Variable Polytropic gas cosmology

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Abstract

We mainly study a cosmological scenario represented by the variable Polytropic gas (VPG) unified energy density proposal. To reach this aim, we start with reconstructing a variable form of the original Polytropic gas (OPG) definition. We show that this model is a generalization of the OPG, cosmological constant plus cold dark matter (ΛCDM) and two different Chaplygin gas models. Later, we fit the auxiliary parameters given in the model and discuss essential cosmological features of the VPG proposal. Besides, we compare the VPG with the OPG by focusing on recent observational dataset given in literature including Planck 2018 results. We see that the VPG model yields better results than the OPG description and it fits very well with the recent experimental data. Moreover, we discuss some thermodynamical features of the VPG and conclude that the model describes a thermodynamically stable system.

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I. INTRODUCTION

There are plenty of astrophysical data that signal us to a universe entered in a speedy expansion phase[1–10]. In order to understand the accelerated expansion phase of our universe, a mysterious type of energy yclept dark energy is to be needed. The dark energy dominates approximately 68.3% of the space-time tissue. Assumption of a dominant component which resembles familiar forms of matter or energy has not been justified yet, because the exotic dark content cannot be detected directly. Diverse proposals have been introduced in literature to express the speedy expansion behavior of our universe. The earliest and simplest description for the mysterious type of energy mentioned above is the cosmological constant[11]. Subsequently, different ideas to identify the dark energy have been introduced: braneworld models[12, 13], scalar fields[14–18], modified gravity theories[19, 20], assuming extra dimensions[21–23] and so on. For a convenient brief about theoretical dark energy models, one can read Ref.[24] and references therein.

An energy-momentum tensor must be defined on the right-hand side of a gravitational field equation when the compactification is due to matter fields. In order to add dark content's contributions to the field equation, various new descriptions, like ghost dark energy[25–28], holographic energy density[29, 30], Hobbit model[31], new agegraphic dark energy[32], Chaplygin gas[33, 34], Polytropic gas[35, 36], etc., have been introduced in literature. Among these formulations, Hobbit[31], Chaplygin gas[33, 34] and Polytropic gas[35, 36] models have a significant feature which leads interesting cosmological conclusions. These three proposals unify different fluids of the standard cosmological model and can express both dark matter and dark energy with a single fluid (consequently automatically removing the coincidence issue).

Constraining the auxiliary parameters given a theoretical model is one of the significant tasks in contemporary theoretical cosmology. The most often considered technique is analyzing the luminosity distance measurements for a specific family of objects[37–40]. In some recent research papers, a new way including observational values of the Hubble parameter (OHV) has been considered to check some cosmological tests[41–46].

In this study, we mainly demonstrate the reliability of the VPG model and show that the model gives better results than other models such as the OPG and the ΛCDM. On this purpose, we (i) define a variable form of the OPG unified dark energy model in the first step, (ii) perform an original fitting analysis including most recent observational dataset in order to determine best values of the auxiliary parameters given in the model, (iii) study the OHV data analysis technique, which is new for both of the VPG and VGCG models, (iv) give a statistical analysis, which is completely new for both of the VPG and VGCG proposals, including the correlation parameter, (v) analyze our results by plotting original graphics, (vi) discuss the theoretical results thermodynamically. The layout of the paper is as follows. In the next section, we give some preliminary relations and definitions. Next, in the third section, we introduce the VPG model with discussions of cosmological parameters. In the fourth section of the paper, we fit the free parameters of the model by considering the recent astrophysical observations. In the fifth section, we test the VPG model and compare it with the OPG proposal. In the sixth section, we investigate thermodynamical features and discuss the stability of the VPG model. The final section is devoted to the closing remarks. All numerical calculations and analyzes are performed by using MATHEMATICA sofware[47].

II. PRELIMINARIES: COSMOLOGICAL SCENARIO

Here, as a first step, we assume the Friedmann-Robertson-Walker (FRW) universe is filled with the VPG and the baryonic matter and its line-element is written as

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{1}$$

where the time-dependent function $a(t)$ indicates the cosmic scale factor while k implies the curvature parameter for the flat ($k=0$), closed ($k=-1$) and open ($k=+1$) universe types. We also suppose that the FRW universe is filled with perfect fluid which is defined by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - g_{\mu\nu}p. \tag{2}$$

Here, $\rho = \rho_{vpg} + \rho_{bm}$ and $p = p_{vpg} + p_{bm}$ are total energy density and pressure, respectively. The subscripts vpg and bm denote the VPG and the baryonic matter, respectively. It is important to mention here that the VPG is a unification of the dark matter and dark energy. Thus, we can write $\rho_{vpg} = \rho_{dm} + \rho_{de}$ and $p_{vpg} = p_{de}$. Note that, now, the subscripts dm and de mean the dark matter and the dark energy, respectively. Moreover, u_μ is the four-velocity vector and we have $u^\mu u_\mu = 1$.

