

THE COLLATZ CONJECTURE
Order and harmony in the sequence numbers

by

Miquel Cerdà Bennassar
August 2019

Abstract: I propose a numerical table that demonstrates visually that the sequences formed with Collatz's algorithm always reach 1.

Keywords: Collatz Conjecture, 3n+1 problem, number theory.

Introduction: The conjecture was raised by the mathematician Lothar Collatz in 1937. It is also known as other names: The 3n + 1 problem, the Ulam conjecture, the Kakutani problem, the conjecture of Thwaites, the Hasse algorithm or the Syracuse problem.

The conjecture reads as follows:

- 1 - Any natural number is chosen, n.
- 2 - If it is even divide by 2, (n / 2).
- 3 - If odd, multiply by 3 and add 1 to the result, (3n + 1).

The process is repeated with each result and a sequence is obtained that always ends in 1 and always It is so, whatever the initial number. The unknown is why it happens and if it happens with all natural numbers.

Overview: For the Collatz conjecture, I identify two types of odd numbers: Those of the form 4n + 3 and those of the 4n + 1 form.

Applying $(3m + 1) / 2$ to those of the form 4n + 3, an odd number is found that is greater than the previous one and the sequence is ascending.

Applying the same operation to those of the 4n + 1 form, it is an even number that requires more divisions by 2, so the odd number they arrive at is always smaller than the previous one and the sequence is descending.

A sequence of Collatz will be more or less descending and more or less long as obtained more or less odd numbers of the form 4n + 1.

If in the set of odd numbers there are the same number of both, do they have the same probability of going out in the Collatz sequences? Can you predict how many will there be and at what moment the numbers of the form 4n + 1 will appear and the sequence will descend?

I also identify two types of even numbers: Those of the form 4n + 2 and those of the form 4n + 4. Those of the form 4n + 2 admit a single division n / 2 and the odd number that results is greater than the previous one and the sequence is ascending.

Those of the form 4n + 4 admit two or more divisions n / 2 and the odd number they arrive at is smaller than the previous one and the sequence is descending.

A sequence of Collatz will be more or less descending and more or less long as more or less even numbers of the form $4n + 4$ are obtained.

If in the set of even numbers there are the same number of both, do they have the same probability of going out in the Collatz sequences? Can you predict how many there will be and at what time the numbers of the $4n + 4$ form will appear and the sequence will descend?

The answers to these two questions may be found in two tables, one for odd numbers and one for even numbers, which I explain below.

TABLE OF THE ODD NUMBERS OF THE SEQUENCES

On a table with k columns, I write in the first row the numbers $2k-1$.

In the successive rows I apply $(3m + 1) / 2$ to the odd numbers, leaving the even number as the last number of each column.

In red they are the odd numbers of the form $4n + 3$ and those of green are the odd numbers of the form $4n + 1$. Even numbers have been left without color.

The value of n of the last number of the columns corresponds to the number of odd numbers and the number of applications $(3m + 1) / 2$ in them.

Example: In column k (48) there are 5 odd numbers and the odd 95 needs 5 steps to reach the even number 728.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	...																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																												
0	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71	73	75	77	79	81	83	85	87	89	91	93	95	97	99	101	...																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																												
1	2	5	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50	53	56	59	62	65	68	71	74	77	80	83	86	89	92	95	98	101	104	107	110	113	116	119	122	125	128	131	134	137	140	143	146	149	152	...																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																												
2	8	17	26	35	44	53	62	71	80	89	98	107	116	125	134	143	152	161	170	179	188	197	206	215	224	233	242	251	260	269	278	287	296	305	314	323	332	341	350	359	368	377	386	395	404	413	422	431	440	449	458	...																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																												
3	26	53	80	107	134	161	188	215	242	269	296	323	350	377	404	431	458	485	512	539	566	593	620	647	674	701	728	755	782	809	836	863	890	917	944	971	1000	1027	1054	1081	1108	1135	1162	1189	1216	1243	1270	1297	1324	1351	1378	1405	1432	1459	1486	1513	1540	1567	1594	1621	1648	1675	1702	1729	1756	1783	1810	1837	1864	1891	1918	1945	1972	2009	2036	2063	2090	2117	2144	2171	2208	2235	2262	2289	2316	2343	2370	2407	2434	2461	2488	2515	2542	2569	2606	2633	2660	2687	2724	2751	2778	2815	2842	2879	2906	2933	2960	2987	3014	3041	3068	3095	3122	3149	3176	3203	3230	3257	3284	3311	3338	3365	3392	3419	3446	3473	3500	3527	3554	3581	3608	3635	3662	3689	3716	3743	3770	3807	3834	3861	3888	3915	3942	3969	3996	4023	4050	4077	4104	4131	4158	4185	4212	4239	4266	4293	4320	4347	4374	4401	4428	4455	4482	4509	4536	4563	4590	4617	4644	4671	4708	4735	4762	4789	4816	4843	4870	4907	4934	4961	4988	5015	5042	5069	5096	5123	5150	5177	5204	5231	5258	5285	5312	5339	5366	5393	5420	5447	5474	5501	5528	5555	5582	5609	5636	5663	5690	5717	5744	5771	5808	5835	5862	5889	5916	5943	5970	6000	6030	6060	6090	6120	6150	6180	6210	6240	6270	6300	6330	6360	6390	6420	6450	6480	6510	6540	6570	6600	6630	6660	6690	6720	6750	6780	6810	6840	6870	6900	6930	6960	6990	7020	7050	7080	7110	7140	7170	7200	7230	7260	7290	7320	7350	7380	7410	7440	7470	7500	7530	7560	7590	7620	7650	7680	7710	7740	7770	7800	7830	7860	7890	7920	7950	7980	8010	8040	8070	8100	8130	8160	8190	8220	8250	8280	8310	8340	8370	8400	8430	8460	8490	8520	8550	8580	8610	8640	8670	8700	8730	8760	8790	8820	8850	8880	8910	8940	8970	9000	9030	9060	9090	9120	9150	9180	9210	9240	9270	9300	9330	9360	9390	9420	9450	9480	9510	9540	9570	9600	9630	9660	9690	9720	9750	9780	9810	9840	9870	9900	9930	9960	9990	10020	10050	10080	10110	10140	10170	10200	10230	10260	10290	10320	10350	10380	10410	10440	10470	10500	10530	10560	10590	10620	10650	10680	10710	10740	10770	10800	10830	10860	10890	10920	10950	10980	11010	11040	11070	11100	11130	11160	11190	11220	11250	11280	11310	11340	11370	11400	11430	11460	11490	11520	11550	11580	11610	11640	11670	11700	11730	11760	11790	11820	11850	11880	11910	11940	11970	12000	12030	12060	12090	12120	12150	12180	12210	12240	12270	12300	12330	12360	12390	12420	12450	12480	12510	12540	12570	12600	12630	12660	12690	12720	12750	12780	12810	12840	12870	12900	12930	12960	12990	13020	13050	13080	13110	13140	13170	13200	13230	13260	13290	13320	13350	13380	13410	13440	13470	13500	13530	13560	13590	13620	13650	13680	13710	13740	13770	13800	13830	13860	13890	13920	13950	13980	14010	14040	14070	14100	14130	14160	14190	14220	14250	14280	14310	14340	14370	14400	14430	14460	14490	14520	14550	14580	14610	14640	14670	14700	14730	14760	14790	14820	14850	14880	14910	14940	14970	15000	15030	15060	15090	15120	15150	15180	15210	15240	15270	15300	15330	15360	15390	15420	15450	15480	15510	15540	15570	15600	15630	15660	15690	15720	15750	15780	15810	15840	15870	15900	15930	15960	15990	16020	16050	16080	16110	16140	16170	16200	16230	16260	16290	16320	16350	16380	16410	16440	16470	16500	16530	16560	16590	16620	16650	16680	16710	16740	16770	16800	16830	16860	16890	16920	16950	16980	17010	17040	17070	17100	17130	17160	17190	17220	17250	17280	17310	17340	17370	17400	17430	17460	17490	17520	17550	17580	17610	17640	17670	17700	17730	17760	17790	17820	17850	17880	17910	17940	17970	18000	18030	18060	18090	18120	18150	18180	18210	18240	18270	18300	18330	18360	18390	18420	18450	18480	18510	18540	18570	18600	18630	18660	18690	18720	18750	18780	18810	18840	18870	18900	18930	18960	18990	19020	19050	19080	19110	19140	19170	19200	19230	19260	19290	19320	19350	19380	19410	19440	19470	19500	19530	19560	19590	19620	19650	19680	19710	19740	19770	19800	19830	19860	19890	19920	19950	19980	20010	20040	20070	20100	20130	20160	20190	20220	20250	20280	20310	20340	20370	20400	20430	20460	20490	20520	20550	20580	20610	20640	20670	20700	20730	20760	20790	20820	20850	20880	20910	20940	20970	21000	21030	21060	21090	21120	21150	21180	21210	21240	21270	21300	21330	21360	21390	21420	21450	21480	21510	21540	21570	21600	21630	21660	21690	21720	21750	21780	21810	21840	21870	21900	21930	21960	21990	22020	22050	22080	22110	22140	22170	22200	22230	22260	22290	22320	22350	22380	22410	22440	22470	22500	22530	22560	22590	22620	22650	22680	22710	22740	22770	22800	22830	22860	22890	22920	22950	22980	23010	23040	23070	23100	23130	23160	23190	23220	23250	23280	23310	23340	23370	23400	23430	23460	23490	23520	23550	23580	23610	23640	23670	23700	23730	23760	23790	23820	23850	23880	23910	23940	23970	24000	24030	24060	24090	24120	24150	24180	24210	24240	24270	24300	24330	24360	24390	24420	24450	24480	24510	24540	24570	24600	24630	24660	24690	24720	24750	24780	24810	24840	24870	24900	24930	24960	24990	25020	25050	25080	25110	25140	25170	25200	25230	25260	25290	25320	25350	25380	25410	25440	25470	25500	25530	25560	25590	25620	25650	25680	25710	25740	25770	25800	25830	25860

