Author Manuel Abarca Hernandez **email** mabarcaher1@gmail.com

1. ABSTRACT

In this work has been calculated two new DM density profiles inside halo region of M31 galaxy and it has been demonstrated that both ones are mathematically equivalents. Its radius dominion is only the halo region because it is needed that baryonic matter density has to be negligible.

The first profile is called direct DM density because it is got directly from rotation curve and represents DM density depending on radius

The second one, called Bernoulli because it is got from a Bernoulli differential equation, represents DM density depending on local gravitational field according a power law. The power of E is called B.

Hypothesis which is the basis to get Bernoulli profile stated that DM is generated locally by the own gravitational field according a power law. DM density = A· E^B where A& B are coefficients and E is gravitational intensity of field. According the theory hypothesis, coefficients A&B are universal so are the same in different galaxies.

To find reasons that author has to do so daring statement, reader can consult [1] Abarca,M.2014. *Dark matter model by quantum vacuum*. [8] Abarca,M.2016. *Dark matter density on big galaxies depend on gravitational field as Universal law* and other papers quoted in bibliography.

Briefly will be explained method followed to develop this paper. Rotation curve data come from [5] Sofue,Y.2015. Thanks this remarkable rotation curve, the regression curve of velocity depending on radius has a correlation coefficient bigger than 0.95

In fourth chapter it is got the function of DM density depending on radius, called direct DM density.

In fifth chapter it is demonstrated that function direct DM density is mathematically equivalent to the function DM density depending on E. Furthermore it is stated the mathematical relation between both functions. Namely, it is got value B =1.6688

In sixth chapter it is got that for radius bigger than 40 kpc the ratio baryonic density versus DM density is under 1% so it is reasonable to consider negligible baryonic matter density in order to simplify calculus.

In seventh chapter it is got a Bernoulli differential equation for field and is solved.

In eighth chapter it is made dimensional analysis for magnitudes Density, field E and universal constants G, h and c. It is demonstrated that it is needed a formula with two Pi monomials. It is found that B = 5/3 is the value coherent with Buckingham theorem and differs only two thousandth regarding B=1.6688 which was got by regression analysis.

In ninth chapter are recalculated parameters a,b and A as a consequence of being B=5/3 instead B=1.688 and are written newly the formulas with these new values. Thanks this change the formulas are now dimensionally right.

In tenth chapter are compared direct formulas and Bernoulli formulas for field and for DM density. The differences range between 0.3% and 0.6% for E on the whole radius dominion. Also it is got a very good approximation for field E. In fact, thanks to such approximation it is demonstrated that Bernoulli formulas become direct formulas.

In the eleventh chapter through the Hubble law it is demonstrated that galaxies in the ancient universe were smaller than at present and as a consequence it is got that proportion of DM versus baryonic matter was lower in the ancient universe. Results got are in agreement with current observational evidences.

2. INTRODUCTION

As reader knows M31 is the twin galaxy of Milky Way in Local Group of galaxies. Its disk radius is approximately 35 kpc and according [5] Sofue, Y. 2015. Its baryonic mass is $M_{BARYONIC} = 1{,}61 \cdot 10^{11}\ M_{SUN}$
The DM theory introduced in [1] Abarca, M.2014. *Dark matter model by quantum vacuum* considers that DM is generated by the own gravitational field. Therefore, in order to study purely the phenomenon it is needed to consider a radius dominion where it is supposed that baryonic matter is negligible.

In previous paper [10] Abarca,M.2016. *A New Dark Matter Density Profile for M31 Galaxy to Demonstrate that Dark Matter is Generated by Gravitational Field,* author has studied DM inside M31 halo through Bernoulli DM profile. However in such paper DM density used it was NFW profile provided by [5] Sofue, Y. 2015 whereas in current paper DM density profile has been got directly from a power regression function on rotation curve in halo region.

This new DM profile has been called direct DM density because this profile is fitted directly from data measures inside halo region. In this work radius dominion begin at 40 kpc because at this distance baryonic density is negligible as it will be shown in chapter six. Therefore the only one kind of matter in halo region it is supposed to be non baryonic dark matter and it is quite simple to state the differential equation for field in these conditions.

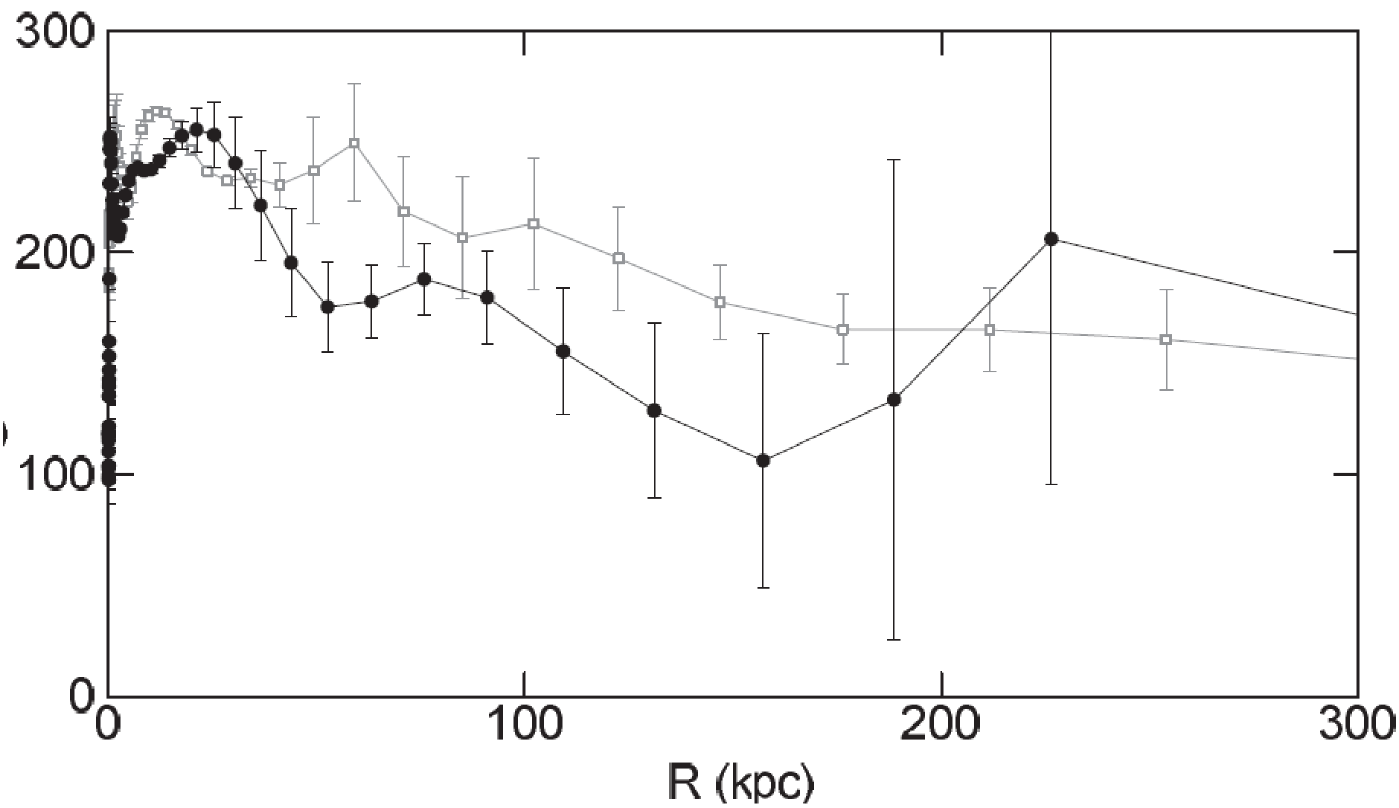
It is known that there is baryonic dark matter such us giant planets, cold gas clouds, brown dwarfs but this kind of DM is more probable to be placed inside galactic disk. Reader can consult: [11] Nieuwenhuizen,T.M. 2010. [12] Nieuwenhuizen,T.M. 2012. [13] Nieuwenhuizen,T.M. 2010 [14] Wyrzykowski,L.2010. [15] M.R.S. Hawkins 2015.In fact there are an important amount of researchers in this way because baryonic DM and non baryonic DM are open problems still.

