

# Negative mass can be positively thought-provoking

## Abstract

Following up previous discussions, suggestions, and insights about negative mass by other authors, this article further elaborates on systems of combined mass and negative mass. These configurations can be massless and stationary. The external physical effects of such systems are analyzed in detail. Some surprising results arise within the frameworks of Newtonian mechanics and Electromagnetism.

## 1. Background

The concept of mass has been studied in science for centuries. It appears in many fundamental physics equations like Newton's laws of motion and universal gravitation. However, the concept of "negative mass" is relatively new. Negative mass derives from negative matter and is of opposite sign to normal mass. There has not been any conclusive physical evidence that it ever existed. But in 2017, a team from Washington State University<sup>1</sup> claimed to have observed negative mass behavior in rubidium atoms. In 2018, University of Rochester<sup>2</sup> researchers reported success in creating particles with negative mass in an atomically thin semiconductor. Nonetheless, the lack of convincing evidence supporting the existence of negative mass has not hindered people's interest in studying it.

In his 1928 hypothesis of electrons and positrons, Dirac<sup>3</sup> speculated the existence of negative energy in a vacuum. Later modern investigations of negative mass began in 1950s. Ferrell,<sup>4</sup> in his 1950 prize winning essay, discussed a possible way to shield the gravitational effect by applying a negative mass. The idea behind it assumed that negative mass had the same gravitational field strength as mass, but repelled matter instead of attracting it. In other words,  $m_1$  and  $m_2$  have opposite signs in Newton's universal gravitational equation  $F = G \frac{m_1 m_2}{r^2}$ . So,  $F$  has a negative sign, which can be interpreted as repulsion (as opposed to normal attraction).

Following up Ferrell's article, Luttinger<sup>5</sup> analyzed the negative mass in context of the principle of equivalence in general relativity. Specifically, he discussed the scenario of two particles at a distance, where one had normal mass while the other had negative mass. He concluded that a) if the quantity of mass and negative mass were equal then they would maintain that distance while the pair moved forever in the direction from negative mass toward mass; b) if the quantity of negative mass was greater than that of mass, then the distance between them would increase—both would move away from each other; c) if the quantity of negative mass was less than that of mass, then the distance between them would decrease—both would move toward each other.

In 1957 Bondi<sup>6</sup>, within the framework of Einstein's general theory of relativity, studied the uniform acceleration in a two-body system consisting of a body of positive mass and a body of negative mass separated by empty space. He concluded that the uniform acceleration did not violate the general theory of relativity.

Forward<sup>7</sup> later explored whether the existence of negative matter led to any contradictions in Newtonian mechanics, and he concluded that the laws of conservation of linear momentum and total energy were not violated. In another article, Price<sup>8</sup> gave several examples to demonstrate

that negative mass could be quite counter intuitive but still be compatible with known mechanics laws.

Bonner<sup>9</sup> went another route, considering a hypothetical universe containing only negative mass. His investigation did not suggest any reasons for why mass in our real universe should be positive. One reason why he suggested a universe of only negative mass was to exclude so-called runaway (or self-accelerating) motion.

## 2. Introduction

So far based on the papers referenced, the existence of negative mass does not violate Newtonian mechanics. Furthermore, most of the previous discussions focused on the interaction between a particle with mass and a particle with negative mass. I.e. how do mass and negative mass interact and move?

In this article, the author considers configurations which consist of a mass and a negative mass of equal magnitudes, and whose center of mass and center of negative mass are aligned at the same position. Therefore, stationary massless systems can be achieved. Finally, the external physical effects of such systems are analyzed within the frameworks of Newton's laws and Coulomb's law.

## 3. Assumption and Terminology

Based on the aforementioned researches we can make two assumptions regarding the fundamental attributes of negative mass.

*Assumption 1:* Negative mass can exist.

*Assumption 2:* Physical laws are the same for negative mass as for mass—i.e. negative mass can be substituted for mass in physical laws (with the same magnitude but opposite sign).

From *Assumption 2*, it can be deduced that negative mass generates a gravitational field similar to that of mass, but with opposite sign; negative mass accelerates in the opposite direction as that of the force which acts on it; there are three kinds of negative masses: inertial, active gravitational, passive gravitational.

