Formula of ζ even-numbers

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abstract
I published the odd value formula for ζ, but I realized that this was true even when it was even. Therefore, it will be announced.

key words
ζ even-numbers, formula

1 Introduction

I made official previous paper[1].

$$\zeta(2m-1) = \frac{2^{2m-1}}{2^{2m-1}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m-1}} \tag{1}$$

m is a positive integer.
This formula holds for odd numbers, but it may hold for even numbers.

In this case, the formula is transformed as follows:

$$\zeta(2m) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{2m}} = \frac{1}{2^{2m}} \sum_{n=1}^{\infty} \frac{1}{n^{2m}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m}} = \frac{1}{2^{2m}}\zeta(2m) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m}} \tag{2}$$

$$(1 - \frac{1}{2^{2m}})\zeta(2m) = (\frac{2^{2m}-1}{2^{2m}})\zeta(2m) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m}} \tag{3}$$

$$\zeta(2m) = \frac{2^{2m}}{2^{2m}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m}} \tag{4}$$

m is a positive integer.

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If m=1

$$\zeta(2) = \frac{2^2}{2^2-1}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} = \frac{4}{3}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} \tag{5}$$

If ∞=10
$\zeta(2) \approx 1.611628414828517525404433776164201206744643981790542327531220969423 8....$
If ∞=20
$\zeta(2) \approx 1.628270869371572772206131131338932788062642746620939996574291345189...$
If ∞=30
$\zeta(2) \approx 1.633823984143959233629940235399597938132955573725470128821490355376 3...$
$\zeta(2) = 1.644934066848226436472415166646025189218949901206798437735558229 37...$
∞=30 seemed to be the limit of the calculator.

If m=2

$$\zeta(4) = \frac{16}{15}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4} \tag{6}$$

If ∞=10
$\zeta(4) \approx 1.082301121641836587579915382686294511311212243488075865497221359 4...$
If ∞=20
$\zeta(4) \approx 1.082320459398015020659441963503798624039413318397956577979066575070...$
If ∞=30
$\zeta(4) \approx 1.082322411122674253866274278996655718012453768878275375647387105547...$
$\zeta(4) = 1.082323233711138191516003696541167902774750951918726907682976215444 12...$
∞=30 seemed to be the limit of the calculator.

If m=3

$$\zeta(6) = \frac{64}{63}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^6} \tag{7}$$

If ∞=10
$\zeta(6) \approx 1.017343030628910108384026458983050254478715065673754098957885929 7...$
If ∞=20
$\zeta(6) \approx 1.017343060995473261375938647877506866869711618696998641526975803359...$
$\zeta(6) = 1.017343061984449139714517929790920527901817490032853561842408664004 3...$
∞=20 seemed to be the limit of the calculator.

If m=4

$$\zeta(8) = \frac{256}{255}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^8} \tag{8}$$

If ∞=10
$\zeta(8) \approx 1.004077356143195947721033674961435105092755543601654685044870088 24...$
If ∞=20
$\zeta(8) \approx 1.004077356197509200802560215297349422835798740016360034810273443 61...$
$\zeta(8) = 1.004077356197944339378685238508652465258960790649850020329110202652 5...$
∞=20 seemed to be the limit of the calculator.

If m=5

$$\zeta(10) = \frac{1024}{1023}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{10}} \tag{9}$$

If ∞=10
$\zeta(10) \approx 1.000994575127713395596883654801393423304300758901415405226780066 00...$
$\zeta(10) = 1.000994575127818085337145958900319017006019531564477517257788994...$
∞=10 seemed to be the limit of the calculator.

If m=6

$$\zeta(12) = \frac{4096}{4095} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{12}} \tag{10}$$

If ∞=10
$\zeta(12) \approx 1.000246086553307837898350728937608769939875657839099331899582091846...$
$\zeta(12) = 1.000246086553308048298637998047739670960416088458003404533...$
∞=10 seemed to be the limit of the calculator.

If m=7

$$\zeta(14) = \frac{16384}{16383} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{14}} \tag{11}$$

If ∞=10
$\zeta(14) \approx 1.000061248135058704393025782997730970862675825510279576412953804 92...$
$\zeta(14) = 1.000061248135058704829258545105135333747481696169154549482755202 2...$
∞=10 seemed to be the limit of the calculator.

If m=8

$$\zeta(16) = \frac{65536}{65535} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{16}} \tag{12}$$

If ∞=8
$\zeta(16) \approx 1.000015282259408651846791233879881425608206629264990534917564141014...$
$\zeta(16) = 1.000015282259408651871732571487636722023237388990471531153105203588 7...$
∞=8 seemed to be the limit of the calculator.

If m=9

$$\zeta(18) = \frac{262144}{262143} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{18}} \tag{13}$$

If ∞=8
$\zeta(18) \approx 1.000003817293264999839773763393535022953556904949205416698268694485...$
$\zeta(18) = 1.000003817293264999839856461644621939730454697218953331143174429...$
∞=8 seemed to be the limit of the calculator.

If m=10

$$\zeta(20) = \frac{1048576}{1048575} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{20}} \tag{14}$$

If ∞=8

$\zeta(20) \approx 1.00000095396203387279611287506829811114176615017147477552008915142...$
$\zeta(20)$ $=1.00000095396203387279611315203868344934594379418741059575005648 98...$
∞=8 seemed to be the limit of the calculator.

If m=11

$$\zeta(22) = \frac{4194304}{4194303} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{22}} \tag{15}$$

If ∞=8
$\zeta(22) \approx 1.00000023845050272773299000271364495850341506148937158420714270742 9...$
$\zeta(22)$ $=1.00000023845050272773299000364818675299493504182177965826984960...$
∞=8 seemed to be the limit of the calculator.

If m=12

$$\zeta(24) = \frac{16777216}{16777215} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{24}} \tag{16}$$

If ∞=8
$\zeta(24) \approx 1.00000005960818905125947961243703694927126322746933684956907465721...$
$\zeta(24)$ $=1.00000005960818905125947961244020793580122750391883730279586424 6...$
∞=8 seemed to be the limit of the calculator.

If m=13

$$\zeta(26) = \frac{67108864}{67108863} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{26}} \tag{17}$$

If ∞=6
$\zeta(26) \approx 1.00000001490155482836504123464733064773756513297654870572919271493...$
$\zeta(26)$ $=1.00000001490155482836504123465850663069862886478816788591054743...$
∞=6 seemed to be the limit of the calculator.

If m=14

$$\zeta(28) = \frac{268435456}{268435455} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{28}} \tag{18}$$

If ∞=6
$\zeta(28) \approx 1.00000000372533402478845705481913830414006066822530605696055267430...$
$\zeta(28)$ $=1.00000000372533402478845705481920401840242323289305929581151976 9...$
∞=6 seemed to be the limit of the calculator.

If m=15

$$\zeta(30) = \frac{1073741824}{1073741823} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{30}} \tag{19}$$

If ∞=6
$\zeta(30) \approx 1.00000000093132743241966818287176434799948212432883470226054770 24...$

$\zeta(30)$ =1.000000000931327432419668182871764735021219813567955136816185008 6...
∞=6 seemed to be the limit of the calculator.

2 Conclusion

The odd formula for ζ has been released, but it has become clear that this is true even when it is even.

3 Appendices

I use WolframAlpha for calculation.

References

[1] T. takami.: Formula for odd values of ζ, viXra:1909.0385

[2] T. takami.: $\zeta(5), \zeta(7), ...\zeta(331), \zeta(333)$ are Irrational Number, viXra:1909.0315