

# **Further mathematical connections between the Dark Matter candidate particles, some Ramanujan's Mock Theta Functions and the Physics of Black Holes. II**

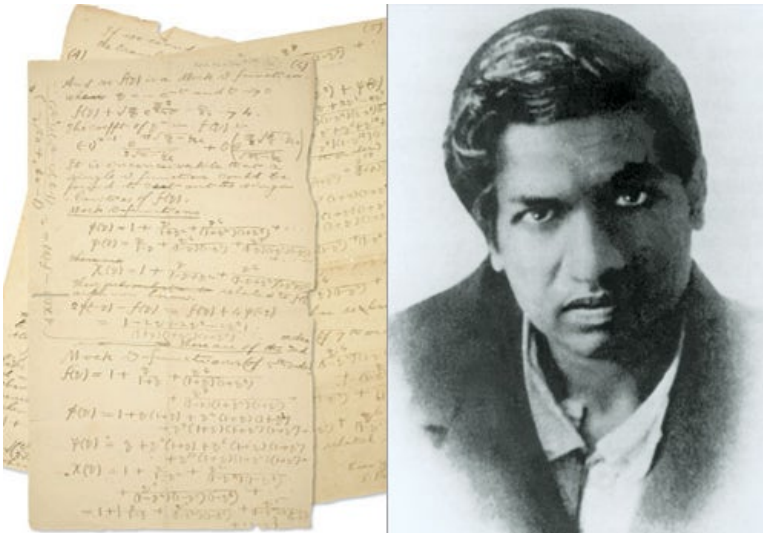
**Michele Nardelli<sup>1</sup>, Antonio Nardelli**

## **Abstract**

*In the present research thesis, we have obtained further interesting new possible mathematical connections concerning the mathematics of Ramanujan mock theta functions, some sectors of Particle Physics, concerning principally the Dark Matter candidate particles and the physics of black holes.*

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<http://judge.smartprinting.co/srinivasa-ramanujan-essay.html>

Now, we have that:

From:

### **Baryogenesis and Dark Matter from B Mesons**

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U.S.A. - From 09/18: Department of Physics, King's College London, Strand,  
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arXiv:1810.00880v3 [hep-ph] 21 Feb 2019

To begin to explore the parameter space of our model we note that the particle masses must be subject to several constraints. For the decay  $\psi \rightarrow \phi \xi$  to be kinematically allowed we have the following:

$$m_\phi + m_\xi < m_\psi. \quad (5)$$

Note that there is also a kinematic upper bound on the mass of the  $\psi$  such that it is light enough for the decay  $B/\bar{B} \rightarrow \psi/\bar{\psi} + \text{Baryon/anti-Baryon} + \text{Mesons}$  to be allowed. This bound depends on the specific process under consideration and the final state visible sector hadrons produced; for instance in the example of Figure 2 it must be the case that  $m_\psi < m_{B_d^0} - m_\Lambda \simeq 4.16 \text{ GeV}$ . A comprehensive list of the possible decay processes and the corresponding constraint on the  $\psi$  mass are itemized in Appendix 4.

Here we categorize the lightest final states for all the quark combinations that allow for  $B$  mesons to decay into a visible baryon plus DM, and for  $\Lambda_b$  baryons decaying to mesons and DM. Note that the mass difference between final and initial states for the  $B$ -mesons will give an upper bound on the dark Dirac fermion  $\psi$  mass. In Table III we list the minimum hadronic mass states for each operator.

| Operator     | Initial State | Final state                     | $\Delta M$ (MeV) |
|--------------|---------------|---------------------------------|------------------|
| $\psi b u s$ | $B_d$         | $\psi + \Lambda(uds)$           | 4163.95          |
|              | $B_s$         | $\psi + \Xi^0(uss)$             | 4025.03          |
|              | $B^+$         | $\psi + \Sigma^+(uus)$          | 4089.95          |
|              | $\Lambda_b$   | $\bar{\psi} + K^0$              | 5121.9           |
| $\psi b u d$ | $B_d$         | $\psi + n(udd)$                 | 4340.07          |
|              | $B_s$         | $\psi + \Lambda(uds)$           | 4251.21          |
|              | $B^+$         | $\psi + p(duu)$                 | 4341.05          |
|              | $\Lambda_b$   | $\bar{\psi} + \pi^0$            | 5484.5           |
| $\psi b c s$ | $B_d$         | $\psi + \Xi_c^0(csd)$           | 2807.76          |
|              | $B_s$         | $\psi + \Omega_c(css)$          | 2671.69          |
|              | $B^+$         | $\psi + \Xi_c^+(csu)$           | 2810.36          |
|              | $\Lambda_b$   | $\bar{\psi} + D^- + K^+$        | 3256.2           |
| $\psi b c d$ | $B_d$         | $\psi + \Lambda_c + \pi^-(cdd)$ | 2853.60          |
|              | $B_s$         | $\psi + \Xi_c^0(cds)$           | 2895.02          |
|              | $B^+$         | $\psi + \Lambda_c(dcu)$         | 2992.86          |
|              | $\Lambda_b$   | $\bar{\psi} + \bar{D}^0$        | 3754.7           |

TABLE III. Here we itemize the lightest possible initial and final states for the  $B$  decay process to visible and dark sector states resulting from the four possible operators. The diagram in Figure 2 corresponds to the first line. The mass difference between initial and final visible sector states corresponds to the kinematic upper bound on the mass of the dark sector  $\psi$  baryon.

in the example of Figure 2 it must

be the case that  $m_\psi < m_{B_d^0} - m_\Lambda \simeq 4.16 \text{ GeV}$ .

Indeed, we have a value of  $4163.95 \text{ MeV} = 4.16395 \text{ GeV}$ ;

**Input interpretation:**

convert 4.16395 GeV (gigaelectronvolts) to megaelectronvolts

**Result:**

4163.95 MeV (megaelectronvolts)

Now, we obtain:

$$4.16395 \text{ GeV} = \text{Kg}$$

**Input interpretation:**

convert  $4.16395 \text{ GeV}/c^2$  to kilograms

**Result:**

$7.4229 \times 10^{-27} \text{ kg}$  (kilograms)

$$\text{Mass} = 7.4229\text{e-}27$$

$$\text{Radius} = 1.102190\text{e-}53$$

$$\text{Temperature} = 1.653267\text{e+}49$$

We remember that the development of the formula concerning the ratio between charge and mass of a black hole, provides the value of the golden ratio for any mass, temperature and radius. To keep in mind that for this mathematical application, we have equated the mass of the dark matter candidate particles, to that of small black holes, or quantum black holes.

Applying the Ramanujan-Nardelli mock formula, we obtain:

$$\text{sqrt}(\frac{1}{\frac{1.962364415 * 10^{19}}{(0.0864055^2)} * (\frac{\text{Pi}}{2 * 1.9632648)} * \frac{1}{(7.4229 * 10^{-27}) * \text{sqrt}[-\frac{1.653267 * 10^{49} * 4 * \text{Pi} * (1.102190 * 10^{-53})^3 - (1.102190 * 10^{-53})^2]}{(6.67 * 10^{-11})}]})})$$

**Input interpretation:**

$$\sqrt{\left(1/\left(\frac{1.962364415 \times 10^{19}}{0.0864055^2} \left(\frac{\pi}{2 \times 1.9632648} \times \frac{1}{7.4229 \times 10^{-27}}\right)\right)\right. \\ \left.\sqrt{-\frac{1.653267 \times 10^{49} \times 4\pi(1.102190 \times 10^{-53})^3 - (1.102190 \times 10^{-53})^2}{6.67 \times 10^{-11}}}\right)\right)}$$

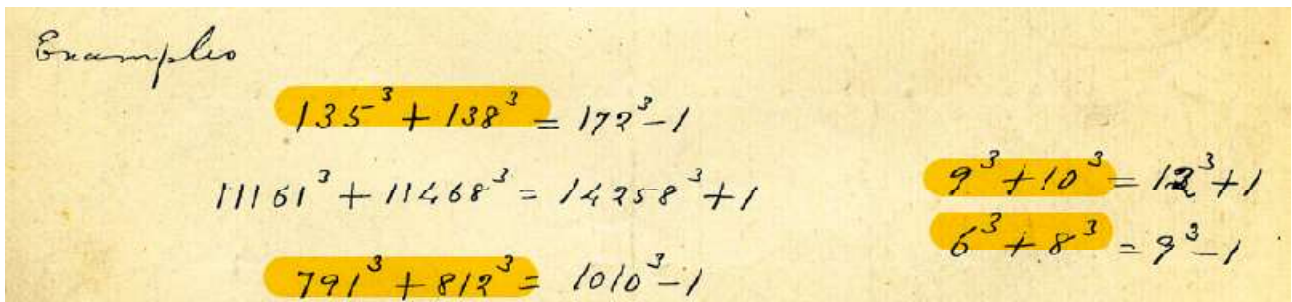
**Result:**

1.618154529084253942870332446218211486837211241520307450613...

1.61815452....

Now, we take the average of all values: 3741,240625 MeV.

From the Ramanujan's sum of two cubes:



We have that:

$$(14258 - 11161) + (135 + 138 + 172) + (1010 - 812) = 3740$$

Thence, we take 3740 MeV

**Input interpretation:**

convert 3740 MeV/c<sup>2</sup> to kilograms

**Result:**

$$6.667 \times 10^{-27} \text{ kg (kilograms)}$$

$$6.667 * 10^{-27} \text{ Kg}$$

$$\text{Mass} = 6.667000\text{e-}27$$

$$\text{Radius} = 9.899505\text{e-}54$$

$$\text{Temperature} = 1.840713\text{e+}49$$

From the Ramanujan-Nardelli mock formula, we obtain:

$$\sqrt{\left( \frac{1}{\left( \frac{1.962364415 \times 10^{19}}{0.0864055^2} \right) \left( \frac{\pi}{2 \times 1.9632648} \times \frac{1}{6.667 \times 10^{-27}} \right)} \sqrt{\frac{1.840713 \times 10^{49} \times 4 \pi (9.899505 \times 10^{-54})^3 - (9.899505 \times 10^{-54})^2}{6.67 \times 10^{-11}}} \right)}$$

**Input interpretation:**

$$\sqrt{\left( \frac{1}{\left( \frac{1.962364415 \times 10^{19}}{0.0864055^2} \right) \left( \frac{\pi}{2 \times 1.9632648} \times \frac{1}{6.667 \times 10^{-27}} \right)} \sqrt{\frac{1.840713 \times 10^{49} \times 4 \pi (9.899505 \times 10^{-54})^3 - (9.899505 \times 10^{-54})^2}{6.67 \times 10^{-11}}} \right)}$$

**Result:**

1.618154216983666601157667174812170907824377622197336102001...  
1.6181542..

It is possible to obtain a similar result also by the following multiplication:

$4/5 (1.962364415 \times 10^{19} / 0.0864055^2)$ . Indeed:

$$\sqrt{\left( \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \left( \frac{1}{6.667 \times 10^{-27}} \right)} \sqrt{\frac{1.840713 \times 10^{49} \times 4 \pi (9.899505 \times 10^{-54})^3 - (9.899505 \times 10^{-54})^2}{6.67 \times 10^{-11}}} \right)}$$

**Input interpretation:**

$$\sqrt{\left( \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \left( \frac{1}{6.667 \times 10^{-27}} \right)} \sqrt{\frac{1.840713 \times 10^{49} \times 4 \pi (9.899505 \times 10^{-54})^3 - (9.899505 \times 10^{-54})^2}{6.67 \times 10^{-11}}} \right)}$$

**Result:**

1.618249249795608133861937700335532726854961140572569308242...  
1.6182492...

This is another mode for the application of the Ramanujan-Nardelli mock formula

Now, from:

### Glueball dark matter in non-standard cosmologies

Bobby S. Acharya,<sup>a;b</sup> Malcolm Fairbairn,<sup>a</sup> Edward Hardy<sup>b;c</sup>

arXiv:1704.01804v2 [hep-ph] 14 Aug 2017

The contribution of glueballs from an  $SU(N)$  gauge sector to the DM relic abundance in this scenario is

$$\frac{(\Omega h^2)_G}{(\Omega h^2)_{\text{DM}}} \simeq (N^2 - 1) \left( \frac{B}{3 \times 10^{-5}} \right) \left( \frac{m_X}{10^5 \text{ GeV}} \right)^{1/2} \left( \frac{\Lambda}{10^4 \text{ GeV}} \right), \quad (3.8)$$

Therefore, by the time the visible sector temperature reaches its present day value  $T_t$ , the glueball velocity  $v_G$  (the idealised velocity left over from the Early Universe, not including the velocity which would subsequently be obtained through gravitational clustering) must satisfy

$$\begin{aligned} v_G &\simeq \frac{T_t}{T_{\text{RH}}} \frac{m_X}{\Lambda} \lesssim 3 \times 10^{-8} \\ \Rightarrow \left( \frac{\Lambda}{10^5 \text{ GeV}} \right) \left( \frac{m_X}{10^5 \text{ GeV}} \right)^{1/2} &\gtrsim 10^{-3.5}. \end{aligned} \quad (3.9)$$

This is because the glueball number density is much smaller than  $\Lambda^3$ , so begins tracking the equilibrium value once the glueball temperature is fairly low, and  $3 \rightarrow 2$  processes only reduce the yield for a short time. Using the approximation  $n \simeq 2n_{\text{nr}}$ , the late time yield is

$$y_\infty \simeq B \frac{g_g}{g_v^{5/4}} \frac{m_X^{3/2}}{M_{\text{Pl}}^{1/2} \Lambda}, \quad (3.13)$$

and unlike in other parts of parameter space, the relic density is independent of  $\Lambda$ ,

$$\frac{(\Omega h^2)_G}{(\Omega h^2)_{\text{DM}}} \simeq (N^2 - 1) \left( \frac{B}{3 \times 10^{-6}} \right) \left( \frac{m_X}{10^5 \text{ GeV}} \right)^{3/2}, \quad (3.14)$$



In models in which glueballs never have efficient scattering, the same is also true for glueballinos. From Eq. (3.6), this scenario typically occurs when  $\Lambda$  is large and  $B$  small. The rate of glueballino annihilation is suppressed initially since they are highly relativistic, and even interactions with small center of mass energy have a cross section parametrically the same as for glueball glueball scattering. Consequently, the glueballino yield remains fixed at the value immediately after modulus decay, similarly to Eq. (3.7), and glueballinos make up a fraction of the observed DM relic abundance

$$\frac{(\Omega h^2)_{\tilde{G}}}{(\Omega h^2)_{\text{DM}}} \simeq g_{\tilde{G}} \left( \frac{B_{\tilde{G}}}{6 \times 10^{-5}} \right) \left( \frac{m_X}{10^5 \text{ GeV}} \right)^{1/2} \left( \frac{m_{\tilde{G}}}{10^4 \text{ GeV}} \right) \quad (4.4)$$

In this scenario the glueballino contribution to the relic abundance is

$$\frac{(\Omega h^2)_{\tilde{G}}}{(\Omega h^2)_{\text{DM}}} \simeq 10^{-8} \left( \frac{\Lambda}{\text{GeV}} \right)^{3/2} \left( \frac{m_{\tilde{G}}}{\text{GeV}} \right)^{3/2} \left( \frac{10^5 \text{ GeV}}{m_X} \right)^{3/2}, \quad (4.8)$$

Despite these caveats, we note that for large values of  $\Lambda$  the glueball abundance produced during matter domination can match the observed relic abundance. In most such models  $\Lambda \gg m_X$ , so no glueballs are produced from the subsequent decay of the modulus, and Eq. (5.4) is the only contribution to the yield. In Fig. 3 the parameter space that leads to the correct relic abundance is plotted, assuming a single modulus with mass  $10^5 \text{ GeV}$  or  $10^6 \text{ GeV}$ . For motivated values of the UV gauge coupling, not far from the visible sector unification value, the correct relic abundance can be obtained for  $B_i \simeq 10^{-6}$ .

After some calculations we find that:

$$(10^{-8} * (10^5)^{1.5} * (10^4)^{1.5} * (10^5/10^5)^{1/5} * ((\text{GeV})^{1.5}))$$

**Input interpretation:**

$$\frac{(10^5)^{1.5} (10^4)^{1.5} \sqrt[5]{\frac{10^5}{10^5}}}{10^8} (\text{GeV (gigaelectronvolt)})^{1.5}$$

**Result:**

$$316228 \text{ GeV}^{1.5} \text{ (gigaelectronvolts to the 1.5)}$$

**Input interpretation:**

$$(316228 \text{ GeV (gigaelectronvolts)})^{1.5}$$

**Result:**

$$177.828.138,371530$$

$$1.778 \times 10^8 \text{ GeV}^{1.5} \text{ (gigaelectronvolts to the 1.5)}$$

$$1.7782813837153 * 10^8 \text{ GeV}$$

**Input interpretation:**

$$1778 \times 10^5$$

$$1778.28138371530 * 10^5 \text{ GeV}$$

Result is a multiple in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

**Scientific notation:**

$$1.778 \times 10^8$$

From the Ramanujan's sums of two cubes, we have:

$$(14258 - 11161 - 1010 - 135 - 138 - 10 - 9 - 8 - 6) = 1781$$

Now:

$$1.7782813837153 * 10^8 \text{ GeV} = \text{Kg}$$

**Input interpretation:**

convert  $1.7782813837153 \times 10^8 \text{ GeV}/c^2$  to kilograms

**Result:**

$$3.170073926979 \times 10^{-19} \text{ kg (kilograms)}$$

$$3.170073926979 * 10^{-19} \text{ Kg}$$

$$\text{Mass} = 3.170073926979e-19 = 3.170074e-19$$

$$\text{Radius} = 4.707089e-46$$

$$\text{Temperature} = 3.871213e+41$$

$$\text{Entropy} = 1.157527e-21$$

From the third GENERAL FORMULA (Ramanujan-Nardelli mock formula), we obtain:

$$\sqrt{\left[ \frac{1}{\left( \frac{1.962364415 * 10^{19}}{(0.0864055^2)} \right) * \left( \frac{\pi}{2 * 1.9632648} \right) * \frac{1}{(3.170074 * 10^{-19}) * \sqrt{\left[ -\left( \left( (3.871213 * 10^{41} * 4 * \pi * (4.707089 * 10^{-46})^3 - (4.707089 * 10^{-46})^2 \right) \right) \right]}} \right]}} \right]}$$

**Input interpretation:**

$$\sqrt{\left(1 / \left(\frac{1.962364415 \times 10^{19}}{0.0864055^2} \left(\frac{\pi}{2 \times 1.9632648} \times \frac{1}{3.170074 \times 10^{-19}}\right.\right.\right. \\ \left.\left.\left.\sqrt{-\frac{3.871213 \times 10^{41} \times 4 \pi (4.707089 \times 10^{-46})^3 - (4.707089 \times 10^{-46})^2}{6.67 \times 10^{-11}}}\right)\right)\right)}$$

**Result:**

1.618154243889548781121868461486319392492586762343755777105...

1.61815424...

And for the fourth GENERAL FORMULA (Ramanujan-Nardelli mock formula), we obtain:

$$\text{sqrt}[\left[\left[\left[\frac{1}{\left(\left(\left(\left(4 \times 1.962364415 \times 10^{19}\right) / \left(5 \times 0.0864055^2\right)\right) \times \frac{1}{3.170074 \times 10^{-19}}\right) \times \text{sqrt}\left[-\left(\left(\left(\left(3.871213 \times 10^{41} \times 4 \pi (4.707089 \times 10^{-46})^3 - (4.707089 \times 10^{-46})^2\right)\right)\right)\right] / \left(6.67 \times 10^{-11}\right)\right)\right]\right]\right]\right]$$

**Input interpretation:**

$$\sqrt{\left(1 / \left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{3.170074 \times 10^{-19}}\right.\right. \\ \left.\left.\sqrt{-\frac{3.871213 \times 10^{41} \times 4 \pi (4.707089 \times 10^{-46})^3 - (4.707089 \times 10^{-46})^2}{6.67 \times 10^{-11}}}\right)\right)}$$

**Result:**

1.618249276703070473253771207129214038161151241816055391401...

1.61824927...

From the Entropy =  $1.157527e-21$ , we obtain:

$$\left(\frac{13}{\pi} - \pi\right) \times \text{colog}^{(1/8)}(1.157527e-21)$$

**Input interpretation:**

$$\left(\frac{13}{\pi} - \pi\right) \sqrt[8]{-\log(1.157527 \times 10^{-21})}$$

$\log(x)$  is the natural logarithm

**Result:**

1.617481202...  
1.617481202...

$1/3 * \ln^2(1.157527e-21)$

**Input interpretation:**

$$\frac{1}{3} \log^2(1.157527 \times 10^{-21})$$

$\log(x)$  is the natural logarithm

**Result:**

774.67046...