The amount of numbers in each column is always the same, depending on the value of k:

		n	valor de k													cantidad de números por columna
2k-1		0	1	3	5	7	9	11	13	15	17	...			0 impares rojos, 1 número impar verde y un número par	
2(2k-1)		1	2	6	10	14	18	22	26	30	34	...			1 número impar rojo, 1 número impar verde y un número par	
4(2k-1)		2	4	12	20	28	36	44	52	60	68	...			2 números impares rojos, 1 número impar verde y un número par	
8(2k-1)		3	8	24	40	56	72	88	104	120	136	...			3 números impares rojos, 1 número impar verde y un número par	
16(2k-1)		4	16	48	80	112	144	176	208	240	272	...			4 números impares rojos, 1 número impar verde y un número par	
32(2k-1)		5	32	96	160	224	288	352	416	480	544	...			5 números impares rojos, 1 número impar verde y un número par	
64(2k-1)		6	64	192	320	448	576	704	832	960	1088	...			6 números impares rojos, 1 número impar verde y un número par	
...		

In each of the columns $k = 2^n * (2k-1)$ there are n odd red numbers, one last green odd number and the even number that closes the column.

Example: in column $k = 2^6 * 50$ there are 50 red odd numbers, the first is number 2251799813685247 and the 50th is 957197316922470118360331 and the last odd of the column, which is green, is number 1435795975383705177540497. The even number that closes the column is 2153693963075557766310746. In Annex 1, the sequence development started with this first column number.

They will also have the same amount of numbers each of the columns $k = (2k-1) * 2^6 * 50$.

A sequence obtained with the Collatz conjecture algorithm is formed by an indeterminate number of columns in the table or cycles.

A sequence started with the first odd number of any column $k = (2k-1) * 2^{10} * 1000000$ would have in that column, 1000000 odd red, one last green odd and an even number closing the column. There will be the same odd number in all columns, for every value of $(2k-1)$.

If the sequence were started with a power number of 2 close to infinity, the first column of the table or cycle of that sequence would have that same amount of odd numbers.

The table is infinite but all its columns or cycles are bounded geometric progressions.

Example of a sequence starting with the number 279:

279, 838, 419, 1258, 629, 1888, 944, 472, 236, 118, 59, 178, 89, 268, 134, 67, 202, 101, 304, 152,
k(140) k(30) k(34)

$$76, 38, \underline{19}, 58, 29, 88, 44, 22, \underline{11}, 34, 17, 52, 26, \underline{13}, 40, 20, 10, \underline{5}, \underline{16}, 8, 4, 2, \underline{1}.$$

$k(10)$ $k(6)$ $k(7)$ $k(3)$ $k(1)$

The columns involved in the sequence:

140	30	34	10	6	7	3	1
279	59	67	19	11	13	5	1
419	89	101	29	17	20	8	2
629	134	152	44	26			
944							

The sequence obtained starting with the number 27 and the 18 columns or cycles that form it:

The same columns without the odd ones of the form $4n + 3$:

k	14	16	61	46	52	88	223	84	142	160	456	289	217	163	31	12	3	1
n																		
0			121				445					577	433	325	61		5	1
1	41		182	137			668		425			866	650	488	92		8	2
2	62			206	233			377	638							53		
3						350	593		566		3077					80		
4		161					890				4616							
5		242								2429								
6										3644								
7																		
8																		

The odd green ones followed by the even number at the end of each column k (even) are also in the columns k (odd).

K	21	81	61	69	117	297	223	189	213	1215	1539	289	217	163	31	27	3	1
	41	161	121	137	233	593	445	377	425	2429	3077	577	433	325	61	53	5	1
	62	242	182	206	350	890	668	566	638	3644	4616	866	650	488	92	80	8	2

Equivalence of k even and K odd : $k(3/2)^n = K$ Example: $k160*(3/2)^5 = K1215$

At each repetition of the algorithm operation $(3m + 1) / 2$, applied to the first odd number of the column, the sequence “jumps” from one column to another, leaving it at a single step to reach the even number that can be divided at least 2^2 . This will always occur in any sequence until the green odd is in row n (0).

An example of the evolution of the columns of the table in which “loses” an odd number in each jump, $k * 3/2$:

$$32*3/2=48, 48*3/2=72, 72*3/2=108, 108*3/2=162 \text{ y } 162*3/2=243$$

$$32*(3/2)^5=243$$

$$k = 32, K = 243$$

	k	32	48	72	108	162	243
n							
0		63	95	143	215	323	485
1		95	143	215	323	485	728
2		143	215	323	485	728	
3		215	323	485	728		
4		323	485	728			
5		485	728				
6		728					
7							

Column k (32) contains the odd numbers and an even number of a Collatz sequence from number 63 to number 728, occupying rows n (0) through n (6).

The same numbers are in columns k (32), k (48), k (72), k (108), k (162) and k (243), occupying row n (0), because in a sequence there may be the numbers of any of these columns and not necessarily all of them.

The cycle that follows the previous one is that of the column of the number 91, because $728/2^3 = 91$

	k	46	69
n			
0		91	137
1		137	206
2		206	
3			
4			
5			
6			
7			

The next cycle: $206/2 = 103$

	k	52	78	117
n				
0		103	155	233
1		155	233	350
2		233	350	
3		350		
4				
5				
6				
7				

It is followed by eleven more cycles until the number 5 results, which heads the last column before reaching 1.

Odd numbers of the form $4n + 3$ always arrive at an odd number of the form $4n + 1$.

THE EVEN NUMBERS OF SEQUENCES IN A TABLE

On a table with k columns, we write in the first row the numbers $2k$.

In the successive rows, the even numbers are divided by 2, leaving the odd number as the last number in each column.

In red they are even numbers that divide them by 2 produce another even number and green ones are even numbers that divide by 2 produce an odd number.

Odd numbers have been left without color.

The value of n of the last number of the columns corresponds to the number of even numbers and the number of divisions between 2 in them.

Example: In column k (48) there are 5 even numbers and the pair 96 needs 5 steps to reach the odd number 3.

The amount of numbers in each column is always the same, depending on the value of k:

		n		valor de k														cantidad de números por columna											
2k-1		0		1	3	5	7	9	11	13	15	17	...					0 pares rojos, 1 número par verde y un número impar											
2(2k-1)		1		2	6	10	14	18	22	26	30	34	...					1 número par rojo, 1 número par verde y un número impar											
4(2k-1)		2		4	12	20	28	36	44	52	60	68	...					2 números pares rojos, 1 número par verde y un número impar											
8(2k-1)		3		8	24	40	56	72	88	104	120	136	...					3 números pares rojos, 1 número par verde y un número impar											
16(2k-1)		4		16	48	80	112	144	176	208	240	272	...					4 números pares rojos, 1 número par verde y un número impar											
32(2k-1)		5		32	96	160	224	288	352	416	480	544	...					5 números pares rojos, 1 número par verde y un número impar											
64(2k-1)		6		64	192	320	448	576	704	832	960	1088	...					6 números pares rojos, 1 número par verde y un número impar											
...												

In each of the columns $k = 2^{\wedge} n * (2k-1)$ there are n red even numbers, one last green pair and the odd number that closes the column.

Examples: In column $k = 2^{\wedge} 50$ there are 50 red even numbers, the first one is number 2251799813685248 and the 50th is 4 and the last pair of the column, which is green, is number 2. The odd number that closes the column is 1.

They would also have the same amount of numbers each of the columns $k = (2k-1) * 2^{\wedge} 50$.

Having both tables the same amount of numbers in their columns with the same value of k and the same value of n and the same amount of red numbers, green numbers and numbers without color, could a single table be formed to develop the sequence path from Collatz ?.

