left( \begin{array}{l} I_{21} \ll \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leq p^{1-\varepsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \\ \ll H \left( \sum_{\lambda \leq p^{1-\varepsilon_2}} \frac{|a(\lambda)|^2}{\lambda} \cdot \frac{4^r}{(\log P \lambda^{-1})^{2r}} + |W_2| \right) \ll \\ \ll H \left( \left( \frac{4}{\varepsilon_2 \log T} \right)^{2r} \sum_{\lambda \leq P} \frac{|a(\lambda)|^2}{\lambda} + |W_2| \right), \\ I_{21} \ll H \left\{ \left( \frac{4}{\varepsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + \right. \\ \left. + (\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r}) T^{-\varepsilon_1} \right\}. \end{array} \right) / 10^4 - 4096 - 2048 + 322$$

= 73491.976505275

$$\begin{aligned} A(r) \times \frac{1}{B(r)} \left( -\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}} &= \\ = -0.000029211892 \times \frac{1}{0.0003644621} \left( -\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} &= \\ = 73491.7883254... & \end{aligned}$$

$$\left( \begin{array}{l}
I_{21} \ll \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leq p^{1-\varepsilon_2}} \frac{a(\lambda)}{V\lambda} B(\lambda) \lambda^{-i(T+t)} \right|^3 dt \ll \\
\ll H \left( \sum_{\lambda \leq p^{1-\varepsilon_2}} \frac{|a(\lambda)|^2}{\lambda} \cdot \frac{4^r}{(\log P \lambda^{-1})^{2r}} + |W_2| \right) \ll \\
\ll H \left( \left(\frac{4}{\varepsilon_2 \log T}\right)^{2r} \sum_{\lambda \leq p} \frac{|a(\lambda)|^2}{\lambda} + |W_2| \right), \\
I_{21} \ll H \left\{ \left(\frac{4}{\varepsilon_2 \log T}\right)^{2r} (\log T) (\log X)^{-2\beta} + \right. \\
\left. + (\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r}) T^{-\varepsilon_1} \right\}. \end{array} \right) / 10^4 - 4096 - 2048 + 322$$

$$\approx A(r) \times \frac{1}{B(r)} \left( -\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}}$$

$$73491.976505275 \approx 73491.7883254\dots$$

$$\left( \begin{array}{l}
A(r) = \frac{(7-p)(3-p)c_1}{64} h(r) - \frac{7-p}{16} \ln [\cosh(kh(r)) - c_2 \sinh(kh(r))] , \\
B(r) = \frac{1}{7-p} \ln [f_-(r) f_+(r)] \\
- \frac{(p+1)(3-p)c_1}{64} h(r) + \frac{p+1}{16} \ln [\cosh(kh(r)) - c_2 \sinh(kh(r))] , \\
\phi(r) = \frac{(p+1)(7-p)c_1}{16} h(r) + \frac{3-p}{4} \ln [\cosh(kh(r)) - c_2 \sinh(kh(r))] , \\
e^{\Lambda(r)} = -\eta(c_2^2 - 1)^{1/2} \frac{\sinh(kh(r))}{\cosh(kh(r)) - c_2 \sinh(kh(r))}, \end{array} \right) \Rightarrow$$

$$\begin{aligned}
I_{21} &\ll \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leq P^{1-\varepsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \\
&\ll H \left( \sum_{\lambda \leq P^{1-\varepsilon_2}} \frac{|a(\lambda)|^2}{\lambda} \cdot \frac{4^r}{(\log P \lambda^{-1})^{2r}} + |W_2| \right) \ll \\
&\ll H \left( \left( \frac{4}{\varepsilon_2 \log T} \right)^{2r} \sum_{\lambda \leq P} \frac{|a(\lambda)|^2}{\lambda} + |W_2| \right), \\
I_{21} &\ll H \left\{ \left( \frac{4}{\varepsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + \right. \\
&\quad \left. + (\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r}) T^{-\varepsilon_1} \right\}.
\end{aligned}
\right)$$

/10<sup>4</sup>-4096-2048+322 =

$$= 73491.7883254... \approx 73491.976505275$$

Note that  $73491 = 64079 + 9349 + 55 + 8$ , where 64079 and 9349 are Lucas numbers, while 55 and 8 are Fibonacci numbers

We have that:

$$7/10^3 + (73491.7883254)^{1/22}$$

### Input interpretation:

$$\frac{7}{10^3} + \sqrt[22]{73491.7883254}$$

### Result:

$$1.671150782077...$$

$$1.671150782077...$$

We note that  $1.67115078...$  is a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-2} \text{ gm}$$

that is the holographic proton mass

$$\frac{7}{10^3} + \sqrt[22]{A(r) \times \frac{1}{B(r)} \left( -\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}}} = 1.671150782077 \dots$$

From the algebraic sum of the various results, we obtain:

$$1 / (((-0.000029211892 + 0.0003644621 - 0.0005946833 + 0.00183393))^1/13)$$

**Input interpretation:**

$$\frac{1}{\sqrt[13]{-0.000029211892 + 0.0003644621 - 0.0005946833 + 0.00183393}}$$

**Result:**

$$1.642875\dots$$

$$1.642875\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

And:

$$2 * \text{sqrt}(((6 * 1 / (((-0.000029211892 + 0.0003644621 - 0.0005946833 + 0.00183393))^1/13))))$$

**Input interpretation:**

$$\frac{1}{2 \sqrt[6]{\sqrt[13]{-0.000029211892 + 0.0003644621 - 0.0005946833 + 0.00183393}}}$$

**Result:**

$$6.279251\dots$$

$6.279251\dots = 2\pi r$  for  $r = 0.9993739\dots$ , radius that is very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{\frac{1}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{e^{-2\pi\sqrt{5}}} \approx 0.9991104684$$

From the following data

$$c_1 \geq 0 \text{ and } k \geq 0$$

$$(r_0^{7-p} \geq 0, c_2 \geq 0) \text{ for } p = 3, \dots, 6$$

$$p = 6; c_1 = 1/4 = 0.25$$

$$p = 6, c_1 = 1/4 = 0.25 \quad k = 1.9931523 \quad c_2 = 1/3 = 0.333333\dots$$

$$h(r) = -0.00097656257761\dots \quad \kappa = 4,1666\dots e-19$$

$$\omega_d = \frac{2\pi^{(d+1)/2}}{\Gamma((d+1)/2)}$$

$$2 * \text{Pi}^{1.5} / \text{gamma}(1.5) = 12.5664\dots$$

**Input:**

$$2 \times \frac{\pi^{1.5}}{\Gamma(1.5)}$$

$\Gamma(x)$  is the gamma function

**Result:**

$$12.5664\dots$$

$$4\pi \approx 12.56637061$$

$$12.5664\dots$$

**Alternative representations:**

$$\frac{2\pi^{1.5}}{\Gamma(1.5)} = \frac{2\pi^{1.5}}{\frac{1}{e^{0.120782}}}$$

•

$$\frac{2\pi^{1.5}}{\Gamma(1.5)} = \frac{2\pi^{1.5}}{\frac{0.947574}{1.06922}}$$

•

$$\frac{2\pi^{1.5}}{\Gamma(1.5)} = \frac{2\pi^{1.5}}{0.5!}$$

$n!$  is the factorial function

### Series representations:

$$\frac{2\pi^{1.5}}{\Gamma(1.5)} = \frac{2\pi^{1.5}}{\sum_{k=0}^{\infty} \frac{(1.5-z_0)^k \Gamma^{(k)}(z_0)}{k!}} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{2\pi^{1.5}}{\Gamma(1.5)} = 2\pi^{0.5} \sum_{k=0}^{\infty} (1.5-z_0)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right) \Gamma^{(j)}(1-z_0)}{j! (-j+k)!}$$

$\mathbb{Z}$  is the set of integers

### Integral representations:

$$\frac{2\pi^{1.5}}{\Gamma(1.5)} = \frac{2\pi^{1.5}}{\int_0^{\infty} e^{-t} t^{0.5} dt}$$

$$\frac{2\pi^{1.5}}{\Gamma(1.5)} = \frac{2\pi^{1.5}}{\int_0^1 \log^{0.5}\left(\frac{1}{t}\right) dt}$$

$$\frac{2\pi^{1.5}}{\Gamma(1.5)} = 2 \exp\left(-\int_0^1 \frac{0.5 - 1.5x + x^{1.5}}{(-1+x) \log(x)} dx\right) \pi^{1.5}$$

$\log(x)$  is the natural logarithm

From:

### Open String Tachyon in Supergravity Solution

*Shinpei Kobayashi, Tsuguhiko Asakawa and So Matsuura*

hep-th/0409044

From the view point of the gravity theory, the three-parameter solution describes a charged dilatonic black object. Thus, the RR-charge  $Q$  and the ADM mass  $M$  are natural quantities to characterize the solution.<sup>5</sup> For convenience, we consider wrapping the spatial world-volume directions on a torus  $T^p$  of volume  $V_p$ . The RR-charge is given by an appropriate surface integral over the sphere-at-infinity in the transverse directions [17, 23];

$$Q = 2\eta N_p(c_2^2 - 1)^{1/2} k r_0^{7-p}, \quad (2.18)$$

where

$$N_p \equiv \frac{(8-p)(7-p)\omega_{8-p}V_p}{16\kappa^2}, \quad (2.19)$$

and  $\omega_d = \frac{2\pi^{(d+1)/2}}{\Gamma((d+1)/2)}$  is the volume of the unit sphere  $S^d$ . The ADM mass is defined as [24, 25]

$$g_{00} = -1 + \frac{2\tilde{\kappa}^2 M}{(8-p)\omega_{8-p}r^{7-p}} + \mathcal{O}\left(\frac{1}{r^{2(7-p)}}\right), \quad (2.20)$$

where the metric is written in Einstein frame and  $\tilde{\kappa}^2 \equiv \kappa^2/V_p$ . Using this definition, we see that the ADM mass of the three-parameter solution is [17]

$$M = N_p \left( \frac{3-p}{2} c_1 + 2c_2 k \right) r_0^{7-p}. \quad (2.21)$$

$$N_p = \frac{(8-p)(7-p)\omega_{8-p}V_p}{16\kappa^2}, \quad (2.19)$$

$V_p = 2\pi^2$  (admitting a torus of unitary radii)

$$(2*12.5664 * 2*\text{Pi}^2) / (16*4.1666e-19^2) = 1.7860228654147...*10^{38} = N_p$$

### Input interpretation:

$$\frac{2 \times 12.5664 \times 2 \pi^2}{16 (4.1666 \times 10^{-19})^2}$$

### Result:

$$1.7860228654147015694312513651614336014199258757930940... \times 10^{38}$$

$$1.7860228654147...*10^{38}$$

see that the ADM mass of the three-parameter solution is [17]

$$M = N_p \left( \frac{3-p}{2} c_1 + 2c_2 k \right) r_0^{7-p}. \quad (2.21)$$

$$1.7860228654147 \times 10^{38} * (-\frac{3}{2} \times \frac{1}{4} + 2 \times \frac{1}{3} \times 1.9931523) \times 2 = M$$

**Input interpretation:**

$$1.7860228654147 \times 10^{38} \left( -\frac{3}{2} \times \frac{1}{4} + 2 \times \frac{1}{3} \times 1.9931523 \right) \times 2$$

**Result:**

$$3406903627010841345080000000000000000000$$

**Scientific notation:**

$$3.40690362701084134508 \times 10^{38}$$

$$3.406903627... \times 10^{38}$$

Inserting the value of the ADM mass  $3.406904e+38$  in the Hawking radiation calculator, we obtain:

$$\text{Mass} = 3.406904e+38$$

$$\text{Radius} = 5.058746e+11$$

$$\text{Temperature} = 3.602107e-16$$

From the Ramanujan-Nardelli mock formula, we obtain:

$$\sqrt{\left[ \left[ \left[ \left[ \left[ 1 / (((((4 * 1.962364415e+19) / (5 * 0.0864055^2)) * 1 / (3.406904e+38)) * \sqrt{[-((3.602107e-16 * 4 * \pi * (5.058746e+11)^3 - (5.058746e+11)^2))]) / ((6.67 * 10^{-11})]]] \right] \right] \right] \right]}$$

**Input interpretation:**

$$\sqrt{\left( \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{3.406904 \times 10^{38}} \right) \sqrt{\frac{-3.602107 \times 10^{-16} \times 4 \pi (5.058746 \times 10^{11})^3 - (5.058746 \times 10^{11})^2}{6.67 \times 10^{-11}}} \right)}$$

### Result:

1.618249336012945517815305624597541789224923815339885247692...

1.618249336...

And:

$$\frac{1}{\sqrt{[1/(((4*1.962364415e+19)/(5*0.0864055^2))*1/(3.406904e+38)*\sqrt{[-((3.602107e-16 * 4*\pi*(5.058746e+11)^3-(5.058746e+11)^2))]/((6.67*10^-11))]}]]}}$$

### Input interpretation:

$$1/\left(\sqrt{\left(1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{3.406904 \times 10^{38}}\right.\right.\right.\right. \\ \left.\left.\left.\left.\left.-\frac{3.602107 \times 10^{-16} \times 4 \pi (5.058746 \times 10^{11})^3 - (5.058746 \times 10^{11})^2}{6.67 \times 10^{-11}}\right)\right)\right)\right)}$$

### Result:

0.617951744360858056724317620844565903058624888618379241810...

0.617951744...

Now:

$x^\mu (\mu = 0, 1, \dots, p)$ $x^i (i = p+1, \dots, 9-p)$ $x^\mu (\mu = 0, 1, \dots, p)$ $x^i (i = p+1, \dots, 9-p)$ $M = \left( \frac{3-p}{2} c_1 + 2c_2 k \right) N_p r_0^{7-p}, Q = \pm 2(c_2^2 - 1)^{1/2} k N_p r_0^{7-p}.$	<b>New Parametrization</b> <div style="border: 1px solid blue; padding: 10px; margin-bottom: 10px;"> <math>r_0^{7-p} \equiv \frac{v \mu_0}{2k}, c_2^2 - 1 \equiv \frac{1}{v^2} (0 \leq v \leq \infty).</math> </div> <div style="border: 2px solid red; padding: 10px;"> <math>M = N_p \left( \sqrt{1+v^2} + \frac{3-p}{4k} c_1 v \right) \mu_0, Q = \pm N_p \mu_0.</math> </div> <div style="margin-top: 20px;"> <small>During the tachyon condensation, the RR-charge does not change its value. We need a new parametrization.</small> </div>
--	---

We have:

$$2(1/3^2 - 1)^{1/2} * 1.9931523 * 1.7860228654147e+38 * 2 = Q$$

**Input interpretation:**

$$2 \sqrt{\frac{1}{3^2} - 1} \times 1.9931523 \times 1.7860228654147 \times 10^{38} \times 2$$

**Result:**

$$1.3424905... \times 10^{39} i$$

**Polar coordinates:**

$$r = 1.34249 \times 10^{39} \text{ (radius)}, \theta = 90^\circ \text{ (angle)}$$

$$1.34249 * 10^{39}$$

$$47 + (((2(1/3^2 - 1)^{1/2} * 1.9931523 * 1.7860228654147e+38 * 2)))^{1/12}$$

Where 47 is a Lucas number

**Input interpretation:**

$$47 + \sqrt[12]{2 \sqrt{\frac{1}{3^2} - 1} \times 1.9931523 \times 1.7860228654147 \times 10^{38} \times 2}$$

**Result:**

$$1853.87386... +$$

$$237.879457... i$$

**Polar coordinates:**

$$r = 1869.07 \text{ (radius)}, \theta = 7.31194^\circ \text{ (angle)}$$

1869.07 result very near to the rest mass of D meson 1869.62

We note that:

$$M = 3.406903627... * 10^{38}$$

$$Q = 1.34249 * 10^{39}$$

$$(((3.406903627e+38 / 1.34249e+39)^{1/3}))$$

**Input interpretation:**

$$\sqrt[3]{\frac{3.406903627 \times 10^{38}}{1.34249 \times 10^{39}}}$$

**Result:**

0.633115476157163040355265609915937185840018535076327708420...

0.633115476157163...

And:

$$(1.34249e+39 / 3.406903627e+38)^{1/e}$$

**Input interpretation:**

$$\sqrt[e]{\frac{1.34249 \times 10^{39}}{3.406903627 \times 10^{38}}}$$

**Result:**

1.656117119728742727591993303619667194597807076633853589694...

1.656117119728.... We note that, the result 1,6561171... is practically equal to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

Indeed:

$$\sqrt[14]{\left( \sqrt{\frac{113+5\sqrt{505}}{8}} + \sqrt{\frac{105+5\sqrt{505}}{8}} \right)^3} = 1,65578 \dots$$

And:

$$(11/10^3+4/10^3)+(1.34249e+39 / 3.406903627e+38)^{1/e}$$

Where 4 and 11 are Lucas numbers

**Input interpretation:**

$$\left( \frac{11}{10^3} + \frac{4}{10^3} \right) + \sqrt[e]{\frac{1.34249 \times 10^{39}}{3.406903627 \times 10^{38}}}$$

**Result:**

1.671117119728742727591993303619667194597807076633853589694...

1.671117119....

We note that 1.671117... is a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

Further:

$$(-55/10^3 + 2/10^3 + 11/10^3 + 4/10^3) + (1.34249e+39 / 3.406903627e+38)^{1/e}$$

Where 2 and 55 are Fibonacci numbers, 4 and 11 are Lucas numbers

**Input interpretation:**

$$\left( -\frac{55}{10^3} + \frac{2}{10^3} + \frac{11}{10^3} + \frac{4}{10^3} \right) + \sqrt[e]{\frac{1.34249 \times 10^{39}}{3.406903627 \times 10^{38}}}$$

**Result:**

1.618117119728742727591993303619667194597807076633853589694...

1.6181171197....

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

Via the boundary state

$$h_{MN}^{(1)}(r) = -\frac{1}{8}(A+B)CN_p((7-p)\eta_{\mu\nu}, -(p+1)\delta_{ij})\frac{1}{r^{7-p}}$$

$$\phi^{(1)}(r) = -\frac{1}{4}CN_p[(p+1)A - (7-p)B]\frac{1}{r^{7-p}}$$

From the 3-parameter solution

$$h_{MN}^{(1)}(r) = -\frac{\mu_0}{8}\left(\sqrt{1+v^2} + \frac{3-p}{4}\frac{c_1}{k}v\right)((7-p)\eta_{\mu\nu}, -(p+1)\delta_{ij})\frac{1}{r^{7-p}}$$

$$\phi^{(1)}(r) = -\frac{\mu_0}{4}\left[\frac{3-p}{4}\sqrt{1+v^2} + \frac{(p+1)(7-p)}{16}\frac{c_1}{k}v\right]\frac{1}{r^{7-p}}$$

$$-3/8(((\sqrt{26})-3/4*(0.25*5)/1.9931523)) * (1-7)*1/4096 = h_{MN}^{(1)}(r)$$

**Input interpretation:**

$$-\frac{3}{8}\left(\sqrt{26} - \frac{3}{4} \times \frac{0.25 \times 5}{1.9931523}\right)(1-7) \times \frac{1}{4096}$$

**Result:**

0.002542598...

0.002542598...

We have also that:

$$\exp(((((-3/8(((\sqrt{26})-3/4*(0.25*5)/1.9931523))) * (1-7)*1/4096))))$$

**Input interpretation:**

$$\exp\left(-\frac{3}{8}\left(\sqrt{26} - \frac{3}{4} \times \frac{0.25 \times 5}{1.9931523}\right)(1-7) \times \frac{1}{4096}\right)$$

**Result:**

1.002545834...

1.002545834...

And:

$$-7/10^4 + \exp(((((-3/8(((\sqrt{26})-3/4*(0.25*5)/1.9931523))) * (1-7)*1/4096))))$$

### Input interpretation:

$$-\frac{7}{10^4} + \exp\left(-\frac{3}{8}\left(\sqrt{26} - \frac{3}{4} \times \frac{0.25 \times 5}{1.9931523}\right)(1-7) \times \frac{1}{4096}\right)$$

### Result:

1.001845834...

**1.001845834...** result practically equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \cfrac{e^{-6\pi}}{1 + \cfrac{e^{-8\pi}}{1 + \dots}}}}$$

$$\approx 1.0018674362$$

### Series representations:

$$-\frac{7}{10^4} + \exp\left(\frac{\left(\sqrt{26} - \frac{3(0.25 \times 5)}{4 \times 1.99315}\right)(-3)(1-7)}{8 \times 4096}\right) =$$

$$-\frac{7}{10000} + \exp\left(-0.000258377 + 0.000549316 \sqrt{25} \sum_{k=0}^{\infty} 25^{-k} \binom{\frac{1}{2}}{k}\right)$$

•

$$-\frac{7}{10^4} + \exp\left(\frac{\left(\sqrt{26} - \frac{3(0.25 \times 5)}{4 \times 1.99315}\right)(-3)(1-7)}{8 \times 4096}\right) =$$

$$-\frac{7}{10000} + \exp\left(-0.000258377 + 0.000549316 \sqrt{25} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{25}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

•

$$-\frac{7}{10^4} + \exp\left(\frac{\left(\sqrt{26} - \frac{3(0.25 \times 5)}{4 \times 1.99315}\right)(-3)(1-7)}{8 \times 4096}\right) =$$

$$-\frac{7}{10000} + \exp\left(-0.000258377 + \frac{0.000274658 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 25^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{\sqrt{\pi}}\right)$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\operatorname{Res}_{z=z_0} f$  is a complex residue

$$-3/4((((-3/4*\sqrt{1+5^2})+7/16*(0.25*5)/1.9931523)))*1/4096 = \phi^{(1)}(\mathbf{r})$$

**Input interpretation:**

$$-\frac{3}{4} \left( -\frac{3}{4} \sqrt{1+5^2} + \frac{7}{16} \times \frac{0.25 \times 5}{1.9931523} \right) \times \frac{1}{4096}$$

**Result:**

0.0006500039...

0.0006500039...

And the inverse:

$$1/(((-3/4((((-3/4*\sqrt{1+5^2})+7/16*(0.25*5)/1.9931523)))*1/4096)))$$

**Input interpretation:**

$$\frac{1}{-\frac{3}{4} \left( -\frac{3}{4} \sqrt{1+5^2} + \frac{7}{16} \times \frac{0.25 \times 5}{1.9931523} \right) \times \frac{1}{4096}}$$

**Result:**

1538.452...

1538.452... result very near to the rest mass of Xi baryon 1535

And:

$$-13/10^3 + [1/(((-3/4((((-3/4*\sqrt{1+5^2})+7/16*(0.25*5)/1.9931523)))*1/4096)))]^{1/15}$$

**Input interpretation:**

$$-\frac{13}{10^3} + \sqrt[15]{\frac{1}{-\frac{3}{4} \left(-\frac{3}{4} \sqrt{1+5^2} + \frac{7}{16} \times \frac{0.25 \times 5}{1.9931523}\right) \times \frac{1}{4096}}}$$

### **Result:**

1.6180688...

1.6180688...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

From the two results, we have also:

$$1 + (0.002542598 - 0.0006500039)$$

### **Input interpretation:**

$1 + (0.002542598 - 0.0006500039)$

### **Result:**

1.0018925941

**1.0018925941** result practically equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \cfrac{e^{-6\pi}}{1 + \cfrac{e^{-8\pi}}{1 + \dots}}}} \approx 1.0018674362$$

We have also, from 3-parameter solution, that:

$$(4267.24+11+3+2)+1/\text{Pi}*1/(0.002542598 * 0.0006500039)$$

Where 4267.24 are a result of a Ramanujan mock theta function and 2, 3 and 11 are Lucas numbers

### **Input interpretation:**

$$(4267.24 + 11 + 3 + 2) + \frac{1}{\pi \times \frac{1}{0.002542598 \times 0.0006500039}}$$

### Result:

196883.3...

**196883.3...** result practically equal to the value of the following partition function, concerning the j-invariants:

$$\begin{aligned} Z_{24}(\tau) &= j(\tau) - 744 \\ &= q^{-1} + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \dots \end{aligned}$$

### Alternative representations:

$$(4267.24 + 11 + 3 + 2) + \frac{1}{(0.0025426 \times 0.000650004)\pi} =$$

$$4283.24 + \frac{1}{1.6527 \times 10^{-6} (180^\circ)}$$

$$(4267.24 + 11 + 3 + 2) + \frac{1}{(0.0025426 \times 0.000650004)\pi} =$$

$$4283.24 + \frac{1}{1.6527 \times 10^{-6} (-i \log(-1))}$$

$$(4267.24 + 11 + 3 + 2) + \frac{1}{(0.0025426 \times 0.000650004)\pi} =$$

$$4283.24 + \frac{1}{1.6527 \times 10^{-6} \cos^{-1}(-1)}$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$\cos^{-1}(x)$  is the inverse cosine function

### Series representations:

$$(4267.24 + 11 + 3 + 2) + \frac{1}{(0.0025426 \times 0.000650004)\pi} = 4283.24 + \frac{151268.}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$(4267.24 + 11 + 3 + 2) + \frac{1}{(0.0025426 \times 0.000650004) \pi} = 4283.24 + \frac{302535.}{-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}$$

$$(4267.24 + 11 + 3 + 2) + \frac{1}{(0.0025426 \times 0.000650004) \pi} = 4283.24 + \frac{605071.}{\sum_{k=0}^{\infty} \frac{2^{-k} (-6+50k)}{\binom{3k}{k}}}$$

$\binom{n}{m}$  is the binomial coefficient

### Integral representations:

$$(4267.24 + 11 + 3 + 2) + \frac{1}{(0.0025426 \times 0.000650004) \pi} = 4283.24 + \frac{302535.}{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

$$(4267.24 + 11 + 3 + 2) + \frac{1}{(0.0025426 \times 0.000650004) \pi} = 4283.24 + \frac{151268.}{\int_0^1 \sqrt{1-t^2} dt}$$

$$(4267.24 + 11 + 3 + 2) + \frac{1}{(0.0025426 \times 0.000650004) \pi} = 4283.24 + \frac{302535.}{\int_0^{\infty} \frac{\sin(t)}{t} dt}$$

### Note

From:

### Rotating strings confronting PDG mesons

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arXiv:1402.5603v1 [hep-ph] 23 Feb 2014

The slopes that were found, on the other hand, are quite uniform for the light quark trajectories. The  $(J, M^2)$  trajectories have a slope in the range  $0.80 - 0.90 \text{ GeV}^{-2}$ , and this slightly decreases to  $0.78 - 0.84 \text{ GeV}^{-2}$  in the  $(n, M^2)$  plane fits. The slope for the strange meson is also in this range, while for the  $s\bar{s}$  states the optimum is found with a higher slope of around  $1.1 \text{ GeV}^{-2}$ , in both planes. The charmed and  $c\bar{c}$  mesons are best fitted in the  $(J, M^2)$  plane with a similar value of approximately  $1 \text{ GeV}^{-2}$  for the slope - and only when adding the appropriate mass for the  $c$  quark. This is what allows the universal slope fit in section 4.1.5, which had an optimum for the slope  $\alpha' = 0.884 \text{ GeV}^{-2}$ .

#### 4.4 Summary of results for the mesons

Table (1) summarizes the results of the fits for the mesons in the  $(J, M^2)$  plane. Tables (2) and (3) likewise summarize the results of the two types of fits for the  $(n, M^2)$  trajectories, that of the rotating string and of the WKB approximation.

The higher values of  $\alpha'$  and  $a$  always correspond to higher values of the endpoint masses, and the ranges listed are those where  $\chi^2$  is within 10% of its optimal value.

Traj.	$N$	$m$	$\alpha'$	$a$
$\pi/b$	4	$m_{u/d} = 90 - 185$	0.808 – 0.863	$(-0.23) - 0.00$
$\rho/a$	6	$m_{u/d} = 0 - 180$	0.883 – 0.933	$0.47 - 0.66$
$\eta/h$	5	$m_{u/d} = 0 - 70$	0.839 – 0.854	$(-0.25) - (-0.21)$
$\omega$	6	$m_{u/d} = 0 - 60$	0.910 – 0.918	$0.45 - 0.50$
$K^*$	5	$m_{u/d} = 0 - 240 \quad m_s = 0 - 390$	0.848 – 0.927	$0.32 - 0.62$
$\phi$	3	$m_s = 400$	1.078	0.82
$D$	3	$m_{u/d} = 80 \quad m_c = 1640$	1.073	-0.07
$D_s^*$	3	$m_s = 400 \quad m_c = 1580$	1.093	0.89
$\Psi$	3	$m_c = 1500$	0.979	-0.09
$\Upsilon$	3	$m_b = 4730$	0.635	1.00

**Table 1.** The results of the meson fits in the  $(J, M^2)$  plane. For the uneven  $K^*$  fit the higher values of  $m_s$  require  $m_{u/d}$  to take a correspondingly low value.  $m_{u/d} + m_s$  never exceeds 480 MeV, and the highest masses quoted for the  $s$  are obtained when  $m_{u/d} = 0$ . The ranges listed are those where  $\chi^2$  is within 10% of its optimal value.  $N$  is the number of data points in the trajectory.

Traj.	$N$	$m$	$\alpha'$	$a$
$\pi/\pi_2$	$4 + 3$	$m_{u/d} = 110 - 250$	0.788 – 0.852	$a_0 = (-0.22) - (-0.00) \quad a_2 = (-0.00) - 0.26$ $(-0.18) - 0.21$
$a_1$	4	$m_{u/d} = 0 - 390$	0.783 – 0.849	$(-0.14) - (-0.02)$
$h_1$	4	$m_{u/d} = 0 - 235$	0.833 – 0.850	
$\omega/\omega_3$	$5 + 3$	$m_{u/d} = 255 - 390$	0.988 – 1.18	$a_1 = 0.81 - 1.00 \quad a_3 = 0.95 - 1.15$
$\phi$	3	$m_s = 510 - 520$	1.072 – 1.112	1.00
$\Psi$	4	$m_c = 1380 - 1460$	0.494 – 0.547	0.71 – 0.88
$\Upsilon$	6	$m_b = 4725 - 4740$	0.455 – 0.471	1.00
$\chi_b$	3	$m_b = 4800$	0.499	0.58

**Table 2.** The results of the meson fits in the  $(n, M^2)$  plane. The ranges listed are those where  $\chi^2$  is within 10% of its optimal value.  $N$  is the number of data points in the trajectory.

The tensor structure of the closed string amplitude (6.6.19) is just two copies of that in the open string amplitude (6.5.15), if one sets  $\alpha' = 2$  in the closed string and  $\alpha' = 1/2$  in the open.

This is what allows the universal slope fit in section 4.1.5, which had an optimum for the slope  $\alpha' = 0.884 \text{ GeV}^2$ . Other values are 0.90904; 0.979

$$(1.07 + 0.98 + 0.884 + 0.90904) \div 4 = 0.96076$$

$$0.884 + 0.97 = 1.854 \div 2 = 0.927$$

$$(0.979 + 0.96076 + 0.927) / 2 = 0.95558666....$$

We take an average value for  $\alpha'$  that is **0.95686** that correspond also with the result of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

$$T^8, T^9 \rightarrow \infty (u^8, u^9 \rightarrow \infty)$$

$$S = 2T_{D9} \int d^{10}x \exp\left(-\frac{\pi}{2}(u^8 x^8)^2 + (u^9 x^9)^2\right) F(\pi \alpha' u^8) F(\pi \alpha' u^9)$$

$$= 2T_{D9} \int d^8x \sqrt{\frac{2}{u^8}} F(\pi \alpha' u^8) \sqrt{\frac{2}{u^9}} F(\pi \alpha' u^9)$$

$$\xrightarrow{u^8, u^9 \rightarrow \infty} 4\pi^2 \alpha'^2 T_{D9} \int d^8x$$

$$(2\pi\sqrt{\alpha'})^2 T_{D9} = T_{D7}$$

*D9  $\overline{D9}$  action  $\rightarrow$  BPS D7 action*

$$((2\pi * \sqrt{0.95686}))^2$$

**Input:**

$$(2\pi \sqrt{0.95686})^2$$

**Result:**

$$37.7753\dots$$

$$37.7753\dots = 37.7753 T_{D9} = T_{D7}$$

$$\boxed{T^9 \rightarrow \infty (u^9 \rightarrow \infty)}$$

$$S = 2T_{D9} \int d^{10}x \exp\left(-\frac{\pi}{2}(u^9 x^9)^2\right) F(\pi \alpha' u^9)$$

$$= 2T_{D9} \int d^9x \sqrt{\frac{2}{u^9}} F(\pi \alpha' u^9)$$

$$\xrightarrow{u^9 \rightarrow \infty} 2\pi \sqrt{2\alpha'} T_{D9} \int d^9x$$

$$\sqrt{2}(2\pi \alpha') T_{D9} = T_{D8}$$

$$D9\overline{D9} \text{ action} \rightarrow \text{non-BPS D8 action}$$

$$\sqrt{2} * ((2\pi * (0.95686)))$$

**Input:**

$$\sqrt{2} (2\pi * 0.95686)$$

**Result:**

$$8.502433936412428654239231768298950744912773819392045893722\dots$$

$$8.5024339364\dots T_{D9} = T_{D8}$$

• **Series representations:**

$$\sqrt{2} 2(\pi 0.95686) = 1.91372 \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\sqrt{2} \cdot 2(\pi \cdot 0.95686) = 1.91372 \pi \exp\left(i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

for ( $x \in \mathbb{R}$  and  $x < 0$ )

$$\sqrt{2} \cdot 2(\pi \cdot 0.95686) = 1.91372 \pi \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}$$

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\mathbb{R}$  is the set of real numbers

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

We note that:

$$37.7753 T_{D9} = T_{D7}$$

$$8.5024339364\dots T_{D9} = T_{D8}$$

$$37.7753 / 8.5024339364$$

**Input interpretation:**

$$\frac{37.7753}{8.5024339364}$$

**Result:**

$$4.442880742451775011712280359353689879262182608071384485118\dots$$

$$4.442880742451775\dots = T_{D7} = T_{D8}$$

And:

$$((((37.7753) / (8.5024339364))))^{1/3}$$

**Input interpretation:**

$$\sqrt[3]{\frac{37.7753}{8.5024339364}}$$

### Result:

1.643948539232212161595302367284254048192904618961750283923...

$$1.64394853923\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

### Asymptotic Behavior of the Solution

$$\begin{aligned} e^{2A(r)} &= 1 - \frac{7-p}{8} \left( \sqrt{1+v^2} + \frac{3-p}{4} \frac{c_1}{k} v \right) \frac{\mu_0}{r^{7-p}} + O\left(\frac{1}{r^{2(7-p)}}\right), \\ e^{2B(r)} &= 1 + \frac{p+1}{8} \left( \sqrt{1+v^2} + \frac{3-p}{4} \frac{c_1}{k} v \right) \frac{\mu_0}{r^{7-p}} + O\left(\frac{1}{r^{2(7-p)}}\right), \\ \phi(r) &= \left( \frac{3-p}{4} \sqrt{1+v^2} - \frac{(p+1)(7-p)}{16} \frac{c_1}{k} v \right) \frac{\mu_0}{r^{7-p}} + O\left(\frac{1}{r^{2(7-p)}}\right), \\ C(r) &= \pm \frac{\mu_0}{r^{7-p}} + O\left(\frac{1}{r^{2(7-p)}}\right). \end{aligned}$$

We have the following results:

$$1 - 1/8(((\sqrt{26}) - 3/4 * (0.25 * 5) / 1.9931523)) * 1/4096 + 1/(4096)^2 = e^{2A(r)}$$

### Input interpretation:

$$1 - \frac{1}{8} \left( \sqrt{26} - \frac{3}{4} \times \frac{0.25 \times 5}{1.9931523} \right) \times \frac{1}{4096} + \frac{1}{4096^2}$$

### Result:

0.99985880414...

**0.99985880414**.... result very near to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3} - 1}}} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}}$$

$$1 + 7/8(((\text{sqrt}(26) - 3/4 * (0.25 * 5) / 1.9931523)) * 1/4096 + 1/(4096)^2) = e^{2B(r)}$$

**Input interpretation:**

$$1 + \frac{7}{8} \left( \sqrt{26} - \frac{3}{4} \times \frac{0.25 \times 5}{1.9931523} \right) \times \frac{1}{4096} + \frac{1}{4096^2}$$

**Result:**

$$1.000988847858121204799682038926411964230504955860488061233\dots$$

$$1.0009888478581212\dots$$

The inverse is:

$$1 / (((((1 + 7/8(((\text{sqrt}(26) - 3/4 * (0.25 * 5) / 1.9931523)) * 1/4096 + 1/(4096)^2))))$$

**Input interpretation:**

$$\frac{1}{1 + \frac{7}{8} \left( \sqrt{26} - \frac{3}{4} \times \frac{0.25 \times 5}{1.9931523} \right) \times \frac{1}{4096} + \frac{1}{4096^2}}$$

**Result:**

$$0.9990121290\dots$$

**0.9990121290....** result very near to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3} - 1}}} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}}$$

$$(((((-3/4*\sqrt{1+5^2})-7/16*(0.25*5)/1.9931523)))*1/4096+1/(4096)^2) = \phi(r)$$

**Input interpretation:**

$$\left( -\frac{3}{4} \sqrt{1+5^2} - \frac{7}{16} \times \frac{0.25 \times 5}{1.9931523} \right) \times \frac{1}{4096} + \frac{1}{4096^2}$$

**Result:**

-0.001000585...