774.67046.... result very near to the rest mass of Charged rho meson 775.4

From:

**Gravitational Waves From SU(N) Glueball Dark Matter**

Amarjit Soni and Yue Zhang - arXiv:1610.06931v3 [hep-ph] 25 Jun 2017

Understanding the nature of dark matter is an open question of central importance for particle physics and cosmology. One of the simplest models of dark matter is a hidden sector with a non-abelian gauge symmetry. In the case of pure gauge theory, its intrinsic scale where the gauge coupling goes strong dictates the mass scale of the dark matter candidate — the lightest dark glueball state. The glueball dark matter scenario has been considered within various contexts [1–10]. As emphasized in [8], a hidden sector with pure  $SU(N)$  gauge group without any fermions or any other intricacy is motivated by its elegance and simplicity. In the CP conserving case, such a hidden sector contains only two parameters: the intrinsic scale  $\Lambda$  (or the lightest scalar glueball mass  $m$  which we use more often) and the number of colors  $N$ . In spite of the very few parameters, we have shown that the model can have a number of non-standard and interesting implications in cosmology [8]. In particular, the dark glueball could be a self-interacting and warm dark matter candidate if  $0.01 \text{ keV} < m < 10 \text{ keV}$  and  $10^6 > N > 10^3$ . In this case, the self-gravitation of the dark glueball field is allowed to form boson stars that are much more massive than the sun,  $\sim 10^6 - 10^8 M_\odot$ . We will investigate the consequence of such a possibility in this work.

In this note, we explore a natural consequence of glueball dark matter from  $SU(N)$  gauge theory: dark  $SU(N)$  stars (DSS), *i.e.*, self-gravitating and compact configurations of the lightest scalar glueball field. It was shown that with non-abelian gauge interactions the “geon-like” configuration does not form by itself [14] but with gravity included a glueball star becomes possible. Our goal is to

For 0.01 and 10 keV = Kg

**Input interpretation:**

convert  $0.01 \text{ keV}/c^2$  to kilograms

**Result:**

$$1.78 \times 10^{-35} \text{ kg (kilograms)}$$

$$1.78 * 10^{-35} \text{ kg}$$

**Input interpretation:**

convert  $10 \text{ keV}/c^2$  to kilograms

**Result:**

$$1.783 \times 10^{-32} \text{ kg (kilograms)}$$

$$1.783 * 10^{-32} \text{ kg}$$

From the inverse formula of Surface area

$$A = M^2 \cdot \frac{16\pi G^2}{c^4}$$

we obtain:

$$1/16 * (((8.778411\text{e-}123)*(3*10^8)^4)) / (((1.78\text{e-}35)^2*(6.6743015*10^{-11})^2))$$

**Input interpretation:**

$$\frac{1}{16} \times \frac{\frac{8.778411}{10^{123}} (3 \times 10^8)^4}{(1.78 \times 10^{-35})^2 (6.6743015 \times 10^{-11})^2}$$

**Result:**

$$3.148685610835212275778977275504506277874187665183783104966\dots$$

3.14868561..... a good approximation to  $\pi$

And:

$$1/6((((1/16 * (((8.778411\text{e-}123)*(3*10^8)^4)) / (((1.78\text{e-}35)^2*(6.6743015*10^{-11})^2))))))^2$$

**Input interpretation:**

$$\frac{1}{6} \left( \frac{1}{16} \times \frac{\frac{8.778411}{10^{123}} (3 \times 10^8)^4}{(1.78 \times 10^{-35})^2 (6.6743015 \times 10^{-11})^2} \right)^2$$

**Result:**

1.652370179313452308129802353567997867515979783591656683256...

1.652370179..... is very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

From the inverse formula of Entropy

$$S = M^2 \cdot \frac{4\pi G}{\hbar c \ln 10}$$

$$((3.649493e-54 * 1.054571726e-34 * 3*10^8 \ln 10)) / ((4*6.6743015e-11)*(1.78e-35)^2))$$

**Input interpretation:**

$$\frac{3.649493 \times 10^{-54} \times 1.054571726 \times 10^{-34} \times 3 \times 10^8 \log(10)}{(4 \times 6.6743015 \times 10^{-11})(1.78 \times 10^{-35})^2}$$

log(x) is the natural logarithm

**Result:**

3.142966735221650950426173746598337744403424453605571096610...

3.1429667.... another good approximation to  $\pi$

And:

$$1/6 * (((((3.649493e-54 * 1.054571726e-34 * 3*10^8 \ln 10)) / ((4*6.6743015e-11)*(1.78e-35)^2))))^2$$

**Input interpretation:**

$$\frac{1}{6} \left( \frac{3.649493 \times 10^{-54} \times 1.054571726 \times 10^{-34} \times 3 \times 10^8 \log(10)}{(4 \times 6.6743015 \times 10^{-11})(1.78 \times 10^{-35})^2} \right)^2$$

log(x) is the natural logarithm

**Result:**

1.646373316451640558831720910267102667614762779460189692268...

1.646373316....  $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

Now, we have:

Mass = 1.780000e-35; 1.783000e-32

Radius= 2.643036e-62; 2.647490e-59

Temperature = 6.894400e+57; 6.882800e+54

Surface gravity = 1.700233e+78

Surface area = 8.778411e-123

Entropy = 3.649493e-54

From the fourth GENERAL FORMULA (Ramanujan-Nardelli mock formula), we obtain:

$$\text{sqrt}[\left[ \left[ \left[ \left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{1.78 \times 10^{-35}} \right) \times \left( \frac{6.894400 \times 10^{57} \times 4 \times \pi \times (2.643036 \times 10^{-62})^3 - (2.643036 \times 10^{-62})^2}{6.67 \times 10^{-11}} \right) \right] \right] \right] \right]$$

**Input interpretation:**

$$\sqrt{\left( 1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.78 \times 10^{-35}} \right) \times \sqrt{\frac{6.894400 \times 10^{57} \times 4 \pi (2.643036 \times 10^{-62})^3 - (2.643036 \times 10^{-62})^2}{6.67 \times 10^{-11}}} \right)}$$

**Result:**

1.618249152809454810599944506845794526058712694103272460809...

1.61824915....

And

$$\sqrt{\left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{1.783 \times 10^{-32}}} \right] \sqrt{\left[ -\frac{(((((6.882800 \times 10^{54} \times 4 \times \pi \times (2.647490 \times 10^{-59})^3 - (2.647490 \times 10^{-59})^2))))}{(6.67 \times 10^{-11})}} \right]}}$$

**Input interpretation:**

$$\sqrt{\left( \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.783 \times 10^{-32}} \right)} \sqrt{-\frac{6.882800 \times 10^{54} \times 4 \pi (2.647490 \times 10^{-59})^3 - (2.647490 \times 10^{-59})^2}{6.67 \times 10^{-11}}} \right)}$$

**Result:**

1.618249322281036615536110296392444708572935782264897943653...

1.61824932...

We note that the masses of these candidates dark glueballs  $1.78 \times 10^{-35}$  and  $1.783 \times 10^{-32}$  kg are sub-multiples of 1783, very near of our average of the hypothetical gluino mass = **1785.16 GeV** practically about  $10^3$  times greater than the upper bound mass value of candidate “glueball”, the scalar meson  $f_0(1710)$ , that have a mass including between  $1723 \pm 5$ ;  $1723 \pm 6$  and  **$1760 \pm 15$  MeV**

Furthermore, from the Ramanujan’s sums of two cubes, we have

$$1010 + 791 + 138 - 135 - 19 = 1785 \text{ value very near to } 1783$$

Now, from:

*...In particular, the dark glueball could be a self-interacting and warm dark matter candidate if  $0,01 \text{ keV} < m < 10 \text{ keV}$  and  $10^6 > N > 10^3$ . In this case, the self-gravitation of the dark glueball field is allowed to form **boson stars** that are much more massive than the sun  $\approx 10^6 - 10^9 M$  ( $M = \text{solar masses}$ )*



maximum, with  $\mathcal{M}_*(x_R) = 0.74$  and  $x_R = 4.7$ . The corresponding mass and radius of DSS are,

$$\begin{aligned} M &= \sqrt{\frac{1}{4\pi}} \frac{M_{pl}^3}{Nm^2} \mathcal{M}_*(x_R) = \left(\frac{1}{N}\right) \left(\frac{0.6 \text{ GeV}}{m}\right)^2 M_\odot, \\ R &= \sqrt{\frac{1}{4\pi}} \frac{M_{pl}}{Nm^2} x_R = \left(\frac{1}{N}\right) \left(\frac{0.6 \text{ GeV}}{m}\right)^2 \times 10 \text{ km}. \end{aligned} \quad (26)$$

Interestingly, if  $m \sim 1 \text{ GeV}$  and  $N \sim \mathcal{O}(1)$ , the DSS has the typical mass as a massive compact halo object (MACHO). On the other hand, [8] showed that, for the dark glueball to be both self-interacting and warm dark matter candidate, the favored ranges of parameters are  $m \sim 0.01 - 10 \text{ keV}$ ,  $N \sim 10^6 - 10^3$ . Following (26), this corresponds to the highest DSS mass in the range  $10^6 - 10^9 M_\odot$  and the lowest DSS radius in the range  $10^2 - 10^5 R_\odot$ , where the solar radius is  $R_\odot = 7 \times 10^5 \text{ km}$ .

From:

$$M = \sqrt{\frac{1}{4\pi}} \frac{M_{pl}^3}{Nm^2} \mathcal{M}_*(x_R) = \left(\frac{1}{N}\right) \left(\frac{0.6 \text{ GeV}}{m}\right)^2 M_\odot$$

For  $N = 10^6$ , and  $N = 10^3$ ,  $m = 0.01$  and  $10 \text{ keV}$ , we obtain the following interesting formulas:

$$(1/(10^6)) * (0.6 \text{ GeV}/0.01 \text{ keV})^2 * (1.9891 * 10^{30})$$

**Input interpretation:**

$$\frac{1}{10^6} \left( \frac{0.6 \text{ GeV (gigaelectronvolts)}}{0.01 \text{ keV (kilolectronvolts)}} \right)^2 \times 1.9891 \times 10^{30}$$

**Result:**

$$\begin{aligned} &7.161 \times 10^{39} \\ &7.161 * 10^{39} \text{ Kg} \end{aligned}$$

$$(1/(10^3)) * (0.6 \text{ GeV}/10 \text{ keV})^2 * (1.9891 * 10^{30})$$

**Input interpretation:**

$$\frac{1}{10^3} \left( \frac{0.6 \text{ GeV (gigaelectronvolts)}}{10 \text{ keV (kilolectronvolts)}} \right)^2 \times 1.9891 \times 10^{30}$$

**Result:**

$$\begin{aligned} &7.161 \times 10^{36} \\ &7.161 * 10^{36} \text{ Kg} \end{aligned}$$

We want to highlight that the formulas, and the relative results, are interesting because, inserted in the Hawking Radiation Calculator, to obtain the various physical parameters of a black hole, they provide the temperature and radius of these "boson stars". The masses, temperatures and radii, finally inserted in the GENERAL FORMULA, deriving from that of the ratio between charge and mass of a black hole (we are treating the particles as little black holes or quantum black holes), provide as a result, also in this case, a value very close to the "golden ratio"!

Indeed, we have:

$$\text{Mass} = 7.161e+39$$

$$\text{Radius} = 1.063302e+13$$

$$\text{Temperature} = 1.713732e-17$$

From the fourth GENERAL FORMULA (Ramanujan-Nardelli mock formula), we obtain:

$$\sqrt{\left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{7.161 \times 10^{39}} \right) \sqrt{\left[ -\left( \frac{1.713732 \times 10^{-17} \times 4 \pi (1.063302 \times 10^{13})^3 - (1.063302 \times 10^{13})^2 \right)}{6.67 \times 10^{-11}} \right]}} \right]}$$

**Input interpretation:**

$$\sqrt{\left( 1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{7.161 \times 10^{39}} \right) \sqrt{\left( \frac{1.713732 \times 10^{-17} \times 4 \pi (1.063302 \times 10^{13})^3 - (1.063302 \times 10^{13})^2}{6.67 \times 10^{-11}} \right)} \right)}$$

**Result:**

1.618249374900668542367178003243040682994733618897936302450...

1.61824937...

And for the mass =  $7.161 \times 10^{36}$ :

$$\text{Mass} = 7.161000e+36$$

$$\text{Radius} = 1.063302e+10$$

Temperature = 1.713732e-14

$$\sqrt{\left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{7.161000 \times 10^{36}} \right)} \sqrt{\frac{1.713732 \times 10^{-14} \times 4 \pi (1.063302 \times 10^{10})^3 - (1.063302 \times 10^{10})^2}{6.67 \times 10^{-11}}} \right]}$$

**Input interpretation:**

$$\sqrt{\left( \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{7.161000 \times 10^{36}} \right)} \sqrt{\frac{1.713732 \times 10^{-14} \times 4 \pi (1.063302 \times 10^{10})^3 - (1.063302 \times 10^{10})^2}{6.67 \times 10^{-11}}} \right)}$$

**Result:**

1.618249374900668542367178003243040682994733618897936302450...  
1.61824937...

We have also:

$$\ln^{(0.27 \times 0.42)} \left( \left( \frac{1}{10^3} \right) \times (0.6 \text{ GeV} / 10 \text{ keV})^2 \times (1.9891 \times 10^{30}) \right)$$

**Input interpretation:**

$$\log^{0.27 \times 0.42} \left( \frac{1}{10^3} \left( \frac{0.6 \text{ GeV (gigaelectronvolts)}}{10 \text{ keV (kiloelectronvolts)}} \right)^2 \times 1.9891 \times 10^{30} \right)$$

log(x) is the natural logarithm

**Result:**

1.655

1.655 is very near to the 14th root of the following Ramanujan's class invariant

$$Q = \left( G_{505} / G_{101/5} \right)^3 = 1164,2696 \text{ i.e. } 1,65578...$$

And:

$$\ln^{(0.27 \times 0.413)} \left( \left( \frac{1}{10^6} \right) \times (0.6 \text{ GeV} / 0.01 \text{ keV})^2 \times (1.9891 \times 10^{30}) \right)$$

**Input interpretation:**

$$\log^{0.27 \times 0.413} \left( \frac{1}{10^6} \left( \frac{0.6 \text{ GeV (gigaelectronvolts)}}{0.01 \text{ keV (kiloelectronvolts)}} \right)^2 \times 1.9891 \times 10^{30} \right)$$

$\log(x)$  is the natural logarithm

**Result:**

1.655

1.655 is very near to the 14th root of the following Ramanujan’s class invariant

$$Q = \left( G_{505}/G_{101/5} \right)^3 = 1164,2696 \text{ i.e. } 1,65578...$$

We note that 7161, from the Ramanujan’s sums of two cubes, we have

$$(14258 \div 2) + 138 - 135 + (10 + 9) + (9 + 1) = 7161$$

With regard the Mass = 7.161e+39, applying the Ramanujan-Nardelli mock formula:

$$\sqrt{\left( 1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{7.161 \times 10^{39}} \right) \sqrt{-\frac{1.713732 \times 10^{-17} \times 4 \pi (1.063302 \times 10^{13})^3 - (1.063302 \times 10^{13})^2}{6.67 \times 10^{-11}}} \right)}$$

$$= 1.618249374900668542367178003243040682994733618897936302450...$$

we obtain also, with the Ramanujan mock theta function  $F(\mathbf{q}) = 1.897512108....$ , the following interesting expressions:

$$\text{sqrt}[\left[ \left[ \left[ \left[ \left[ 1 / \left( \left( \left( \left( \left( \left( 4 * 1.897512108 * 10^{19} \right) / \left( 5 * 0.0864055^2 \right) \right) * 1 / \left( 7.161 * 10^{39} \right) * \text{sqrt} \left[ - \left( \left( \left( \left( \left( 1.713732 * 10^{-17} * 4 * \text{Pi} * \left( 1.063302 * 10^{13} \right)^3 - \left( 1.063302 * 10^{13} \right)^2 \right) \right) \right) \right] / \left( 6.67 * 10^{-11} \right) \right] \right] \right] \right] \right] \right]$$

**Input interpretation:**

$$\sqrt{\left( 1 / \left( \frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{7.161 \times 10^{39}} \right) \sqrt{-\frac{1.713732 \times 10^{-17} \times 4 \pi (1.063302 \times 10^{13})^3 - (1.063302 \times 10^{13})^2}{6.67 \times 10^{-11}}} \right)}$$

**Result:**

1.645670938989220735779582455212860786179956182117339551569...

1.6456709...

$$\sqrt{\sqrt{\sqrt{6 \cdot \sqrt{\frac{1}{\left(\frac{4 \cdot 1.897512108 \cdot 10^{19}}{5 \cdot 0.0864055^2}\right) \cdot \frac{1}{7.161 \cdot 10^{39}} \cdot \sqrt{-\frac{1.713732 \cdot 10^{-17} \cdot 4 \cdot \pi \cdot (1.063302 \cdot 10^{13})^3 - (1.063302 \cdot 10^{13})^2}}{6.67 \cdot 10^{-11}}}}}}}}}}^{1/2}$$

**Input interpretation:**

$$\sqrt{\sqrt{\sqrt{6 \cdot \sqrt{\frac{1}{\left(\frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2}\right) \times \frac{1}{7.161 \times 10^{39}} \cdot \sqrt{-\frac{1.713732 \times 10^{-17} \times 4 \pi (1.063302 \times 10^{13})^3 - (1.063302 \times 10^{13})^2}}{6.67 \times 10^{-11}}}}}}}}}}}$$

**Result:**

3.142296235865632770127443862490999778327444783106432430287...

3.14229623...

Thence:

$$2 \sqrt{\sqrt{\sqrt{6 \cdot \sqrt{\frac{1}{\left(\frac{4 \cdot 1.897512108 \cdot 10^{19}}{5 \cdot 0.0864055^2}\right) \cdot \frac{1}{7.161 \cdot 10^{39}} \cdot \sqrt{-\frac{1.713732 \cdot 10^{-17} \cdot 4 \cdot \pi \cdot (1.063302 \cdot 10^{13})^3 - (1.063302 \cdot 10^{13})^2}}{6.67 \cdot 10^{-11}}}}}}}}}}^{1/2}$$

**Input interpretation:**

$$2 \sqrt{\sqrt{\sqrt{6 \cdot \sqrt{\frac{1}{\left(\frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2}\right) \times \frac{1}{7.161 \times 10^{39}} \cdot \sqrt{-\frac{1.713732 \times 10^{-17} \times 4 \pi (1.063302 \times 10^{13})^3 - (1.063302 \times 10^{13})^2}}{6.67 \times 10^{-11}}}}}}}}}}}$$

**Result:**

6.284592471731265540254887724981999556654889566212864860574...

6.28459247....

**We have, in conclusion, for the Mass = 7.161e+39, the following new mathematical connection:**

From the fourth GENERAL FORMULA (Ramanujan-Nardelli mock formula) for M = 7.161e+39 Kg of the Boson Star - DSS, i.e. dark SU(N) stars (DSS), a natural consequence of glueball dark matter from SU(N) gauge theory. We obtain:

$$\begin{aligned} & \sqrt{\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{M} \sqrt{-\frac{T \times 4 \pi r^3 - r^2}{G}}}} \Rightarrow \\ & \Rightarrow \sqrt{\left( \frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{7.161 \times 10^{39}}} \sqrt{-\frac{1.713732 \times 10^{-17} \times 4 \pi (1.063302 \times 10^{13})^3 - (1.063302 \times 10^{13})^2}{6.67 \times 10^{-11}}} \right)} = \\ & = 1.618249374900668542367178003243040682994733618897936302450... \\ & = 1.61824937... \Rightarrow \\ & \Rightarrow \sqrt[5]{\left( \frac{1}{\frac{1}{32}(-1 + \sqrt{5})^5 + 5e^{(-\sqrt{5}\pi)^5}} - \frac{11 \times 5e^{(-\sqrt{5}\pi)^5}}{2\left(\frac{1}{32}(-1 + \sqrt{5})^5 + 5e^{(-\sqrt{5}\pi)^5}\right)} - \frac{5\sqrt{5} \times 5e^{(-\sqrt{5}\pi)^5}}{2\left(\frac{1}{32}(-1 + \sqrt{5})^5 + 5e^{(-\sqrt{5}\pi)^5}\right)} \right)} = \\ & = \sqrt[5]{\left( \frac{1}{\frac{1}{32}(-1 + \sqrt{5})^5 + 5e^{(-\sqrt{5}\pi)^5}} \right) + \frac{1.6382898797095665677239458827012056245798314722584}{10^{7429}}} = \\ & = 1.618033988749894848204586834365638117720309179805762862135... \\ & = 1.61803398... \end{aligned}$$

Now, we have:

potentials. We study how the two types of boson star potentials can be discriminated by the FRB and GW measurements. Throughout this paper, we will adopt the natural unit  $c = \hbar = 1$ . We denote the Planck mass by  $m_P = G_N^{-1/2}$  and the reduced Planck mass by  $M_P = (8\pi G_N)^{-1/2}$ , where  $G_N$  is the Newton gravitational constant. For the cosmological parameters, we use values based on Planck TT,TE,EE+lowE+lensing at the 68% confidence level in Ref. [10].

Stability properties of boson stars were studied in the literature both analytically [13, 35–37] and numerically [38–41]. Mass and compactness of the ground state of a boson star depend on the central value of the scalar field ( $\phi_0$ ) and the scalar potential  $U(|\Phi|^2)$ . For the case of the mini boson stars with free scalar field, there exists a critical point for the central value of the scalar field,  $\phi_0^*$ , beyond which the ground state is unstable under small radial perturbations. As  $\phi_0$  increases, stable configurations have larger masses but smaller effective radius, leading to a greater compactness. This behavior continues until the central value of the scalar field reaches  $\phi_0^*$ , where the total mass of the star encounters a turnaround. This turnaround implies a maximum allowed mass for a boson star in the ground state, which is found to be  $M_{\max} = 0.633 m_P^2/m_\phi$  [1, 2]. More generic potentials which include one or more self-interaction terms added to the mass-term show the same stability features as the case of the free-field potential [3, 6, 7, 14].