As it is known, NFW profile is fitted over bulge, disk and galactic halo and taking in consideration that there is an unknown amount of baryonic DM in bulge and galactic disk it is needed concluded that NFW profile is more imprecise than direct DM profile in order to study non baryonic DM in halo because direct DM density has been fitted exclusively with data of DM non baryonic in halo region.

In fact according [5] Sofue, Y. 2015 data, in chapter six will be got that for radius bigger than 40 kpc baryonic matter density is under 1% versus DM density. This is the reason why radius dominion in this work is from 40 kpc to 250 kpc. In chapter seven it will be got a simple Bernoulli differential equation for gravitational field. However to get a so simple differential equation it is needed that $M'(r) = 4\pi r^2 \varphi_{DM}(r)$.In other words, it is needed that density of baryonic matter would be negligible versus D.M. density. In addition it is supposed hypothesis $\varphi_{DM}(r) = A \cdot E^B$

Several previous papers such as [2] Abarca,M.2015 and others have studied DM density as power of gravitational field in several galaxies: Milky Way, M33, NGC3198 and others galaxies. The results got support hypothesis that $\varphi_{DM}(r) = A \cdot E^B$ being A&B quite similar for different giant galaxies.

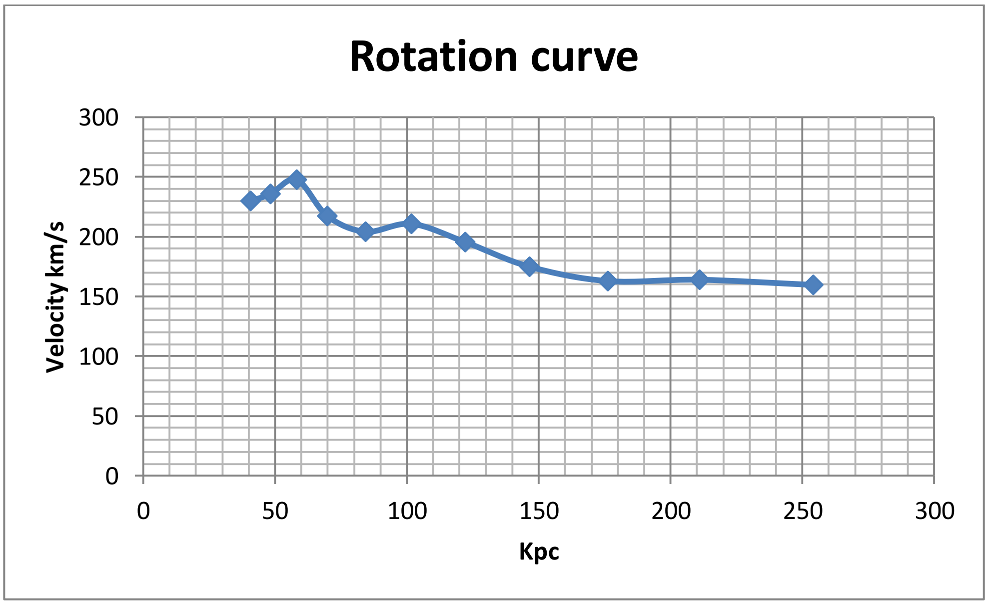
3. OBSERVATIONAL DATA FROM SOFUE. 2015 PAPER



Graphic come from [5] Sofue,Y. 2015.

Grey line belong to M31 rotation curve and black line to Milky Way.

M31 point data	
Radius	Velocity
kpc	km/s
40,7	230
48,25	235,9
58,2	247,8
69,8	217,4
84,3	204,3
101,7	210,9
122,1	195,6
146,5	175
176,2	163
211	164,1
254,1	159,8



From graphic it is clear there is a high correlation between spin radius and velocity.

In chapter six will be shown reason why dominion data begin at 40 kpc in this work. It is accepted that disk radius of M31 is approximately 35 kpc.

3.1 POWER REGRESSION TO ROTATION CURVE

It is seen that experimental measures of rotation curve has a very good fitted curve by power regression.

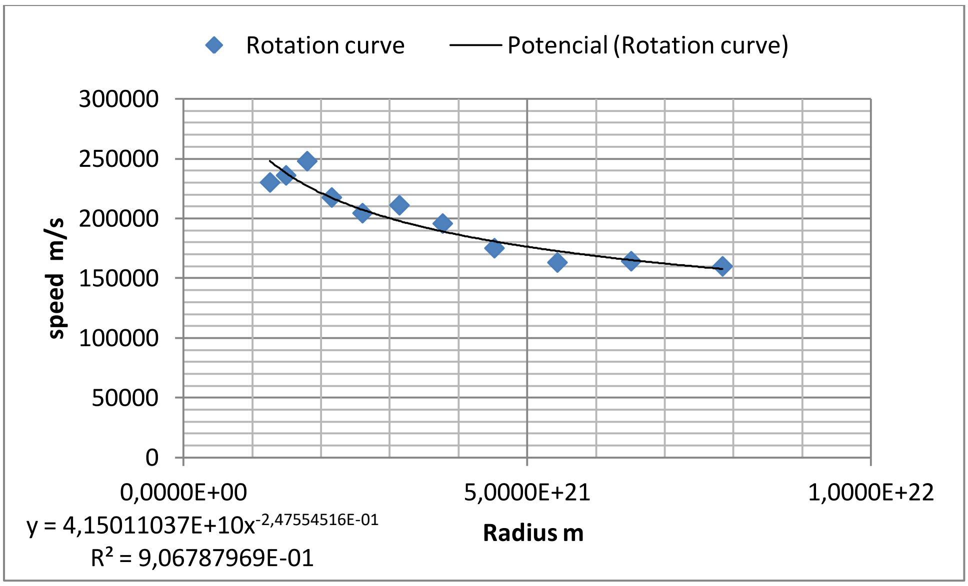
In particular coefficients of $v = a{\cdot}r^b$ are in table below. Units are into I.S.