-0.001000585....

The inverse is:

$$1 / ((((((((-3/4*\sqrt{1+5^2})-7/16*(0.25*5)/1.9931523)))*1/4096+1/(4096)^2)))$$

**Input interpretation:**

$$\frac{1}{\left( -\frac{3}{4} \sqrt{1+5^2} - \frac{7}{16} \times \frac{0.25 \times 5}{1.9931523} \right) \times \frac{1}{4096} + \frac{1}{4096^2}}$$

**Result:**

-999.4150...

-999.4150....

We can to obtain:

$$-(47+18-4) - 1 / ((((((((-3/4*\sqrt{1+5^2})-7/16*(0.25*5)/1.9931523)))*1/4096+1/(4096)^2)))$$

where 4, 18 and 47 are Lucas numbers

**Input interpretation:**

$$-(47+18-4) - \frac{1}{\left( -\frac{3}{4} \sqrt{1+5^2} - \frac{7}{16} \times \frac{0.25 \times 5}{1.9931523} \right) \times \frac{1}{4096} + \frac{1}{4096^2}}$$

**Result:**

938.4150...

938.4150.... result practically equal to the proton rest mass in MeV 938.272

$$1/4096 + 1/4096^2 = \mathbf{C(r)}$$

**Input:**

$$\frac{1}{4096} + \frac{1}{4096^2}$$

**Exact result:**

$$\frac{4097}{16777216}$$

**Decimal form:**

$$0.000244200229644775390625$$

$$0.00024420022964\dots$$

The inverse is:

$$1/(((1/4096+ 1/(4096)^2)))$$

**Input:**

$$\frac{1}{\frac{1}{4096} + \frac{1}{4096^2}}$$

**Exact result:**

$$\frac{16777216}{4097}$$

**Decimal approximation:**

$$4095.000244081034903587991213082743470832316329021235050036\dots$$

$$4095.00024408\dots$$

And:

$$-4+1/4*1/(((1/4096+ 1/(4096)^2)))$$

**Input:**

$$-4 + \frac{1}{4} \times \frac{1}{\frac{1}{4096} + \frac{1}{4096^2}}$$

**Exact result:**

$$\begin{array}{r} 4177916 \\ \hline 4097 \end{array}$$

### Decimal approximation:

1019.750061020258725896997803270685867708079082255308762509...

1019.75006102.... result practically equal to the rest mass of Phi meson 1019.445

The algebraic sum of the results is:

$$0.99985880414 + 1.0009888478581212 - 0.001000585 + 0.00024420022964$$

$$(0.99985880414 + 1.0009888478581212 - 0.001000585 + 0.00024420022964)$$

### Input interpretation:

$$0.99985880414 + 1.0009888478581212 - 0.001000585 + 0.00024420022964$$

### Result:

$$2.0000912672277612$$

$$2.0000912672277612$$

The inverse is:

$$1/(0.99985880414 + 1.0009888478581212 - 0.001000585 + 0.00024420022964)$$

### Input interpretation:

$$\frac{1}{0.99985880414 + 1.0009888478581212 - 0.001000585 + 0.00024420022964}$$

### Result:

$$0.499977184234225545741654252536279854401881080172529088355...$$

$$0.49997718423422.... \approx 0.5 = 1/2$$

Note:

From Wikipedia

Apart from the trivial zeros, the Riemann zeta function has no zeros to the right of  $\sigma = 1$  and to the left of  $\sigma = 0$  (neither can the zeros lie too close to those lines). Furthermore, the non-trivial zeros are symmetric about the real axis and the line  $\sigma = 1/2$  and, according to the Riemann hypothesis, they all lie on the line  $\sigma = 1/2$

From the multiplication of the results, we obtain:

$$-(0.99985880414 * 1.0009888478581212 * -0.001000585 * 0.00024420022964)$$

**Input interpretation:**

$$-(0.99985880414 \times 1.0009888478581212 \times (-0.001000585) \times 0.00024420022964)$$

**Result:**

$$2.445501705645899835428343404735872067240592 \times 10^{-7}$$

**Repeating decimal:**

$$2.4455017056458998354283434047358720672405920000000 \times 10^{-7}$$

$$2.44550170564589\dots \times 10^{-7}$$

From the inverse, we obtain:

$$\frac{1}{-(0.99985880414 * 1.0009888478581212 * -0.001000585 * 0.00024420022964)}$$

**Input interpretation:**

$$\frac{1}{0.99985880414 \times 1.0009888478581212 \times (-0.001000585) \times 0.00024420022964}$$

**Result:**

$$4.08914047244911850993853057081518139940255597915679117\dots \times 10^6$$

$$4.0891404724491185\dots \times 10^6$$

From the ratio, we obtain:

$$(0.99985880414 * 1 / 1.0009888478581212 * 1 / -0.001000585 * 1 / 0.00024420022964)$$

**Input interpretation:**

$$0.99985880414 \times \frac{1}{1.0009888478581212} \left( -\frac{1}{0.001000585} \right) \times \frac{1}{0.00024420022964}$$

**Result:**

$$-4.0879858145599941205268133058325493813834949177292476\dots \times 10^6$$

$$-4.087985814559\dots \times 10^6$$

We have that:

$$11/10^3 + (4.087985814559 \times 10^6)^{1/31}$$

Where 11 is a Lucas number:

**Input interpretation:**

$$\frac{11}{10^3} + \sqrt[3]{4.087985814559 \times 10^6}$$

**Result:**

1.6450844457876...

$$1.6450844457876\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

The ratio between the two results, is:

$$-(4.0891404724491185e+6 / -4.08798581455999412 \times 10^6)$$

**Input interpretation:**

$$-\left(-\frac{4.0891404724491185 \times 10^6}{4.08798581455999412 \times 10^6}\right)$$

**Result:**

1.000282451540074304376823712036953868202129874390888064579...

1.00028245154.....

$$-11/10^3 + ((((-(4.0891404724491185e+6 / -4.08798581455999412 \times 10^6))))^{1728})$$

**Input interpretation:**

$$-\frac{11}{10^3} + \left(-\left(-\frac{4.0891404724491185 \times 10^6}{4.08798581455999412 \times 10^6}\right)\right)^{1728}$$

**Result:**

1.618066815879239949269228345354389673035579356651229485707...

1.618066815879....

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

From:

## **Open String Tachyon in Supergravity Solution**

*Shinpei Kobayashi, Tsuguhiko Asakawa and So Matsuura*

arXiv:hep-th/0409044v4 13 Jun 2005

We have also that:

Next, from the graviton and the dilaton, we see that the parameters must satisfy the following relations simultaneously;

$$1 + \frac{2}{N - \bar{N}} \text{tr } e^{-|T|^2} = \sqrt{1 + v^2} + \frac{3 - p}{4} \frac{c_1 v}{k} = \sqrt{1 + v^2} - \frac{(p + 1)(7 - p)}{4(3 - p)} \frac{c_1 v}{k}, \quad (3.31)$$

$$\text{sqrt}(1+5^2)+[((((7/(12)*(0.25*5)/1.9931523))))]$$

### **Input interpretation:**

$$\sqrt{1+5^2} + \frac{7}{12} \times \frac{0.25 \times 5}{1.9931523}$$

### **Result:**

5.464855...

5.464855....

And:

$$\text{sqrt}(1+5^2)+(((3/(4)*(0.25*5)/1.9931523)))$$

### **Input interpretation:**

$$\sqrt{1+5^2} + \frac{3}{4} \times \frac{0.25 \times 5}{1.9931523}$$

### **Result:**

5.569380...

5.569380

# Effective action for unstable D-brane

Kraus-Larsen ('01)

$$S = 2T_{D9} \int d^{10}x \exp(-2\pi\alpha' |T|^2) \prod_{I=1}^k F(\pi\alpha'^2 (\partial_I T^I)^2)$$

$$F(x) \equiv \frac{4^x x \Gamma(x)}{2 \Gamma(2x)} \sim \begin{cases} 1 + 2(\ln 2)x + O(x^2) & (x \rightarrow 0) \\ \sqrt{\pi x} + O(x^{-1/2}) & (x \rightarrow \infty) \end{cases}$$

$$T^I(x^I) = \alpha'^{-1/2} u_G^I X^I : \text{linear tachyon}$$

$$\exp(-2\pi\alpha' |T|^2) \rightarrow \underline{\exp(-2u_I^2(X^I)^2)}$$

Gaussian brane

Thus, when  $T = 0$  (i.e.  $\text{tr } e^{-|T|^2} = \bar{N}$ ), the factor is the tension of the sum of  $N$  D-branes and  $\bar{N}$   $\bar{D}$ -branes, while, when  $|T| \sim \infty$ , the tension becomes that of  $(N - \bar{N})$  D-branes.

We have the following equation concerning the Gaussian brane:

$$\exp(-2\pi\alpha' |T|^2) \rightarrow \exp(-2u_I^2(X^I)^2)$$

From:

**Modular equations and approximations to  $\pi$**  - Srinivasa Ramanujan  
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

But we know that

$$\begin{aligned} 64e^{-\pi\sqrt{n}} g_n^{24} &= 1 - 24e^{-\pi\sqrt{n}} + 276e^{-2\pi\sqrt{n}} - \dots, \\ 64g_n^{24} &= e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \dots, \\ 64a - 64bg_n^{-24} + \dots &= e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \dots, \\ 64a - 4096be^{-\pi\sqrt{n}} + \dots &= e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \dots, \end{aligned}$$

We have that:

$$1 - 24 \exp(-\pi \sqrt{5}) + 276 \exp(-2\pi \sqrt{5})$$

**Input:**

$$1 - 24 \exp(-\pi \sqrt{5}) + 276 \exp(-2\pi \sqrt{5})$$

**Exact result:**

$$1 + 276 e^{-2\sqrt{5}\pi} - 24 e^{-\sqrt{5}\pi}$$

**Decimal approximation:**

$$0.978869614522638054627885737812212890555439677240268836037\dots$$

$$0.978869614522638\dots$$

**Alternate forms:**

$$1 + e^{-2\sqrt{5}\pi} (276 - 24 e^{\sqrt{5}\pi})$$

$$e^{-2\sqrt{5}\pi} (276 - 24 e^{\sqrt{5}\pi} + e^{2\sqrt{5}\pi})$$

**Series representations:**

$$1 - 24 \exp(-\pi \sqrt{5}) + 276 \exp(-2\pi \sqrt{5}) = \\ 1 + 276 \exp\left(-2\pi \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right) - 24 \exp\left(-\pi \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)$$

$$1 - 24 \exp(-\pi \sqrt{5}) + 276 \exp(-2\pi \sqrt{5}) = \\ 1 + 276 \exp\left(-2\pi \sqrt{4} \sum_{k=0}^{\infty} \frac{(-\frac{1}{4})^k (-\frac{1}{2})_k}{k!}\right) - 24 \exp\left(-\pi \sqrt{4} \sum_{k=0}^{\infty} \frac{(-\frac{1}{4})^k (-\frac{1}{2})_k}{k!}\right)$$

$$1 - 24 \exp(-\pi \sqrt{5}) + 276 \exp(-2\pi \sqrt{5}) = \\ 1 + 276 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}\right) - \\ 24 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\operatorname{Res}_{z=z_0} f$  is a complex residue

We have that:

$$\exp(-2\pi i \sqrt{5})$$

**Input:**

$$\exp(-2\pi i \sqrt{5})$$

**Exact result:**

$$e^{-2\sqrt{5}\pi}$$

**Decimal approximation:**

$$7.9126772531148564617976195047972304547693255522093231\dots \times 10^{-7}$$

$$7.9126772531\dots \times 10^{-7}$$

**Property:**

$e^{-2\sqrt{5}\pi}$  is a transcendental number

**Series representations:**

$$e^{-2\pi i \sqrt{5}} = e^{-2\pi i \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{1/2}{k}}$$

•

$$e^{-2\pi i \sqrt{5}} = \exp \left( -2\pi i \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)$$

•

$$e^{-2\pi i \sqrt{5}} = \exp \left( -\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right)$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\operatorname{Res}_{z=z_0} f$  is a complex residue

and we observe that obtain:

$$1/276 * (((0.978869614522638 - 1 + 24 * \exp(-\pi * \sqrt{5})))$$

**Input interpretation:**

$$\frac{1}{276} \left( 0.978869614522638 - 1 + 24 \exp(-\pi \sqrt{5}) \right)$$

**Result:**

$$7.9126772531... \times 10^{-7}$$

$$7.9126772531.... * 10^{-7}$$

Thence, we set  $-2\pi\alpha'|T|^2 = -2u_l^2(X^l)^2 = -2\pi\sqrt{5}$  and obtain:

$$\exp(-2\pi\alpha'|T|^2) \rightarrow \exp(-2u_l^2(X^l)^2) = \exp(-2\pi\sqrt{5}) = 7.9126772531.... * 10^{-7}$$

In conclusion, we have for S:

$$2 * 4.442880742451775 * \int [ \exp(-2\pi\sqrt{5}) * (((3 * 4^3 \gamma(3))) / ((2 * \gamma(6)))) ] x dx$$

**Input interpretation:**

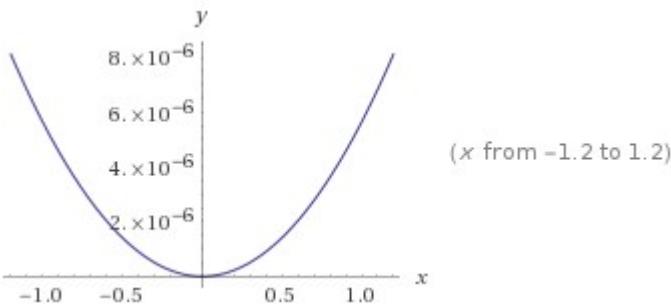
$$2 * 4.442880742451775 \int \left( \exp(-2\pi\sqrt{5}) * \frac{3 * 4^3 \gamma(3)}{2 \gamma(6)} \right) x dx$$

$\Gamma(x)$  is the gamma function

**Result:**

$$5.624813022256033 \times 10^{-6} x^2$$

$$5.624813022256033 * 10^{-6} x^2$$

**Plot:**

And for  $x = 0.9502566373$

$$(5.624813022256033 \times 10^{-6} * 0.9502566373^2)$$

**Input interpretation:**

$$5.624813022256033 \times 10^{-6} \times 0.9502566373^2$$

**Result:**

$$5.07913684302283311736114563793403257 \times 10^{-6}$$

$$5.07913684302283.... \times 10^{-6}$$

And:

$$1/(5.624813022256033 \times 10^{-6} * 0.9502566373^2)$$

**Input interpretation:**

$$\frac{1}{5.624813022256033 \times 10^{-6} \times 0.9502566373^2}$$

**Result:**

$$196883.8467846542578415527390192438982632066243000478961182...$$

196883.84678465.... result practically equal to the value of the following partition function, concerning the j-invariants:

$$\begin{aligned} Z_{24}(\tau) &= j(\tau) - 744 \\ &= q^{-1} + 196884 q + 21493760 q^2 + 864299970 q^3 + 20245856256 q^4 + \dots \end{aligned}$$

With regard the value **0.9502566373** is very near to the result of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}-\varphi+1}} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}$$

We have indeed:

$$0.9568666373 - 0.00661 = 0.9502566373; \quad 0.00661 = 661/10^5 \text{ and}$$

$661 = 610 + 47 + 4$  where 610 is a Fibonacci number, 4 and 47 are Lucas numbers

From:

**On certain trigonometrical sums and their applications in the theory of numbers**  
*Srinivasa Ramanujan* - Transactions of the Cambridge Philosophical Society, XXII,  
 No.13, 1918, 259 – 276

In all these equations the series on the right hand are finite Dirichlet's series and therefore absolutely convergent.

$$\begin{aligned} \{1^{-s} + (-3)^{-s} + 5^{-s} + (-7)^{-s} + \dots\} \delta_{2s}(n) &= \frac{\pi^s n^{s-1}}{(s-1)!} \left\{ 1^{-s} \left( \frac{\sin n\pi}{\sin n\pi} \right) \right. \\ &\quad \left. + 2^{-s} \left( \frac{\sin n\pi}{\sin(\frac{1}{2}n\pi + \frac{1}{2}s\pi)} \right) + 3^{-s} \left( \frac{\sin n\pi}{\sin(\frac{1}{3}n\pi + s\pi)} \right) + 4^{-s} \left( \frac{\sin n\pi}{\sin(\frac{1}{4}n\pi + \frac{3}{2}s\pi)} \right) + \dots \right\} \end{aligned}$$

$$s > 1; \quad s = 2, n = 5 \quad (14.4)$$

$$\text{Pi}^2 * 5 / 1! (((((1^2 - 2 * (\sin(5\pi) / \sin(5\pi)) + 2^2 - 2 * ((\sin(5\pi)) / (\sin(5\pi/2 + 2\pi/2)))) + 3^2 - 2 * ((\sin(5\pi)) / (\sin(5\pi/3 + 2\pi/3)))) + 4^2 - 2 * ((\sin(5\pi)) / (\sin(5\pi/4 + 6\pi/2)))))))$$

$$\text{Pi}^2 * 5 / 1!$$

**Input:**

$$\pi^2 * \frac{5}{1!}$$

$n!$  is the factorial function

**Exact result:**

$$5\pi^2$$

**Decimal approximation:**

49.34802200544679309417245499938075567656849703620395313206...

49.348022005446793...

**Property:**

$5\pi^2$  is a transcendental number

**Alternative representations:**

$$\frac{\pi^2 5}{1!} = \frac{5\pi^2}{\Gamma(2)}$$

$$\frac{\pi^2 5}{1!} = \frac{5\pi^2}{\Gamma(2, 0)}$$

$$\frac{\pi^2 5}{1!} = \frac{5\pi^2}{(1)_1}$$

$\Gamma(x)$  is the gamma function

$\Gamma(a, x)$  is the incomplete gamma function

$(a)_n$  is the Pochhammer symbol (rising factorial)

**Series representations:**

$$\frac{\pi^2 5}{1!} = 30 \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$\frac{\pi^2 5}{1!} = -60 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

$$\frac{\pi^2 5}{1!} = 40 \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

**Integral representations:**

$$\frac{\pi^2 5}{1!} = 80 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2$$

$$\frac{\pi^2 5}{1!} = 20 \left( \int_0^\infty \frac{1}{1+t^2} dt \right)^2$$

$$\frac{\pi^2 5}{1!} = 20 \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2$$

$$1^{-2} * ((\sin(5\pi)) / (\sin(5\pi))) + 2^{-2} (((\sin(5\pi)) / (\sin(5\pi/2+2\pi/2))))$$

**Input:**

$$\frac{\frac{\sin(5)\pi}{\sin(5)\pi}}{1^2} + \frac{\frac{\sin(5\pi/2+2\pi/2)}{\sin(5\pi/2+2\pi/2)}}{2^2}$$

**Exact result:**

$$1 - \frac{1}{4}\pi \sin(5)$$

**Decimal approximation:**

1.753137364157659229297604172981025529744569214617537371534...

1.7531373641576592...

**Alternate forms:**

$$\frac{1}{4}(4 - \pi \sin(5))$$

$$1 - \frac{1}{8}i e^{-5i}\pi + \frac{1}{8}i e^{5i}\pi$$

**Alternative representations:**

$$\frac{\sin(5)\pi}{1^2(\sin(5)\pi)} + \frac{\sin(5)\pi}{2^2 \sin\left(\frac{5\pi}{2} + \frac{2\pi}{2}\right)} = \frac{\pi}{\csc(5)\pi} + \frac{\pi}{\frac{4\csc(5)}{\csc\left(\frac{7\pi}{2}\right)}}$$

$$\frac{\sin(5)\pi}{1^2(\sin(5)\pi)} + \frac{\sin(5)\pi}{2^2 \sin\left(\frac{5\pi}{2} + \frac{2\pi}{2}\right)} = \frac{\pi \cos(-5 + \frac{\pi}{2})}{\pi \cos(-5 + \frac{\pi}{2})} + \frac{\pi \cos(-5 + \frac{\pi}{2})}{4 \cos(-3\pi)}$$

$$\frac{\sin(5)\pi}{1^2(\sin(5)\pi)} + \frac{\sin(5)\pi}{2^2 \sin\left(\frac{5\pi}{2} + \frac{2\pi}{2}\right)} = \frac{-\pi \cos(5 + \frac{\pi}{2})}{-\pi \cos(5 + \frac{\pi}{2})} - \frac{\pi \cos(5 + \frac{\pi}{2})}{4(-\cos(4\pi))}$$

$\csc(x)$  is the cosecant function

### Series representations:

$$\frac{\sin(5)\pi}{1^2(\sin(5)\pi)} + \frac{\sin(5)\pi}{2^2 \sin\left(\frac{5\pi}{2} + \frac{2\pi}{2}\right)} = 1 - \frac{1}{4}\pi \sum_{k=0}^{\infty} \frac{(-1)^k 5^{1+2k}}{(1+2k)!}$$

$$\frac{\sin(5)\pi}{1^2(\sin(5)\pi)} + \frac{\sin(5)\pi}{2^2 \sin\left(\frac{5\pi}{2} + \frac{2\pi}{2}\right)} = 1 - \frac{1}{2}\pi \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(5)$$

$$\frac{\sin(5)\pi}{1^2(\sin(5)\pi)} + \frac{\sin(5)\pi}{2^2 \sin\left(\frac{5\pi}{2} + \frac{2\pi}{2}\right)} = 1 - \frac{1}{4}\pi \sum_{k=0}^{\infty} \frac{(-1)^k \left(5 - \frac{\pi}{2}\right)^{2k}}{(2k)!}$$

$n!$  is the factorial function  
 $J_n(z)$  is the Bessel function of the first kind

### Integral representations:

$$\frac{\sin(5)\pi}{1^2(\sin(5)\pi)} + \frac{\sin(5)\pi}{2^2 \sin\left(\frac{5\pi}{2} + \frac{2\pi}{2}\right)} = 1 - \frac{5\pi}{4} \int_0^1 \cos(5t) dt$$

$$\frac{\sin(5)\pi}{1^2(\sin(5)\pi)} + \frac{\sin(5)\pi}{2^2 \sin\left(\frac{5\pi}{2} + \frac{2\pi}{2}\right)} = 1 + \frac{5i\sqrt{\pi}}{16} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-25/(4s)+s}}{s^{3/2}} ds \quad \text{for } \gamma > 0$$

$$\frac{\sin(5)\pi}{1^2(\sin(5)\pi)} + \frac{\sin(5)\pi}{2^2 \sin\left(\frac{5\pi}{2} + \frac{2\pi}{2}\right)} = 1 + \frac{i\sqrt{\pi}}{8} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(\frac{5}{2}\right)^{1-2s} \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} ds \quad \text{for } 0 < \gamma < 1$$

$\Gamma(x)$  is the gamma function

$$3^{-2}((\sin(5\pi))/(\sin(5\pi/3+2\pi)))+4^{-2}((\sin(5\pi))/(\sin(5\pi/4+6\pi/2)))$$

### Input:

$$\frac{\frac{\sin(5)\pi}{\sin(5\times\frac{\pi}{3}+2\pi)}}{3^2} + \frac{\frac{\sin(5)\pi}{\sin(5\times\frac{\pi}{4}+6\times\frac{\pi}{2})}}{4^2}$$

### Exact result:

$$\frac{\pi \sin(5)}{8\sqrt{2}} - \frac{2\pi \sin(5)}{9\sqrt{3}}$$

## Decimal approximation:

0.120236006815747125666503745990367693844778481112933310261...

0.1202360068157471256665....

## Alternate forms:

$$\frac{1}{432} \left( 27\sqrt{2} - 32\sqrt{3} \right) \pi \sin(5)$$

$$\left( \frac{1}{8\sqrt{2}} - \frac{2}{9\sqrt{3}} \right) \pi \sin(5)$$

$$\frac{1}{432} \left( 27\sqrt{2} \pi \sin(5) - 32\sqrt{3} \pi \sin(5) \right)$$

## Alternative representations:

$$\frac{\sin(5)\pi}{3^2 \sin(\frac{5\pi}{3} + 2\pi)} + \frac{\sin(5)\pi}{4^2 \sin(\frac{5\pi}{4} + \frac{6\pi}{2})} = \frac{\pi \cos(-5 + \frac{\pi}{2})}{9 \cos(-\frac{19\pi}{6})} + \frac{\pi \cos(-5 + \frac{\pi}{2})}{4^2 \cos(-\frac{5\pi}{2} - \frac{5\pi}{4})}$$

$$\frac{\sin(5)\pi}{3^2 \sin(\frac{5\pi}{3} + 2\pi)} + \frac{\sin(5)\pi}{4^2 \sin(\frac{5\pi}{4} + \frac{6\pi}{2})} = \frac{\pi}{\frac{9 \csc(5)}{\csc(\frac{11\pi}{3})}} + \frac{\pi}{\frac{4^2 \csc(5)}{\csc(3\pi + \frac{5\pi}{4})}}$$

$$\frac{\sin(5)\pi}{3^2 \sin(\frac{5\pi}{3} + 2\pi)} + \frac{\sin(5)\pi}{4^2 \sin(\frac{5\pi}{4} + \frac{6\pi}{2})} = -\frac{\pi \cos(5 + \frac{\pi}{2})}{9 \left( -\cos(\frac{25\pi}{6}) \right)} - \frac{\pi \cos(5 + \frac{\pi}{2})}{4^2 \left( -\cos(\frac{7\pi}{2} + \frac{5\pi}{4}) \right)}$$

$\csc(x)$  is the cosecant function

## Series representations:

$$\frac{\sin(5)\pi}{3^2 \sin(\frac{5\pi}{3} + 2\pi)} + \frac{\sin(5)\pi}{4^2 \sin(\frac{5\pi}{4} + \frac{6\pi}{2})} = \frac{1}{432} \left( 27\sqrt{2} - 32\sqrt{3} \right) \pi \sum_{k=0}^{\infty} \frac{(-1)^k 5^{1+2k}}{(1+2k)!}$$

$$\frac{\sin(5)\pi}{3^2 \sin(\frac{5\pi}{3} + 2\pi)} + \frac{\sin(5)\pi}{4^2 \sin(\frac{5\pi}{4} + \frac{6\pi}{2})} = \sum_{k=0}^{\infty} \frac{1}{216} (-1)^k \left( 27\sqrt{2} - 32\sqrt{3} \right) \pi J_{1+2k}(5)$$

$$\frac{\sin(5)\pi}{3^2 \sin(\frac{5\pi}{3} + 2\pi)} + \frac{\sin(5)\pi}{4^2 \sin(\frac{5\pi}{4} + \frac{6\pi}{2})} = \frac{1}{432} \left( 27\sqrt{2} - 32\sqrt{3} \right) \pi \sum_{k=0}^{\infty} \frac{(-1)^k \left( 5 - \frac{\pi}{2} \right)^{2k}}{(2k)!}$$

$n!$  is the factorial function  
 $J_n(z)$  is the Bessel function of the first kind

### Integral representations:

$$\frac{\sin(5)\pi}{3^2 \sin\left(\frac{5\pi}{3} + 2\pi\right)} + \frac{\sin(5)\pi}{4^2 \sin\left(\frac{5\pi}{4} + \frac{6\pi}{2}\right)} = \int_0^1 \frac{5}{432} \left(27\sqrt{2} - 32\sqrt{3}\right) \pi \cos(5t) dt$$

$$\begin{aligned} & \frac{\sin(5)\pi}{3^2 \sin\left(\frac{5\pi}{3} + 2\pi\right)} + \frac{\sin(5)\pi}{4^2 \sin\left(\frac{5\pi}{4} + \frac{6\pi}{2}\right)} = \\ & -\frac{5i(27\sqrt{2} - 32\sqrt{3})\sqrt{\pi}}{1728} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-25/(4s)+s}}{s^{3/2}} ds \quad \text{for } \gamma > 0 \end{aligned}$$

$$\begin{aligned} & \frac{\sin(5)\pi}{3^2 \sin\left(\frac{5\pi}{3} + 2\pi\right)} + \frac{\sin(5)\pi}{4^2 \sin\left(\frac{5\pi}{4} + \frac{6\pi}{2}\right)} = \\ & -\frac{1}{864} i(27\sqrt{2} - 32\sqrt{3})\sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(\frac{5}{2}\right)^{1-2s} \Gamma(s)}{\Gamma\left(\frac{3}{2} - s\right)} ds \quad \text{for } 0 < \gamma < 1 \end{aligned}$$

$\Gamma(x)$  is the gamma function

$$49.348022005446793 (1.7531373641576592 + 0.1202360068157471256665)$$

### Input interpretation:

$$49.348022005446793 (1.7531373641576592 + 0.1202360068157471256665)$$

### Result:

$$92.4472703352136937373339594237960125345$$

### Repeating decimal:

$$92.4472703352136937373339594237960125345$$

$$\color{red}92.447270335213693\dots$$

We note that:

$$(((49.3480220054 (1.7531373641 + 0.1202360068))))^{1/9}$$

### Input interpretation:

$$\sqrt[9]{49.3480220054 (1.7531373641 + 0.1202360068)}$$

### Result:

$$1.65360842733\dots$$

1.65360842733....

And:

$$18/10^3 + \sqrt[9]{49.3480220054 (1.7531373641 + 0.1202360068)})^{1/9}$$

**Input interpretation:**

$$\frac{18}{10^3} + \sqrt[9]{49.3480220054 (1.7531373641 + 0.1202360068)}$$

**Result:**

$$1.67160842733\dots$$

$$1.67160842733\dots$$

We note that 1.67160842733... is a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

$$\begin{aligned} r_2(n) &= \delta_2(n) = 4 \left\{ \left( \frac{\sin n\pi}{\sin n\pi} \right) - \frac{1}{3} \left( \frac{\sin n\pi}{\sin \frac{1}{3}n\pi} \right) + \frac{1}{5} \left( \frac{\sin n\pi}{\sin \frac{1}{5}n\pi} \right) - \dots \right\} \\ &= 4 \left\{ \frac{1}{2} \left( \frac{\sin n\pi}{\cos \frac{1}{2}n\pi} \right) - \frac{1}{4} \left( \frac{\sin n\pi}{\cos \frac{1}{4}n\pi} \right) + \frac{1}{6} \left( \frac{\sin n\pi}{\cos \frac{1}{6}n\pi} \right) - \dots \right\}; \end{aligned} \quad (14.5)$$

$$s = 8, n = 5$$

$$4(((1/2(\sin 5\pi)/((\cos 5\pi)/2) - 1/4(\sin 5\pi)/((\cos 5\pi)/4) + 1/6(\sin 5\pi)/((\cos 5\pi)/6))))$$

**Input:**

$$4 \left( \frac{1}{2} \times \frac{\sin(5)\pi}{\frac{1}{2}(\cos(5)\pi)} - \frac{1}{4} \times \frac{\sin(5)\pi}{\frac{1}{4}(\cos(5)\pi)} + \frac{1}{6} \times \frac{\sin(5)\pi}{\frac{1}{6}(\cos(5)\pi)} \right)$$

**Exact result:**

$$4 \tan(5)$$

**Decimal approximation:**

$$-13.5220600249863425479308235177893756348382768331418395087\dots$$

-13.5220600249863...