In Fig. 3, we present the boson stars mass  $M$  as a function of the central value of the scalar field  $\phi_0$  for the scalar potentials  $U_{\text{Liouville}}$  (red curve) and  $U_{\text{Log}}$  (blue curve). In both cases, we have set the coupling strength parameter to be  $f = M_P$ . We

compute the mass of the ground state of these boson stars by using Eq.(2.16), which agrees with the ADM-mass in Eq.(2.17). The maximum mass for both potentials is obtained at the critical central value of the field  $\phi_0^* \approx 0.27$ . The total mass of the star shows a turnaround at this critical central value. All configurations on the left-hand (right-hand) side of the critical point  $\phi_0^*$  are stable (unstable). For the two scalar potentials  $U_{\text{Liouville}}$  and  $U_{\text{Log}}$ , we have  $M_{\max} = 0.666 m_P^2/m_\phi$  and  $M_{\max} = 0.602 m_P^2/m_\phi$ , respectively. In comparison to the maximum mass for a free-field potential, the repulsive self-interaction terms present in the expansion of the  $U(1)$  Liouville potential enhance the value of  $M_{\max}$ . In contrast, the lower value of  $M_{\max}$  for the case of the  $U(1)$  logarithmic potential is caused by the presence of the net effective attractive self-interactions from its expansion series. For the case of  $U_{\text{Liouville}}$  potential (as shown by the blue curve in Fig. 3), the value of  $M_{\max}$  and the  $M(\phi_0)$  curve are in full agreement with [14].

| $\xi_{\text{DM}}$ | Liouville Potential ( $\Lambda = 100$ )                            | Logarithmic Potential ( $\Lambda = 100$ )                          |
|-------------------|--|--|
| 0.1               | $3.6 \times 10^{-12} \lesssim m_\phi \lesssim 3.6 \times 10^{-10}$ | $8.4 \times 10^{-13} \lesssim m_\phi \lesssim 8.4 \times 10^{-11}$ |
| 0.01              | $3.6 \times 10^{-13} \lesssim m_\phi \lesssim 3.6 \times 10^{-3}$  | $8.4 \times 10^{-14} \lesssim m_\phi \lesssim 8.4 \times 10^{-4}$  |

**Table 2:** Allowed mass range of the scalar particle under two benchmark potentials, as inferred from the MACHO mass constraints [48–52] for the DM fraction  $\xi_{\text{DM}} = 0.1$  and 0.01. Here the unit of the scalar particle mass is eV.

We have:

$$m_\phi \gtrsim \xi_{\text{DM}}^2 \times 4.875 \times 10^{-6} \times \left(\frac{g_*}{106.75}\right) \times \left(\frac{r}{0.064}\right)^{-2} \text{ eV}.$$

For  $10.75 \leq g_* \leq 106.75$ ,<sup>8</sup> we may expect the allowed minimum scalar mass for the boson stars with  $\xi_{\text{DM}} = 10^{-2}$  (or  $\xi_{\text{DM}} = 10^{-1}$ ) to be  $m_\phi \gtrsim \mathcal{O}(10^{-10})\text{eV}$  (or  $m_\phi \gtrsim \mathcal{O}(10^{-8})\text{eV}$ ). In comparison with the MACHO constraints in Table 2, we find that the fraction  $\xi_{\text{DM}} = 0.01$  is consistent with the current cosmological and astrophysical data for the scalar mass-range  $\mathcal{O}(10^{-10})\text{eV} < m_\phi < \mathcal{O}(10^{-3})\text{eV}$ , while the fraction  $\xi_{\text{DM}} = 0.1$  is excluded because the MACHO and CMB constraints on  $m_\phi$  do not overlap. Hence, in the following analyses, we focus on the benchmark case where the boson stars are responsible for 1% of the DM population in the Universe with the mass range  $\mathcal{O}(10^{-10})\text{eV} < m_\phi < \mathcal{O}(10^{-3})\text{eV}$ .

For the case of Liouville potential, we find that boson stars with scalar particle mass  $m_\phi = 10^{-10}\text{eV}$  (for any  $\Lambda$ ) or  $m_\phi = 10^{-9}\text{eV}$  (for  $\Lambda > 100$ ) are lying in the overlapping region between the cyan and yellow areas, so their merger will produce GW signals whose frequency can be probed by the LIGO detector. For  $m_\phi = 10^{-10}\text{eV}$ , if the self-interaction strength  $\Lambda \leq 20$ , the boson star merger at the luminosity distance  $D_L < 250\text{Mpc}$  may produce detectable GWs by LIGO. For  $m_\phi = 10^{-9}\text{eV}$ , if  $\Lambda > 100$ , then those mergers that occurred at the luminosity distance  $D_L < 100\text{Mpc}$  may produce detectable GWs. For boson stars with logarithmic potential, the scalar particle mass also needs to be  $m_\phi = 10^{-10}\text{eV}$  so that it can lie in the overlapping region of the  $(C_{\text{max}}, M_{\text{max}})$  plane. We find that even the case with strong self-interaction strength  $\Lambda > 100$  requires the merger of boson stars to occur at  $D_L < 250\text{Mpc}$  for the LIGO detection. In comparison, for the same scalar particle mass  $m_\phi$  and coupling strength  $\Lambda$ , the Liouville potential achieves a higher compactness  $C_{\text{max}}$  and boson star mass  $M_{\text{max}}$ . Hence, the farther merging event can be probed for boson stars with Liouville potential rather than the logarithmic potential. In addition, the mass  $m_\phi = 10^{-10}\text{eV}$  which can be probed by the GW detection lies in the range  $\mathcal{O}(10^{-10})\text{eV} \lesssim m_\phi \lesssim \mathcal{O}(10^{-3})\text{eV}$  for both potentials. This implies that GW detection can compensate the FRB lensing probe for searching boson stars with the logarithmic potential.



$$(10^{-2})^2 4.875e-6(106.75)(0.064)^2 \text{ eV}$$

**Input interpretation:**

$$\left(\frac{1}{10^2}\right)^2 \times 4.875 \times 10^{-6} \times 106.75 \times 0.064^2 \text{ eV (electronvolts)}$$

**Result:**

$$2.132 \times 10^{-10} \text{ eV (electronvolts)}$$

$$2.132 * 10^{-10} \text{ eV}$$

$$(10^{-1})^2 4.875e-6(106.75)(0.064)^2 \text{ eV}$$

**Input interpretation:**

$$\left(\frac{1}{10}\right)^2 \times 4.875 \times 10^{-6} \times 106.75 \times 0.064^2 \text{ eV (electronvolts)}$$

**Result:**

$$2.132 \times 10^{-8} \text{ eV (electronvolts)}$$

$$2.132 \times 10^{-10} \text{ electronvolts} = \text{Kg}$$

**Input interpretation:**

convert  $2.132 \times 10^{-10} \text{ eV (electronvolts)}$  to kilograms

**Result:**

$$3.801 \times 10^{-46} \text{ kg (kilograms)}$$

(using  $E = mc^2$ )

We have:

$$\text{Mass} = 3.801000e-46$$

$$\text{Radius} = 5.643921e-73$$

$$\text{Temperature} = 3.228632e+68$$

From the fourth GENERAL FORMULA (Ramanujan-Nardelli mock formula), we obtain:

$$\text{sqrt}\left[\left[\left[\left[\frac{1}{\left(\left(\left(\left(4 * 1.962364415 * 10^{19}\right) / \left(5 * 0.0864055^2\right)\right)\right) * \frac{1}{\left(3.801 * 10^{-46}\right)} * \text{sqrt}\left[\left(-\left(\left(\left(3.228632 * 10^{68} * 4 * \text{Pi} * \left(5.643921 * 10^{-73}\right)^3 - \left(5.643921 * 10^{-73}\right)^2\right)\right)\right) / \left(\left(6.67 * 10^{-11}\right)\right)\right]\right]\right]\right]\right]\right]$$

**Input interpretation:**

$$\sqrt{\left(1/\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{3.801 \times 10^{-46}}\right)\sqrt{-\frac{3.228632 \times 10^{68} \times 4 \pi (5.643921 \times 10^{-73})^3 - (5.643921 \times 10^{-73})^2}{6.67 \times 10^{-11}}}\right)}\right)$$

**Result:**

1.618249189360019444130533713602261515870207425948249952475...  
1.6182491...

And with the Ramanujan mock theta function  $F(q) = 1.897512108\dots$ , the following interesting expressions:

$$\text{sqrt}\left[\left[\left[\left[\frac{1}{\left(\frac{4 * 1.897512108 * 10^19}{5 * 0.0864055^2}\right) * 1/(3.801 * 10^{-46})}\right] * \text{sqrt}\left[-\left(\frac{3.228632 * 10^68 * 4 * \text{Pi} * (5.643921 * 10^{-73})^3 - (5.643921 * 10^{-73})^2}{6.67 * 10^{-11}}\right)\right]\right]\right]\right]$$

**Input interpretation:**

$$\sqrt{\left(1/\left(\frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{3.801 \times 10^{-46}}\right)\sqrt{-\frac{3.228632 \times 10^{68} \times 4 \pi (5.643921 \times 10^{-73})^3 - (5.643921 \times 10^{-73})^2}{6.67 \times 10^{-11}}}\right)}\right)$$

**Result:**

1.645670750304547682278830397476926124949569204290200205417...  
1.64567...  $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

$$2\text{sqrt}\left(\left(\left(6 * \text{sqrt}\left[\left[\left[\left[\frac{1}{\left(\frac{4 * 1.897512108 * 10^19}{5 * 0.0864055^2}\right) * 1/(3.801 * 10^{-46})}\right] * \text{sqrt}\left[-\left(\frac{3.228632 * 10^68 * 4 * \text{Pi} * (5.643921 * 10^{-73})^3 - (5.643921 * 10^{-73})^2}{6.67 * 10^{-11}}\right)\right]\right]\right]\right]\right)\right)$$

**Input interpretation:**

$$2 \sqrt{\left(6 \sqrt{\left(1/\left(\frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{3.801 \times 10^{-46}}\right)\sqrt{\left(-\frac{1}{6.67 \times 10^{-11}}\left(3.228632 \times 10^{68} \times 4 \pi (5.643921 \times 10^{-73})^3 - (5.643921 \times 10^{-73})^2\right)\right)}\right)}\right)}$$

**Result:**

6.284592111450761795946310010473286241169657323509788123560...

**Input interpretation:**

6.2845921114507617959463100104732862411696573235097883

**Rational approximation:**

$$\frac{546\,750\,552\,160\,970\,016\,992\,587\,321}{86\,998\,574\,046\,638\,614\,620\,276\,104} = 6 + \frac{24\,759\,107\,881\,138\,329\,270\,930\,697}{86\,998\,574\,046\,638\,614\,620\,276\,104}$$

**Possible closed forms:**

$$\frac{345\,830\,208\,\pi}{172\,876\,397} \approx 6.2845921114507617967913$$

$$\sec\left(\tan\left(\frac{767\,715\,071}{106\,075\,671}\right)\right) \approx 6.2845921114507617961467$$

$$\sqrt{\frac{1}{545} (2336 + 4315 e + 2240 \pi + 610 \log(2))} \approx 6.28459211145076179584023$$

sec(x) is the secant function  
log(x) is the natural logarithm

Difference:

$$6.2845921114507617959463100104732862411696573235097881 / (2\pi)$$

**Input interpretation:**

$$\frac{6.2845921114507617959463100104732862411696573235097881}{2\pi}$$

**Result:**

1.0002238998537203431926585259343710150900524417679534...

**Input interpretation:**

1.0002238998537203431926585259343710150900524417679534

**Rational approximation:**

$$\frac{362\,187\,832\,773\,777\,101\,604\,833\,946}{362\,106\,757\,123\,825\,929\,138\,992\,335} = 1 + \frac{81\,075\,649\,951\,172\,465\,841\,611}{362\,106\,757\,123\,825\,929\,138\,992\,335}$$

**Continued fraction:**

- Linear form

$$\begin{array}{c}
1 \\
1 + \frac{1}{4466 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{5 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{4 + \frac{1}{35 + \frac{1}{1 + \frac{1}{247 + \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}} \\
\end{array}$$

**Possible closed forms:**

$$\sqrt[3]{\frac{1}{30} (-5373 + 916 e + 948 \pi - 94 \log(2))} \approx 1.0002238998537203472271$$

$$2 \pi \cos^{-1} \left( \frac{37277018}{40454517} \right)^2 \approx 1.000223899853720343172336$$

$$\frac{1}{25} \left( \frac{965429}{7422731} \right)^{2/3} \pi^4 \approx 1.00022389985372026855$$

$\log(x)$  is the natural logarithm

$\cos^{-1}(x)$  is the inverse cosine function

$$C = 2\pi r = 2\pi \cdot 1.0002238998537203431926 \dots = 6.28459211145076 \dots$$

**Input interpretation:**

$$2\pi \times 1.0002238998537203431926$$

**Result:**

- More digits

6.2845921114507617959459...

Now, we calculate the ellipse perimeter (from: <https://www.mathsisfun.com/geometry/ellipse-perimeter.html>):

The famous Indian mathematician **Ramanujan** came up with this better approximation:

$$p \approx \pi \left[ 3(a + b) - \sqrt{(3a + b)(a + 3b)} \right]$$

We have, for a = b:

$$\text{Pi}(3*(1.00022389985372+1.00022389985372)-\text{sqrt}(((3*1.00022389985372+1.00022389985372)*(1.00022389985372+3*1.00022389985372))))$$

**Input interpretation:**

$$\pi (3 (1.00022389985372 + 1.00022389985372) - \sqrt{((3 \times 1.00022389985372 + 1.00022389985372) (1.00022389985372 + 3 \times 1.00022389985372))})$$

**Result:**

6.2845921114508...

**Series representations:**

- More

$$\pi (3 (1.000223899853720000 + 1.000223899853720000) - \sqrt{((3 \times 1.000223899853720000 + 1.000223899853720000) (1.000223899853720000 + 3 \times 1.000223899853720000))}) = 6.00134339912232000 \pi - 1.00000000000000000000 \pi \sqrt{15.00716559741735193} \sum_{k=0}^{\infty} e^{-2.708527793531277839k} \left( \frac{1}{2} \right)_k$$

$$\pi (3 (1.000223899853720000 + 1.000223899853720000) - \sqrt{((3 \times 1.000223899853720000 + 1.000223899853720000) (1.000223899853720000 + 3 \times 1.000223899853720000))}) = 6.00134339912232000 \pi - 1.00000000000000000000 \pi \sqrt{15.00716559741735193} \sum_{k=0}^{\infty} \frac{(-0.0666348347733361673)^k \left( -\frac{1}{2} \right)_k}{k!}$$

- $$\pi (3 (1.000223899853720000 + 1.000223899853720000) - \sqrt{((3 \times 1.000223899853720000 + 1.000223899853720000) (1.000223899853720000 + 3 \times 1.000223899853720000))}) = 6.001343399122320000 \pi - \frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} e^{-2.708527793531277839s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{2 \sqrt{\pi}}$$

From the initial result:

$$2.132 \times 10^{-10} \text{ electronvolts} = \text{Kg}$$

**Input interpretation:**

convert  $2.132 \times 10^{-10}$  eV (electronvolts) to kilograms

**Result:**

$$3.801 \times 10^{-46} \text{ kg (kilograms)}$$

(using  $E = mc^2$ )

We obtain also these expressions:

$$55 + \left( \left( \frac{1}{2.132 \times 10^{-10}} \right) \right)^{1/3}$$

**Input interpretation:**

$$55 + \sqrt[3]{\frac{1}{2.132 \times 10^{-10}}}$$

**Result:**

$$1728.93\dots$$

$$1728.93\dots$$

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$55 \times 2 + \left( \left( \frac{1}{2.132 \times 10^{-10}} \right) \right)^{1/3}$$

**Input interpretation:**

$$55 \times 2 + \sqrt[3]{\frac{1}{2.132 \times 10^{-10}}}$$

**Result:**

$$1783.931\dots$$

1783.931... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

$$\left(\left(\frac{1}{2.132 \times 10^{-10}}\right)\right)^{1/3}$$

**Input interpretation:**

$$\sqrt[3]{\frac{1}{2.132 \times 10^{-10}}}$$

**Result:**

1673.93...

1673.93... result very near to the rest mass of Omega baryon 1672.45

$$\frac{1}{2.73911418 + \frac{0.898893179}{26}} \left( \left( \left( \frac{1}{2.132 \times 10^{-10}} \right) \right)^{1/3} \right)^{\left( \frac{1.8975121 + 1.8236681 - (1 - (1.08753454 + 1.08185)/2)}{2} \right)}$$

Where 2.73911418, 0.898893179, 1.8975121, 1.8236681, 1.08753454 and 1.08185 are Ramanujan's mock theta functions, that developed give us the following values:

$$0.36053094741951311052971; \quad 0.2627518414982517793088$$

**Input interpretation:**

$$\frac{1}{2.73911418 + \frac{0.898893179}{26}} \left( \left( \frac{1}{2.132 \times 10^{-10}} \right) \right)^{\left( 1 - \frac{1.08753454 + 1.08185}{2} \right)}$$

**Result:**

125.335...

125.335... result very near to the mass of Higgs boson 125.18

Or:

$$0.3605309474 \left( \left( \frac{1}{2.132 \times 10^{-10}} \right) \right)^{0.262751841}$$

**Input interpretation:**

$$0.3605309474 \left( \frac{1}{2.132 \times 10^{-10}} \right)^{0.262751841}$$

**Result:**

125.3348...

125.3348...

Now, we have that:

The microlensing survey provides constraints on  $\xi_{\text{DM}}$  and  $M_{\text{max}}$  for MACHO mass range  $10^{-11} < M/M_{\odot} < 30$  [48–50, 52], which can be applied to the primordial black holes (PBHs) and the exotic compact objects including boson stars. For the heavier mass range of  $M \gtrsim 100M_{\odot}$ , the cosmic microwave background (CMB) anisotropy excludes PBHs as the dominant component of DM [53]. On the other hand, the survival of a star cluster near the core of Eridanus II and of a sample of compact ultra-faint dwarfs places constraints on  $\xi_{\text{DM}}$  and  $M_{\text{max}}$  for MACHO mass range  $M \gtrsim 5M_{\odot}$  [51]. For exemplary fractions  $\xi_{\text{DM}} = 0.1$  and  $0.01$ , we find the allowed MACHO mass ranges to be  $1 \lesssim M/M_{\odot} \lesssim 100$  and  $10^{-7} \lesssim M/M_{\odot} \lesssim 10^3$ , respectively. In Table 2,

In this Appendix, following [61], we explain the procedure of determining the parameter space of the physical quantities  $(C, M_{\text{max}})$  of the boson star which can be probed by the LIGO GW detector. For the simplicity of illustration, we consider the situation where the two merging boson stars have the same mass and compactness as described by the same the scalar potential. The GW emissions from the merger of the binary boson stars are characterized by the frequency,

$$\nu^{\text{BS}} = \frac{(C/3)^{3/2}}{2\pi M}, \quad (\text{C.1})$$

where the parameters  $(C, M)$  are the common (compactness, mass) of the binary boson stars. Requiring  $\nu^{\text{BS}}$  to lie within the GW frequency range (50 – 1000Hz) to which LIGO detection is sensitive, one obtains the following relation

$$C^{3/2} \times 6.149M_{\odot} \leq M \leq C^{3/2} \times 124.451M_{\odot}, \quad (\text{C.2})$$

which must be satisfied by  $(C, M_{\text{max}})$  of the binary boson stars to be probed with the low level noise. We draw the corresponding region of the parameter space by the cyan color as in Fig. 11. The signal to noise ratio (SNR) of the GW signals with strain  $h(t)$  reads

$$\rho^2 = \int_0^{\nu^{\text{BS}}} d\nu \frac{4|\tilde{h}(\nu)|^2}{S_n(\nu)}, \quad (\text{C.3})$$



obtained important insights. For the boson star with Liouville potential, the maximum compactness can reach as high as  $C_{\max} \sim 0.18$  for large coupling, and is larger than the case with a repulsive quartic interaction or a logarithmic interaction. On the other hand, the case of the logarithmic potential showed a slight deficit as compared to the case of a repulsive quartic potential. In the last part of this section, we applied the Swampland conjecture and found that the maximum compactness  $C_{\max}$  obtained by the full numerical computation for both potentials arises from low energy effective scalar field theories which could be UV-completed by a consistent quantum gravity theory.

| $V(\Phi)$   | $f=1/\sqrt{10}$ | $f=1/\sqrt{20}$ | $f=1/\sqrt{40}$ | $f=1/\sqrt{60}$ | $f=1/\sqrt{80}$ | $f=1/\sqrt{100}$ |
|-------------|-----------------|-----------------|-----------------|-----------------|-----------------|------------------|
| Liouville   | 1.866 (0.59)    | 2.996 (0.67)    | 4.427 (0.70)    | 5.593 (0.722)   | 6.494 (0.726)   | 7.3 (0.73)       |
| Logarithmic | 0.727 (0.23)    | 0.827 (0.185)   | 0.885 (0.14)    | 0.891 (0.115)   | 0.939 (0.105)   | 0.900 (0.09)     |

Table 1: The values of  $\phi(r=0)/f$  corresponding to  $M_{\max}$  ( $C_{\max}$ ) are presented for each potential and coupling strength parameter  $f$  (in the unit of  $M_{\text{p}}$ ). In each parentheses the value of  $\phi(0)$  is also shown. For the Liouville potential, all the  $\phi(0)$  values are less than 1 which is the intersection of the two criteria (see the text). For the logarithmic potential,  $\phi(r=0)/f < 2.6555$  can be checked for  $f$  considered in this study, and so does  $\phi(0) < 1$ . This shows that all the  $C_{\max}$  values shown in Fig. 5 can be regarded as arising from effective scalar theories UV-completed by a consistent quantum gravity.