Power regression for M31 rot. curve	
V=a*r^b	
a	4,15011040E+10
b	-2,47554520E-01
Correlation coeff.	0,952254

Data fitted are in grey columns below.
In third column is shown results of fitted velocity and fourth column shows relative difference between measures and fitted results.
Correlation coefficient is above 0.95 which is very good correlation.

radius	velocity measures	veloc. Fitted.	Rel. Diff.	Radius
m	m/s		%	kpc
1,2559E+21	230000	2,4827E+05	7,36	40,7
1,4889E+21	235900	2,3803E+05	0,89	48,25
1,7959E+21	247800	2,2723E+05	-9,05	58,2
2,1538E+21	217400	2,1723E+05	-0,08	69,8
2,6012E+21	204300	2,0732E+05	1,46	84,3
3,1382E+21	210900	1,9791E+05	-6,57	101,7
3,7676E+21	195600	1,8915E+05	-3,41	122,1
4,5206E+21	175000	1,8081E+05	3,21	146,5
5,4370E+21	163000	1,7273E+05	5,63	176,2
6,5108E+21	164100	1,6519E+05	0,66	211
7,8408E+21	159800	1,5777E+05	-1,29	254,1

Below is shown a graphic with measures data and power regression function.



In my opinion a correlation coefficient of 0,952254 is a very high correlation if it is considered that M31 is 770 kpc away and errors in measures are not negligible. Therefore this value support strongly hypothesis that rotation curve of M31 follow a law $v = a{\cdot}r^b$ where a & b are written above.

4. DIRECT FORMULA FOR DM DENSITY ON M31 HALO GOT FROM ROTATION CURVE

4.1 THEORETICAL DEVELOPPMENT FOR GALACTIC HALOS

Outside disk region, rotation curve it is fitted by power regression with a high correlation coefficient according formula $v = a{\cdot}r^b$. As $M(<r) = \frac{v^2 \cdot R}{G}$ represents total mass enclosed by a sphere with radius r, by substitution of velocity results $M = \frac{v^2 \cdot R}{G} = \frac{a^2 \cdot r^{2b+1}}{G}$

If it is considered outside region of disk where baryonic matter is negligible regarding dark matter it is possible to calculate DM density by a simple derivative. In next chapter will be show that for r > 40 kpc baryonic matter is negligible.

As density of D.M. is $D_{DM} = \frac{dm}{dV}$ where $d\,m = \frac{a^2 \cdot (2b+1) \cdot r^{2b} dr}{G}$ and $d\,V = 4\pi r^2 dr$ results

$$D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$$

Writing $L = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot$ results $D_{DM}(r) = L{\cdot}r^{2b-2}$. In case b = -1/2 DM density is cero which is Keplerian rotation.

4.2 DIRECT DM DENSITY FOR M31 HALO

Parameters a & b from power regression of M31 rotation curve allow calculate easily direct DM density

Direct DM density for M31 halo 40 < r < 300 kpc
$D_{DM}(r) = L{\cdot}r^{2b-2}$ kg/m^3
L = 1,03701707086078E $\cdot 10^{30}$
2b -2 = -2,49510904

5. DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

As independent variable for this function is E, gravitational field, previously will be studied formula for E in the following paragraph.

5.1 GRAVITATIONAL FIELD E THROUGH VIRIAL THEOREM

As it is known total gravitational field may be calculated through Virial theorem, formula $E = v^2/R$ whose I.S. unit is m/s^2 is well known. Hereafter, virial gravitational field, E, got through this formula will be called E.

By substitution of $v = a{\cdot}r^b$ in formula $E = \frac{v^2}{r}$ it is right to get $E = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ briefly $E = a^2 \cdot r^{2b-1}$

5.2 DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

According hypothesis dark matter by quantum vacuum $D_{DM} = A{\cdot}E^B$. Where A & B are parameters to be calculated. This hypothesis has been widely studied by author in previous papers. [1] Abarca,M. [2] Abarca,M.

[8] Abarca,M. [9] Abarca,M. [10] Abarca,M.

As it is known direct DM density $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ depend on a & b parameters which come from power regression formula for velocity. In previous paragraph has been shown formula for gravitational field $E = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ which depend on a & b as well. Through a simple mathematical treatment it is possible to get A & B to find function of DM density depending on E. Specifically formulas are $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$.

M31 galaxy	$D_{DM} = A \cdot E^B$
A	$3{,}766521943774 \cdot 10^{-6}$
B	1,668847537702

According parameters a & b got in previous chapter, A& B parameters are:

As conclusion, in this chapter has been demonstrated that a power law for velocity

$v = a \cdot r^b$ *is mathematically equivalent to a power law for DM density depending on E.* $D_{DM} = A \cdot E^B$

5.3 THE IMPORTANCE OF $D_{DM} = A \cdot E^B$

This formula is vital for theory of dark matter generated by gravitational field because it is supposed that DM is generated locally according an unknown quantum gravity mechanism. In other words, the propagation of gravitational field has this additional effect on the space as the gravitational wave goes by.

The formulas $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ and $E = a^2 \cdot r^{2b-1}$ have been got rightly from rotation curve. Therefore it can be considered more specific for each galaxy. However the formula $D_{DM} = A \cdot E^B$ is much more essential.

The basis of this theory is that such formula is right for different gravitational systems. Therefore A & B parameters have to be the same for different galaxies. This is the initial hypothesis of this theory. However, there is an important fact to highlight. It is clear that A depend on a and b, both parameters are global parameters.

As the gravitational interaction time between masses is proportional to distance, it is right to think that DM generated by a gravitational field has a bigger proportion as the system increase its size. For example inside the Solar system it is clear that Newton and General Relativity Theory is able to explain with total accuracy every gravitational phenomenon without DM hypothesis. Therefore it is right to conclude that DM arises when gravitational interaction takes a longer time to link the matter. Namely, for galaxy scale or bigger systems.

Furthermore, there are clear observational evidences that inside cluster of galaxies the proportion of DM is bigger than inside galaxies. In other words, it is right to think that the bigger the gravitational system is, the bigger will be the parameter A or even B.

However, there are observational evidences of DM inside dwarf and medium size galaxies that show a bigger proportion of DM than inside giant galaxies.

In my opinion this fact could be explained by other reasons. For example dwarf galaxies are always orbiting near giant galaxies, so it is possible that the proportion of baryonic matter cold, which is unobservable, could be bigger. Anyway this is an open problem for current cosmology.

To sum up, regarding theory of DM generated by gravitational field, parameters A&B has to be the same for different gravitational system on condition they have the same size. i.e. two similar giant galaxies should have the same parameters A&B. However, a bigger gravitational system. i.e. galaxy cluster should have bigger parameter in order to produce a bigger fraction of D.M. Nonetheless, in chapter 9, it will be shown that total DM increase with the square root of distance. For example, the proportion of DM inside galactic disk of M31 is lower than the proportion when it is considered the whole halo whose radius is 350 kpc, so the maximum proportion goes up to 90% of DM versus baryonic matter.

6. RATIO BARYONIC MASS VERSUS DARK MATTER MASS DEPENDING ON RADIUS FOR M31

In this paragraph will be estimated radius which is needed to consider negligible baryonic density regarding DM density in M31 galaxy.

[5] According Sofue, Y. data for M31 disk are

M31 Galaxy	Baryonic Mass at disk	a_d	Σ_0
	$M_d = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$		
	$M_d = 1{,}26 \cdot 10^{11}$ Msun	5,28 kpc	1,5 kg/m^2

Where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ represents superficial density at disk. Total mass disk is given by integration of superficial density from cero to infinite. $M_d = \int_0^\infty 2\pi \cdot r \Sigma(r) \cdot dr = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$

In order to compare baryonic density and DM density it is considered differential baryonic mass and differential DM masses depending on radius.