### Property:

$4 \tan(5)$  is a transcendental number

- **Alternate forms:**

$$\frac{4 \sin(10)}{1 + \cos(10)}$$

$$\frac{4i(e^{-5i} - e^{5i})}{e^{-5i} + e^{5i}}$$

$$\frac{4(1 + 2 \cos(2) + 2 \cos(4)) \tan(1)}{1 - 2 \cos(2) + 2 \cos(4)}$$

### Alternative representations:

$$4 \left( \frac{\sin(5)\pi}{\frac{1}{2}(\cos(5)\pi)2} - \frac{\sin(5)\pi}{\frac{1}{4}(\cos(5)\pi)4} + \frac{\sin(5)\pi}{\frac{1}{6}(\cos(5)\pi)6} \right) = \\ 4 \left( \frac{\pi \cos(-5 + \frac{\pi}{2})}{\frac{2}{2}(\pi \cosh(-5i))} - \frac{\pi \cos(-5 + \frac{\pi}{2})}{\frac{4}{4}(\pi \cosh(-5i))} + \frac{\pi \cos(-5 + \frac{\pi}{2})}{\frac{6}{6}(\pi \cosh(-5i))} \right)$$

$$4 \left( \frac{\sin(5)\pi}{\frac{1}{2}(\cos(5)\pi)2} - \frac{\sin(5)\pi}{\frac{1}{4}(\cos(5)\pi)4} + \frac{\sin(5)\pi}{\frac{1}{6}(\cos(5)\pi)6} \right) = \\ 4 \left( -\frac{\pi \cos(5 + \frac{\pi}{2})}{\frac{2}{2}(\pi \cosh(-5i))} + \frac{\pi \cos(5 + \frac{\pi}{2})}{\frac{4}{4}(\pi \cosh(-5i))} - \frac{\pi \cos(5 + \frac{\pi}{2})}{\frac{6}{6}(\pi \cosh(-5i))} \right)$$

$$4 \left( \frac{\sin(5)\pi}{\frac{1}{2}(\cos(5)\pi)2} - \frac{\sin(5)\pi}{\frac{1}{4}(\cos(5)\pi)4} + \frac{\sin(5)\pi}{\frac{1}{6}(\cos(5)\pi)6} \right) = \\ 4 \left( \frac{\pi \cos(-5 + \frac{\pi}{2})}{\frac{2}{2}(\pi \cosh(5i))} - \frac{\pi \cos(-5 + \frac{\pi}{2})}{\frac{4}{4}(\pi \cosh(5i))} + \frac{\pi \cos(-5 + \frac{\pi}{2})}{\frac{6}{6}(\pi \cosh(5i))} \right)$$

$\cosh(x)$  is the hyperbolic cosine function  
 $i$  is the imaginary unit

### Series representations:

$$4 \left( \frac{\sin(5)\pi}{\frac{1}{2}(\cos(5)\pi)2} - \frac{\sin(5)\pi}{\frac{1}{4}(\cos(5)\pi)4} + \frac{\sin(5)\pi}{\frac{1}{6}(\cos(5)\pi)6} \right) = 4i + 8i \sum_{k=1}^{\infty} (-1)^k q^{2k} \text{ for } q = e^{5i}$$

$$4 \left( \frac{\sin(5)\pi}{\frac{1}{2}(\cos(5)\pi)2} - \frac{\sin(5)\pi}{\frac{1}{4}(\cos(5)\pi)4} + \frac{\sin(5)\pi}{\frac{1}{6}(\cos(5)\pi)6} \right) = 4i \sum_{k=-\infty}^{\infty} (-1)^k e^{10ik} \operatorname{sgn}(k)$$

$$4 \left( \frac{\sin(5)\pi}{\frac{1}{2}(\cos(5)\pi)2} - \frac{\sin(5)\pi}{\frac{1}{4}(\cos(5)\pi)4} + \frac{\sin(5)\pi}{\frac{1}{6}(\cos(5)\pi)6} \right) = 160 \sum_{k=1}^{\infty} \frac{1}{-100 + (1-2k)^2 \pi^2}$$

$\operatorname{sgn}(x)$  is the sign of  $x$

### Integral representation:

$$4 \left( \frac{\sin(5)\pi}{\frac{1}{2}(\cos(5)\pi)2} - \frac{\sin(5)\pi}{\frac{1}{4}(\cos(5)\pi)4} + \frac{\sin(5)\pi}{\frac{1}{6}(\cos(5)\pi)6} \right) = 4 \int_0^5 \sec^2(t) dt$$

$\sec(x)$  is the secant function

We have:

$$1 + 1 / ((((-4(((1/2(\sin 5 \pi)) / ((\cos 5 \pi)) / 2) - 1/4(\sin 5 \pi)) / ((\cos 5 \pi)) / 4) + 1/6(\sin 5 \pi)) / ((\cos 5 \pi)) / 6)))^1/6$$

### Input:

$$1 + \frac{1}{\sqrt[6]{-4 \left( \frac{1}{2} \times \frac{\sin(5)\pi}{\frac{1}{2}(\cos(5)\pi)} - \frac{1}{4} \times \frac{\sin(5)\pi}{\frac{1}{4}(\cos(5)\pi)} + \frac{1}{6} \times \frac{\sin(5)\pi}{\frac{1}{6}(\cos(5)\pi)} \right)}}$$

### Exact result:

$$1 + \frac{1}{\sqrt[3]{2} \sqrt[6]{-\tan(5)}}$$

### Decimal approximation:

$$1.647877438942306115157004352575917521352272713123485853653\dots$$

$$1.6478774389\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\dots$$

### Property:

$$1 + \frac{1}{\sqrt[3]{2} \sqrt[6]{-\tan(5)}} \text{ is a transcendental number}$$

### Alternate forms:

$$1 + \frac{\sqrt[6]{-\cot(5)}}{\sqrt[3]{2}}$$

$$1 - \frac{(-\tan(5))^{5/6} \cot(5)}{\sqrt[3]{2}}$$

$$1 + \frac{\sqrt[6]{-(1 + \cos(10)) \csc(10)}}{\sqrt[3]{2}}$$

$\cot(x)$  is the cotangent function

$\csc(x)$  is the cosecant function

### Alternative representations:

$$1 + \frac{1}{1 + \sqrt[6]{-4 \left( \frac{\sin(5)\pi}{2(\cos(5)\pi)2} - \frac{\sin(5)\pi}{4(\cos(5)\pi)4} + \frac{\sin(5)\pi}{6(\cos(5)\pi)6} \right)}} =$$

$$1 + \frac{1}{1 + \sqrt[6]{-4 \left( \frac{\pi \cos(-5+\frac{\pi}{2})}{2(\pi \cosh(-5i))} - \frac{\pi \cos(-5+\frac{\pi}{2})}{4(\pi \cosh(-5i))} + \frac{\pi \cos(-5+\frac{\pi}{2})}{6(\pi \cosh(-5i))} \right)}}$$

$$1 + \frac{1}{1 + \sqrt[6]{-4 \left( \frac{\sin(5)\pi}{2(\cos(5)\pi)2} - \frac{\sin(5)\pi}{4(\cos(5)\pi)4} + \frac{\sin(5)\pi}{6(\cos(5)\pi)6} \right)}} =$$

$$1 + \frac{1}{1 + \sqrt[6]{-4 \left( -\frac{\pi \cos(5+\frac{\pi}{2})}{2(\pi \cosh(-5i))} + \frac{\pi \cos(5+\frac{\pi}{2})}{4(\pi \cosh(-5i))} - \frac{\pi \cos(5+\frac{\pi}{2})}{6(\pi \cosh(-5i))} \right)}}$$

$$1 + \frac{1}{1 + \sqrt[6]{-4 \left( \frac{\sin(5)\pi}{2(\cos(5)\pi)2} - \frac{\sin(5)\pi}{4(\cos(5)\pi)4} + \frac{\sin(5)\pi}{6(\cos(5)\pi)6} \right)}} =$$

$$1 + \frac{1}{1 + \sqrt[6]{-4 \left( \frac{\pi \cos(-5+\frac{\pi}{2})}{2(\pi \cosh(5i))} - \frac{\pi \cos(-5+\frac{\pi}{2})}{4(\pi \cosh(5i))} + \frac{\pi \cos(-5+\frac{\pi}{2})}{6(\pi \cosh(5i))} \right)}}$$

$\cosh(x)$  is the hyperbolic cosine function

$i$  is the imaginary unit

## Series representations:

$$1 + \frac{1}{\sqrt[6]{-4 \left( \frac{\sin(5)\pi}{2(\cos(5)\pi)2} - \frac{\sin(5)\pi}{4(\cos(5)\pi)4} + \frac{\sin(5)\pi}{6(\cos(5)\pi)6} \right)}} = \\ 1 + \frac{1}{\sqrt[3]{2} \sqrt[6]{-i \left( 1 + 2 \sum_{k=1}^{\infty} (-1)^k q^{2k} \right)}} \quad \text{for } q = e^{5i}$$

$$1 + \frac{1}{\sqrt[6]{-4 \left( \frac{\sin(5)\pi}{2(\cos(5)\pi)2} - \frac{\sin(5)\pi}{4(\cos(5)\pi)4} + \frac{\sin(5)\pi}{6(\cos(5)\pi)6} \right)}} = \\ 1 + \frac{1}{\sqrt[3]{2} \sqrt[6]{-i \sum_{k=-\infty}^{\infty} (-1)^k e^{10ik} \operatorname{sgn}(k)}}$$

$$1 + \frac{1}{\sqrt[6]{-4 \left( \frac{\sin(5)\pi}{2(\cos(5)\pi)2} - \frac{\sin(5)\pi}{4(\cos(5)\pi)4} + \frac{\sin(5)\pi}{6(\cos(5)\pi)6} \right)}} = \\ 1 + \frac{1}{2^{5/6} \sqrt[6]{5} \sqrt[6]{-\sum_{k=1}^{\infty} \frac{1}{-100+(-1+2k)^2 \pi^2}}}$$

$\operatorname{sgn}(x)$  is the sign of  $x$

## Integral representation:

$$1 + \frac{1}{\sqrt[6]{-4 \left( \frac{\sin(5)\pi}{2(\cos(5)\pi)2} - \frac{\sin(5)\pi}{4(\cos(5)\pi)4} + \frac{\sin(5)\pi}{6(\cos(5)\pi)6} \right)}} = \\ -\frac{2^{2/3} \left( -\int_0^5 \sec^2(t) dt \right)^{5/6} - 2 \int_0^5 \sec^2(t) dt}{2 \int_0^5 \sec^2(t) dt}$$

$\sec(x)$  is the secant function

$$(1^{-s} - 3^{-s} + 5^{-s} - \dots) \delta'_{2s}(n) = \frac{(\frac{1}{2}\pi)^s}{(s-1)!} (n + \frac{1}{4}s)^{s-1} \left\{ 1^{-s} \left( \frac{\sin(4n+s)\pi}{\sin(4n+s)\pi} \right) - 3^{-s} \left( \frac{\sin(4n+s)\pi}{\sin \frac{1}{3}(4n+s)\pi} \right) + 5^{-s} \left( \frac{\sin(4n+s)\pi}{\sin \frac{1}{5}(4n+s)\pi} \right) - \dots \right\}$$

$$s = 3, n = 5 \quad (14.8)$$

$$(((\text{Pi}/2)^3 * (5+3/4)^2)) / 2! * (((1^3(\sin 23\text{Pi})/(\sin 23\text{Pi}) - 3^3(\sin 23\text{Pi})/((\sin 23\text{Pi})/3) + 5^3(\sin 23\text{Pi})/((\sin 23\text{Pi})/5))))$$

**Input:**

$$\frac{\left(\frac{\pi}{2}\right)^3 \left(5 + \frac{3}{4}\right)^2}{2!} \left( \frac{\frac{\sin(23)\pi}{\sin(23)\pi}}{1^3} - \frac{\frac{\sin(23)\pi}{\sin(23)\pi}}{3^3} + \frac{\frac{\sin(23)\pi}{\sin(23)\pi}}{5^3} \right)$$

$n!$  is the factorial function

**Exact result:**

$$\frac{110561\pi^3}{57600}$$

**Decimal approximation:**

$$59.51536382032341004202841788426731519016024529744485055160\dots$$

**59.51536382032341...**

**Property:**

$\frac{110561\pi^3}{57600}$  is a transcendental number

**Alternative representations:**

$$\frac{\left( \frac{\sin(23)\pi}{1^3(\sin(23)\pi)} - \frac{\sin(23)\pi}{3^3(\sin(23)\pi)} + \frac{\sin(23)\pi}{5^3(\sin(23)\pi)} \right) \left( \left(\frac{\pi}{2}\right)^3 \left(5 + \frac{3}{4}\right)^2 \right)}{\left( \frac{\pi}{2}\right)^3 \left(5 + \frac{3}{4}\right)^2 \left( -\frac{\pi \cos(-23+\frac{\pi}{2})}{\frac{27}{3}(\pi \cos(-23+\frac{\pi}{2}))} + \frac{\pi \cos(-23+\frac{\pi}{2})}{\pi \cos(-23+\frac{\pi}{2})} + \frac{\pi \cos(-23+\frac{\pi}{2})}{\frac{1}{5} \times 5^3 (\pi \cos(-23+\frac{\pi}{2}))} \right)} = \\ (1)_2$$

$$\frac{\left(\frac{\sin(23)\pi}{1^3(\sin(23)\pi)} - \frac{\sin(23)\pi}{\frac{1}{3}\times 3^3(\sin(23)\pi)} + \frac{\sin(23)\pi}{\frac{1}{5}\times 5^3(\sin(23)\pi)}\right)\left(\left(\frac{\pi}{2}\right)^3\left(5+\frac{3}{4}\right)^2\right)}{\left(\frac{\pi}{2}\right)^3\left(5+\frac{3}{4}\right)^2\left(-\frac{\pi \cos(-23+\frac{\pi}{2})}{\frac{27}{3}(\pi \cos(-23+\frac{\pi}{2}))} + \frac{\pi \cos(-23+\frac{\pi}{2})}{\pi \cos(-23+\frac{\pi}{2})} + \frac{\pi \cos(-23+\frac{\pi}{2})}{\frac{1}{5}\times 5^3(\pi \cos(-23+\frac{\pi}{2}))}\right)} = \\ 1!! \times 2!!$$

$$\frac{\left(\frac{\sin(23)\pi}{1^3(\sin(23)\pi)} - \frac{\sin(23)\pi}{\frac{1}{3}\times 3^3(\sin(23)\pi)} + \frac{\sin(23)\pi}{\frac{1}{5}\times 5^3(\sin(23)\pi)}\right)\left(\left(\frac{\pi}{2}\right)^3\left(5+\frac{3}{4}\right)^2\right)}{2!} = \\ \left(\frac{\pi}{2}\right)^3\left(5+\frac{3}{4}\right)^2\left(\frac{-\pi \cos(23+\frac{\pi}{2})}{-\pi \cos(23+\frac{\pi}{2})} + \frac{\pi \cos(23+\frac{\pi}{2})}{\frac{27}{3}(-\pi \cos(23+\frac{\pi}{2}))} - \frac{\pi \cos(23+\frac{\pi}{2})}{\frac{1}{5}\times 5^3(-\pi \cos(23+\frac{\pi}{2}))}\right) \\ 1!! \times 2!!$$

$(a)_n$  is the Pochhammer symbol (rising factorial)

$n!!$  is the double factorial function

### Series representations:

$$\frac{\left(\frac{\sin(23)\pi}{1^3(\sin(23)\pi)} - \frac{\sin(23)\pi}{\frac{1}{3}\times 3^3(\sin(23)\pi)} + \frac{\sin(23)\pi}{\frac{1}{5}\times 5^3(\sin(23)\pi)}\right)\left(\left(\frac{\pi}{2}\right)^3\left(5+\frac{3}{4}\right)^2\right)}{2!} = -\frac{110561 \sum_{k=1}^{\infty} \frac{(-1)^k}{(-1+2k)^3}}{1800}$$

$$\frac{\left(\frac{\sin(23)\pi}{1^3(\sin(23)\pi)} - \frac{\sin(23)\pi}{\frac{1}{3}\times 3^3(\sin(23)\pi)} + \frac{\sin(23)\pi}{\frac{1}{5}\times 5^3(\sin(23)\pi)}\right)\left(\left(\frac{\pi}{2}\right)^3\left(5+\frac{3}{4}\right)^2\right)}{2!} = \frac{110561}{900} \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^3$$

$$\frac{\left(\frac{\sin(23)\pi}{1^3(\sin(23)\pi)} - \frac{\sin(23)\pi}{\frac{1}{3}\times 3^3(\sin(23)\pi)} + \frac{\sin(23)\pi}{\frac{1}{5}\times 5^3(\sin(23)\pi)}\right)\left(\left(\frac{\pi}{2}\right)^3\left(5+\frac{3}{4}\right)^2\right)}{2!} = \\ \frac{110561}{900} \left( \sum_{k=0}^{\infty} \frac{(-1)^{1+k} 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^3$$

### Integral representations:

$$\frac{\left(\frac{\sin(23)\pi}{1^3(\sin(23)\pi)} - \frac{\sin(23)\pi}{\frac{1}{3}\times 3^3(\sin(23)\pi)} + \frac{\sin(23)\pi}{\frac{1}{5}\times 5^3(\sin(23)\pi)}\right)\left(\left(\frac{\pi}{2}\right)^3\left(5+\frac{3}{4}\right)^2\right)}{2!} = \frac{110561}{900} \left( \int_0^1 \sqrt{1-t^2} dt \right)^3$$

$$\frac{\left(\frac{\sin(23)\pi}{1^3 (\sin(23)\pi)} - \frac{\sin(23)\pi}{\frac{1}{3} \times 3^3 (\sin(23)\pi)} + \frac{\sin(23)\pi}{\frac{1}{5} \times 5^3 (\sin(23)\pi)}\right) \left(\left(\frac{\pi}{2}\right)^3 \left(5 + \frac{3}{4}\right)^2\right)}{2!} = \frac{110561 \left(\int_0^\infty \frac{1}{1+t^2} dt\right)^3}{7200}$$

$$\frac{\left(\frac{\sin(23)\pi}{1^3 (\sin(23)\pi)} - \frac{\sin(23)\pi}{\frac{1}{3} \times 3^3 (\sin(23)\pi)} + \frac{\sin(23)\pi}{\frac{1}{5} \times 5^3 (\sin(23)\pi)}\right) \left(\left(\frac{\pi}{2}\right)^3 \left(5 + \frac{3}{4}\right)^2\right)}{2!} = \frac{110561 \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt\right)^3}{7200}$$

$$(1^{-s} + 3^{-s} + 5^{-s} + \dots) \delta'_{2s}(n) = \frac{(\frac{1}{2}\pi)^s}{(s-1)!} (n + \frac{1}{4}s)^{s-1}$$

$$\left\{ 1^{-s} \left( \frac{\sin(n + \frac{1}{4}s)\pi}{\sin(n + \frac{1}{4}s)\pi} \right) + 3^{-s} \left( \frac{\sin(n + \frac{1}{4}s)\pi}{\sin \frac{1}{3}(n + \frac{1}{4}s)\pi} \right) + 5^{-s} \left( \frac{\sin(n + \frac{1}{4}s)\pi}{\sin \frac{1}{5}(n + \frac{1}{4}s)\pi} \right) + \dots \right\}$$

$$s = 8, n = 5 \quad (14.6)$$

$$(((\text{Pi}/2)^8 * (5+8/4)^7)) / 7! * (((1^8 - 8(\sin 7\text{Pi})/(\sin 7\text{Pi})) + 3^{8-8(\sin 7\text{Pi})/((\sin 7\text{Pi})/3)} + 5^{8-8(\sin 7\text{Pi})/((\sin 7\text{Pi})/5)})))$$

**Input:**

$$\frac{\left(\frac{\pi}{2}\right)^8 \left(5 + \frac{8}{4}\right)^7}{7!} \left( \frac{\frac{\sin(7)\pi}{\sin(7)\pi}}{1^8} + \frac{\frac{\sin(7)\pi}{\frac{1}{3}(\sin(7)\pi)}}{3^8} + \frac{\frac{\sin(7)\pi}{\frac{1}{5}(\sin(7)\pi)}}{5^8} \right)$$

$n!$  is the factorial function

**Exact result:**

$$\frac{20110883235863\pi^8}{31492800000000}$$

**Decimal approximation:**

$$6059.249712444108642091454077093882218276566830849034501029\dots$$

6059.2497124441

### Property:

$\frac{20110883235863\pi^8}{31492800000000}$  is a transcendental number

### Alternative representations:

$$\frac{\left(\frac{\sin(7)\pi}{1^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{3}\times 3^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{5}\times 5^8(\sin(7)\pi)}\right)\left(\left(\frac{\pi}{2}\right)^8\left(5+\frac{8}{4}\right)^7\right)}{\left(\frac{\pi}{2}\right)^8\left(5+\frac{8}{4}\right)^7\left(\frac{\pi \cos(-7+\frac{\pi}{2})}{\frac{1}{3}\times 3^8(\pi \cos(-7+\frac{\pi}{2}))} + \frac{\pi \cos(-7+\frac{\pi}{2})}{1^8(\pi \cos(-7+\frac{\pi}{2}))} + \frac{\pi \cos(-7+\frac{\pi}{2})}{\frac{1}{5}\times 5^8(\pi \cos(-7+\frac{\pi}{2}))}\right)} = (1)_7$$

$$\frac{\left(\frac{\sin(7)\pi}{1^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{3}\times 3^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{5}\times 5^8(\sin(7)\pi)}\right)\left(\left(\frac{\pi}{2}\right)^8\left(5+\frac{8}{4}\right)^7\right)}{\left(\frac{\pi}{2}\right)^8\left(5+\frac{8}{4}\right)^7\left(\frac{\pi \cos(-7+\frac{\pi}{2})}{\frac{1}{3}\times 3^8(\pi \cos(-7+\frac{\pi}{2}))} + \frac{\pi \cos(-7+\frac{\pi}{2})}{1^8(\pi \cos(-7+\frac{\pi}{2}))} + \frac{\pi \cos(-7+\frac{\pi}{2})}{\frac{1}{5}\times 5^8(\pi \cos(-7+\frac{\pi}{2}))}\right)} = \frac{7!}{6!! \times 7!!}$$

$$\frac{\left(\frac{\sin(7)\pi}{1^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{3}\times 3^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{5}\times 5^8(\sin(7)\pi)}\right)\left(\left(\frac{\pi}{2}\right)^8\left(5+\frac{8}{4}\right)^7\right)}{\left(\frac{\pi}{2}\right)^8\left(5+\frac{8}{4}\right)^7\left(\frac{\pi \cos(-7+\frac{\pi}{2})}{\frac{1}{3}\times 3^8(\pi \cos(-7+\frac{\pi}{2}))} + \frac{\pi \cos(-7+\frac{\pi}{2})}{1^8(\pi \cos(-7+\frac{\pi}{2}))} + \frac{\pi \cos(-7+\frac{\pi}{2})}{\frac{1}{5}\times 5^8(\pi \cos(-7+\frac{\pi}{2}))}\right)} = e^{\log\Gamma(8)}$$

$(a)_n$  is the Pochhammer symbol (rising factorial)

$n!!$  is the double factorial function

$\log\Gamma(x)$  is the logarithm of the gamma function

### Series representations:

$$\frac{\left(\frac{\sin(7)\pi}{1^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{3}\times 3^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{5}\times 5^8(\sin(7)\pi)}\right)\left(\left(\frac{\pi}{2}\right)^8\left(5+\frac{8}{4}\right)^7\right)}{7!} = \frac{321774131773808\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^8}{7688671875}$$

$$\frac{\left(\frac{\sin(7)\pi}{1^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{3} \times 3^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{5} \times 5^8(\sin(7)\pi)}\right) \left(\left(\frac{\pi}{2}\right)^8 \left(5 + \frac{8}{4}\right)^7\right)}{7!} =$$

$$\frac{321774131773808 \left(\sum_{k=0}^{\infty} \frac{(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}\right)^8}{7688671875}$$

$$\frac{\left(\frac{\sin(7)\pi}{1^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{3} \times 3^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{5} \times 5^8(\sin(7)\pi)}\right) \left(\left(\frac{\pi}{2}\right)^8 \left(5 + \frac{8}{4}\right)^7\right)}{7!} =$$

$$\frac{20110883235863 \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)\right)^8}{31492800000000}$$

### Integral representations:

$$\frac{\left(\frac{\sin(7)\pi}{1^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{3} \times 3^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{5} \times 5^8(\sin(7)\pi)}\right) \left(\left(\frac{\pi}{2}\right)^8 \left(5 + \frac{8}{4}\right)^7\right)}{7!} =$$

$$\frac{321774131773808 \left(\int_0^1 \sqrt{1-t^2} dt\right)^8}{7688671875}$$

$$\frac{\left(\frac{\sin(7)\pi}{1^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{3} \times 3^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{5} \times 5^8(\sin(7)\pi)}\right) \left(\left(\frac{\pi}{2}\right)^8 \left(5 + \frac{8}{4}\right)^7\right)}{7!} =$$

$$\frac{20110883235863 \left(\int_0^\infty \frac{1}{1+t^2} dt\right)^8}{123018750000}$$

$$\frac{\left(\frac{\sin(7)\pi}{1^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{3} \times 3^8(\sin(7)\pi)} + \frac{\sin(7)\pi}{\frac{1}{5} \times 5^8(\sin(7)\pi)}\right) \left(\left(\frac{\pi}{2}\right)^8 \left(5 + \frac{8}{4}\right)^7\right)}{7!} =$$

$$\frac{20110883235863 \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt\right)^8}{123018750000}$$

$$(1^{-s} + 3^{-s} + 5^{-s} + \dots) \delta'_{2s}(n) = \frac{(\frac{1}{2}\pi)^s}{(s-1)!} (n + \frac{1}{4}s)^{s-1}$$

$$\left\{ 1^{-s} \left( \frac{\sin(2n + \frac{1}{2}s)\pi}{\sin(2n + \frac{1}{2}s)\pi} \right) + 3^{-s} \left( \frac{\sin(2n + \frac{1}{2}s)\pi}{\sin \frac{1}{3}(2n + \frac{1}{2}s)\pi} \right) \right.$$

$$\left. + 5^{-s} \left( \frac{\sin(2n + \frac{1}{2}s)\pi}{\sin \frac{1}{5}(2n + \frac{1}{2}s)\pi} \right) + \dots \right\}$$

$$s = 6, n = 5 \quad (14.7)$$

$$(((\text{Pi}/2)^6 * (5+6/4)^5)) / 5! * (((1^{-6}(\sin 13\text{Pi})/(\sin 13\text{Pi}) + 3^{-6}(\sin 13\text{Pi})/((\sin 13\text{Pi})/3)+5^{-6}(\sin 13\text{Pi})/((\sin 13\text{Pi})/5))))$$

**Input:**

$$\frac{\left(\frac{\pi}{2}\right)^6 \left(5 + \frac{6}{4}\right)^5}{5!} \left( \frac{\frac{\sin(13)\pi}{\sin(13)\pi}}{1^6} + \frac{\frac{\sin(13)\pi}{3(\sin(13)\pi)}}{3^6} + \frac{\frac{\sin(13)\pi}{5(\sin(13)\pi)}}{5^6} \right)$$

$n!$  is the factorial function

**Exact result:**

$$\frac{283\,201\,136\,699\,\pi^6}{186\,624\,000\,000}$$

**Decimal approximation:**

$$1458.904066093649072048270395317733494198187941635852590407\dots$$

1458.90406609...

**Property:**

$\frac{283\,201\,136\,699\,\pi^6}{186\,624\,000\,000}$  is a transcendental number

**Alternative representations:**

$$\frac{\left( \frac{\sin(13)\pi}{1^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{3 \times 3^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{5 \times 5^6(\sin(13)\pi)} \right) \left( \left(\frac{\pi}{2}\right)^6 \left(5 + \frac{6}{4}\right)^5 \right)}{5!} =$$

$$\frac{\left(\frac{\pi}{2}\right)^6 \left(5 + \frac{6}{4}\right)^5 \left( \frac{\pi \cos(-13+\frac{\pi}{2})}{\frac{1}{3} \times 3^6 (\pi \cos(-13+\frac{\pi}{2}))} + \frac{\pi \cos(-13+\frac{\pi}{2})}{1^6 (\pi \cos(-13+\frac{\pi}{2}))} + \frac{\pi \cos(-13+\frac{\pi}{2})}{\frac{1}{5} \times 5^6 (\pi \cos(-13+\frac{\pi}{2}))} \right)}{(1)_5}$$

•

$$\frac{\left( \frac{\sin(13)\pi}{1^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{3} \times 3^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{5} \times 5^6(\sin(13)\pi)} \right) \left( \left(\frac{\pi}{2}\right)^6 \left(5 + \frac{6}{4}\right)^5 \right)}{4!! \times 5!!} =$$

$$\frac{\left( \frac{\sin(13)\pi}{1^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{3} \times 3^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{5} \times 5^6(\sin(13)\pi)} \right) \left( \left(\frac{\pi}{2}\right)^6 \left(5 + \frac{6}{4}\right)^5 \right)}{\frac{5!}{\left( \frac{\pi}{2}\right)^6 \left(5 + \frac{6}{4}\right)^5 \left( \frac{\pi \cos(-13+\frac{\pi}{2})}{\frac{1}{3} \times 3^6(\pi \cos(-13+\frac{\pi}{2}))} + \frac{\pi \cos(-13+\frac{\pi}{2})}{1^6(\pi \cos(-13+\frac{\pi}{2}))} + \frac{\pi \cos(-13+\frac{\pi}{2})}{\frac{1}{5} \times 5^6(\pi \cos(-13+\frac{\pi}{2}))} \right)}}{4!! \times 5!!}$$

$$\frac{\left( \frac{\sin(13)\pi}{1^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{3} \times 3^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{5} \times 5^6(\sin(13)\pi)} \right) \left( \left(\frac{\pi}{2}\right)^6 \left(5 + \frac{6}{4}\right)^5 \right)}{e^{\log\Gamma(6)}} =$$

$(a)_n$  is the Pochhammer symbol (rising factorial)

$n!!$  is the double factorial function

$\log\Gamma(x)$  is the logarithm of the gamma function

### Series representations:

$$\frac{\left( \frac{\sin(13)\pi}{1^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{3} \times 3^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{5} \times 5^6(\sin(13)\pi)} \right) \left( \left(\frac{\pi}{2}\right)^6 \left(5 + \frac{6}{4}\right)^5 \right)}{5!} =$$

$$\frac{1982407956893 \sum_{k=1}^{\infty} \frac{1}{k^6}}{1382400000}$$

$$\frac{\left( \frac{\sin(13)\pi}{1^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{3} \times 3^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{5} \times 5^6(\sin(13)\pi)} \right) \left( \left(\frac{\pi}{2}\right)^6 \left(5 + \frac{6}{4}\right)^5 \right)}{5!} =$$

$$\frac{283201136699 \sum_{k=0}^{\infty} \frac{1}{(1+2k)^6}}{194400000}$$

$$\frac{\left( \frac{\sin(13)\pi}{1^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{3} \times 3^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{5} \times 5^6(\sin(13)\pi)} \right) \left( \left(\frac{\pi}{2}\right)^6 \left(5 + \frac{6}{4}\right)^5 \right)}{5!} =$$

$$\frac{283201136699 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^6}{45562500}$$

### Integral representations:

$$\frac{\left( \frac{\sin(13)\pi}{1^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{3} \times 3^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{5} \times 5^6(\sin(13)\pi)} \right) \left( \left(\frac{\pi}{2}\right)^6 \left(5 + \frac{6}{4}\right)^5 \right)}{5!} =$$

$$\frac{283\,201\,136\,699 \left( \int_0^1 \sqrt{1-t^2} dt \right)^6}{45\,562\,500}$$

$$\frac{\left( \frac{\sin(13)\pi}{1^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{3} \times 3^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{5} \times 5^6(\sin(13)\pi)} \right) \left( \left(\frac{\pi}{2}\right)^6 \left(5 + \frac{6}{4}\right)^5 \right)}{5!} =$$

$$\frac{283\,201\,136\,699 \left( \int_0^\infty \frac{1}{1+t^2} dt \right)^6}{2\,916\,000\,000}$$

$$\frac{\left( \frac{\sin(13)\pi}{1^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{3} \times 3^6(\sin(13)\pi)} + \frac{\sin(13)\pi}{\frac{1}{5} \times 5^6(\sin(13)\pi)} \right) \left( \left(\frac{\pi}{2}\right)^6 \left(5 + \frac{6}{4}\right)^5 \right)}{5!} =$$

$$\frac{283\,201\,136\,699 \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^6}{2\,916\,000\,000}$$

92.447270335213693

-13.5220600249863...

6059.2497124441

1458.90406609...

59.51536382032341...

$$-(843+29+4) + 0.64 * -(-6059.2497124441 * 1/1458.90406609) *$$

$$(92.447270335213693 * -59.51536382032341 * -13.5220600249863)$$

Where 843, 29 and 4 are Lucas numbers

**Input interpretation:**

$$-(843 + 29 + 4) + 0.64 \times (-1) \left( -6059.2497124441 \times \frac{1}{1458.90406609} \right)$$

$$(92.447270335213693 \times (-59.51536382032341) \times (-13.5220600249863))$$

**Result:**

196883.8549388760524496170508460679685211925628748461938486...

196883.854938...

$$-(32^2+47+4)+196884*(47-2) / \frac{(-6059.2497124441 * 1458.90406609) * (92.447270335213693 * -59.51536382032341 * -13.5220600249863)}$$

Where 47 and 4 are Lucas numbers

**Input interpretation:**

$$\frac{-(32^2 + 47 + 4) + 196884 \left( \frac{47 - 2}{-6059.2497124441 \times 1458.90406609} \right)}{(92.447270335213693 \times (-59.51536382032341) \times (-13.5220600249863))}$$

**Result:**

73491.43789502874491362049018034879924370285424729960575759...

73491.437895...

We note that:

$$196883,854938 \div 73491,437895 = 2,6790039843$$

We have:

$$\begin{aligned} & \left( \frac{1}{1 - 0.449329} + \frac{0.449329}{(1 - 0.449329^2)(1 - 0.449329^3)} \right) + \\ & \frac{0.449329^2}{(1 - 0.449329^3)(1 - 0.449329^4)(1 - 0.449329^5)} = \\ & = \chi(q) = 2.6709253774829 \end{aligned}$$

$$\underline{73491.437895} * 2.6709253774829 = 196290.1465;$$

$$\underline{521 + 76 - 3 = 594}; \text{ where } 521, 76 \text{ and } 3 \text{ are Lucas numbers}$$

$$\underline{196290.1465 + 594 = 196884.1465}$$

Results concerning and linked to the finite Dirichlet's series absolutely convergent of Ramanujan above analyzed.

With regard the previous results, i.e.:

$$\left( \begin{aligned}
 I_{21} &\ll \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leq p^{1-\varepsilon}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \\
 &\ll H \left( \sum_{\lambda \leq p^{1-\varepsilon_2}} \frac{|a(\lambda)|^2}{\lambda} \cdot \frac{4^r}{(\log P \lambda^{-1})^{2r}} + |W_2| \right) \ll \\
 &\ll H \left( \left( \frac{4}{\varepsilon_2 \log T} \right)^{2r} \sum_{\lambda \leq P} \frac{|a(\lambda)|^2}{\lambda} + |W_2| \right), \\
 I_{21} &\ll H \left\{ \left( \frac{4}{\varepsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + \right. \\
 &\quad \left. + (\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r}) T^{-\varepsilon_1} \right\}. 
 \end{aligned} \right) / 10^4 - 4096 - 2048 + 322$$

$$\approx A(r) \times \frac{1}{B(r)} \left( -\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}}$$

$$73491.976505275 \approx 73491.7883254\dots$$

We have that:

$$73491,976505275 \times 2,67092537 = 196291,58453938293632675$$

$$196291,58453938293632675 + 594 = 196885,58453938293632675$$

$$73491,7883254 \times 2,67092537 = 196291,081924980675398$$

$$196291,081924980675398 + 594 = 196885,081924980675398$$

We have the following data:

$$0.99951171875 = f_-(r)$$

$$1.00048828125 = f_+(r)$$

$$p = 6; c_1 = 1/4 = 0.25$$

$$\sqrt{((2^2 - ((7/16) \cdot 0.25^2)))} = k = 1.9931523$$

From:

### Supergravity Description of Non-BPS Branes

*Philippe Brax, Gautam Mandal, and Yaron Oz* - arXiv:hep-th/0005242v2 13 Jul 2000

Figure 2: The two-parameter space of solutions for  $Q = 0$ , as parameterised by  $M, c_1$ . Path II represents decay of the brane-antibrane configuration to flat space.

Now (26) implies  $c_2 = \pm 1$ . As remarked in Section 3 below, the physically relevant choice for  $p > 3$  is  $c_2 = 1$ , while for  $p < 3$  it is  $c_2 = -1$  (for  $p = 3$  the two choices are physically equivalent). To simplify the discussion we will present the formulae in the rest of this section for  $p > 3$ ; it is straightforward to write down the formulae in the other cases.

The solution now reads

$$\begin{aligned} e^{2A} &= \left(\frac{f_-}{f_+}\right)^\alpha, \\ e^{2B} &= f_-^{\beta_-} f_+^{\beta_+}, \\ e^\phi &= (f_-/f_+)^{\gamma}, \\ e^\Lambda &= 0, \end{aligned} \tag{27}$$

where

$$\begin{aligned} \alpha &= (7-p) \left( \frac{(3-p)c_1 + 4k}{32} \right), \\ \beta_\pm &= \frac{2}{7-p} \mp \left( \frac{(p+1)((p-3)c_1 - 4k)}{32} \right), \\ \gamma &= \frac{1}{16} ((7-p)(p+1)c_1 - 4(3-p)k). \end{aligned} \tag{28}$$

These represent the most general 2-parameter  $(r_0, c_1)$  solution of Type II supergravity with no gauge field and  $\text{SO}(p,1) \times \text{SO}(9-p)$  symmetry.

$$e^{2A} = \left(\frac{f_-}{f_+}\right)^\alpha$$

$$\alpha = (7-p) \left( \frac{(3-p)c_1 + 4k}{32} \right)$$

For p = 6

$$(7-6) (((((3-6)*1/4+4*1.9931523))/32))$$

**Input interpretation:**

$$(7-6) \left( \frac{1}{32} \left( (3-6) \times \frac{1}{4} + 4 \times 1.9931523 \right) \right)$$

**Result:**

$$0.2257065375$$

$$0.2257065375 = \alpha$$

For:

$$0.99951171875 = f_-(r)$$

$$1.00048828125 = f_+(r)$$

And:

$$e^{2A} = \left( \frac{f_-}{f_+} \right)^\alpha$$

We obtain:

$$(0.99951171875 / 1.00048828125)^{0.2257065375}$$

**Input interpretation:**

$$\left( \frac{0.99951171875}{1.00048828125} \right)^{0.2257065375}$$

**Result:**

$$0.999779607732\dots$$

**0.999779607732....** result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{\frac{1}{1+\sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}-1}}-\varphi+1}}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+...}}}\approx 0.9991104684$$

And:

$$e^{2B} = f_-^{\beta_-} f_+^{\beta_+}$$

$$\beta_{\pm} = \frac{2}{7-p} \mp \left( \frac{(p+1)((p-3)c_1 - 4k)}{32} \right)$$

$$p = 6; c_1 = 1/4 = 0.25$$

$$k = 1.9931523$$

$$2 + (((((6+1)*((6-3)*0.25-4*1.9931523))/32))))$$

**Input interpretation:**

$$2 + \frac{1}{32} ((6 + 1) ((6 - 3) \times 0.25 + 4 \times (-1.9931523)))$$

**Result:**

$$0.4200542375$$

$$0.4200542375$$

$$2 - (((((6+1)*((6-3)*0.25-4*1.9931523))/32))))$$

**Input interpretation:**

$$2 - \frac{1}{32} ((6 + 1) ((6 - 3) \times 0.25 + 4 \times (-1.9931523)))$$

**Result:**

$$3.5799457625$$

$$3.5799457625$$

$$c^{2B} = f_-^{\beta_-} f_+^{\beta_+}$$

For:

$$0.99951171875 = f_-(r)$$

$$1.00048828125 = f_+(r)$$

$$(0.99951171875^{3.5799457625} * 1.00048828125^{0.4200542375})$$

**Input interpretation:**

$$0.99951171875^{3.5799457625} \times 1.00048828125^{0.4200542375}$$

**Result:**

$$0.9984577977\dots$$

$$0.9984577977\dots$$

$$e^\phi = (f_-/f_+)^{\gamma}$$

$$\gamma = \frac{1}{16} ((7-p)(p+1)c_1 - 4(3-p)k)$$

$$1/16(((7-6)(6+1)*0.25-4(3-6)*1.9931523))$$

**Input interpretation:**

$$\frac{1}{16} ((7-6)(6+1)\times 0.25 + 4(3-6)\times (-1.9931523))$$

**Result:**

$$1.604239225$$

$$1.604239225$$

For:

$$0.99951171875 = f_-(r)$$

$$1.00048828125 = f_+(r)$$

$$(0.99951171875/1.00048828125)^{1.604239225}$$

**Input interpretation:**

$$\left(\frac{0.99951171875}{1.00048828125}\right)^{1.604239225}$$

**Result:**

0.99843458655...

0.99843458655...