From the Table 1, we obtain the following mean:

$$0,23 + 0,185 + 0,14 + 0,115 + 0,105 + 0,09 = 0.865; \quad 0.865 / 6 =$$

$$= 0,1441666....$$

Now, from the (C2), we have the following expressions for  $C = 0.18$  and  $C = 0.14416$

$$(0.18^{1.5}) * (1.9891 * 10^{30}) * 6.149$$

**Input interpretation:**

$$0.18^{1.5} \times 1.9891 \times 10^{30} \times 6.149$$

**Result:**

$$9.34049... \times 10^{29}$$

$$9.34049... * 10^{29}$$

$$(0.18^{1.5}) * (1.9891 * 10^{30}) * 5$$

**Input interpretation:**

$$0.18^{1.5} \times 1.9891 \times 10^{30} \times 5$$

**Result:**

$$7.59513... \times 10^{29}$$

$$7.59513... * 10^{29}$$

$$(0.1441666^{1.5}) * (1.9891 * 10^{30}) * 6.149$$

**Input interpretation:**

$$0.1441666^{1.5} \times 1.9891 \times 10^{30} \times 6.149$$

**Result:**

$$6.69512... \times 10^{29}$$

$$6.69512... * 10^{29}$$

$$(0.1441666^{1.5}) * (1.9891 * 10^{30}) * 5$$

**Input interpretation:**

$$0.1441666^{1.5} \times 1.9891 \times 10^{30} \times 5$$

**Result:**

$$5.44407... \times 10^{29}$$

$$5.44407... * 10^{29}$$

$$(0.18^{1.5}) * (1.9891 * 10^{30}) * 124.451$$

**Input interpretation:**

$$0.18^{1.5} \times 1.9891 \times 10^{30} \times 124.451$$

**Result:**

$$1.89044... \times 10^{31}$$

$$1.89044... * 10^{31}$$

$$(0.1441666^{1.5}) * (1.9891 * 10^{30}) * 124.451$$

**Input interpretation:**

$$0.1441666^{1.5} \times 1.9891 \times 10^{30} \times 124.451$$

**Result:**

$$1.35504... \times 10^{31}$$

$$1.35504... * 10^{31}$$

Or, less the value of C, for 6.149 and 5 solar masses, we obtain:

$$(1.9891 * 10^{30}) * 6.149$$

**Input interpretation:**

$$1.9891 \times 10^{30} \times 6.149$$

**Result:**

122309759000000000000000000000

**Scientific notation:**

$$1.22309759 \times 10^{31}$$

$$1.22309759 * 10^{31}$$

$$(1.9891 * 10^{30}) * 5$$

**Input interpretation:**

$$1.9891 \times 10^{30} \times 5$$

**Result:**

99455000000000000000000000000

**Scientific notation:**

$$9.9455 \times 10^{30}$$

$$9.9455 * 10^{30}$$

For 124.451 solar masses, we obtain:

$$(1.9891 * 10^{30}) * 124.451$$

**Input interpretation:**

$$1.9891 \times 10^{30} \times 124.451$$

**Result:**

247545484100000000000000000000

**Scientific notation:**

$$2.475454841 \times 10^{32}$$

$$2.475454841 * 10^{32}$$

We take the values corresponding to the 5 and 6.149 solar masses. We have:

$$9.34049... * 10^{29} \quad 7.59513... * 10^{29} \quad 6.69512... * 10^{29} \quad 5.44407... * 10^{29}$$

$$1.22309759 * 10^{31} \quad 9.9455 * 10^{30}$$

The averages are:  $7,2687025 * 10^{29}$  Kg ,  $9.9455 * 10^{30}$  Kg and  $1.22309759 * 10^{31}$

With regard the values corresponding to 5 solar masses, we have that:  
 $6.5196 * 10^{29}$  kg and  $9.9455 * 10^{30}$  Kg

The mean is: 5.298.730.000.000.000.000.000.000.000

Indeed:

$$1/2(6.5196 * 10^{29} + 9.9455 * 10^{30}) \text{ kg}$$

**Input interpretation:**

$$\frac{1}{2} (6.5196 \times 10^{29} + 9.9455 \times 10^{30}) \text{ kg (kilograms)}$$

**Result:**

$$5.2987 \times 10^{30} \text{ kg (kilograms)}$$

**Scientific notation:**

$$5.29873 \times 10^{30}$$

**Comparisons as mass:**

$$2.665 M_{\odot} \text{ (solar masses)}$$

Note that:


$$\sqrt{2.665} = 1.63248277173145 \text{ and from the following result}$$

$$(5.29873/1.9891) = 2,663883163;$$

$$\sqrt{2.663883163} = 1.6321406689497$$

- ≈ ( 0.3 ≈ 1/4 ) ×  
mass below which massive stars pass through a red giant stage (≈10  $M_{\odot}$ )
- ≈ (0.1 to 1.3) × mass of a B-type main-sequence star ( 2 to 16  $M_{\odot}$  )
- ≈ (1.1 to 1.6) × mass of an A-type main-sequence star ( 2  $M_{\odot}$  )

**Corresponding main-sequence star properties:**

|                       |   |
|-----------------------|---|
| color                 |  |
| temperature class     | B9  |
| effective temperature | 11 000 K  |
| absolute magnitude    | + 0.35 (bolometric)   |
| radius                | 2.3 $R_{\odot}$   |
| luminosity            | 58 $L_{\odot}$  |
| B-V color index       | -0.061  |
| lifetime              | 860 million yr  |
| end state             | carbon-oxygen white dwarf   |

Units»

**Interpretation:**

mass

**Corresponding quantities:**

Weight  $w$  of a body from  $w = mg$ :

$5.196 \times 10^{31}$  N (newtons)

$5.196 \times 10^{36}$  dynes

$5.299 \times 10^{33}$  ponds

$1.168 \times 10^{31}$  lbf (pounds-force)

Luminosity  $L$  for a main-sequence star from  $L = L_{\odot}(M/M_{\odot})^{3.5}$ :

$1.189 \times 10^{28}$  W (watts)

31  $L_{\odot}$  (solar luminosities)

Now, from  $5.29873 * 10^{30}$ , we obtain:

Mass =  $5.298730e+30$

Radius = 7867.827

Temperature =  $2.316033e-8$

From the fourth GENERAL FORMULA (Ramanujan-Nardelli mock formula), we obtain:

$$\sqrt{\left[ \left[ \left[ \left[ \left[ \left[ \left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{5.298730 \times 10^{30}} \right] \sqrt{\frac{-2.316033 \times 10^{-8} \times 4 \pi \times 7867.827^3 - 7867.827^2}{6.67 \times 10^{-11}}} \right] \right] \right] \right] \right] \right] \sqrt{-\left( \frac{2.316033 \times 10^{-8} \times 4 \pi \times 7867.827^3 - 7867.827^2}{6.67 \times 10^{-11}} \right)} \right] \right] \right] \right] \right] \right] \right]$$

**Input interpretation:**

$$\sqrt{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{5.298730 \times 10^{30}} \sqrt{\frac{-2.316033 \times 10^{-8} \times 4 \pi \times 7867.827^3 - 7867.827^2}{6.67 \times 10^{-11}}}}$$

**Result:**

1.618249273207088689512781866634297184888546159698791077606...

1.618249...

And for the value of Ramanujan mock theta function 1.897512108, we obtain:

$$\sqrt{\left[ \left[ \left[ \left[ \left[ \left[ \left[ \frac{1}{\left( \frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{5.298730 \times 10^{30}} \right] \sqrt{\frac{-2.316033 \times 10^{-8} \times 4 \pi \times 7867.827^3 - 7867.827^2}{6.67 \times 10^{-11}}} \right] \right] \right] \right] \right] \right] \sqrt{-\left( \frac{2.316033 \times 10^{-8} \times 4 \pi \times 7867.827^3 - 7867.827^2}{6.67 \times 10^{-11}} \right)} \right] \right] \right] \right] \right] \right] \right]$$

**Input interpretation:**

$$\sqrt{\frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{5.298730 \times 10^{30}} \sqrt{\frac{-2.316033 \times 10^{-8} \times 4 \pi \times 7867.827^3 - 7867.827^2}{6.67 \times 10^{-11}}}}$$

**Result:**

1.645670835572422533157236154079440652780828762143223481545...

1.64567...  $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

$$2\sqrt{\left( \left( \left( \left( \left( \left( \left( \frac{1}{\left( \frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{5.298730 \times 10^{30}} \right) \sqrt{\frac{-2.316033 \times 10^{-8} \times 4 \pi \times 7867.827^3 - 7867.827^2}{6.67 \times 10^{-11}}} \right) \right] \right] \right] \right] \right] \right] \sqrt{-\left( \frac{2.316033 \times 10^{-8} \times 4 \pi \times 7867.827^3 - 7867.827^2}{6.67 \times 10^{-11}} \right)} \right) \right) \right) \right) \right) \right) \right]$$

**Input interpretation:**

$$2 \sqrt{6 \sqrt{\frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{5.298730 \times 10^{30}} \sqrt{\frac{-2.316033 \times 10^{-8} \times 4 \pi \times 7867.827^3 - 7867.827^2}{6.67 \times 10^{-11}}}}}$$

**Result:**

6.28459...

6.28459... =  $2\pi r$

Note that:

**Input interpretation:**

6.2845922742639510707344892689664319430522535115783135

$$2\pi$$

**Result:**

1.0002239257662442168315552289031143684437788799325338...

1.0002239257662442168315

Now, we calculate the ellipse perimeter from the better approximation formula of

**Ramanujan:**

$$p \approx \pi \left[ 3(a + b) - \sqrt{(3a + b)(a + 3b)} \right]$$

We have, for a = b:

$$\pi(3*(1.0002239257662442168315+1.0002239257662442168315)-\sqrt{((3*1.0002239257662442168315+1.0002239257662442168315)*(1.0002239257662442168315+3*1.0002239257662442168315)))})$$

**Input interpretation:**

$$\pi(3(1.0002239257662442168315 + 1.0002239257662442168315) - \sqrt{((3 \times 1.0002239257662442168315 + 1.0002239257662442168315)(1.0002239257662442168315 + 3 \times 1.0002239257662442168315)))})$$

**Result:**

6.284592274263951070734...

6.28459...

**Series representations:**

$$\pi(3(1.00022392576624421683150000 + 1.00022392576624421683150000) - \sqrt{((3 \times 1.00022392576624421683150000 + 1.00022392576624421683150000)(1.00022392576624421683150000 + 3 \times 1.00022392576624421683150000)))}) = 6.0013435545974653009890000 \pi - 1.00000000000000000000000000000000$$

$$\frac{\pi}{\sum_{k=0}^{\infty} e^{-2.70852784879730494512752219k} \binom{\frac{1}{2}}{k}} \sqrt{15.0071664268037955475622110}$$

$$\pi (3 (1.00022392576624421683150000 + 1.00022392576624421683150000) - \sqrt{((3 \times 1.00022392576624421683150000 + 1.00022392576624421683150000) (1.00022392576624421683150000 + 3 \times 1.00022392576624421683150000)))}) = 6.0013435545974653009890000 \pi - 1.00000000000000000000000000000000$$

$$\frac{\pi \sqrt{15.0071664268037955475622110} \sum_{k=0}^{\infty} \frac{(-0.066634831090693684258655561)^k \left(-\frac{1}{2}\right)_k}{k!}}{2 \sqrt{\pi}}$$

$$\pi (3 (1.00022392576624421683150000 + 1.00022392576624421683150000) - \sqrt{((3 \times 1.00022392576624421683150000 + 1.00022392576624421683150000) (1.00022392576624421683150000 + 3 \times 1.00022392576624421683150000)))}) = 6.00134355459746530098900000$$

$$\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} e^{-2.70852784879730494512752219s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2 \sqrt{\pi}}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\operatorname{Res}_f$  is a complex residue  $z=z_0$

Utilizing the formula of torus surface  $S = 4\pi^2 r * d$ , we obtain, from the radius of BH = 7867.827:

$$((4 * \pi^2 * 1 / (2 * 89 - 2)) * 7867.827)$$

**Input interpretation:**

$$4 \pi^2 \times \frac{1}{2 \times 89 - 2} \times 7867.827$$

**Result:**

1764.826...



1764.826...

result in the range of the mass of candidate “glueball”  $f_0(1710)$

### Alternative representations:

$$\frac{4\pi^2 7867.83}{2 \times 89 - 2} = \frac{1}{176} \times 31471.3 (180^\circ)^2$$

$$\frac{4\pi^2 7867.83}{2 \times 89 - 2} = \frac{1}{176} \times 31471.3 (-i \log(-1))^2$$

$$\frac{4\pi^2 7867.83}{2 \times 89 - 2} = \frac{188828. \zeta(2)}{176}$$

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

$\zeta(s)$  is the Riemann zeta function

[More information »](#)

### Series representations:

$$\frac{4\pi^2 7867.83}{2 \times 89 - 2} = 2861.03 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k} \right)^2$$

$$\frac{4\pi^2 7867.83}{2 \times 89 - 2} = 715.257 \left( -1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}} \right)^2$$

$$\frac{4\pi^2 7867.83}{2 \times 89 - 2} = 178.814 \left( \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}} \right)^2$$

$\binom{n}{m}$  is the binomial coefficient

[More information »](#)

**Integral representations:**

$$\frac{4\pi^2 7867.83}{2 \times 89 - 2} = 715.257 \left( \int_0^\infty \frac{1}{1+t^2} dt \right)^2$$

$$\frac{4\pi^2 7867.83}{2 \times 89 - 2} = 2861.03 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2$$

$$\frac{4\pi^2 7867.83}{2 \times 89 - 2} = 715.257 \left( \int_0^\infty \frac{\sin(t)}{t} dt \right)^2$$

From the value of the entropy  $3.233971e+77$ , we obtain:

$$10 \ln(3.233971e+77)$$

**Input interpretation:**

$$10 \log(3.233971 \times 10^{77})$$

$\log(x)$  is the natural logarithm

**Result:**

1784.72763...

1784.72763... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

Note that, from the Ramanujan sum of two cubes, we obtain 1780. Indeed:

$$14258 - 11468 - 1010 = 1780$$

From the surface gravity  $5.711584e+12$ , we obtain:

$$(5.711584e+12)^{1/4}$$

**Input interpretation:**

$$\sqrt[4]{5.711584 \times 10^{12}}$$

**Result:**

1545.928...

1545.928... result very near to the  $f_2(1565)$  mass  $1542 \pm 19$

We have that, from:

| $\xi_{\text{DM}}$ | Liouville Potential ( $\Lambda = 100$ )                              | Logarithmic Potential ( $\Lambda = 100$ )                            |
|-------------------|--|--|
| 0.1               | $3.6 \times 10^{-12} \lesssim m_{\Phi} \lesssim 3.6 \times 10^{-10}$ | $8.4 \times 10^{-13} \lesssim m_{\Phi} \lesssim 8.4 \times 10^{-11}$ |
| 0.01              | $3.6 \times 10^{-13} \lesssim m_{\Phi} \lesssim 3.6 \times 10^{-3}$  | $8.4 \times 10^{-14} \lesssim m_{\Phi} \lesssim 8.4 \times 10^{-4}$  |

Table 2: Allowed mass range of the scalar particle under two benchmark potentials, as inferred from the MACHO mass constraints [48–52] for the DM fraction  $\xi_{\text{DM}} = 0.1$  and 0.01. Here the unit of the scalar particle mass is eV.

$$3.6 * 10^{-13} \text{ eV}$$

$$\chi(q) = 2.6709253774829... \text{ and } 0.9243408674589$$

From the sum of the above mock theta functions, we obtain:

$$3.5952662449418 \approx 3.6$$

We take this value and obtain:

$$3.5952662449418e-13 \text{ eV}$$

### Input interpretation:

convert  $3.5952662449418 \times 10^{-13} \text{ eV}/c^2$  to kilograms

### Result:

$$6.409143056892 \times 10^{-49} \text{ kg (kilograms)}$$

$$6.40914305689 * 10^{-49} \text{ kg}$$

$$\text{Mass} = 6.409143e-49$$

$$\text{Radius} = 9.516626e-76$$

$$\text{Temperature} = 1.914770e+71$$

$$\text{Entropy} = 4.731430e-81$$

From the following fourth GENERAL FORMULA (Ramanujan-Nardelli mock formula), we obtain:

sqrt[[[1/((((((4\*1.962364415e+19)/(5\*0.0864055^2)))\*1/(6.409143e-49)\* sqrt[[-(((1.914770e+71 \* 4\*Pi\*(9.516626e-76)^3-(9.516626e-76)^2)))) / ((6.67\*10^-11))]]]]]]

**Input interpretation:**

$$\sqrt{\left(1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{6.409143 \times 10^{-49}} \sqrt{-\frac{1.914770 \times 10^{71} \times 4 \pi (9.516626 \times 10^{-76})^3 - (9.516626 \times 10^{-76})^2}{6.67 \times 10^{-11}}} \right) \right)}$$

**Result:**

1.618249219723227730953646546941504630502892462832980882653...  
1.618249...

And for the mock theta function 1.897512108, we obtain:

sqrt[[[1/((((((4\*1.897512108e+19)/(5\*0.0864055^2)))\*1/(6.409143e-49)\* sqrt[[-(((1.914770e+71 \* 4\*Pi\*(9.516626e-76)^3-(9.516626e-76)^2)))) / ((6.67\*10^-11))]]]]]]

**Input interpretation:**

$$\sqrt{\left(1 / \left( \frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{6.409143 \times 10^{-49}} \sqrt{-\frac{1.914770 \times 10^{71} \times 4 \pi (9.516626 \times 10^{-76})^3 - (9.516626 \times 10^{-76})^2}{6.67 \times 10^{-11}}} \right) \right)}$$

**Result:**

1.645670781182266695869433030422643782063882284478017756517...  
1.64567...  $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

And:



sqrt[[[1/(((((((4\*1.962364415e+19)/(5\*0.0864055^2))))\*1/(1.507934e-49)\* sqrt[[-(((8.138310e+71 \* 4\*Pi\*(2.239058e-76)^3-(2.239058e-76)^2)))) / ((6.67\*10^-11))]]]]]]

**Input interpretation:**

$$\sqrt{\left(1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.507934 \times 10^{-49}} \sqrt{-\frac{8.138310 \times 10^{71} \times 4 \pi (2.239058 \times 10^{-76})^3 - (2.239058 \times 10^{-76})^2}{6.67 \times 10^{-11}}} \right) \right)}$$

**Result:**

1.618249276203142445357760087938552676720668241543654601222...  
1.6182492...

And inserting the mock theta function 1.897512108:

sqrt[[[1/(((((((4\*1.897512108e+19)/(5\*0.0864055^2))))\*1/(1.507934e-49)\* sqrt[[-(((8.138310e+71 \* 4\*Pi\*(2.239058e-76)^3-(2.239058e-76)^2)))) / ((6.67\*10^-11))]]]]]]

**Input interpretation:**

$$\sqrt{\left(1 / \left( \frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.507934 \times 10^{-49}} \sqrt{-\frac{8.138310 \times 10^{71} \times 4 \pi (2.239058 \times 10^{-76})^3 - (2.239058 \times 10^{-76})^2}{6.67 \times 10^{-11}}} \right) \right)}$$

**Result:**

1.645670838619245027957354406800819598743646732943027254410...  
1.64567...  $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

And:

1/2\*

((((3.817978468883578995+sqrt[[[1/(((((((4\*1.962364415e+19)/(5\*0.0864055^2))))\*1/(1.507934e-49)\* sqrt[[-(((8.138310e+71 \* 4\*Pi\*(2.239058e-76)^3-(2.239058e-76)^2)))) / ((6.67\*10^-11))]]]]]]

**Input interpretation:**

$$\frac{1}{2} \left( 3.817978468883578995 + \sqrt{\left( 1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.507934 \times 10^{-49}} \sqrt{-\frac{8.138310 \times 10^{71} \times 4 \pi (2.239058 \times 10^{-76})^3 - (2.239058 \times 10^{-76})^2}{6.67 \times 10^{-11}}} \right) \right) \right)}$$

**Result:**

2.718113872543360720178880043969276338360334120771827300611...