The Einstein field equations can be written in the following form

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \left[(\rho + p) u_\mu u_\nu - g_{\mu\nu} p \right] \tag{3}$$

where $R_{\mu\nu}$, $g_{\mu\nu}$, R and G show the Ricci tensor, metric tensor, Ricci scalar and the gravitational constant, respectively.

The recent observational data obtained by SNe-Ia[1], WMAP[4–6], SDSS[7], X-ray[48] and Planck-results[8–10] have strongly suggested that the geometry of the universe is spatially flat. From this point of view, we take $k = 0$ in the further calculations. Hence, making use of equations (1) and (3), it follows that

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho, \tag{4}$$

where H is the Hubble expansion parameter.

Next, the continuity equation, i.e. $T^{\mu\nu}_{;\nu} = 0$, yields

$$\dot{\rho}_{bm} + 3H\rho_{bm} = 0, \tag{5}$$

$$\dot{\rho}_{vpg} + 3H(\rho_{vpg} + p_{vpg}) = 0. \tag{6}$$

Additionally, we can also write

$$\dot{\rho}_{dm} + 3H\rho_{dm} = 0, \tag{7}$$

$$\dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = 0, \tag{8}$$

with the fact that equation-of-state (EoS) parameters of the baryonic matter and the dark matter are taken as $\omega_{bm} = \omega_{dm} = 0$. From equations (5) and (7), one can easliy obtain that

$$\rho_{bm} = \rho_0^b a^{-3}, \tag{9}$$

$$\rho_{dm} = \rho_0^m a^{-3}, \tag{10}$$

where ρ_0^b and ρ_0^m represent present values of the baryonic matter and the dark matter, respectively.

In addition to the above calculations, introducing dimensionless density parameters helps us to rewrite the Friedmann equation (4) in a very useful and elegant form. On this purpose, firstly, we define

$$\Omega_{bm} = \frac{8\pi G}{3H^2} \rho_{bm}, \qquad \Omega_{vpg} = \frac{8\pi G}{3H^2} \rho_{vpg}, \tag{11}$$

$$\Omega_{dm}=\frac{8\pi G}{3H^2}\rho_{dm},\qquad \Omega_{de}=\frac{8\pi G}{3H^2}\rho_{de}. \tag{12}$$

Subsequently, in the second step, it can be concluded that

$$\sum_{i=bm,dm,de}\Omega_i\equiv 1, \tag{13}$$

where $\Omega_i\equiv(\Omega_{bm},\Omega_{dm},\Omega_{de})$ and $\Omega_{vpg}=\Omega_{dm}+\Omega_{de}$.

It is known that we can fix the free parameters given in the model according to the current cosmological measurements by using the red shift parameter in corresponding calculations. The red shift parameter z is related to the cosmic scale factor $a(t)$ by

$$z+1=\frac{1}{a}, \tag{14}$$

with $a_0=1$ which is the present value of the cosmic scale factor. So, we get

$$\rho_{bm}=\rho_0^b(1+z)^3, \tag{15}$$

$$\rho_{dm}=\rho_0^m(1+z)^3. \tag{16}$$

III. THE VPG PROPOSAL

The OPG model is described by the following EoS[35, 36]

$$p_{pg}=\beta\rho_{pg}^{1+\frac{1}{\xi}}, \tag{17}$$

where both β and the polytropic index ξ denote real constants. Here, we define another form of the PG model in order to check whether we can get better conclusions. So, we assume

$$p_{vpg}=\beta a^{-n}\rho_{vpg}^{1+\frac{1}{\xi}}, \tag{18}$$

where n is a constant, then, it can be seen that we have three free parameters. It is significant to mention here that this new proposal can be reduced some of other unified dark matter-energy models. In the case $n=0$, the above relation recovers the OPG model[35, 36]. Assuming $n=0$, $\beta=-\kappa$ and $\xi=-\frac{1}{2}$ reduces the above expression into the form of the original Chaplygin (OCG)[33, 34] which is given by $p_{ocg}=-\frac{\kappa}{\rho_{ocg}}$. Next, considering the limiting case including $n=0$, $\beta=-\kappa$ and $\xi=-\frac{1}{1+\alpha}$ transforms the expression of the VPG model into the form of the generalized Chaplygin (GCG)[49–51] model, i.e. $p_{gcg}=-\frac{\kappa}{\rho_{gcg}^{\alpha}}$.