Comparison of the two tables:

Odd numbers:

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	...																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																										
n																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																														
0	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71	73	75	77	79	81	83	85	87	89	91	93	95	97	99	101	...																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																										
1	2	5	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50	53	56	59	62	65	68	71	74	77	80	83	86	89	92	95	98	101	104	107	110	113	116	119	122	125	128	131	134	137	140	143	146	149	152	...																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																										
2	8	17	26	35	44	53	62	71	80	89	98	107	116	125	134	143	152	161	170	179	188	197	206	215	224	233	242	251	269	278	286	295	304	313	322	331	340	349	358	367	376	385	394	403	412	421	430	439	448	457	466	475	484	493	497	506	515	524	533	542	551	560	569	578	587	596	595	594	593	592	591	590	589	588	587	586	585	584	583	582	581	580	579	578	577	576	575	574	573	572	571	570	569	568	567	566	565	564	563	562	561	560	559	558	557	556	555	554	553	552	551	550	549	548	547	546	545	544	543	542	541	540	539	538	537	536	535	534	533	532	531	530	529	528	527	526	525	524	523	522	521	520	519	518	517	516	515	514	513	512	511	510	509	508	507	506	505	504	503	502	501	500	499	498	497	496	495	494	493	492	491	490	489	488	487	486	485	484	483	482	481	480	479	478	477	476	475	474	473	472	471	470	469	468	467	466	465	464	463	462	461	460	459	458	457	456	455	454	453	452	451	450	449	448	447	446	445	444	443	442	441	440	439	438	437	436	435	434	433	432	431	430	429	428	427	426	425	424	423	422	421	420	419	418	417	416	415	414	413	412	411	410	409	408	407	406	405	404	403	402	401	400	399	398	397	396	395	394	393	392	391	390	389	388	387	386	385	384	383	382	381	380	379	378	377	376	375	374	373	372	371	370	369	368	367	366	365	364	363	362	361	360	359	358	357	356	355	354	353	352	351	350	349	348	347	346	345	344	343	342	341	340	339	338	337	336	335	334	333	332	331	330	329	328	327	326	325	324	323	322	321	320	319	318	317	316	315	314	313	312	311	310	309	308	307	306	305	304	303	302	301	300	299	298	297	296	295	294	293	292	291	290	289	288	287	286	285	284	283	282	281	280	279	278	277	276	275	274	273	272	271	270	269	268	267	266	265	264	263	262	261	260	259	258	257	256	255	254	253	252	251	250	249	248	247	246	245	244	243	242	241	240	239	238	237	236	235	234	233	232	231	230	229	228	227	226	225	224	223	222	221	220	219	218	217	216	215	214	213	212	211	210	209	208	207	206	205	204	203	202	201	200	199	198	197	196	195	194	193	192	191	190	189	188	187	186	185	184	183	182	181	180	179	178	177	176	175	174	173	172	171	170	169	168	167	166	165	164	163	162	161	160	159	158	157	156	155	154	153	152	151	150	149	148	147	146	145	144	143	142	141	140	139	138	137	136	135	134	133	132	131	130	129	128	127	126	125	124	123	122	121	120	119	118	117	116	115	114	113	112	111	110	109	108	107	106	105	104	103	102	101	100	99	98	97	96	95	94	93	92	91	90	89	88	87	86	85	84	83	82	81	80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	61	60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15	-16	-17	-18	-19	-20	-21	-22	-23	-24	-25	-26	-27	-28	-29	-30	-31	-32	-33	-34	-35	-36	-37	-38	-39	-40	-41	-42	-43	-44	-45	-46	-47	-48	-49	-50	-51	-52	-53	-54	-55	-56	-57	-58	-59	-60	-61	-62	-63	-64	-65	-66	-67	-68	-69	-70	-71	-72	-73	-74	-75	-76	-77	-78	-79	-80	-81	-82	-83	-84	-85	-86	-87	-88	-89	-90	-91	-92	-93	-94	-95	-96	-97	-98	-99	-100	-101	-102	-103	-104	-105	-106	-107	-108	-109	-110	-111	-112	-113	-114	-115	-116	-117	-118	-119	-120	-121	-122	-123	-124	-125	-126	-127	-128	-129	-130	-131	-132	-133	-134	-135	-136	-137	-138	-139	-140	-141	-142	-143	-144	-145	-146	-147	-148	-149	-150	-151	-152	-153	-154	-155	-156	-157	-158	-159	-160	-161	-162	-163	-164	-165	-166	-167	-168

With the columns of the table of even numbers, we can form the column with the same value of k from the table of odd numbers.

Example: column k (4)

From even to odd:

From odd to even:

$$\begin{aligned}1*8-1 &= 7 \\3*4-1 &= 11 \\9*2-1 &= 17 \\27*1-1 &= 26\end{aligned}$$

$$\begin{aligned}(7+1)/1 &= 8 \\(11+1)/3 &= 4 \\(17+1)/9 &= 2 \\(26+1)/27 &= 1\end{aligned}$$

$$3^n * \text{even} - 1 = \text{odd}$$

(odd+1)/3^n=even

A TRIANGLE WITH THE COLUMNS OF THE TABLE

A triangle T (n):

A number n is written at the top vertex and the triangle is formed as follows:

For the columns, multiply n by 2 and add 1 to the result, $2n + 1$. Repeat with the results obtained.

For the rows, multiply n by 3, add 1 to the result and divide by 2, $(3n + 1) / 2$. Repeat until you reach an even number.

A triangle $T(n)$ for each value of $(n) = 0, 4, 6, 10, 12, 16, 18, 22, 24, \dots$ and we can write on them all odd numbers.

The triangle T (0):

Each row k of the triangle is column k of the table and the value of k is any number of the triangle plus 1.

0											
1	2										
3	5	8									
7	11	17	26								
15	23	35	53	80							
31	47	71	107	161	242						
63	95	143	215	323	485	728					
127	191	287	431	647	971	1457	2186				
255	383	575	863	1295	1943	2915	4373	6560			
511	767	1151	1727	2591	3887	5831	8747	13121	19682		
...

Examples:

Column k (8) of the table has the numbers 15, 23, 35, 53 and 80.

Column k (12) of the table has the numbers 23, 35, 53 and 80.

Column k (18) of the table has the numbers 35, 53 and 80.

Column k (27) of the table has the numbers 53 and 80.

8		12		18		27	
15		23		35		53	
23		35		53		80	
35		53		80			
53		80					
80							

The triangle T(4):

4										
9	14									
19	29	44								
39	59	89	134							
79	119	179	269	404						
159	239	359	539	809	1214					
319	479	719	1079	1619	2429	3644				
639	959	1439	2159	3239	4859	7289	10934			
1279	1919	2879	4319	6479	9719	14579	21869	32804		
2559	3839	5759	8639	12959	19439	29159	43739	65609	98414	
5119	7679	11519	17279	25919	38879	58319	87479	131219	196829	295244
...

The triangle T(6):

6										
13	20									
27	41	62								
55	83	125	188							
111	167	251	377	566						
223	335	503	755	1133	1700					
447	671	1007	1511	2267	3401	5102				
895	1343	2015	3023	4535	6803	10205	15308			
1791	2687	4031	6047	9071	13607	20411	30617	45926		
3583	5375	8063	12095	18143	27215	40823	61235	91853	137780	
7167	10751	16127	24191	36287	54431	81647	122471	183707	275561	413342
...

The triangle T (n) that contains the first odd number of the Collatz sequence is what will be the beginning or first cycle of it.

In the columns of the table and in the rows of the triangles, the numbers:
 $a(n + 1) = (a(n) * 3 + 1) / 2$.

MULTIPLY BY 3 AND DIVIDE BETWEEN 2

If in triangle T (0) I add 1 to their numbers, another triangle T (1) results:

In each column of this triangle, the even numbers of the sequences and the odd numbers they reach after being subjected to n divisions by 2, which coincide with the columns in the table of even numbers.

A triangle $T(n)$ for each value of $(n) = 1, 5, 7, 11, 13, 17, 19, 23, 25, \dots (6n-1, 6n + 1)$ and we can write on them all even numbers.

In the rows of the triangle, each number: $a(n+1) = a(n) * 3/2$. All have as prime factors 3 and 2.

Multiplying by 3 and dividing by 2 is the script with which a sequence obtained with the Collatz algorithm is developed and this keeps the odd ones in the sequence in the same row.

Example: $8 * 3 = 24$, $24/2 = 12$, $12 * 3 = 36$, $36/2 = 18$, $18 * 3 = 54$, $54/2 = 27$. (numbers 7, 11, 17, 26 of the sequence).

To develop a sequence of Collatz in this triangle and to see its entire route, from its beginning until it reaches 1, we form a table with the triangle as a base.

Starting with each last number of the rows of the triangle, we apply these two operations: To the odd we add 1 and to the even we divide it by 2.

We write each result above the previous one, in the same column, until we reach 1.

In this extension of the triangle, the numbers are in red.