$dM_{DISK} = 2\pi r \Sigma(r) dr$ where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ and

$dM_{DM} = 4\pi r^2 D_{DM}(r) dr$ where $D_{DM}(r) = \dfrac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$

It is defined ratio function as quotient of both differential quantities $Ratio = \dfrac{dM_{DISK}}{dM_{DM}} = \dfrac{\Sigma(r)}{2 \cdot r \cdot D_{DM}(r)}$

Radius	Radius	Ratio (r)	$\Sigma(r)$	Direct DM
Kpc	m	Ratio	kg/m^2	kg/m^3
36	1,110852E+21	2,310614E-02	1,64056151250E-03	3,1957946476E-23
38	1,172566E+21	1,715255E-02	1,12327743139E-03	2,7924857817E-23
40	1,234280E+21	1,268028E-02	7,69097762116E-04	2,4570213865E-23
42	1,295994E+21	9,339073E-03	5,26594188719E-04	2,1754010061E-23
44	1,357708E+21	6,854954E-03	3,60554214629E-04	1,9370002366E-23

For a radius 40 kpc ratio baryonic matter versus DM is only 1,2 % therefore is a good approximation to consider negligible baryonic mass density regarding DM density when radius is bigger than 40 kpc.
This is the reason why in this work dominion for radius begin at 40 kpc.

7. A DIFFERENTIAL EQUATION FOR A GRAVITATIONAL FIELD

7.1 INTRODUCTION

This formula $D_{DM} = \frac{a^2\cdot(2b+1)}{4\pi G}\cdot r^{2b-2}$ is a local formula because it has been got by differentiation. However E, which represents a local magnitude $E = \frac{G\cdot M(<r)}{r^2} = \frac{a^2\cdot r^{2b}}{r} = a^2\cdot r^{2b-1}$ has been got through $v = a\cdot r^b$ whose parameters a & b were got by a regression process on the whole dominion of rotation speed curve. Therefore, D_{DM} formula has a character more local than E formula because the former was got by a differentiation process whereas the later involves M(<r) which is the mass enclosed by the sphere of radius r.

In other words, the process of getting D_{DM} involves a derivative whereas the process to get E(r) involves M(r) which is a global magnitude. This is a not suitable situation because the formula $D_{DM} = A\cdot E^B$ involves two local magnitudes. Therefore it is needed to develop a new process with a more local nature or character.

It is clear that a differential equation for E is the best method to study locally such magnitude.

7.2 A DIFFERENTIAL BERNOULLI EQUATION FOR GRAVITATIONAL FIELD IN A GALACTIC HALO

As it is known in this formula $\vec{E} = -G\frac{M(r)}{r^2}\hat{r}$, M(r) represents mass enclosed by a sphere with radius r. If it is considered a region where does not exit any baryonic matter, such as any galactic halo, then the derivative of M(r) depend on dark matter density essentially and therefore $M'(r) = 4\pi r^2\varphi_{DM}(r)$.

If $E = G\frac{M(r)}{r^2}$, vector modulus, is differentiated then it is got $E'(r) = G\frac{M'(r)\cdot r^2 - 2rM(r)}{r^4}$

If $M'(r) = 4\pi r^2\varphi_{DM}(r)$ is replaced above then it is got $E'(r) = 4\pi G\varphi_{DM}(r) - 2G\frac{M(r)}{r^3}$ As $\varphi_{DM}(r) = A\cdot E^B(r)$ it is right to get $E'(r) = 4\pi\cdot G\cdot A\cdot E^B(r) - 2\frac{E(r)}{r}$ which is a Bernoulli differential equation.

$E'(r) = K\cdot E^B(r) - 2\frac{E(r)}{r}$ being $K = 4\pi\cdot G\cdot A$

Calling y to E, the differential equation is written this way $y` = K\cdot y^B - \frac{2\cdot y}{r}$

Bernoulli family equations $y` = K\cdot y^B - \frac{2\cdot y}{r}$ may be converted into a differential linear equation with this variable change $u = y^{1-B}$.

General solution is $E(r) = \left(Cr^{2B-2} + \frac{Kr(1-B)}{3-2B}\right)^{\frac{1}{1-B}}$ $with\ B \neq 1\ and\ B \neq 3/2$ where C is the parameter of initial condition of gravitational field at a specific radius.

Calling $\alpha = 2B - 2$ $\beta = \frac{1}{1-B}$ and $D = \left(\frac{K(1-B)}{3-2B}\right)$ formula may be written as

$$E(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$$

Calculus of parameter C through initial conditions R_0 and E_0

Suppose R_0 and E_0 are the specific initial conditions for radius and gravitational field, then $C = \frac{E_0^{1/\beta} - D \cdot R_0}{R_0^{\alpha}}$

Final comment

It is clear that the Bernoulli solution contains implicitly the fact that the only matter added in region without baryonic matter is given by $\varphi_{DM}(r) = A \cdot E^{B}(r)$. Therefore it would be right to considerate that such differential equation for E with specific initial conditions is the more suitable method to calculate D_{DM} in different gravitational system.

8. DIMENSIONAL ANALYSIS FOR D.M. DENSITY AS POWER E FORMULA

8.1 POWER OF E THROUGH BUCKINGHAM THEOREM

As it is supposed that DM density as power of E come from a quantum gravity theory, it is right to think that constant Plank h should be considered and universal constant of gravitation G as well.

So the elements for dimensional analysis are D, density of DM whose units are Kg/m^3, E gravitational field whose units are m/s^2, G and finally h.

In table below are developed dimensional expression for these four elements D, E, G and h.

	G	h	E	D
M	-1	1	0	1
L	3	2	1	-3
T	-2	-1	-2	0

According Buckingham theorem it is got the following formula for Density

$D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ being K a dimensionless number which may be understood as a coupling constant between field E and DM density.

As it is shown in previous epigraph, results got for M31 parameters are B = 1,6688475377 and A= $3,766521943 \cdot 10^{-6}$

In this case relative difference between B = 1,668847 and 10/7 is 16,8 %. This relative difference is acceptable because of errors of rotation curve measures.

8.2 POWER E FORMULA FOR DM DENSITY WITH TWO PI MONOMIALS

As this formula come from a quantum gravitation theory it is right to consider that Universal constants involved are G, h and c. So elements to make dimensional analysis are D, E, G,h and c =$2.99792458 \cdot 10^{8}$ m/s.

	G	h	E	D	c
M	-1	1	0	1	0
L	3	2	1	-3	1
T	-2	-1	-2	0	-1

According Buckingham theorem, as matrix rank is three, there are two pi monomials. The first one was calculated in previous paragraph and the second one involves G, h, E and c.

These are both pi monomials $\pi_1 = D \cdot \sqrt[7]{G^9 \cdot h^2} \cdot E^{-\frac{10}{7}}$ and $\pi_2 = \frac{c}{\sqrt[7]{G \cdot h}} E^{-\frac{2}{7}}$. So formula for DM density through two pi monomials will be $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ being J a dimensionless number and $f(\pi_2)$ an unknown function, which can not be calculated by dimensional analysis theory.