From:

**"Boundary States and Black p-branes"** Shinpei Kobayashi (RESCEU) in collaboration with Tsuguhiko Asakawa (RIKEN) So Matsuura (RIKEN)" 2004/05/19

**dilaton (10-dim.)**

$$\begin{aligned}
\hat{\phi}^{(2)} &= \frac{3-p}{2\sqrt{2}} k T_p^2 G(r)^2 \\
&= \kappa \frac{(p-3)\Gamma\left(\frac{9-p}{2}\right)^2 T_p^2}{8\sqrt{2}\pi^{9-p}(7-p)^2} \frac{1}{r^{14-2p}} \\
&= \kappa \frac{T_p^2}{2^{7-p}\sqrt{2}\pi^{\frac{9-p}{2}}(7-p)^2} \frac{p-3}{(7-p)(p-5)} \frac{\Gamma\left(\frac{9-p}{2}\right)^2 \Gamma\left(\frac{p-3}{2}\right)}{\Gamma(8-p)} \int \frac{d^{9-p}k_i}{(2\pi)^{9-p}} \frac{e^{ik_i \cdot x}}{|k_i|^{p-5}}
\end{aligned}$$

(a)

From:

**D–Brane Primer**

*Clifford V. Johnson*

arXiv:hep-th/0007170v3 24 Aug 2000

This agrees with the recursion relation (183). The actual D–brane tension includes a factor of the string coupling from the action (179),

$$\tau_p = \frac{\sqrt{\pi}}{16\kappa} (4\pi^2 \alpha')^{(11-p)/2} \quad (192)$$

$$\kappa^{-1} = 2.4e+18; \quad \kappa = 4.1666\dots e-19$$

$\alpha'$  that is  $0.95686 \approx 1 \text{ (GeV)}^{-2}$

$$(((\sqrt{\pi}(4\pi^2 \times 0.95686)^{2.5})) / ((16 \times 4.1666 \times 10^{-19}))$$

**Input interpretation:**

$$\frac{\sqrt{\pi} (4 \pi^2 \times 0.95686)^{2.5}}{16 \times 4.1666 \times 10^{-19}}$$

**Result:**

$$2.33181\dots \times 10^{21}$$

$$2.33181\dots \times 10^{21} = T_P$$

$$(((4.1666e-19 * (((3(\gamma 3/2)^2 * (2.33181e+21)^2))) / (((8\sqrt{2}*\pi^3)))) * ((1 / 4096^2)))$$

**Input interpretation:**

$$\left(4.1666 \times 10^{-19} \times \frac{3 \Gamma\left(\frac{3}{2}\right)^2 (2.33181 \times 10^{21})^2}{8 \sqrt{2} \pi^3}\right) \times \frac{1}{4096^2}$$

$\Gamma(x)$  is the gamma function

**Result:**

$$9.06995\dots \times 10^{14}$$

$$9.06995\dots \times 10^{14}$$

For  $r = 5$ ,

**Input interpretation:**

we obtain:

$$(((4.1666e-19 * (((3(\gamma 3/2)^2 * (2.33181e+21)^2))) / (((8\sqrt{2}*\pi^3)))) * ((1 / 5^2)))$$

$$\left(4.1666 \times 10^{-19} \times \frac{3 \Gamma\left(\frac{3}{2}\right)^2 (2.33181 \times 10^{21})^2}{8 \sqrt{2} \pi^3}\right) \times \frac{1}{5^2}$$

$\Gamma(x)$  is the gamma function

**Result:**

$$6.08674\dots \times 10^{20}$$

$$6.08674\dots * 10^{20}$$

The mass of the dilaton is of the order of one hundred MeV, or about two hundred times (we take  $181 = 199-18$ ) the mass of an electron, which weighs about  $10^{-27}$  grams (or  $10^{-31}$  kg)

$$\text{electron mass} = 9,1093837015(28) \times 10^{-31} \text{ kg}$$

$$181 * 0.91093837015 \times 10^{-27} \text{ grams}$$

Where  $181 = 199-18$  that are Lucas numbers

**Input interpretation:**

$$181 \times 0.91093837015 \times 10^{-27} \text{ grams}$$

**Unit conversion:**

$$1.64879845 \times 10^{-28} \text{ kg (kilograms)}$$

$$1.64879845 * 10^{-28} \text{ kg}$$

Now, we have that:

$$13 * \sqrt{1 / 1.64879845 * 10^{-28}}$$

Where 13 is a Fibonacci number

**Input interpretation:**

$$13 \sqrt{\frac{1}{1.64879845 \times 10^{-28}}}$$

**Result:**

$$1.01241732\dots \times 10^{15}$$

$$1.01241732\dots * 10^{15}$$

With regard the Regge slope, we have that:

Table (1) summarizes the results of the fits for the mesons in the  $(J, M^2)$  plane. Tables (2) and (3) likewise summarize the results of the two types of fits for the  $(n, M^2)$  trajectories, that of the rotating string and of the WKB approximation.

The higher values of  $\alpha'$  and  $a$  always correspond to higher values of the endpoint masses, and the ranges listed are those where  $\chi^2$  is within 10% of its optimal value.

Traj.	$N$	$m$	$\alpha'$	$a$
$\pi/\pi_2$	$4 + 3$	$m_{u/d} = 110 - 250$	$0.788 - 0.852$	$a_0 = (-0.22) - (-0.00)$ $a_2 = (-0.00) - 0.26$
$a_1$	$4$	$m_{u/d} = 0 - 390$	$0.783 - 0.849$	$(-0.18) - 0.21$
$h_1$	$4$	$m_{u/d} = 0 - 235$	$0.833 - 0.850$	$(-0.14) - (-0.02)$
$\omega/\omega_3$	$5 + 3$	$m_{u/d} = 255 - 390$	$0.988 - 1.18$	$a_1 = 0.81 - 1.00$ $a_3 = 0.95 - 1.15$
$\phi$	$3$	$m_s = 510 - 520$	$1.072 - 1.112$	$1.00$
$\Psi$	$4$	$m_c = 1380 - 1460$	$0.494 - 0.547$	$0.71 - 0.88$
$\Upsilon$	$6$	$m_b = 4725 - 4740$	$0.455 - 0.471$	$1.00$
$\chi b$	$3$	$m_b = 4800$	$0.499$	$0.58$

**Table 2.** The results of the meson fits in the  $(n, M^2)$  plane. The ranges listed are those where  $\chi^2$  is within 10% of its optimal value.  $N$  is the number of data points in the trajectory.

Now, we take the following values: 0.988 and the result of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}} - 1}}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

We obtain:

$(0.988 + 0.9991104684)/2 = 0.9935552342$ . Thence, in conclusion.

$((\text{sqrt}(\text{Pi})(4\text{Pi}^2 * 0.9935552342)^{2.5})) / ((16 * 4.1666\text{e}-19))$

**Input interpretation:**

$$\frac{\sqrt{\pi} (4\pi^2 \times 0.9935552342)^{2.5}}{16 \times 4.1666 \times 10^{-19}}$$

**Result:**

$$2.56184\dots \times 10^{21}$$

$$2.56184\dots \times 10^{21}$$

$$(((4.1666e-19 * (((3(\text{gamma } 3/2)^2 * (2.56184e+21)^2))) / (((8\sqrt{2}*\text{Pi}^3)))) * ((1/4096^2)))$$

**Input interpretation:**

$$\left(4.1666 \times 10^{-19} \times \frac{3 \Gamma\left(\frac{3}{2}\right)^2 (2.56184 \times 10^{21})^2}{8 \sqrt{2} \pi^3}\right) \times \frac{1}{4096^2}$$

$\Gamma(x)$  is the gamma function

**Result:**

$$1.09477\dots \times 10^{15}$$

$$1.09477\dots \times 10^{15} = \hat{\phi}^{(2)}$$

Thence the following possible mathematical connection:

$$\begin{aligned} & \left( 13 \sqrt{\frac{1}{1.64879845 \times 10^{-28}}} \right) = 1.01241732 \times 10^{15} \cong \\ & \cong \left( \left( 4.1666 \times 10^{-19} \times \frac{3 \Gamma\left(\frac{3}{2}\right)^2 (2.56184 \times 10^{21})^2}{8 \sqrt{2} \pi^3} \right) \times \frac{1}{4096^2} \right) = 1.09477 \times 10^{15} \end{aligned}$$

$$1.01241732\dots \times 10^{15} \approx 1.09477\dots \times 10^{15}$$

For  $r = 961 \times 10^{22}$

$$(((4.1666e-19 * (((3(\text{gamma } 3/2)^2 * (2.33181e+21)^2))) / (((8\sqrt{2}*\text{Pi}^3)))) * 1/(961 \times 10^{22})^2)))$$

Where  $961 = 987-21-5$  that are Fibonacci numbers

**Input interpretation:**

$$\left( 4.1666 \times 10^{-19} \times \frac{3 \Gamma\left(\frac{3}{2}\right)^2 (2.33181 \times 10^{21})^2}{8 \sqrt{2} \pi^3} \right) \times \frac{1}{(961 \times 10^{22})^2}$$

$\Gamma(x)$  is the gamma function

**Result:**

$$1.64770\dots \times 10^{-28}$$

$1.64770\dots \times 10^{-28}$  result very near to the value of dilaton mass  $1.64879845 \times 10^{-28}$  kg

With  $r = 4096$ , after the following calculation, we obtain the following expression:

$$(199-2)*1/((((4.1666e-19*(((3(gamma 3/2)^2 * (2.56184e+21)^2))) / (((8sqrt(2)*Pi^3)))) * ((1 / 4096^2))))^2$$

Where 2 and 199 are Lucas numbers

**Input interpretation:**

$$(199 - 2) \times \frac{1}{\left( \left( 4.1666 \times 10^{-19} \times \frac{3 \Gamma\left(\frac{3}{2}\right)^2 (2.56184 \times 10^{21})^2}{8 \sqrt{2} \pi^3} \right) \times \frac{1}{4096^2} \right)^2}$$

$\Gamma(x)$  is the gamma function

**Result:**

$$1.64370\dots \times 10^{-28}$$

$1.64370\dots \times 10^{-28}$  result that is very near to the value of dilaton mass like those obtained previously.

Thence, we have this further mathematical connection:

$$\left( \left( 4.1666 \times 10^{-19} \times \frac{3 \Gamma\left(\frac{3}{2}\right)^2 (2.33181 \times 10^{21})^2}{8 \sqrt{2} \pi^3} \right) \times \frac{1}{(961 \times 10^{22})^2} \right) = 1.64770 \times 10^{-28} \cong$$

$$\approx \left( \frac{(199 - 2) \times \frac{1}{\left( 4.1666 \times 10^{-19} \times \frac{3 \Gamma(\frac{3}{2})^2 (2.56184 \times 10^{21})^2}{8 \sqrt{2} \pi^3} \right) \times \frac{1}{4096^2}}}{2} \right) = 1.64370 \times 10^{-28}$$

$$1.64770\dots * 10^{-28} \approx 1.64370\dots * 10^{-28}$$

Note that, the result of the hypothetical dilaton mass is a sub-multiple very near to  $\zeta(2) = 1.64493\dots$

From:

### **Excited D-branes and Supergravity Solutions**

Tsuguhiko Asakawa, Shinpei Kobayashi and So Matsuura  
hep-th/0506221 - June 2005

**The boundary state corresponding to the NSNS-sector of  $N$  Dp-branes in the limit of  $u \rightarrow \infty$**

Now, we have:

$$1.7860228654147\dots * 10^{38} = N_p \quad 2.33181\dots * 10^{21} = T_P$$

$$\alpha' \text{ that is } 0.9935552342 \approx 1 (\text{GeV})^{-2} \quad r = 4096; \quad n = 843 \text{ (Lucas number)}$$

$y = 4\pi\alpha'u^2$  is a dimensionless parameter  $0 < u < \infty$   $u = 9$

$$(4*\pi*0.9935552342*9^2)$$

### **Input interpretation:**

$$4\pi \times 0.9935552342 \times 9^2$$

### **Result:**

$$1011.316047\dots$$

$$1011.316047\dots$$

Now, we have:

$$|Bp'; u\rangle_{\text{NS}} = N \exp \left[ \int d\tilde{\sigma} \left( -\frac{1}{4u^2} P_i D P_i \right) \right] |Bp\rangle_{\text{NS}}, \quad (\text{A.7})$$

where we determine the constant factor of (A.1) so that  $|Bp'; u\rangle_{\text{NS}}$  becomes the boundary state corresponding to the NSNS-sector of  $N$  Dp-branes in the limit of  $u \rightarrow \infty$ .

Using the explicit expression of  $P_i(\tilde{\sigma})$  by the string oscillators and the zeta-function regularization,

$$\prod_{m=1}^{\infty} \frac{b}{m+a} = \frac{\Gamma(a+1)}{\sqrt{2\pi b}}, \quad \prod_{r=1/2}^{\infty} \frac{b}{r+a} = \frac{\Gamma(a+1/2)}{\sqrt{2\pi}}, \quad (\text{A.8})$$

we obtain

$$\begin{aligned} |Bp'; u\rangle_{\text{NS}} &= N \frac{T_p}{2} \left[ \frac{4^y y^{1/2} \Gamma^2(y)}{\sqrt{4\pi} \Gamma(2y)} \right]^{9-p} \exp \left[ - \sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n}^M S_{MN}^{(n)} \tilde{\alpha}_{-n}^N + i \sum_{r=1/2}^{\infty} b_{-r}^M S_{MN}^{(r)} \tilde{b}_{-r}^N \right] \times \\ &\times \int \frac{\prod_{i=p+1}^9 dp_i}{(2\pi)^{9-p}} \exp \left( - \frac{p_i^2}{8\pi u^2} \right) |p_\mu = 0, p_i\rangle_{\text{NS}}, \quad (y = 4\pi\alpha' u^2) \end{aligned} \quad (\text{A.9})$$

with

$$S_{MN}^{(n)} = \left( \eta_{\mu\nu}, -\frac{y-n}{y+n} \delta_{ij} \right), \quad S_{MN}^{(r)} = \left( \eta_{\mu\nu}, -\frac{y-r}{y+r} \delta_{ij} \right). \quad (\text{A.10})$$

$$-(1011.316047-843)/(1011.316047+843) = S_{MN}^{(n)}$$

### Input interpretation:

$$-\frac{1011.316047 - 843}{1011.316047 + 843}$$

### Result:

$$-0.09076988104175102357834473294616319523227423162131541430\dots$$

$$-0.090769881041751\dots$$

$$-(1011.316047-4096)/(1011.316047+4096) = S_{MN}^{(r)}$$

### Input interpretation:

$$-\frac{1011.316047 - 4096}{1011.316047 + 4096}$$

## Result:

0.603973579197614124074589504251214003635753462115832911172...

0.6039735791976...

We have:

$$1.7860228654147... * 10^{38} = N_p \quad 2.33181... * 10^{21} = T_P$$

$\alpha'$  that is  $0.9935552342 \approx 1$   $(GeV)^{-2}$   $r = 4096$ ;  $n = 843$  (Lucas number)

$$y = 1011.316047; u = 9; p_i = 8$$

$$\langle Bp'; u \rangle_{\text{NS}} = N \frac{T_p}{2} \left[ \frac{4^y y^{1/2} \Gamma^2(y)}{\sqrt{4\pi} \Gamma(2y)} \right]^{9-p} \exp \left[ - \sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n}^M S_{MN}^{(n)} \tilde{\alpha}_{-n}^N + i \sum_{r=1/2}^{\infty} b_{-r}^M S_{MN}^{(r)} \tilde{b}_{-r}^N \right] \times \\ \times \int \frac{\prod_{i=p+1}^9 dp_i}{(2\pi)^{9-p}} \exp \left( - \frac{p_i^2}{8\pi u^2} \right) |p_\mu = 0, p_i\rangle_{\text{NS}}, \quad (y = 4\pi \alpha' u^2) \quad (\text{A.9})$$

with

$$S_{MN}^{(n)} = \left( \eta_{\mu\nu}, -\frac{y-n}{y+n} \delta_{ij} \right), \quad S_{MN}^{(r)} = \left( \eta_{\mu\nu}, -\frac{y-r}{y+r} \delta_{ij} \right). \quad (\text{A.10})$$

Now:

$$N \frac{T_p}{2} \left[ \frac{4^y y^{1/2} \Gamma^2(y)}{\sqrt{4\pi} \Gamma(2y)} \right]^{9-p}$$

$$(1.78602286e+38 * 2.33181e+21) / 2 * (((((4^1011.316047 * 1011.316047^{1/2}) * \\ \text{gamma}^2(1011.316047))) / (((\text{sqrt}(4\text{Pi}) * \text{gamma}(2 * 1011.316047))))))^3$$

## Input interpretation:

$$\left( \frac{1}{2} (1.78602286 \times 10^{38} \times 2.33181 \times 10^{21}) \right) \\ \left( \frac{4^{1011.316047} \sqrt{1011.316047} \Gamma(1011.316047)^2}{\sqrt{4\pi} \Gamma(2 \times 1011.316047)} \right)^3$$

$\Gamma(x)$  is the gamma function

## Result:

$$2.08311... \times 10^{59}$$

**2.08311...\*10<sup>59</sup>**

Now, we have:

where  $T_p$  is the tension for a single D $p$ -brane,  $\mu = 0, 1, \dots, p$  are the directions longitudinal to the worldvolume,  $i = p+1, \dots, 9$  are the directions transverse to the D $p$ -branes, and  $\alpha_{-n}^M$  and  $b_{-r}^M$  ( $\tilde{\alpha}_{-n}^M$  and  $\tilde{b}_{-r}^M$ ) are the creation operators of the modes of the left-moving (right-moving) worldsheet bosons and fermions, respectively.  $S_{MN} = \text{diag}(\eta_{\mu\nu}, -\delta_{ij})$  gives Neumann (Dirichlet) boundary conditions on the string worldsheet in  $\mu(i)$  directions, respectively. Here the number

Thence:

$$-0.090769881041751\dots \quad 0.6039735791976\dots$$

We take: 26 left-movers and 10 right-movers (see Polchinski book Vol. II pag.73)

$$\exp \left[ - \sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n}^M S_{MN}^{(n)} \tilde{\alpha}_{-n}^N + i \sum_{r=1/2}^{\infty} b_{-r}^M S_{MN}^{(r)} \tilde{b}_{-r}^N \right]$$

$$\exp(-1/843*26*-0.090769881041751*26 + i * 10*0.6039735791976*10)$$

**Input interpretation:**

$$\exp\left(-\frac{1}{843} \times 26 \times (-0.090769881041751) \times 26 + i \times 10 \times 0.6039735791976 \times 10\right)$$

*i* is the imaginary unit

**Result:**

$$-0.81765160391\dots - \\ 0.69867869758\dots i$$

**Polar coordinates:**

$$r = 1.07550270471 \text{ (radius)}, \quad \theta = -139.486297457^\circ \text{ (angle)}$$

**1.07550270471**

And:

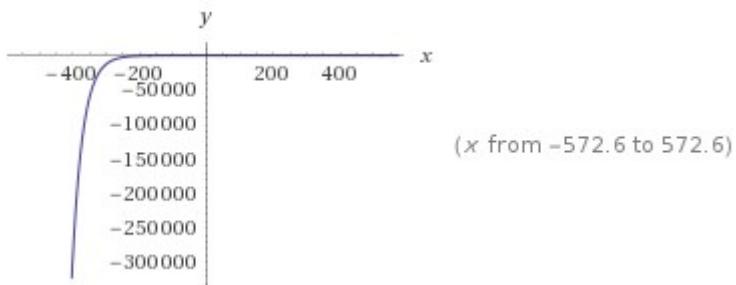
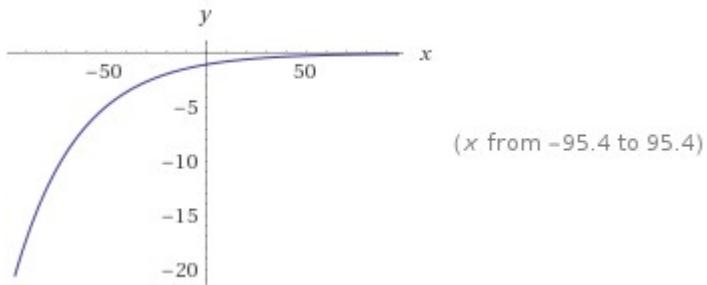
$$\int \frac{\prod_{i=p+1}^9 dp_i}{(2\pi)^{9-p}} \exp\left(-\frac{p_i^2}{8\pi u^2}\right) |p_\mu = 0, p_i\rangle_{\text{NS}}$$

integrate  $[8/(2\pi)^3 \exp(-64/(81\pi^8))x]$

### Indefinite integral:

$$\int \frac{8 \exp\left(-\frac{64x}{81\pi^8}\right)}{(2\pi)^3} dx \approx \text{constant} + -1.02588 \times 2.71828^{-0.031438x}$$

### Plots of the integral:



### Alternate form assuming $x$ is real:

$$-\frac{81}{8\pi^2(e^{x/\pi})^{8/81}} + \text{constant}$$

### Series expansion of the integral at $x = 0$ :

$$-\frac{81}{8\pi^2} + \frac{x}{\pi^3} - \frac{4x^2}{81\pi^4} + \frac{32x^3}{19683\pi^5} - \frac{64x^4}{1594323\pi^6} + O(x^5)$$

(Taylor series)

### Definite integral:

- More digits

$$\int_0^\infty \frac{e^{-(8x)/(81\pi)}}{\pi^3} dx = \frac{81}{8\pi^2} \approx 1.02588$$

**1.02588**

• **Definite integral over a half-period:**

- More digits

$$\int_0^{-\frac{81i\pi^2}{8}} \frac{e^{-(8x)/(81\pi)}}{\pi^3} dx = \frac{81}{4\pi^2} \approx 2.05175$$

Thence:

$$(2.08311 \times 10^{59} * 1.07550270471 * 1.02588)$$

**Input interpretation:**

$$2.08311 \times 10^{59} \times 1.07550270471 \times 1.02588$$

**Result:**

$$229837174377516273682800$$

**Scientific notation:**

$$2.298371743775162736828 \times 10^{59}$$

$$2.2983717437... \times 10^{59}$$

And:

$\Gamma^\mu$ . In the last line  $\mathbf{x}^\mu$  is identified with the superfield  $\mathbf{X}^\mu$  on the string worldsheet. Then the state corresponding to this solution is

$$\begin{aligned} |Bp'; v\rangle_{\text{NS}} &\equiv \int [d\mathbf{X}^\mu] \exp \left\{ \int d\hat{\sigma} \left( -i\mathbf{X}^\mu \mathbf{P}_\mu - \frac{1}{4v^2} D\mathbf{X}^\mu D^2\mathbf{X}^\mu \right) \right\} |\mathbf{X}^M = 0\rangle_{\text{NS}} \\ &= \int [d\mathbf{X}^\mu] \exp \left\{ \int d\hat{\sigma} \left( -\frac{1}{4v^2} D\mathbf{X}^\mu D^2\mathbf{X}^\mu \right) \right\} |\mathbf{X}^\mu, \mathbf{X}^i = 0\rangle_{\text{NS}}, \end{aligned} \quad (\text{A.18})$$

where the measure has been fixed so that this state expresses the boundary state of  $N$  Dp-branes in the  $v \rightarrow \infty$  limit. This expression can again be evaluated using the zeta-function regularization as

$$\begin{aligned} |Bp'; v\rangle_{\text{NS}} &= \frac{NT_p}{2} \left( \frac{4^y y^{1/2} \Gamma^2(y)}{\sqrt{4\pi} \Gamma(2y)} \right)^{p+1} \\ &\times \exp \left[ - \sum_{n=1}^{\infty} \frac{1}{n} a_{-n}^M S_{MN}^{(n)} \tilde{a}_{-n}^N + i \sum_{r=1/2}^{\infty} b_{-r}^M S_{MN}^{(r)} \tilde{b}_{-r}^N \right] |p_\mu = 0, x^i = 0\rangle_{\text{NS}}, \end{aligned} \quad (\text{A.19})$$

where  $y = v^2/\pi\alpha'$  and

$$S_{MN}^{(n)} = \left( \frac{y-n}{y+n} \eta_{\mu\nu}, -\delta_{ij} \right), \quad S_{MN}^{(r)} = \left( \frac{y-r}{y+r} \eta_{\mu\nu}, -\delta_{ij} \right), \quad (\text{A.20})$$

In the limit of  $y \rightarrow \infty$ ,

$$\frac{4^y y^{1/2} \Gamma^2(y)}{\sqrt{4\pi} \Gamma(2y)} \rightarrow 1 \quad (\text{A.21})$$

For  $y \rightarrow \infty$ , we obtain:

$$(1.78602286e+38 * 2.33181e+21)/2$$

### Input interpretation:

$$\frac{1}{2} (1.78602286 \times 10^{38} \times 2.33181 \times 10^{21})$$

### Result:

$$208\,233\,298\,258\,830\,000$$

### Scientific notation:

$$2.0823329825883 \times 10^{59}$$

$$2.0823329825883 * 10^{59}$$

We have:

$$|Bp'; u\rangle_{\text{NS}} = N \exp \left[ \int d\hat{\sigma} \left( -\frac{1}{4u^2} \mathbf{P}_i D\mathbf{P}_i \right) \right] |Bp\rangle_{\text{NS}}$$

$$= 2.2983717437... * 10^{59}$$

$$\left| Bp'; v \right\rangle_{\text{NS}} = \int [d\mathbf{X}^\mu] \exp \left\{ \int d\hat{\sigma} \left( -\frac{1}{4v^2} D\mathbf{X}^\mu D^2\mathbf{X}^\mu \right) \right\} \left| \mathbf{X}^\mu, \mathbf{X}^i = 0 \right\rangle_{\text{NS}}$$

$$= 2.0823329825883 * 10^{59}$$

We have the following expressions:

$$(2.2983717437 * 10^{59}) / (2.0823329825883 * 10^{59})$$

**Input interpretation:**

$$\frac{2.2983717437 \times 10^{59}}{2.0823329825883 \times 10^{59}}$$

**Result:**

$$1.103748422043033655081434665916266021109135320693631356426...$$

1.103748422043.... result that is very near to the sum of the following two Ramanujan continued fraction:

$$0.5683000031 + 0.5269391135 = 1.0952391166$$

$$4 \int_0^\infty \frac{tdt}{e^{\sqrt{5}t} \cosh t} = \frac{1}{1 + \frac{1^2}{1 + \frac{1^2}{1 + \frac{2^2}{1 + \frac{2^2}{1 + \frac{3^2}{1 + \frac{3^2}{1 + \dots}}}}}} \approx 0.5683000031$$

$$2 \int_0^\infty \frac{t^2 dt}{e^{\sqrt{3}t} \sinh t} = \frac{1}{1 + \frac{1^3}{1 + \frac{1^3}{1 + \frac{2^3}{3 + \frac{2^3}{1 + \frac{3^3}{5 + \frac{3^3}{1 + \frac{7}{\dots}}}}}}} \approx 0.5269391135$$

$$(10178) + 2 * (((2.2983717437 * 10^{59}) + (2.0823329825883 * 10^{59})))^{1/12}$$

We have  $10178 = 6765 + 2584 + 610 + 199 + 18 + 2$ , where 6765, 2584, 610 are Fibonacci numbers, while 199, 18 and 2 are Lucas numbers

### **Input interpretation:**

$$10178 + 2 \sqrt[12]{2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59}}$$

### **Result:**

$$196884.165976\dots$$

**196884.165976**.... ... result practically equal to the value of the following partition function:

$$\begin{aligned} Z_{24}(\tau) &= j(\tau) - 744 \\ &= q^{-1} + 196884 q + 21493760 q^2 + 864299970 q^3 + 20245856256 q^4 + \dots \end{aligned}$$

From:

$$\begin{aligned}
|Bp'; u\rangle_{\text{NS}} &= N \exp \left[ \int d\hat{\sigma} \left( -\frac{1}{4u^2} \mathbf{P}_i D \mathbf{P}_i \right) \right] |Bp\rangle_{\text{NS}} \\
&= 2.2983717437... * 10^{59} \\
|\mathbf{B}p'; v\rangle_{\text{NS}} &= \int [d\mathbf{X}^\mu] \exp \left\{ \int d\hat{\sigma} \left( -\frac{1}{4v^2} D\mathbf{X}^\mu D^2 \mathbf{X}^\mu \right) \right\} |\mathbf{X}^\mu, \mathbf{X}^i = 0\rangle_{\text{NS}} \\
&= 2.0823329825883 * 10^{59}
\end{aligned}$$

$-(4181 - 233 - 21) + 2 * (((2.2983717437 * 10^{59}) + (2.0823329825883 * 10^{59})))^{1/13}$

Where 4181, 233 and 21 are Fibonacci numbers

### Input interpretation:

$$-(4181 - 233 - 21) + 2 \sqrt[13]{2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59}}$$

### Result:

$$73490.8437525...$$

73490.8437525... result very near to the following ratio (A. Nardelli) concerning the general asymptotically flat solution of the equations of motion of the p-brane:

$$\begin{aligned}
A(r) &\times \frac{1}{B(r)} \left( -\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}} \\
&-0.000029211892 \times \frac{1}{0.0003644621} \left( -\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} \\
&= 73491.78832548118710549159572042220548025195726563413398700... = \\
&= 73491.7883254...
\end{aligned}$$

We note that:

$$(((2.2983717437 * 10^{59}) + (2.0823329825883 * 10^{59})))^{1/276}$$

Where  $276 = 233 + 47 - 4$  (233 is a Fibonacci number, 47 and 4 are Lucas numbers)

### Input interpretation:

$$\sqrt[276]{2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59}}$$

### Result:

1.6447221856470...

$$1.6447221856470\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

In conclusion, we have also:

$$\ln(((2.2983717437 * 10^{59}) + (2.0823329825883 * 10^{59})))$$

### Input interpretation:

$$\log(2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59})$$

$\log(x)$  is the natural logarithm

### Result:

$$137.3297300945\dots$$

137.32973.... result very near to the rest mass of Pion mesons 139.570 and 134.976 and practically equal to the average is 137.273

### Alternative representations:

$$\log(2.29837174370000 \times 10^{59} + 2.08233298258830000 \times 10^{59}) = \\ \log_e(4.38070472628830 \times 10^{59})$$

$$\bullet \quad \log(2.29837174370000 \times 10^{59} + 2.08233298258830000 \times 10^{59}) = \\ \log(a) \log_a(4.38070472628830 \times 10^{59})$$

$$\bullet \quad \log(2.29837174370000 \times 10^{59} + 2.08233298258830000 \times 10^{59}) = \\ -\text{Li}_1(1 - 4.38070472628830 \times 10^{59})$$

$\log_b(x)$  is the base-  $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

## Series representations:

$$\log(2.29837174370000 \times 10^{59} + 2.08233298258830000 \times 10^{59}) = \\ \log(4.38070472628830 \times 10^{59}) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-137.329730094507464 k}}{k}$$

$$\log(2.29837174370000 \times 10^{59} + 2.08233298258830000 \times 10^{59}) = \\ 2 i \pi \left\lfloor \frac{\arg(4.38070472628830 \times 10^{59} - x)}{2 \pi} \right\rfloor + \log(x) - \\ \sum_{k=1}^{\infty} \frac{(-1)^k (4.38070472628830 \times 10^{59} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(2.29837174370000 \times 10^{59} + 2.08233298258830000 \times 10^{59}) = \\ \left[ \frac{\arg(4.38070472628830 \times 10^{59} - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) + \\ \log(z_0) + \left[ \frac{\arg(4.38070472628830 \times 10^{59} - z_0)}{2 \pi} \right] \log(z_0) - \\ \sum_{k=1}^{\infty} \frac{(-1)^k (4.38070472628830 \times 10^{59} - z_0)^k z_0^{-k}}{k}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

## Integral representations:

$$\log(2.29837174370000 \times 10^{59} + 2.08233298258830000 \times 10^{59}) = \\ \int_1^{4.38070472628830 \times 10^{59}} \frac{1}{t} dt$$

$$\log(2.29837174370000 \times 10^{59} + 2.08233298258830000 \times 10^{59}) = \\ \frac{1}{2 i \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-137.329730094507464 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$  is the gamma function

From:

**"Boundary States and Black p-branes"** Shinpei Kobayashi (RESCEU) in collaboration with Tsuguhiko Asakawa (RIKEN) So Matsuura (RIKEN)" 2004/05/19

$$\hat{\phi}(r) = -\frac{p-3}{2\sqrt{2}} T_p G(r) + \frac{p-3}{2\sqrt{2}} \kappa T_p^2 G(r)^2 + \dots$$

**From the following data:**

$$1.09477\dots * 10^{15} = \hat{\phi}^{(2)}$$

$\Omega_2 = 1$ , is a gauge transformation parameter,  $\kappa = 4.1666\dots \times 10^{-19}$

$$1.7860228654147\dots * 10^{38} = N_p \quad 2.33181\dots * 10^{21} = T_p$$

$\alpha'$  that is  $0.9935552342 \approx 1$  (GeV) $^{-2}$   $r = 4096$ ;  $n = 843$  (Lucas number)

$$y = 1011.316047; u = 9; p_i = 8, p = 6$$

$$e^{-2\phi} = e^{-2\sqrt{2}\kappa\hat{\phi}} = H(r)^{-\sqrt{2}\kappa\frac{p-3}{2}},$$

$$H(r) = 1 + \frac{2\kappa T_p}{(7-p)\Omega_{8-p} r^{7-p}} \equiv 1 + 2\kappa T_p G(r)$$

$$1 + \frac{2 \times 4.1666 \times 10^{-19} \times 2.33181 \times 10^{21}}{4096}$$

**Input interpretation:**

$$1 + \frac{2 \times 4.1666 \times 10^{-19} \times 2.33181 \times 10^{21}}{4096}$$

**Result:**

$$1.47440036845703125$$

$$1.4744003684\dots = H(r)$$

$$(1.4744003684 - 1) / (2 * 4.1666e-19 * 2.33181e+21)$$

**Input interpretation:**

$$\frac{1.4744003684 - 1}{2 \times 4.1666 \times 10^{-19} \times 2.33181 \times 10^{21}}$$

**Result:**

$$0.000244140624970650012214751510489926198205227608985575384\dots$$

•

**Repeating decimal:**

$$0.000244140624970650012214751510489926198205227608985575384\dots$$

(period 2041800)

$$0.00024414062497065\dots = G(r)$$

We observe that:

$$1 / (((1.4744003684 - 1) / (2 * 4.1666e-19 * 2.33181e+21)))$$

**Input interpretation:**

$$\frac{1}{\frac{1.4744003684 - 1}{2 \times 4.1666 \times 10^{-19} \times 2.33181 \times 10^{21}}}$$

**Result:**

$$4096.000000492411084729671976367714810568852838184263096369\dots$$

4096.0000004924.... result practically equal to the value of r, that is 4096.

Note that  $\sqrt{4096} = 64 = 8^2$ , where 8 is a Fibonacci numbers and are the "modes" corresponding to the physical vibrations of a superstring.

Thence:

$$G(r) = 1 / r = 1 / 4096$$

Thence:

$$\hat{\phi}(r) = -\frac{p-3}{2\sqrt{2}} T_p G(r) + \frac{p-3}{2\sqrt{2}} \kappa T_p^2 G(r)^2 + \dots$$

$$-(((3/(2*\text{sqrt}(2)))*(2.33181\text{e}+21)*((1/(4096)))))+(((3/(2*\text{sqrt}(2)))*(4.1666\text{e}-19)*(2.33181\text{e}+21)^2)*((1/(4096))^2)))$$

**Input interpretation:**

$$-\left(\frac{3}{2\sqrt{2}} \times 2.33181 \times 10^{21} \times \frac{1}{4096} + \left(\frac{3}{2\sqrt{2}} \times 4.1666 \times 10^{-19} (2.33181 \times 10^{21})^2\right)\left(\frac{1}{4096}\right)^2\right)$$

**Result:**

$$-7.47050\dots \times 10^{17}$$

$$-7.47050\dots * 10^{17}$$

We observe that from the result of this expression, we can obtain:

$$(3571+233+11)+1/4[(((-(((3/(2*\text{sqrt}(2)))*(2.33181\text{e}+21)*((1/(4096)))))+(((3/(2*\text{sqrt}(2)))*(4.1666\text{e}-19)*(2.33181\text{e}+21)^2)*((1/(4096))^2))))]^1/3$$

Where 11 and 3571 are Lucas numbers and 233 is a Fibonacci number

**Input interpretation:**

$$(3571 + 233 + 11) + \frac{1}{4} \left( -\left( -\frac{3}{2\sqrt{2}} \left( 2.33181 \times 10^{21} \times \frac{1}{4096} \right) + \left( \frac{3}{2\sqrt{2}} \times 4.1666 \times 10^{-19} (2.33181 \times 10^{21})^2 \right) \left( \frac{1}{4096} \right)^2 \right) \right)^{(1/3)}$$

**Result:**

$$196884.3612355790262930818210234562962529826220937890171828\dots$$

**196884.36...**

And:

$$(2207+843+233)+1/11[(((\left(-\left(\left(\left(3/(2*\text{sqrt}(2))\right)*(2.33181e+21)*((1/(4096))))\right)+\left(\left(\left(3/(2*\text{sqrt}(2))\right)*(4.1666e-19)*(2.33181e+21)^2)*((1/(4096))^2)\right)\right)))])^{1/3}$$

Where 2207 and 843 are Lucas numbers and 233 is a Fibonacci number

### **Input interpretation:**

$$(2207 + 843 + 233) + \frac{1}{11} \left( -\left( -\frac{3}{2\sqrt{2}} \left( 2.33181 \times 10^{21} \times \frac{1}{4096} \right) + \left( \frac{3}{2\sqrt{2}} \times 4.1666 \times 10^{-19} (2.33181 \times 10^{21})^2 \right) \left( \frac{1}{4096} \right)^2 \right) \right)^{(1/3)}$$

### **Result:**

73490.04044930146410657520764489319863744822621592327897559...