We can further deduce the following attributes of negative mass. They are important in later arguments and are listed below:

*Attribute 1:* Negative mass repels negative mass—i.e. negative masses move away from each other.

*Attribute 2:* Negative mass repels mass.

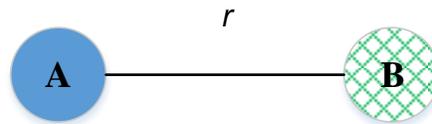
*Attribute 3:* Mass attracts negative mass—i.e. negative mass moves toward mass.

From *Attribute 1*, it follows that a plural quantity of negative masses is unstable if no other forces bind them together. However, the smallest unit of negative mass is stable. The unit will be referred to as a *Yinon* in this article. There could be numerous kinds of yinons with different magnitudes.

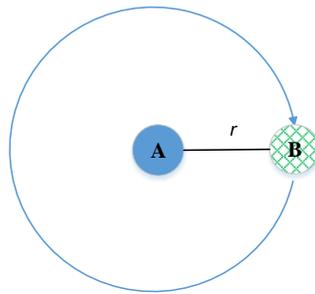
Let us suppose that there exists a unit of “ordinary” mass that has an inertial mass with equal magnitude to that of a yinon. Instead of saying “a mass with the magnitude of a yinon” we simply call that unit of mass a **Masson** in the rest of this article.

#### 4. Internal Configuration with an Equal Number of Yinons and Massons

Let’s carry out some thought experiments to construct massless configurations. To simplify the experiments, we restrict the configurations to be on a two dimensional plane and treat them as systems of point particles or point charges.



In Fig.1, let particle **A** (blue) be a masson  $m_a$ , and particle **B** (green) be a yinon  $-m_b$ . Let  $r$  be the distance between A and B. The gravitational force exerted on B by A is  $F_{ab} = G \frac{m_a \cdot -m_b}{r^2}$ , and points from A to B. However, the acceleration of B is  $a_b = G \frac{m_a}{r^2}$ , and points from B to A. The center of mass of this configuration is at infinity. The pair is massless and forms a runaway motion in the direction from B toward A.



In Fig.2, a yinon B rotates around a masson A at the center of a circle. This configuration is not stationary because its center of mass is at infinity. The pair will spiral outward from its original position, which is a variation of linear runaway motion.



In Fig.3, if a masson A can be hollow and a yinon B can be embedded inside A, then A's center of mass and B's center of negative mass are aligned at the same position. Thus the configuration is stationary. Likewise, the configuration of A inside a hollowed B is also stationary.

We have not had success in constructing a stationary massless configuration consisting of a yinon and a masson— unless we consider that a masson or a yinon can be hollow. But we find success in constructing a stationary massless configuration of two yinons and two massons.



Figure 4 shows a configuration of two yinons and two massons stuck together in a line. There are four permutations in which these yinons and massons can be arranged in a line. Only the alignment of yinon-masson-masson-yinon is stationary. Since the attraction on one yinon generated by the two massons is greater than the repulsion exerted on it by the other yinon, the yinons do not move away from the center massons (which also attract each other). The center of mass of the yinons is aligned at the center of mass of the massons. The configuration is stationary, and we can treat it as a system of point particles.

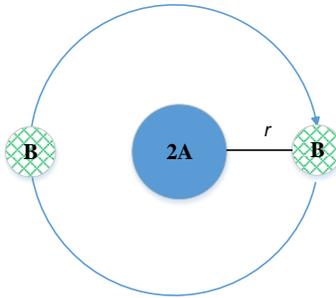
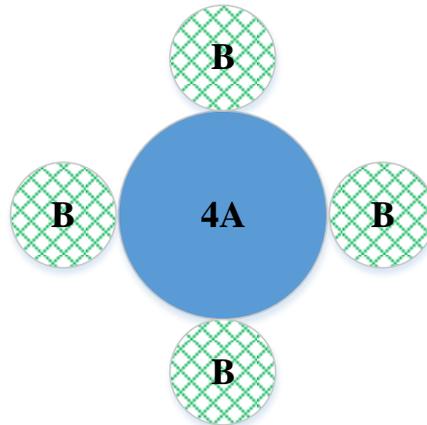