2.7181138...  $\approx e = 2.71828$

Now, from the result of the multiplication of the following two mock theta functions

$33021.1(-5.74968e-40) = -1.89860758248 \times 10^{-35}$  divided by this other mock theta function  $0.8988931+1$ , we obtain, from this further Ramanujan-Nardelli mock formula:

$$(1.898607e-35)/(1.8988931)$$

$$\text{sqrt}[[1/(((((((4*1.962364415e+19)/(5*0.0864055^2))))*1/(1.507934e-49)* \text{sqrt}[-(((8.138310e+71 * 4*Pi*(2.239058e-76)^3-(2.239058e-76)^2)))))/((6.67*10^-11))]]]$$

**Input interpretation:**

$$\frac{1.898607 \times 10^{-35}}{1.8988931} \sqrt{\left( 1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.507934 \times 10^{-49}} \sqrt{-\frac{8.138310 \times 10^{71} \times 4 \pi (2.239058 \times 10^{-76})^3 - (2.239058 \times 10^{-76})^2}{6.67 \times 10^{-11}}} \right) \right) \right)}$$

**Result:**

$1.61801... \times 10^{-35}$

$1.61801... * 10^{-35}$

that is a good approximation to the value of Planck length  $1.6162 * 10^{-35}$

From the entropy  $2.619130e-82$ , we obtain:

$$\text{sqrt}(2.619130e-82)$$

**Input interpretation:**

$$\sqrt{2.619130 \times 10^{-82}}$$

**Result:**

$$1.618373... \times 10^{-41}$$

$$1.618373... * 10^{-41}$$

And:

$$8 \operatorname{colog}(2.619130e-82)$$

**Input interpretation:**

$$8(-\log(2.619130 \times 10^{-82}))$$

$\log(x)$  is the natural logarithm

**Result:**

$$1502.79308...$$

1502.79308... result very near to the  $f_0(1500)$  mass  $1506 \pm 6$

Note that, from the Ramanujan sum of two cubes, we obtain:

$$14258 - 11468 - 1010 - 135 - 138 = 1507$$

$$14258 / 8 = 1782.25 \text{ very near to gluino mass} = 1785.16 \text{ GeV}$$

$$11468 / 8 = 1433.5 \approx f_2(1430) \text{ mass} = 1430$$

From this three entropy:

1.664134e-75    3.233971e+77    and    2.619130e-82, we obtain:

$$1/8 * \operatorname{colog}(1.664134e-75)$$

**Input interpretation:**

$$\frac{1}{8} (-\log(1.664134 \times 10^{-75}))$$

$\log(x)$  is the natural logarithm

**Result:**

$$21.5230721...$$

$$21.5230721...$$

$$1/8 \ln(3.233971e+77)$$



**Input interpretation:**

$$\frac{1}{8} \log(3.233971 \times 10^{77})$$

$\log(x)$  is the natural logarithm

**Result:**

22.3090954...

22.3090954...

$$1/8 * \text{colog}(2.619130e-82)$$

**Input interpretation:**

$$\frac{1}{8} (-\log(2.619130 \times 10^{-82}))$$

$\log(x)$  is the natural logarithm

**Result:**

23.4811419...

23.4811419...

Results very near to a Black Hole entropies (see previous our papers)

Furthermore, from the two entropies  $4.731430e-81$   $2.619131e-82$

$$8 * ((\text{colog}(4.731430e-81) + \text{colog}(2.619131e-82)))$$

**Input interpretation:**

$$8 (-\log(4.731430 \times 10^{-81}) - \log(2.619131 \times 10^{-82}))$$

$\log(x)$  is the natural logarithm

**Result:**

2982.43440...

2982.43440... result very near to the rest mass of Charmed eta meson 2980.3

Now, we have that:

**Figure 1:** Radial profile of the scalar field as a function of the radius for different values of  $\phi_0$  and the coupling strength parameter, in the case of  $U(1)$  Liouville potential. The field and the radius are shown in units of  $M_P$  and  $1/m_\phi$ , respectively. The red, blue, and green curves correspond to the ground state solutions for  $(0.273, M_P)$ ,  $(0.185, 1/\sqrt{20} M_P)$ , and  $(0.140, 1/\sqrt{40} M_P)$  values of  $(\phi_0, f)$ , respectively. In particular, the red curve is obtained under the suitable values  $\omega = 0.84732123346818 m_\phi$  and  $\gamma_0 = -0.80267626206664$ .

unit  $c = \hbar = 1$ . We denote the Planck mass by  $m_P = G_N^{-1/2}$  and the reduced Planck mass by  $M_P = (8\pi G_N)^{-1/2}$ , where  $G_N$  is the Newton gravitational constant. For the cosmological parameters, we use values based on Planck TT,TE,EE+lowE+lensing at the 68% confidence level in Ref. [10].

We have the following values: 0.84732123346818 and -0.80267626206664 and  $1/\sqrt{20} * 1/\sqrt{8\pi * 6.67e-11}$  and  $1/\sqrt{40} * 1/\sqrt{8\pi * 6.67e-11}$

We have:

$$(((1/\sqrt{20}) * 1/\sqrt{8\pi * 6.67e-11})) + (((1/\sqrt{40}) * 1/\sqrt{8\pi * 6.67e-11})))$$

**Input interpretation:**

$$\frac{1}{\sqrt{20}} \times \frac{1}{\sqrt{8\pi * 6.67 * 10^{-11}}} + \frac{1}{\sqrt{40}} \times \frac{1}{\sqrt{8\pi * 6.67 * 10^{-11}}}$$

**Result:**

9323.15...

Note that:

$$-21 + (((1/\sqrt{20}) * 1/\sqrt{8\pi * 6.67e-11})) + (((1/\sqrt{40}) * 1/\sqrt{8\pi * 6.67e-11})))$$

**Input interpretation:**

$$-21 + \left( \frac{1}{\sqrt{20}} \times \frac{1}{\sqrt{8\pi * 6.67 * 10^{-11}}} + \frac{1}{\sqrt{40}} \times \frac{1}{\sqrt{8\pi * 6.67 * 10^{-11}}} \right)$$

**Result:**

9302.15...

9302.15... result very near to the rest mass of Bottom eta meson 9300

$$(((1/\sqrt{20}) * 1/\sqrt{8\pi*6.67e-11})))$$

**Input interpretation:**

$$\frac{1}{\sqrt{20}} \times \frac{1}{\sqrt{8\pi \times 6.67 \times 10^{-11}}}$$

**Result:**

5461.38...

5461.38...

$$-34-13+(((1/\sqrt{20}) * 1/\sqrt{8\pi*6.67e-11})))$$

**Input interpretation:**

$$-34 - 13 + \frac{1}{\sqrt{20}} \times \frac{1}{\sqrt{8\pi \times 6.67 \times 10^{-11}}}$$

**Result:**

5414.38...

5414.38... result very near to the rest mass of Strange B meson 5412.8

$$(((1/\sqrt{40}) * 1/\sqrt{8\pi*6.67e-11})))$$

**Input interpretation:**

$$\frac{1}{\sqrt{40}} \times \frac{1}{\sqrt{8\pi \times 6.67 \times 10^{-11}}}$$

**Result:**

3861.78...

3861.78...

$$-233-8+(((1/\sqrt{40}) * 1/\sqrt{8\pi*6.67e-11})))$$

**Input interpretation:**

$$-233 - 8 + \frac{1}{\sqrt{40}} \times \frac{1}{\sqrt{8\pi \times 6.67 \times 10^{-11}}}$$

**Result:**

3620.78...

3620.78... result very near to the rest mass of double charmed Xi baryon 3621.40

$$(((((((1/\sqrt{20}) * 1/\sqrt{8\pi*6.67e-11}))) + (((1/\sqrt{40}) * 1/\sqrt{8\pi*6.67e-11}))))))^{1/19}$$

**Input interpretation:**

$$\sqrt[19]{\frac{1}{\sqrt{20}} \times \frac{1}{\sqrt{8\pi \times 6.67 \times 10^{-11}}} + \frac{1}{\sqrt{40}} \times \frac{1}{\sqrt{8\pi \times 6.67 \times 10^{-11}}}}$$

**Result:**

1.617798245732611704017737283949870853425420697024552986004...

1.6177982... result very near to the value of golden ratio

$$2\sqrt{((((6*(((1/\sqrt{20}) * 1/\sqrt{8\pi*6.67e-11})) + ((1/\sqrt{40}) * 1/\sqrt{8\pi*6.67e-11}))))^1/18))}$$

**Input interpretation:**

$$2 \sqrt[6]{\sqrt[18]{\frac{1}{\sqrt{20}} \times \frac{1}{\sqrt{8\pi \times 6.67 \times 10^{-11}}} + \frac{1}{\sqrt{40}} \times \frac{1}{\sqrt{8\pi \times 6.67 \times 10^{-11}}}}}$$

**Result:**

6.3149695...

6.3149695...  $\approx 2\pi r$

$$1/(2\pi) * 2\sqrt{((((6*(((1/\sqrt{20}) * 1/\sqrt{8\pi*6.67e-11})) + ((1/\sqrt{40}) * 1/\sqrt{8\pi*6.67e-11}))))^1/18))}$$

**Input interpretation:**

$$\frac{1}{2\pi} \times 2 \sqrt[6]{\sqrt[18]{\frac{1}{\sqrt{20}} \times \frac{1}{\sqrt{8\pi \times 6.67 \times 10^{-11}}} + \frac{1}{\sqrt{40}} \times \frac{1}{\sqrt{8\pi \times 6.67 \times 10^{-11}}}}}$$

**Result:**

1.0050586...

1.0050586...

From the Ramanujan better approximate formula for ellipse perimeter,

$$p \approx \pi \left[ 3(a + b) - \sqrt{(3a + b)(a + 3b)} \right]$$

we obtain:

$$\text{Pi}(3*(1.0050586+1.0050586)-\text{sqrt}(((3*1.0050586+1.0050586)*(1.0050586+3*1.0050586))))$$

**Input interpretation:**

$$\pi \left( 3 (1.0050586 + 1.0050586) - \sqrt{(3 \times 1.0050586 + 1.0050586) (1.0050586 + 3 \times 1.0050586)} \right)$$

**Result:**

6.314969...

6.314969... that is the ellipse perimeter

**Series representations:**

$$\pi \left( 3 (1.00506 + 1.00506) - \sqrt{(3 \times 1.00506 + 1.00506) (1.00506 + 3 \times 1.00506)} \right) = 6.03035 \pi - \pi \left( \sqrt{15.1623} \sum_{k=0}^{\infty} e^{-2.71881k} \binom{\frac{1}{2}}{k} \right)$$

$$\pi \left( 3 (1.00506 + 1.00506) - \sqrt{(3 \times 1.00506 + 1.00506) (1.00506 + 3 \times 1.00506)} \right) = 6.03035 \pi - \pi \left( \sqrt{15.1623} \sum_{k=0}^{\infty} \frac{(-0.0659531)^k \left(-\frac{1}{2}\right)_k}{k!} \right)$$

$$\pi \left( 3 (1.00506 + 1.00506) - \sqrt{(3 \times 1.00506 + 1.00506) (1.00506 + 3 \times 1.00506)} \right) = 6.03035 \pi - \frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} e^{-2.71881s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{2 \sqrt{\pi}}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(a)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function

$\text{Res}_{z=z_0} f$  is a complex residue

From the following values: 0.84732123346818 and -0.80267626206664, we obtain:

$$-((( -0.80267626206664 - (0.84732123346818))))$$

**Input interpretation:**

$$-(-0.80267626206664 - 0.84732123346818)$$

**Result:**

$$1.64999749553482$$

$$1.64999749\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934$$

From the following data (see previous formula)

$$\text{Mass} = 7.161e+39$$

$$\text{Radius} = 1.063302e+13$$

$$\text{Temperature} = 1.713732e-17$$

$$\hbar = 1.054571726 * 10^{-34} \text{ J s}$$

$$c = 3 * 10^8$$

$$G = 6.67 * 10^{-11}$$

Energy dispersion P:

$$P = \frac{\hbar c^6}{15\,360 \pi G^2 M^2}$$

$$(((((((1.054571726 * 10^{-34}) * (3 * 10^8)^6)))))) / (((32 * 480 * \pi) * (6.674 * 10^{-11})^2 * (7.161 * 10^{39})^2))))))$$

**Input interpretation:**

$$\frac{1.054571726 \times 10^{-34} \times 3 \times 10^8 \times 6}{(32 \times 480 \pi) (6.674 \times 10^{-11})^2 (7.161 \times 10^{39})^2}$$

**Result:**

$$1.72222... \times 10^{-89}$$

$$1.72222... * 10^{-89}$$

Note that:

$$10^3 * 10^{89} \left( \frac{(((((1.054571726 * 10^{-34}) * (3 * 10^8) * 6))) / (((32 * 480 * \pi) * (6.674 * 10^{-11})^2 * (7.161 * 10^{39})^2)))))) \right)$$

**Input interpretation:**

$$10^3 \times 10^{89} \times \frac{1.054571726 \times 10^{-34} \times 3 \times 10^8 \times 6}{(32 \times 480 \pi) (6.674 \times 10^{-11})^2 (7.161 \times 10^{39})^2}$$

**Result:**

$$1722.22...$$

$$1722.22...$$

this result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

And, from the sum of the following three Ramanujan mock theta functions:

$$1,897512108 + 1,8236681145196 + 2,6709253774829 + 0,50970737445 =$$

$$= 6.9018129744525$$

we obtain:

$$6.9018129744525 + 10^3 * 10^{89} \left( \frac{(((((1.054571726 * 10^{-34}) * (3 * 10^8) * 6))) / (((32 * 480 * \pi) * (6.674 * 10^{-11})^2 * (7.161 * 10^{39})^2)))))) \right)$$

**Input interpretation:**

$$6.9018129744525 + 10^3 \times 10^{89} \times \frac{1.054571726 \times 10^{-34} \times 3 \times 10^8 \times 6}{(32 \times 480 \pi) (6.674 \times 10^{-11})^2 (7.161 \times 10^{39})^2}$$

**Result:**

$$1729.12...$$

$$1729.12...$$

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic

curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

And:

$$(2.67092537 \times 3) * 1.0061571663 * 1.08185 \operatorname{colog} \left( \frac{(1.054571726 \times 10^{-34}) * (3 \times 10^8)^6}{((32 * 480 * \pi) * (6.674 \times 10^{-11})^2 * (7.161 \times 10^{39})^2)} \right)$$

Where 2.67092537, 1.0061571663 and 1.08185 are Ramanujan mock theta functions

**Input interpretation:**

$$(2.67092537 \times 3) \times 1.0061571663 \times 1.08185 \left( -\log \left( \frac{1.054571726 \times 10^{-34} \times 3 \times 10^8 \times 6}{(32 \times 480 \pi) (6.674 \times 10^{-11})^2 (7.161 \times 10^{39})^2} \right) \right)$$

log(x) is the natural logarithm

**Result:**

1782.66...

1782.66... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

Furthermore, we have:

$$-21 - \left( \frac{1}{27} \times 10^3 \right)^2 + \int_0^{\frac{\pi^2}{6}} (2.67092537 \times 3) * 1.0061571663 * 1.08185 \operatorname{colog} \left( \frac{(1.054571726 \times 10^{-34}) * (3 \times 10^8)^6}{((32 * 480 * \pi) * (6.674 \times 10^{-11})^2 * (7.161 \times 10^{39})^2)} \right) dx, [0, \pi^2/6]$$

**Input interpretation:**

$$-21 - \left( \frac{1}{27} \times 10^3 \right)^2 + \int_0^{\frac{\pi^2}{6}} (2.67092537 \times 3) \times 1.0061571663 \times 1.08185 \left( -\log \left( \frac{1.054571726 \times 10^{-34} \times 3 \times 10^8 \times 6}{(32 \times 480 \pi) (6.674 \times 10^{-11})^2 (7.161 \times 10^{39})^2} \right) \right) dx$$

log(x) is the natural logarithm

**Result:**

1019.02

1019.02 result practically equal to the rest mass of Phi meson 1019.445



Now, we calculate the following integral:

integrate (2.67092537\*3) \* 1.0061571663 \* 1.08185 colog (((((((1.054571726\*10^-34)\* (3\*10^8)6))) / (((32\*480\*Pi) \* (6.674\*10^-11)^2 \* (7.161\*10^39)^2))))))x, [0, Pi^2/5^2]

**Definite integral:**

$$\int_0^{\frac{\pi^2}{5^2}} (2.67092537 \times 3) 1.0061571663 \times 1.08185 \left( -\log \left( \frac{1.054571726 \times 10^{-34} \times 3 \times 10^8 \times 6}{(32 \times 480 \pi) (6.674 \times 10^{-11})^2 (7.161 \times 10^{39})^2} \right) x \right) dx = 138.918$$

log(x) is the natural logarithm

138.918 result very near to the rest mass of Pion meson 139.57

And:

-2^2+integrate (2.67092537\*3) \* 1.0061571663 \* 1.08185 colog (((((((1.054571726\*10^-34)\* (3\*10^8)6))) / (((32\*480\*Pi) \* (6.674\*10^-11)^2 \* (7.161\*10^39)^2))))))x, [0, Pi^2/5^2]

**Input interpretation:**

$$-2^2 + \int_0^{\frac{\pi^2}{5^2}} (2.67092537 \times 3) \times 1.0061571663 \times 1.08185 \left( -\log \left( \frac{1.054571726 \times 10^{-34} \times 3 \times 10^8 \times 6}{(32 \times 480 \pi) (6.674 \times 10^{-11})^2 (7.161 \times 10^{39})^2} \right) x \right) dx$$

log(x) is the natural logarithm

**Result:**

134.918

134.918 result practically equal to the rest mass of Pion meson 134.9766

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**Self-interacting dark matter from a non-Abelian hidden sector**

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around 100 MeV. Alternatively, the hidden sector may be a supersymmetric pure gauge theory with a  $\sim 10$  TeV gluino thermal relic. In this case, the dark matter is largely composed of glueballinos that strongly

In fact, however, we will show that all of these features are present in a supersymmetric version of the hidden glueball scenario, in which the hidden sector is a supersymmetric pure gauge theory. In this model, the dark matter is a  $\sim 10$  TeV hidden gluino, which freezes out in the early Universe when the temperature is high. At freeze-out, the

The thermally averaged transfer cross section, then, depends on four parameters:  $m_X$ ,  $\Lambda$ ,  $\alpha$ , and  $V_{\max}$ . In Fig. 4, we plot the ratio  $\langle\sigma_T\rangle/m_X$  in the  $(m_X, \Lambda)$  plane for  $\alpha = 1$  and three representative characteristic velocities:  $V_{\max} = 40$  km/s for dwarfs,  $V_{\max} = 100$  km/s for LSBs, and  $V_{\max} = 1000$  km/s for clusters. For masses  $m_X \sim 1$  TeV and  $\Lambda \sim 10$  MeV, we achieve transfer cross sections around the targeted range between  $0.1 \text{ cm}^2/\text{g}$  and

The particle spectrum in  $\Lambda$ MSB models is completely specified by quantum numbers, dimensionless couplings, and the gravitino mass. In the visible sector, the wino mass limit  $m_{\tilde{W}} \gtrsim 100$  GeV implies

$$m_{3/2} \gtrsim 37 \text{ TeV}. \quad (17)$$

preferred range. The red, shaded region is excluded by null searches for visible-sector winos at LEP2. The yellow dot in the top panel defines a representative model with  $m_X \sim 14$  TeV,  $\Lambda \simeq 0.35$  MeV,  $N = 2$ , and  $\xi_f \simeq 0.02$ .

Of course, the goal is not simply to obtain a multi-component model of dark matter with the correct relic densities, but to obtain self-interacting dark matter. The regions with the preferred self-interaction cross sections are also shown in Fig. 5. For values of  $m_X \sim 10$  TeV,  $\Lambda \sim 1$  MeV,  $2 \leq N \lesssim 10$ , and  $10^{-3} \lesssim \xi_f \lesssim 10^{-2}$ , we find models that satisfy the relic density constraints and also satisfy the scattering constraints for dwarfs and LSBs. Viable models also exist for the lower values of  $m_X$  down to the LEP2 limit for the larger values of  $N$  and lower values of  $\xi_f$ . A representative model is one with  $m_X \approx 14$  TeV,  $\Lambda \approx 0.35$  MeV,  $N = 2$ , and  $\xi_f = 0.02$ ; this is shown as a yellow dot in Fig. 5. For these parameters, Fig. 1 shows how the dark matter coupling behaves from the scale  $m_X$  down to confinement.

self-interaction cross sections are in the preferred range. The red, shaded region is excluded by null searches for visible-sector winos at LEP2. The yellow dot defines a representative model with  $m_X \approx 2.5$  TeV,  $\Lambda \approx 1.4$  MeV, and  $N = 2$ .

To give a concrete example, consider the following parameters:  $N = 2$ ,  $m_X = 2.5$  TeV,  $\Lambda \approx 1.4$  MeV,  $m_C = 0.5$  TeV,  $m_R = 1$  GeV,  $g_h = 1.1$ ,  $\lambda_R = 1.6$ , and  $g_n = 0.1$ . The output glueball relic density is  $\sim 5\%$  of the total dark matter abundance. We find this result by numerically solving the coupled Boltzmann equations for the gluons and right-handed neutrinos:

The average is:  $(10+14+2.5) \div 3 = 8,833333333333333$ . From the sum of two values of Ramanujan mock theta functions, we obtain:

$$8,044256216625 + 0,8094974 = 8.853753616625 \text{ TeV}$$

We insert this value, converted in Kg, in the Hawking Radiation Calculator.