															1	1	...																
															1	1	2	2	...														
															1	2	2	3	4	...													
															1	1	2	3	4	5	7	...											
															1	1	2	3	4	5	10	14	...										
															1	2	3	4	5	6	9	13	19	28	...								
															1	1	2	3	4	5	8	11	17	25	37	55	...						
															1	1	2	3	4	5	7	10	15	22	33	49	73	110	...				
															1	2	3	4	5	7	10	15	22	33	49	73	110	219	...				
1	2	3	4	6	8	12	18	26	39	58	87	130	195	292	438	...	1	2	3	4	6	9	13	20	29	44	65	98	146	219	...		
2	3	5	7	11	16	23	35	52	77	116	173	260	390	584	876	...	2	3	5	7	11	16	23	35	52	77	116	173	260	390	584	876	
4	6	9	14	21	31	46	69	103	154	231	346	519	779	1168	1752	...	4	6	9	14	21	31	46	69	103	154	231	346	519	779	1168	1752	
8	12	18	27	41	61	92	137	206	308	462	692	1038	1557	2336	3504	...	8	12	18	27	41	61	92	137	206	308	462	692	1038	1557	2336	3504	
16	24	36	54	81	122	183	274	411	616	923	1384	2076	3114	4671	7007	...	16	24	36	54	81	122	183	274	411	616	923	1384	2076	3114	4671	7007	
32	48	72	108	162	243	365	547	821	1231	1846	2768	4152	6228	9342	14013	...	32	48	72	108	162	243	365	547	821	1231	1846	2768	4152	6228	9342	14013	...
64	96	144	216	324	486	729	1094	1641	2461	3691	5536	8304	12456	18684	28026	...	64	96	144	216	324	486	729	1094	1641	2461	3691	5536	8304	12456	18684	28026	...
128	192	288	432	648	972	1458	2187	3281	4921	7382	11072	16608	24912	37367	56051	...	128	192	288	432	648	972	1458	2187	3281	4921	7382	11072	16608	24912	37367	56051	...
256	384	576	864	1296	1944	2916	4374	6561	9842	14763	22144	33216	49823	74734	112101	...	256	384	576	864	1296	1944	2916	4374	6561	9842	14763	22144	33216	49823	74734	112101	...
512	768	1152	1728	2592	3888	5832	8748	13122	19683	29525	44287	66431	99646	149468	224202	...	512	768	1152	1728	2592	3888	5832	8748	13122	19683	29525	44287	66431	99646	149468	224202	...
1024	1536	2304	3456	5184	7776	11664	17496	26244	39366	59049	88574	132861	199291	298936	448404	...	1024	1536	2304	3456	5184	7776	11664	17496	26244	39366	59049	88574	132861	199291	298936	448404	...
2048	3072	4608	6912	10368	15552	23328	34992	52488	78732	118098	177147	265721	398581	597872	896807	...	2048	3072	4608	6912	10368	15552	23328	34992	52488	78732	118098	177147	265721	398581	597872	896807	...
4096	6144	9216	13824	20736	31104	46656	69984	104976	157464	236196	354294	531441	797162	1195743	1793614	...	4096	6144	9216	13824	20736	31104	46656	69984	104976	157464	236196	354294	531441	797162	1195743	1793614	...
8192	12288	18432	27648	41472	62208	93312	139968	209952	314928	472392	708588	1062882	1594323	2391485	3587227	...	8192	12288	18432	27648	41472	62208	93312	139968	209952	314928	472392	708588	1062882	1594323	2391485	3587227	...
16384	24576	36864	55296	82944	124416	186624	279936	419904	629856	944784	1417176	2125764	3188646	4782969	7174454	...	16384	24576	36864	55296	82944	124416	186624	279936	419904	629856	944784	1417176	2125764	3188646	4782969	7174454	...
32768	49152	73728	110592	165888	248832	373248	559872	839808	1259712	1889568	2834352	4251528	6377292	9565938	14348907	...	32768	49152	73728	110592	165888	248832	373248	559872	839808	1259712	1889568	2834352	4251528	6377292	9565938	14348907	...
...				

In this table we can visualize any sequence of Collatz, whose first odd number of it, is in the triangle. If it is not, it can be displayed in the table whose base or triangle contains that number. All sequences started with the odd numbers of a triangle row will have the same trajectory.

Like the triangles, the tables obtained from them, are unique for each $T(n)$ and the representation of the sequences will be made in the table formed from the triangle that contains the first odd number of the same.

For example, to visualize the sequence started with the number 508, whose first odd number is 127, we will represent it in the table of triangle $T(1)$, because in row 2^8 there is the number 127.

The sequence obtained with the Collatz algorithm, started with the number 508:

508, 254, 127, 382, 191, 574, 287, 862, 431, 1294, 647, 1942, 971, 2914, 1457, 4372, 2186, 1093, 3280, 1640, 820, 410, 205, 616, 308, 154, 77, 232, 116, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.

The black numbers are in the base triangle and the red ones are in the extension of this one.

The odd ones of the triangle: 127, 191, 287, 431, 647, 971, 1457 are in the same row, that is, none of them moves away from the 1, descending to lower rows.

The same happens with the odd ones of the red zone, none descends to lower rows and only when changing the column it descends to the lower row, but goes back up with the division between 2.

Each column change occurs when the odd number is multiplied by 3. The more odd the sequence has, the more column changes there will be and the longer it will reach 1.

Example: In the following table the odd 19 descends a row to the number 57 when it is multiplied by 3, but goes back up to 29 when the number 58 is divided by 2.

The development of a Collatz sequence started with the number 39:

												...
										1	1	...
								1	1	2	2	...
							1	2	2	3	4	...
						1	1	2	3	4	5	...
					1	1	2	2	3	5	7	...
				1	2	2	3	4	6	9	13	...
1	1	2	3	4	5	8	11	17	25	37	55	...
2	2	3	5	7	10	15	22	38	49	73	109	...
3	4	6	9	13	19	29	43	65	97	145	217	...
5	8	12	17	26	38	51	76	114	171	257	385	577
10	15	23	34	51	76	102	152	228	342	513	769	1154
20	30	45	68	102	152	205	304	456	684	1026	1538	2307
40	60	90	135	205	304	405	608	912	1367	2051	3076	4614
80	120	180	270	405	608	1215	1823	2734	4101	6151	9227	13840
160	240	360	540	810	1215	19440	29160	43740	65610	98415	147623	221434
320	480	720	1080	1620	2430	3645	5468	8202	12302	18453	27680	...
640	960	1440	2160	3240	4860	7290	10935	16403	24604	36906	55359	...
1280	1920	2880	4320	6480	9720	14580	21870	32805	49208	73812	110717	...
2560	3840	5760	8640	12960	19440	29160	43740	65610	98415	147623	221434	...
5120	7680	11520	17280	25920	38880	58320	87480	131220	196830	295245	442868	...
10240	15360	23040	34560	51840	77760	116640	174960	262440	393660	590490	885735	...
...

The sequence obtained with the Collatz algorithm, started with the number 39:

39, 118, 59, 178, 89, 268, 134, 67, 202, 101, 304, 152, 76, 38, 19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.

The sequence runs through the table developed from triangle T (5), because the first odd number in the sequence, 39, is in this triangle. The sequences started with the numbers 59, 89 and 134 will have the same route.

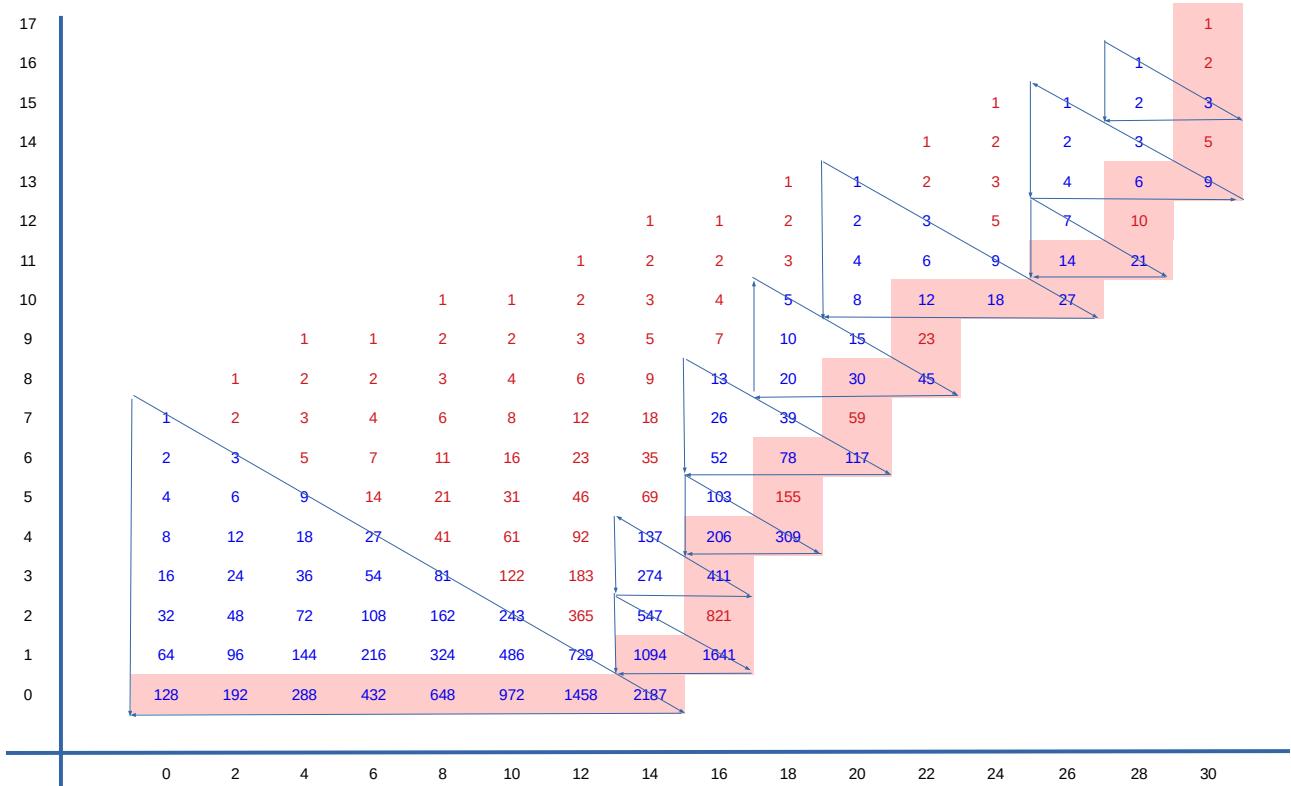
For triangle extension numbers, (in red), if 1 is subtracted from the odd number instead of adding 1, the table is equally valid:

Impar-1							
11	10	9	8	7	6	5	4
							1
						1	2
				1	1	2	4
			1	2	3	5	8
	1	1	2	4	6	10	16
1	2	3	5	8	13	20	
3	4	7	11	17	26	40	
6	9	14	22	34	52		
12	19	29	44				
25	38	58	88				
51	77	116					
	154	232					
0	1	2	3	4	5	6	7

Impar+1							
11	10	9	8	7	6	5	4
							1
						1	2
				1	2	2	3
			1	1	2	3	4
		1	2	2	3	5	8
	1	2	2	3	5	7	10
2	3	4	6	9	13	20	
4	5	8	11	17	26	40	
7	10	15	22	34	52		
13	20	29	44				
26	39	58	88				
51	77	116					
	154	232					
0	1	2	3	4	5	6	7

Next, the sequence table started with the number 127, considering as sequences cycles the columns of the odd numbers table and the rows of the triangles, since both are formed by the same numbers.

