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1. ABSTRACT

In this work has been calculated two new DM density profiles inside halo region of M31 galaxy and it has been demonstrated that both ones are mathematically equivalents. Its radius dominion is only the halo region because it is needed that baryonic matter density has to be negligible.

The first profile is called direct DM density because it is got directly from rotation curve and represents DM density depending on radius

The second one, called Bernoulli because it is got from a Bernoulli differential equation, represents DM density depending on local gravitational field according a power law. The power of E is called B.

Hypothesis which is the basis to get Bernoulli profile stated that DM is generated locally by the own gravitational field according this formula. DM density = A· E^B where A& B are coefficients and E is gravitational intensity of field.

Briefly will be explained method followed to develop this paper. Rotation curve data come from [5] Sofue,Y.2015. Thanks this remarkable rotation curve, the regression curve of velocity depending on radius has a correlation coefficient bigger than 0.96 and data range from 40 kpc up to 300 kpc.

In fourth chapter it is got the function of DM density depending on radius, called direct DM density.

In fifth chapter it is demonstrated that function direct DM density is mathematically equivalent to the function DM density depending on E. Furthermore it is stated the mathematical relation between both functions. Namely, it is got value B =1.6682

In sixth chapter it is got that for radius bigger than 40 kpc the ratio baryonic density versus DM density is under 1% so it is reasonable to consider negligible baryonic matter density in order to simplify calculus.

In seventh chapter it is got a Bernoulli differential equation for field and is solved.

In eighth chapter it is made dimensional analysis for magnitudes Density, field E and universal constants G, h and c. It is demonstrated that it is needed a formula with two Pi monomials. It is found that B = 5/3 is the value coherent with Buckingham theorem and differs only two thousandth regarding B=1.6682 which was got by regression analysis.

In ninth chapter are recalculated parameters a,b and A as a consequence of being B=5/3 instead B=1.688 and are written newly the formulas with these new values. Thanks this change the formulas are now dimensionally right.

In tenth chapter are calculated some different type of DM masses in M31 to be compared with DM calculated by Sofue. There is a good agreement between both results.

In the eleventh chapter through the Hubble law it is demonstrated that galaxies in the ancient universe were smaller than at present and as a consequence it is got that proportion of DM versus baryonic matter was lower in the ancient universe. Results got are in agreement with current observational evidences.

In the twelfth chapter is estimated total mass of Local Group of galaxies as 5*10^12 Msun which is in perfect agreement with current accepted value based on measures of relative velocity between Milky Way and M31.

2. INTRODUCTION

As reader knows M31 is the twin galaxy of Milky Way in Local Group of galaxies. Its disk radius is approximately 35 kpc and according [5] Sofue, Y. 2015. Its baryonic mass is $M_{BARYONIC} = 1{,}61 \cdot 10^{11}\ M_{SUN}$
The DM theory introduced in [1] Abarca, M.2014. *Dark matter model by quantum vacuum* considers that DM is generated by the own gravitational field. Therefore, in order to study purely the phenomenon it is needed to consider a radius dominion where it is supposed that baryonic matter is negligible.

In previous paper [10] Abarca,M.2016. *A New Dark Matter Density Profile for M31 Galaxy to Demonstrate that Dark Matter is Generated by Gravitational Field,* author has studied DM inside M31 halo through Bernoulli DM profile. However in such paper DM density used it was NFW profile provided by [5] Sofue, Y. 2015 whereas in current paper DM density profile has been got directly from a power regression function on rotation curve in halo region.

This new DM profile has been called direct DM density because this profile is fitted directly from data measures inside halo region. In this work radius dominion begin at 40 kpc because at this distance baryonic density is negligible as it will be shown in chapter six. Therefore the only type of matter in halo region it is supposed to be non baryonic dark matter and it is quite simple to state the differential equation for field in these conditions.

It is known that there is baryonic dark matter such us giant planets, cold gas clouds, brown dwarfs but this kind of DM is more probable to be placed inside galactic disk. Reader can consult: [11] Nieuwenhuizen,T.M. 2010. [12] Nieuwenhuizen,T.M. 2012. [13] Nieuwenhuizen,T.M. 2010 [14] Wyrzykowski,L.2010. [15] M.R.S. Hawkins 2015.In fact there are an important amount of researchers in this way because baryonic DM and non baryonic DM are open problems still.

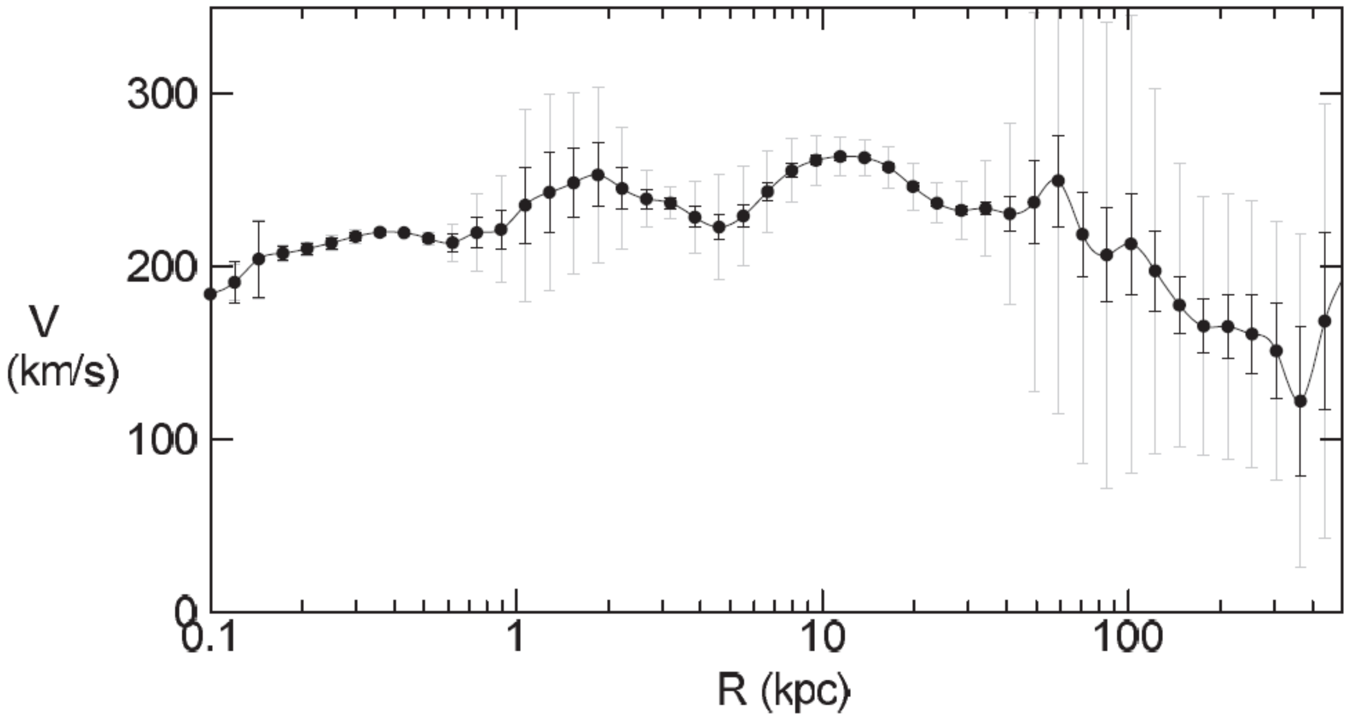
As it is known, NFW profile is fitted over bulge, disk and galactic halo and taking in consideration that there is an unknown amount of baryonic DM in bulge and galactic disk it is needed concluded that NFW profile is more imprecise than direct DM profile in order to study non baryonic DM in halo because direct DM density has been fitted exclusively with data of DM non baryonic in halo region.