8.3 MATHEMATICAL ANALYSIS TO DISCARD FORMULA WITH ONLY ONE PI MONOMIAL

As it was shown in paragraph **5.2** $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ and $B = \frac{2b-2}{2b-1}$. Being a, b parameters got to fit rotation curve of velocities $v = a \cdot r^b$

Conversely, it is right to clear up parameters a and b from above formulas.

Therefore $b = \frac{B-2}{2B-2}$ and $a = \left[\frac{4\pi GA(B-1)}{2B-3}\right]^{\frac{2b-1}{2}}$ being $B \neq 1 \; and \; B \neq 3/2$.

As A is a positive quantity then $2b+1 > 0$. As $2b+1 = \frac{2B-3}{B-1} > 0$ Therefore $B \in (-\infty, 1) \cup (3/2, \infty)$.

If B=3/2 then 2b+1=0 and A=0 so dark matter density is cero which is Keplerian rotation curve.

In every galactic rotation curve studied, B parameter has been bigger than 3/2. See Abarca papers quoted in Bibliographic references. Therefore experimental data got in several galaxies fit perfectly with mathematical findings made in this paragraph especially for $B \in (3/2, \infty)$.

The main consequence this mathematical analysis is that formula $D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ got with only a pi monomial is wrong because B=10/7 = 1.428.

Therefore formula $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ got thorough dimensional analysis by two pi monomials it is more suitable formula.

This formula is physically more acceptable because it is got considering G, h and c as universal constant involved in formula of density. As according my theory, DM is generated through a quantum gravitation mechanism it is right to consider not only **G** and **h** but also **c** as well.

8.4 LOOKING FOR A D.M. DENSITY FUNCTION COHERENT WITH DIMENSIONAL ANALYSIS

It is right to think that $f(\pi_2)$ should be a power of π_2, because it is supposed that density of D.M. is a power of E.

M31 galaxy	$D_{DM} = A \cdot E^{B}$
A	3,766521943774E $\cdot 10^{-6}$
B	1,668847537702

Taking in consideration A &B parameters on the left, power for π_2 must be -5/6. This way, power of E in formula $D_{DM} = A \cdot E^{B}$ will be 5/3 = 1.666666 , which is the best approximation to B= 1.668847.

Finally $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ becomes $D = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}} E^{\frac{5}{3}}$ being M a dimensionless number.

CALCULUS OF DIMENSIONLESS NUMBER INCLUDED IN FORMULA OF DARK MATTER DENSITY

By equation of $D = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}} E^{\frac{5}{3}}$ and $D=A*E^B$

It is right that $A = \frac{a^{\frac{-4}{3}}}{8\pi G} = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}}$ and then $M = \frac{\sqrt[6]{G \cdot h \cdot c^5}}{8 \cdot \pi \cdot \sqrt[4]{a^3}} =$ 2.9819168*10^{-10}

9. RECALCULATING FORMULAS IN M31 HALO WITH B = 5/3

Findings got through Buckingham theorem are crucial. It is clear that a physic formula has to be dimensionally coherent .Therefore it is a magnificent support to the theory of DM generated by gravitational field that statistical value got by regression analysis in M31, differs only 2 thousandth regarding value got by Buckingham theorem.

Now it is needed to rewrite all the formulas considering B=5/3. Furthermore, with B= 5/3, a lot of parameters of the theory become simple fraction numbers. In other words, theory gains simplicity and credibility.

In chapter 5 was shown the relation between a&b parameters and A&B parameters. Now considering B= 5/3

as $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$. It is right to get $b = \frac{B-2}{2B-2} = -\frac{1}{4}$ and $A = \frac{a^{\frac{-4}{3}}}{8\pi G}$

Therefore, the central formula of theory become $D_{DM} = A \cdot E^{\frac{5}{3}} = \frac{a^{\frac{-4}{3}}}{8 \cdot \pi \cdot G} \cdot E^{\frac{5}{3}}$

9.1 RECALCULATING THE PARAMETER a IN M31 HALO

Table below comes from chapter 3 and represents regression curve of velocity depending on radius.

Power regression for M31 rot. curve	
$V=a*r^b$	
a old	$4{,}1501104*10^{10}$
b (old)	-0,24755452
Correlation coeff.	0,952254

Due to Buckingham theorem it is needed that b= -1/4 . Therefore it is needed to recalculate parameter **a** in order to find a new couple of values a&b that fit perfectly to experimental measures of rotation curve in M31 halo.

Probably there will be another more sophisticated way to recalculate parameter a. However it will be used a simple way to do it. This way consist on recalculate a new parameter **a** for every value of speed got by regression curve. The new value is got with this formula: **new a** = vel. with old a&b (column 4)* radius^0.25.

This new value is tabulated in column 5. Afterwards it is got its average value equal to $a_{NEW} = 4{,}683972*10^{10}$. In column 6 is tabulated the new velocity regression curve. In column 7 is tabulated the relative difference percentage between velocity regression curve with old parameters a&b and with the new ones. Reader can see that maximum relative difference is 0.22 % which is quite acceptable because the method to recalculate it has been quite simple.

1	2	3	4	5	6	7
Radius kpc	Radius m	V measures	Vel with old a&b	New value of a	V with new a&b	% of Relative difference
40,7	1,26E+21	230000	2,4826E+05	4,673531E+10	2,4882E+05	-2,23E-01
48,25	1,49E+21	235900	2,3802E+05	4,675477E+10	2,3845E+05	-1,82E-01
58,2	1,80E+21	247800	2,2722E+05	4,677621E+10	2,2753E+05	-1,36E-01
69,8	2,15E+21	217400	2,1723E+05	4,679700E+10	2,1743E+05	-9,13E-02
84,3	2,60E+21	204300	2,0731E+05	4,681861E+10	2,0740E+05	-4,51E-02
101,7	3,14E+21	210900	1,9790E+05	4,684010E+10	1,9790E+05	8,03E-04
122,1	3,77E+21	195600	1,8914E+05	4,686104E+10	1,8906E+05	4,55E-02
146,5	4,52E+21	175000	1,8080E+05	4,688192E+10	1,8064E+05	9,00E-02
176,2	5,44E+21	163000	1,7273E+05	4,690309E+10	1,7249E+05	1,35E-01
211	6,51E+21	164100	1,6519E+05	4,692377E+10	1,6489E+05	1,79E-01
254,1	7,84E+21	159800	1,5776E+05	4,694510E+10	1,5741E+05	2,24E-01
			Average a=	4,683972E+10		

Velocity regression curve with new a &b	
V = a*r^(-1/4)	
New b	-1 /4
New a	$4{,}683972*10^{10}$

RECALCULATING a WITH MINUMUN SQUARE METHOD

When it is searched the parameter a, a method widely used is called the minimum squared method. So it is searched a new parameter **a** for the formula V= a*r $^{-0.25}$ on condition that $\sum_e (v-v_e)^2$ has a minimum value. Where v represents the value fitted for velocity formula and v_e represents each measure of velocity. It is right to calculate the formula for a.

$$a = \frac{\sum_e Ve \cdot r_e^{-0.25}}{\sum_e r_e^{-0.5}} = 4.6847 \cdot 10^{10}$$

Where r_e represents each radius measure and v_e represents its velocity associated.