### Input interpretation:

convert  $8.853753616625 \text{ TeV}/c^2$  to kilograms

### Result:

$$1.57832465396 \times 10^{-23} \text{ kg (kilograms)}$$

$$\text{Mass} = 1.578325e-23 \text{ Kg}$$



2sqrt((((6\*sqrt[[[1/((((((4\*1.897512108e+19)/(5\*0.0864055^2)))\*1/(1.578325e-23)\* sqrt[-((((((7.775353e+45 \* 4\*Pi\*(2.343578e-50)^3-(2.343578e-50)^2)))))) / ((6.67\*10^-11))]]]]))))))

**Input interpretation:**

$$2 \sqrt{\left( 6 \sqrt{\left( 1 / \left( \frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.578325 \times 10^{-23}} \right. \right. \right. \right. \left. \left. \left. \sqrt{\left( -\frac{1}{6.67 \times 10^{-11}} \left( 7.775353 \times 10^{45} \times 4 \pi (2.343578 \times 10^{-50})^3 - (2.343578 \times 10^{-50})^2 \right) \right) \right) \right) \right) \right)$$

**Result:**

6.284592399932952288547858966291804552251412716870451183788...

6.28459... ≈ C = 2πr

Note that:

**Input interpretation:**

$$\frac{6.2845923999329522885478589662918045522514127168704511}{2 \pi}$$

**Result:**

1.0002239457670869540679677303700834699466301995720933...

1.000223945767...

From the Ramanujan better approximate formula for ellipse perimeter,

$$p \approx \pi \left[ 3(a + b) - \sqrt{(3a + b)(a + 3b)} \right]$$

we obtain:

$$\text{Pi}(3*(1.000223945767+1.000223945767)-\text{sqrt}(((3*1.000223945767+1.000223945767)*(1.000223945767+3*1.000223945767))))$$

**Input interpretation:**

$$\pi (3 (1.000223945767 + 1.000223945767) - \sqrt{((3 \times 1.000223945767 + 1.000223945767) (1.000223945767 + 3 \times 1.000223945767))})$$

**Result:**

6.28459239993...

6.28459... that is the ellipse perimeter

**Series representations:**

$$\frac{\pi (3 (1.0002239457670000 + 1.0002239457670000) - \sqrt{((3 \times 1.0002239457670000 + 1.0002239457670000) (1.0002239457670000 + 3 \times 1.0002239457670000)))}}{6.001343674602000 \pi - 1.0000000000000000 \pi \sqrt{15.007167066971305} \sum_{k=0}^{\infty} e^{-2.7085278914547579k} \binom{\frac{1}{2}}{k}}$$

$$\frac{\pi (3 (1.0002239457670000 + 1.0002239457670000) - \sqrt{((3 \times 1.0002239457670000 + 1.0002239457670000) (1.0002239457670000 + 3 \times 1.0002239457670000)))}}{6.001343674602000 \pi - 1.0000000000000000 \pi \sqrt{15.007167066971305} \sum_{k=0}^{\infty} \frac{(-0.06663482824822157)^k \left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\pi (3 (1.0002239457670000 + 1.0002239457670000) - \sqrt{((3 \times 1.0002239457670000 + 1.0002239457670000) (1.0002239457670000 + 3 \times 1.0002239457670000)))}}{6.001343674602000 \pi - \frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} e^{-2.7085278914547579s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2 \sqrt{\pi}}}$$

$\binom{n}{m}$  is the binomial coefficient

$n!$  is the factorial function

$(\alpha)_n$  is the Pochhammer symbol (rising factorial)

$\Gamma(x)$  is the gamma function  
 $\text{Res}_{s=z_0} f$  is a complex residue

With regard the value 8.853753616625 TeV, we obtain also:

$$(-89 - 21) + 10^4 * 1 / (8.853753616625)$$

**Input interpretation:**

$$(-89 - 21) + 10^4 \times \frac{1}{8.853753616625}$$







$1.783 \times 10^{-23}$  kg (kilograms)

$1.783 * 10^{-23}$  kg

Mass =  $1.783000e-23$

Radius =  $2.647490e-50$

Temperature =  $6.882800e+45$

Now, we insert these values in the Ramanujan-Nardelli mock formula:

$\text{sqrt}\left[\left[\left[\left[\left[\left[\left[\frac{1}{\left(\frac{4 * 1.962364415e+19}{5 * 0.0864055^2}\right) * \frac{1}{1.783000e-23}\right]} * \text{sqrt}\left[\frac{-\left(\left(\left(6.882800e+45 * 4 * \text{Pi} * (2.647490e-50)^3 - (2.647490e-50)^2\right)\right)}{\left(6.67 * 10^{-11}\right)}\right]}{\right]\right]\right]\right]\right]\right]$

**Input interpretation:**

$$\sqrt{\left(1 / \left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.783000 \times 10^{-23}}\right)\right) \times \sqrt{\frac{-\left(6.882800 \times 10^{45} \times 4 \pi (2.647490 \times 10^{-50})^3 - (2.647490 \times 10^{-50})^2\right)}{6.67 \times 10^{-11}}}}$$

**Result:**

1.618249322281036615536110296392444708572935782264897943653...

1.61824932.....

For the same formula, but with the Ramanujan mock theta function  $F(q) =$

**1.897512108...** , we obtain (Ramanujan-Nardelli second mock formula):

$\text{sqrt}\left[\left[\left[\left[\left[\left[\left[\frac{1}{\left(\frac{4 * 1.897512108e+19}{5 * 0.0864055^2}\right) * \frac{1}{1.783000e-23}\right]} * \text{sqrt}\left[\frac{-\left(\left(\left(6.882800e+45 * 4 * \text{Pi} * (2.647490e-50)^3 - (2.647490e-50)^2\right)\right)}{\left(6.67 * 10^{-11}\right)}\right]}{\right]\right]\right]\right]\right]\right]$

**Input interpretation:**

$$\sqrt{\left(1 / \left(\frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.783000 \times 10^{-23}}\right)\right) \times \sqrt{\frac{-\left(6.882800 \times 10^{45} \times 4 \pi (2.647490 \times 10^{-50})^3 - (2.647490 \times 10^{-50})^2\right)}{6.67 \times 10^{-11}}}}$$

**Result:**

1.645670885477938466561579365060100391624301687965069006670...

$$1.6456708... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

We have also that:

$$(-5+144)+10^3*\text{sqrt}\left[\left[\left[\frac{1}{\left(\frac{4*1.897512108e+19}{5*0.0864055^2}\right)*1/(1.783e-23)}\right]*\text{sqrt}\left[-\left(\frac{6.882800e+45 * 4*\text{Pi}*(2.647490e-50)^3-(2.647490e-50)^2}{6.67*10^{-11}}\right)\right]\right]\right]$$

**Input interpretation:**

$$(-5 + 144) + 10^3 \sqrt{\left(1 / \left(\frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.783 \times 10^{-23}}\right) \sqrt{-\frac{6.882800 \times 10^{45} \times 4 \pi (2.647490 \times 10^{-50})^3 - (2.647490 \times 10^{-50})^2}{6.67 \times 10^{-11}}}\right)}$$

**Result:**

1784.67...

1784.67....

result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

And:

$$-(\text{sqrt}729)+10^2*\text{sqrt}\left[\left[\left[\frac{1}{\left(\frac{4*1.897512108e+19}{5*0.0864055^2}\right)*1/(1.783e-23)}\right]*\text{sqrt}\left[-\left(\frac{6.882800e+45 * 4*\text{Pi}*(2.647490e-50)^3-(2.647490e-50)^2}{6.67*10^{-11}}\right)\right]\right]\right]$$

**Input interpretation:**

$$-\sqrt{729} + 10^2 \sqrt{\left(1 / \left(\frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.783 \times 10^{-23}}\right) \sqrt{-\frac{6.882800 \times 10^{45} \times 4 \pi (2.647490 \times 10^{-50})^3 - (2.647490 \times 10^{-50})^2}{6.67 \times 10^{-11}}}\right)}$$

**Result:**

137.567...

137.567.... result very near to the mean of the rest masses of two Pion mesons 134.9766 and 139.57 that is 137.2733

Now, for 2.5 TeV, we have:

**Input interpretation:**

convert 2.5 TeV /c<sup>2</sup> to kilograms

**Result:**

4.46 × 10<sup>-24</sup> kg (kilograms)

4.46 \* 10<sup>-24</sup> kg

Mass = 4.460000e-24

Radius = 6.622438e-51

Temperature = 2.751577e+46

Now, we insert these values in the Ramanujan-Nardelli mock formula:

sqrt[[[1/(((((((4\*1.962364415e+19)/(5\*0.0864055^2))))\*1/(4.46e-24)\* sqrt[[-(((2.751577e+46 \* 4\*Pi\*(6.622438e-51)^3-(6.622438e-51)^2)))) / ((6.67\*10^-11))]]]]]]

**Input interpretation:**

$$\sqrt{\left(1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{4.46 \times 10^{-24}} \sqrt{-\frac{2.751577 \times 10^{46} \times 4 \pi (6.622438 \times 10^{-51})^3 - (6.622438 \times 10^{-51})^2}{6.67 \times 10^{-11}}} \right) \right)}$$

**Result:**

1.618249215993256829425609049987943215739586241434081926654...

1.618249215...

For the same formula, but with the Ramanujan mock theta function **F(q) = 1.897512108...** , we obtain (Ramanujan-Nardelli second mock formula):

$$\sqrt{\left[ \frac{1}{\left( \frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{4.46 \times 10^{-24}}} \right] \sqrt{\left[ -\frac{2.751577 \times 10^{46} \times 4 \pi (6.622438 \times 10^{-51})^3 - (6.622438 \times 10^{-51})^2}{6.67 \times 10^{-11}} \right]}}$$

**Input interpretation:**

$$\sqrt{\left( 1 / \left( \frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{4.46 \times 10^{-24}} \right) \sqrt{-\frac{2.751577 \times 10^{46} \times 4 \pi (6.622438 \times 10^{-51})^3 - (6.622438 \times 10^{-51})^2}{6.67 \times 10^{-11}}} \right)}$$

**Result:**

1.645670777389090680400928762821067485039313445417999994323...

$$1.64567077... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

We have also that:

$$(-5+144)+10^3*\sqrt{\left[ \frac{1}{\left( \frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{4.46 \times 10^{-24}}} \right] \sqrt{\left[ -\frac{2.751577 \times 10^{46} \times 4 \pi (6.622438 \times 10^{-51})^3 - (6.622438 \times 10^{-51})^2}{6.67 \times 10^{-11}} \right]}}$$

**Input interpretation:**

$$(-5 + 144) + 10^3 \sqrt{\left( \frac{1}{\left( \frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{4.46 \times 10^{-24}}} \right) \sqrt{-\frac{2.751577 \times 10^{46} \times 4 \pi (6.622438 \times 10^{-51})^3 - (6.622438 \times 10^{-51})^2}{6.67 \times 10^{-11}}}}$$

**Result:**

1784.67...

1784.67... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

And:

$$-(\sqrt{729})+10^2*\sqrt{\left[ \frac{1}{\left( \frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{4.46 \times 10^{-24}}} \right] \sqrt{\left[ -\frac{2.751577 \times 10^{46} \times 4 \pi (6.622438 \times 10^{-51})^3 - (6.622438 \times 10^{-51})^2}{6.67 \times 10^{-11}} \right]}}$$

**Input interpretation:**

$$-\sqrt{729} + 10^2 \sqrt{\left( 1 / \left( \frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{4.46 \times 10^{-24}} \right) \sqrt{\frac{2.751577 \times 10^{46} \times 4 \pi (6.622438 \times 10^{-51})^3 - (6.622438 \times 10^{-51})^2}{6.67 \times 10^{-11}}} \right)}$$

**Result:**

137.567...

137.567... result very near to the mean of the rest masses of two Pion mesons  
134.9766 and 139.57 that is 137.2733

We note these further interesting mathematical connections:

Entropy = 2.291196e-31

Surface area = 5.511193e-100

Lifetime = 7.458561e-87

Now, we insert these values in the Ramanujan-Nardelli mock formula:

**Entropy**

$$\text{sqrt}[1 / ((((((4 * 1.962364415e+19) / (5 * 0.0864055^2))) * 1 / (4.46e-24)) * \text{sqrt}[-(((2.291196e-31 * 4 * \pi * (6.622438e-51)^3 - (6.622438e-51)^2))]) / ((6.67 * 10^{-11})))]))]]]$$

**Input interpretation:**

$$\sqrt{\left( 1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{4.46 \times 10^{-24}} \right) \sqrt{\frac{2.291196 \times 10^{-31} \times 4 \pi (6.622438 \times 10^{-51})^3 - (6.622438 \times 10^{-51})^2}{6.67 \times 10^{-11}}} \right)}$$

**Result:**

1.617322027323014169471458123727563956969262089739670591863...

1.617322027...

### Surface area

$\sqrt{\left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{4.46 \times 10^{-24}} \right]} \times \sqrt{\frac{-\frac{5.511193}{10^{100}} \times 4 \pi (6.622438 \times 10^{-51})^3 - (6.622438 \times 10^{-51})^2}{6.67 \times 10^{-11}}}$

### Input interpretation:

$$\sqrt{\left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{4.46 \times 10^{-24}}} \right]} \times \sqrt{\frac{-\frac{5.511193}{10^{100}} \times 4 \pi (6.622438 \times 10^{-51})^3 - (6.622438 \times 10^{-51})^2}{6.67 \times 10^{-11}}}$$

### Result:

1.617322027323014169471458123727563956969262089739670591863...

1.617322027.....

### Lifetime

$\sqrt{\left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{4.46 \times 10^{-24}} \right]} \times \sqrt{\frac{-\frac{7.458561 \times 10^{-87}}{10^{100}} \times 4 \pi (6.622438 \times 10^{-51})^3 - (6.622438 \times 10^{-51})^2}{6.67 \times 10^{-11}}}$

### Input interpretation:

$$\sqrt{\left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{4.46 \times 10^{-24}}} \right]} \times \sqrt{\frac{-\frac{7.458561 \times 10^{-87}}{10^{100}} \times 4 \pi (6.622438 \times 10^{-51})^3 - (6.622438 \times 10^{-51})^2}{6.67 \times 10^{-11}}}$$

### Result:

1.617322027323014169471458123727563956969262089739670591863...

1.617322027...

Return to the following previous value:

8.4588897096614e-14 eV

**Input interpretation:**

convert 8.4588897096614 × 10<sup>-14</sup> eV/c<sup>2</sup> to kilograms

**Result:**

1.5079337817599 × 10<sup>-49</sup> kg (kilograms)

1.5079337817599 \* 10<sup>-49</sup> kg

Mass = 1.507934e-49

Radius = 2.239058e-76

Temperature = 8.138310e+71

Surface area = 6.299999e-151

Entropy = 2.619130e-82

Lifetime = 2.882686e-163

From the following fourth Ramanujan-Nardelli mock GENERAL FORMULA, we obtain for these other values:

**Surface area**

sqrt[[[1/(((((((4\*1.962364415e+19)/(5\*0.0864055^2))))\*1/(1.507934e-49)\* sqrt[[-(((6.299999e-151 \* 4\*Pi\*(2.239058e-76)^3-(2.239058e-76)^2)))) / ((6.67\*10^-11))]]]]]]

**Input interpretation:**

$$\sqrt{\left(1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.507934 \times 10^{-49}} \right) \sqrt{-\frac{\frac{6.299999}{10^{151}} \times 4 \pi (2.239058 \times 10^{-76})^3 - (2.239058 \times 10^{-76})^2}{6.67 \times 10^{-11}}}\right)}$$

**Result:**

1.617322087497014110863805772804110302752512432028583797462...

1.61732208...

## Entropy

$\sqrt{\left[ \left[ \left[ \left[ \left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{1.507934 \times 10^{-49}} \right) \times \sqrt{\left[ - \left( \frac{2.619130 \times 10^{-82} \times 4 \pi (2.239058 \times 10^{-76})^3 - (2.239058 \times 10^{-76})^2 \right)}{6.67 \times 10^{-11}} \right]} \right]} \right]} \right]}$

### Input interpretation:

$$\sqrt{\left( 1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.507934 \times 10^{-49}} \right) \times \sqrt{- \frac{2.619130 \times 10^{-82} \times 4 \pi (2.239058 \times 10^{-76})^3 - (2.239058 \times 10^{-76})^2}{6.67 \times 10^{-11}}} \right)}$$

### Result:

1.617322087497014110863805772804110302752512432028583797462...

1.61732208...

## Lifetime

$\sqrt{\left[ \left[ \left[ \left[ \left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{1.507934 \times 10^{-49}} \right) \times \sqrt{\left[ - \left( \frac{2.882686 \times 10^{-163} \times 4 \pi (2.239058 \times 10^{-76})^3 - (2.239058 \times 10^{-76})^2 \right)}{6.67 \times 10^{-11}} \right]} \right]} \right]} \right]}$

### Input interpretation:

$$\sqrt{\left( 1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.507934 \times 10^{-49}} \right) \times \sqrt{- \frac{\frac{2.882686}{10^{163}} \times 4 \pi (2.239058 \times 10^{-76})^3 - (2.239058 \times 10^{-76})^2}{6.67 \times 10^{-11}}} \right)}$$

### Result:

1.617322087497014110863805772804110302752512432028583797462...

1.61732208...



The most significant and interesting result is that inserting indifferently the values of the temperature, the Entropy, the Surface area or Lifetime, the result is always very close to the golden ratio!

## Appendix A

**On the numbers 1.61803398, 1.644934..., 2.71828... and  $1.616252 * 10^{-35}$**

We obtain a good approximation to  $e = 2.71828$  utilizing the following formula:

$$1/2((((1.6183348395+(1.0061571663)^{15} + 1.6183348395+(1.0061571663)^{16}))))$$

**Input interpretation:**

$$\frac{1}{2} (1.6183348395 + 1.0061571663^{15} + 1.6183348395 + 1.0061571663^{16})$$

**Result:**

2.718156654191789497519278090498051508572117994088209051885...

2.7181566...  $\approx e = 2.71828...$

We have obtained this result utilizing the second GENERAL FORMULA (Ramanujan-Nardelli mock formula):

$$\text{sqrt}(((((((1/ (((((((((1.962364415 * 10^{19})/(0.0864055^2)))) * 1/(M)* \text{sqrt}[-((((T * (4*Pi*r)^3-(r^2)))) / ((G))]])))))))))$$

$$\sqrt{\frac{1}{\frac{1.962364415 \times 10^{19}}{0.0864055^2} \times \frac{1}{M} \sqrt{-\frac{T(4\pi r)^3 - r^2}{G}}}}$$

Note that:

$$(1.0061571663)^{15} + 1.6183348395 + (1.0061571663)^{16} =$$

$$= 3.817978468883578995038556180996103017144235988176418103771$$

Indeed, for example, from the previous formula concerning the Mass = 5.298730e+30 kg, we obtain:

$$\sqrt{\left[ \frac{1}{\left( \frac{1.962364415 \times 10^{19}}{0.0864055^2} \right) \times \frac{1}{5.298730 \times 10^{30}}} \right] \sqrt{\left[ - \frac{2.316033 \times 10^{-8} (4\pi \times 7867.827)^3 - 7867.827^2}{6.67 \times 10^{-11}} \right]}}$$

**Input interpretation:**

$$\sqrt{\frac{1}{\frac{1.962364415 \times 10^{19}}{0.0864055^2} \times \frac{1}{5.298730 \times 10^{30}}} \sqrt{-\frac{2.316033 \times 10^{-8} (4\pi \times 7867.827)^3 - 7867.827^2}{6.67 \times 10^{-11}}}}$$

**Result:**

1.618334870724927223008588450274649183809674855381549826645...

1.61833487...

$$\sqrt{\left[ \frac{1}{\left( \frac{1.897512108 \times 10^{19}}{0.0864055^2} \right) \times \frac{1}{5.298730 \times 10^{30}}} \right] \sqrt{\left[ - \frac{2.316033 \times 10^{-8} (4\pi \times 7867.827)^3 - 7867.827^2}{6.67 \times 10^{-11}} \right]}}$$

**Input interpretation:**

$$\sqrt{\frac{1}{\frac{1.897512108 \times 10^{19}}{0.0864055^2} \times \frac{1}{5.298730 \times 10^{30}}} \sqrt{-\frac{2.316033 \times 10^{-8} (4\pi \times 7867.827)^3 - 7867.827^2}{6.67 \times 10^{-11}}}}$$

**Result:**

1.645757883557573235034327624648959640169562276819937582430...