On the other hand, the case including $n \neq 0$, $\xi = -\frac{1}{1+\alpha}$ and $\beta = -\kappa$ yields the variable generalized Chaplygin gas (VGCG)[52, 53] model, i.e. $p_{vgcg} = -\frac{\kappa a^{-n}}{\rho_{vgcg}^{\alpha}}$. Although the variable form of the PG model, namely the VPG, is new in literature, it should be underlined here that the VPG and the VGCG models are two equivalent perspectives of the same thing:

$$\beta a^{-n} \rho_{vpg}^{1+\frac{1}{\xi}} \Longleftrightarrow \frac{-\kappa a^{-n}}{\rho_{vgcg}^{\alpha}}. \tag{19}$$

Now, we can focus on the equation (6) to find an exact expression for the corresponding energy density. We can rewrite the equation (6) in a more convenient form as given below

$$d(\rho_{vpg} a^3) + p_{vpg} d(a^3) = 0. \tag{20}$$

Consequently, using the above equation, we can derive the energy density of the VPG model:

$$\rho_{vpg} = \rho_0^p \left[\Delta(1+z)^n + (1-\Delta)(1+z)^{-\frac{3}{\xi}}\right]^{-\xi}, \tag{21}$$

where ρ_0^p indicates the present value of VPG energy density and Δ is defined as

$$\Delta = -\frac{\beta}{1+\frac{n\xi}{3}} \sqrt[\xi]{\rho_0^p}. \tag{22}$$

For an expanding spacetime model, it must be $n > 0$ and $\xi < 0$ or the vice versa. Otherwise, $a \to \infty$ yields $\rho_{vpg} \to \infty$ which cannot define an expanding spacetime. Now, in order to investigate the evolution propensity of the dark matter and the dark energy and study cosmological features of the dark energy, one can consider the decomposition of the VPG fluid: $\rho_{vpg} = \rho_{dm} + \rho_{de}$ and $p_{vpg} = p_{de}$. Then, using equations (15), (16) and (21), we get

$$\begin{aligned} \rho_{de} &= \rho_{vpg} - \rho_{dm} \\ &= \rho_0^p \left[\Delta(1+z)^n + (1-\Delta)(1+z)^{-\frac{3}{\xi}}\right]^{-\xi} - \rho_0^m (1+z)^3. \end{aligned} \tag{23}$$

Next, making use of equations (4), (11), (12), (15), (16) and (21), we obtain the following result

$$H^2 = H_0^2 \left\{\Omega_{vpg}^0 \left[\Delta(1+z)^n + (1-\Delta)(1+z)^{-\frac{3}{\xi}}\right]^{-\xi} + \Omega_{bm}^0 (1+z)^3\right\}, \tag{24}$$

where H_0 denotes the present value of the Hubble parameter and

$$\Omega_{vpg}^0 + \Omega_{bm}^0 = 1, \tag{25}$$

with

$$\Omega_{vpg}^0 = \frac{8\pi G}{3H_0^2} \rho_{vpg}, \tag{26}$$

and

$$\Omega^0_{bm} = \frac{8\pi G}{3H_0^2}\rho_{bm}. \tag{27}$$

On the other hand, with the help of equations (18) and (21), the EoS parameter of the VPG is calculated as

$$\omega \approx -1 - \frac{n\xi}{3}. \tag{28}$$

Here, it seems that the auxiliary parameters n and ξ have prominent influences. Depending on signatures of these parameters, the above result indicates three different cases for the EoS parameter. One can conclude that the universe tends to be (i) phantom dominated[17], i.e. $\omega < -1$ when $n\xi < -3$ except for $n\xi = 0$, (ii) quintessence dominated[54, 55], i.e. $\omega > -1$ for $n\xi < -3$, (iii) ΛCDM dominated (which is a parametrization of the Big Bang idea), i.e. $\omega = -1$ for $n\xi = 0$. We should emphasize here that the choice $n = 0$ represents not only the ΛCDM model but also the OPG proposal.

IV. FITTING THE MODEL PARAMETERS

In this section, on the basis of the result (21), we consider the recent observational datasets such as the Supernova type Ia (SN Ia) sample, the OHV and the baryon acoustic oscillations (BAO) data to constrain the VPG proposal and investigate the evolutionary behavior of our universe.

A. Analysis methods

1. SN Ia data

Here, we consider the observational SN Ia dataset which consists information about the luminosity distance. The Hubble-free definition of the luminosity parameter is written as

$$d_L = (1+z)\int_0^z \frac{H_0 dz'}{H(z')}. \tag{29}$$

Additionally, for the SN Ia dataset, χ^2 function is given by[56]

$$\chi^2_{SN} = \sum_i^{580} \frac{[\mu_{obs}(z_i) - \mu_{theo}(z_i)]^2}{\sigma_i^2}, \tag{30}$$

where the theoretical distance modulus is written as

$$\mu_{theo} = 5\log_{10} d_L(z_i) + \mu_0, \tag{31}$$

with $\mu_0 = 42.38 - 5\log_{10} h$. Note that $\mu_{obs}(z_i)$, σ_i and $h = H_0/100/[kmsec^{-1}Mpc^{-1}]$ show the observed distance modulus, the uncertainty in the distance modulus and the then-favored dimensionless Hubble parameter, respectively. For the minimization of χ^2_{SN} with respect to μ_0 for 580 data points of the SN Ia measurements[57], we get[58]

$$\tilde{\chi}^2_{SN} = P - \frac{Q^2}{R}, \tag{32}$$

where

$$P = \sum_i^{580} \frac{[\mu_{obs}(z_i) - \mu_{theo}(z_i; \mu_0 = 0)]^2}{\sigma_i^2}, \tag{33}$$