In fact NFW density profile produce a bigger DM density throughout the whole halo region where is possible to compare direct DM density and NFW density and consequently the total amount of DM at a specific radius always is a bit bigger when ii is calculated with NFW density, as it will be shown in chapter 10.

In chapter seven it will be got a simple Bernoulli differential equation for gravitational field. However to get a so simple differential equation it is needed that $M'(r) = 4\pi r^2 \varphi_{DM}(r)$ .In other words, it is needed that density of baryonic matter would be negligible versus D.M. density.

Several previous papers such as [2] Abarca,M.2015 and others have studied DM density as power of gravitational field in several galaxies: Milky Way, M33, NGC3198 and others galaxies. The results got support the formula for DM density depending on gravitational field $\varphi_{DM}(r) = A \cdot E^B$ being A&B quite similar for different giant galaxies.

3. OBSERVATIONAL DATA FROM SOFUE. 2015 PAPER



Graphic come from [5] Sofue,Y. 2015. The axis for radius has logarithmic scale. In previous version V2 of this work, dominion extended up to 252 kpc, whereas in this version dominion reach up to 303 kpc. In the following epigraph it will explained the reason for this extension.

kpc	km/s
40,5	229,9
49,1	237,4
58,4	250,5
70,1	219,2
84,2	206,9
101,1	213,5
121,4	197,8
145,7	178,8
175	165,6
210,1	165,6
252,3	160,7
302,9	150,8

As in previous version, in chapter six will be shown reason why dominion data begin at 40 kpc in this work, although it is accepted that disk radius of M31 is approximately 35 kpc.

3.1 POWER REGRESSION TO ROTATION CURVE

The measures of rotation curve have a very good fitted curve by power regression.

In particular coefficients of $v = a{\cdot}r^{b}$ are in table below. Units are into I.S.

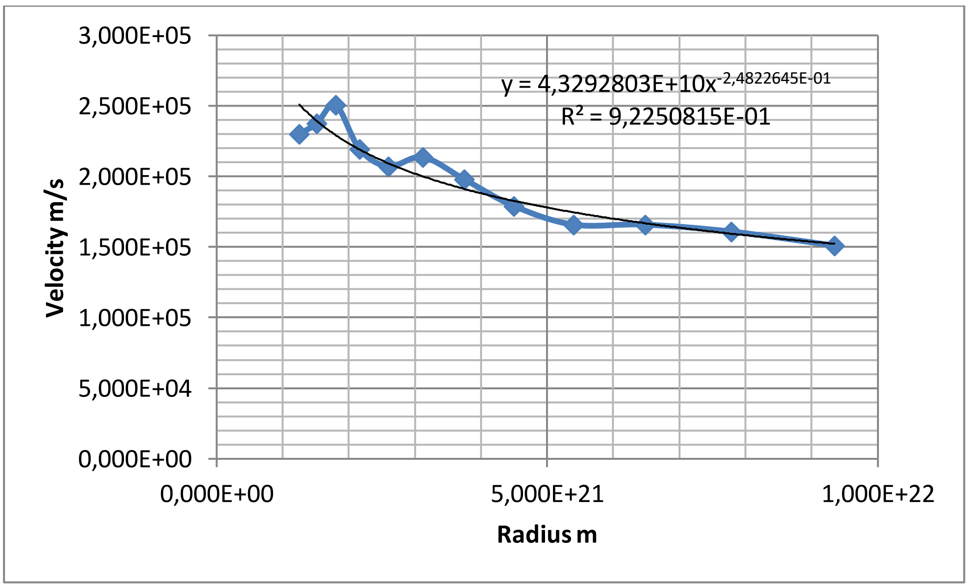
Power regression for M31 rot. curve	
V=a*r^b	
a	$4{,}32928{*}10^{10}$
b	-0.24822645
Correlation coeff.	0,96

Data fitted are in grey columns below.

In fifth column is shown results of fitted velocity and sixth column shows relative difference between measures and fitted results.

Radius kpc	Vel. km/s	Radius m	Vel. m/s	Vel. fitted	Relative Diff.
40,5	229,9	1,250E+21	2,299E+05	2,510E+05	8,397E-02
49,1	237,4	1,515E+21	2,374E+05	2,393E+05	7,777E-03
58,4	250,5	1,802E+21	2,505E+05	2,292E+05	-9,304E-02
70,1	219,2	2,163E+21	2,192E+05	2,190E+05	-8,154E-04
84,2	206,9	2,598E+21	2,069E+05	2,093E+05	1,138E-02
101,1	213,5	3,120E+21	2,135E+05	2,000E+05	-6,755E-02
121,4	197,8	3,746E+21	1,978E+05	1,911E+05	-3,500E-02
145,7	178,8	4,496E+21	1,788E+05	1,826E+05	2,107E-02
175	165,6	5,400E+21	1,656E+05	1,745E+05	5,115E-02
210,1	165,6	6,483E+21	1,656E+05	1,668E+05	7,100E-03
252,3	160,7	7,785E+21	1,607E+05	1,594E+05	-8,307E-03
302,9	150,8	9,347E+21	1,508E+05	1,523E+05	9,891E-03

Below is shown a graphic with measures data and power regression function.



Correlation coefficient equal to 0,96 is a hundredth bigger than data published in previous version V2 of this work, because this time it has been considerate a wide dominion up to 303 kpc. According theory of DM generated by field, halo extend up to a half of distance to Milky Way, 375 kpc, consequently the data for radius 303 kpc is trustworthy.

Furthermore, it has been calculated regression curve with another data placed at 363 kpc, but power regression is -0.28 and correlation coefficient is 0.954. This result shows that such data is not trustworthy because according dimensional analysis power has to be -0.25. As 363 kpc is placed in the border of M31 halo it is possible that such data might be influenced by a different field. Therefore it is better to study data only up to 303 kpc.

4. DIRECT FORMULA FOR DM DENSITY ON M31 HALO GOT FROM ROTATION CURVE

4.1 THEORETICAL DEVELOPPMENT FOR GALACTIC HALOS

Outside disk region, rotation curve it is fitted by power regression with a high correlation coefficient according formula $v = a{\cdot}r^b$. As $M(<r) = \frac{v^2{\cdot}R}{G}$ represents total mass enclosed by a sphere with radius r, by substitution of velocity results $M = \frac{v^2{\cdot}R}{G} = \frac{a^2{\cdot}r^{2b+1}}{G}$

If it is considered outside region of disk where baryonic matter is negligible regarding dark matter it is possible to calculate DM density by a simple derivative. In next chapter will be show that for r > 40 kpc baryonic matter is negligible.

