# The generalized De Rham cohomology

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**Abstract**

Here is defined a genaralization of the de Rham cohomology by mean of a smooth function over the differentiable manifold.

## 1 The de Rham cohomology

Let $M$ be a differentiable manifold, then we can construct the exterior algebra of differentiable forms:

$$\Lambda^*(M)$$

The differentiable operator $d$ is defined over the forms with help of the Leibnitz rule. It has the property that:

$$d(a \wedge b) = d(a) \wedge b + (-1)^{deg(a)} a \wedge d(b)$$

$$d \circ d = 0$$

Then the de Rham cohomology is:

$$H^*(M, \mathbf{R}) = Ker(d)/Im(d)$$

## 2 The generalization of the de Rham cohomology

We take a smooth function $f$ and we define: 0) over the functions:

$$d_f(g) = f dg$$

1) over the 1-forms:

$$d_f(a) = f da - df \wedge a$$

2) over the $k$-forms:

$$d_f(a \wedge b) = d_f(a) \wedge b + (-1)^{deg(a)} a \wedge d_f(b)$$

We can verify that it is a differential operator:

$$d_f \circ d_f = 0$$

Then the generalized de Rahm cohomology is:

$$H_f^*(M, \mathbf{R}) = Ker(d_f)/Im(d_f)$$

## References

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