The sequence table started with the number 127 and the triangles involved or cycles:



The sequence has gone through 30 horizontal steps or iterations of odd numbers and 16 vertical steps or iterations of even numbers, divisions by 2.

Sequence started with the number 27 and the triangles or cycles that form it:

1									
2	3								
4	6	9							
8	12	18	27						
16	24	36	54	81					
32	48	72	108	162	243				
64	96	144	216	324	486	729			
128	192	288	432	648	972	1458	2187		
256	384	576	864	1296	1944	2916	4374	6561	
512	768	1152	1728	2592	3888	5832	8748	13122	19683
...

547									
1094	1641								
2188	3282	4923							
4376	6564	9846	14769						
8752	13128	19692	29538	44307					
17504	26256	39384	59076	88614	132921				
...	

127, 191, 287, 431, 647, 971, 1457, 2186

1093, 1640

103								
206	309							
412	618	927						
824	1236	1854	2781					
1648	2472	3708	5562	8343				
3296	4944	7416	11124	16686	25029			
...		

205, 308

13								
26	39							
52	78	117						
104	156	234	351					
208	312	468	702	1053				
416	624	936	1404	2106	3159			
...		

77, 116

5								
10	15							
20	30	45						
40	60	90	135					
80	120	180	270	405				
160	240	360	540	810	1215			
...		

29, 44

1								
2	3							
4	6	9						
8	12	18	27					
16	24	36	54	81				
32	48	72	108	162	243			
...		

11, 17, 26

7								
14	21							
28	42	63						
56	84	126	189					
112	168	252	378	567				
224	336	504	756	1134	1701			
...		

13, 20

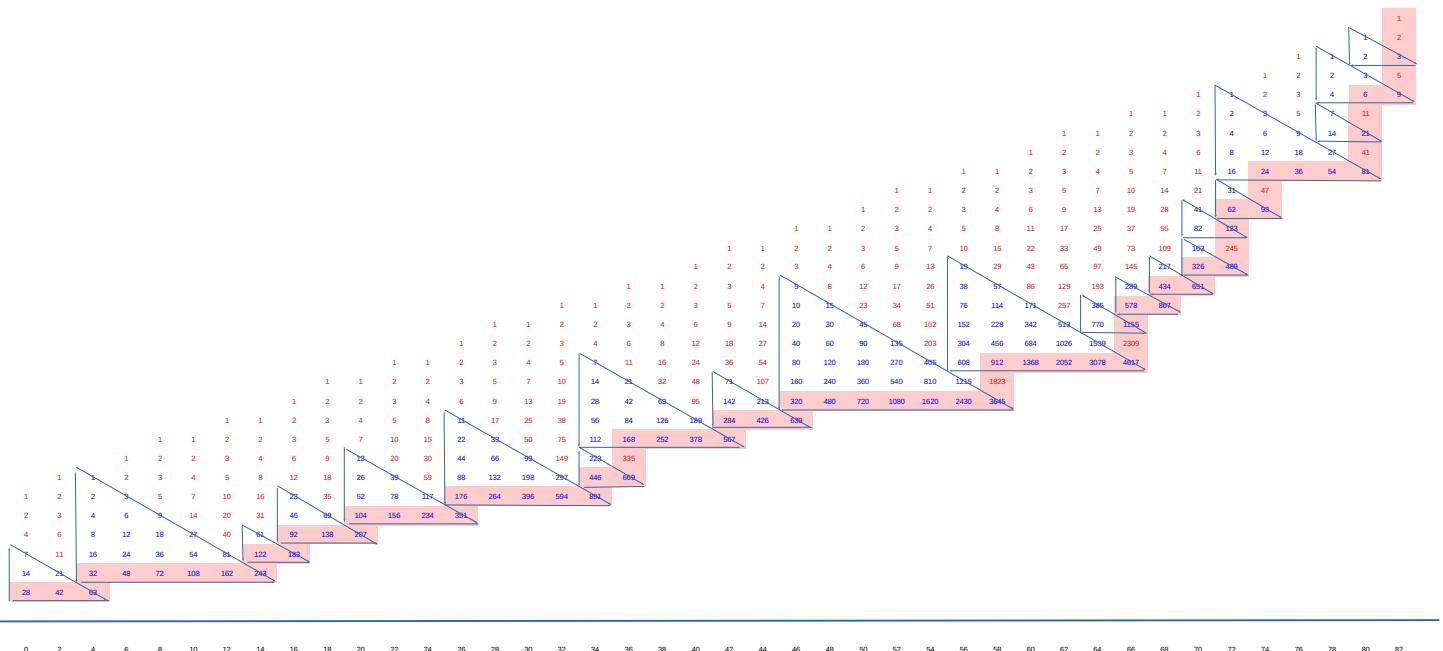
1								
2	3							
4	6	9						
8	12	18	27					
16	24	36	54	81				
32	48	72	108	162	243			
...		

5, 8

The sequence started with the number 27 in the 18 columns of the table or cycles:

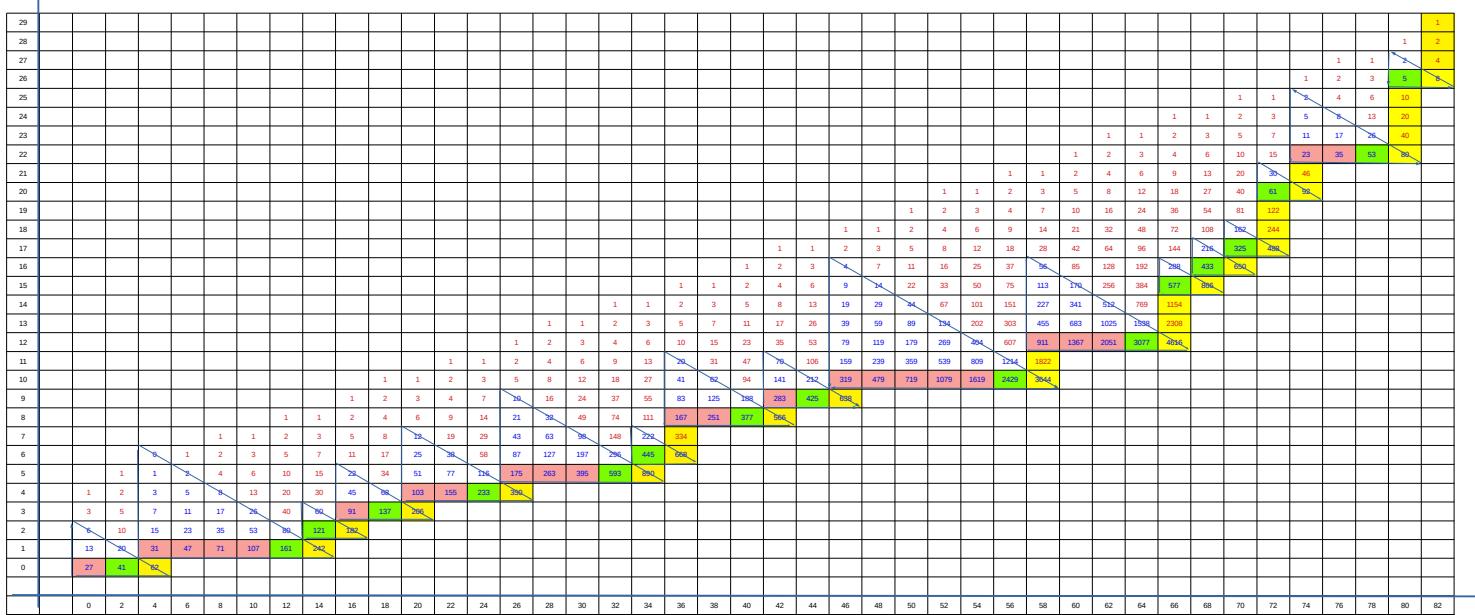
k	14	16	61	46	52	88	223	84	142	160	456	289	217	163	31	12	3	1
n																		
0	27	31	121	91	103	175	445	167	283	319	911	577	433	325	61	23	5	1
1	41	47	182	137	155	263	668	251	425	479	1367	866	650	488	92	35	8	2
	62	71		206	233	395		377	638	719	2051					53		
3		107			350	593		566		1079	3077					80		
4		161				890				1619	4616							
5		242								2429								
6										3644								
7																		
8																		
9																		

The same sequence in the table of triangles:



The sequence started with the number 27 has 82 horizontal iterations of the odd numbers and 29 vertical iterations of the even numbers.

We also see the development of the sequence with the columns of the odd number table, such as rows of triangles:



The rows of the triangles that form the cycles of the sequence, with the odd ones of the $4n + 3$ form in red, the odd ones of the $4n + 1$ form of green and the even numbers that join the cycles in yellow.