As density of D.M. is $D_{DM} = \frac{dm}{dV}$ where $d\,m = \frac{a^2\cdot(2b+1){\cdot}r^{2b}dr}{G}$ and $d\,V = 4\pi r^2 dr$ results

$$D_{DM} = \frac{a^2\cdot(2b+1)}{4\pi G}{\cdot}r^{2b-2}$$

Writing $L = \frac{a^2\cdot(2b+1)}{4\pi G}\cdot$ results $D_{DM}(r) = L{\cdot}r^{2b-2}$. In case b = -1/2 DM density is cero which is Keplerian rotation.

4.2 DIRECT DM DENSITY FOR M31 HALO

Parameters a & b from power regression of M31 rotation curve allow calculate easily direct DM density

Direct DM density for M31 halo 40 < r < 300 kpc
$D_{DM}(r) = L{\cdot}r^{2b-2}$ kg/m^3

5. DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

As independent variable for this function is E, gravitational field, previously will be studied formula for E in the following paragraph.

5.1 GRAVITATIONAL FIELD E THROUGH VIRIAL THEOREM

As it is known total gravitational field may be calculated through Virial theorem, formula $E = v^2/R$ whose I.S. unit is m/s^2 is well known. Hereafter, virial gravitational field, E, got through this formula will be called E.

By substitution of $v = a{\cdot}r^b$ in formula $E = \dfrac{v^2}{r}$ it is right to get $E = \dfrac{a^2{\cdot}r^{2b}}{r} = a^2{\cdot}r^{2b-1}$ briefly $E = a^2{\cdot}r^{2b-1}$

5.2 DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

According hypothesis dark matter by quantum vacuum $D_{DM} = A{\cdot}E^B$. Where A & B are parameters to be calculated. This hypothesis has been widely studied by author in previous papers. [1] Abarca,M. [2] Abarca,M.

[8] Abarca,M. [9] Abarca,M. [10] Abarca,M.

As it is known direct DM density $D_{DM} = \dfrac{a^2\cdot(2b+1)}{4\pi G}{\cdot}r^{2b-2}$ depend on a & b parameters which come from power regression formula for velocity. In previous paragraph has been shown formula for gravitational field $E = \dfrac{a^2{\cdot}r^{2b}}{r} = a^2{\cdot}r^{2b-1}$ which depend on a & b as well. Through a simple mathematical treatment it is possible to get A & B to find function of DM density depending on E. Specifically formulas are $A = \dfrac{a^{\frac{2}{2b-1}}\cdot(2b+1)}{4\pi G}$ & $B = \dfrac{2b-2}{2b-1}$.

According parameters a & b got in previous chapter, A& B parameters are:

M31 galaxy	$D_{DM} = A{\cdot}E^B$
A	3,6559956 $\cdot 10^{-6}$
B	1,6682469

As conclusion, in this chapter has been demonstrated that a power law for velocity

$v = a{\cdot}r^b$ *is mathematically equivalent to a power law for DM density depending on E.* $D_{DM} = A{\cdot}E^B$

5.3 THE IMPORTANCE OF $D_{DM} = A{\cdot}E^B$

This formula is vital for theory of dark matter generated by gravitational field because it is supposed that DM is generated locally according an unknown quantum gravity mechanism. In other words, the propagation of gravitational field has this additional effect on the space as the gravitational wave goes by.

The formulas $D_{DM} = \dfrac{a^2\cdot(2b+1)}{4\pi G}{\cdot}r^{2b-2}$ and $E = a^2{\cdot}r^{2b-1}$ have been got rightly from rotation curve. Therefore it can be considered more specific for each galaxy. However the formula $D_{DM} = A{\cdot}E^B$ is much more essential.

The basis of this theory is that such formula is right for different gravitational systems. Therefore A & B parameters have to be the same for different galaxies. This is the initial hypothesis of this theory. However, there is an important fact to highlight. It is clear that A depend on a and b, both parameters are global parameters.

As the gravitational interaction time between masses is proportional to distance, it is right to think that DM generated by a gravitational field has a bigger proportion as the system increase its size. For example inside the Solar system it is clear that Newton and General Relativity Theory is able to explain with total accuracy every gravitational phenomenon without DM hypothesis. Therefore it is right to conclude that DM arises when gravitational interaction takes a longer time to link the matter. Namely, for galaxy scale or bigger systems.

Furthermore, there are clear observational evidences that inside cluster of galaxies the proportion of DM is bigger than inside galaxies. In other words, it is right to think that A&B might be different a bigger scale. Namely galaxy cluster scale.

However, there are observational evidences of DM inside dwarf and medium size galaxies that show a bigger proportion of DM than inside giant galaxies.

In my opinion this fact could be explained by other reasons. For example dwarf galaxies are always orbiting near giant galaxies, so it is possible that the proportion of baryonic matter cold, which is unobservable, could be bigger. Anyway this is an open problem for current cosmology.

To sum up, regarding theory of DM generated by gravitational field, parameters A&B has to be the same for different gravitational system on condition they have the same size. i.e. two similar giant galaxies should have the same parameters A&B. However, a bigger gravitational system. i.e. galaxy cluster should have bigger parameter in order to produce a bigger fraction of D.M. Nonetheless, in chapter 9, it will be shown that total DM increase with the square root of distance. For example, the proportion of DM inside galactic disk of M31 is lower than the proportion when it is considered the whole halo whose radius is 350 kpc, so the maximum proportion goes up to 90% of DM versus baryonic matter.

6. RATIO BARYONIC MASS VERSUS DARK MATTER MASS DEPENDING ON RADIUS FOR M31

In this paragraph will be estimated radius which is needed to consider negligible baryonic density regarding DM density in M31 galaxy.

[5] According Sofue, Y. data for M31 disk are

M31 Galaxy	Baryonic Mass at disk	a_d	Σ_0
	$M_d = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$		
	$M_d = 1{,}26 \cdot 10^{11}$ Msun	5,28 kpc	1,5 kg/m^2

Where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ represents superficial density at disk. Total mass disk is given by integration of superficial density from cero to infinite. $M_d = \int_0^\infty 2\pi \cdot r\Sigma(r) \cdot dr = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$

In order to compare baryonic density and DM density it is considered differential baryonic mass and differential DM masses depending on radius.

$dM_{DISK} = 2\pi r\Sigma(r)dr$ where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ and

$dM_{DM} = 4\pi r^2 D_{DM}(r)dr$ where $D_{DM}(r) = \dfrac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$

It is defined ratio function as quotient of both differential quantities $Ratio = \dfrac{dM_{DISK}}{dM_{DM}} = \dfrac{\Sigma(r)}{2 \cdot r \cdot D_{DM}(r)}$

Radius	Radius	Ratio (r)	$\Sigma(r)$	Direct DM
Kpc	m	Ratio	kg/m^2	kg/m^3
36	1,110852E+21	2,310614E-02	1,64056151250E-03	3,1957946476E-23

38	1,172566E+21	1,715255E-02	1,12327743139E-03	2,7924857817E-23
40	1,234280E+21	1,268028E-02	7,69097762116E-04	2,4570213865E-23
42	1,295994E+21	9,339073E-03	5,26594188719E-04	2,1754010061E-23
44	1,357708E+21	6,854954E-03	3,60554214629E-04	1,9370002366E-23

For a radius 40 kpc ratio baryonic matter versus DM is only 1,2 % therefore is a good approximation to consider negligible baryonic mass density regarding DM density when radius is bigger than 40 kpc.
This is the reason why in this work dominion for radius begin at 40 kpc.