1.64575788...

$$\frac{1}{2} * \left( \left( \left( \left( \left( \left( 3.817978468883578995 + \sqrt{\left[ \frac{1}{\left( \frac{1.962364415 \times 10^{19}}{0.0864055^2} \right) \times \frac{1}{5.298730 \times 10^{30}}} \right] \sqrt{\left[ - \frac{2.316033 \times 10^{-8} (4\pi \times 7867.827)^3 - 7867.827^2}{6.67 \times 10^{-11}} \right]} \right) \right) \right) \right) \right) \right)$$

**Input interpretation:**

$$\frac{1}{2} \left( 3.817978468883578995 + \sqrt{\frac{1}{\frac{1.962364415 \times 10^{19}}{0.0864055^2} \times \frac{1}{5.298730 \times 10^{30}} \sqrt{-\frac{2.316033 \times 10^{-8} (4\pi \times 7867.827)^3 - 7867.827^2}{6.67 \times 10^{-11}}}}}} \right)$$

**Result:**

2.718156669804253109004294225137324591904837427690774913322...

2.718156...  $\approx e$

And, the new mathematical connection (second Ramanujan-Nardelli mock formula) with the Planck's length:

$$(1.898607e-35)/(1.8988931) \text{ sqrt}[\frac{1}{\frac{1.962364415 \times 10^{19}}{0.0864055^2} \times \frac{1}{5.298730 \times 10^{30}} \sqrt{-\frac{2.316033 \times 10^{-8} (4\pi \times 7867.827)^3 - 7867.827^2}{6.67 \times 10^{-11}}}}}]$$

**Input interpretation:**

$$\frac{1.898607 \times 10^{-35}}{1.8988931} \sqrt{\frac{1}{\frac{1.962364415 \times 10^{19}}{0.0864055^2} \times \frac{1}{5.298730 \times 10^{30}} \sqrt{-\frac{2.316033 \times 10^{-8} (4\pi \times 7867.827)^3 - 7867.827^2}{6.67 \times 10^{-11}}}}}}$$

**Result:**

1.61809...  $\times 10^{-35}$

1.61809...  $\times 10^{-35}$  that is a good approximation to the value of Planck length

**Input:**

$e$

**Decimal approximation:**

2.718281828459045235360287471352662497757247093699959574966...

2.71828...

**Property:**

$e$  is a transcendental number

**Continued fraction:**

$$\begin{array}{c}
 2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{6 + \frac{1}{1 + \frac{1}{8 + \frac{1}{1 + \frac{1}{10 + \frac{1}{1 + \frac{1}{12 + \frac{1}{1 + \frac{1}{14 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}} \\
 \end{array}$$

**Alternative representation:**

$$e = e^z \text{ for } z = 1$$

**Series representations:**

$$e = \sum_{k=0}^{\infty} \frac{1}{k!}$$

$$e = \sum_{k=0}^{\infty} \frac{(k-1)^2}{k!}$$

$$e = \sum_{k=0}^{\infty} \frac{2k+1}{(2k)!}$$

$n!$  is the factorial function

For the golden ratio, we have that:

**Input:**

$$\phi$$

$\phi$  is the golden ratio

**Decimal approximation:**

1.618033988749894848204586834365638117720309179805762862135...

1.61803398...

**Alternate forms:**

$$\frac{1}{2}(1+\sqrt{5})$$

$$\frac{1}{2} + \frac{\sqrt{5}}{2}$$

**Continued fraction:**

$$\begin{array}{c}
 1 \\
 \hline
 1 + \frac{1}{\hline} \\
 1 + \frac{1}{1 + \frac{1}{\hline}} \\
 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\hline}}} \\
 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\hline}}}} \\
 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\hline}}}}} \\
 \dots
 \end{array}$$

**Alternative representations:**

$$\phi = 2 \sin(54^\circ)$$

$$\phi = -2 \sin(666^\circ)$$

$$\phi = 2 \cos\left(\frac{\pi}{5}\right)$$

For  $\zeta(2)$ , we have:

**Input:**

$$\zeta(2)$$

$\zeta(s)$  is the Riemann zeta function

**Exact result:**

$$\frac{\pi^2}{6}$$

**Decimal approximation:**

1.644934066848226436472415166646025189218949901206798437735...

1.644934...

**Property:**

$\frac{\pi^2}{6}$  is a transcendental number

**Continued fraction:**

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

**Alternative representations:**

$$\zeta(2) = \zeta(2, 1)$$

$$\zeta(2) = S_{1,1}(1)$$

$$\zeta(2) = -\frac{\text{Li}_2(-1)}{\frac{1}{2}}$$

$\zeta(s, \alpha)$  is the generalized Riemann zeta function

$S_{n,p}(x)$  is the Nielsen generalized polylogarithm function

$\text{Li}_n(x)$  is the polylogarithm function

### Integral representations:

$$\zeta(2) = \frac{8}{3} \left( \int_0^1 \sqrt{1-t^2} dt \right)^2$$

$$\zeta(2) = \frac{2}{3} \left( \int_0^\infty \frac{1}{1+t^2} dt \right)^2$$

$$\zeta(2) = \frac{2}{3} \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2$$

From Wikipedia:

The **Schwarzschild radius** (sometimes historically referred to as the **gravitational radius**) is a physical parameter that shows up in the Schwarzschild solution to [Einstein's field equations](#), corresponding to the [radius](#) defining the [event horizon](#) of a Schwarzschild [black hole](#). It is a characteristic radius associated with every quantity of mass. The *Schwarzschild radius* was named after the [German](#) astronomer [Karl Schwarzschild](#), who calculated this exact solution for the theory of [general relativity](#) in 1916.

The Schwarzschild radius is given as

$$r_s = \frac{2GM}{c^2}$$

where G is the [gravitational constant](#), M is the object mass, and c is the [speed of light](#).

| Object                            | Mass: M                 | Schwarzschild radius:  | Schwarzschild density: |
|-----------------------------------|-------------------------|------------------------|------------------------|
| SMBH in Messier 87 <sup>[9]</sup> | $1.3 \times 10^{40}$ kg | $1.9 \times 10^{13}$ m | 0.44 kg/m <sup>3</sup> |

From:

<http://aesop.phys.utk.edu/ads-cft/L3.pdf>

## Reissner-Nordstrom BlackHoles

### 3.1 The holes



Let us combine gravity with electromagnetism to find a charged black hole. The action is

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + \frac{1}{8\pi G} \int d^3x \sqrt{h} K - \frac{1}{16\pi} \int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu}$$

so there is now a source (electromagnetic field energy corresponds to mass). We have

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$

where (from electromagnetism)

$$T_{\mu\nu} = \frac{1}{4\pi} \left( g^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right)$$

which has trace zero so there is no scale: no mass, no distance, no time, just photons.  $R = 0$ , just as in the Schwarzschild black hole, but we now have the Maxwell equations with no charges,

$$\nabla_{\mu} F^{\mu\nu} = 0$$

The most symmetric solution is the 4-vector with time component  $A_0 = \frac{Q}{r}$ ; however, we want the potential to be zero at the horizon so we set

$$A_0 = \frac{Q}{r} - \frac{Q}{r_+}$$

and

$$f(r) = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}$$

This is the Reissner-Nördstrom black hole. The horizon  $r_+$  ( $0 = f(r_+)$ ) is the solution to

$$r^2 - 2GMr + GQ^2 = 0$$

so

$$r_{\pm} = GM \pm \sqrt{G^2M^2 - GQ^2}$$

( $r_-$  is inside  $r_+$  so we cannot observe it). When  $Q = 0$ , we get  $r_+ = 2GM$  as expected. The minimal  $r_+$  occurs when  $G^2M^2 - GQ^2 = 0$  so there is a minimal mass

$$M_{\min} = \frac{Q^2}{G}$$

Below this mass, we would have a naked singularity, but we stick with dressed singularities. The temperature is given by

$$\begin{aligned} T &= \frac{1}{4\pi} f'(r_+) \\ &= \frac{1}{4\pi} \left( \frac{2GM}{r_+^2} - \frac{2GQ^2}{r_+^3} \right) \\ &= \frac{1}{4\pi} \left( \frac{1}{r_+} - \frac{GQ^2}{r_+^3} \right) \end{aligned}$$

The entropy is

$$S = \frac{A_+}{4G} = \frac{\pi r_+^2}{G}.$$

From

$$T = \frac{1}{4\pi} \left( \frac{1}{r_+} - \frac{GQ^2}{r_+^3} \right)$$

For  $r_+ = r$ , we obtain:

$$\frac{1}{4\pi} \left( \frac{r^2 - GQ^2}{r^3} \right) = T;$$

we raise the fraction  $1/4\pi$  to the cube and insert it as a factor into the expression in parentheses

$$\frac{r^2 - GQ^2}{4\pi r^3} = T; \quad r^2 - GQ^2 = T \times 4\pi r^3;$$

$$-GQ^2 = T \times 4\pi r^3 - r^2; \quad Q^2 = \frac{T \times 4\pi r^3 - r^2}{-G}; \quad Q = \sqrt{\frac{T \times 4\pi r^3 - r^2}{-G}}$$

From the ratio between Q and M, we obtain:

$$\frac{Q}{M} = \frac{1}{M} \sqrt{-\frac{T \times 4\pi r^3 - r^2}{G}}$$

Now, we multiply the result for  $(1.962364415 * 10^{19} / 0.0864055^2)$  and obtain:

$$\frac{1.962364415 \times 10^{19}}{0.0864055^2} \frac{1}{M} \sqrt{-\frac{T \times 4\pi r^3 - r^2}{G}} \quad (1)$$

Finally, we invert the expression, extracting the square root and multiplying by 1.1180931. We obtain the first GENERAL FORMULA (Ramanujan-Nardelli mock formula):

$$1.1180931 \sqrt{\frac{1}{\frac{1.962364415 \times 10^{19}}{0.0864055^2} \times \frac{1}{M} \sqrt{-\frac{T \times 4\pi r^3 - r^2}{G}}}} \cong \phi = \frac{\sqrt{5}+1}{2} = 1.61803398... \quad (2)$$

Or, inserting  $16\pi^2$  inside the square root of (1), i.e.  $16\pi^2 \times 4\pi = 64\pi^3 = (4\pi)^3$ , inverting the expression and extracting the square root, we obtain the second GENERAL FORMULA (Ramanujan-Nardelli mock formula):

$$\sqrt{\frac{1}{\frac{1.962364415 \times 10^{19}}{0.0864055^2} \times \frac{1}{M} \sqrt{-\frac{T \times (4\pi r)^3 - r^2}{G}}}} \cong \phi = \frac{\sqrt{5}+1}{2}$$

$$= 1.61803398 ... \quad (3)$$

With regard the [second GENERAL FORMULA](#), we have that:

**General formula:**

$$\sqrt{\left(\frac{1}{\left(\frac{1.962364415 \times 10^{19}}{0.0864055^2}\right) \times \frac{1}{M} \sqrt{\frac{-T(4\pi r)^3 - r^2}{G}}}\right)}$$

$$\sqrt{\frac{1}{\frac{1.962364415 \times 10^{19}}{0.0864055^2} \times \frac{1}{M} \sqrt{\frac{-T(4\pi r)^3 - r^2}{G}}}}$$

$$1.95053 \times 10^{-11} \sqrt{\frac{M}{\sqrt{\frac{r^2 - 64\pi^3 r^3 T}{G}}}}$$

**Alternate forms assuming G, M, r, and T are positive:**

$$\frac{1.95053 \times 10^{-11} \sqrt[4]{G} \sqrt{M}}{\sqrt[4]{r^2 - 64\pi^3 r^3 T}}$$

$$\frac{1.95053 \times 10^{-11} \sqrt{\frac{M}{r}}}{\sqrt[4]{\frac{1 - 64\pi^3 r T}{G}}}$$

**Real roots:**

$$G < 0, \quad M = 0, \quad r < 0, \quad T < \frac{0.00050393}{r}$$

$$G < 0, \quad M = 0, \quad r > 0, \quad T > \frac{0.00050393}{r}$$

$$G > 0, \quad M = 0, \quad r < 0, \quad T > \frac{0.00050393}{r}$$

$$G > 0, \quad M = 0, \quad r > 0, \quad T < \frac{0.00050393}{r}$$

**Series expansion at  $r = 0$ :**

$$\begin{aligned} & 1.95053 \times 10^{-11} \sqrt{\frac{M}{\sqrt{\frac{r^2}{G}}}} + \\ & 9.67656 \times 10^{-9} r T \sqrt{\frac{M}{\sqrt{\frac{r^2}{G}}}} + 0.0000120014 r^2 T^2 \sqrt{\frac{M}{\sqrt{\frac{r^2}{G}}}} + \\ & 0.0178617 r^3 T^3 \sqrt{\frac{M}{\sqrt{\frac{r^2}{G}}}} + 28.7988 r^4 T^4 \sqrt{\frac{M}{\sqrt{\frac{r^2}{G}}}} + O(r^5) \end{aligned}$$

(generalized Puiseux series)

Big-O notation »

**Series expansion at  $r = \infty$ :**

$$\begin{aligned} & 2.92243 \times 10^{-12} \sqrt{\frac{M}{\sqrt{\frac{-r^3 T}{G}}}} + \frac{3.68175 \times 10^{-16} \sqrt{\frac{M}{\sqrt{\frac{-r^3 T}{G}}}}}{r T} + \\ & \frac{1.15959 \times 10^{-19} \sqrt{\frac{M}{\sqrt{\frac{-r^3 T}{G}}}}}{r^2 T^2} + \frac{4.38265 \times 10^{-23} \sqrt{\frac{M}{\sqrt{\frac{-r^3 T}{G}}}}}{r^3 T^3} + O\left(\left(\frac{1}{r}\right)^4\right) \end{aligned}$$

(generalized Puiseux series)

Big-O notation »

**Derivative:**

$$\frac{\partial}{\partial r} \left( 1.95053 \times 10^{-11} \sqrt{\frac{M}{\sqrt{r^2 - 64 \pi^3 r^3 T / G}}} \right) =$$

$$\frac{(9.75263 \times 10^{-12} - 2.90297 \times 10^{-8} r T) \sqrt{\frac{M}{\sqrt{r^2 - 64 \pi^3 r^3 T / G}}}}{r (1984.4 r T - 1)}$$

**Indefinite integral:**

$$\int \sqrt{\frac{1}{\frac{(1.962364415 \times 10^{19}) \sqrt{-\frac{T(4\pi r^3 - r^2)}{G}}}{0.0864055^2 M}}} dr = 3.90105 \times 10^{-11} r \sqrt[4]{1 - 1984.4 r T}$$

$${}_2F_1(0.25, 0.5; 1.5; 1984.4 r T) \sqrt{\frac{M}{\sqrt{r^2 - 1984.4 r^3 T}}} + \text{constant}$$

${}_2F_1(a, b; c; x)$  is the hypergeometric function

**Third GENERAL FORMULA (Ramanujan-Nardelli mock formula):**

$$\text{sqrt}(\left(\frac{1}{\left(\frac{1.962364415 \times 10^{19}}{0.0864055^2}\right)}\right) * \left(\frac{\pi}{2 \times 1.9632648}\right) * \frac{1}{(M)} * \text{sqrt}\left[\frac{-\left(\left(T * 4\pi(r^3 - r^2)\right)\right)}{\left(6.67 * 10^{-11}\right)}\right]\right)$$

**Input interpretation:**

$$\sqrt{\frac{1}{\frac{1.962364415 \times 10^{19}}{0.0864055^2} \left(\frac{\pi}{2 \times 1.9632648} \times \frac{1}{M}\right) \sqrt{\frac{-T \times 4\pi r^3 - r^2}{6.67 \times 10^{-11}}}}}$$

**Result:**

$$6.23179 \times 10^{-14} \sqrt{\frac{M}{\sqrt{r^2 - 4\pi r^3 T}}}$$

**Alternate form:**

$$6.23179 \times 10^{-14} \sqrt{\frac{M}{\sqrt{-r^2 (4\pi r T - 1)}}}$$

**Alternate forms assuming M, r, and T are positive:**

$$\frac{6.23179 \times 10^{-14} \sqrt{M}}{\sqrt[4]{r^2 - 4\pi r^3 T}}$$

$$\frac{6.23179 \times 10^{-14} \sqrt{\frac{M}{r}}}{\sqrt[4]{1 - 4\pi r T}}$$

**Root:**

$$12.5664 r^2 T - r \neq 0, \quad M = 0$$

**Series expansion at  $r = 0$ :**

$$6.23179 \times 10^{-14} \sqrt{\frac{M}{\sqrt{r^2}}} + 1.95777 \times 10^{-13} r T \sqrt{\frac{M}{\sqrt{r^2}}} +$$

$$1.53763 \times 10^{-12} r^2 T^2 \sqrt{\frac{M}{\sqrt{r^2}}} + 1.44918 \times 10^{-11} r^3 T^3 \sqrt{\frac{M}{\sqrt{r^2}}} +$$

$$1.47964 \times 10^{-10} r^4 T^4 \sqrt{\frac{M}{\sqrt{r^2}}} + O(r^5)$$

(generalized Puiseux series)

Big-O notation »

**Series expansion at  $r = \infty$ :**

$$3.30986 \times 10^{-14} \sqrt{\frac{M}{\sqrt{-r^3 T}}} + \frac{6.58476 \times 10^{-16} \sqrt{\frac{M}{\sqrt{-r^3 T}}}}{r T} +$$

$$\frac{3.27499 \times 10^{-17} \sqrt{\frac{M}{\sqrt{-r^3 T}}}}{r^2 T^2} + \frac{1.95462 \times 10^{-18} \sqrt{\frac{M}{\sqrt{-r^3 T}}}}{r^3 T^3} + O\left(\left(\frac{1}{r}\right)^4\right)$$

(generalized Puiseux series)

Big-O notation »

**Derivative:**

$$\frac{\partial}{\partial r} \left( 6.23179 \times 10^{-14} \sqrt{\frac{M}{\sqrt{r^2 - 4\pi r^3 T}}} \right) = \frac{(3.11589 \times 10^{-14} - 5.87332 \times 10^{-13} r T) \sqrt{\frac{M}{\sqrt{r^2 - 4\pi r^3 T}}}}{r (12.5664 r T - 1)}$$

**Indefinite integral:**

$$\int \sqrt{\frac{1}{\frac{(1.962364415 \times 10^{19})\pi \sqrt{-\frac{T 4\pi r^3 - r^2}{6.67 \times 10^{-11}}}}{0.0864055^2 ((2 \times 1.9632648)M)}}} dr = 1.24636 \times 10^{-13} r \sqrt[4]{1 - 12.5664 r T} {}_2F_1(0.25, 0.5; 1.5; 12.5664 r T) \sqrt{\frac{M}{\sqrt{r^2 - 12.5664 r^3 T}}} + \text{constant}$$

${}_2F_1(a, b; c; x)$  is the hypergeometric function

**Limit:**

$$\lim_{r \rightarrow \pm\infty} 6.23179 \times 10^{-14} \sqrt{\frac{M}{\sqrt{r^2 - 4\pi r^3 T}}} = 0 \approx 0$$

**Fourth GENERAL FORMULA (Ramanujan-Nardelli mock formula)**

(we obtain the formula multiplying for 4 the Mock theta function  $1.962364415 * 10^{19}$  in the numerator and for 5 the Mock theta function  $0.0864055^2$  in the denominator)

$$\text{sqrt}(\frac{1}{\frac{(4 * 1.962364415 * 10^{19})}{(5 * 0.0864055^2)}} * \frac{1}{M} * \text{sqrt}[-\frac{(T * 4 * \text{Pi} * (r)^3 - (r)^2)}{G}]})$$

**Input interpretation:**

$$\sqrt{\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{M} \sqrt{-\frac{T \times 4\pi r^3 - r^2}{G}}}}$$

**Result:**



$$2.18075 \times 10^{-11} \sqrt{\frac{M}{\sqrt{\frac{r^2 - 4\pi r^3 T}{G}}}}$$

**Alternate forms assuming G, M, r, and T are positive:**

$$\frac{2.18075 \times 10^{-11} \sqrt[4]{G} \sqrt{M}}{\sqrt[4]{r^2 - 4\pi r^3 T}}$$

$$\frac{2.18075 \times 10^{-11} \sqrt{\frac{M}{r}}}{\sqrt[4]{\frac{1 - 4\pi r T}{G}}}$$

**Real roots:**

$$G < 0, \quad M = 0, \quad r < 0, \quad T < \frac{0.0795775}{r}$$

$$G < 0, \quad M = 0, \quad r > 0, \quad T > \frac{0.0795775}{r}$$

$$G > 0, \quad M = 0, \quad r < 0, \quad T > \frac{0.0795775}{r}$$

$$G > 0, \quad M = 0, \quad r > 0, \quad T < \frac{0.0795775}{r}$$

**Series expansion at  $r = 0$ :**

$$\begin{aligned} & 2.18075 \times 10^{-11} \sqrt{\frac{M}{\sqrt{\frac{r^2}{G}}}} + 6.85104 \times 10^{-11} r T \sqrt{\frac{M}{\sqrt{\frac{r^2}{G}}}} + \\ & 5.38079 \times 10^{-10} r^2 T^2 \sqrt{\frac{M}{\sqrt{\frac{r^2}{G}}}} + 5.07128 \times 10^{-9} r^3 T^3 \sqrt{\frac{M}{\sqrt{\frac{r^2}{G}}}} + \\ & 5.17786 \times 10^{-8} r^4 T^4 \sqrt{\frac{M}{\sqrt{\frac{r^2}{G}}}} + O(r^5) \end{aligned}$$

(generalized Puiseux series)

Big-O notation »

Series expansion at  $r = \infty$ :

$$\begin{aligned}
 & 1.15825 \times 10^{-11} \sqrt{\frac{M}{\sqrt{-\frac{r^3 T}{G}}}} + \frac{2.30427 \times 10^{-13} \sqrt{\frac{M}{\sqrt{-\frac{r^3 T}{G}}}}}{r T} + \\
 & \frac{1.14605 \times 10^{-14} \sqrt{\frac{M}{\sqrt{-\frac{r^3 T}{G}}}}}{r^2 T^2} + \frac{6.84 \times 10^{-16} \sqrt{\frac{M}{\sqrt{-\frac{r^3 T}{G}}}}}{r^3 T^3} + O\left(\left(\frac{1}{r}\right)^4\right)
 \end{aligned}$$

(generalized Puiseux series)

Big-O notation »

Derivative:

$$\begin{aligned}
 & \frac{\partial}{\partial r} \left( 2.18075 \times 10^{-11} \sqrt{\frac{M}{\sqrt{\frac{r^2 - 4\pi r^3 T}{G}}}} \right) = \\
 & \frac{(1.09038 \times 10^{-11} - 2.05531 \times 10^{-10} r T) \sqrt{\frac{M}{\sqrt{\frac{r^2 - 4\pi r^3 T}{G}}}}}{r (12.5664 r T - 1)}
 \end{aligned}$$

Indefinite integral:

$$\begin{aligned}
 & \int \sqrt{\frac{1}{\frac{(4 \times 1.962364415 \times 10^{19}) \sqrt{-\frac{T 4\pi r^3 - r^2}{G}}}{(5 \times 0.0864055^2) M}}} dr = 4.36151 \times 10^{-11} r \sqrt[4]{1 - 12.5664 r T} \\
 & {}_2F_1(0.25, 0.5; 1.5; 12.5664 r T) \sqrt{\frac{M}{\sqrt{\frac{r^2 - 12.5664 r^3 T}{G}}}} + \text{constant}
 \end{aligned}$$

${}_2F_1(a, b; c; x)$  is the hypergeometric function

Limit:

$$\lim_{r \rightarrow \pm\infty} 2.18075 \times 10^{-11} \sqrt{\frac{M}{\sqrt{\frac{r^2 - 4\pi r^3 T}{G}}}} = 0 \approx 0$$

For the values obtained very near to  $\phi$ , we make some important considerations.