[73490.0404....](#)

With regard the inverse of this expression, we obtain:

$$1/ ((((((((-(((3/(2*\text{sqrt}(2)))*(2.33181e+21)*((1/(4096)))))+(((3/(2*\text{sqrt}(2)))*(4.1666e-19)*(2.33181e+21)^2)*((1/(4096))^2)))))))$$

### **Input interpretation:**

$$-\frac{1}{\frac{3}{2\sqrt{2}} \times 2.33181 \times 10^{21} \times \frac{1}{4096} + \left( \frac{3}{2\sqrt{2}} \times 4.1666 \times 10^{-19} (2.33181 \times 10^{21})^2 \right) \left( \frac{1}{4096} \right)^2}$$

### **Result:**

$-1.33860... \times 10^{-18}$

$-1.33860... * 10^{-18}$

### **Result:**

$-1.338599166652011286129329335178616807048245060521807... \times 10^{-18}$

$-1.338599166652.... * 10^{-18}$

And:

$$1/(1.897512108+1.8236681145196)* (-1.338599166652011 \times 10^{-18})^2 * 1729 * 196884$$

Where 1.897512108 and 1.8236681145196 are two results of Ramanujan mock theta functions, 1729 is the Hardy-Ramanujan number and 196884 is a fundamental number of the following ***j*-invariant**

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \dots$$

(In mathematics, Felix Klein's ***j*-invariant** or ***j* function**, regarded as a function of a complex variable  $\tau$ , is a modular function of weight zero for  $SL(2, \mathbf{Z})$  defined on the upper half plane of complex numbers. Several remarkable properties of  $j$  have to do with its  $q$  expansion (Fourier series expansion), written as a Laurent series in terms of  $q = e^{2\pi i\tau}$  (the square of the nome), which begins:

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \dots$$

Note that  $j$  has a simple pole at the cusp, so its  $q$ -expansion has no terms below  $q^{-1}$ .

All the Fourier coefficients are integers, which results in several almost integers, notably Ramanujan's constant:

$$e^{\pi\sqrt{163}} \approx 640320^3 + 744.$$

The asymptotic formula for the coefficient of  $q^n$  is given by

$$\frac{e^{4\pi\sqrt{n}}}{\sqrt{2}n^{3/4}},$$

as can be proved by the Hardy–Littlewood circle method)

### Input interpretation:

$$\frac{1}{1.897512108 + 1.8236681145196} (-1.338599166652011 \times 10^{-18})^2 \times 1729 \times 196884$$

•

### Result:

$$1.6391768575611497127000918854364778314913891635633535\dots \times 10^{-28}$$

$1.6391768575611497\dots \times 10^{-28}$  result very near to the value of dilaton mass  
 $1.64879845 \times 10^{-28}$  kg

We now, analyze this other equation:

$$= K \frac{T_p^2}{2^{7-p} \sqrt{2} \pi^{\frac{9-p}{2}}} \frac{p-3}{(7-p)(p-5)} \frac{\Gamma\left(\frac{9-p}{2}\right)^2 \Gamma\left(\frac{p-3}{2}\right)}{\Gamma(8-p)} \int \frac{d^{9-p} k_i}{(2\pi)^{9-p}} \frac{e^{ik_i \cdot x}}{|k_i|^{p-5}}$$

For  $r = 5$ , from:

$$\kappa \frac{(p-3)\Gamma\left(\frac{9-p}{2}\right)^2 T_p^2}{8\sqrt{2}\pi^{9-p}(7-p)^2} \frac{1}{r^{14-2p}}$$

we obtain:

$$(((4.1666e-19 * (((3(gamma 3/2)^2 * (2.33181e+21)^2))) / (((8sqrt(2)*Pi^3)))) * ((1 / 5^2))$$

$$\left(4.1666 \times 10^{-19} \times \frac{3 \Gamma\left(\frac{3}{2}\right)^2 (2.33181 \times 10^{21})^2}{8 \sqrt{2} \pi^3}\right) \times \frac{1}{5^2}$$

$$6.08674... \times 10^{20}$$

$$\textcolor{red}{6.08674... * 10^{20}}$$

For:

$$\kappa = 4.1666...e-19; \ k = 1.9931523; \ 2.33181... * 10^{21} = T_P; \ r = 4096; \ n = 843 \text{ (Lucas number)}; \ y = 1011.316047; \ u = 9; \ p_i = 8, p = 6; \ x = 0.9502566373,$$

we obtain:

$$(((4.1666e-19 * (2.33181e+21)^2 * 3 * gamma(3/2)^2 * gamma(3/2))) / (((2sqrt(2)*Pi^1.5 * gamma(2))))$$

### **Input interpretation:**

$$\frac{4.1666 \times 10^{-19} (2.33181 \times 10^{21})^2 \times 3 \Gamma\left(\frac{3}{2}\right)^2 \Gamma\left(\frac{3}{2}\right)}{2 \sqrt{2} \pi^{1.5} \Gamma(2)}$$

$\Gamma(x)$  is the gamma function

### **Result:**

$$3.0036851307320539608604160394482007026163460017212449... \times 10^{23}$$

$$3.003685130732... * 10^{23}$$

$$3.003685130732 \times 10^{23} * \text{integrate} [(1.9931523)/(2Pi)^3 * ((exp(i*1.9931523*0.9502566373)) / (1.9931523)]x$$

### **Input interpretation:**

$$3.003685130732 \times 10^{23} \int \left( \frac{1.9931523}{(2\pi)^3} \times \frac{\exp(i \times 1.9931523 \times 0.9502566373)}{1.9931523} \right) x dx$$

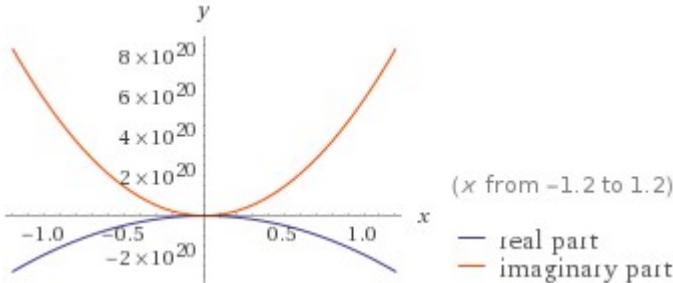
$i$  is the imaginary unit

### Result:

$$(-1.92301 \times 10^{20} + 5.74109 \times 10^{20} i) x^2$$

$$(-1.92301 \times 10^{20} + 5.74109 \times 10^{20} i) x^2$$

### Plot:



$$\text{For } x = \zeta(2e/0.63165)$$

$$(-1.92301 \times 10^{20} + 5.74109 \times 10^{20} i) (((\zeta(2e/0.63165))))^2$$

### Input interpretation:

$$(-1.92301 \times 10^{20} + 5.74109 \times 10^{20} i) \zeta\left(2 \times \frac{e}{0.63165}\right)^2$$

$\zeta(s)$  is the Riemann zeta function

$i$  is the imaginary unit

### Result:

$$-1.93322 \dots \times 10^{20} + 5.77157 \dots \times 10^{20} i$$

### Polar coordinates:

$$r = 6.08673 \times 10^{20} \text{ (radius)}, \quad \theta = 108.519^\circ \text{ (angle)}$$

6.08673 \* 10<sup>20</sup> result practically equal to the previous expression

We have also that:

$$-5778-18+76+1/2((((-1.92301 \times 10^{20} + 5.74109 \times 10^{20} i) (((\zeta(2e/0.63165))))^2)))^{1/4}$$

### Input interpretation:

$$-5778 - 18 + 76 + \frac{1}{2} \sqrt[4]{(-1.92301 \times 10^{20} + 5.74109 \times 10^{20} i) \zeta\left(2 \times \frac{e}{0.63165}\right)^2}$$

$\zeta(s)$  is the Riemann zeta function

$i$  is the imaginary unit

## Result:

$$64174.8\dots + \\ 35812.6\dots i$$

## Polar coordinates:

$$r = 73491.1 \text{ (radius), } \theta = 29.1636^\circ \text{ (angle)}$$

$$\color{blue}73491.1$$

From:

## D-branes, Matrix Theory and K-homology

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It is important to note that unlike the non-BPS D-instanton state without boundary interaction (5.1), the RR-sector may not be projected out, since the boundary interaction carries the fermion zero modes. Let us check that the boundary state (5.52) becomes (5.1) when the boundary interaction is turned off, i.e.  $T = \Phi_\mu = 0$ . In this case, the boundary interaction for RR-sector vanishes since the zero mode of  $\eta$  is not saturated in the  $\eta$  integral. The boundary interaction for NS-sector becomes

$$e^{-S_b(P,\Pi)} / (\text{Tr}_H 1) = \int [d\eta] e^{\oint d\sigma \left( \frac{1}{4} \dot{\eta} \eta \right)} = \int \prod_{r>0} d\eta_r d\eta_{-r} e^{(\pi r \eta_r \eta_{-r})} \quad (5.53)$$

$$= \prod_{r=1/2,1/3,\dots} \pi r \quad (5.54)$$

$$= \sqrt{2}. \quad (5.55)$$

In (5.68), we used a  $\zeta$ -function regularization trick

$$\prod_{r=1/2,3/2,\dots} u^2 = 1, \quad (5.70)$$

and took a naive limit. Let us justify (5.68) using oscillator expansion and  $\zeta$ -function regularization. First, we consider the bosonic part. We adopt the  $p = 0$  case for simplicity.

$$\int [dx] \exp \left( - \oint d\sigma \frac{\dot{x}^2}{4u^2} \right) |x\rangle \quad (5.71)$$

$$= \int dx_0 \prod_{n=1}^{\infty} \left( \int dx_{-n} dx_n e^{-\frac{n}{2u^2} x_{-n} x_n - \frac{1}{2} x_{-n} x_n - u_r^\dagger \tilde{a}_n^\dagger + a_n^\dagger x_n + x_{-n} \tilde{a}_n^\dagger} \right) |x_0\rangle \quad (5.72)$$

$$= \frac{u\Gamma(u^2)}{\sqrt{2\pi}} e^{-\sum_{n=1}^{\infty} \left( 1 - \frac{2}{1+n/u^2} \right) a_n^\dagger \tilde{a}_n^\dagger} |0\rangle. \quad (5.73)$$

We used the  $\zeta$ -function regularization to obtain (5.73) (see Appendix B).

Similarly the fermionic part can also be calculated.

$$\int [ud\theta] \exp \left( - \oint d\sigma \frac{\theta \dot{\theta}}{4u^2} \right) |\theta; \pm\rangle_{\text{NS}} \quad (5.74)$$

$$= \prod_{r>0}^{\infty} \left( \int u^2 d\theta_{-r} d\theta_r e^{-\frac{r}{2u^2} \theta_{-r} \theta_r - \frac{1}{2} \theta_{-r} \theta_r \pm i\psi_r^\dagger \tilde{\psi}_r^\dagger + \psi_r^\dagger \theta_r \mp i\theta_{-r} \tilde{\psi}_r^\dagger} \right) |0\rangle_{\text{NS}} \quad (5.75)$$

$$= \frac{\sqrt{2\pi}}{\Gamma(u^2 + \frac{1}{2})} e^{\pm \sum_{r>0}^{\infty} \left( 1 - \frac{2}{1+r/u^2} \right) i\psi_r^\dagger \tilde{\psi}_r^\dagger} |0\rangle_{\text{NS}}. \quad (5.76)$$

Combining (5.73) and (5.76) together, we obtain

$$e^{-S_b(P, \Pi_{\pm})} |D; \pm\rangle_{\text{NS}} \quad (5.77)$$

$$= \frac{u\Gamma(u^2)}{\Gamma(u^2 + \frac{1}{2})} e^{-\sum_{n=1}^{\infty} \left( 1 - \frac{2}{1+n/u^2} \right) a_n^\dagger \tilde{a}_n^\dagger \pm \sum_{r>0}^{\infty} \left( 1 - \frac{2}{1+r/u^2} \right) i\psi_r^\dagger \tilde{\psi}_r^\dagger} |0\rangle_{\text{NS}} \quad (5.78)$$

$$\rightarrow |B0; \pm\rangle_{\text{NS}} \quad (\text{as } u \rightarrow \infty), \quad (5.79)$$

which actually agrees with the previous estimation (5.69). This calculation is precisely analogous to that given in [15, 50, 51]. In fact the coefficient  $\frac{u\Gamma(u^2)}{\Gamma(u^2 + 1/2)} = \frac{4u^2 \Gamma(u^2)^2}{2\sqrt{\pi} \Gamma(2u^2)}$  in (5.78) is exactly the same as the factor which plays a crucial role to obtain the exact D-brane tension in BSFT [13, 15, 16].

Now, we have that:

$$\begin{aligned}
e^{-S_b(P,\Pi)} / (\text{Tr}_{\mathcal{U}} 1) &= \int [d\eta] e^{\oint d\sigma \left( \frac{i}{4} \bar{m} \right)} = \int \prod_{r>0} d\eta_r d\eta_{-r} e^{(\pi r \eta_r \eta_{-r})} \\
&= \prod_{r=1/2, 1/3, \dots} \pi r \\
&= \sqrt{2}.
\end{aligned} \tag{5.53-5.55}$$

$$\begin{aligned}
\int dx_0 \prod_{n=1}^{\infty} \left( \int dx_{-n} dx_n e^{-\frac{n}{2u^2} (x_{-n} x_n - \frac{1}{2} x_{-n} x_n - a_n^\dagger \tilde{a}_n^\dagger + a_n^\dagger x_n + x_{-n} \tilde{a}_n^\dagger)} \right) |x_0\rangle &= \\
= \frac{u \Gamma(u^2)}{\sqrt{2\pi}} e^{-\sum_{n=1}^{\infty} \left( 1 - \frac{2}{1+n/u^2} \right) a_n^\dagger \tilde{a}_n^\dagger} |0\rangle &
\end{aligned} \tag{5.72-5.73}$$

For  $u = -1$ ,  $u^2 = 1$  and  $n = 76$ , that is a Lucas number. we obtain:

$$((-1 * \text{gamma}(1) * \exp(-(1-2/(1+76))))) / (\sqrt{2*\pi})$$

**Input:**

$$-\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}$$

$\Gamma(x)$  is the gamma function

**Exact result:**

$$-\frac{1}{e^{75/77} \sqrt{2\pi}}$$

**Decimal approximation:**

$$-0.15062461854366025119000966396613316426978057953678458626\dots$$

$$-0.15062461854366\dots$$

**Alternative representations:**

$$-\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}} = -\frac{\exp\left(-1 + \frac{2}{77}\right)}{\sqrt{2\pi}}$$

$$-\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}} = -\frac{\exp\left(-1 + \frac{2}{77}\right) e^0}{\sqrt{2\pi}}$$

$$-\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}} = -\frac{\exp\left(-1 + \frac{2}{77}\right)(1)_0}{\sqrt{2\pi}}$$

$(a)_n$  is the Pochhammer symbol (rising factorial)

### Series representations:

$$-\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}} = -\frac{\exp\left(-\frac{75}{77}\right) \sum_{k=0}^{\infty} \frac{(1-z_0)^k \Gamma^{(k)}(z_0)}{k!}}{\exp\left(i\pi \left\lfloor \frac{\arg(2\pi-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2\pi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

for ( $x \in \mathbb{R}$  and ( $z_0 \notin \mathbb{Z}$  or  $z_0 > 0$ ) and  $x < 0$ )

$$-\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}} = -\frac{\exp\left(-\frac{75}{77}\right) \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(2\pi-z_0)/(2\pi) \rfloor} z_0^{-1/2-1/2 \lfloor \arg(2\pi-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(1-z_0)^k \Gamma^{(k)}(z_0)}{k!}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2\pi-z_0)^k z_0^{-k}}{k!}}$$

for ( $z_0 \notin \mathbb{Z}$  or  $z_0 > 0$ )

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$n!$  is the factorial function

$i$  is the imaginary unit

$\mathbb{R}$  is the set of real numbers

$\mathbb{Z}$  is the set of integers

### Integral representations:

$$-\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}} = -\frac{\exp\left(-\frac{75}{77}\right)}{\sqrt{2\pi}} \int_0^1 1 dt$$

$$-\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}} = -\frac{e^{\int_0^1 dx} \exp\left(-\frac{75}{77}\right)}{\sqrt{2\pi}}$$

$$-\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}} = -\frac{\exp\left(-\frac{75}{77}\right)}{\sqrt{2\pi}} \int_0^\infty e^{-t} dt$$

From the ratio between the two results, we obtain:

$$-\text{sqrt}(2) / (((((-1 * \text{gamma}(1) * \exp(-(1-2/(1+76)))))) / (\text{sqrt}(2*\text{Pi}))))$$

**Input:**

$$-\frac{\sqrt{2}}{\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}}$$

$\Gamma(x)$  is the gamma function

**Exact result:**

$$2 e^{75/77} \sqrt{\pi}$$

**Decimal approximation:**

$$9.388993486235248023358766237557825779873676817334217629463\dots$$

9.388993486.... result very near to the value of black hole entropy 9.3664

**Alternative representations:**

$$-\frac{-\sqrt{2}}{\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}} = -\frac{-\sqrt{2}}{-\frac{\exp\left(-1 + \frac{2}{77}\right)}{\sqrt{2\pi}}}$$

$$-\frac{-\sqrt{2}}{\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}} = -\frac{-\sqrt{2}}{-\frac{\exp\left(-1 + \frac{2}{77}\right) e^0}{\sqrt{2\pi}}}$$

$$-\frac{-\sqrt{2}}{\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}} = -\frac{-\sqrt{2}}{-\frac{\exp\left(-1 + \frac{2}{77}\right) (1)_0}{\sqrt{2\pi}}}$$

$(a)_n$  is the Pochhammer symbol (rising factorial)

## Series representations:

$$\frac{-\sqrt{2}}{\frac{\Gamma(1)\exp\left(-\left(1-\frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}} = \left( \exp\left(i\pi\left[\frac{\arg(2-x)}{2\pi}\right]\right) \exp\left(i\pi\left[\frac{\arg(2\pi-x)}{2\pi}\right]\right) \sqrt{x}^{-2} \right. \\ \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} (2-x)^{k_1} (2\pi-x)^{k_2} x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!} \right) / \\ \left( \exp\left(-\frac{75}{77}\right) \sum_{k=0}^{\infty} \frac{(1-z_0)^k \Gamma^{(k)}(z_0)}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0) \text{ and } x < 0)$$

$$\frac{-\sqrt{2}}{\frac{\Gamma(1)\exp\left(-\left(1-\frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}} = \\ \left( \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor + 1/2 \lfloor \arg(2\pi-z_0)/(2\pi) \rfloor} z_0^{1+1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor + 1/2 \lfloor \arg(2\pi-z_0)/(2\pi) \rfloor} \right. \\ \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (2\pi-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!} \right) / \\ \left( \exp\left(-\frac{75}{77}\right) \sum_{k=0}^{\infty} \frac{(1-z_0)^k \Gamma^{(k)}(z_0)}{k!} \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$n!$  is the factorial function

$i$  is the imaginary unit

$\mathbb{R}$  is the set of real numbers

$\mathbb{Z}$  is the set of integers

## Integral representations:

$$\frac{-\sqrt{2}}{\frac{\Gamma(1)\exp\left(-\left(1-\frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}} = \frac{\sqrt{2} \sqrt{2\pi}}{\exp\left(-\frac{75}{77}\right) \int_0^1 1 dt}$$

•

$$\frac{-\sqrt{2}}{\frac{\Gamma(1) \exp\left(-\left(1-\frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}} = \frac{e^{-\int_0^1 dx} \sqrt{2} \sqrt{2\pi}}{\exp\left(-\frac{75}{77}\right)}$$

$$\frac{-\sqrt{2}}{\frac{\Gamma(1) \exp\left(-\left(1-\frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}} = \frac{\sqrt{2} \sqrt{2\pi}}{\exp\left(-\frac{75}{77}\right) \int_0^\infty e^{-t} dt}$$

Furthermore. We have also that:

$$(29/10^3 + 4/10^3) + 1 + 1/((-sqrt(2) / ((((((-1 * gamma(1) * exp(-(1-2/(1+76))))))) / (sqrt(2*Pi)))))^1/5$$

**Input:**

$$\left(\frac{29}{10^3} + \frac{4}{10^3}\right) + 1 + \frac{1}{\sqrt[5]{\frac{-\sqrt{2}}{\frac{\Gamma(1) \exp\left(-\left(1-\frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}}}}$$

$\Gamma(x)$  is the gamma function

**Exact result:**

$$\frac{1033}{1000} + \frac{1}{\sqrt[5]{2} e^{15/77} \sqrt[10]{\pi}}$$

**Decimal approximation:**

- More digits

$$1.671963709091062656449751763652621241257209162259880967645\dots$$

$$1.671963709\dots$$

We note that 1.671963709... is a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

### Alternate forms:

$$\frac{500 \times 2^{4/5} + 1033 e^{15/77} \sqrt[10]{\pi}}{1000 e^{15/77} \sqrt[10]{\pi}}$$

$$\frac{1000 + 1033 \sqrt[5]{2} e^{15/77} \sqrt[10]{\pi}}{1000 \sqrt[5]{2} e^{15/77} \sqrt[10]{\pi}}$$

### Alternative representations:

$$\left(\frac{29}{10^3} + \frac{4}{10^3}\right) + 1 + \frac{1}{\sqrt[5]{\frac{-\sqrt{2}}{\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2 \pi}}}}} = 1 + \frac{33}{10^3} + \frac{1}{\sqrt[5]{\frac{-\sqrt{2}}{\frac{\exp\left(-1 + \frac{2}{77}\right)}{\sqrt{2 \pi}}}}}$$

$$\left(\frac{29}{10^3} + \frac{4}{10^3}\right) + 1 + \frac{1}{\sqrt[5]{\frac{-\sqrt{2}}{\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2 \pi}}}}} = 1 + \frac{33}{10^3} + \frac{1}{\sqrt[5]{\frac{-\sqrt{2}}{\frac{\exp\left(-1 + \frac{2}{77}\right) e^0}{\sqrt{2 \pi}}}}}$$

$$\left(\frac{29}{10^3} + \frac{4}{10^3}\right) + 1 + \frac{1}{\sqrt[5]{\frac{-\sqrt{2}}{\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2 \pi}}}}} = 1 + \frac{33}{10^3} + \frac{1}{\sqrt[5]{\frac{-\sqrt{2}}{\frac{\exp\left(-1 + \frac{2}{77}\right) (1)_0}{\sqrt{2 \pi}}}}}$$

( $a)_n$  is the Pochhammer symbol (rising factorial)

### Series representations:

$$\left(\frac{29}{10^3} + \frac{4}{10^3}\right) + 1 + \frac{1}{\sqrt[5]{\frac{-\sqrt{2}}{\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2 \pi}}}}} = \frac{1033}{1000} + \frac{1}{2^{2/5} e^{15/77} \sqrt[10]{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}}$$

$$\left(\frac{29}{10^3} + \frac{4}{10^3}\right) + 1 + \frac{1}{\sqrt[5]{\frac{-\sqrt{2}}{\frac{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)}{\sqrt{2 \pi}}}}} = \frac{1033}{1000} + \frac{1}{\sqrt[5]{2} \sqrt[10]{\pi} \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{15/77}}$$

$$\left(\frac{29}{10^3} + \frac{4}{10^3}\right) + 1 + \frac{1}{\sqrt[5]{\frac{-\sqrt{2}}{\frac{\Gamma(1)\exp\left(-\left(1-\frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}}}} = \frac{1033}{1000} + \frac{1}{\sqrt[5]{2} \sqrt[10]{\pi} \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{15/77}}$$

$n!$  is the factorial function

### Integral representations:

$$\left(\frac{29}{10^3} + \frac{4}{10^3}\right) + 1 + \frac{1}{\sqrt[5]{\frac{-\sqrt{2}}{\frac{\Gamma(1)\exp\left(-\left(1-\frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}}}} = \frac{1033}{1000} + \frac{1}{2^{2/5} e^{15/77} \sqrt[10]{\int_0^1 \sqrt{1-t^2} dt}}$$

$$\left(\frac{29}{10^3} + \frac{4}{10^3}\right) + 1 + \frac{1}{\sqrt[5]{\frac{-\sqrt{2}}{\frac{\Gamma(1)\exp\left(-\left(1-\frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}}}} = \frac{1033}{1000} + \frac{1}{2^{3/10} e^{15/77} \sqrt[10]{\int_0^\infty \frac{1}{1+t^2} dt}}$$

$$\left(\frac{29}{10^3} + \frac{4}{10^3}\right) + 1 + \frac{1}{\sqrt[5]{\frac{-\sqrt{2}}{\frac{\Gamma(1)\exp\left(-\left(1-\frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}}}} = \frac{1033}{1000} + \frac{1}{2^{3/10} e^{15/77} \sqrt[10]{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}}$$

We have that:

$$\begin{aligned} & \prod_{r>0}^{\infty} \left( \int u^2 d\theta_{-r} d\theta_r e^{-\frac{r}{2u^2} \theta_{-r} \theta_r - \frac{1}{2} \theta_{-r} \theta_r \pm i \psi_r^\dagger \tilde{\psi}_r^\dagger + \psi_r^\dagger \theta_r \mp i \theta_{-r} \tilde{\psi}_r^\dagger} \right) |0\rangle_{\text{NS}} \\ &= \frac{\sqrt{2\pi}}{\Gamma(u^2 + \frac{1}{2})} e^{\pm \sum_{r>0}^{\infty} \left(1 - \frac{2}{1+r/u^2}\right) i \psi_r^\dagger \tilde{\psi}_r^\dagger} |0\rangle_{\text{NS}}. \end{aligned}$$

For  $u = -1$ ,  $u^2 = 1$  and  $r = 21$ , that is a Fibonacci number, we obtain:

$$(((\text{sqrt}(2\pi)*\text{exp}(1-2/(1+21)))) / (((\text{gamma}(1+1/2))))$$

**Input:**

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}$$

$\Gamma(x)$  is the gamma function

### Exact result:

$$2\sqrt{2} e^{10/11}$$

### Decimal approximation:

7.020340210733174778869048128213632009022809623908429215962...

7.020340210733174....

### Property:

$2\sqrt{2} e^{10/11}$  is a transcendental number

### Alternative representations:

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2\pi}}{e^{-\log G(3/2) + \log G(5/2)}}$$

- $$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2\pi}}{(1)_2}$$

- $$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2\pi}}{\frac{\sqrt[8]{e} \pi^{3/4}}{2^{23/24} A^{3/2} \left(\frac{24}{\sqrt[24]{2}} \sqrt[8]{e} \sqrt[4]{\pi}\right)}}$$

$\log G(z)$  gives the logarithm of the Barnes G-function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$A$  is the Glaisher-Kinkelin constant

### Series representations:

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \frac{\exp\left(\frac{10}{11}\right) \exp\left(i\pi \left\lfloor \frac{\arg(2\pi-x)}{2\pi} \right\rfloor\right) \sqrt{x}}{\sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}-z_0\right)^k \Gamma^{(k)}(z_0)}{k!}}$$

for ( $x \in \mathbb{R}$  and ( $z_0 \notin \mathbb{Z}$  or  $z_0 > 0$ ) and  $x < 0$ )

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \frac{\exp\left(\frac{10}{11}\right) \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(2\pi-z_0)/(2\pi) \rfloor} z_0^{1/2 + 1/2 \lfloor \arg(2\pi-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2\pi-z_0)^k z_0^{-k}}{k!}}{\sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}-z_0\right)^k \Gamma^{(k)}(z_0)}{k!}}$$

for ( $z_0 \notin \mathbb{Z}$  or  $z_0 > 0$ )

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$n!$  is the factorial function

$i$  is the imaginary unit

$\mathbb{R}$  is the set of real numbers

$\mathbb{Z}$  is the set of integers

## Integral representations:

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \frac{\exp\left(\frac{10}{11}\right) \sqrt{2\pi}}{\int_0^\infty e^{-t} \sqrt{t} dt}$$

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \frac{\exp\left(\frac{10}{11}\right) \sqrt{2\pi}}{\int_0^1 \sqrt{\log\left(\frac{1}{t}\right)} dt}$$

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \exp\left(-\int_0^1 \frac{\frac{1}{2} - \frac{3x}{2} + x^{3/2}}{(-1+x) \log(x)} dx\right) \exp\left(\frac{10}{11}\right) \sqrt{2\pi}$$

$\log(x)$  is the natural logarithm

We obtain also:

$$-9/10^3 + (((((\sqrt{2\pi} \exp(1 - \frac{2}{1+21}))) / (((\Gamma(1 + \frac{1}{2})))))))^{1/4}$$

**Input:**

$$-\frac{9}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp(1 - \frac{2}{1+21})}{\Gamma(1 + \frac{1}{2})}}$$

$\Gamma(x)$  is the gamma function

**Exact result:**

$$2^{3/8} e^{5/22} - \frac{9}{1000}$$

**Decimal approximation:**

- More digits  
1.618756880266165150116585859913027296090949476243250209554...

1.6187568802...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

**Property:**

$-\frac{9}{1000} + 2^{3/8} e^{5/22}$  is a transcendental number

•

**Alternate form:**

$$\frac{1000 \times 2^{3/8} e^{5/22} - 9}{1000}$$

**Alternative representations:**

$$-\frac{9}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp(1 - \frac{2}{1+21})}{\Gamma(1 + \frac{1}{2})}} = -\frac{9}{10^3} + \sqrt[4]{\frac{\exp(1 - \frac{2}{22}) \sqrt{2\pi}}{(1)_{\frac{1}{2}}}}$$

•

$$-\frac{9}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = -\frac{9}{10^3} + \sqrt[4]{\frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2\pi}}{e^{-\log G(3/2) + \log G(5/2)}}}$$

$$-\frac{9}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = -\frac{9}{10^3} + \sqrt[4]{\frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2\pi}}{\frac{\frac{8\sqrt[e]{\pi^{3/4}}}{2^{23/24} A^{3/2} \left(\frac{24}{\sqrt[2]{2}} \frac{8\sqrt[e]{4\sqrt{\pi}}}{\sqrt[4]{\pi}}\right)}}{A^{3/2}}}}$$

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\log G(z)$  gives the logarithm of the Barnes G-function

$A$  is the Glaisher-Kinkelin constant

### Series representations:

$$-\frac{9}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = -\frac{9}{1000} + 2^{3/8} \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{5/22}$$

$$-\frac{9}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = -\frac{9}{1000} + 2^{13/88} \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{5/22}$$

$$-\frac{9}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = -\frac{9}{1000} + 2^{3/8} \left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{5/22}$$

$n!$  is the factorial function

### Integral representations:

$$-\frac{9}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = -\frac{9}{1000} + \sqrt[4]{\frac{\exp\left(\frac{10}{11}\right) \sqrt{2\pi}}{\int_0^1 \sqrt{\log\left(\frac{1}{t}\right)} dt}}$$

$$-\frac{9}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = -\frac{9}{1000} + \sqrt[4]{\frac{\exp\left(\frac{10}{11}\right) \sqrt{2\pi}}{\int_0^\infty e^{-t} \sqrt{t} dt}}$$

$$\begin{aligned} -\frac{9}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} &= \\ -\frac{9}{1000} + \sqrt[4]{\exp\left(-\int_0^1 \frac{\frac{1}{2} - \frac{3x}{2} + x^{3/2}}{(-1+x) \log(x)} dx\right) \exp\left(\frac{10}{11}\right) \sqrt{2\pi}} \end{aligned}$$

$\log(x)$  is the natural logarithm

And:

$$(47/10^3 - 3/10^3) + (((((\sqrt{2\pi}) * \exp(1-2/(1+21)))) / (((\Gamma(1+1/2)))))))^{1/4}$$

Where 47 and 3 are Lucas numbers

**Input:**

$$\left(\frac{47}{10^3} - \frac{3}{10^3}\right) + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}}$$

$\Gamma(x)$  is the gamma function

**Exact result:**

$$\frac{11}{250} + 2^{3/8} e^{5/22}$$

**Decimal approximation:**

$$1.671756880266165150116585859913027296090949476243250209554\dots$$

$$1.6717568802\dots$$

We note that 1.6717568802... is a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

**Property:**

$\frac{11}{250} + 2^{3/8} e^{5/22}$  is a transcendental number

**Alternate form:**

$$\frac{1}{250} (11 + 250 \times 2^{3/8} e^{5/22})$$

**Alternative representations:**

$$\left(\frac{47}{10^3} - \frac{3}{10^3}\right) + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{44}{10^3} + \sqrt[4]{\frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2\pi}}{(1)_2}}$$

$$\left(\frac{47}{10^3} - \frac{3}{10^3}\right) + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{44}{10^3} + \sqrt[4]{\frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2\pi}}{e^{-\log G(3/2)+\log G(5/2)}}}$$

$$\left(\frac{47}{10^3} - \frac{3}{10^3}\right) + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{44}{10^3} + \sqrt[4]{\frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2\pi}}{\frac{\frac{8\sqrt{e}\pi^{3/4}}{2^{23/24}A^{3/2}(24\sqrt{2}\sqrt[8]{e}\sqrt[4]{\pi})}}{A^{3/2}}}}$$

( $a)_n$  is the Pochhammer symbol (rising factorial)

$\log G(z)$  gives the logarithm of the Barnes G-function

$A$  is the Glaisher-Kinkelin constant

**Series representations:**

$$\left(\frac{47}{10^3} - \frac{3}{10^3}\right) + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{11}{250} + 2^{3/8} \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{5/22}$$

$$\left(\frac{47}{10^3} - \frac{3}{10^3}\right) + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{11}{250} + 2^{13/88} \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{5/22}$$

$$\left(\frac{47}{10^3} - \frac{3}{10^3}\right) + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{11}{250} + 2^{3/8} \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{5/22}$$

$n!$  is the factorial function

### Integral representations:

$$\left(\frac{47}{10^3} - \frac{3}{10^3}\right) + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{11}{250} + \sqrt[4]{\frac{\exp\left(\frac{10}{11}\right) \sqrt{2\pi}}{\int_0^1 \sqrt{\log\left(\frac{1}{t}\right)} dt}}$$

$$\left(\frac{47}{10^3} - \frac{3}{10^3}\right) + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{11}{250} + \sqrt[4]{\frac{\exp\left(\frac{10}{11}\right) \sqrt{2\pi}}{\int_0^\infty e^{-t} \sqrt{t} dt}}$$

$$\begin{aligned} \left(\frac{47}{10^3} - \frac{3}{10^3}\right) + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} &= \\ \frac{11}{250} + \sqrt[4]{\exp\left(-\int_0^1 \frac{\frac{1}{2} - \frac{3x}{2} + x^{3/2}}{(-1+x)\log(x)} dx\right) \exp\left(\frac{10}{11}\right) \sqrt{2\pi}} & \end{aligned}$$

$\log(x)$  is the natural logarithm

In conclusion, multiplying the two result, we obtain:

$$[-\sqrt{2} / (((((-1 * \text{gamma}(1) * \exp(-(1-2/(1+76)))))) / (\sqrt{2*\pi}))) * (((((\sqrt{2*\pi}) * \exp(1-2/(1+21)))))) / (((\text{gamma}(1+1/2)))))]$$

**Input:**

$$\frac{-\sqrt{2}}{\frac{\Gamma(1) \exp(-1 - \frac{2}{1+76})}{\sqrt{2\pi}}} \times \frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}$$

