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Irrational number problem

A solution to the irrational numbers problem is proposed

The irrationality problem was first discovered in the geometry when extracting the root.

It was known in the Age of Antiquity, associated with Pythagoras.

The logical contradiction revealed is the following. On the one hand, there is proof that all points on the line are integers or fractions, i.e. rational numbers.

This proof is as follows.

The line segment with the coordinates of its ends 0 and 1 is taken. Both these coordinates are integers.

The segment is divided in half and each of the newly received segments is considered.

The ends of these segments have coordinates of 0 and 0.5 or 0.5 and 1, which are integers or fractions, i.e. rational numbers.

The division into halves continues, bringing the ends of the following segments closer together while keeping them constant rational numbers.

In the extreme case, at infinite division, the ends of the segments are merged into one point, *remaining* rational numbers.

The logical deduction is that the initial segment is filled with *only* rational numbers, in other words, there is no place for any irrationality.

On the contrary, another proof leads to the fact that some points on the line cannot be set by either integers or fractions, i.e. they are not rational.

This proof is as follows: a right triangle with two equal legs each 1 is taken. According to the Pythagorean theorem, the length of the hypotenuse is $\sqrt{2}$. This cannot be either an integer or an irreducible fraction $\frac{a}{b}$, because in this case $a^2 = 2b^2$. Consequently, a is an even number represented as $a = 2k$. But then $a^2 = (2k)^2 = 4k^2 = 2b^2$. It means that $b^2 = 2k^2$, i.e. b is also an even number. We get a logical contradiction: on the one hand, $\frac{a}{b}$ should be irreducible fraction (otherwise it can be reduced by a common factor), on the other hand, both its parts a and b are even numbers, i.e. have a common factor 2, and therefore the fraction is reducible.

So, the first *logically not contradictory* proof is opposed to the second one, *logically contradictory* proof.

Since the first proof does not contain a logical contradiction, it cannot cause any doubts and must be regarded as absolutely correct.

The second proof, on the other hand, contains a logical contradiction. Therefore, it cannot in any way serve as a rebuttal of the first, logically not contradictory proof. And it must, as a logical contradiction, be regarded as highly doubtful and requiring further consideration.

The proposed consideration is as follows.

First, what does it mean that the leg length is 1? This means that both legs were measured with some standard and that the result of this measurement is one. The reasonable question for any measurement is: what the accuracy is? The answer is as follows: the absolute error of measuring with any standard is equal to the standard itself, and the accuracy of measuring is determined by the ratio of the absolute error (equal to the standard) to the very measurable value (the relative error).

The value of the standard relative to itself is equal to one *with infinite degree of accuracy*, which can be expressed as decimal fraction: $s = 1,(0)$. But the values of both legs a and b, measured by this standard, should look like this: $a = 1 \pm \Delta a = 1 \pm s$, $b = 1 \pm \Delta b = 1 \pm s$, where s is the standard value.

In this case, we obtain: absolute error $\Delta a = s = 1$, $\Delta b = s = 1$, $a = 1 \pm 1$, $b = 1 \pm 1$. The relative error determining the accuracy of each measurement is equal to

$$\delta\, a(\%) = \frac{\Delta a 100}{a} = \frac{100}{1} = 100\% \text{ and } \delta b(\%) = \frac{\Delta b 100}{b} = \frac{100}{1} = 100\%$$

And even if we take one leg as a standard, for example, a, which means $\delta\, a(\%) = 0,(0)$, i.e. the infinite accuracy of its measurements and its zero relative error, the relative error of measurement of the second leg still remains $\delta b = 100\%$.

This is what this carelessly thrown equality of the lengths of both legs to one means in practice.

And what do we get from measuring hypotenuse with the standard s?

There are two possible answers: $c = 1 \pm \Delta s = 1 \pm s = 1 \pm 1$ or $c = 2 \pm \Delta c = 2 \pm s = 2 \pm 1$.

In the first case, the error of hypotenuse measurement is 100%, as in the case of the leg, and in the second case it is 50%. The second answer is obviously more accurate, although not very good either.

What do we have on the Pythagorean theorem now? The legs are equal to 1 ± 1, i.e. they can be considered equal to 1 or 2, and the hypotenuse can be equal to 1 or 2, or even 3. And each of these answers is correct in its own way with a certain degree of accuracy.

But at the same time $1^2 + 1^2 \neq 1^2$ or 2^2 and of course $\neq 3^2$.

And the second possible option also gives: $2^2 + 2^2 \neq 1^2$ or 2^2 or 3^2.

And even the leg as a standard also gives: $1^2 + 2^2 \neq 1^2$ or 2^2 or 3^2. In other words, the required equality is not achieved with *any* option of such measurements.

Accuracy is improved by reducing the value of the standard s, for example, by a factor of 10.

In this case, $a = 10 \pm 1$, $b = 10 \pm 1$, $c = 14 \pm 1$, $\delta\, a = 10\%$, $\delta\, b = 10\%$, $\delta\, c = \frac{100}{14} = 7\%$.

Or by a factor of 100, when $a = 100 \pm 1$, $b = 100 \pm 1$, $c = 141 \pm 1$, $\delta\, a = 1\%$, $\delta\, b = 1\%$, $\delta\, c = \frac{100}{141} = 0.7\%$, etc.

However, $a^2 + b^2 \neq c^2$ still, i.e. the Pythagoras theorem is still not suitable.

This can only be achieved at *infinite accuracy* of measurements, when the standard $s = 0,(0)$, $a = 10000... = \infty$, $b = 10000... = \infty$,

$c = 14142135623730950488016887242097141... = \infty$,

Or in the case of an expression through the reference standard s: $a = 1,(0)$, $b = 1,(0)$, $c = 1,41421356237309504880168872420971 41...$

In this, and only in this case, the Pythagorean theorem is fair, but it looks as follows: $a^2 + b^2 = c^2 \;\rightarrow\; \infty^2 + \infty^2 = \infty^2$.

It may look complicated in the usual sense, but no longer contains any logical contradiction.

What does all this mean?

That is: the Pythagorean theorem, as well as all geometry theorems in general are fair *without exception*, provided that one more theorem is fair.

In view of its universality and exceptional importance, it can be called the *Great Geometric Theorem (GGT).*

Its content is as follows: *all geometric theorems are correct only if the accuracy of measurements is infinite.*

It means that *there are no* such integers of 1 of both legs and *there can be no* such integers, and there can be only infinite decimal fraction in the particular case we consider: $a = 1,(0)$, and $b = 1,(0)$.

In this case, all arguments about the reducibility or irreducibility of infinite fractions and, accordingly, evenness or oddness immediately disappear, because it is possible only *when the fraction under consideration is finite*. It is easily achieved by a simple break of infinity, i.e. by *violation of infinite accuracy*. But then the Pythagorean theorem is immediately violated, i.e. ceases to be performed.

It means that the whole problem under consideration is immediately and irrevocably removed!

All this leads to the fact that any point on the geometric line is set as an *infinite fraction*, and there is no difference or peculiarity neither of the leg nor of the hypotenuse.

Moreover, it means all numbers without exception, setting the position of any geometric points, should be considered irrational because of the simple infinity of their fractions, or we must accept that there are no irrational numbers at all.

It is precisely because the consideration performed leads to the complete removal of the logical contradiction that forced to invent them.