The difference with the value got in previous epigraph is only 7 ten-thousandth.

9.2 RECALCULATING PARAMETER A IN M31 HALO

At the beginning of this chapter was got that $A = \frac{a^{\frac{-4}{3}}}{8\pi G}$.

In previous epigraph has been recalculated the parameter a. Therefore A has to change according this new value.

The beside table shows the value of new parameters.

New parameter a&b and A&B	
B	5/3
$b = \frac{B-2}{2B-2}$	b = -1/4
a new	4,683972*10 ^{10}
$A = \frac{a^{\frac{-4}{3}}}{8\pi G}$	New parameter A A= 3.531*10^{-6}
$D_{NEW} = 8 \cdot \pi \cdot G \cdot A$	5,9226*10 $^{-15}$

9.3 FORMULAS OF DIRECT D.M. IN M31 HALO

Function of Density DM depending on radius.

$$D_{DM}(r) = L \cdot r^{2b-2} = L \cdot r^{\frac{-5}{2}} \quad \text{being} \quad L = \frac{a^2 \cdot (2b+1)}{4\pi G} = \frac{a^2}{8 \cdot \pi \cdot G} = 1.308 \cdot 10^{30}$$

Function of E depending on radius $E = a^2 \cdot r^{2b-1} = a^2 \cdot r^{\frac{-3}{2}}$ being a^2 = 2.193959*10 ^{21}

Mass enclosed by a sphere of radius r. $M(<r) = \frac{v^2 \cdot R}{G} = \frac{a^2 \cdot r^{2b+1}}{G} = \frac{a^2 \cdot \sqrt{r}}{G}$ being $\frac{a^2}{G}$ = 3.2874*10 ^{31}

9.4 BERNOULLI SOLUTION FOR E IN M31 HALO

$$E(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$$

being $\alpha = 2B - 2 = \dfrac{4}{3}$ being $\beta = \dfrac{1}{1-B} = \dfrac{-3}{2}$ and $D = \left(\dfrac{4\cdot\pi\cdot G\cdot A(1-B)}{3-2B}\right) = 8\cdot\pi\cdot G\cdot A$ Therefore

$E(r) = \left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-3}{2}}$ being $D = 8\pi GA = a^{\frac{-4}{3}}$ = 5.9226*10^{-15} being $C = \dfrac{E_0^{\frac{-2}{3}} - D\cdot R_0}{R_0^{\frac{4}{3}}}$ the initial condition of differential equation solution for E.

CALCULUS OF PARAMETER C

Radius	Velocity	E	C
48.25 kpc = 1.48885E21 m	2.380198*10^{5}	3.80518*10^{-11}	1,256162*10^{-24}

The radius chosen for C has been 48.25 kpc because for this radius the experimental measure of velocity match very well with the value got by regression curve of velocity depending on radius, see epigraph 3.1 to check this statement. This way Eo calculated by regression speed match very well with Eo calculated by measure of velocity. According the formula of C and these values it is got C = 1,256162*10^{-24}

9.5 DARK MATTER DENSITY DEPENDING ON FIELD E.

As D_{DM}= A*E^{B} Using field got by Bernoulli solution it is right to get

$D_{DM}(r) = A\left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-5}{2}}$ Being **A=** 3,531*10^{-6} **D=**5,9226*10^{-15} and **C** = 1,256*10^{-24}

It is clear that theory has won simplicity and elegancy.

10. COMPARISON BETWEEN DIRECT DM DENSITY AND BERNOULLI FORMULAS

10.1 COMPARISON FOR FORMULAS OF GRAVITTIONAL FIELD AND FOR DENSITY

In this epigraph it will be compared direct DM field E with Bernoulli field E by one side and direct DM density with Bernoulli DM density by other side throughout the whole radius dominion.

The first and second columns show the radius measured in kpc and meters. In the third column is shown field E calculated with direct E formula, 9.5.1 epigraph, the fourth column show Bernoulli field, epigraph 9.5.2, the fifth column show direct DM, epigraph 9.5.1, and the sixth column show calculus of DM density as power of E, epigraph 9.53. Reader can check that there is a perfect agreement between either direct E and Bernoulli E or direct DM and Bernoulli DM. Namely the difference arises at the third or fourth decimal digit. The seventh and eighth columns show the relative difference percentage. This differences range between 0.3% and 0.6% for E and between 0,6 % and 1% for DM density all over radius dominion.

1	2	3	4	5	6	7	8
Radius kpc	Radius m	direct E	Berni E	Direct DM	Berni DM	Relt.Diff.% E	Relt. diff.% Dens. DM
4,8252E+01	1,4889E+21	3,8188E-11	3,8050E-11	1,5293E-23	1,5199E-23	3,62E-01	6,15E-01
6,8150E+01	2,1029E+21	2,2751E-11	2,2659E-11	6,4509E-24	6,4065E-24	4,06E-01	6,88E-01
8,8048E+01	2,7169E+21	1,5492E-11	1,5424E-11	3,4001E-24	3,3746E-24	4,42E-01	7,48E-01
1,0795E+02	3,3309E+21	1,1413E-11	1,1359E-11	2,0430E-24	2,0267E-24	4,73E-01	8,00E-01
1,2784E+02	3,9449E+21	8,8547E-12	8,8104E-12	1,3384E-24	1,3271E-24	5,00E-01	8,45E-01
1,4774E+02	4,5589E+21	7,1275E-12	7,0901E-12	9,3222E-25	9,2396E-25	5,25E-01	8,86E-01
1,6764E+02	5,1729E+21	5,8970E-12	5,8647E-12	6,7973E-25	6,7345E-25	5,47E-01	9,23E-01
1,8754E+02	5,7869E+21	4,9838E-12	4,9555E-12	5,1352E-25	5,0860E-25	5,68E-01	9,58E-01
2,0744E+02	6,4009E+21	4,2842E-12	4,2590E-12	3,9909E-25	3,9514E-25	5,87E-01	9,90E-01
2,2734E+02	7,0149E+21	3,7342E-12	3,7116E-12	3,1741E-25	3,1417E-25	6,05E-01	1,02E+00
2,4723E+02	7,6289E+21	3,2926E-12	3,2721E-12	2,5734E-25	2,5465E-25	6,22E-01	1,05E+00

10.2 COMPARISON BETWEEN DIRECT AND BERNOULLI FORMULAS OF MASS

In epigraph 9.5.1 it is shown the formula $M(<r) = \frac{v^2 \cdot R}{G} = \frac{a^2 \cdot r^{2b+1}}{G} = \frac{a^2 \cdot \sqrt{r}}{G}$ so $M(<r) = 3.2874 \cdot 10^{31} \cdot \sqrt{r}$ is the dynamical mass enclosed by a sphere of radius r.