$\Gamma(x)$  is the gamma function

**Exact result:**

$$4 e^{145/77} \sqrt{2\pi}$$

**Decimal approximation:**

$$65.91392850972916644078202711384134437667928162288015453455\dots$$

$$65.913928509729$$

**Alternative representations:**

$$\frac{(\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right))(-\sqrt{2})}{\frac{\Gamma\left(1 + \frac{1}{2}\right)(-\Gamma(1) \exp(-1 - \frac{2}{1+76}))}{\sqrt{2\pi}}} = -\frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2} \sqrt{2\pi}}{\frac{e^{-\log G(3/2) + \log G(5/2)} \left(-\exp\left(-1 + \frac{2}{77}\right) e^0\right)}{\sqrt{2\pi}}}$$

$$\frac{(\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right))(-\sqrt{2})}{\frac{\Gamma\left(1 + \frac{1}{2}\right)(-\Gamma(1) \exp(-1 - \frac{2}{1+76}))}{\sqrt{2\pi}}} = -\frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2} \sqrt{2\pi}}{\frac{(1)_1 \left(-\exp\left(-1 + \frac{2}{77}\right) (1)_0\right)}{\sqrt{2\pi}}}$$

$$\frac{(\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right))(-\sqrt{2})}{\frac{\Gamma\left(1 + \frac{1}{2}\right)(-\Gamma(1) \exp(-1 - \frac{2}{1+76}))}{\sqrt{2\pi}}} = -\frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2} \sqrt{2\pi}}{\frac{\left(\frac{8}{\sqrt[8]{e}} \pi^{3/4}\right) \left(-\exp\left(-1 + \frac{2}{77}\right)\right)}{\frac{\left(2^{23/24} A^{3/2} \left(\frac{24}{\sqrt[24]{2}} \frac{8}{\sqrt[8]{e}} \frac{4}{\sqrt[4]{\pi}}\right)\right) \sqrt{2\pi}}{A^{3/2}}}}$$

$\log G(z)$  gives the logarithm of the Barnes G-function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$A$  is the Glaisher-Kinkelin constant

**Series representations:**

$$\frac{(\sqrt{2\pi} \exp(1 - \frac{2}{1+21}))(-\sqrt{2})}{\frac{\Gamma(1+\frac{1}{2})(-\Gamma(1)\exp(-(1-\frac{2}{1+76})))}{\sqrt{2\pi}}} =$$

$$\left( \exp\left(\frac{10}{11}\right) \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \exp^2\left(i\pi \left\lfloor \frac{\arg(2\pi-x)}{2\pi} \right\rfloor\right) \sqrt{x}^3 \right.$$

$$\left. \left( \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \left( \sum_{k=0}^{\infty} \frac{(-1)^k (2\pi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^2 \right) /$$

$$\left( \exp\left(-\frac{75}{77}\right) \left( \sum_{k=0}^{\infty} \frac{(1-z_0)^k \Gamma^{(k)}(z_0)}{k!} \right) \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}-z_0\right)^k \Gamma^{(k)}(z_0)}{k!} \right)$$

for ( $x \in \mathbb{R}$  and ( $z_0 \notin \mathbb{Z}$  or  $z_0 > 0$ ) and  $x < 0$ )

$$\frac{(\sqrt{2\pi} \exp(1 - \frac{2}{1+21}))(-\sqrt{2})}{\frac{\Gamma(1+\frac{1}{2})(-\Gamma(1)\exp(-(1-\frac{2}{1+76})))}{\sqrt{2\pi}}} =$$

$$\left( \exp\left(\frac{10}{11}\right) \left( \frac{1}{z_0} \right)^{1/2 [\arg(2-z_0)/(2\pi)] + [\arg(2\pi-z_0)/(2\pi)]} z_0^{3/2 + 1/2 [\arg(2-z_0)/(2\pi)] + [\arg(2\pi-z_0)/(2\pi)]} \right.$$

$$\left. \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2\pi-z_0)^k z_0^{-k}}{k!} \right)^2 \right) /$$

$$\left( \exp\left(-\frac{75}{77}\right) \left( \sum_{k=0}^{\infty} \frac{(1-z_0)^k \Gamma^{(k)}(z_0)}{k!} \right) \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}-z_0\right)^k \Gamma^{(k)}(z_0)}{k!} \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$n!$  is the factorial function

$i$  is the imaginary unit

$\mathbb{R}$  is the set of real numbers

$\mathbb{Z}$  is the set of integers

## Integral representations:

$$\frac{(\sqrt{2\pi} \exp(1 - \frac{2}{1+21}))(-\sqrt{2})}{\frac{\Gamma(1+\frac{1}{2})(-\Gamma(1)\exp(-(1-\frac{2}{1+76})))}{\sqrt{2\pi}}} = \frac{\exp\left(-\int_0^1 \frac{1 - \frac{3x}{2} + x^{3/2}}{(-1+x)\log(x)} dx\right) \exp\left(\frac{10}{11}\right) \sqrt{2} \sqrt{2\pi}^2}{\exp\left(-\frac{75}{77}\right)}$$

$$\frac{\left(\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)\right)(-\sqrt{2})}{\frac{\Gamma\left(1+\frac{1}{2}\right)\left(-\Gamma(1)\exp\left(-\left(1-\frac{2}{1+76}\right)\right)\right)}{\sqrt{2\pi}}} =$$

$$\frac{\exp\left(\frac{5\gamma}{2} - \int_0^1 \frac{2-x-x^{3/2}+\log(x)+\log(x^{3/2})}{\log(x)-x\log(x)} dx\right) \exp\left(\frac{10}{11}\right) \sqrt{2} \sqrt{2\pi}^2}{\exp\left(-\frac{75}{77}\right)}$$

•

$$\frac{\left(\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)\right)(-\sqrt{2})}{\frac{\Gamma\left(1+\frac{1}{2}\right)\left(-\Gamma(1)\exp\left(-\left(1-\frac{2}{1+76}\right)\right)\right)}{\sqrt{2\pi}}} = \frac{\exp\left(\frac{10}{11}\right) \sqrt{2} \sqrt{2\pi}^2}{\exp\left(-\frac{75}{77}\right) \left(\int_0^1 1 dt\right) \int_0^1 \sqrt{\log\left(\frac{1}{t}\right)} dt}$$

$\log(x)$  is the natural logarithm

$\gamma$  is the Euler-Mascheroni constant

$$(3571+123+8)+(2.739911-0.5447171)[-sqrt(2) / (((((-1* gamma (1) * exp-((1-2/(1+76)))))) / (sqrt(2*Pi)))) * (((((sqrt(2Pi)*exp(1-2/(1+21)))))) / (((gamma (1+1/2))))))]^e$$

Where 8 is a Fibonacci number, 123 and 3571 are Lucas numbers. Furthermore, 2.739911 and 0.5447171 are two Ramanujan mock theta functions

**Input interpretation:**

$$(3571 + 123 + 8) + (2.739911 - 0.5447171) \left( \frac{-\sqrt{2}}{-\frac{\Gamma(1)\exp\left(-\left(1-\frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}} \times \frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} \right)^e$$

$\Gamma(x)$  is the gamma function

**Result:**

196883.5...

**196883.5**

**Alternative representations:**

$$(3571 + 123 + 8) + (2.73991 - 0.544717) \left( \frac{\left( \sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right) \right) (-\sqrt{2})}{\frac{\Gamma\left(1+\frac{1}{2}\right) \left(-\Gamma(1) \exp\left(-1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}} \right)^e = \\ 3702 + 2.19519 \left( -\frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2} \sqrt{2\pi}}{\frac{e^{-\log G(3/2) + \log G(5/2)} \left(-\exp\left(-1 + \frac{2}{77}\right) e^0\right)}{\sqrt{2\pi}}} \right)^e$$

$$(3571 + 123 + 8) + (2.73991 - 0.544717) \left( \frac{\left( \sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right) \right) (-\sqrt{2})}{\frac{\Gamma\left(1+\frac{1}{2}\right) \left(-\Gamma(1) \exp\left(-1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}} \right)^e = \\ 3702 + 2.19519 \left( -\frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2} \sqrt{2\pi}}{\frac{\frac{(1)_1 \left(-\exp\left(-1 + \frac{2}{77}\right) (1)_0\right)}{2}}{\sqrt{2\pi}}} \right)^e$$

$$(3571 + 123 + 8) + (2.73991 - 0.544717) \left( \frac{\left( \sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right) \right) (-\sqrt{2})}{\frac{\Gamma\left(1+\frac{1}{2}\right) \left(-\Gamma(1) \exp\left(-1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}} \right)^e = \\ 3702 + 2.19519 \left( -\frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2} \sqrt{2\pi}}{\frac{\left(\sqrt[8]{e} \pi^{3/4}\right) \left(-\exp\left(-1 + \frac{2}{77}\right)\right)}{\frac{\left(2^{23/24} A^{3/2} \left(\sqrt[24]{2} \sqrt[8]{e} \sqrt[4]{\pi}\right)\right) \sqrt{2\pi}}{A^{3/2}}}} \right)^e$$

$\log G(z)$  gives the logarithm of the Barnes G-function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$A$  is the Glaisher-Kinkelin constant

## Integral representations:

$$(3571 + 123 + 8) + (2.73991 - 0.544717) \left( \frac{\left( \sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right) \right) (-\sqrt{2})}{\frac{\Gamma\left(1 + \frac{1}{2}\right) \left(-\Gamma(1) \exp\left(-1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}} \right)^e = \\ 2.19519 \left( 1686.41 + \left( \frac{\exp\left(\frac{10}{11}\right) \sqrt{2} \sqrt{2\pi}^2}{\exp\left(-\frac{75}{77}\right) \left(\int_0^1 1 dt\right) \int_0^1 \sqrt{\log\left(\frac{1}{t}\right)} dt} \right)^e \right)$$

$$(3571 + 123 + 8) + (2.73991 - 0.544717) \left( \frac{\left( \sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right) \right) (-\sqrt{2})}{\frac{\Gamma\left(1 + \frac{1}{2}\right) \left(-\Gamma(1) \exp\left(-1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}} \right)^e = \\ 3702 + 2.19519 \left( \frac{\mathcal{A}^{-\int_0^1 \left(\frac{1}{2} - \frac{3x}{2} + x^{3/2}\right)/(-1+x) \log(x) dx} \exp\left(\frac{10}{11}\right) \sqrt{2} \sqrt{2\pi}^2}{\exp\left(-\frac{75}{77}\right)} \right)^e$$

$$(3571 + 123 + 8) + (2.73991 - 0.544717) \left( \frac{\left( \sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right) \right) (-\sqrt{2})}{\frac{\Gamma\left(1 + \frac{1}{2}\right) \left(-\Gamma(1) \exp\left(-1 - \frac{2}{1+76}\right)\right)}{\sqrt{2\pi}}} \right)^e = \\ 2.19519 \left( 1686.41 + \left( \frac{\exp\left(\frac{10}{11}\right) \sqrt{2} \sqrt{2\pi}^2}{\exp\left(-\frac{75}{77}\right) \left(\int_0^\infty \mathcal{A}^{-t} dt\right) \int_0^\infty \sqrt{t} \mathcal{A}^{-t} dt} \right)^e \right)$$

$\log(x)$  is the natural logarithm

From the three results, we obtain also:

$$(((\text{sqrt}(2\pi)*\exp(1-2/(1+21)))) / (((\text{gamma}(1+1/2))))$$

**Input:**

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}$$

$\Gamma(x)$  is the gamma function

**Exact result:**

$$2\sqrt{2} e^{10/11}$$

**Decimal approximation:**

$$7.020340210733174778869048128213632009022809623908429215962\dots$$

$$7.02034021\dots$$

**Property:**

$2\sqrt{2} e^{10/11}$  is a transcendental number

- Alternative representations:**

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \frac{\exp\left(1 - \frac{2}{22}\right)\sqrt{2\pi}}{e^{-\log G(3/2) + \log G(5/2)}}$$

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \frac{\exp\left(1 - \frac{2}{22}\right)\sqrt{2\pi}}{(1)_L}_2$$

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \frac{\exp\left(1 - \frac{2}{22}\right)\sqrt{2\pi}}{\frac{\sqrt[8]{e}\pi^{3/4}}{2^{23/24}A^{3/2}\left(\sqrt[24]{2}\sqrt[8]{e}\sqrt[4]{\pi}\right)}}_A^{3/2}$$

**Series representations:**

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \frac{\exp\left(\frac{10}{11}\right) \exp\left(i\pi \left\lfloor \frac{\arg(2\pi-x)}{2\pi} \right\rfloor\right) \sqrt{x}}{\sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}-z_0\right)^k \Gamma^{(k)}(z_0)}{k!}} \sum_{k=0}^{\infty} \frac{(-1)^k (2\pi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

for ( $x \in \mathbb{R}$  and ( $z_0 \notin \mathbb{Z}$  or  $z_0 > 0$ ) and  $x < 0$ )

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \frac{\exp\left(\frac{10}{11}\right) \left(\frac{1}{z_0}\right)^{1/2 [\arg(2\pi - z_0)/(2\pi)]} z_0^{1/2 + 1/2 [\arg(2\pi - z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2\pi - z_0)^k z_0^{-k}}{k!}}{\sum_{k=0}^{\infty} \frac{\left(\frac{3}{2} - z_0\right)^k \Gamma(k)(z_0)}{k!}}$$

for ( $z_0 \notin \mathbb{Z}$  or  $z_0 > 0$ )

### Integral representations:

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \frac{\exp\left(\frac{10}{11}\right) \sqrt{2\pi}}{\int_0^\infty e^{-t} \sqrt{t} dt}$$

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \frac{\exp\left(\frac{10}{11}\right) \sqrt{2\pi}}{\int_0^1 \sqrt{\log\left(\frac{1}{t}\right)} dt}$$

$$\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)} = \exp\left(-\int_0^1 \frac{\frac{1}{2} - \frac{3x}{2} + x^{3/2}}{(-1+x)\log(x)} dx\right) \exp\left(\frac{10}{11}\right) \sqrt{2\pi}$$

$\log(x)$  is the natural logarithm

And:

$$18/10^3 + (((((\sqrt{2\pi}) * \exp(1-2/(1+21)))) / (((\Gamma(1+1/2)))))))^{1/4}$$

Where 18 is a Lucas number

### Input:

$$\frac{18}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}}$$

$\Gamma(x)$  is the gamma function

**Exact result:**

$$\frac{9}{500} + 2^{3/8} e^{5/22}$$

**Decimal approximation:**

1.645756880266165150116585859913027296090949476243250209554...

$$1.64575688\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

**Property:**

$\frac{9}{500} + 2^{3/8} e^{5/22}$  is a transcendental number

• **Alternate form:**

$$\frac{1}{500} (9 + 500 \times 2^{3/8} e^{5/22})$$

**Alternative representations:**

$$\frac{18}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{18}{10^3} + \sqrt[4]{\frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2\pi}}{(1)_L}_2}$$

$$\frac{18}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{18}{10^3} + \sqrt[4]{\frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2\pi}}{e^{-\log G(3/2) + \log G(5/2)}}}$$

$$\frac{18}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{18}{10^3} + \sqrt[4]{\frac{\exp\left(1 - \frac{2}{22}\right) \sqrt{2\pi}}{\frac{8\sqrt{e} \pi^{3/4}}{2^{23/24} A^{3/2} \left(\frac{24}{\sqrt{2}} \frac{8}{\sqrt{e}} \sqrt[4]{\pi}\right)}}}$$

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\log G(z)$  gives the logarithm of the Barnes G-function

$A$  is the Glaisher-Kinkelin constant

### Series representations:

$$\frac{18}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{9}{500} + 2^{3/8} \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{5/22}$$

$$\frac{18}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{9}{500} + 2^{13/88} \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{5/22}$$

$$\frac{18}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{9}{500} + 2^{3/8} \left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{5/22}$$

$n!$  is the factorial function

### Integral representations:

$$\frac{18}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{9}{500} + \sqrt[4]{\frac{\exp\left(\frac{10}{11}\right) \sqrt{2\pi}}{\int_0^1 \sqrt{\log\left(\frac{1}{t}\right)} dt}}$$

$$\frac{18}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{9}{500} + \sqrt[4]{\frac{\exp\left(\frac{10}{11}\right) \sqrt{2\pi}}{\int_0^\infty e^{-t} \sqrt{t} dt}}$$

$$\frac{18}{10^3} + \sqrt[4]{\frac{\sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right)}{\Gamma\left(1 + \frac{1}{2}\right)}} = \frac{9}{500} + \sqrt[4]{\exp\left(-\int_0^1 \frac{\frac{1}{2} - \frac{3x}{2} + x^{3/2}}{(-1+x)\log(x)} dx\right) \exp\left(\frac{10}{11}\right) \sqrt{2\pi}}$$

$\log(x)$  is the natural logarithm

$$(-1.0018674362)*1/(2\pi) * \sqrt{2} * 1 / (((-1 * \text{gamma}(1) * \exp(-(1-2/(1+76)))) * 1 / (\sqrt{2*\pi}) * (((\sqrt{2}\pi)\exp(1-2/(1+21)))) * 1 / ((\text{gamma}(1+1/2))))$$

Where **1.0018674362** is the following Rogers-Ramanujan continued fraction

$$\frac{e^{\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \cfrac{e^{-6\pi}}{1 + \cfrac{e^{-8\pi}}{1 + \dots}}}}$$

### Input interpretation:

$$-1.0018674362 \times \frac{1}{2\pi} \sqrt{2} \left( -\frac{1}{\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)} \right) \times \\ \frac{1}{\sqrt{2\pi}} \left( \sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right) \right) \times \frac{1}{\Gamma\left(1 + \frac{1}{2}\right)}$$

$\Gamma(x)$  is the gamma function

### Result:

1.6727372213...

1.6727372213.... result very near to the proton mass

### Alternative representations:

$$\frac{-1.00186743620000 \sqrt{2} \left( \sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right) \right)}{\left( -\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right) \right) \sqrt{2\pi} \Gamma\left(1 + \frac{1}{2}\right) (2\pi)} = \\ -\frac{1.00186743620000 \exp\left(1 - \frac{2}{22}\right) \sqrt{2} \sqrt{2\pi}}{(2\pi) \left( -\exp\left(-1 + \frac{2}{77}\right) e^0 \right) e^{-\log G(3/2) + \log G(5/2)} \sqrt{2\pi}}$$

•

$$\frac{-1.00186743620000 \sqrt{2} \left(\sqrt{2 \pi} \exp\left(1-\frac{2}{1+21}\right)\right)}{\left(\left(-\Gamma(1) \exp\left(-\left(1-\frac{2}{1+76}\right)\right)\right) \sqrt{2 \pi} \Gamma\left(1+\frac{1}{2}\right)(2 \pi)\right.} =$$

$$-\frac{1.00186743620000 \exp\left(1-\frac{2}{22}\right) \sqrt{2} \sqrt{2 \pi}}{(2 \pi) \left(-\exp\left(-1+\frac{2}{77}\right) (1)_0\right) (1)_{\frac{1}{2}} \sqrt{2 \pi}}$$

$$\frac{-1.00186743620000 \sqrt{2} \left(\sqrt{2 \pi} \exp\left(1-\frac{2}{1+21}\right)\right)}{\left(\left(-\Gamma(1) \exp\left(-\left(1-\frac{2}{1+76}\right)\right)\right) \sqrt{2 \pi} \Gamma\left(1+\frac{1}{2}\right)(2 \pi)\right.} =$$

$$-\frac{1.00186743620000 \exp\left(1-\frac{2}{22}\right) \sqrt{2} \sqrt{2 \pi}}{(2 \pi) \left(-\exp\left(-1+\frac{2}{77}\right) \left(\frac{8}{\sqrt[8]{e}} \pi^{3/4}\right) \sqrt{2 \pi}\right.}$$

$$\left.\left.\frac{2^{23/24} A^{3/2} \left(\frac{24}{\sqrt{2}} \frac{8}{\sqrt[8]{e}} \frac{4}{\sqrt{\pi}}\right)}{A^{3/2}}\right)\right.$$

$\log G(z)$  gives the logarithm of the Barnes G-function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$A$  is the Glaisher-Kinkelin constant

## Series representations:

$$\frac{-1.00186743620000 \sqrt{2} \left(\sqrt{2 \pi} \exp\left(1-\frac{2}{1+21}\right)\right)}{\left(\left(-\Gamma(1) \exp\left(-\left(1-\frac{2}{1+76}\right)\right)\right) \sqrt{2 \pi} \Gamma\left(1+\frac{1}{2}\right)(2 \pi)\right.} =$$

$$\frac{0.500933718100000 \exp\left(\frac{10}{11}\right) \exp\left(i \pi \left[\frac{\arg(2-x)}{2 \pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\pi \exp\left(-\frac{75}{77}\right) \left(\sum_{k=0}^{\infty} \frac{(1-z_0)^k \Gamma^{(k)}(z_0)}{k!}\right) \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}-z_0\right)^k \Gamma^{(k)}(z_0)}{k!}}$$

for ( $x \in \mathbb{R}$  and ( $z_0 \notin \mathbb{Z}$  or  $z_0 > 0$ ) and  $x < 0$ )

$$\frac{-1.00186743620000 \sqrt{2} \left(\sqrt{2 \pi} \exp\left(1-\frac{2}{1+21}\right)\right)}{\left(\left(-\Gamma(1) \exp\left(-\left(1-\frac{2}{1+76}\right)\right)\right) \sqrt{2 \pi} \Gamma\left(1+\frac{1}{2}\right)(2 \pi)\right.} =$$

$$\left. \left(0.500933718100000 \exp\left(\frac{10}{11}\right) \left(\frac{1}{z_0}\right)^{1/2 [\arg(2-z_0)/(2 \pi)]} z_0^{1/2 + 1/2 [\arg(2-z_0)/(2 \pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}\right) / \right.$$

$$\left. \left(\pi \exp\left(-\frac{75}{77}\right) \left(\sum_{k=0}^{\infty} \frac{(1-z_0)^k \Gamma^{(k)}(z_0)}{k!}\right) \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}-z_0\right)^k \Gamma^{(k)}(z_0)}{k!}\right) \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$n!$  is the factorial function

$i$  is the imaginary unit

$\mathbb{R}$  is the set of real numbers

$\mathbb{Z}$  is the set of integers

## Integral representations:

$$\frac{-1.00186743620000 \sqrt{2} \left( \sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right) \right)}{\left( (-\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)) \sqrt{2\pi} \Gamma\left(1 + \frac{1}{2}\right) (2\pi) \right)} =$$

$$\frac{0.500933718100000 \exp\left(-\int_0^1 \frac{\frac{1}{2} - \frac{3x}{2} + x^{3/2}}{(-1+x)\log(x)} dx\right) \exp\left(\frac{10}{11}\right) \sqrt{2}}{\pi \exp\left(-\frac{75}{77}\right)}$$

$$\frac{-1.00186743620000 \sqrt{2} \left( \sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right) \right)}{\left( (-\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)) \sqrt{2\pi} \Gamma\left(1 + \frac{1}{2}\right) (2\pi) \right)} =$$

$$\frac{0.500933718100000 \exp\left(\frac{5\gamma}{2} - \int_0^1 \frac{2-x-x^{3/2}+\log(x)+\log(x^{3/2})}{\log(x)-x\log(x)} dx\right) \exp\left(\frac{10}{11}\right) \sqrt{2}}{\pi \exp\left(-\frac{75}{77}\right)}$$

$$\frac{-1.00186743620000 \sqrt{2} \left( \sqrt{2\pi} \exp\left(1 - \frac{2}{1+21}\right) \right)}{\left( (-\Gamma(1) \exp\left(-\left(1 - \frac{2}{1+76}\right)\right)) \sqrt{2\pi} \Gamma\left(1 + \frac{1}{2}\right) (2\pi) \right)} = \frac{0.500933718100000 \exp\left(\frac{10}{11}\right) \sqrt{2}}{\pi \exp\left(-\frac{75}{77}\right) \left( \int_0^1 1 dt \right) \int_0^1 \sqrt{\log\left(\frac{1}{t}\right)} dt}$$

$\log(x)$  is the natural logarithm

$\gamma$  is the Euler-Mascheroni constant

From:

## Commutative Geometry for Non-commutative D-branes by Tachyon Condensation

*Tsuguhiko Asakawa, Goro Ishiki, Takaki Matsumoto, So Matsuura and Hisayoshi Muraki*

<https://arxiv.org/abs/1804.00161v1>

Now, we have that:

A NC D2-brane on a fuzzy sphere can be made out of  $k$  D0-branes, if the scalar field has the profile

$$\Phi^i = \rho L_i, \quad [L_i, L_j] = i \epsilon_{ij}^k L_k, \quad (3.23)$$

where  $\rho$  is a real parameter and  $L_i$  ( $i = 1, 2, 3$ ) are  $su(2)$  generators in the spin- $\ell$  irreducible representation [31]. Thus it is possible for  $k \geq 2$ . We denote corresponding  $k = 2\ell + 1$  states as  $|m\rangle$  ( $m = -\ell, -\ell+1, \dots, \ell-1, \ell$ ). Because  $\Phi^2 = \rho^2 L^2 = \rho^2 \ell(\ell+1) \mathbf{1}_k = \rho^2 \frac{k^2-1}{4} \mathbf{1}_k$ , a naive guess of the radius of this fuzzy sphere is  $\rho \sqrt{\frac{k^2-1}{4}}$ . We will compare it with the radius of  $S^2$  obtained from the tachyon condensation below.

$$T^{(m)} \begin{pmatrix} a_m \\ b_m \end{pmatrix} = u \begin{pmatrix} |x| - \rho m & -\rho \sqrt{(\ell-m)(\ell+m+1)} \\ -\rho \sqrt{(\ell-m)(\ell+m+1)} & -|x| + \rho(m+1) \end{pmatrix} \begin{pmatrix} a_m \\ b_m \end{pmatrix}. \quad (3.44)$$

This matrix is diagonalized in a standard way (see Appendix A.3 for more detail) and the eigenvalues at each point (i.e., functions) are found to be

$$\lambda_{\pm}^{(m)}(|x|) = u \left[ \frac{\rho}{2} \pm \sqrt{|M^{(m)}|} \right], \quad (3.45)$$

with

$$|M^{(m)}| \equiv \sqrt{\rho^2(\ell-m)(\ell+m+1) \left( |x| - \rho(m+\frac{1}{2}) \right)^2}. \quad (3.46)$$

For all  $m = -\ell, -\ell+1, \dots, \ell-1$ ,  $|M^{(m)}|^2$  satisfies  $|M^{(m)}|^2 \geq 2\rho^2\ell$ , and thus  $|M^{(m)}| > \rho/2$  for all spin  $\ell \geq 1/2$ . This implies that for any  $\ell$  and  $m$ , two eigenvalues  $\lambda_{\pm}^{(m)}$  are always non-zero at any point  $x \in \mathcal{U}_N$ . Therefore, the tachyon condensation annihilates the corresponding eigenstates.

$x \in \mathcal{U}_N$  with the radius  $|x| = \rho\ell$

For  $\rho = 3$ ,  $\ell = 1$ ,  $m = -1.8$

$\text{sqrt}(((9(1+1.8)(1-1.8+1)(3*1 - 3(-1.8+1/2))^2)))$

**Input:**

$$\sqrt{9(1+1.8)(1-1.8+1)\left(3 \times 1 - 3\left(-1.8 + \frac{1}{2}\right)\right)^2}$$

**Result:**

15.4905...

15.4905.... result very near to the value of the black hole entropy 15.6730

$$(15127+843-47-11)+(((\text{sqrt}(((9(1+1.8)(1-1.8+1)(3*1 - 3(-1.8+1/2))^2))))))^4$$

Where 11, 47, 843 and 15127 are Lucas numbers

**Input:**

$$(15127+843-47-11)+\sqrt{9(1+1.8)(1-1.8+1)\left(3 \times 1 - 3\left(-1.8 + \frac{1}{2}\right)\right)^2}^4$$

**Result:**

73490.11407936

**73490.114....**

And:

$$(15127+843+29-2)+\pi * (((\text{sqrt}(((9(1+1.8)(1-1.8+1)(3*1 - 3(-1.8+1/2))^2))))))^4$$

**Input:**

$$(15127+843+29-2)+\pi\sqrt{9(1+1.8)(1-1.8+1)\left(3 \times 1 - 3\left(-1.8 + \frac{1}{2}\right)\right)^2}^4$$

**Result:**

196883.9801992724173095172568682082898266947670444187352565...

**196883.98019....**

**Series representations:**

$$(15127+843+29-2)+\pi\sqrt{9(1+1.8)(1-1.8+1)\left(3 \times 1 - 3\left(-1.8 + \frac{1}{2}\right)\right)^2}^4 = \\ 15997+\pi\sqrt{238.954}^4 \left(\sum_{k=0}^{\infty} e^{-5.47627k} \binom{\frac{1}{2}}{k}\right)^4$$

$$(15127 + 843 + 29 - 2) + \pi \sqrt{9(1 + 1.8)(1 - 1.8 + 1)} \left( 3 \times 1 - 3 \left( -1.8 + \frac{1}{2} \right) \right)^2 = \\ 15997 + \pi \sqrt{238.954}^4 \left( \sum_{k=0}^{\infty} \frac{(-0.0041849)^k \left( -\frac{1}{2} \right)_k}{k!} \right)^4$$

$$(15127 + 843 + 29 - 2) + \pi \sqrt{9(1 + 1.8)(1 - 1.8 + 1)} \left( 3 \times 1 - 3 \left( -1.8 + \frac{1}{2} \right) \right)^2 = \\ 15997 + \frac{\pi \left( \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} e^{-5.47627s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right)^4}{16 \sqrt{\pi}^4}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

From:

## Spherical D-brane by Tachyon Condensation

Tsuguhiko Asakawa and So Matsuura - arXiv:1703.10352v1 [hep-th] 30 Mar 2017

Now, we have that:

Combining (3.27) and (3.29), we obtain the effective potential for the transverse scalar field in the radial direction  $R$  in the constant RR 4-form field strength:

$$V(R) = 4\pi \sqrt{R^4 + \frac{\lambda^2}{4} T_2} - \mu_2 \frac{8\pi}{3} f R^3 = 4\pi T_2 \left( \sqrt{R^4 + \frac{\lambda^2}{4}} - \frac{2}{3} f R^3 \right), \quad (3.30)$$

whose shape is drawn in the Fig.1. When  $f$  satisfies

$$\lambda f^2 < 1, \quad (3.31)$$

$V(R)$  has a local minimum at

$$R = R_- \equiv \frac{\sqrt{1 - \sqrt{1 - \lambda^2 f^4}}}{\sqrt{2} f}, \quad (3.32)$$

while there is no local minimum for  $\lambda f^2 \geq 1$ . In particular, if  $\lambda f^2 \ll 1$ , the local minimum  $R_-$  and its energy is given approximately as

$$R_- \sim \frac{\lambda}{2} f, \quad V(R_-) \sim T_0 \left( 1 - \frac{\lambda^2 f^4}{24} \right). \quad (3.33)$$

This means that the spherical D2-brane is stabilized at a very small radius with the size of the string length. It is more stable than a single D0-brane ( $R = 0$ ) but the difference of the energy is small.

One way to stabilize the spherical D2-brane in a macroscopic radius is to construct a bound state of a D2-brane and a large number  $k$  of D0-branes as discussed in [9]. In fact, when the  $U(1)$  gauge flux is  $F = \frac{k}{2} \sin \theta$ , the effective potential for  $R$  becomes

$$V_k(R) = 4\pi T_2 \left( \sqrt{R^4 + \frac{\lambda^2 k^2}{4}} - \frac{2}{3} f R^3 \right). \quad (3.34)$$

$$\mu_2 = \sqrt{2\pi\lambda}\mu_3$$

$$\mu_3 = \frac{2\pi}{g_s(2\pi\sqrt{\alpha'})^4}, \quad \lambda = 2\pi\alpha'$$

our case is not a familiar type mentioned above. However, it is in accordance with the charge quantization since our tachyon profile can be regarded as a topological soliton in the region  $0 \leq r < \infty$  only in the limit of  $u \rightarrow \infty$  as argued in the previous section.

From Wikipedia:

In a non-Abelian gauge theory, the **gauge coupling parameter**,  $g$ , appears in the **Lagrangian** as

$$\frac{1}{4g^2} \text{Tr } G_{\mu\nu} G^{\mu\nu},$$

(where  $G$  is the gauge **field tensor**) in some conventions. In another widely used convention,  $G$  is rescaled so that the coefficient of the kinetic term is 1/4 and  $g$  appears in the **covariant derivative**. This should be understood to be similar to a dimensionless version of the **elementary charge** defined as

$$\frac{e}{\sqrt{\epsilon_0 \hbar c}} = \sqrt{4\pi\alpha} \approx 0.30282212 .$$

For  $R = 0.97536759$ ;  $g_s = 0.30282212$ ;  $f = 0.724963711183$ ;

$\lambda = 1.90268749507$ ;  $\alpha' = 0.95686$

$$((((\text{sqrt}((((((1-(\text{sqrt}(1-1.90268749507^2 \times 0.724963711183^4))))))))))) * 1 / (((\text{sqrt}(2) * 0.724963711183)))) = R$$

**Input interpretation:**

$$\sqrt{1 - \sqrt{1 - 1.90268749507^2 \times 0.724963711183^4}} \times \frac{1}{\sqrt{2} \times 0.724963711183}$$

**Result:**

0.975367597944288579416422630667484285804623559782025939381...

0.97536759...

Numerator:

$$((((\text{sqrt}((((((1-(\text{sqrt}(1-1.90268749507^2 \times 0.724963711183^4)))))))))))$$

**Input interpretation:**

$$\sqrt{1 - \sqrt{1 - 1.90268749507^2 \times 0.724963711183^4}}$$

**Result:**

0.999999055852347061482946996585770631285140977149251366527...

0.999999055852347.....