7. A DIFFERENTIAL EQUATION FOR A GRAVITATIONAL FIELD

7.1 INTRODUCTION

This formula $D_{DM} = \frac{a^2\cdot(2b+1)}{4\pi G}\cdot r^{2b-2}$ is a local formula because it has been got by differentiation. However E, which represents a local magnitude $E = \frac{G\cdot M(<r)}{r^2} = \frac{a^2\cdot r^{2b}}{r} = a^2\cdot r^{2b-1}$ has been got through $v = a\cdot r^b$ whose parameters a & b were got by a regression process on the whole dominion of rotation speed curve. Therefore, D_{DM} formula has a character more local than E formula because the former was got by a differentiation process whereas the later involves M(<r) which is the mass enclosed by the sphere of radius r.

In other words, the process of getting D_{DM} involves a derivative whereas the process to get E(r) involves M(r) which is a global magnitude. This is a not suitable situation because the formula $D_{DM} = A\cdot E^B$ involves two local magnitudes. Therefore it is needed to develop a new process with a more local nature or character.

It is clear that a differential equation for E is the best method to study locally such magnitude.

7.2 A DIFFERENTIAL BERNOULLI EQUATION FOR GRAVITATIONAL FIELD IN A GALACTIC HALO

As it is known in this formula $\vec{E} = -G\frac{M(r)}{r^2}\hat{r}$, M(r) represents mass enclosed by a sphere with radius r. If it is considered a region where does not exit any baryonic matter, such as any galactic halo, then the derivative of M(r) depend on dark matter density essentially and therefore $M'(r) = 4\pi r^2\varphi_{DM}(r)$.

If $E = G\frac{M(r)}{r^2}$, vector modulus, is differentiated then it is got $E'(r) = G\frac{M'(r)\cdot r^2 - 2rM(r)}{r^4}$

If $M'(r) = 4\pi r^2\varphi_{DM}(r)$ is replaced above then it is got $E'(r) = 4\pi G\varphi_{DM}(r) - 2G\frac{M(r)}{r^3}$ As $\varphi_{DM}(r) = A\cdot E^B(r)$ it is right to get $E'(r) = 4\pi\cdot G\cdot A\cdot E^B(r) - 2\frac{E(r)}{r}$ which is a Bernoulli differential equation.

$$E'(r) = K\cdot E^B(r) - 2\frac{E(r)}{r} \text{ being } K = 4\pi\cdot G\cdot A$$

Calling ***y*** to E, the differential equation is written in this simple way $y' = K\cdot y^B - \frac{2\cdot y}{r}$

Bernoulli family equations $y` = K\cdot y^{B} - \frac{2\cdot y}{r}$ may be converted into a differential linear equation with this variable change $u = y^{1-B}$. Which is $\frac{u`}{1-B} + \frac{2u}{r} = K$

The homogenous equation is $\frac{u`}{1-B} + \frac{2u}{r} = 0$ Whose general solution is $u = C\cdot r^{2B-2}$ being C the integration constant.

If it is searched a particular solution for the complete differential equation with a simple linear function u=z*r then it is got that $z = \frac{K\cdot(1-B)}{3-2B}$. Therefore the general solution for u- equation is $u = C\cdot r^{2B-2} + z\cdot r$

When it is inverted the variable change it is got the general solution for field E.

General solution is $E(r) = \left(Cr^{2B-2} + \frac{Kr(1-B)}{3-2B}\right)^{\frac{1}{1-B}}$ $with\ B \neq 1\ and\ B \neq 3/2$ where C is the parameter of initial condition of gravitational field at a specific radius.

Calling $\alpha = 2B-2$ $\beta = \frac{1}{1-B}$ and $D = \left(\frac{K(1-B)}{3-2B}\right)$ formula may be written as

$$E(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$$

Calculus of parameter C through initial conditions $R_0\ and\ E_0$

Suppose $R_0\ and\ E_0$ are the specific initial conditions for radius and gravitational field, then $C = \frac{E_0^{\ 1/\beta} - D\cdot R_0}{R_0^{\ \alpha}}$

Final comment

It is clear that the Bernoulli solution contains implicitly the fact that it is supposed there is not any baryonic matter inside the radius dominion and the only DM matter is added by $\varphi_{DM}(r) = A\cdot E^{B}(r)$. Therefore this solution for field works only in the halo region and $R_0\ and\ E_0$ could be the border radius of galactic disk where it is supposed begins the halo region and the baryonic density is negligible.

8. DIMENSIONAL ANALYSIS FOR D.M. DENSITY AS POWER OF E FORMULA

8.1 POWER OF E THROUGH BUCKINGHAM THEOREM

As it is supposed that DM density as power of E come from a quantum gravity theory, it is right to think that constant Plank h should be considered and universal constant of gravitation G as well.

So the elements for dimensional analysis are D, density of DM whose units are Kg/m^3, E gravitational field whose units are m/s^2, G and finally h.

In table below are developed dimensional expression for these four elements D, E, G and h.

	G	h	E	D
M	-1	1	0	1

L	3	2	1	-3
T	-2	-1	-2	0

According Buckingham theorem it is got the following formula for Density

$D = \dfrac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ being K a dimensionless number which may be understood as a coupling constant between field E and DM density.

As it is shown in previous epigraph, parameters for M31 is B = 1,6682469

In this case relative difference between B = 1,6682469 and 10/7 is 16,7 %. A 17% of error in cosmology could be acceptable. However by the end of the chapter it will be found a better solution.

8.2 POWER E FORMULA FOR DM DENSITY WITH TWO PI MONOMIALS

As this formula come from a quantum gravitation theory it is right to consider that Universal constants involved are G, h and c. So elements to make dimensional analysis are D, E, G,h and c =$2.99792458 \cdot 10^8$ m/s.

	G	h	E	D	c
M	-1	1	0	1	0
L	3	2	1	-3	1
T	-2	-1	-2	0	-1

According Buckingham theorem, as matrix rank is three, there are two pi monomials. The first one was calculated in previous paragraph and the second one involves G, h, E and c.

These are both pi monomials $\pi_1 = D \cdot \sqrt[7]{G^9 \cdot h^2} \cdot E^{-\frac{10}{7}}$ and $\pi_2 = \dfrac{c}{\sqrt[7]{G \cdot h}} E^{-\frac{2}{7}}$. So formula for DM density through two pi monomials will be $D = \dfrac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ being J a dimensionless number and $f(\pi_2)$ an unknown function, which can not be calculated by dimensional analysis theory.

8.3 MATHEMATICAL ANALYSIS TO DISCARD FORMULA WITH ONLY ONE PI MONOMIAL

As it was shown in paragraph **5.2** $A = \dfrac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ and $B = \dfrac{2b-2}{2b-1}$. Being a, b parameters got to fit rotation curve of velocities $v = a \cdot r^b$

Conversely, it is right to clear up parameters a and b from above formulas.