Through that formula it is right to calculate the mass enclosed by the spherical corona defined by 48,25 kpc and 254 kpc getting a mass m=1.6425 $*10^{42}$ kg =8.25$*10^{11}$ M sun

With formula below is calculated the mass enclosed by a spherical corona defined by a $R_{MINIMUM}$ and $R_{MAXIMUM}$

$$M_{DM} = \int_{R1}^{R2} 4\pi r^2 \cdot \rho(r) dr = \int_{R1}^{R2} 4\pi r^2 A E^B dr = 4\pi A \int_{R1}^{R2} r^2 \left[C \cdot r^{\frac{4}{3}} + D \cdot r \right]^{\frac{-5}{2}} \cdot dr = 4 \cdot \pi \cdot A \cdot I$$

Where $4\pi A = 4.4372 \cdot 10^5$ **D**=5,9226*10^{-15} and C = 1,2562*10^{-24} and I symbolise the definite integral.

$$\int_{1.4889 \times 10^{21}}^{7.841 \times 10^{21}} \frac{x^2}{(1.2562 \times 10^{-24}\, x^{1.3333333} + 5.9226 \times 10^{-15}\, x)^{2.5}}\, dx = 3.67079 \times 10^{46}$$

This complex integral has been calculated by the remarkable software on line Wolfram alpha.
The above definite integral has calculated the mass contained inside spherical corona defined by Rmin= 48.25kpc and Rmax = 254 kpc. $M_{DM} = 4 \cdot \pi \cdot A \cdot I$ = 1.6285* 10^{42} kg =8.18*10^{11} Msun
The difference between two ways to calculate the spherical corona mass is only 7 hundredth which is an error quite small. Namely, the relative error is 0.0085

Mass contained inside spherical corona of M31 galaxy between radius 48.25 kpc and 254 kpc
Mass of spherical corona by integration of DM density = 8.18*10^{11} Msun
Mass of spherical corona by direct formula of mass = 8.25*10^{11} Msun

It would seem impossible that both values match so perfectly, because direct formula was got rightly whereas the mass calculated by integration involves a long way as the reader can check.

10.3 ANALYSIS OF TWO TERMS IN BERNOULLI SOLUTION OF FIELD E

In this epigraph will be compared the addends $C*r^{4/3}$ and $D*r$ throughout the whole dominion.

Solution for field E is $E(r)=\left(Cr^{\frac{4}{3}}+Dr\right)^{\frac{-3}{2}}$ being $D=5{,}9226*10^{-15}$ and $C = 1{,}2562*10^{-24}$

In table below has been made the comparison between both terms and the last column shows that C addend is approximately 3 thousandth of D addend.

Radius m	Radius kpc	Bernoulli E	C term	D term	C term/D term
1,489E+21	4,825E+01	3,805E-11	2,136E+04	8,818E+06	2,4219E-03
2,103E+21	6,815E+01	2,266E-11	3,384E+04	1,245E+07	2,7173E-03
2,717E+21	8,805E+01	1,542E-11	4,762E+04	1,609E+07	2,9595E-03
3,331E+21	1,079E+02	1,136E-11	6,249E+04	1,973E+07	3,1675E-03
3,945E+21	1,278E+02	8,810E-12	7,830E+04	2,336E+07	3,3513E-03
4,559E+21	1,477E+02	7,090E-12	9,496E+04	2,700E+07	3,5168E-03
5,173E+21	1,676E+02	5,865E-12	1,124E+05	3,064E+07	3,6681E-03
5,787E+21	1,875E+02	4,955E-12	1,305E+05	3,427E+07	3,8079E-03
6,401E+21	2,074E+02	4,259E-12	1,493E+05	3,791E+07	3,9380E-03
7,015E+21	2,273E+02	3,712E-12	1,687E+05	4,155E+07	4,0601E-03
7,629E+21	2,472E+02	3,272E-12	1,887E+05	4,518E+07	4,1753E-03

Therefore when C term is neglected it is got a good approximation of field E.

10.4 GETTING DIRECT FORMULAS THROUGH BERNOULLI FIELD APROXIMATION

As it has been shown in the previous epigraph if it is neglected C addend it is got a good approximation of Bernoulli field which is quite simple. Furthermore, thanks to this more simple formula for E, it will be got the initial formulas of theory calculated in chapter 4.

$E(r)=\left(Cr^{\frac{4}{3}}+Dr\right)^{\frac{-3}{2}}$ When in previous formula is neglected the addend C, it is got $E=a^2\cdot r^{\frac{-3}{2}}$ which is precisely direct formula for E.

Similarly it is got a good approximation of DM density $D_{DM}(r)=L\cdot r^{\frac{-5}{2}}$ being $L=\frac{a^2}{8\cdot\pi\cdot G}$ =1.308*10 ^{30} which is precisely the formula of direct DM density.

Similarly, when it is made the same approximation for field E in the integral of total mass, then the integral becomes

$$M_{DM} = \int_{R1}^{R2} 4\pi r^2 \cdot \rho(r) dr = \int_{R1}^{R2} 4\pi r^2 AE^B dr = 4\pi A \int_{R1}^{R2} r^2 [D \cdot r]^{\frac{-5}{2}} \cdot dr$$ whose indefinite integral is $M(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$

which is direct formula of mass enclosed by a sphere of radius r.

10.5 MONOTONOUS GROWING OF MASS FORMULA DEPENDING ON RADIUS

The formula $M(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ is a very simple formula which raises a right query. Which is the radius limit for this formula?

The answer is right because according theory of DM generated by gravitational field, the radius limit is the limit of galactic gravitational influence. For example the Local Group of galaxies is composed by Milky Way and M31 as two giant galaxies and thirty dwarf o medium size galaxies which orbiting around the two giants ones. Therefore the limit for radius of gravitational influence for M31 could be the half distance between Milky Way and M31, which is approximately 350 kpc. For this radius the formula of mass gives rightly a total mass of $1.7*10^{12}$ Msun.

By integration of complete Bernoulli field thanks Wolfram alpha software it is got this result.

$$\int_{1.489\times10^{21}}^{1.08\times10^{22}} \frac{x^2}{(1.2562\times10^{-24}\, x^{1.3333333} + 5.9226\times10^{-15}\, x)^{2.5}}\, dx = 4.79711\times10^{46}$$

So mass of spherical corona defined by 48 kpc and 350 kpc is equivalent to $1.07*10^{12}$ Msun which added to dynamical mass of sphere with radius 48 kpc equivalent to $6.38*10^{11}$ Msun is equal to $1.7*10^{12}$ Msun. Reader can check that both methods agree perfectly even for this enormous distances. Do not forget that dynamical mass at radius 48 kpc include baryonic and DM mass, whereas mass contained into the spherical corona include only DM.

The parameter **a** has been got through a statistical study in halo from 40 kpc to 250 kpc, so it is clear that this formula is not acceptable for far more distances. Furthermore, it is supposed that theory of DM generated by G.F. is an approximation for a further quantum gravity theory.

In addition this type of formula could be checked in galaxy clusters, see [9] Abarca,M.2016. A study about Coma cluster.

11. PROPORTION OF GALACTIC DM IN THE ANCIENT UNIVERSE IS LOWER THAN AT PRESENT

According the Hubble law, the expansion of the space in the Universe follows this simple differential equation.

$\frac{\partial R}{\partial t} = H * R$ being H= Constant Hubble = 70 Km/s/ Mpc being 1 Mpc = $3.0857*10^{22}$ m H= $2.2685*10^{-18}$ s^{-1}

And being R a specific distance, for example the radius of some galaxy.