Figure 5 shows a configuration of two massons ( $m_a$ ) at the center while two yinons rotate around them in a circular orbit. The two yinons ( $-m_b$ ) are always on opposite sides in the orbit. The repulsion exerted on one yinon by the other is  $F_{bb} = G \frac{-m_b \cdot -m_b}{4r^2}$ , and the attraction exerted on each yinon by the massons is  $F_{ab} = G \frac{2m_a \cdot -m_b}{r^2}$ . A centripetal acceleration  $a_b = \frac{v_b^2}{r}$  of each yinon toward the massons keeps each yinon in a circular motion. Since  $m_a = -m_b$  we can calculate the uniform circular speed is  $v_b = \sqrt{G \frac{7m_a}{4r}}$ . The repulsions exerted on the center massons by each

Yinons cancel each other out. The center of mass of yinons is aligned at the center of mass of massons. Thus this configuration is stationary. Again we can treat this configuration as a system of point particles.

There are other configurations which consist of an equal number of yinons and massons, and those configurations can additionally be stationary.



For instance, Fig.6 shows four massons and four yinons stuck together. Two yinons are aligned horizontally while the other two are aligned vertically.

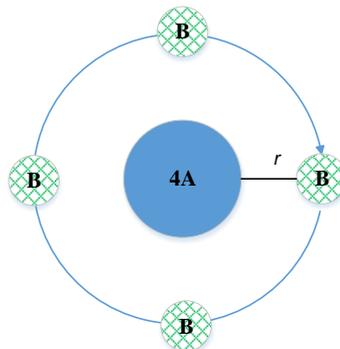


Figure 7 shows four massons are at the center and four yinons orbit around them, while at any time the four yinons are separated evenly in the circle.

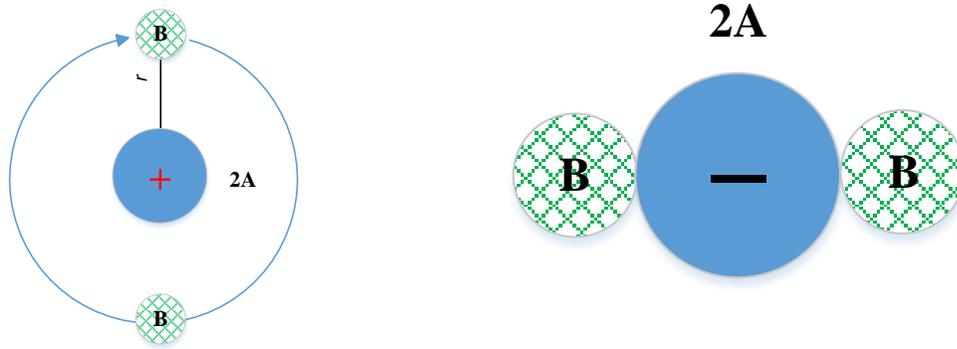


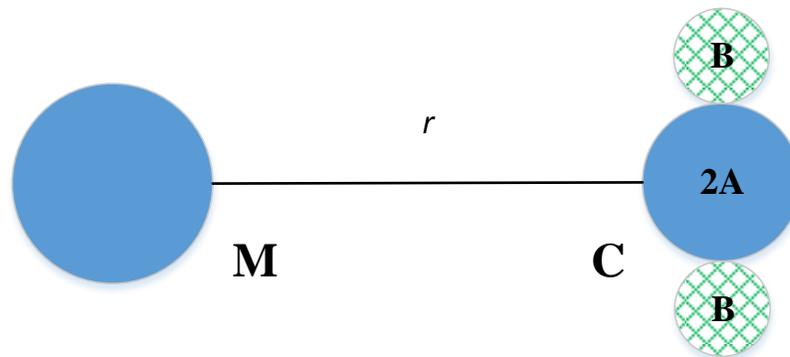
Figure 8 shows two possible configurations where the two massons at the center have an electric charge. In addition to being a stationary configuration, each of them is still massless but has an electric charge. Since there is only one electric charge, there is no electrostatic interaction inside the configuration. Therefore we can isolate an electric charge from its host matter. I.e., a system can have zero net mass yet have electric charge. We can treat it as a system of point charge.