In the following table, a sequence started with the number 31:

0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
27	41	65	91	117	143	169	195	221	247	273	299	325	351	377	403	429	455	481	507	533	559	585	611	637	663	689	715	741	767	793	819	845	871	897	923	949	975	991	1017	1043	1069	1095	1121	1147	1173	1199	1225	1251	1277	1303	1329	1355	1381	1407	1433	1459	1485	1511	1537	1563	1589	1615	1641	1667	1693	1719	1745	1771	1797	1823	1849	1875	1901	1927	1953	1979	2005	2031	2057	2083	2109	2135	2161	2187	2213	2239	2265	2291	2317	2343	2369	2395	2421	2447	2473	2500	2526	2552	2578	2604	2630	2656	2682	2708	2734	2760	2786	2812	2838	2864	2890	2916	2942	2968	2994	3020	3046	3072	3098	3124	3150	3176	3202	3228	3254	3280	3306	3332	3358	3384	3410	3436	3462	3488	3514	3540	3566	3592	3618	3644	3670	3696	3722	3748	3774	3800	3826	3852	3878	3904	3930	3956	3982	4008	4034	4060	4086	4112	4138	4164	4190	4216	4242	4268	4294	4320	4346	4372	4410	4448	4486	4524	4562	4600	4638	4676	4714	4752	4790	4828	4866	4904	4942	4980	5018	5056	5094	5132	5170	5208	5246	5284	5322	5360	5398	5436	5474	5512	5550	5588	5626	5664	5702	5740	5778	5816	5854	5892	5930	5968	6006	6044	6082	6120	6158	6196	6234	6272	6310	6348	6386	6424	6462	6500	6538	6576	6614	6652	6690	6728	6766	6804	6842	6880	6918	6956	6994	7032	7070	7108	7146	7184	7222	7260	7298	7336	7374	7412	7450	7488	7526	7564	7602	7640	7678	7716	7754	7792	7830	7868	7906	7944	7982	8020	8058	8096	8134	8172	8210	8248	8286	8324	8362	8400	8438	8476	8514	8552	8590	8628	8666	8704	8742	8780	8818	8856	8894	8932	8970	9008	9046	9084	9122	9160	9198	9236	9274	9312	9350	9388	9426	9464	9502	9540	9578	9616	9654	9692	9730	9768	9806	9844	9882	9920	9958	9996	10034	10072	10110	10148	10186	10224	10262	10300	10338	10376	10414	10452	10490	10528	10566	10604	10642	10680	10718	10756	10794	10832	10870	10908	10946	10984	11022	11060	11098	11136	11174	11212	11250	11288	11326	11364	11402	11440	11478	11516	11554	11592	11630	11668	11706	11744	11782	11820	11858	11896	11934	11972	12010	12048	12086	12124	12162	12200	12238	12276	12314	12352	12390	12428	12466	12504	12542	12580	12618	12656	12694	12732	12770	12808	12846	12884	12922	12960	13018	13056	13094	13132	13170	13208	13246	13284	13322	13360	13408	13446	13484	13522	13560	13608	13646	13684	13722	13760	13808	13846	13884	13922	13960	14008	14046	14084	14122	14160	14208	14246	14284	14322	14360	14408	14446	14484	14522	14560	14608	14646	14684	14722	14760	14808	14846	14884	14922	14960	15008	15046	15084	15122	15160	15208	15246	15284	15322	15360	15408	15446	15484	15522	15560	15608	15646	15684	15722	15760	15808	15846	15884	15922	15960	16008	16046	16084	16122	16160	16208	16246	16284	16322	16360	16408	16446	16484	16522	16560	16608	16646	16684	16722	16760	16808	16846	16884	16922	16960	17008	17046	17084	17122	17160	17208	17246	17284	17322	17360	17408	17446	17484	17522	17560	17608	17646	17684	17722	17760	17808	17846	17884	17922	17960	18008	18046	18084	18122	18160	18208	18246	18284	18322	18360	18408	18446	18484	18522	18560	18608	18646	18684	18722	18760	18808	18846	18884	18922	18960	19008	19046	19084	19122	19160	19208	19246	19284	19322	19360	19408	19446	19484	19522	19560	19608	19646	19684	19722	19760	19808	19846	19884	19922	19960	20008	20046	20084	20122	20160	20208	20246	20284	20322	20360	20408	20446	20484	20522	20560	20608	20646	20684	20722	20760	20808	20846	20884	20922	20960	21008	21046	21084	21122	21160	21208	21246	21284	21322	21360	21408	21446	21484	21522	21560	21608	21646	21684	21722	21760	21808	21846	21884	21922	21960	22008	22046	22084	22122	22160	22208	22246	22284	22322	22360	22408	22446	22484	22522	22560	22608	22646	22684	22722	22760	22808	22846	22884	22922	22960	23008	23046	23084	23122	23160	23208	23246	23284	23322	23360	23408	23446	23484	23522	23560	23608	23646	23684	23722	23760	23808	23846	23884	23922	23960	24008	24046	24084	24122	24160	24208	24246	24284	24322	24360	24408	24446	24484	24522	24560	24608	24646	24684	24722	24760	24808	24846	24884	24922	24960	25008	25046	25084	25122	25160	25208	25246	25284	25322	25360	25408	25446	25484	25522	25560	25608	25646	25684	25722	25760	25808	25846	25884	25922	25960	26008	26046	26084	26122	26160	26208	26246	26284	26322	26360	26408	26446	26484	26522	26560	26608	26646	26684	26722	26760	26808	26846	26884	26922	26960	27008	27046	27084	27122	27160	27208	27246	27284	27322	27360	27408	27446	27484	27522	27560	27608	27646	27684	27722	27760	27808	27846	27884	27922	27960	28008	28046	28084	28122	28160	28208	28246	28284	28322	28360	28408	28446	28484	28522	28560	28608	28646	28684	28722	28760	28808	28846	28884	28922	28960	29008	29046	29084	29122	29160	29208	29246	29284	29322	29360	29408	29446	29484	29522	29560	29608	29646	29684	29722	29760	29808	29846	29884	29922	29960	30008	30046	30084	30122	30160	30208	30246	30284	30322	30360	30408	30446	30484	30522	30560	30608	30646	30684	30722	30760	30808	30846	30884	30922	30960	31008	31046	31084	31122	31160	31208	31246	31284	31322	31360	31408	31446	31484	31522	31560	31608	31646	31684	31722	31760	31808	31846	31884	31922	31960	32008	32046	32084	32122	32160	32208	32246	32284	32322	32360	32408	32446	32484	32522	32560	32608	32646	32684	32722	32760	32808	32846	32884	32922	32960	33008	33046	33084	33122	33160	33208	33246	33284	33322	33360	33408	33446	33484	33522	33560	33608	33646	33684	33722	33760	33808	33846	33884	33922	33960	34008	34046	34084	34122	34160	34208	34246	34284	34322	34360	34408	34446	34484	34522	34560	34608	34646	34684	34722	34760	34808	34846	34884	34922	34960	35008	35046	35084	35122	35160	35208	35246	35284	35322	35360	35408	35446	35484	35522	35560	35608	35646	35684	35722	35760	35808	35846	35884	35922	35960	36008	36046	36084	36122	36160	36208	36246	36284	36322	36360	36408	36446	36484	36522	36560	36608	36646	36684	36722	36760	36808	36846	36884	36922	36960	37008	37046	37084	37122	37160	37208	37246	37284	37322	37360	37408	37446	37484	37522	37560	37608	37646	37684	37722	37760	37808	37846	37884	37922	37960	38008	38046	38084	38122	38160	38208	38246	38284	38322	38360	38408	38446	38484	38522	38560	38608	38646	38684	38722	38760	3880

1	2	3	4	5	6	7	8	9	10
2	3	5	7	11	16	23	35	52	77
4	6	9	14	21	31	46	69	103	154
8	12	18	27	41	61	92	137	206	308
16	24	36	54	81	122	183	274	411	616
32	48	72	108	162	243	368	547	821	1231
64	96	144	216	324	496	729	1094	1641	2461
128	192	288	432	648	972	1458	2187	3281	4921
256	384	576	864	1296	1944	2916	4374	6561	9842
512	768	1152	1728	2592	3888	5832	8748	13122	19683
1024	1536	2304	3456	5184	7776	11664	17496	26244	39366
2048	3072	4608	6912	10368	15952	23328	34992	52488	78732

Table and the triangles of the sequence started with the number 27. You could complete all the numbers of each triangle, but they are not necessary to see the development of the sequence.

Applying $(3m + 1) / 2$ to an odd number causes the sequence to move to the next column of the table, but in the same row and ascends to higher rows when at the end of each cycle, the even number is divided by 2. In this table, ascending means approaching number 1.

In the table, each cycle or triangle is above the previous one and the sequence never descends to a lower row. Neither will it go parallel to the line of numbers 1 nor will it be separated, because the cycles or columns with the largest number of odd numbers (longer rows) are very scarce:

In the first 10,000 columns, the one with the largest amount of odd numbers is column k(8192), with 14 numbers. It is followed by column k(4096) with 13 numbers, two columns have 12 numbers, five columns have 11 numbers, ten columns with 10 numbers, twenty columns with 9 numbers, etc.

In the first 50,000, the column with the most numbers k(32768) has 16.

In the first 300,000, the column with the most numbers k(262144) has 19.