Therefore $b = \dfrac{B-2}{2B-2}$ and $a = \left[\dfrac{4\pi G A(B-1)}{2B-3}\right]^{\frac{2b-1}{2}}$ being $B \neq 1 \; and \; B \neq 3/2$.

As A is a positive quantity then $2b+1 > 0$. As $2b+1 = \dfrac{2B-3}{B-1} > 0$ Therefore $B \in (-\infty, 1) \cup (3/2, \infty)$.

If B=3/2 then 2b+1=0 and A=0 so dark matter density is cero which is Keplerian rotation curve.

In every galactic rotation curve studied, B parameter has been bigger than 3/2. See Abarca papers quoted in Bibliographic references. Therefore experimental data got in several galaxies fit perfectly with mathematical findings made in this paragraph especially for $B \in (3/2, \infty)$.

The main consequence this mathematical analysis is that formula $D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ got with only a pi monomial is wrong because B=10/7 = 1.428. Therefore formula $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ got thorough dimensional analysis by two pi monomials it is more suitable formula.

This formula is physically more acceptable because it is got considering G, h and c as universal constant involved in formula of density. As according my theory, DM is generated through a quantum gravitation mechanism it is right to consider not only **G** and **h** but also **c** as well.

8.4 LOOKING FOR A D.M. DENSITY FUNCTION COHERENT WITH DIMENSIONAL ANALYSIS

It is right to think that $f(\pi_2)$ should be a power of π_2, because it is supposed that density of D.M. is a power of E.

M31 galaxy	$D_{DM} = A \cdot E^B$
A	$3,6559956 \cdot 10^{-6}$
B	1,6682469

Taking in consideration A &B parameters on the left, power for π_2 must be -5/6. This way, power of E in formula $D_{DM} = A \cdot E^B$ will be 5/3 = 1.666666 , which is the best approximation to B= 1.6682469.

Finally $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ becomes $D = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}} E^{\frac{5}{3}}$ being M a dimensionless number.

CALCULUS OF DIMENSIONLESS NUMBER INCLUDED IN FORMULA OF DARK MATTER DENSITY

By equation of $D = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}} E^{\frac{5}{3}}$ and D=A*E^B

It is right that $A = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}}$ and then $M = A \cdot \sqrt[6]{G^7 \cdot c^5 \cdot h}$

9. RECALCULATING FORMULAS IN M31 HALO WITH B = 5/3

Findings got through Buckingham theorem are crucial. It is clear that a physic formula has to be dimensionally coherent .Therefore it is a magnificent support to the theory of DM generated by gravitational field that statistical value got by regression analysis in M31, differs less than 2 thousandth regarding value got by Buckingham theorem.

Now it is needed to rewrite all the formulas considering B=5/3. Furthermore, with B= 5/3, a lot of parameters of the theory become simple fraction numbers. In other words, theory gains simplicity and credibility.

In chapter 5 was shown the relation between a&b parameters and A&B parameters. Now considering B= 5/3 as $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$. It is right to get $b = \frac{B-2}{2B-2} = -\frac{1}{4}$ and $A = \frac{a^{\frac{-4}{3}}}{8\pi G}$

Therefore, the central formula of theory becomes $D_{DM} = A \cdot E^{\frac{5}{3}} = \frac{a^{\frac{-4}{3}}}{8 \cdot \pi \cdot G} \cdot E^{\frac{5}{3}}$

9.1 RECALCULATING THE PARAMETER a IN M31 HALO

Table below comes from chapter 3 and represents regression curve of velocity depending on radius.

Regression for M31 dominion 40-303 kpc	
$V=a*r^b$	
a	$4,32928*10^{10}$
b	-0.24822645
Correlation coeff.	0,96

Due to Buckingham theorem it is needed that b= -1/4 . Therefore it is needed to recalculate parameter **a** in order to find a new couple of values a&b that fit perfectly to experimental measures of rotation curve in M31 halo.

RECALCULATING a WITH MINUMUN SQUARE METHOD

When it is searched the parameter a, a method widely used is called the minimum squared method. So it is searched a new parameter **a** for the formula $V= a*r^{-0.25}$ on condition that $\sum_e (v - v_e)^2$ has a minimum value. Where v represents the value fitted for velocity formula and v_e represents each measure of velocity. It is right to calculate the formula for a.

$$a = \frac{\sum_e Ve \cdot r_e^{-0.25}}{\sum_e r_e^{-0.5}} = 4.727513 \cdot 10^{10}$$

New parameter a&b and A&B	
B	5/3
$b = \frac{B-2}{2B-2}$	b = -1/4
a new	$4,727513*10^{10}$

Where r_e represents each radius measure and v_e represents its velocity associated.

$A = \frac{a^{\frac{-4}{3}}}{8\pi G}$	New parameter A
	$3.488152 \cdot 10^{-6}$

9.2 RECALCULATING PARAMETER A IN M31 HALO

At the beginning of this chapter was got that $A = \frac{a^{\frac{-4}{3}}}{8\pi G}$.

In previous epigraph has been recalculated the parameter a. Therefore A has to change according this new value.

The beside table shows the value of new parameters.

9.3 FORMULAS OF DIRECT D.M.

With these new parameters recalculated it is going to get the direct formulas got at the beginning of paper.

Function of Density DM depending on radius.

$$D_{DM}(r) = L \cdot r^{2b-2} = L \cdot r^{\frac{-5}{2}} \text{ being } L = \frac{a^2 \cdot (2b+1)}{4\pi G} = \frac{a^2}{8 \cdot \pi \cdot G} = 1{,}3326 \cdot 10^{30}$$

Function of E depending on radius $E = a^2 \cdot r^{2b-1} = a^2 \cdot r^{\frac{-3}{2}}$ being $a^2 = 2{,}235 \cdot 10^{21}$

Mass enclosed by a sphere of radius r. $M(<r) = \frac{v^2 \cdot R}{G} = \frac{a^2 \cdot r^{2b+1}}{G} = \frac{a^2 \cdot \sqrt{r}}{G}$ being $\frac{a^2}{G} = 3{,}349 \cdot 10^{31}$

9.4 BERNOULLI SOLUTION FOR E IN M31 HALO

In chapter 7 was got the solution for field in the halo region, now thanks dimensional analysis it is possible to get formulas far simple because some parameters are simple fractions.

$$E(r) = \left(Cr^{\alpha} + Dr\right)^{\beta} \text{ being } \alpha = 2B - 2 = \frac{4}{3} \text{ being } \beta = \frac{1}{1-B} = \frac{-3}{2} \text{ and } D = \left(\frac{4 \cdot \pi \cdot G \cdot A(1-B)}{3-2B}\right) = 8 \cdot \pi \cdot G \cdot A$$

Therefore $E(r) = \left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-3}{2}}$ being $D = 8\pi GA = a^{\frac{-4}{3}} = 5{,}85 \cdot 10^{-15}$ being $C = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$ the initial condition of differential equation solution for E.