### 5. External Effect of Equal Number of Yinons and Massons

So far we have proposed massless configurations consisting of an equal number of yinons and massons. Some of them, like Fig. 1 and Fig. 2, are not stationary because their centers of mass are at infinity. Thus, they may not be viable systems of point particles.

Let us examine the external physical effects of stationary configurations within the frameworks of Newtonian mechanics and Electromagnetism.

(1) Interaction between a mass and a massless configuration



Let's call the configuration C and let there exist a mass M separated by a spatial space  $r$ , as shown in Fig.9.

The inertial mass of C is zero since it has an equal number of massons and yinons; and the centers of massons and yinons are aligned at the same position. The active and passive gravitational masses of C are zero since its mass cancels out its negative mass exactly.

Therefore, the gravitational force exerted by C on M is zero according to  $F_{cm} = G \frac{m_c m_m}{r^2} = 0$ , where  $m_c = 0$ . Based on Newton's 2<sup>nd</sup> law,  $F_{cm} = m_m a_m$ , where  $m_m > 0$ . So M has a zero acceleration  $a_m = 0$ ; and M is standstill.

M's motion obeys Newton's 1<sup>st</sup> law, 2<sup>nd</sup> law and gravitational law.

On the other hand, the gravitational force exerted by M on C is also zero according to  $F_{mc} = G \frac{m_m m_c}{r^2} = 0$ , where  $m_c = 0$ . Based on Newton's 2<sup>nd</sup> law,  $F_{mc} = m_c a_c$ , where  $m_c = 0$ . So C's acceleration  $a_c$  can be positive, negative or zero.

However, inside configuration C, yinons are attracted by M—according to *attribute 3*—with an acceleration of  $a_y = G \frac{m_M}{r^2}$ ; and the massons are attracted to M with an acceleration of  $a_m = G \frac{m_M}{r^2}$ . Thus, both yinons and massons move toward C at the same acceleration. So the center of mass of C can move toward M at the acceleration of  $a_c = a_y = a_m$ .

This implies that a mass can attract a massless object. Even though it is counter intuitive, Newton's 2<sup>nd</sup> law and gravitational law still hold for C. The non-zero acceleration of C does not contradict the previous conclusion of  $a_c$  being of positive, negative or zero.

The motions of M and C obey Newton's 3<sup>rd</sup> law. Linear momentum is conserved as is mechanical energy.

But Newton's 1<sup>st</sup> law does not hold for C—if we consider how C can accelerate toward M without a net force acting on it.

(2) Interaction between a mass and a massless configuration with an electric charge

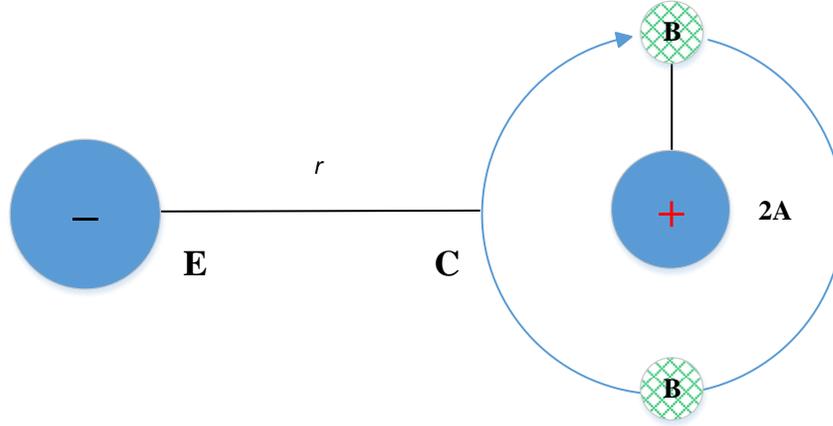
Since there is no electrostatic interaction between them this case is a variation of case (1). The massless configuration carrying an electric charge moves toward mass.

(3) Interaction between an electric charge and a massless configuration

If the charge has no mass, such as in Fig. 8, then there is no motion for either the charge or the configuration since there is neither gravitational nor electrostatic interaction between them.

If the charge has mass, then the massless configuration moves toward the charge, creating a variation of case (1).