$$Q = \sum_i^{580} \frac{[\mu_{obs}(z_i) - \mu_{theo}(z_i; \mu_0 = 0)]}{\sigma_i^2}, \tag{34}$$

$$R = \sum_i^{580} \frac{1}{\sigma_i^2}. \tag{35}$$

2. OHV

Considering some observed[59–72] $H(z)$ values given in TABLE I, we can investigate the validity of the constraints on the auxiliary parameters given in the VPG definition. One can minimize χ^2_{OHD} which can be defined as

$$\chi^2_{OHV} = \sum_i^{27} \frac{[H_{obs}(z_i) - H_{theo}(z_i)]^2}{\sigma_i^2} \tag{36}$$

where H_{obs} and H_{theo} describe observational and theoretical values of the cosmic Hubble parameter, respectively.

3. BAO data

It is known that our universe includes a fraction of baryon, so the acoustic oscillations in the relativistic plasma may be stamped on the late-time power spectrum of the non-relativistic matter[53, 73]. We can minimize the χ^2_{BAO} which is written as

$$\chi^2_{BAO} = \frac{[\Gamma(\theta) - \Gamma_{obs}]^2}{\sigma_A^2}, \tag{37}$$

TABLE I: The recent observable $H(z)$ dataset.

z	H_{obs}	σ	Ref.	z	H_{obs}	σ	Ref.
0.0708	69.00	$\mp$19.68	[60]	0.5700	92.40	$\mp$4.500	[68]
0.1200	68.60	$\mp$26.20	[60]	0.5930	104.0	$\mp$13.00	[62]
0.1700	83.00	$\mp$8.000	[61]	0.6800	92.00	$\mp$8.000	[62]
0.1990	75.00	$\mp$5.000	[62]	0.7300	97.30	$\mp$7.000	[69]
0.2400	79.69	$\mp$2.650	[63]	0.7810	105.0	$\mp$12.00	[62]
0.2800	88.80	$\mp$36.60	[60]	0.8750	125.0	$\mp$17.50	[62]
0.3500	84.40	$\mp$7.000	[64]	0.9000	117.0	$\mp$23.00	[61]
0.3802	83.00	$\mp$13.50	[62]	1.3000	168.0	$\mp$17.00	[70]
0.4000	95.00	$\mp$17.00	[61]	1.4300	177.0	$\mp$18.00	[70]
0.4247	87.10	$\mp$11.20	[65]	1.5300	140.0	$\mp$14.00	[61]
0.4300	86.45	$\mp$3.680	[63]	1.7500	202.0	$\mp$40.00	[71]
0.4497	92.80	$\mp$12.90	[65]	1.9650	186.5	$\mp$50.40	[62]
0.4783	80.90	$\mp$9.000	[65]	2.3400	222.0	$\mp$7.000	[72]
0.4800	97.00	$\mp$62.00	[66, 67]				

where

$$\Gamma(\theta) = \frac{\sqrt{\Omega_{dm}^0}}{\sqrt[3]{E(z_{BAO})}} \left\{ \frac{1}{z_{BAO}} \int_0^z \frac{dz'}{E(z';\theta)} \right\}^{\frac{2}{3}}, \tag{38}$$

with $E(z) = \frac{H(z)}{H_0}$. Remember that Ω_{dm}^0 is not explicitly included in the VPG proposal. According to the Planck results[9, 10], here we take $H_0 = 67.4 \pm 0.5 km sec^{-1} Mpc^{-1}$ and $\Omega_{dm}^0 = 0.278$. Next, in the 1σ confidence region, it was measured that $\Gamma_{obs} = 0.469(\frac{0.98}{n_s})^{0.35} \pm 0.017$ from the SDSS observations at $z_{BAO} = 0.35$. Here, $n_s = 0.96$ indicates the scalar spectral index[74].

B. Constraints on the free parameters

We focus on a combined constraint on the VPG model. So, we have

$$\chi^2 = \tilde{\chi}_{SN}^2 + \chi_{OHV}^2 + \chi_{BAO}^2 \tag{39}$$

where $\tilde{\chi}^2_{SN}$, χ^2_{OHV} and χ^2_{BAO} are defined by equations (32), (36) and (37). From this point of view, making us of the observational datasets, one can calculate the best fit values of auxiliary parameters:

$$(\beta, n, \xi) = (-0.031, -1.869, 2.990) \tag{40}$$

with

$$\chi^2_{min} = 324.691. \tag{41}$$

In FIGs. 1 and 2, we depict confidence contours on $\beta - n$, $\xi - n$ and $\beta - \xi$ parameter spaces for SN Ia+OHV+BAO datasets.