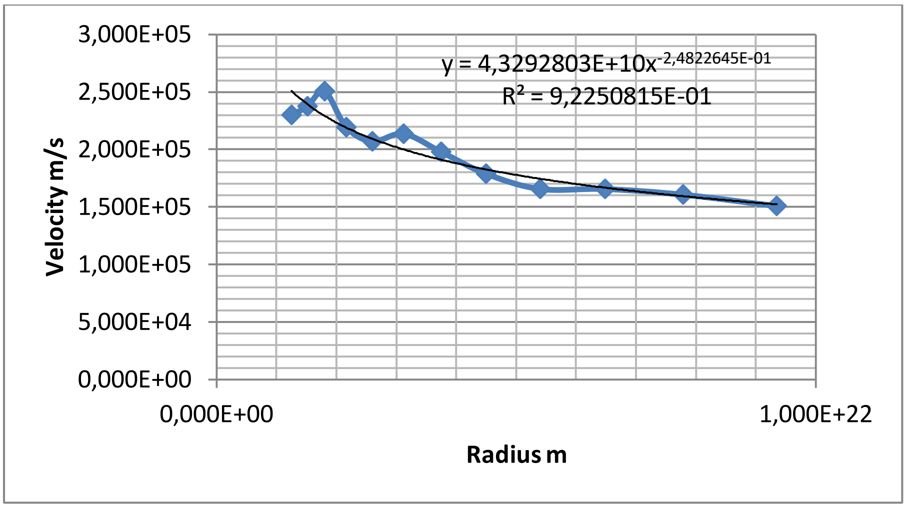
CALCULUS OF PARAMETER C

As it was pointed in the epigraph 7.2 C is calculated through the initial condition in the halo region. As it was shown in the chapter 6 at 40.5 kpc (below point P) radius the baryonic matter may be considerate negligible so it is reasonable to calculate C at this point with its formula $C = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$

Similarly it is possible to calculate C for different points inside the halo region. See in graph below points P, Q, R. They are the three first points to the left.

Points → Radius	Velocity m/s	E field.	Parameter C
P→ 40.5 kpc = $1.25*10^{21}$ m	$2.29945*10^{5}$	$4{,}2293071*10^{-11}$	$6{,}88783573*10^{-23}$
Q → 49.1 kpc	2.374 E5	3,719857E-11	6,36196464E-24
R → 58.4 kpc	2.505E5	3,482162E-11	-5,308924E-23

It is clear that there is a high difference between these three values for C. The reason is simple through the graph. There are two positives because such points (points P and Q) are below de regression curve, whereas the third (point R) is the above one.
The value of C associated to the second point, Q, placed at 49.1 kpc is far smaller than the other ones because it is very close to the regression curve. In the following epigraph it will be a bit clear the reason why Q is so small.



Studying case C = 0

Now It will be investigated the conditions to get C= 0. Then formula $C = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$ leds to

$E_0^{\frac{-2}{3}} = D \cdot R_0 = a^{\frac{-4}{3}} \cdot R_0$ and as $E = a^2 \cdot r^{2b-1}$ then $E_0^{\frac{-2}{3}} = a^{\frac{-4}{3}} \cdot R_0^{\frac{2-4b}{3}} = D * R_0 = a^{\frac{-4}{3}} \cdot R_0$ and by equation of power of R_0 $\frac{2-4b}{3} = 1$ it is got b= -1/4 .

At the beginning of chapter was shown that B= 5/3 leds rightly to b = -1/4. So b= -1/4 is rigorously the power of radius on the rotation curve of galaxy in the halo region, where there is not any baryonic matter. Namely formula is $V = a*r^{-0.25}$. Therefore C= 0 for every point belonging to regression curve whose power is -1 /4.

In the graph above, the point Q, at 49.1 kpc is very close to the regression curve so this is the reason why C is far smaller than the other two points. Unfortunately, measures of rotation curves might have considerable errors.

Summarising, in this epigraph has been demonstrated that C= 0 is the right option when it is calculated field E, or DM density inside the halo region. Namely , it is right to consider C=0 even for point P, at 40 kpc because at this point was demonstrated that baryonic density is negligible, despite the fact that if it is considered the measures of point P, C calculated is far bigger than C calculated for point Q.

In brief , it is right to consider C = 0 inside halo region from 40 kpc and bigger values of radius.

In the epigraphs 9.5 and 9.6 it will be shown that for C= 0 the Bernoulli solution for field becomes direct formula for field, and the same happens with Bernoulli DM density and mass formulas.

9.5 GETTING DIRECT FORMULAS THROUGH BERNOULLI FIELD WHEN PARAMETER C = 0

Thanks demonstration made in previous epigraph it is trustworthy to consider C= 0 in halo región.

FOR FIELD E

When in formula $E(r)=\left(Cr^{\frac{4}{3}}+Dr\right)^{\frac{-3}{2}}$ C= 0 then it is got $E=a^2\cdot r^{\frac{-3}{2}}$ being a^2 = 2.235*10[21] which is precisely direct formula for E.

FOR DM DENSITY

As D_{DM}= A*E[B] Using field got by Bernoulli solution it is right to get

$$D_{DM}(r)=A\left(Cr^{\frac{4}{3}}+Dr\right)^{\frac{-5}{2}}$$

Being **A=** 3,488*10[-6] **D**=5,85*10[-15] if **C** = 0 then formula becomes

$$D_{DM}(r)=A\cdot D^{\frac{-5}{2}}\cdot r^{\frac{-5}{2}}=L\cdot r^{\frac{-5}{2}}$$

being $L=\frac{a^2}{8\cdot\pi\cdot G}$ = 1.3326*10[30] which is direct DM density formula.

FOR TOTAL MASS INSIDE A SPHERICAL CORONA

$$M_{DM}=\int_{R1}^{R2}4\pi r^2\cdot\rho(r)dr=\int_{R1}^{R2}4\pi r^2 AE^B dr=4\pi A\int_{R1}^{R2}\mathrm{r}^2[D\cdot r]^{\frac{-5}{2}}\cdot dr$$

whose indefinite integral is $M(<r)=\frac{a^2\cdot\sqrt{r}}{G}$ which is direct formula of mass enclosed by a sphere of radius r. Being $\frac{a^2}{G}=3.349\cdot 10^{31}$.

Such formula is only right for radius belonging to halo. Therefore it is only possible to calculate the DM inside a spherical corona defined by two radius R_1 and R_2 so $R_1 < M_{DM} < R_2 = \frac{a^2}{G}\cdot\left[\sqrt{R_2}-\sqrt{R_1}\right]$

10. MASSES IN M31

In this chapter it will calculated some different types of masses related to M31 and will be compared with got by [5] Sofue.

10.1 DYNAMICAL MASSES

Radius kpc	Velocity	M dynamic	M Dynamic
kpc	Km/s	kg	M sun
40,5	229,9	9,898E+41	4,974E+11

49,1	237,4	1,280E+42	6,430E+11
58,4	250,5	1,695E+42	8,515E+11
70,1	219,2	1,558E+42	7,827E+11
84,2	206,9	1,667E+42	8,376E+11
101,1	213,5	2,131E+42	1,071E+12
121,4	197,8	2,196E+42	1,104E+12
145,7	178,8	2,154E+42	1,082E+12
175	165,6	2,219E+42	1,115E+12
210,1	165,6	2,664E+42	1,339E+12
252,3	160,7	3,013E+42	1,514E+12
302,9	150,8	3,185E+42	1,601E+12

As it is known, this type of mass represents the mass enclosed by a sphere with a radius r in order to produce a balanced rotation with a specific velocity at such radius.