In the first 1,000,000, the column with the most numbers k(524288) has 20.

All the rows of the triangles or columns in the table are finite and their last number is even, so it will inevitably end in a number 1.

It is a visual demonstration that, although the numbers suffer increasing and decreasing oscillations, all sequences will ascend to number 1.

THE AMOUNT OF ODD NUMBERS in each column of the odd table and the amount of even numbers in the even table: 1, 2, 1, 3, 1, 2, 1, 4, 1, 2, . . . matches the number of digits 1 to the right of the last 0 in the binary representation of the odd numbers and the number of digits 0 to the right of the last 1 in the binary representation of the even numbers. Example:

1	3	5	7	9	11	13	15	17	19	21
1	11	101	111	1001	1011	1101	1111	10001	10011	10101
2	4	6	8	10	12	14	16	18	20	22
10	100	110	1000	1010	1100	1110	10000	10010	10100	10110
1	2	1	3	1	2	1	4	1	2	1

The last row corresponds to the amount of numbers in the columns of the tables and forms a fractal sequence with infinite sequences of the natural numbers, as follows: The numbers 1 are in the values of odd k and the following terms are in $2k$.

Example: The terms of the first sequence are in $k(1), k(2), k(4), k(8), k(16), \dots$

The terms of the second sequence are in $k(3), k(6), k(12), k(24), k(48), \dots$

The terms of the third sequence are in $k(5), k(10), k(20), k(40), k(80), \dots$

"**Impares**" is the amount of those numbers in every column.

Impars is the amount of those numbers in every column.
It is also the value of n of the even number at the end of each column.
It is also the number of steps or iterations that the first number of the column needs to reach the even number, applying the Collatz algorithm.

It is also the number of digits 1 on the right after 0, of odd numbers written in binary. Example: Column k (24) has 4 even numbers and the number 47 in binary is 101111.

This is a fractal sequence and forms infinite sequences of natural numbers, as for example, the sequence of odd terms is 1, 3, 5, 7, 9, 11, 13, ... and the following terms are in 2k.

The terms of the second sequence are $l(2), l(6), l(12), l(24), l(48)$

The terms of the third sequence are in $k(5), k(10), k(20), k(40), k(80)$.

"Pares" is the amount of those numbers in every column. It is also the value of n of the odd number at the end of each row.

It is also the number of steps or iterations that the first number of the column needs to reach the odd number, applying the Collatz algorithm.

It is also the number of digits 0 on the right after 1, of the even numbers written in binary. Example: Column k (24) has 4 even numbers and the number 48 in binary is 110000.

This is a fractal sequence and forms infinite sequences of natural numbers, as in the odd numbers table.

In the first column there is only 1 odd number.

In the first 3 columns, there are 4 odd numbers and one that has 2 odd numbers.

In the first 7 columns, there are 11 odd numbers and one that has 3 odd numbers.

In the first 15 columns, there are 26 odd numbers and one that has 4 odd numbers.

In the first 31 columns, there are 57 odd numbers and one that has 5 odd numbers.

In the first 63 columns, there are 120 odd numbers and one that has 6 odd numbers.

In the first k ($2^n - 1$), there are $2^n - n - 1$ odd numbers and the column k (2^{n-1}) has n odd numbers and is the one with the largest amount.

In each “triangle” $T(n)$ of the fractal there are $2 * T(n) + n$ odd numbers and the even number is $3^{\wedge} n-1$.

Example:

T(1)	$2*T(0)+1=$	1	odd number.
T(2)	$2*T(1)+2=$	4	odd numbers and the even 8.
T(3)	$2*T(2)+3=$	11	odd numbers and the even 26.
T(4)	$2*T(3)+4=$	26	odd numbers and the even 80.
T(5)	$2*T(4)+5=$	57	odd numbers and the even 242.
T(6)	$2*T(5)+6=$	120	odd numbers and the even 728.
T(7)	...		

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	...
	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71	73	75	77	79	81	83	85	87	89	91	93	95	97	99	101	103	105	107	109	111	113	115	117	119	121	123	125	...
2	5	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50	53	56	59	62	65	68	71	74	77	80	83	86	89	92	95	98	101	104	107	110	113	116	119	122	125	128	131	134	137	140	143	146	149	152	155	158	161	164	167	170	173	176	179	182	185	188	...	
	8		17		26		35		44		53		62		71		80		89		98		107		116		125		134		143		152		161		170		179		188		197		206		215		224		233		242		251		260		269		278	...		
	26				53				80							107		134			161			188				218			242			323				404			485				566			404	...															
							80																																					728				...																
	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	5	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	5	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	...																
1	3		7				15													31																						63																						
1		4																			11																						26																					
																						57																						120																				

1	1																																																
2	2	1																																															
4	3	1	2	1																																													
8	4	1	2	1	3	1	2	1	1																																								
16	5	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1																																	
32	6	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	5	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1																	
64	7	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	5	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	4	1	2	1	3	1	2	1	...

APPENDIX 1

Sequence started with the number 2251799813685247, which is the first number in column k (2^k) of the table.

In this column are the first 50 odd ones of the sequence, which are of the form $4n-1$ and occupy the even steps from 0 to 98. The odd 51^o, which is of the form $4n-3$ in step 100 and the pair 52^o which is the last number of the column, in step 102.

step: value

0:	2251799813685247
1:	6755399441055742
2:	3377699720527871
3:	10133099161583614
4:	5066549580791807
5:	15199648742375422
6:	7599824371187711
7:	22799473113563134
8:	11399736556781567
9:	34199209670344702
10:	17099604835172351
11:	51298814505517054
12:	25649407252758527
13:	76948221758275582
14:	38474110879137791
15:	115422332637413374
16:	57711166318706687
17:	173133498956120062
18:	86566749478060031
19:	259700248434180094
20:	129850124217090047
21:	389550372651270142
22:	194775186325635071
23:	584325558976905214
24:	292162779488452607
25:	876488338465357822
26:	438244169232678911
27:	1314732507698036734
28:	657366253849018367
29:	1972098761547055102
30:	986049380773527551
31:	2958148142320582654
32:	1479074071160291327
33:	4437222213480873982
34:	2218611106740436991
35:	6655833320221310974
36:	3327916660110655487
37:	9983749980331966462
38:	4991874990165983231
39:	14975624970497949694
40:	7487812485248974847
41:	22463437455746924542
42:	11231718727873462271
43:	33695156183620386814
44:	16847578091810193407
45:	50542734275430580222
46:	25271367137715290111
47:	75814101413145870334
48:	37907050706572935167
49:	113721152119718805502
50:	56860576059859402751

51: 170581728179578208254
52: 85290864089789104127
53: 255872592269367312382
54: 127936296134683656191
55: 383808888404050968574
56: 191904444202025484287
57: 575713332606076452862
58: 287856666303038226431
59: 863569998909114679294
60: 431784999454557339647
61: 1295354998363672018942
62: 647677499181836009471
63: 1943032497545508028414
64: 971516248772754014207
65: 2914548746318262042622
66: 1457274373159131021311
67: 4371823119477393063934
68: 2185911559738696531967
69: 6557734679216089595902
70: 3278867339608044797951
71: 9836602018824134393854
72: 4918301009412067196927
73: 14754903028236201590782
74: 7377451514118100795391
75: 22132354542354302386174
76: 11066177271177151193087
77: 33198531813531453579262
78: 16599265906765726789631
79: 49797797720297180368894
80: 24898898860148590184447
81: 74696696580445770553342
82: 37348348290222885276671
83: 112045044870668655830014
84: 56022522435334327915007
85: 168067567306002983745022
86: 84033783653001491872511
87: 252101350959004475617534
88: 126050675479502237808767
89: 378152026438506713426302
90: 189076013219253356713151
91: 567228039657760070139454
92: 283614019828880035069727
93: 850842059486640105209182
94: 425421029743320052604591
95: 1276263089229960157813774
96: 638131544614980078906887
97: 1914394633844940236720662
98: 957197316922470118360331
99: 2871591950767410355080994
100: 1435795975383705177540497
101: 4307387926151115532621492
102: 2153693963075557766310746
103: 1076846981537778883155373
104: 3230540944613336649466120
105: 1615270472306668324733060
106: 807635236153334162366530
107: 403817618076667081183265
108: 1211452854230001243549796
109: 605726427115000621774898
110: 302863213557500310887449
111: 908589640672500932662348
112: 454294820336250466331174
113: 227147410168125233165587
114: 681442230504375699496762

115: 340721115252187849748381
116: 1022163345756563549245144
117: 511081672878281774622572
118: 255540836439140887311286
119: 127770418219570443655643
120: 383311254658711330966930
121: 191655627329355665483465
122: 574966881988066996450396
123: 287483440994033498225198
124: 143741720497016749112599
125: 431225161491050247337798
126: 215612580745525123668899
127: 646837742236575371006698
128: 323418871118287685503349
129: 970256613354863056510048
130: 485128306677431528255024
131: 242564153338715764127512
132: 121282076669357882063756
133: 60641038334678941031878
134: 30320519167339470515939
135: 90961557502018411547818
136: 45480778751009205773909
137: 136442336253027617321728
138: 68221168126513808660864
139: 34110584063256904330432
140: 17055292031628452165216
141: 8527646015814226082608
142: 4263823007907113041304
143: 2131911503953556520652
144: 1065955751976778260326
145: 532977875988389130163
146: 1598933627965167390490
147: 799466813982583695245
148: 2398400441947751085736
149: 1199200220973875542868
150: 599600110486937771434
151: 299800055243468885717
152: 899400165730406657152
153: 449700082865203328576
154: 224850041432601664288
155: 112425020716300832144
156: 56212510358150416072
157: 28106255179075208036
158: 14053127589537604018
159: 7026563794768802009
160: 21079691384306406028
161: 10539845692153203014
162: 5269922846076601507
163: 15809768538229804522
164: 7904884269114902261
165: 23714652807344706784
166: 11857326403672353392
167: 5928663201836176696
168: 2964331600918088348
169: 1482165800459044174
170: 741082900229522087
171: 2223248700688566262
172: 1111624350344283131
173: 3334873051032849394
174: 1667436525516424697
175: 5002309576549274092
176: 2501154788274637046
177: 1250577394137318523
178: 3751732182411955570