Denominator:

$$(((\sqrt{2} * 0.724963711183)))$$

**Input interpretation:**

$$\sqrt{2} \times 0.724963711183$$

**Result:**

$$1.02525351258\dots$$

$$1.02525351258\dots$$

$$(2\pi) / (((0.30282212((2\pi * \sqrt{0.95686})^4))) = \mu_3$$

**Input interpretation:**

$$\frac{2\pi}{0.30282212(2\pi\sqrt{0.95686})^4}$$

**Result:**

$$0.0145404\dots$$

$$0.0145404\dots = \mu_3$$

**Series representations:**

$$\frac{2\pi}{0.302822(2\pi\sqrt{0.95686})^4} = \frac{0.412784}{\pi^3 \left( \sum_{k=0}^{\infty} \frac{(-1)^k (-0.04314)^k \left(\frac{-1}{2}\right)_k}{k!} \right)^4}$$

$$\frac{2\pi}{0.302822(2\pi\sqrt{0.95686})^4} = \frac{6.60454\sqrt{\pi}^4}{\pi^3 \left( \sum_{j=0}^{\infty} \text{Res}_{s=-j} (-0.04314)^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) \right)^4}$$

$$\frac{2\pi}{0.302822(2\pi\sqrt{0.95686})^4} = \frac{0.412784}{\pi^3 \sqrt{z_0}^4 \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{-1}{2}\right)_k (0.95686 - z_0)^k z_0^{-k}}{k!} \right)^4}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

$\mathbb{R}$  is the set of real numbers

$$(((\sqrt{2\pi \times 1.90268749507}) \times 0.0145404) = \mu_2)$$

**Input interpretation:**

$$\sqrt{2\pi \times 1.90268749507} \times 0.0145404$$

**Result:**

0.0502748...

$$0.0502748... = \mu_2$$

**Series representations:**

$$\sqrt{2\pi \times 1.902687495070000} \times 0.0145404 = \\ 0.0145404 \sqrt{-1 + 3.805374990140000\pi} \sum_{k=0}^{\infty} (-1 + 3.805374990140000\pi)^{-k} \binom{\frac{1}{2}}{k}$$

$$\sqrt{2\pi \times 1.902687495070000} \times 0.0145404 = 0.0145404 \\ \sqrt{-1 + 3.805374990140000\pi} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 + 3.805374990140000\pi)^{-k} \left(\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{2\pi \times 1.902687495070000} \times 0.0145404 = \\ 0.0145404 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1}{2}\right)_k (3.805374990140000\pi - z_0)^k z_0^{-k}}{k!}$$

for  $\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\mathbb{R}$  is the set of real numbers

$$\begin{aligned} (((4*\pi*\sqrt(((0.97536759^4+(1.90268749507^2)/4))T)))- \\ 4*\pi*T\sqrt(((0.97536759^4+(1.90268749507^2)/4))) \\ 2/3*0.724963711183*0.97536759^3)))) = 0.0502748*((8*\pi)/3)* \\ 0.724963711183*0.97536759^3 \end{aligned}$$

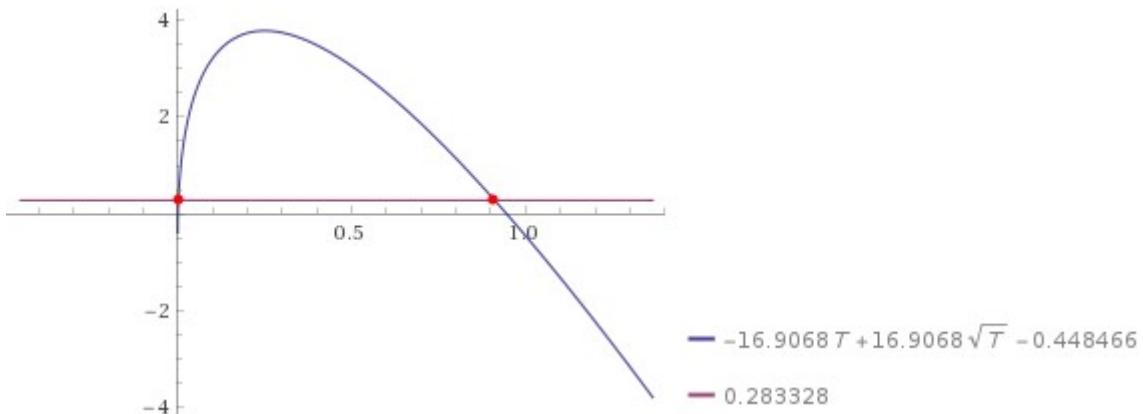
### Input interpretation:

$$\begin{aligned} & 4\pi\sqrt{\left(0.97536759^4 + \frac{1.90268749507^2}{4}\right)T} - \\ & 4\pi T\sqrt{0.97536759^4 + \frac{1.90268749507^2}{4}} + \\ & \frac{2}{3} \times 0.97536759^3 \times (-0.724963711183) = \\ & 0.0502748 \times \frac{8\pi}{3} \times 0.724963711183 \times 0.97536759^3 \end{aligned}$$

### Result:

$$-16.9068 T + 16.9068 \sqrt{T} - 0.448466 = 0.283328$$

### Plot:



### Alternate form:

$$-16.9068(\sqrt{T} - 0.972731)(\sqrt{T} - 0.0272694) = 0.283328$$

### Alternate form assuming T is positive:

$$T + 0.043284 = \sqrt{T}$$

**Solutions:**

$$T \approx 0.00205569$$

•

$$T \approx 0.911376$$

$$T = 0.907673$$

For

$$R = 0.97536759; g_s = 0.30282212; f = 0.724963711183;$$

$$\lambda = 1.90268749507; \alpha' = 0.95686; \mu_2 = 0.0502748$$

$$4\pi T_2 \left( \sqrt{R^4 + \frac{\lambda^2}{4}} - \frac{2}{3} f R^3 \right)$$

$$4*\text{Pi}*0.907673 (((((\text{sqrt}(((0.97536759^4+(1.90268749507^2)/4))))))-2/3*0.724963711183*0.97536759^3$$

**Input interpretation:**

$$4\pi \times 0.907673$$

$$\left( \sqrt{0.97536759^4 + \frac{1.90268749507^2}{4}} + \frac{2}{3} \times 0.97536759^3 \times (-0.724963711183) \right)$$

**Result:**

$$10.2306\dots$$

$$10.2306\dots$$

**Series representations:**

$$4\pi 0.907673 \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{2 \times 0.7249637111830000 \times 0.975368^3}{3} \right) = \\ -1.62824\pi + 3.63069\pi \sum_{k=0}^{\infty} \frac{(-0.810106)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$4\pi 0.907673 \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{2 \times 0.7249637111830000 \times 0.975368^3}{3} \right) = \\ -1.62824\pi - \frac{1.81535\pi \sum_{j=0}^{\infty} \text{Res}_{s=-j} e^{0.21059s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{\sqrt{\pi}}$$

$$4\pi 0.907673 \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{2 \times 0.7249637111830000 \times 0.975368^3}{3} \right) = \\ -1.62824\pi + 3.63069\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1.81011 - z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

$\mathbb{R}$  is the set of real numbers

We obtain also:

$$18/10^3 + 1/(2\pi) (((((4*\pi*0.907673 (((\sqrt{((0.97536759^4 + (1.90268749507^2)/4))))) - 2/3*0.724963711183*0.97536759^3))))))$$

Where 18 is a Lucas number

### Input interpretation:

$$\frac{18}{10^3} + \frac{1}{2\pi} \left( 4\pi \times 0.907673 \left( \sqrt{0.97536759^4 + \frac{1.90268749507^2}{4}} + \frac{2}{3} \times 0.97536759^3 \times (-0.724963711183) \right) \right)$$

### Result:

1.64625...

$$1.64625\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\dots$$

### Series representations:

$$\begin{aligned} & \frac{18}{10^3} + \\ & \frac{4\pi \times 0.907673 \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{2 \times 0.7249637111830000 \times 0.975368^3}{3} \right)}{= -0.796122 + 1.81535 \sum_{k=0}^{\infty} \frac{(-0.810106)^k \binom{2\pi}{2k}}{k!}} \end{aligned}$$

$$\begin{aligned} & \frac{18}{10^3} + \\ & \frac{4\pi \times 0.907673 \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{2 \times 0.7249637111830000 \times 0.975368^3}{3} \right)}{= -0.796122 - \frac{0.907673 \sum_{j=0}^{\infty} \text{Res}_{s=-j} e^{0.21059s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{\sqrt{\pi}}} \end{aligned}$$

$$\frac{18}{10^3} + \frac{1}{2\pi} 4\pi 0.907673 \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{\frac{2 \times 0.7249637111830000 \times 0.975368^3}{3}}{-0.796122 + 1.81535 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1.81011 - z_0)^k z_0^{-k}}{k!}} \right) =$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\operatorname{Res}_{z=z_0} f$  is a complex residue

$\mathbb{R}$  is the set of real numbers

And:

$$(47/10^3 - 4/10^3) + 1/(2\pi) (((((4*\pi*0.907673 (((((\sqrt{((0.97536759^4 + (1.90268749507^2)/4))/4}))))) - 2/3 * 0.724963711183 * 0.97536759^3))))))$$

**Input interpretation:**

$$\left( \frac{47}{10^3} - \frac{4}{10^3} \right) + \frac{1}{2\pi} \left( 4\pi \times 0.907673 \left( \sqrt{0.97536759^4 + \frac{1.90268749507^2}{4}} + \frac{\frac{2}{3} \times 0.97536759^3 \times (-0.724963711183)}{3} \right) \right)$$

**Result:**

1.671248248273008083632001148140858655592507517622144737179...

1.671248248...

We note that 1.671248248... is a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

### Series representations:

$$\begin{aligned} & \left( \frac{47}{10^3} - \frac{4}{10^3} \right) + \\ & \frac{4\pi 0.907673 \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{2 \times 0.7249637111830000 \times 0.975368^3}{3} \right)}{= -0.771122 + 1.81535 \sum_{k=0}^{\infty} \frac{(-0.810106)^k \left( -\frac{1}{2} \right)_k}{k!}} \end{aligned}$$

$$\begin{aligned} & \left( \frac{47}{10^3} - \frac{4}{10^3} \right) + \\ & \frac{4\pi 0.907673 \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{2 \times 0.7249637111830000 \times 0.975368^3}{3} \right)}{= -0.771122 - \frac{0.907673 \sum_{j=0}^{\infty} \text{Res}_{s=-j} e^{0.21059 s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{\sqrt{\pi}}} \end{aligned}$$

$$\begin{aligned} & \left( \frac{47}{10^3} - \frac{4}{10^3} \right) + \frac{1}{2\pi} 4\pi 0.907673 \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \right. \\ & \left. \frac{2 \times 0.7249637111830000 \times 0.975368^3}{3} \right) = \\ & -0.771122 + 1.81535 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k (1.81011 - z_0)^k z_0^{-k}}{k!} \end{aligned}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=0} f$  is a complex residue

$\mathbb{R}$  is the set of real numbers

Now, from the result:

$$-16.9068 T + 16.9068 \sqrt{T} - 0.448466 = 0.283328$$

and the solution  $T_1 = 0.907673$ , we have  $0.907673 - 0.283328 = T_2 = 0.624345$

$$4\pi \sqrt{R^4 + \frac{\lambda^2}{4} T_2} - \mu_2 \frac{8\pi}{3} f R^3$$

$$4*\text{Pi} (((((((\text{sqrt}(((0.97536759^4 + (1.90268749507^2)/4)))))*0.624345))))) - (((((0.0502748*((8\pi)/3)* 0.724963711183*0.97536759^3))))))$$

**Input interpretation:**

$$4\pi \left( \sqrt{0.97536759^4 + \frac{1.90268749507^2}{4}} \times 0.624345 \right) - \\ 0.0502748 \times \frac{8\pi}{3} \times 0.724963711183 \times 0.97536759^3$$

**Result:**

10.2724...

10.2724...

**Percent decrease:**

$$4\pi \left( \sqrt{0.97536759^4 + \frac{1.90268749507^2}{4}} \times 0.624345 \right) - \\ \frac{1}{3} \times 0.0502748 (8\pi) 0.724963711183 \times 0.97536759^3 = 10.2724 \text{ is } 2.68413$$

$$\% \text{ smaller than } 4\pi \left( \sqrt{0.97536759^4 + \frac{1.90268749507^2}{4}} \times 0.624345 \right) = 10.5557.$$

### Series representations:

$$(4\pi) \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} 0.624345 - \frac{1}{3} \times 0.0502748 ((8\pi) 0.7249637111830000 \times 0.975368^3) = \\ -0.0901862\pi + 2.49738\pi \sum_{k=0}^{\infty} \frac{(-0.810106)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$(4\pi) \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} 0.624345 - \frac{1}{3} \times 0.0502748 ((8\pi) 0.7249637111830000 \times 0.975368^3) = \\ -0.0901862\pi - \frac{1.24869\pi \sum_{j=0}^{\infty} \text{Res}_{s=-j} e^{0.21059s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{\sqrt{\pi}}$$

$$(4\pi) \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} 0.624345 - \frac{1}{3} \times 0.0502748 ((8\pi) 0.7249637111830000 \times 0.975368^3) = \\ -0.0901862\pi + 2.49738\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1.81011 - z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\underset{z=z_0}{\text{Res}} f$  is a complex residue

$\mathbb{R}$  is the set of real numbers

Or, for  $T = 1/16$ , we obtain:

$$4*\text{Pi} (((((((\text{sqrt}(((0.97536759^4+(1.90268749507^2)/4)))))*1/16)))) - (((((0.0502748*((8\pi)/3)*0.724963711183*0.97536759^3))))))$$

### Input interpretation:

$$4\pi \left( \sqrt{0.97536759^4 + \frac{1.90268749507^2}{4}} \times \frac{1}{16} \right) - \\ 0.0502748 \times \frac{8\pi}{3} \times 0.724963711183 \times 0.97536759^3$$

**Result:**

$$0.773348\dots$$

$$0.773348\dots$$

**Percent decrease:**

$$\frac{4}{16}\pi \sqrt{0.97536759^4 + \frac{1.90268749507^2}{4}} - \\ \frac{1}{3} \times 0.0502748 (8\pi) 0.724963711183 \times 0.97536759^3 = 0.773348 \text{ is}$$

$$26.8132\% \text{ smaller than } \frac{4}{16}\pi \sqrt{0.97536759^4 + \frac{1.90268749507^2}{4}} = 1.05668.$$

**Series representations:**

$$\frac{1}{16}(4\pi) \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \\ \frac{1}{3} \times 0.0502748 ((8\pi) 0.7249637111830000 \times 0.975368^3) = \\ -0.0901862\pi + 0.25\pi \sum_{k=0}^{\infty} \frac{(-0.810106)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\frac{1}{16}(4\pi) \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \\ \frac{1}{3} \times 0.0502748 ((8\pi) 0.7249637111830000 \times 0.975368^3) = \\ -0.0901862\pi - \frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-j} e^{0.21059s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{8\sqrt{\pi}}$$

$$\begin{aligned} & \frac{1}{16} (4\pi) \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \\ & \frac{1}{3} \times 0.0502748 ((8\pi) 0.7249637111830000 \times 0.975368^3) = \\ & -0.0901862\pi + 0.25\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1.81011 - z_0)^k z_0^{-k}}{k!} \end{aligned}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\operatorname{Res}_{z=z_0} f$  is a complex residue

$\mathbb{R}$  is the set of real numbers

$$4\pi * 1/16 * (((\sqrt(((0.97536759^4 + (1.90268749507^2)/4))) - ((2/3 * 0.724963711183 * 0.97536759^3))))$$

### Input interpretation:

$$4\pi * \frac{1}{16} \left( \sqrt{0.97536759^4 + \frac{1.90268749507^2}{4}} - \frac{2}{3} \times 0.724963711183 \times 0.97536759^3 \right)$$

### Result:

0.70445148...

0.70445148...

### Series representations:

$$\frac{4}{16} \pi \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \right. \\ \left. \frac{2}{3} (0.7249637111830000 \times 0.975368^3) \right) = \\ -0.112117 \pi + 0.25 \pi \sum_{k=0}^{\infty} \frac{(-0.810106)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\frac{4}{16} \pi \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \right. \\ \left. \frac{2}{3} (0.7249637111830000 \times 0.975368^3) \right) = \\ -0.112117 \pi - \frac{0.125 \pi \sum_{j=0}^{\infty} \text{Res}_{s=-j} e^{0.21059 s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{\sqrt{\pi}}$$

$$\frac{4}{16} \pi \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \right. \\ \left. \frac{2}{3} (0.7249637111830000 \times 0.975368^3) \right) = \\ -0.112117 \pi + 0.25 \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1.81011 - z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

$\mathbb{R}$  is the set of real numbers

Thence, we obtain:

$$V(R) = 4\pi \sqrt{R^4 + \frac{\lambda^2}{4}} T_2 - \mu_2 \frac{8\pi}{3} f R^3 = 4\pi T_2 \left( \sqrt{R^4 + \frac{\lambda^2}{4}} - \frac{2}{3} f R^3 \right)$$

$0.773348\dots \approx 0.70445148\dots$

Now, we have that:

$$V_k(R) = 4\pi T_2 \left( \sqrt{R^4 + \frac{\lambda^2 k^2}{4}} - \frac{2}{3} f R^3 \right)$$

For  $k = 1$ , we obtain:

$$4*\text{Pi}*1/16*(((\text{sqrt}(((0.97536759^4+(1.90268749507^2)/4))))-((2/3*0.724963711183 *0.97536759^3))))$$

**Input interpretation:**

$$4\pi \times \frac{1}{16} \left( \sqrt{0.97536759^4 + \frac{1.90268749507^2}{4}} - \frac{2}{3} \times 0.724963711183 \times 0.97536759^3 \right)$$

**Result:**

$0.70445148\dots$

**0.70445148...**

Note that:

$0.70445148 * 2 = 1.40890296$  result very near to  $\sqrt{2} = 1.414\dots$  and to the value of the Ramanujan mock theta function:

$$1 + \frac{0.449329}{1 + 0.449329^2} + \frac{0.449329^4}{(1 + 0.449329^2)(1 + 0.449329^4)}$$

$1.406436589504891048492970141912370852583779342136575571764\dots$

**$\phi(q) = 1.40643658\dots$**

**Series representations:**

$$\frac{4}{16} \pi \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \right. \\ \left. \frac{2}{3} (0.7249637111830000 \times 0.975368^3) \right) = \\ -0.112117 \pi + 0.25 \pi \sum_{k=0}^{\infty} \frac{(-0.810106)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\frac{4}{16} \pi \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \right. \\ \left. \frac{2}{3} (0.7249637111830000 \times 0.975368^3) \right) = \\ -0.112117 \pi - \frac{0.125 \pi \sum_{j=0}^{\infty} \text{Res}_{s=-j} e^{0.21059 s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{\sqrt{\pi}}$$

$$\frac{4}{16} \pi \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \right. \\ \left. \frac{2}{3} (0.7249637111830000 \times 0.975368^3) \right) = \\ -0.112117 \pi + 0.25 \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1.81011 - z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

$\mathbb{R}$  is the set of real numbers

Now, we have:

$$\frac{7}{3}((((((4*\pi*1/16*(((\sqrt(((0.97536759^4+(1.90268749507^2)/4))))-((2/3*0.724963711183 *0.97536759^3)))))))))))$$

Where 7 and 3 are Lucas numbers

### Input interpretation:

$$\frac{7}{3} \left( 4\pi \times \frac{1}{16} \left( \sqrt{0.97536759^4 + \frac{1.90268749507^2}{4}} - \frac{2}{3} \times 0.724963711183 \times 0.97536759^3 \right) \right)$$

### Result:

$$1.6437201\dots$$

$$1.6437201\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

### Series representations:

$$\frac{\left( 4\pi \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{2 \times 0.7249637111830000 \times 0.975368^3}{3} \right) \right) 7}{16 \times 3} = \\ -0.261605 \pi + 0.583333 \pi \sum_{k=0}^{\infty} \frac{(-0.810106)^k \left( -\frac{1}{2} \right)_k}{k!}$$

$$\frac{\left( 4\pi \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{2 \times 0.7249637111830000 \times 0.975368^3}{3} \right) \right) 7}{16 \times 3} = \\ -0.261605 \pi - \frac{0.291667 \pi \sum_{j=0}^{\infty} \text{Res}_{s=-j} e^{0.21059 s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{\sqrt{\pi}}$$

$$\frac{\left( 4\pi \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{2 \times 0.7249637111830000 \times 0.975368^3}{3} \right) \right) 7}{16 \times 3} = \\ -0.261605 \pi + 0.583333 \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k (1.81011 - z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\operatorname{Res}_{z=z_0} f$  is a complex residue

$\mathbb{R}$  is the set of real numbers

And:

$$(21/10^3 + 7/10^3) + 7/3 (((((4*\pi*1/16)*(((\sqrt{((0.97536759^4 + (1.90268749507^2)/4)) - ((2/3 * 0.724963711183 * 0.97536759^3)))))))))))$$

Where 21 is a Fibonacci number

### Input interpretation:

$$\left( \frac{21}{10^3} + \frac{7}{10^3} \right) + \frac{7}{3} \left( 4\pi \times \frac{1}{16} \sqrt{0.97536759^4 + \frac{1.90268749507^2}{4}} - \frac{2}{3} \times 0.724963711183 \times 0.97536759^3 \right)$$

### Result:

1.6717201...

1.6717201...

We note that 1.6717201... is a result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass

### Series representations:

$$\left( \frac{21}{10^3} + \frac{7}{10^3} \right) + \frac{\left[ 4\pi \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{2 \times 0.7249637111830000 \times 0.975368^3}{3} \right) \right] 7}{0.028 - 0.261605\pi + 0.583333\pi \sum_{k=0}^{\infty} \frac{(-0.810106)^k \left( -\frac{1}{2} \right)_k}{k!}} =$$

$$\left( \frac{21}{10^3} + \frac{7}{10^3} \right) + \frac{\left[ 4\pi \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{2 \times 0.7249637111830000 \times 0.975368^3}{3} \right) \right] 7}{0.028 - 0.261605\pi - \frac{0.291667\pi \sum_{j=0}^{\infty} \text{Res}_{s=-j} e^{0.21059s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{\sqrt{\pi}}} =$$

$$\left( \frac{21}{10^3} + \frac{7}{10^3} \right) + \frac{\left[ 4\pi \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{2 \times 0.7249637111830000 \times 0.975368^3}{3} \right) \right] 7}{0.028 - 0.261605\pi + 0.583333\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k (1.81011 - z_0)^k z_0^{-k}}{k!}}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

We have also:

where  $C > 0$  is a positive constant parameter. As in the previous discussion, we obtain the effective potential for  $R$  as

$$V(R) = 4\pi T_2 \left( \sqrt{R^4 + \frac{\lambda^2}{4}} - CR^2 \right). \quad (3.38)$$

Here we assume  $C < 1$  then the shape of the potential can be drawn as in Fig.2. There is a global minimum at  $R_c$  given by

$$R_c^2 = \frac{\lambda}{2} \sqrt{\frac{C^2}{1 - C^2}}. \quad (3.39)$$

In particular, if  $C$  satisfies

$$C^2 = 1 - \epsilon^2 \quad (\epsilon \ll 1), \quad (3.40)$$

then the radius  $R_c$  and the value of the potential becomes

$$R_c \sim \frac{\lambda}{2\epsilon}, \quad V(R_c) \sim 4\pi T_2 \frac{\lambda}{2} \epsilon = T_0 \epsilon. \quad (3.41)$$

Therefore, the spherical D2-brane is stable at the macroscopic radius much larger than the string scale, even if  $k = 1$ , and the energy of the system is much smaller than that of the single D0-brane.

For  $C = 1/64$ , we obtain:

$$4*\text{Pi}*1/16* (((((((sqrt((((0.97536759^4+(1.90268749507^2)/4)))))))-1/64*0.97536759^2))))$$

### **Input interpretation:**

$$4\pi \times \frac{1}{16} \left( \sqrt{0.97536759^4 + \frac{1.90268749507^2}{4}} - \frac{1}{64} \times 0.97536759^2 \right)$$

### **Result:**

1.0450015...

1.0450015...

### **Series representations:**

$$\frac{4}{16} \pi \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{0.975368^2}{64} \right) =$$

$$-0.00371618 \pi + 0.25 \pi \sum_{k=0}^{\infty} \frac{(-0.810106)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\frac{4}{16} \pi \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{0.975368^2}{64} \right) =$$

$$-0.00371618 \pi - \frac{0.125 \pi \sum_{j=0}^{\infty} \text{Res}_{s=-j} e^{0.21059 s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{\sqrt{\pi}}$$

$$\frac{4}{16} \pi \left( \sqrt{0.975368^4 + \frac{1.902687495070000^2}{4}} - \frac{0.975368^2}{64} \right) =$$

$$-0.00371618 \pi + 0.25 \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1.81011 - z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

$\mathbb{R}$  is the set of real numbers

From:

$$R_c^2 = \frac{\lambda}{2} \sqrt{\frac{C^2}{1-C^2}}.$$

And

$$C^2 = 1 - \epsilon^2 \quad (\epsilon \ll 1)$$

$$C^2 = 1 - (1/64)^2 = 0.999755859375$$

$$1.90268749507/2 * (((\sqrt{((0.999755859375/(1-0.999755859375))))}))$$

**Input interpretation:**

$$\frac{1.90268749507}{2} \sqrt{\frac{0.999755859375}{1 - 0.999755859375}}$$

**Result:**

60.8785670...

60.8785670...

From:

$$R_c \sim \frac{\lambda}{2\epsilon}, \quad V(R_c) \sim 4\pi T_2 \frac{\lambda}{2} \epsilon = T_0 \epsilon.$$

And

$$V(R_c) \sim 4\pi T_2 \frac{\lambda}{2} c$$

$$4P_1 * 1/16 * (1.90268749507/2) * 1/64$$

**Input interpretation:**

$$4\pi \times \frac{1}{16} \times \frac{1.90268749507}{2} \times \frac{1}{64}$$

**Result:**

0.0116747442512...

0.0116747442512...

**Alternative representations:**

$$\frac{(4\pi) 1.902687495070000}{16 \times 2 \times 64} = \frac{684.9674982252000^\circ}{16 \times 64}$$

$$\frac{(4\pi) 1.902687495070000}{16 \times 2 \times 64} = -\frac{3.805374990140000 i \log(-1)}{16 \times 64}$$

$$\frac{(4\pi)1.902687495070000}{16 \times 2 \times 64} = \frac{3.805374990140000 \cos^{-1}(-1)}{16 \times 64}$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$\cos^{-1}(x)$  is the inverse cosine function

### Series representations:

$$\frac{(4\pi)1.902687495070000}{16 \times 2 \times 64} = 0.01486474605523438 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\begin{aligned} \frac{(4\pi)1.902687495070000}{16 \times 2 \times 64} &= \\ -0.007432373027617188 + 0.007432373027617188 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}} \end{aligned}$$

$$\frac{(4\pi)1.902687495070000}{16 \times 2 \times 64} = 0.003716186513808594 \sum_{k=0}^{\infty} \frac{2^{-k} (-6+50k)}{\binom{3k}{k}}$$

$\binom{n}{m}$  is the binomial coefficient

### Integral representations:

$$\frac{(4\pi)1.902687495070000}{16 \times 2 \times 64} = 0.007432373027617188 \int_0^\infty \frac{1}{1+t^2} dt$$

$$\frac{(4\pi)1.902687495070000}{16 \times 2 \times 64} = 0.01486474605523438 \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{(4\pi) 1.902687495070000}{16 \times 2 \times 64} = 0.007432373027617188 \int_0^\infty \frac{\sin(t)}{t} dt$$

From:

**Fat Inflatons, Large Turns and the  $\eta$ -problem**

*Dibya Chakraborty, Roberta Chiovonoli, Oscar Loaiza-Brito, Gustavo Niz,  
Ivonne Zavala - arXiv:1908.09797v2 [hep-th] 27 Aug 2019*

#### 4.1 Effective 4D action and cosmological equations

Our starting action is given by (see eq. (3.24))

$$S_4 = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{Pl}}^2}{2} R_4 + \frac{1}{2} g_{ij} v^i v^j - V(r, \theta) \right] \quad (4.1)$$

where the four dimensional metric is the FRW metric (2.2),  $g_{ij}$  is defined in (3.25) and the full expression for the scalar potential is given by (see (3.25), (3.31), (3.33)):

$$V(r, \theta) = V_0 + 4\pi p T_5 \mathcal{H}^{-1} [\mathcal{F}^{1/2} - \ell_s^2 \pi q g_s] + \gamma [\bar{\Phi}_- + \Phi_h], \quad (4.2)$$

where  $\gamma = 4\pi^2 \ell_s^2 p q T_5 g_s$  and (see (3.24),(3.25))

$$\mathcal{F} = \frac{\mathcal{H}}{9} (r^2 + 3u^2)^2 + (\pi \ell_s^2 q)^2 \quad (4.3)$$

$$\bar{\Phi}_- = \frac{5}{72} [81(9\rho^2 - 2)\rho^2 + 162 \log(9(\rho^2 + 1)) - 9 - 160 \log(10)] \quad (4.4)$$

$$\begin{aligned} \Phi_h = a_0 & \left[ \frac{2}{\rho^2} - 2 \log \left( \frac{1}{\rho^2} + 1 \right) \right] + 2a_1 \left[ 6 + \frac{1}{\rho^2} - 2(2 + 3\rho^2) \log \left( 1 + \frac{1}{\rho^2} \right) \right] \cos \theta \\ & + \frac{b_1}{2} (2 + 3\rho^2) \cos \theta. \end{aligned} \quad (4.5)$$

As we explained in the previous section, the coefficients  $a_0, a_1, b_1$  are arbitrary, but small (in [46]  $a_1 = 0$ ). We have also introduced a constant piece  $V_0$ , which we tune in order to downlift the de Sitter minimum of the potential to Minkowski. The reasons behind are twofold. This term will encode any unknown physics that may shift these minima to Minkowski. For example, due to the explicit stabilisation mechanism of the closed string moduli, which we haven't included. Moreover, the recently proposed dS swampland conjectures [5–7] exclude dS minima in string theory, if correct, while Minkowski minima are allowed.

Finally, the four dimensional Planck mass,  $M_{\text{Pl}}$  after compactification is given by (see (3.1))

$$M_{\text{Pl}}^2 \gtrsim \kappa_{10}^{-2} \text{Vol}(T^{1,1}) \int_0^u y^5 \mathcal{H}(y) \sim \frac{N u^2}{4(2\pi)^3 g_s \ell_s^4}. \quad (4.6)$$

where we used that  $\text{Vol}(T^{1,1}) = 16\pi^3/27$  and assumed that most of the volume comes from the throat, approximating  $\mathcal{H} \sim L^4/\rho^4$ . For concreteness, for the cosmological solutions we fix  $M_{\text{Pl}}$  to the lower bound.

$N$	$g_s$	$\ell_s$	$u$	$q$	$a_0$	$a_1$	$b_1$
1000	0.01	501.961	$50\ell_s$	1	0.001	0.0005	0.001

(remember that  $\rho = r/3u$ )

$$n_s = 1 - 2\epsilon - \eta, \quad r = 16\epsilon$$

$r_{min} = 21.414$  and  $\theta_{min} = \pi$ ,

$u = 50 * 501.961 = 25098.05$  Planck units

$\rho = 0.000284404565$

$$\overline{\Phi}_- = \frac{5}{72} [81(9\rho^2 - 2)\rho^2 + 162 \log(9(\rho^2 + 1)) - 9 - 160 \log(10)]$$

$$\frac{5}{72}(((81(9 \times 0.000284404565^2 - 2) \times 0.000284404565^2 + 162 \log(9(0.000284404565^2 + 1)) - 9 - 160 \log(10))))$$

**Input interpretation:**

$$\frac{5}{72} (81(9 \times 0.000284404565^2 - 2) \times 0.000284404565^2 + 162 \log(9(0.000284404565^2 + 1)) - 9 - 160 \log(10))$$

$\log(x)$  is the natural logarithm

**Result:**

-1.49050231601219...

-1.49050231601219....

**Alternative representations:**

$$\begin{aligned} & \frac{1}{72} (81(9 \times 0.000284405^2 - 2) 0.000284405^2 + \\ & \quad 162 \log(9(0.000284405^2 + 1)) - 9 - 160 \log(10)) 5 = \\ & \frac{5}{72} (-9 - 160 \log(a) \log_a(10) + 162 \log(a) \log_a(9(1 + 0.000284405^2)) + \\ & \quad 81 \times 0.000284405^2 (-2 + 9 \times 0.000284405^2)) \end{aligned}$$

•

$$\begin{aligned}
& \frac{1}{72} (81(9 \times 0.000284405^2 - 2) 0.000284405^2 + \\
& \quad 162 \log(9(0.000284405^2 + 1)) - 9 - 160 \log(10)) 5 = \\
& \frac{5}{72} (-9 - 160 \log_e(10) + 162 \log_e(9(1 + 0.000284405^2)) + \\
& \quad 81 \times 0.000284405^2 (-2 + 9 \times 0.000284405^2))
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{72} (81(9 \times 0.000284405^2 - 2) 0.000284405^2 + \\
& \quad 162 \log(9(0.000284405^2 + 1)) - 9 - 160 \log(10)) 5 = \\
& \frac{5}{72} (-9 + 160 \text{Li}_1(-9) - 162 \text{Li}_1(1 - 9(1 + 0.000284405^2)) + \\
& \quad 81 \times 0.000284405^2 (-2 + 9 \times 0.000284405^2))
\end{aligned}$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$\begin{aligned}
& \frac{1}{72} (81(9 \times 0.000284405^2 - 2) 0.000284405^2 + \\
& \quad 162 \log(9(0.000284405^2 + 1)) - 9 - 160 \log(10)) 5 = -0.625001 + \\
& 11.25 \log(8.) - 11.1111 \log(9) + \sum_{k=1}^{\infty} \frac{11.1111 \left(-\frac{1}{9}\right)^k - 11.25 (-1)^k e^{-2.07944 k}}{k}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{72} (81(9 \times 0.000284405^2 - 2) 0.000284405^2 + \\
& \quad 162 \log(9(0.000284405^2 + 1)) - 9 - 160 \log(10)) 5 = \\
& -0.625001 + 22.5 i \pi \left\lfloor \frac{\arg(9. - x)}{2 \pi} \right\rfloor - 22.2222 i \pi \left\lfloor \frac{\arg(10 - x)}{2 \pi} \right\rfloor + \\
& 0.138889 \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^k (-11.25 (9. - x)^k + 11.1111 (10 - x)^k) x^{-k}}{k} \quad \text{for } x < 0
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{72} (81(9 \times 0.000284405^2 - 2) 0.000284405^2 + \\
& 162 \log(9(0.000284405^2 + 1)) - 9 - 160 \log(10)) 5 = \\
& -0.625001 + \frac{45}{4} \left\lfloor \frac{\arg(9 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) - 11.1111 \left\lfloor \frac{\arg(10 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \\
& 0.138889 \log(z_0) + \frac{45}{4} \left\lfloor \frac{\arg(9 - z_0)}{2\pi} \right\rfloor \log(z_0) - 11.1111 \left\lfloor \frac{\arg(10 - z_0)}{2\pi} \right\rfloor \log(z_0) + \\
& \sum_{k=1}^{\infty} \frac{(-1)^k (-11.25(9 - z_0)^k + 11.1111(10 - z_0)^k) z_0^{-k}}{k}
\end{aligned}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

### Integral representations:

$$\begin{aligned}
& \frac{1}{72} (81(9 \times 0.000284405^2 - 2) 0.000284405^2 + 162 \log(9(0.000284405^2 + 1)) - \\
& 9 - 160 \log(10)) 5 = -0.625001 + \int_1^{9 - 1.25 + 0.138889 t} \frac{dt}{(-0.111111 + t)t}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{72} (81(9 \times 0.000284405^2 - 2) 0.000284405^2 + \\
& 162 \log(9(0.000284405^2 + 1)) - 9 - 160 \log(10)) 5 = -0.625001 + \\
& \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{5.625 \times 9^{-s} e^{-2.07944 s} (9^s - 0.987654 e^{2.07944 s}) \Gamma(-s)^2 \Gamma(1+s)}{i\pi \Gamma(1-s)} ds
\end{aligned}$$

for  $-1 < \gamma < 0$

$\Gamma(x)$  is the gamma function

And:

$$\begin{aligned}
& 0.001 * (((((2/0.000284404565^2 - \\
& 2 \ln(1/(0.000284404565^2 + 1)))) + 2 * 0.0005 * (((((6+1/0.000284404565^2 - \\
& 2(2+3*0.000284404565^2) \ln(1+1/(0.000284404565^2))))))) * \cosPi + \\
& 0.001/2(2+3*0.000284404565^2) * \cosPi
\end{aligned}$$

$$\Phi_h = a_0 \left[ \frac{2}{\rho^2} - 2 \log \left( \frac{1}{\rho^2} + 1 \right) \right] + 2a_1 \left[ 6 + \frac{1}{\rho^2} - 2(2 + 3\rho^2) \log \left( 1 + \frac{1}{\rho^2} \right) \right] \cos \theta \\ + \frac{b_1}{2} (2 + 3\rho^2) \cos \theta.$$

### Input interpretation:

$$0.001 \left( \frac{2}{0.000284404565^2} - 2 \log \left( \frac{1}{0.000284404565^2} + 1 \right) \right) + 2 \times 0.0005 \\ \left( 6 + \frac{1}{0.000284404565^2} - 2(2 + 3 \times 0.000284404565^2) \log \left( 1 + \frac{1}{0.000284404565^2} \right) \right) \\ \cos(\pi) + \frac{0.001}{2} (2 + 3 \times 0.000284404565^2) \cos(\pi)$$

$\log(x)$  is the natural logarithm

### Result:

12363.1...

12363.1....