(4) Interaction between an electric charge and a massless configuration with an electric charge



Let us assume there is a massless configuration C which carries a positive electric charge ( $e^+$ ); there is an object E which carries a negative electric charge ( $e^-$ ).

If C and E are separated by a distance  $r$ , as shown in Fig.10, then the electrostatic attractive force exerted by E on C is  $F_{ec} = K \frac{e^- e^+}{r^2}$ , where  $e > 0$ , so  $F_{ec} > 0$ .

The gravitational force exerted by E on C is zero since C is massless. Therefore the net force acting on C is  $F_{ec} > 0$ .

According to Newton's 2<sup>nd</sup> law  $F_{ec} = m_c a_c$ , where  $m_c = 0$ . Thus, we can conclude that  $a_c$  cannot be a finite number.

However, inside the configuration C, the massons with a positive electric charge are attracted by the external negative charge E. The gravitational force exerted on the massons by E can be ignored when compared to that of the electrostatic force by E. Thus, the net force acting on the massons is  $F_{ec} > 0$ . Based on Newton's 2<sup>nd</sup> law, the massons move toward E at an acceleration of  $a_m = \frac{F_{ec}}{2m_a}$ .

The yinons have no electric charge so they are not affected by E. The gravitational force on them exerted by E can be ignored when compared to that exerted by the massons. But the yinons move together with the massons via the gravitational bonding between the yinons and the massons. Thus, the yinons and massons move toward E at the same acceleration, which is determined by the massons' acceleration  $a_m = \frac{F_{ec}}{2m_a}$ .

Therefore, C can move toward E at a finite acceleration  $a_c = a_m$ , which contradicts the previous conclusion of  $a_c$  not being finite.

## 6. Discussion and Conclusion

In the 5.1 thought experiment where it happens in an inertial reference frame, Newton's 2<sup>nd</sup> law, 3<sup>rd</sup> law, and gravitational law hold. The laws of conservation of linear momentum and conservation of mechanical energy also hold. We can deduce that a massless object cannot be

detected by its gravitational effect since it does not exert gravitational force on an external object. On the other hand, a mass can affect a massless object which is not uniformly composed of the same kinds of mass (normal or negative).

However as a system consisting of an equal number of yinons and massons, Newton's 1<sup>st</sup> law does not hold.

Let's rephrase the 1<sup>st</sup> law: "An object continues in a state of rest or in a state of motion at a constant speed along a straight line, unless compelled to change that state by a net force."

Because the net force acting on C is zero, C should be in a state of rest or in a state of motion at a constant speed. However, the 5.1 thought experiment demonstrates that C can move toward M at non-zero acceleration.

In the 5.4 thought experiment where it happens in an inertial reference frame, Newton's 1<sup>st</sup> law, 3<sup>rd</sup> law, gravitational law, and Coulomb's law hold. The laws of conservation of linear momentum and conservation of total energy also hold.

But as a system consisting of an equal number of yinons and massons, if it carries electric charge then Newton's 2<sup>nd</sup> law does not hold.

Let's rephrase the 2<sup>nd</sup> law: "When a net external force  $\sum F$  acts on an object of mass  $m$ , the acceleration  $a$  that results is directly proportional to the net force and has a magnitude that is inversely proportional to the mass. The direction of the acceleration is the same as the direction of the net force,  $\sum F = ma$ ." Since the net force acting on C is non zero, the acceleration of C should be inversely proportional to its mass. But the mass is zero, thus its acceleration should be infinite.

In both laws the object is described to have non-zero mass. I.e., the object cannot be massless.

In this article, the author made two assumptions regarding negative mass, and then went further to construct stationary massless configurations, and presented thought experiments within the frameworks of Newton's laws and Electromagnetism.

Throughout these thought experiments, we have analyzed the external physical effects of stationary massless configurations. The uncovered resulting effects are quite thought-provoking. A few dilemmas have emerged from these thought experiments.

These effects do not contradict the two assumptions, but are inconsistent with Newton's 1<sup>st</sup> law and 2<sup>nd</sup> law since the laws do not consider massless objects. In order to explain the intricacies involving massless object we may need to go beyond Newtonian mechanics and Electromagnetism.

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