179: 1875866091205977785
180: 5627598273617933356
181: 2813799136808966678
182: 1406899568404483339
183: 4220698705213450018
184: 2110349352606725009
185: 6331048057820175028
186: 3165524028910087514
187: 1582762014455043757
188: 4748286043365131272
189: 2374143021682565636
190: 1187071510841282818
191: 593535755420641409
192: 1780607266261924228
193: 890303633130962114
194: 445151816565481057
195: 1335455449696443172
196: 667727724848221586
197: 333863862424110793
198: 1001591587272332380
199: 500795793636166190
200: 250397896818083095
201: 751193690454249286
202: 375596845227124643
203: 1126790535681373930
204: 563395267840686965
205: 1690185803522060896
206: 845092901761030448
207: 422546450880515224
208: 211273225440257612
209: 105636612720128806
210: 52818306360064403
211: 158454919080193210
212: 79227459540096605
213: 237682378620289816
214: 118841189310144908
215: 59420594655072454
216: 29710297327536227
217: 89130891982608682
218: 44565445991304341
219: 133696337973913024
220: 66848168986956512
221: 33424084493478256
222: 16712042246739128
223: 8356021123369564
224: 4178010561684782
225: 2089005280842391
226: 6267015842527174
227: 3133507921263587
228: 9400523763790762
229: 4700261881895381
230: 14100785645686144
231: 7050392822843072
232: 3525196411421536
233: 1762598205710768
234: 881299102855384
235: 440649551427692
236: 220324775713846
237: 110162387856923
238: 330487163570770
239: 165243581785385
240: 495730745356156
241: 247865372678078
242: 123932686339039

243: 371798059017118
244: 185899029508559
245: 557697088525678
246: 278848544262839
247: 836545632788518
248: 418272816394259
249: 1254818449182778
250: 627409224591389
251: 1882227673774168
252: 941113836887084
253: 470556918443542
254: 235278459221771
255: 705835377665314
256: 352917688832657
257: 1058753066497972
258: 529376533248986
259: 264688266624493
260: 794064799873480
261: 397032399936740
262: 198516199968370
263: 99258099984185
264: 297774299952556
265: 148887149976278
266: 74443574988139
267: 223330724964418
268: 111665362482209
269: 334996087446628
270: 167498043723314
271: 83749021861657
272: 251247065584972
273: 125623532792486
274: 62811766396243
275: 188435299188730
276: 94217649594365
277: 282652948783096
278: 141326474391548
279: 70663237195774
280: 35331618597887
281: 105994855793662
282: 52997427896831
283: 158992283690494
284: 79496141845247
285: 238488425535742
286: 119244212767871
287: 357732638303614
288: 178866319151807
289: 536598957455422
290: 268299478727711
291: 804898436183134
292: 402449218091567
293: 1207347654274702
294: 603673827137351
295: 1811021481412054
296: 905510740706027
297: 2716532222118082
298: 1358266111059041
299: 4074798333177124
300: 2037399166588562
301: 1018699583294281
302: 3056098749882844
303: 1528049374941422
304: 764024687470711
305: 2292074062412134
306: 1146037031206067

307: 3438111093618202
308: 1719055546809101
309: 5157166640427304
310: 2578583320213652
311: 1289291660106826
312: 644645830053413
313: 1933937490160240
314: 966968745080120
315: 483484372540060
316: 241742186270030
317: 120871093135015
318: 362613279405046
319: 181306639702523
320: 543919919107570
321: 271959959553785
322: 815879878661356
323: 407939939330678
324: 203969969665339
325: 611909908996018
326: 305954954498009
327: 917864863494028
328: 458932431747014
329: 229466215873507
330: 688398647620522
331: 344199323810261
332: 1032597971430784
333: 516298985715392
334: 258149492857696
335: 129074746428848
336: 64537373214424
337: 32268686607212
338: 16134343303606
339: 8067171651803
340: 24201514955410
341: 12100757477705
342: 36302272433116
343: 18151136216558
344: 9075568108279
345: 27226704324838
346: 13613352162419
347: 40840056487258
348: 20420028243629
349: 61260084730888
350: 30630042365444
351: 15315021182722
352: 7657510591361
353: 22972531774084
354: 11486265887042
355: 5743132943521
356: 17229398830564
357: 8614699415282
358: 4307349707641
359: 12922049122924
360: 6461024561462
361: 3230512280731
362: 9691536842194
363: 4845768421097
364: 14537305263292
365: 7268652631646
366: 3634326315823
367: 10902978947470
368: 5451489473735
369: 16354468421206
370: 8177234210603

371: 24531702631810
372: 12265851315905
373: 36797553947716
374: 18398776973858
375: 9199388486929
376: 27598165460788
377: 13799082730394
378: 6899541365197
379: 20698624095592
380: 10349312047796
381: 5174656023898
382: 2587328011949
383: 7761984035848
384: 3880992017924
385: 1940496008962
386: 970248004481
387: 2910744013444
388: 1455372006722
389: 727686003361
390: 2183058010084
391: 1091529005042
392: 545764502521
393: 1637293507564
394: 818646753782
395: 409323376891
396: 1227970130674
397: 613985065337
398: 1841955196012
399: 920977598006
400: 460488799003
401: 1381466397010
402: 690733198505
403: 2072199595516
404: 1036099797758
405: 518049898879
406: 1554149696638
407: 777074848319
408: 2331224544958
409: 1165612272479
410: 3496836817438
411: 1748418408719
412: 5245255226158
413: 2622627613079
414: 7867882839238
415: 3933941419619
416: 11801824258858
417: 5900912129429
418: 17702736388288
419: 8851368194144
420: 4425684097072
421: 2212842048536
422: 1106421024268
423: 553210512134
424: 276605256067
425: 829815768202
426: 414907884101
427: 1244723652304
428: 622361826152
429: 311180913076
430: 155590456538
431: 77795228269
432: 233385684808
433: 116692842404
434: 58346421202

435: 29173210601
436: 87519631804
437: 43759815902
438: 21879907951
439: 65639723854
440: 32819861927
441: 98459585782
442: 49229792891
443: 147689378674
444: 73844689337
445: 221534068012
446: 110767034006
447: 55383517003
448: 166150551010
449: 83075275505
450: 249225826516
451: 124612913258
452: 62306456629
453: 186919369888
454: 93459684944
455: 46729842472
456: 23364921236
457: 11682460618
458: 5841230309
459: 17523690928
460: 8761845464
461: 4380922732
462: 2190461366
463: 1095230683
464: 3285692050
465: 1642846025
466: 4928538076
467: 2464269038
468: 1232134519
469: 3696403558
470: 1848201779
471: 5544605338
472: 2772302669
473: 8316908008
474: 4158454004
475: 2079227002
476: 1039613501
477: 3118840504
478: 1559420252
479: 779710126
480: 389855063
481: 1169565190
482: 584782595
483: 1754347786
484: 877173893
485: 2631521680
486: 1315760840
487: 657880420
488: 328940210
489: 164470105
490: 493410316
491: 246705158
492: 123352579
493: 370057738
494: 185028869
495: 555086608
496: 277543304
497: 138771652
498: 69385826

499: 34692913
500: 104078740
501: 52039370
502: 26019685
503: 78059056
504: 39029528
505: 19514764
506: 9757382
507: 4878691
508: 14636074
509: 7318037
510: 21954112
511: 10977056
512: 5488528
513: 2744264
514: 1372132
515: 686066
516: 343033
517: 1029100
518: 514550
519: 257275
520: 771826
521: 385913
522: 1157740
523: 578870
524: 289435
525: 868306
526: 434153
527: 1302460
528: 651230
529: 325615
530: 976846
531: 488423
532: 1465270
533: 732635
534: 2197906
535: 1098953
536: 3296860
537: 1648430
538: 824215
539: 2472646
540: 1236323
541: 3708970
542: 1854485
543: 5563456
544: 2781728
545: 1390864
546: 695432
547: 347716
548: 173858
549: 86929
550: 260788
551: 130394
552: 65197
553: 195592
554: 97796
555: 48898
556: 24449
557: 73348
558: 36674
559: 18337
560: 55012
561: 27506
562: 13753

563: 41260
564: 20630
565: 10315
566: 30946
567: 15473
568: 46420
569: 23210
570: 11605
571: 34816
572: 17408
573: 8704
574: 4352
575: 2176
576: 1088
577: 544
578: 272
579: 136
580: 68
581: 34
582: 17
583: 52
584: 26
585: 13
586: 40
587: 20
588: 10
589: 5
590: 16
591: 8
592: 4
593: 2
594: 1