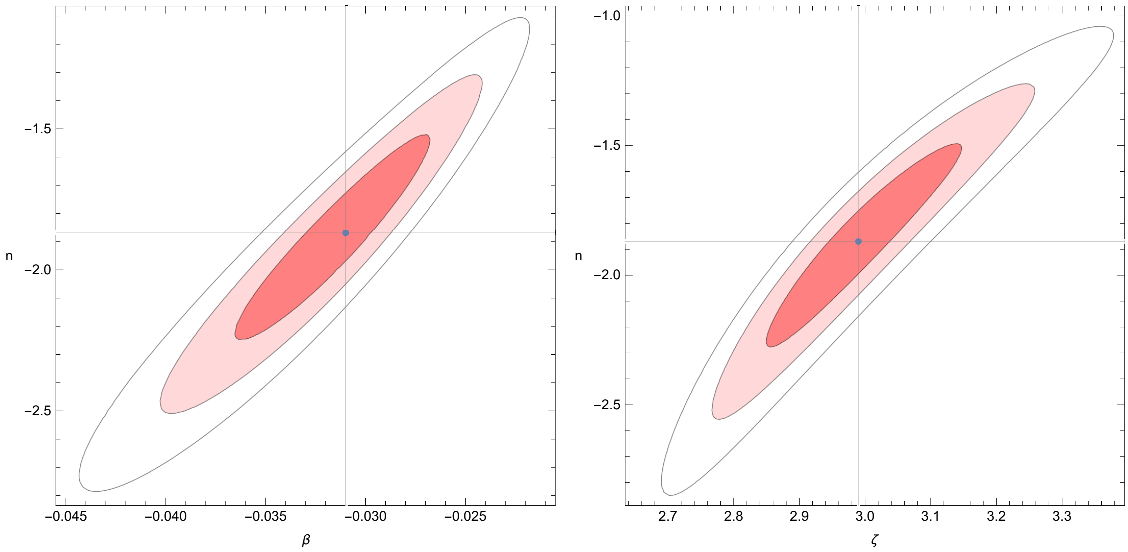

FIG. 1: Plots of 1σ (pink), 2σ (light pink) and 3σ (white) confidence contours on $\beta - n$ and $\xi - n$ parameter spaces for SN Ia+OHV+BAO datasets in the VPG dark energy description.

V. TESTING THE MODEL

FIGs. 3 and 4 illustrate the evolutionary nature of the cosmic Hubble parameter according to the OPG and the VPG models in the 1σ confidence region, respectively. Note that, in FIGs. 3 and 4, the circles show the recent observable values.

In order to check correlation between H_{obs} and H_{theo} datasets, we can calculate the corresponding correlation coefficient r which is used in statistics to measure how strong a

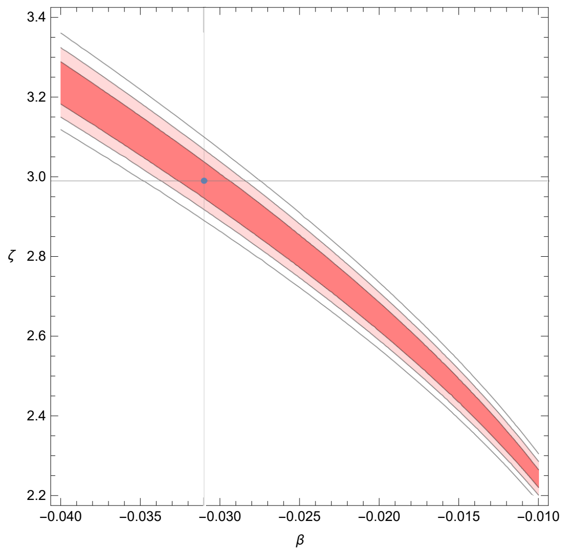

FIG. 2: Plots of 1σ (pink), 2σ (light pink) and 3σ (white) confidence contours on $\beta - \xi$ parameter space for SN Ia+OHV+BAO datasets in the VPG dark energy description.