### Alternative representations:

$$0.001 \left( \frac{2}{0.000284405^2} - 2 \log \left( \frac{1}{0.000284405^2} + 1 \right) \right) + \\ 2 \times 0.0005 \left( 6 + \frac{1}{0.000284405^2} - 2(2 + 3 \times 0.000284405^2) \log \left( 1 + \frac{1}{0.000284405^2} \right) \right) \\ \cos(\pi) + \frac{1}{2} ((2 + 3 \times 0.000284405^2) \cos(\pi)) 0.001 = \\ 0.0005 \cosh(-i\pi) (2 + 3 \times 0.000284405^2) + 0.001 \cosh(-i\pi) \\ \left( 6 - 2 \log \left( 1 + \frac{1}{0.000284405^2} \right) (2 + 3 \times 0.000284405^2) + \frac{1}{0.000284405^2} \right) + \\ 0.001 \left( -2 \log \left( 1 + \frac{1}{0.000284405^2} \right) + \frac{2}{0.000284405^2} \right)$$

$$0.001 \left( \frac{2}{0.000284405^2} - 2 \log \left( \frac{1}{0.000284405^2} + 1 \right) \right) + \\ 2 \times 0.0005 \left( 6 + \frac{1}{0.000284405^2} - 2(2 + 3 \times 0.000284405^2) \log \left( 1 + \frac{1}{0.000284405^2} \right) \right) \\ \cos(\pi) + \frac{1}{2} ((2 + 3 \times 0.000284405^2) \cos(\pi)) 0.001 = \\ 0.0005 \cosh(i\pi) (2 + 3 \times 0.000284405^2) + 0.001 \cosh(i\pi) \\ \left( 6 - 2 \log(a) \log_a \left( 1 + \frac{1}{0.000284405^2} \right) (2 + 3 \times 0.000284405^2) + \frac{1}{0.000284405^2} \right) + \\ 0.001 \left( -2 \log(a) \log_a \left( 1 + \frac{1}{0.000284405^2} \right) + \frac{2}{0.000284405^2} \right)$$

$$\begin{aligned}
& 0.001 \left( \frac{2}{0.000284405^2} - 2 \log \left( \frac{1}{0.000284405^2} + 1 \right) \right) + \\
& 2 \times 0.0005 \left( 6 + \frac{1}{0.000284405^2} - 2(2 + 3 \times 0.000284405^2) \log \left( 1 + \frac{1}{0.000284405^2} \right) \right) \\
& \cos(\pi) + \frac{1}{2} ((2 + 3 \times 0.000284405^2) \cos(\pi)) 0.001 = \\
& 0.0005 \cosh(i\pi)(2 + 3 \times 0.000284405^2) + 0.001 \cosh(i\pi) \\
& \left( 6 - 2 \log_e \left( 1 + \frac{1}{0.000284405^2} \right) (2 + 3 \times 0.000284405^2) + \frac{1}{0.000284405^2} \right) + \\
& 0.001 \left( -2 \log_e \left( 1 + \frac{1}{0.000284405^2} \right) + \frac{2}{0.000284405^2} \right)
\end{aligned}$$

$\cosh(x)$  is the hyperbolic cosine function

$\log_b(x)$  is the base- $b$  logarithm

$i$  is the imaginary unit

### Series representations:

$$\begin{aligned}
& 0.001 \left( \frac{2}{0.000284405^2} - 2 \log \left( \frac{1}{0.000284405^2} + 1 \right) \right) + \\
& 2 \times 0.0005 \left( 6 + \frac{1}{0.000284405^2} - 2(2 + 3 \times 0.000284405^2) \log \left( 1 + \frac{1}{0.000284405^2} \right) \right) \\
& \cos(\pi) + \frac{1}{2} ((2 + 3 \times 0.000284405^2) \cos(\pi)) 0.001 = 24726.2 - \\
& 0.002 \log(1.23631 \times 10^7) + (12363.1 - 0.004 \log(1.23631 \times 10^7)) \sum_{k=0}^{\infty} \frac{(-1)^k \pi^{2k}}{(2k)!}
\end{aligned}$$

$$\begin{aligned}
& 0.001 \left( \frac{2}{0.000284405^2} - 2 \log \left( \frac{1}{0.000284405^2} + 1 \right) \right) + 2 \times 0.0005 \\
& \left( 6 + \frac{1}{0.000284405^2} - 2(2 + 3 \times 0.000284405^2) \log \left( 1 + \frac{1}{0.000284405^2} \right) \right) \cos(\pi) + \\
& \frac{1}{2} ((2 + 3 \times 0.000284405^2) \cos(\pi)) 0.001 = 24726.2 - 0.002 \log(1.23631 \times 10^7) + \\
& (12363.1 - 0.004 \log(1.23631 \times 10^7)) \sum_{k=0}^{\infty} \frac{\cos\left(\frac{k\pi}{2} + z_0\right) (\pi - z_0)^k}{k!}
\end{aligned}$$

$$\begin{aligned}
& 0.001 \left( \frac{2}{0.000284405^2} - 2 \log \left( \frac{1}{0.000284405^2} + 1 \right) \right) + 2 \times 0.0005 \\
& \left( 6 + \frac{1}{0.000284405^2} - 2(2 + 3 \times 0.000284405^2) \log \left( 1 + \frac{1}{0.000284405^2} \right) \right) \cos(\pi) + \\
& \frac{1}{2} ((2 + 3 \times 0.000284405^2) \cos(\pi)) 0.001 = 24726.2 + 12363.1 J_0(\pi) - \\
& 0.002 \log(1.23631 \times 10^7) - 0.004 J_0(\pi) \log(1.23631 \times 10^7) + \\
& \sum_{k=1}^{\infty} (-1)^k J_{2k}(\pi) (24726.2 - 0.008 \log(1.23631 \times 10^7))
\end{aligned}$$

$n!$  is the factorial function

$J_n(z)$  is the Bessel function of the first kind

### Integral representations:

$$\begin{aligned}
& 0.001 \left( \frac{2}{0.000284405^2} - 2 \log \left( \frac{1}{0.000284405^2} + 1 \right) \right) + \\
& 2 \times 0.0005 \left( 6 + \frac{1}{0.000284405^2} - 2(2 + 3 \times 0.000284405^2) \log \left( 1 + \frac{1}{0.000284405^2} \right) \right) \\
& \cos(\pi) + \frac{1}{2} ((2 + 3 \times 0.000284405^2) \cos(\pi)) 0.001 = 37089.3 + \\
& \int_0^1 \pi (-12363.1 + 0.004 \log(1.23631 \times 10^7)) \sin(\pi t) dt - 0.006 \log(1.23631 \times 10^7)
\end{aligned}$$

$$\begin{aligned}
& 0.001 \left( \frac{2}{0.000284405^2} - 2 \log \left( \frac{1}{0.000284405^2} + 1 \right) \right) + \\
& 2 \times 0.0005 \left( 6 + \frac{1}{0.000284405^2} - 2(2 + 3 \times 0.000284405^2) \log \left( 1 + \frac{1}{0.000284405^2} \right) \right) \\
& \cos(\pi) + \frac{1}{2} ((2 + 3 \times 0.000284405^2) \cos(\pi)) 0.001 = 24726.2 + \\
& \int_{\frac{\pi}{2}}^{\pi} (-12363.1 + 0.004 \log(1.23631 \times 10^7)) \sin(t) dt - 0.002 \log(1.23631 \times 10^7)
\end{aligned}$$

$$\begin{aligned}
& 0.001 \left( \frac{2}{0.000284405^2} - 2 \log \left( \frac{1}{0.000284405^2} + 1 \right) \right) + \\
& 2 \times 0.0005 \left( 6 + \frac{1}{0.000284405^2} - 2(2 + 3 \times 0.000284405^2) \log \left( 1 + \frac{1}{0.000284405^2} \right) \right) \\
& \cos(\pi) + \frac{1}{2} ((2 + 3 \times 0.000284405^2) \cos(\pi)) 0.001 = \\
& 37089.3 + \int_1^{1.23631 \times 10^7} \left( -\frac{0.006}{t} - 0.001 \pi \sin(8.0886 \times 10^{-8} \pi (-1+t)) \right) dt + \\
& 8.0886 \times 10^{-8} \int_0^1 \int_0^1 \frac{\sin(\pi t_2)}{8.0886 \times 10^{-8} + t_1} dt_2 dt_1
\end{aligned}$$

Multiplying the two results, we obtain:

$$((-12363.11119623 * (-1.49050231601219))))$$

**Input interpretation:**

$$-(12363.11119623 \times (-1.49050231601219))$$

**Result:**

$$18427.2458710970517941620437$$

**Repeating decimal:**

$$18427.2458710970517941620437$$

$$18427.245871\dots$$

$$-(199+18)+4*((-12363.11119623 * (-1.49050231601219))))$$

**Input interpretation:**

$$-(199 + 18) + 4 * ((-12363.11119623 \times (-1.49050231601219)))$$

**Result:**

$$73491.9834843882071766481748$$

$$73491.98348\dots$$

We can obtain, the following mathematical connection, with the previous equation:

$$-(199+18)+4 \left( \begin{array}{l} \frac{5}{72} [81 (9\rho^2 - 2) \rho^2 + 162 \log (9 (\rho^2 + 1)) - 9 - 160 \log (10)] \\ \times -a_0 \left[ \frac{2}{\rho^2} - 2 \log \left( \frac{1}{\rho^2} + 1 \right) \right] + 2a_1 \left[ 6 + \frac{1}{\rho^2} - 2(2 + 3\rho^2) \log \left( 1 + \frac{1}{\rho^2} \right) \right] \cos \theta \\ + \frac{b_1}{2} (2 + 3\rho^2) \cos \theta . \end{array} \right)$$

$$= 73491.98438 \Rightarrow$$

$$-3927 + 2 \left( \begin{array}{l} {}^{13} \sqrt{N \exp \left[ \int d\hat{\sigma} \left( -\frac{1}{4u^2} P_i D P_i \right) \right] |B_P\rangle_{NS}} + \\ \int [dX^\mu] \exp \left\{ \int d\hat{\sigma} \left( -\frac{1}{4v^2} D X^\mu D^2 X^\mu \right) \right\} |X^\mu, X^i = 0\rangle_{NS} \end{array} \right)$$

= 73490.8437525....

And:

$$-((5778+29+8)-11*((-(12363.11119623*(-1.49050231601219)))))$$

**Input interpretation:**

$$-((5778 + 29 + 8) - 11 \cdot (-12363.11119623 \times (-1.49050231601219)))$$

**Result:**

$$196884.7045820675697357824807$$

196884.7....

$$-(5778+29+8)-11\left(\begin{array}{l} \frac{5}{72}[81(9\rho^2-2)\rho^2+162\log(9(\rho^2+1))-9-160\log(10)] \\ \times -a_0\left[\frac{2}{\rho^2}-2\log\left(\frac{1}{\rho^2}+1\right)\right]+2a_1\left[6+\frac{1}{\rho^2}-2(2+3\rho^2)\log\left(1+\frac{1}{\rho^2}\right)\right]\cos\theta \\ +\frac{b_1}{2}(2+3\rho^2)\cos\theta. \end{array}\right)$$

= 196884.704582...

that can be connected with another previous equation:

$$\left( (3571 + 233 + 11) + \frac{1}{4} \left( -\left( -\frac{3}{2\sqrt{2}} \left( 2.33181 \times 10^{21} \times \frac{1}{4096} \right) + \left( \frac{3}{2\sqrt{2}} \times 4.1666 \times 10^{-19} (2.33181 \times 10^{21})^2 \right) \left( \frac{1}{4096} \right)^2 \right) \right)^{(1/3)} \right)$$

$$\begin{aligned} &= 196884.3612355790262930818210234562962529826220937890171828... \\ &= 196884.36 \end{aligned}$$

Thence:

$$-(5778+29+8)-11 \left( \frac{5}{72} [81 (9\rho^2 - 2) \rho^2 + 162 \log(9 (\rho^2 + 1)) - 9 - 160 \log(10)] \right. \\ \left. \times -a_0 \left[ \frac{2}{\rho^2} - 2 \log \left( \frac{1}{\rho^2} + 1 \right) \right] + 2a_1 \left[ 6 + \frac{1}{\rho^2} - 2(2 + 3\rho^2) \log \left( 1 + \frac{1}{\rho^2} \right) \right] \cos \theta \right. \\ \left. + \frac{b_1}{2} (2 + 3\rho^2) \cos \theta. \right)$$

$$= 196884.704582... \Rightarrow$$

$$\Rightarrow \left( (3571 + 233 + 11) + \frac{1}{4} \left( - \left( - \frac{3}{2\sqrt{2}} \left( 2.33181 \times 10^{21} \times \frac{1}{4096} \right) + \right. \right. \right. \\ \left. \left. \left. \left( \frac{3}{2\sqrt{2}} \times 4.1666 \times 10^{-19} (2.33181 \times 10^{21})^2 \right) \left( \frac{1}{4096} \right)^2 \right) \right) \right)^{(1/3)} = \\ = 196884.3612355790262930818210234562962529826220937890171828... \\ = 196884.36$$

Now, we have that:

Finally, the four dimensional Planck mass,  $M_{\text{Pl}}$  after compactification is given by (3.1))

$$M_{\text{Pl}}^2 \gtrsim \kappa_{10}^{-2} \text{Vol}(T^{1,1}) \int_0^u y^5 \mathcal{H}(y) \sim \frac{N u^2}{4(2\pi)^3 g_s \ell_s^4}.$$

From:

$$M_{\text{Pl}}^2 \gtrsim \kappa_{10}^{-2} \text{Vol}(T^{1,1}) \int_0^u y^5 \mathcal{H}(y) \sim \frac{N u^2}{4(2\pi)^3 g_s \ell_s^4}$$

We obtain:

$$((1000*(50*501.961)^2)) / (((4*(2\pi)^3 * 0.01 * 501.961^4))))$$

### **Input interpretation:**

$$\frac{1000 (50 \times 501.961)^2}{4 (2 \pi)^3 \times 0.01 \times 501.961^4}$$

**Result:**

1.000001060617684410186205575315043304529922624690292755028...

1.000001060617...

**Alternative representations:**

$$\frac{1000 (50 \times 501.961)^2}{4 (2\pi)^3 0.01 \times 501.961^4} = \frac{1000 \times 25098.1^2}{0.04 \times 501.961^4 (360^\circ)^3}$$

$$\frac{1000 (50 \times 501.961)^2}{4 (2\pi)^3 0.01 \times 501.961^4} = \frac{1000 \times 25098.1^2}{0.04 \times 501.961^4 (-2i \log(-1))^3}$$

$$\frac{1000 (50 \times 501.961)^2}{4 (2\pi)^3 0.01 \times 501.961^4} = \frac{1000 \times 25098.1^2}{0.04 \times 501.961^4 (2 \cos^{-1}(-1))^3}$$

 $\log(x)$  is the natural logarithm $i$  is the imaginary unit $\cos^{-1}(x)$  is the inverse cosine function**Series representations:**

$$\frac{1000 (50 \times 501.961)^2}{4 (2\pi)^3 0.01 \times 501.961^4} = \frac{0.484474}{\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^3}$$

$$\frac{1000 (50 \times 501.961)^2}{4 (2\pi)^3 0.01 \times 501.961^4} = \frac{3.87579}{\left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}\right)^3}$$

$$\frac{1000 (50 \times 501.961)^2}{4 (2\pi)^3 0.01 \times 501.961^4} = \frac{31.0063}{\left(\sum_{k=0}^{\infty} \frac{2^{-k} (-6+50k)}{\binom{3k}{k}}\right)^3}$$

 $\binom{n}{m}$  is the binomial coefficient

### Integral representations:

$$\frac{1000(50 \times 501.961)^2}{4(2\pi)^3 0.01 \times 501.961^4} = \frac{3.87579}{\left(\int_0^\infty \frac{1}{1+t^2} dt\right)^3}$$

$$\frac{1000(50 \times 501.961)^2}{4(2\pi)^3 0.01 \times 501.961^4} = \frac{0.484474}{\left(\int_0^1 \sqrt{1-t^2} dt\right)^3}$$

$$\frac{1000(50 \times 501.961)^2}{4(2\pi)^3 0.01 \times 501.961^4} = \frac{3.87579}{\left(\int_0^\infty \frac{\sin(t)}{t} dt\right)^3}$$

From the inverse, we obtain:

$$1 / (((((1000 * (50 * 501.961)^2)) / (((4 * (2\pi)^3 * 0.01 * 501.961^4))))))$$

### Input interpretation:

$$\frac{1}{\frac{1000(50 \times 501.961)^2}{4(2\pi)^3 \times 0.01 \times 501.961^4}}$$

### Result:

0.99998939383440498493180011324995557295649969784689765221...

**0.9999893938344...** result that is a good approximation to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{\frac{1}{1+\sqrt[5]{\sqrt{\varphi^5 \sqrt{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

### Alternative representations:

$$\frac{1}{\frac{1000(50 \times 501.961)^2}{4(2\pi)^3 0.01 \times 501.961^4}} = \frac{1}{\frac{1000 \times 25098.1^2}{0.04 \times 501.961^4 (360^\circ)^3}}$$

$$\frac{1}{\frac{1000(50 \times 501.961)^2}{4(2\pi)^3 0.01 \times 501.961^4}} = \frac{1}{\frac{1000 \times 25098.1^2}{0.04 \times 501.961^4 (-2i \log(-1))^3}}$$

$$\frac{1}{\frac{1000(50 \times 501.961)^2}{4(2\pi)^3 0.01 \times 501.961^4}} = \frac{1}{\frac{1000 \times 25098.1^2}{0.04 \times 501.961^4 (2 \cos^{-1}(-1))^3}}$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$\cos^{-1}(x)$  is the inverse cosine function

### Series representations:

$$\frac{1}{\frac{1000(50 \times 501.961)^2}{4(2\pi)^3 0.01 \times 501.961^4}} = 2.0641 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^3$$

$$\frac{1}{\frac{1000(50 \times 501.961)^2}{4(2\pi)^3 0.01 \times 501.961^4}} = 0.258012 \left( -1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}} \right)^3$$

$$\frac{1}{\frac{1000(50 \times 501.961)^2}{4(2\pi)^3 0.01 \times 501.961^4}} = 0.0322515 \left( \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}} \right)^3$$

$\binom{n}{m}$  is the binomial coefficient

### Integral representations:

$$\frac{1}{\frac{1000(50 \times 501.961)^2}{4(2\pi)^3 0.01 \times 501.961^4}} = 0.258012 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^3$$

$$\frac{1}{\frac{1000(50 \times 501.961)^2}{4(2\pi)^3 0.01 \times 501.961^4}} = 2.0641 \left( \int_0^1 \sqrt{1-t^2} dt \right)^3$$

$$\frac{1}{\frac{1000(50 \times 501.961)^2}{4(2\pi)^3 0.01 \times 501.961^4}} = 0.258012 \left( \int_0^\infty \frac{\sin(t)}{t} dt \right)^3$$

For this other values:

$N$	$g_s$	$\ell_s$	$u$	$q$	$a_0$	$a_1$	$b_1$
1000	0.01	501.961	$50\ell_s$	1	0.001	0.0005	0.001

(remember that  $\rho = r/3u$ )

$$n_s = 1 - 2\epsilon - \eta, \quad r = 16\epsilon$$

$$r_{min} = 456.797 \text{ and } \theta_{min} = 33\pi,$$

$$u = 50 * 501.961 = 25098.05 \text{ Planck units}$$

$$\rho = 456.797/(3*25098.05) = 0.00606683254940788892$$

we obtain:

$$\frac{5}{72}(((81(9*0.006066832549^2-2)*0.006066832549^2+162 \ln(9*(0.006066832549^2+1))-9-160 \ln(10))))))$$

**Input interpretation:**

$$\frac{5}{72} (81 (9 \times 0.006066832549^2 - 2) \times 0.006066832549^2 + 162 \log(9 (0.006066832549^2 + 1)) - 9 - 160 \log(10))$$

$\log(x)$  is the natural logarithm

**Result:**

$$-1.490502255050\dots$$

-1.490502255050...

### Alternative representations:

$$\begin{aligned} & \frac{1}{72} (81(9 \times 0.00606683^2 - 2) 0.00606683^2 + \\ & \quad 162 \log(9(0.00606683^2 + 1)) - 9 - 160 \log(10)) 5 = \\ & \frac{5}{72} (-9 - 160 \log(a) \log_a(10) + 162 \log(a) \log_a(9(1 + 0.00606683^2)) + \\ & \quad 81 \times 0.00606683^2 (-2 + 9 \times 0.00606683^2)) \end{aligned}$$

- 
$$\begin{aligned} & \frac{1}{72} (81(9 \times 0.00606683^2 - 2) 0.00606683^2 + \\ & \quad 162 \log(9(0.00606683^2 + 1)) - 9 - 160 \log(10)) 5 = \\ & \frac{5}{72} (-9 - 160 \log_e(10) + 162 \log_e(9(1 + 0.00606683^2)) + \\ & \quad 81 \times 0.00606683^2 (-2 + 9 \times 0.00606683^2)) \end{aligned}$$

- 
$$\begin{aligned} & \frac{1}{72} (81(9 \times 0.00606683^2 - 2) 0.00606683^2 + \\ & \quad 162 \log(9(0.00606683^2 + 1)) - 9 - 160 \log(10)) 5 = \\ & \frac{5}{72} (-9 + 160 \text{Li}_1(-9) - 162 \text{Li}_1(1 - 9(1 + 0.00606683^2)) + \\ & \quad 81 \times 0.00606683^2 (-2 + 9 \times 0.00606683^2)) \end{aligned}$$

$\log_b(x)$  is the base- $b$  logarithm

$\text{Li}_n(x)$  is the polylogarithm function

### Series representations:

$$\begin{aligned} & \frac{1}{72} (81(9 \times 0.00606683^2 - 2) 0.00606683^2 + \\ & \quad 162 \log(9(0.00606683^2 + 1)) - 9 - 160 \log(10)) 5 = \\ & -0.625414 + 11.25 \log(8.00033) - 11.1111 \log(9) + \\ & \sum_{k=1}^{\infty} \frac{11.1111 \left(-\frac{1}{9}\right)^k - 11.25 (-1)^k e^{-2.07948 k}}{k} \end{aligned}$$

$$\begin{aligned}
& \frac{1}{72} (81(9 \times 0.00606683^2 - 2) 0.00606683^2 + \\
& 162 \log(9(0.00606683^2 + 1)) - 9 - 160 \log(10)) 5 = -0.625414 + \\
& 22.5 i \pi \left\lfloor \frac{\arg(9.00033 - x)}{2\pi} \right\rfloor - 22.2222 i \pi \left\lfloor \frac{\arg(10 - x)}{2\pi} \right\rfloor + 0.138889 \log(x) + \\
& \sum_{k=1}^{\infty} \frac{(-1)^k (-11.25 (9.00033 - x)^k + 11.1111 (10 - x)^k) x^{-k}}{k} \quad \text{for } x < 0
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{72} (81(9 \times 0.00606683^2 - 2) 0.00606683^2 + \\
& 162 \log(9(0.00606683^2 + 1)) - 9 - 160 \log(10)) 5 = \\
& -0.625414 + 22.5 i \pi \left\lfloor -\frac{-\pi + \arg\left(\frac{9.00033}{z_0}\right) + \arg(z_0)}{2\pi} \right\rfloor - \\
& 22.2222 i \pi \left\lfloor -\frac{-\pi + \arg\left(\frac{10}{z_0}\right) + \arg(z_0)}{2\pi} \right\rfloor + 0.138889 \log(z_0) + \\
& \sum_{k=1}^{\infty} \frac{(-1)^k (-11.25 (9.00033 - z_0)^k + 11.1111 (10 - z_0)^k) z_0^{-k}}{k}
\end{aligned}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

## Integral representations:

$$\begin{aligned}
& \frac{1}{72} (81(9 \times 0.00606683^2 - 2) 0.00606683^2 + \\
& 162 \log(9(0.00606683^2 + 1)) - 9 - 160 \log(10)) 5 = \\
& -0.625414 + \int_1^{9.00033} \frac{-1.24959 + 0.138889 t}{(-0.111074 + t) t} dt
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{72} (81(9 \times 0.00606683^2 - 2) 0.00606683^2 + \\
& 162 \log(9(0.00606683^2 + 1)) - 9 - 160 \log(10)) 5 = -0.625414 + \\
& \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{5.625 \times 9^{-s} e^{-2.07948 s} (9^s - 0.987654 e^{2.07948 s}) \Gamma(-s)^2 \Gamma(1+s)}{i\pi \Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0
\end{aligned}$$

And:

$$0.001 * (((2/0.006066832549^2 - 2\ln(1/(0.006066832549^2 + 1)))) + 2 * 0.0005 * (((((6+1/0.006066832549^2 - 2(2+3*0.006066832549^2)\ln(1+1/(0.006066832549^2)))))*\cos33\pi + 0.001/2(2+3*0.006066832549^2)*\cos33\pi)$$

### Input interpretation:

$$\begin{aligned} & 0.001 \left( \frac{2}{0.006066832549^2} - 2 \log \left( \frac{1}{0.006066832549^2} + 1 \right) \right) + 2 \times 0.0005 \\ & \left( 6 + \frac{1}{0.006066832549^2} - 2(2 + 3 \times 0.006066832549^2) \log \left( 1 + \frac{1}{0.006066832549^2} \right) \right) \\ & \cos(33)\pi + \frac{0.001}{2}(2 + 3 \times 0.006066832549^2) \cos(33)\pi \end{aligned}$$

$\log(x)$  is the natural logarithm

### Result:

53.1861...

53.1861...

### Alternative representations:

$$\begin{aligned} & 0.001 \left( \frac{2}{0.00606683^2} - 2 \log \left( \frac{1}{0.00606683^2} + 1 \right) \right) + \\ & 2 \times 0.0005 \left( 6 + \frac{1}{0.00606683^2} - 2(2 + 3 \times 0.00606683^2) \log \left( 1 + \frac{1}{0.00606683^2} \right) \right) \\ & \cos(33)\pi + \frac{1}{2}(2 + 3 \times 0.00606683^2) 0.001 \cos(33)\pi = \\ & 0.0005\pi \cosh(-33i)(2 + 3 \times 0.00606683^2) + 0.001\pi \cosh(-33i) \\ & \left( 6 - 2 \log \left( 1 + \frac{1}{0.00606683^2} \right) \right) (2 + 3 \times 0.00606683^2) + \frac{1}{0.00606683^2} + \\ & 0.001 \left( -2 \log \left( 1 + \frac{1}{0.00606683^2} \right) + \frac{2}{0.00606683^2} \right) \end{aligned}$$

$$\begin{aligned} & 0.001 \left( \frac{2}{0.00606683^2} - 2 \log \left( \frac{1}{0.00606683^2} + 1 \right) \right) + \\ & 2 \times 0.0005 \left( 6 + \frac{1}{0.00606683^2} - 2(2 + 3 \times 0.00606683^2) \log \left( 1 + \frac{1}{0.00606683^2} \right) \right) \\ & \cos(33)\pi + \frac{1}{2}(2 + 3 \times 0.00606683^2) 0.001 \cos(33)\pi = \\ & 0.0005\pi \cosh(33i)(2 + 3 \times 0.00606683^2) + 0.001\pi \cosh(33i) \\ & \left( 6 - 2 \log(a) \log_a \left( 1 + \frac{1}{0.00606683^2} \right) \right) (2 + 3 \times 0.00606683^2) + \frac{1}{0.00606683^2} + \\ & 0.001 \left( -2 \log(a) \log_a \left( 1 + \frac{1}{0.00606683^2} \right) + \frac{2}{0.00606683^2} \right) \end{aligned}$$

$$\begin{aligned}
& 0.001 \left( \frac{2}{0.00606683^2} - 2 \log \left( \frac{1}{0.00606683^2} + 1 \right) \right) + \\
& 2 \times 0.0005 \left( 6 + \frac{1}{0.00606683^2} - 2(2 + 3 \times 0.00606683^2) \log \left( 1 + \frac{1}{0.00606683^2} \right) \right) \\
& \cos(33)\pi + \frac{1}{2}(2 + 3 \times 0.00606683^2) 0.001 \cos(33)\pi = \\
& 0.0005\pi \cosh(33i)(2 + 3 \times 0.00606683^2) + 0.001\pi \cosh(33i) \\
& \left( 6 - 2 \log_e \left( 1 + \frac{1}{0.00606683^2} \right) (2 + 3 \times 0.00606683^2) + \frac{1}{0.00606683^2} \right) + \\
& 0.001 \left( -2 \log_e \left( 1 + \frac{1}{0.00606683^2} \right) + \frac{2}{0.00606683^2} \right)
\end{aligned}$$

$\cosh(x)$  is the hyperbolic cosine function

$\log_b(x)$  is the base- $b$  logarithm

$i$  is the imaginary unit

### Series representations:

$$\begin{aligned}
& 0.001 \left( \frac{2}{0.00606683^2} - 2 \log \left( \frac{1}{0.00606683^2} + 1 \right) \right) + \\
& 2 \times 0.0005 \left( 6 + \frac{1}{0.00606683^2} - 2(2 + 3 \times 0.00606683^2) \log \left( 1 + \frac{1}{0.00606683^2} \right) \right) \\
& \cos(33)\pi + \frac{1}{2}(2 + 3 \times 0.00606683^2) 0.001 \cos(33)\pi = \\
& 54.3383 - 0.002 \log(27170.1) + \sum_{k=0}^{\infty} \frac{(-1089)^k \pi (27.1761 - 0.00400022 \log(27170.1))}{(2k)!}
\end{aligned}$$

$$\begin{aligned}
& 0.001 \left( \frac{2}{0.00606683^2} - 2 \log \left( \frac{1}{0.00606683^2} + 1 \right) \right) + 2 \times 0.0005 \\
& \left( 6 + \frac{1}{0.00606683^2} - 2(2 + 3 \times 0.00606683^2) \log \left( 1 + \frac{1}{0.00606683^2} \right) \right) \cos(33)\pi + \\
& \frac{1}{2}(2 + 3 \times 0.00606683^2) 0.001 \cos(33)\pi = 54.3383 - 0.002 \log(27170.1) + \\
& \pi (27.1761 - 0.00400022 \log(27170.1)) \sum_{k=0}^{\infty} \frac{\cos \left( \frac{k\pi}{2} + z_0 \right) (33 - z_0)^k}{k!}
\end{aligned}$$

$$\begin{aligned}
& 0.001 \left( \frac{2}{0.00606683^2} - 2 \log \left( \frac{1}{0.00606683^2} + 1 \right) \right) + 2 \times 0.0005 \\
& \left( 6 + \frac{1}{0.00606683^2} - 2(2 + 3 \times 0.00606683^2) \log \left( 1 + \frac{1}{0.00606683^2} \right) \right) \cos(33)\pi + \\
& \frac{1}{2}(2 + 3 \times 0.00606683^2) 0.001 \cos(33)\pi = 54.3383 + 27.1761\pi J_0(33) - \\
& 0.002 \log(27170.1) - 0.00400022\pi J_0(33) \log(27170.1) + \\
& \sum_{k=1}^{\infty} (-1)^k \pi J_{2k}(33) (54.3523 - 0.00800044 \log(27170.1))
\end{aligned}$$

$n!$  is the factorial function

$J_n(z)$  is the Bessel function of the first kind

### Integral representations:

$$\begin{aligned}
& 0.001 \left( \frac{2}{0.00606683^2} - 2 \log \left( \frac{1}{0.00606683^2} + 1 \right) \right) + \\
& 2 \times 0.0005 \left( 6 + \frac{1}{0.00606683^2} - 2(2 + 3 \times 0.00606683^2) \log \left( 1 + \frac{1}{0.00606683^2} \right) \right) \\
& \cos(33)\pi + \frac{1}{2}(2 + 3 \times 0.00606683^2) 0.001 \cos(33)\pi = 54.3383 + \\
& \int_{\frac{\pi}{2}}^{33} \pi (-27.1761 + 0.00400022 \log(27170.1)) \sin(t) dt - 0.002 \log(27170.1)
\end{aligned}$$

$$\begin{aligned}
& 0.001 \left( \frac{2}{0.00606683^2} - 2 \log \left( \frac{1}{0.00606683^2} + 1 \right) \right) + \\
& 2 \times 0.0005 \left( 6 + \frac{1}{0.00606683^2} - 2(2 + 3 \times 0.00606683^2) \log \left( 1 + \frac{1}{0.00606683^2} \right) \right) \\
& \cos(33)\pi + \frac{1}{2}(2 + 3 \times 0.00606683^2) 0.001 \cos(33)\pi = \\
& 54.3383 + 27.1761\pi + \int_0^1 \pi (-896.813 + 0.132007 \log(27170.1)) \sin(33t) dt - \\
& 0.002 \log(27170.1) - 0.00400022\pi \log(27170.1)
\end{aligned}$$

$$\begin{aligned}
& 0.001 \left( \frac{2}{0.00606683^2} - 2 \log \left( \frac{1}{0.00606683^2} + 1 \right) \right) + \\
& 2 \times 0.0005 \left( 6 + \frac{1}{0.00606683^2} - 2(2 + 3 \times 0.00606683^2) \log \left( 1 + \frac{1}{0.00606683^2} \right) \right) \\
& \cos(33)\pi + \frac{1}{2}(2 + 3 \times 0.00606683^2) 0.001 \cos(33)\pi = \\
& 0.132007 \left( 411.631 + 205.869\pi + 7.57534 \int_1^{27170.1} \frac{-0.002 - 0.00400022\pi - 0.0330085\pi t \sin(-0.00121461 + 0.00121461t)}{t} dt + \right. \\
& \left. 0.0000368065 \int_0^1 \int_0^1 \frac{\sin(33t_2)}{0.0000368065 + t_1} dt_2 dt_1 \right)
\end{aligned}$$

Multiplying the two results, we obtain:

$$(-2207+199+89)+12*(53.1860546717078120379056 * -1.490502255050)^2$$

Where 2207 and 199 are Lucas numbers, while 89 is a Fibonacci number

### Input interpretation:

$$(-2207 + 199 + 89) + 12 (53.1860546717078120379056 \times (-1.490502255050))^2$$

### Result:

$$73493.28015137826735292028933722943158470896857611755023431\dots$$

$$73493.28015\dots$$

Thence, we have the following mathematical connection:

$$\begin{aligned}
& -2207+199+89)+12 \left( \times -a_0 \left[ \frac{2}{\rho^2} - 2 \log \left( \frac{1}{\rho^2} + 1 \right) \right] + 2a_1 \left[ 6 + \frac{1}{\rho^2} - 2(2 + 3\rho^2) \log \left( 1 + \frac{1}{\rho^2} \right) \right] \cos \theta \right. \\
& \left. + \frac{b_1}{2} (2 + 3\rho^2) \cos \theta. \right)
\end{aligned}$$

$$= 73493.28015\dots \Rightarrow$$

$$\Rightarrow -3927 + 2 \left( \sqrt{^{13} \left( N \exp \left[ \int d\hat{\sigma} \left( -\frac{1}{4u^2} \mathbf{P}_i D \mathbf{P}_i \right) \right] |Bp\rangle_{NS} + \int [dX^\mu] \exp \left\{ \int d\hat{\sigma} \left( -\frac{1}{4v^2} D X^\mu D^2 X^\mu \right) \right\} |X^\mu, X^i = 0\rangle_{NS}} \right)}$$

= 73490.8437525....

From the simple multiplication of the two results, we obtain:

$$(53.1860546717078120379056 * -1.490502255050)$$

**Input interpretation:**

$$53.1860546717078120379056 \times (-1.490502255050)$$

**Result:**

$$-79.27393442539308127719983287902328$$

$$-79.273934425\dots$$

And from the following algebraic sum:

$$-(-53.1860546717078120379056 - 1.490502255050)$$

**Input interpretation:**

$$-(-53.1860546717078120379056 - 1.490502255050)$$

**Result:**

$$54.6765569267578120379056$$

54.6765569... near to the Fibonacci number 55

And:

$$-7/10^3 + (-53.1860546717078120379056 * -1.490502255050))^{1/9}$$

where 7 is a Lucas number

**Input interpretation:**

$$-\frac{7}{10^3} + \sqrt[9]{-(53.1860546717078120379056 \times (-1.490502255050)))}$$

**Result:**

1.6186029934945...

1.61860299...

This result is a very good approximation to the value of the golden ratio  
1,618033988749...

### Fig.1 – Ramanujan mock theta functions (values)

Now, we have also these other results concerning the Ramanujan mock theta functions:

$$R = -1.08185; \quad R = 1.08753454; \quad R = 1.08094974; \quad \text{SUM} = R_a = 1.08663428$$

$$R_b = -4267.24; \quad R_c = 6.5960861587 * 10^{20}$$

$$R_d = -1.0058343895 * 10^{-12}; \quad R_e = -5.74968 * 10^{-40}; \quad R_f = -4.9290621621 * 10^6;$$

$$R_g = 4.04237000433962 * 10^{14}; \quad R_h = 3.0773505768788923 * 10^{13};$$

$$R_i = -0.0818160338; \quad R_l = -2498.279529; \quad R_m = -0.07609064; \quad R_n = 0.923910279;$$

$$R_o = 33021.1005; \quad R_p = -2122.1867; \quad R_q = 1.63161 * 10^{20}; \quad R_r = 9.39267 * 10^{17};$$

$$R_s = -0.0814135...; \quad R_t = -1.0061571663...; \quad R_u = 0.924340867458.$$

### Table 1

From:

### Three-dimensional AdS gravity and extremal CFTs at

**c = 8m** - Spyros D. Avramis, Alex Kehagias and Constantina Mattheopoulou

Received: September 7, 2007 - Accepted: October 28, 2007 - Published: November 9, 2007

$m$	$L_0$	$d$	$S$	$S_{BH}$	$m$	$L_0$	$d$	$S$	$S_{BH}$
3	1	196883	12.1904	12.5664	6	1	42987519	17.5764	17.7715
	2	21296876	16.8741	17.7715		2	40448921875	24.4233	25.1327
	3	842609326	20.5520	21.7656		3	8463511703277	29.7668	30.7812
4	2/3	139503	11.8458	11.8477	7	2/3	7402775	15.8174	15.6730
	5/3	69193488	18.0524	18.7328		5/3	33934039437	24.2477	24.7812
	8/3	6928824200	22.6589	23.6954		8/3	16953652012291	30.4615	31.3460
5	1/3	20619	9.9340	9.3664	8	1/3	278511	12.5372	11.8477
	4/3	86645620	18.2773	18.7328		4/3	13996384631	23.3621	23.6954
	7/3	24157197490	23.9078	24.7812		7/3	19400406113385	30.5963	31.3460

**Table 1:** Degeneracies, microscopic entropies and semiclassical entropies for the first few values of  $m$  and  $L_0$ .

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