The formula of dynamical mass is $M_{DYN}(<r) = \frac{V^2 \cdot r}{G}$.

Beside is tabulated dynamical masses at different radius in kg and Solar masses, according rotation curve data of [5] Sofue.

10.2 DM MASSES OF SPHERICAL CORONA IN M31 HALO THROUGH DIRECT FORMULA

In chapter 9 was got this formula M $_{\text{DM SPHERICAL CORONA}} = \frac{a^2}{G} \cdot \left[\sqrt{R_2} - \sqrt{R_1}\right]$ being $\frac{a^2}{G} = 3.349 \cdot 10^{31}$

Radius kpc	M corona kg	M corona Msun
40,5	0	0
100	6,764E+41	3,399E+11
200	1,447E+42	7,271E+11
300	2,038E+42	1,024E+12
385	2,466E+42	1,239E+12

Beside is tabulated DM masses of spherical corona from 40,5 kpc up to 100,200, 300 and 385 kpc.

Being 385 kpc the half of the distance between M31 and Milky Way.

10.3 TOTAL MASS IN M31

Radius	M corona kg	M corona	TOTAL
40,5		Msun	
100	6,764E+41	3,399E+11	8,373E+11
200	1,447E+42	7,271E+11	1,225E+12
300	2,038E+42	1,024E+12	1,522E+12
385	2,466E+42	1,239E+12	1,737E+12

Adding the dynamical mass up to 40,5 kpc = 4.97*10 ^{11} Msun to different corona masses, it is got the total mass at a specific radius, through direct formula of mass.

In the following epigraph will be compared data highlight in grey with results got by Sofue.

According Sofue Baryonic matter in M31 = 1.6*10 ^{11} Msun so the proportion Baryonic mass versus Total mass = 9.2%

10.4 COMPARISON OF TOTAL MASS DIRECT FORMULA AND TOTAL MASS NFW METHOD IN SOFUE PAPER

In [5] Sofue, the author has published the following results.

Total Mass up to 200 kpc	13.9 ± 2.6 x10 ^{11} Msun
Total mass up to 385 kpc	19.9 ± 3.9x10 ^{11} Msun

Total mas up to 200 kpc through direct formula	12.2 *10 [11] Msun
Total mass up to 385 kpc through direct formula	17.4 *10 [11] Msun

Total masses through direct formula, above data in grey, with Sofue results match perfectly when it is considered the error in calculus.

It is important to highlight that direct formula give results a bit lower than NFW method. I have explained the reason why this happen in several previous papers. i.e. [10] Abarca.

11. PROPORTION OF GALACTIC DM IN THE ANCIENT UNIVERSE IS LOWER THAN AT PRESENT

According the Hubble law, the expansion of the space in the Universe follows this simple differential equation.

$\frac{\partial R}{\partial t} = H * R$ being H= Constant Hubble = 70 Km/s/ Mpc being 1 Mpc = $3.0857 \cdot 10^{22}$ m H= $2.2685 \cdot 10^{-18}$ s^{-1}

And being R a specific distance, for example the radius of a galaxy.

The inverse of Hubble constant has time units and 1/H = $1.397 \cdot 10^{10}$ years. So 1/H is called Hubble time.

Hubble time is quite close to current Universe age equal to $1.38 \cdot 10^{10}$ years, according current cosmology. So for our proposes it is enough to approximate 1/H equal to the Universe age.

The solution of such simple differential equation is $R = R_0 \cdot e^{Ht}$ where R_0 it is the initial radius of a specific galaxy for t=0. However, this formula is not suitable because at t = 0 there was not any galaxies.

So it is better rewrite the solution including current time as a reference to measure the time, then

$R = R_C \cdot e^{H(t-t_c)}$ Where t_C represents the current time, R_C is the size of a specific galaxy nowadays and t is the age of such galaxy.

With this solution it is clear that for t = tc $R = R_C$.

Going back 1/ 3 of the Universe age then $t-t_C = -1/(3H)$ and $R = R_C \cdot e^{\frac{-1}{3}} = 0.72 \cdot R_C$

In other words 4650 million years ago the size of a galaxy was 0.72 times its current size.

Going back 2/3 of the Universe age then $t-t_C = -2/(3H)$ and $R = R_C \cdot e^{\frac{-2}{3}} = 0.51 \cdot R_C \approx 0.5 \cdot R_C$

In other words 9300 million years ago the size of galaxies was one half of its current sizes.

11.1 COMPARISON BETWEEN DM OF M31 IN THE ANCIENT UNIVERSE AND AT PRESENT

In epigraph 10.2 was got the mass of DM inside the corona from 40 kpc up to 385 kpc = $1.24 \cdot 10^{12}$ Msun through the formula $M_{\text{DM SPHERICAL CORONA}} = \frac{a^2}{G} \cdot \left[\sqrt{R_2} - \sqrt{R_1}\right]$ being $\frac{a^2}{G} = 3.349 \cdot 10^{31}$

Going back 4650 million years, the current distances are reduced by factor 0.72. So 40 kpc becomes R min= 29 kpc and 385 kpc becomes R max= 277 kpc then according the previous formula.

$M_{DM\ SPHERICAL\ CORONA}$ (from 29 up to 277) = 1.052*10 ^{12} M sun

Comparing this result with the current value $M_{DM\ SPHERICAL\ CORONA}$ (40 up to 385) = 1.24*10 ^{12} M sun it can be checked that the amount of DM 4650 million years ago was 15.3% lower.

Going back 9300 million years, the current distances are reduced by a factor 0.5 so Rmin = 20 kpc and Rmax = 192.5 kpc then $M_{DM\ SPHERICAL\ CORONA}$ (from 20 up to 192.5 kpc) = 8.79*10 ^{11} Msun which means a 29% lower of DM regarding the current value.

Although these simple calculus have been made inside the spherical corona where baryonic matter is negligible, inside the galaxy it happens the same, the DM is lower because the galactic size in the ancient Universe is lower. What happens is that calculus of DM inside galactic disk is far more complex because it is needed to know baryonic density function and solve a far more complex differential equation than a simple Bernoulli one.

This previous calculus would be in agreement with majority of cosmologist, because it seems that observational evidences back a lower fraction of DM in measures of ancient galaxies. In other words in galaxies which are far away, and consequently the observational data inform about what happens thousand million years ago in a very far away galaxy.

Furthermore, in the paper [16] Alfred L. Tiley, 2018. After studying some 1,500 galaxies, researchers led by Alfred Tiley of Durham University have determined that the fraction of dark matter in galaxies placed 10 Gys (10^{10} light-years) away is at least 60% regarding current amount of DM for galaxies placed in the close Universe.