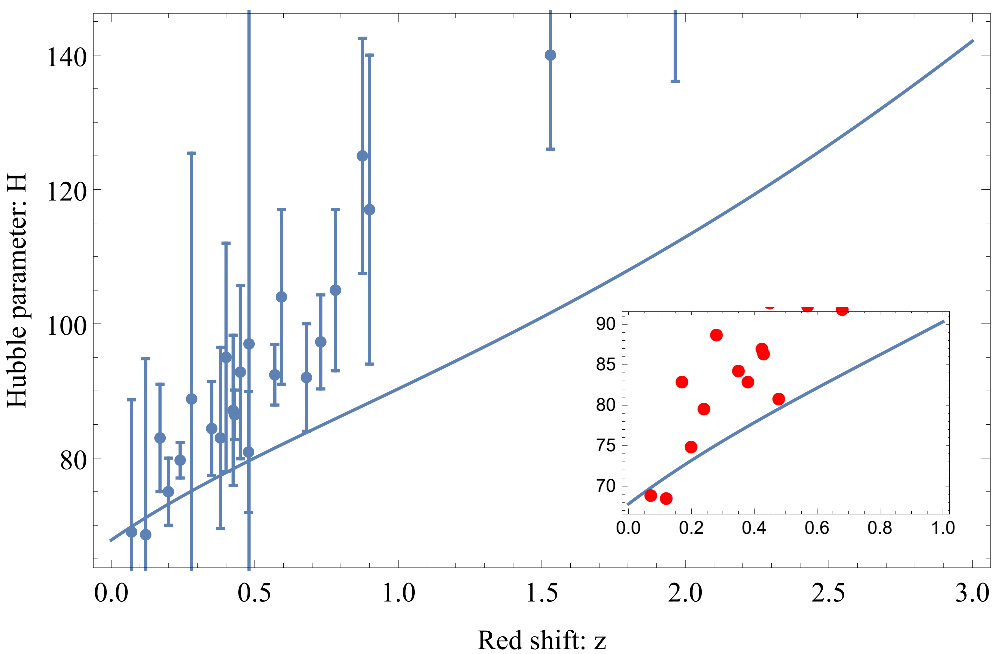

FIG. 3: $H \sim z$ relation for SN Ia+OHV+BAO datasets in the OPG dark energy model.

relationship is between two different variables. Thus, we focus on the following relations:

$$S_x = \sqrt{\frac{\Sigma_{n=1}^{27}\left(H_{obs}(z_i) - \overline{H}_{obs}\right)^2}{n-1}}, \tag{42}$$

$$S_y = \sqrt{\frac{\Sigma_{n=1}^{27}\left(H_{theo}(z_i) - \overline{H}_{theo}\right)^2}{n-1}}, \tag{43}$$

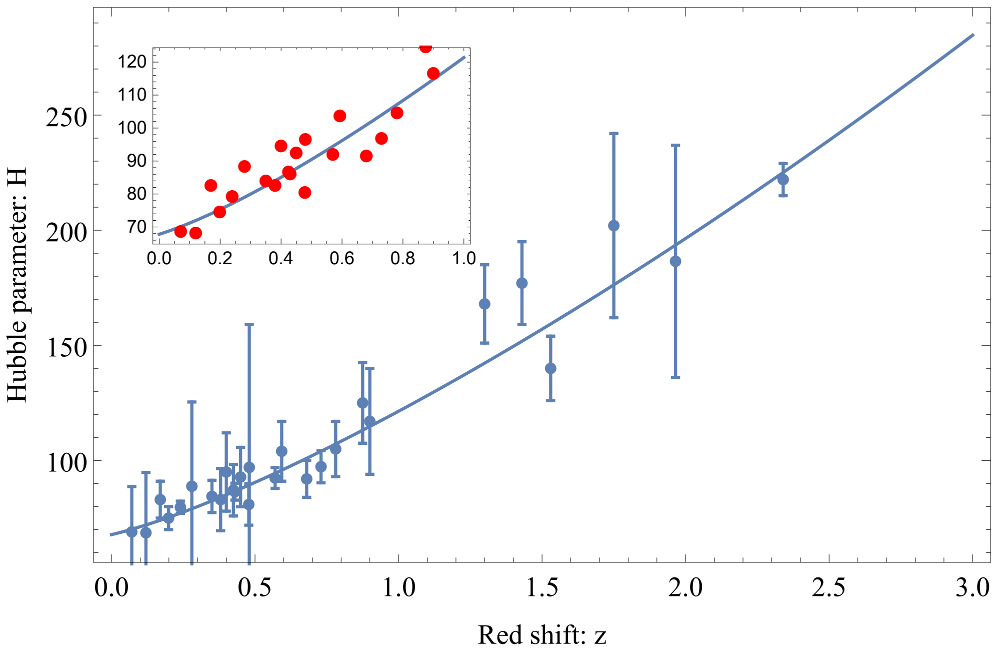

FIG. 4: $H \sim z$ relation for SN Ia+OHV+BAO datasets in the VPG proposal.

where $\overline{H}_{obs}$ and $\overline{H}_{theo}$ indicate mean values of observational and theoretical $H(z)$ datasets, respectively. Making use of the above relations, one can define the correlation coefficient as given below

$$r = \frac{\Sigma_{n=1}^{27}\left(H_{obs}(z_i) - \overline{H}_{obs}\right)\left(H_{theo}(z_i) - \overline{H}_{theo}\right)}{(n-1)S_x S_y}. \tag{44}$$

Note that the correlation coefficient should always take values between -1 and $+1$. The case $r = +1$ ($r = -1$) means the points are on a perfect straight line with positive (negative) slope. The case of zero implies no relationship at all. Absolute value of the correlation coefficient, i.e. $|r|$, indicates the relationship strength. It is known that the larger the absolute value of correlation coefficient, the stronger the linear relationship. Considering numerical values given in TABLE I, we find that $r = 0.968769$ which means there is a strong positive relationship between H_{obs} and H_{theo}.

