Triple Arccosines Theorem

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Theorem. If $\alpha_1 + \beta_1 = \pi/2$,

$\alpha_2 + \beta_2 = \pi/2$,

$\alpha_3 + \beta_3 = \pi/2$, then:

$$\arccos(\cos\alpha_1\cos\beta_3) + \arccos(\cos\alpha_2\cos\beta_1) + \arccos(\cos\alpha_3\cos\beta_2) = \pi$$

Proof. Let $ABCDA_1B_1C_1D_1$ be a cube (see the figure).

Let $M \subseteq [AB]$, $N \subseteq [BB_1]$, $L \subseteq [BC]$.

Let $\alpha_1 = \angle NMB$, $\alpha_2 = \angle BNL$, $\alpha_3 = \angle BLM$,

$\beta_1 = \angle MNB$, $\beta_2 = \angle NLB$, $\beta_3 = \angle BML$,

Then, considering the triangles MNB, NBL and MBL we have:

$\alpha_1 + \beta_1 = \pi/2$,

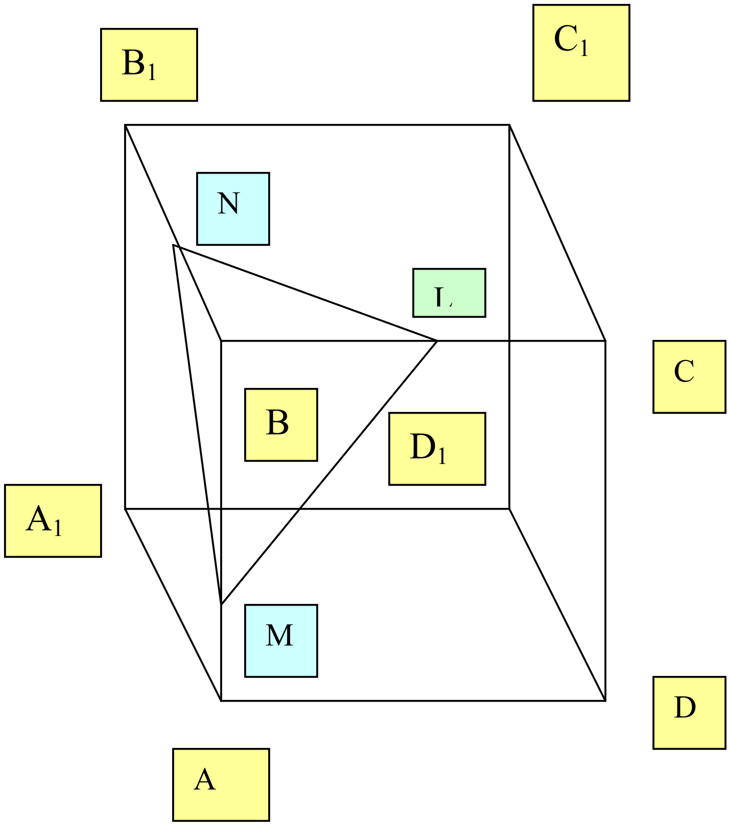
$\alpha_2 + \beta_2 = \pi/2$,

$\alpha_3 + \beta_3 = \pi/2$.

By applying the Triple Cosines Theorem(see http://vixra.org/abs/1909.0028)

to the triangle MNL we finally have:

$$\arccos(\cos\alpha_1\cos\beta_3) + \arccos(\cos\alpha_2\cos\beta_1) + \arccos(\cos\alpha_3\cos\beta_2) = \pi$$



B1
C1
N
L
C
B
D1
A1
M
D
A