12 THE RIGHT MASS OF THE LOCAL GROUP OF GALAXIES THROUGH DIRECT MASS FORMULA

According [5] Sofue the relative velocity between M31 and Milky Way is 170 km/s. Assuming that both galaxies are bounded gravitationally it is possible to calculate the total mass of the Local group through a simple formula because of the Virial theorem.

$M = \frac{v^2 \cdot r}{G}$ As r = 770 kpc and v= 170 km/s then $M_{LOCAL\ GROUP}$ = 5.17 *10^{12} Msun

According [5] Sofue, the total mass of M31 and Milky Way is approximately 3*10^{12} Msun, so there is a mass lack of 2*10^{12} Msun which is a considerable amount of matter. Namely read epigraph 4.6 of [5] Sofue paper.

Up to now I have considered that the border of a M31 is placed at a half the distance to Milky Way because it is supposed that up to such distance gravitational field dominates whereas for bigger distances is Milky Way field which dominates.

This hypothesis is right when it is considered rotation curves of different systems bounded to each galaxy i.e. stars or dwarf galaxies. However when it is considered the gravitational interaction between both giant galaxies it is needed to extend their haloes up to its twin galaxy. Therefore the M31 halo extend up to 770 kpc and reciprocally the Milky Way halo extend up to 770 kpc. It is simply Newtonian Mechanics. For example the Moon orbit is calculated through the Earth field at the Moon radius.

12.1 M31 TOTAL MASS WITH ITS HALO EXTENDED UP TO 770 KPC

Through $M_{\text{DM SPHERICAL CORONA}} = \frac{a^2}{G} \cdot \left[\sqrt{R_2} - \sqrt{R_1}\right]$ being $\frac{a^2}{G} = 3.349 \cdot 10^{31}$ it is right to calculate

$M_{\text{DM SPHERICAL CORONA}}$ (from 40 up to 770) = $2*10^{12}$ Msun that added to M_{DYN} (< 40) = $5*10^{11}$ Msun gives a total mass for M31 = $2.5*10^{12}$ Msun.

Considering that Milky Way is a very similar galaxy of M31, it is right to consider that total mass of Local Group rises up to $M_{\text{LOCAL GROUP}} = 5*10^{12}$ Msun which is a magnificent result of DM theory by gravitational field.

NFW HALO EXTENDED UP TO 770 KPC

According parameters of NFW model got for M31 by [5] Sofue, it is possible to calculate total DM inside an extended halo up to 770 kpc then $M_{\text{TOTAL DM}} = 2{,}54*10^{12}$ Msun which added to Baryonic matter of M31 = $1.6*10^{11}$ Msun gives a total mass for M31 = $2.7*10^{12}$ M sun, which is a bit bigger regarding I have calculated through Direct mass formula.

The big difference between my model, DM generated by gravitational field and standard model of DM is that the former allows a right extension of halo because the origin of DM is the own field whereas the standard model of DM in galaxies is a numerical method ad hoc to fit the extra of mass that shows the measures of galactic rotation curves.

Final discussion

The concept of extended haloes applied to galaxy cluster may explain the reason why the proportion of DM inside a cluster is bigger than inside a galaxy. The gravitational interaction inside the intergalactic space produces an extra of DM.

For example Baryonic mass of Local Group is approximately $3*10^{11}$ Msun and total mass is bigger than $5*10^{12}$ Msun so the proportion of baryonic mass versus total mass is lower than 6% whereas such proportion was 9 % inside M31.

However for bigger scales this effect is compensated by the Dark energy. See [9] Abarca. A study about Coma cluster. Some years ago was checked that super clusters are the bigger structures gravitationally bounded because the universal expansion dominates over gravitational forces between super clusters.

13. CONCLUSION

This work is focused in halo region of M31 where baryonic density is negligible regarding DM non baryonic. The reason is that according the main hypothesis of this theory, the non baryonic DM is generated locally by the gravitational field. Therefore it is needed to study DM on the radius dominion where it is possible to study gravitational field propagation without interference of baryonic mass density or at least where this density is negligible.

In order to defend properly the conclusion of this paper, it is important to emphasize that correlation coefficient of power regression over velocity measures in rotation curve in halo region is bigger than 0,96. See chapter 3 where was got coefficients a& b for $v = a \cdot r^b$ law.

In chapter four was mathematically demonstrated that the power law $v = a{\cdot}r^b$ in halo region is equivalent a DM density called direct DM , whose formula is $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} {\cdot} r^{2b-2}$.

In chapter five was demonstrated mathematically that the power law for velocity $v = a{\cdot}r^b$ on the rotation curve is mathematically equivalent to a power law for DM density depending on E. $D_{DM} = A{\cdot}E^B$.

Where $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$.

Therefore joining chapters 3,4 and 5 it is concluded that the high correlation coefficient bigger than 0.96 at power regression law for rotation curve $v = a{\cdot}r^b$ in halo region support strongly that DM density inside halo region is a power of gravitational field $D_{DM} = A{\cdot}E^B$ whose parameters A & B are written above.

As it was pointed at introduction, it is known that there is baryonic dark matter such as giant planets, cold gas clouds, brown dwarfs but this type of DM is more probable to be placed inside galactic disk and bulge.

Reader can consult these papers about this open problem: [11] Nieuwenhuizen,T.M. 2010. [12] Nieuwenhuizen,T.M. 2012. [13] Nieuwenhuizen,T.M. 2010 [14] Wyrzykowski,L.2010. [15] M.R.S. Hawkins 2015

In chapter seven was got a Bernoulli differential equation for M31 halo in order to look for a local method to calculate the local field E.

The core this paper is chapter eight, where it is made dimensional analysis for magnitudes Density and field E and for universal constants G, h and c. It is demonstrated that formula for DM density need two Pi monomials. Furthermore it is found that coefficient B = 5/3, power of E, is coherent with Buckingham theorem and it differs only two thousandth regarding B value got by statistical regression of rotation curve in M31 halo. As a consequence of this simple fraction for B, the others parameters a&b and A were recalculated and formulas of theory were rewriting. This way, these formulas achieve dimensional coherence and the theory gains simplicity and credibility.

It is important to state that Buckingham theorem allows the value B=5/3 as power for field, although another different fractions would be allowed too. However, it is such fraction which match perfectly with the value got by statistic regression of rotation curve data on rotation curve in M31 halo.

Results got about $D_{DM} = A{\cdot}E^B$ in other galaxies, see [8] Abarca,M.2016, suggest that DM density follows a similar law in different galaxies. Namely the power for E is close to 5/3, on condition that galaxies should be similar giant galaxies i.e. its velocity is bigger than 200 km/s in the disk region of the rotation curves.

In the eleventh chapter, through a simple application of Hubble law has been demonstrated that in the ancient Universe, the galaxies are smaller and consequently the proportion of DM versus baryonic is lower. These calculus are in agreement with current observational evidences.
Finally in the twelfth chapter has been estimated the total mass of Local Group as $5{*}10^{12}$ Msun thanks to the concept of extended halo. This calculus agrees perfectly with measures based on relative velocity of Milky Way and M31.
In my opinion these facts suggest strongly that nature of non baryonic DM is generated by gravitational field according a Universal mechanism. In other words, it is clear that DM it is a phenomenon of quantum gravitation.

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