VI. THERMODYNAMICS OF THE VPG

Here, we consider some thermodynamical relations in order to check thermodynamical features of the model. We start with the energy density relation $\rho = \frac{U}{V}$ where U indicates the internal energy and $V = \frac{4}{3}\pi r_h^3$ with the dynamical apparent horizon $r_h = \frac{1}{\sqrt{H^2 + k(1+z)^2}}$

denotes the volume of the system. Remember that it should be taken $k = 0$ due to the recent astrophysical observations[1, 4–10, 48]. For the flat FRW universe, we can write $r_h = \frac{1}{H}$ yclept the dynamical Hubble horizon. In this section, we also focus on another important relation defining the entropy, i.e. $S = \frac{A}{4G}$ where $A = 4\pi r_h^2$ is the surface area.

Consequently, making use of the equation (24) defining the theoretical Hubble parameter, we get

$$\begin{aligned} S &= \frac{\pi}{GH_0}\left[\Omega^0_{vpg}\left[\Delta(1+z)^n + (1-\Delta)(1+z)^{-\frac{3}{\xi}}\right]^{-\xi} + \Omega^0_{bm}(1+z)^3\right]^{-2} \\ &= \frac{\pi^{\frac{1}{3}}}{G}\left(\frac{3V}{4}\right)^{\frac{2}{3}}. \end{aligned} \tag{45}$$

It is clearly seen from the above result that the entropy is an incensing function of volume which means it is also an increasing function of time. Therefore, the second thermodynamical law is valid as it is expected.

Next, we can also check the thermodynamical stability of the VPG model. In order to reach this aim, one can check the validity of $T\frac{\partial S}{\partial T} > 0$. As a matter of fact, this case leads to the relation of heat capacity which can be rewritten in terms of volume[75]:

$$C = V\frac{\partial \rho}{\partial V}\left[\frac{\partial T}{\partial V}\right]^{-1}. \tag{46}$$

Hence, for the VPG proposal, one can calculate that

$$C = \frac{6\pi V}{3^{\frac{1}{3}}(8\pi)^{\frac{2}{3}}G}\left[\frac{\sqrt{V}}{2} + \beta a^{-n}\left(\frac{1}{\xi} + \frac{1}{2}\right)\left(\frac{3}{8\pi G}\right)^{-\frac{1}{3\xi}} V^{\frac{1}{3}-\frac{2}{3\xi}}\right]^{-1}. \tag{47}$$

In FIG. 5, we plot $S \sim V$ (blue dashed line) and $C \sim V$ (red solid line) relations and conclude that the VPG model is thermodynamically stable.

On the other hand, according to the first thermodynamical law, the temperature is described as

$$T = (\rho + p)\frac{V}{S}. \tag{48}$$

It is significant to write relations among ρ, S and T, because it leads very interesting cosmological implications. Making use of our previous computations, one can find that

$$T(\rho) = \frac{(8\pi G)^{\frac{5}{6}}}{6\pi 3^{-\frac{1}{6}}}\left(1 + \beta a^{-n}\rho^{\frac{1}{\xi}}\right)\sqrt{\rho}. \tag{49}$$

In FIG. 6, we depict the evolution temperature as a function of energy density according to the best fit values of auxiliary parameters β, n and ξ. It is seen that temperature of

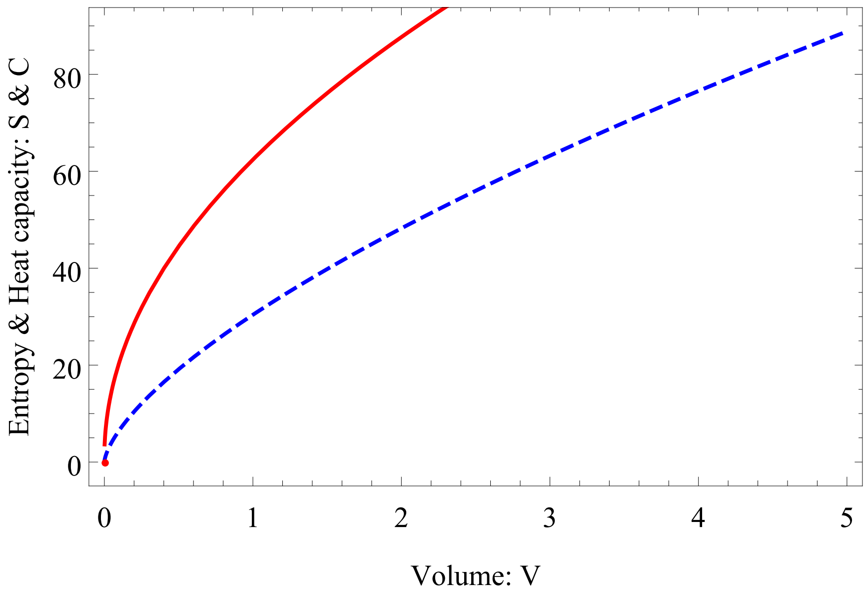

FIG. 5: $S \sim V$ (blue dashed line) and $C \sim V$ (red solid line) relations for SN Ia+OHD+BAO datasets in the VPG proposal. Here, we take $z = 0.35$ and $8\pi G = 1$.

the FRW universe dominated by the VPG increases by increasing energy density just as expected. Thence, we may interpret this conclusion as the third law of thermodynamics is satisfied for the VPG.