The inverse of Hubble constant has time units and 1/H = $1.397*10^{10}$ years. So 1/H is called Hubble time.

Hubble time is quite close to current Universe age equal to $1.38*10^{10}$ years, according current cosmology. So for our proposes it is enough to approximate 1/H equal to the Universe age.

The solution of such simple differential equation is $R = R_0 \cdot e^{Ht}$ where R_0 it is the initial radius of a specific galaxy for t=0. However, this formula is not suitable because at t = 0 there was not any galaxies.

So it is better rewrite the solution considering current time as the reference to measure the time, then $R = R_C \cdot e^{H(t-t_c)}$ Where t_C represents the current time, R_C is the size of some specific galaxy nowadays and t is the age of such galaxy.

With this solution it is clear that for t = tc $R = R_C$.

Going back 1/ 3 of the Universe age then $t-t_C = -1/(3H)$ and $R = R_C \cdot e^{\frac{-1}{3}} = 0.72 \cdot R_C$

In other words 4650 million years ago the size of a galaxy was 0.72 times its current size.

Going back 2/3 of the Universe age then $t-t_C = -2/(3H)$ and $R = R_C \cdot e^{\frac{-2}{3}} = 0.51 \cdot R_C \approx 0.5 \cdot R_C$

In other words 9300 million years ago the size of galaxies was one half of its current sizes.

11.1 COMPARISON BETWEEN DM OF M31 IN THE ANCIENT UNIVERSE AND AT PRESENT

From the epigraph 10.2 the formula $M(<r) = \frac{v^2 \cdot R}{G} = \frac{a^2 \cdot r^{2b+1}}{G} = \frac{a^2 \cdot \sqrt{r}}{G}$ is the dynamical mass enclosed inside a sphere of radius r.

So $M(r_{\min} < r < r_{\max}) = 3.287 \cdot 10^{31} \cdot (\sqrt{r_{\max}} - \sqrt{r_{\min}})$ represents the mass enclosed by a spherical corona whose radius are Rmax and R min.

Thanks this formula is quite right to calculate DM inside a spherical corona.

For example considering the spherical corona for M31 where currently baryonic matter is negligible Rmin = 40 kpc and Rmax= 350 Kpc then Total $DM_{SPHERICAL\ CORONA} = 1.13*10^{12}\ M_{SUN}$

Going back 4650 million years, the current distances are reduced by factor 0.7. So 40 kpc becomes R min= 29 kpc and 350 kpc becomes R max= 252 kpc then Total $DM_{SPHERICAL\ CORONA} = 0.96*10^{12}\ M_{SUN}$. Which means a 15% of reduction of DM regarding total current amount.

Going back 9300 million years, the current distances are reduced by a factor 0.5 so Rmin = 20 kpc and Rmax = 175 kpc then Total $DM_{SPHERICAL\ CORONA} = 8*10^{11}\ M_{SUN}$. Witch means a 29.2 % of reduction of DM regarding total current amount.

Although these simple calculus have been made inside the spherical corona where baryonic matter is negligible, inside the galaxy it happen the same, the DM is lower because the galactic size in the ancient Universe is lower. What happens is that calculus of DM inside galactic disk is far more complex because it is needed to know baryonic density function and solve a far more complex differential equation than a simple Bernoulli one.

This calculus would be in agreement with majority of cosmologist, because it seems that observational evidences back a lower fraction of DM in measures of ancient galaxies. In other words in galaxies which are far away, and consequently the observational data inform about what happens thousand million years ago in a very far away galaxy.

Furthermore, in the paper [16] Alfred L. Tiley, 2108. After studying some 1,500 galaxies, researchers led by Alfred Tiley of Durham University have determined that the fraction of dark matter in galaxies placed 10 Gys (10^{10} light-years) away is at least 60%. This findings could back calculus made above.

12. CONCLUSION

This work is focused in halo region of M31 where baryonic density is negligible regarding DM non baryonic. The reason is that according the main hypothesis of this theory, the non baryonic DM is generated locally by the gravitational field. Therefore it is needed to study DM on the radius dominion where it is possible to study gravitational field propagation without interference of baryonic mass density or at least where this density is negligible.

In order to defend properly the conclusion of this paper, it is important to emphasize that correlation coefficient of power regression over velocity measures in rotation curve in halo region is bigger than 0,95. See chapter 3 where was got coefficients a& b for $v = a \cdot r^b$ law.

In chapter four was mathematically demonstrated that the power law $v = a \cdot r^b$ in halo region is equivalent a DM density called direct DM , whose formula is $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$.

In chapter five was demonstrated mathematically that the power law for velocity $v = a \cdot r^b$ on the rotation curve is mathematically equivalent to a power law for DM density depending on E. $D_{DM} = A \cdot E^B$.

Where $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$.

Therefore joining chapters 3,4 and 5 it is concluded that the high correlation coefficient equal to 0.95 at power regression law for rotation curve $v = a \cdot r^b$ in halo region support strongly that DM density inside halo region is a power of gravitational field $D_{DM} = A \cdot E^B$ whose parameters A & B are written above.

As it was pointed at introduction, it is known that there is baryonic dark matter such as giant planets, cold gas clouds, brown dwarfs but this type of DM is more probable to be placed inside galactic disk and bulge.

Reader can consult these papers about this open problem: [11] Nieuwenhuizen,T.M. 2010. [12] Nieuwenhuizen,T.M. 2012. [13] Nieuwenhuizen,T.M. 2010 [14] Wyrzykowski,L.2010. [15] M.R.S. Hawkins 2015

In chapter seven was got a Bernoulli differential equation for M31 halo in order to look for a local method to calculate the local field E. In fact in chapter 10 was demonstrated that Bernoulli field solution it is a more accurate method than direct formula for E, although both formulas differs less than 0.6%.

The core this paper is chapter eight, where it is made dimensional analysis for magnitudes Density and field E and for universal constants G, h and c. It is demonstrated that formula for DM density need two Pi monomials. Furthermore it is found that coefficient B = 5/3, power of E, is coherent with Buckingham theorem and it differs only two thousandth regarding B value got by statistical regression of rotation curve in M31 halo. As a consequence of this simple fraction for B, the others parameters a&b and A were recalculated and formulas of theory were rewriting. This way these formulas achieve dimensional coherence and the theory gains simplicity and credibility.

It is important to state that Buckingham theorem allow the value B=5/3 as power for field, although another different fractions would be allowed too. However, it is such fraction which match perfectly with the value got by statistic regression of rotation curve data on rotation curve in M31 halo.
Results got about $D_{DM} = A{\cdot}E^{B}$ in other galaxies, see [8] Abarca,M.2016, suggest that DM density follows a similar law in different galaxies. Namely the power for E is close to 5/3, on condition that galaxies should be similar giant galaxies i.e. its velocity is bigger than 200 km/s in the disk region of the rotation curves.

Finally in the last chapter, through a simple application of Hubble law has been demonstrated that in the ancient Universe, the galaxies are smaller and consequently the proportion of DM versus baryonic is lower. These calculus are in agreement with current observational evidences.
In my opinion these facts suggest strongly that nature of non baryonic DM is generated by gravitational field according a Universal mechanism. In other words, it is clear that DM it is a phenomenon of quantum gravitation.

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