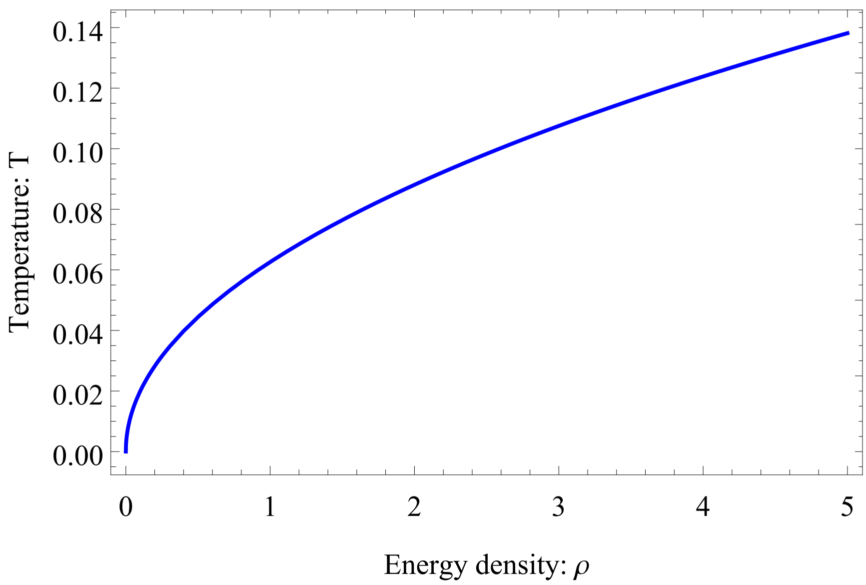

FIG. 6: Graphical analyze of $T \sim \rho$ relation with the best fit values of the free parameters given in the VPG model. Here, we assume that $z = 0.35$ and $8\pi G = 1$.

VII. CLOSING REMARKS

We introduce the VPG model as a unification of the dark matter and the dark energy and investigate its observational constraint by making use of a union case of the SN Ia sample, the OHV and the BAO data. Next, we emphasize in the third section that the VPG proposal can be reduced not only to the original PG model but also the ΛCDM, OCG and the GCG models by making use of suitable limiting cases. Additionally, the VPG model can also be transformed to the VGCG proposal, which is a different perspective of the same thing. The VPG and of course also the OPG, OCG, GCG and the VGCG models unify different fluids (dark matter and dark energy) of the standard cosmological model and can express both of them with a single fluid, therefore automatically removes the cosmic coincidence problem. It is known that we have three significant reasons for the failure of the ΛCDM idea: (i) incompatibility with astrophysical dataset, (ii) even modified Newtonian dynamics (MOND) works better than that and (iii) fundamental theoretical issues.

Based on the best fit values of auxiliary parameters, which are obtained by performing some numerical analyses for the VPG proposal, it is concluded that our universe will not end up with big rip in the future. We also discriminate the VPG model with the other unified energy density descriptions by making use of numerical and statistical analyzes. It is important to emphasize here that there is an obvious difference between the VPG and the OPG (also the ΛCDM) models. Remember, it was concluded from equation (28) that taking $n = 0$ reduces the VPG proposal into the OPG and the ΛCDM models. According to the numerical analysis, we see that compatibility of the VPG definition with the current observations is better than the OPG (of course also the ΛCDM) type formulation. One can also calculate the best fit values of the auxiliary parameters given in the VGCG model by making use of the following correspondence

$$\underbrace{\beta a^{-n}\rho_{vpg}^{1+\frac{1}{\xi}} \Longleftrightarrow \frac{-\kappa a^{-n}}{\rho_{vgcg}^{\alpha}}}_{\beta\to-\kappa,\ \ \xi\to-\frac{1}{1+\alpha},\ \ \rho_{vpg}\to\rho_{vgcg}} \ . \tag{50}$$

So, for the VGCG model, it can be calculated that

$$(\kappa, n, \alpha) = (0.031, -1.869, -1.334), \tag{51}$$

which is different from the set given in Ref.[53]. Thus, we can say that these are the most suitable values of the auxiliary parameters of the VGCG model so far.

Additionally, the VPG model describes an interesting cosmological system via its thermodynamical features. As we discuss in the sixth section, temperature of the system dominated by the VPG increases by increasing energy density, which is also observed in an ideal gas, just as expected. Furthermore, we proved that the variable form of the PG is thermodynamically stable during any expansion process and is consistent with the current astrophysical observations. As in the case of the ideal gas, one can also use some thermodynamical relations in order to obtain some significant expressions for the thermodynamical quantities. On this purpose, some interesting topics can also be in further studies such as thermal efficiency of polytropic Carnot engine and reversible adiabatic process for the VPG model as function of temperature, volume, pressure and the cosmic scale parameter a.

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