We have the following two mock theta functions:

$$\phi(q) = 0.50970737445... F(q) = 1.369955709...$$

We note that, from the result of an expression 1.61824937, we obtain:

$$1.61824937 * 1.369955709 - (0.50970737445/2) =$$

$$= 1.96207627579215333$$

We have that:

$$\text{Mass} = 7.161e+39$$

$$\text{Radius} = 1.063302e+13$$

$$\text{Temperature} = 1.713732e-17$$

From the previous expression (Ramanujan-Nardelli mock formula), we obtain:

$$\sqrt{\left[ \left[ \left[ \frac{1}{\left( \left( \left( \left( \left( \left( 4 * 1.962364415 * 10^{19} \right) / \left( 5 * 0.0864055^2 \right) \right) * 1 / \left( 7.161 * 10^{39} \right) * \sqrt{\left[ - \left( \left( \left( \left( 1.713732 * 10^{-17} * 4 * \pi * \left( 1.063302 * 10^{13} \right)^3 - \left( 1.063302 * 10^{13} \right)^2 \right) \right) \right) \right) \right) \right) \right) \right) \right) / \left( \left( 6.67 * 10^{-11} \right) \right) \right] \right] \right] \right]$$

$$\sqrt{\left( \frac{1}{\left( \frac{4 * 1.962364415 * 10^{19}}{5 * 0.0864055^2} * \frac{1}{7.161 * 10^{39}} \right)} \sqrt{\frac{1.713732 * 10^{-17} * 4 * \pi * \left( 1.063302 * 10^{13} \right)^3 - \left( 1.063302 * 10^{13} \right)^2}{6.67 * 10^{-11}}} \right)}$$

$$1.618249374900668542367178003243040682994733618897936302450...$$

$$1.6182493749006685423671780032430406829947336188979363$$

We have also that:

$$\sqrt{\left[ \frac{1}{\left( \frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{7.161 \times 10^{39}} \right) \sqrt{\left[ -\frac{1.713732 \times 10^{-17} \times 4\pi (1.063302 \times 10^{13})^3 - (1.063302 \times 10^{13})^2}{6.67 \times 10^{-11}} \right]}} \right]}$$

**Input interpretation:**

$$\sqrt{\left( \frac{1}{\left( \frac{4 \times 1.897512108 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{7.161 \times 10^{39}} \right) \sqrt{\left[ -\frac{1.713732 \times 10^{-17} \times 4\pi (1.063302 \times 10^{13})^3 - (1.063302 \times 10^{13})^2}{6.67 \times 10^{-11}} \right]}} \right)}$$

**Result:**

1.645670938989220735779582455212860786179956182117339551569...  
1.64567...

And:

$$\frac{1.898607e-35}{1.8988931} \sqrt{\left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{7.161 \times 10^{39}} \right) \sqrt{\left[ -\frac{1.713732 \times 10^{-17} \times 4\pi (1.063302 \times 10^{13})^3 - (1.063302 \times 10^{13})^2}{6.67 \times 10^{-11}} \right]}} \right]}$$

**Input interpretation:**

$$\frac{1.898607 \times 10^{-35}}{1.8988931} \sqrt{\left( \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{7.161 \times 10^{39}} \right) \sqrt{\left[ -\frac{1.713732 \times 10^{-17} \times 4\pi (1.063302 \times 10^{13})^3 - (1.063302 \times 10^{13})^2}{6.67 \times 10^{-11}} \right]}} \right)}$$

**Result:**

$1.61801... \times 10^{-35}$   
 $1.61801... \times 10^{-35}$  that is a good approximation to the value of Planck length

(second Ramanujan-Nardelli mock formula)

Further, we obtain:

$$(1.6182493749) * 1.369955709 - (0.50970737445/2)$$

Where 1.369955709 and 0.50970737445 are two Ramanujan mock theta functions

## Input interpretation:

$$1.6182493749 \times 1.369955709 - \frac{0.50970737445}{2}$$

## Result:

1.9620762825049363041

**1.9620762825049363041** value very near to a Ramanujan mock theta function and practically near to the value of DM particle that has a Planck scale mass:  $m \approx 10^{19}$  GeV (Planck mass =  $1,2209 \times 10^{19}$  GeV/c<sup>2</sup> = 21,76 μg [Wikipedia](#))

## Appendix B

From Wikipedia:

In physics, the **Planck mass**, denoted by  $m_{\text{P}}$ , is the unit of mass in the system of natural units known as Planck units. It is approximately 0.02 milligrams. Unlike some other Planck units, such as Planck length, Planck mass is not a fundamental lower or upper bound; instead, Planck mass is a unit of mass defined using only what Max Planck considered fundamental and universal units. One Planck mass is roughly the mass of a flea egg.<sup>[1]</sup> For comparison, this value is of the order of  $10^{15}$  (a quadrillion) times larger than the highest energy available to contemporary particle accelerators.<sup>[2]</sup>

It is defined as:

$$m_{\text{P}} = \sqrt{\frac{\hbar c}{G}},$$

where  $c$  is the speed of light in a vacuum,  $G$  is the gravitational constant, and  $\hbar$  is the reduced Planck constant.

Substituting values for the various components in this definition gives the approximate equivalent value of this unit in terms of other units of mass:

$$1 \text{ } m_{\text{P}} \approx 1.220 \text{ } 910 \times 10^{19} \text{ } \frac{\text{GeV}}{c^2} = 2.176 \text{ } 435(24) \times 10^{-8} \text{ } \text{kg}^{[3]} = 21.764 \text{ } 70 \text{ } \mu\text{g} = 1.3107 \times 10^{19} \text{ } \text{u}^{[4]}$$

For the Planck mass  $m_{\text{P}} = \sqrt{\hbar c/G}$ , the Schwarzschild radius ( $r_{\text{S}} = 2l_{\text{P}}$ ) and the Compton wavelength ( $\lambda_{\text{C}} = 2\pi l_{\text{P}}$ ) are of the same order as the Planck length  $l_{\text{P}} = \sqrt{\hbar G/c^3}$ .

The Planck mass is  $\approx 1.220910 * 10^{19} \text{ GeV}$  , very near to the exponent of result  $1.962 * 10^{19} \text{ GeV}$

We know that:

$$1.64375 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

From the following results of Ramanujan mock theta functions:

$$f(q) = 1.22734321771259\dots \text{ and } -0.0814135$$

we have that:

$$1.644934 * 1.22734321771259 - 0.0814135 = 1,93748508848484151906$$

value very nearly to the average  $1.962 * 10^{19} \text{ GeV}$  **practically near to the value of DM particle that has a Planck scale mass:  $m \approx 10^{19} \text{ GeV}$  (Planck mass =  $1,2209 \times 10^{19} \text{ GeV}/c^2 = 21,76 \mu\text{g}$  [Wikipedia](#))**

We have also:

$$f(q) = 1.22734321771259$$

$$\chi(q) = 1.66162973306\dots$$

$$\text{Partial mock} = 0.07612513678$$

$1,66162973306 \times 1,2273432177 - 0,07612513678 = 1,963264846419852467162$  a value that is very important, as we have seen in the paper

Note that:  $(1.937485 + 0.0814135) / 1.227343217 = 1,644933928 = \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

From:

<https://plus.maths.org/content/ramanujan>

Jf

$$(i) \frac{1 + 53x + 9x^2}{1 - 82x - 82x^2 + x^3} = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$$

or  $\frac{\alpha_0}{x} + \frac{\alpha_1}{x^2} + \frac{\alpha_2}{x^3} + \dots$

$$(ii) \frac{2 - 26x - 12x^2}{1 - 82x - 82x^2 + x^3} = b_0 + b_1x + b_2x^2 + b_3x^3 + \dots$$

or  $\frac{\beta_0}{x} + \frac{\beta_1}{x^2} + \frac{\beta_2}{x^3} + \dots$

$$(iii) \frac{2 + 8x - 10x^2}{1 - 82x - 82x^2 + x^3} = c_0 + c_1x + c_2x^2 + c_3x^3 + \dots$$

or  $\frac{\gamma_0}{x} + \frac{\gamma_1}{x^2} + \frac{\gamma_2}{x^3} + \dots$

then

$$\left. \begin{aligned} a_n^3 + b_n^3 &= c_n^3 + (-1)^n \\ \text{and } d_n^3 + \beta_n^3 &= \gamma_n^3 + (-1)^n \end{aligned} \right\}$$

Examples

$$135^3 + 138^3 = 172^3 - 1$$

$$11161^3 + 11468^3 = 14258^3 + 1$$

$$791^3 + 812^3 = 1010^3 - 1$$

$$9^3 + 10^3 = 12^3 + 1$$

$$6^3 + 8^3 = 9^3 - 1$$

$$9^3 + 10^3 = 12^3 + 1$$

**Input:**

$$9^3 + 10^3 = 12^3 + 1$$

**Result**

**Left hand side:**

$$9^3 + 10^3 = 1729$$

**Right hand side:**

$$12^3 + 1 = 1729$$

1729

$$1010^3 = 791^3 + 812^3 + 1$$

$$1010^3 = 791^3 + 812^3 + 1$$

$$(791^3 + 812^3 + 1)^{1/3}$$

**Input:**

$$\sqrt[3]{791^3 + 812^3 + 1}$$

**Exactresult:**

1010

1010

$$(1010^3 - 812^3 - 1)^{1/3}$$

**Input:**

$$\sqrt[3]{1010^3 - 812^3 - 1}$$

**Exactresult:**

791

791

$$(1010^3 - 791^3 - 1)^{1/3}$$

**Input:**

$$\sqrt[3]{1010^3 - 791^3 - 1}$$

**Exactresult:**

812

812

Wehavethat:

$$135^3 + 138^3 = 172^3 - 1$$



**Input:**

$$135^3 + 138^3 = 172^3 - 1$$

$$(172^3 - 138^3 - 1)^{1/3}$$

**Input:**

$$\sqrt[3]{172^3 - 138^3 - 1}$$

**Exactresult:**

135

135

$$(172^3 - 135^3 - 1)^{1/3}$$

**Input:**

$$\sqrt[3]{172^3 - 135^3 - 1}$$

**Exactresult:**

138

138

Now, we observe that:

$1010 - 55 = 955$ ; result equal to the baryonic Dark Matter mass 955 GeV

$1010 + 8 = 1018$ ; result near to the rest mass of Phi meson 1019.445

$(1010 + 135 + 138) - 55 + 5 = 1233$ ; result equal to the rest mass of Delta baryon 1232

$(1729 + 135 + 138) - 34 = 1968$ ; result practically equal to the rest mass of strange D meson 1968.49

$(791 + 812) + 55 + 13 = 1671$ ; result very near to the rest mass of Omega baryon 1672.45

135 and 138; results very near to the rest masses of two Pion mesons 134.9766 and 139.57

We have also that:

$(1729 + 135 + 138) - 34 - 5 = 1963$ ; result very near to the value mean that comes out from the temperature  $1.729 * 10^{14}$  K regarding the Hawking Radiation (black hole) and that is  $1.96286.. * 10^{19}$  GeV .Furthermore, **1.96286095714** is very nearly to the result of the following Ramanujan mock theta function:  $\chi(q) = 1.962364415...$

$$1.96286 * 10^{19} \text{ GeV} = 1962.9 * 10^{19} \text{ MeV}$$

**Input interpretation:**

$1.9629 \times 10^{19}$  GeV (gigaelectronvolts)

**Unit conversions:**

$1.963 \times 10^{28}$  eV (electronvolts)

3.145 GJ (gigajoules)

$3.145 \times 10^9$  J (joules)

**Input interpretation:**

$1962.9 \times 10^{19}$  MeV (megaelectronvolts)

**Unit conversions:**

$1.963 \times 10^{28}$  eV (electronvolts)

Note that:

$$(1729)^{1/11} \cdot 10^{14} \text{ K}$$

**Input interpretation:**

$\sqrt[11]{1729} \times 10^{14}$  K (kelvins)

**Result:**

$1.969 \times 10^{14}$  K (kelvins)

**Conclusion**

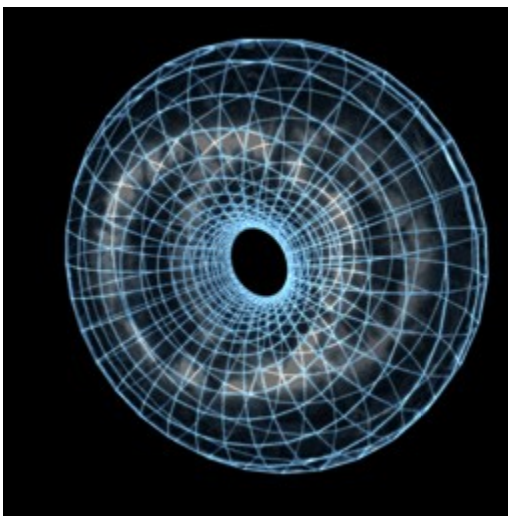
What links the three numbers: 1.61803398, 1.64493 ..., 2.71828 is that they are irrational numbers that can be expressed through infinite continuous fractions. This allows us to deduce that, at least from the mathematical point of view, the results

obtained from the formula derived from the ratio between mass and charge of a particles/quantum black holes (Ramanujan-Nardelli mock formula), are connected to a countably infinite set. It may be the "form" of the string / brane that according to our cosmological vision represents the infinite-dimensional toroidal Hilbert space, in which are included golden dragon curves, whose formula is:

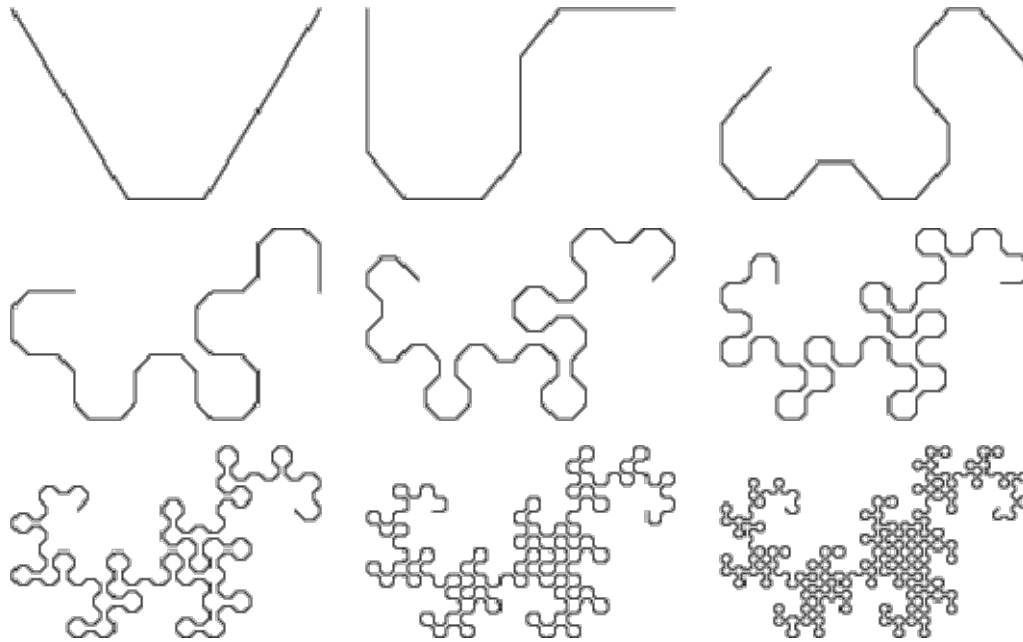
$$\log_{\varphi/\bar{\varphi}}(\varphi) = \varphi \quad 1.61803$$

Built from two similarities of ratios  $r$  and  $r^2$ , with  $r = 1/\varphi^{1/\varphi}$ . Its dimension equals  $\varphi$  because  $(r^2)^\varphi + r^\varphi = 1$ . With  $\varphi = (1 + \sqrt{5})/2$  (Golden number).

[https://en.wikipedia.org/wiki/Four-dimensional\\_space#mediaviewer/File:Clifford-torus.gif](https://en.wikipedia.org/wiki/Four-dimensional_space#mediaviewer/File:Clifford-torus.gif)



## Dragon curve



From:

<http://mathworld.wolfram.com/DragonCurve.html>

From: **S. Ramanujan to G.H. Hardy 12 January 1920 - University of Madras**

I am extremely sorry for not writing you a single letter up to now . . . I discovered very interesting functions recently which I call “Mock”  $\vartheta$ -functions. Unlike the “False”  $\vartheta$ -functions (studied partially by Prof. Rogers in his interesting paper) they enter into mathematics as beautifully as the ordinary  $\vartheta$ -function. I am sending you with this letter some examples . . .

If we consider a  $\vartheta$ -function in the transformed Eulerian form e.g.

$$(A) \quad 1 + \frac{q}{(1-q)^2} + \frac{q^4}{(1-q)^2(1-q^2)^2} + \frac{q^9}{(1-q)^2(1-q^2)^2(1-q^3)^2} + \dots$$

$$(B) \quad 1 + \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^2)} + \frac{q^9}{(1-q)(1-q^2)(1-q^3)} + \dots$$

### Mock $\vartheta$ -functions

$$\begin{aligned}\phi(q) &= 1 + \frac{q}{1+q^2} + \frac{q^4}{(1+q^2)(1+q^4)} + \dots \\ \psi(q) &= \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^9}{(1-q)(1-q^3)(1-q^5)} + \dots \\ \chi(q) &= 1 + \frac{q}{1-q+q^2} + \frac{q^4}{(1-q+q^2)(1-q^2+q^4)} + \dots\end{aligned}$$

These are related to  $f(q)$  as shown below.

$$\begin{aligned}2\phi(-q) - f(q) &= f(q) + 4\psi(-q) \\ &= \frac{1 - 2q + 2q^4 - 2q^9 + \dots}{(1+q)(1+q^2)(1+q^3)\dots} \\ 4\chi(q) - f(q) &= \frac{(1 - 2q^3 + 2q^{12} - \dots)^2}{(1-q)(1-q^2)(1-q^3)\dots}\end{aligned}$$

These are of the 3rd order.

---

Mock  $\vartheta$ -functions (of 5th order)

$$\begin{aligned}
 f(q) &= 1 + \frac{q}{1+q^2} + \frac{q^4}{(1+q)(1+q^2)} + \frac{q^9}{(1+q)(1+q^2)(1+q^3)} + \dots \\
 \phi(q) &= 1 + q(1+q) + q^4(1+q)(1+q^3) + q^9(1+q)(1+q^3)(1+q^5) + \dots \\
 \psi(q) &= q + q^3(1+q) + q^6(1+q)(1+q^2) + q^{10}(1+q)(1+q^2)(1+q^3) + \dots \\
 \chi(q) &= 1 + \frac{q}{1-q^2} + \frac{q^2}{(1-q^3)(1-q^4)} + \frac{q^3}{(1-q^4)(1-q^5)(1-q^6)} + \dots \\
 &= 1 + \left\{ \frac{q}{1-q} + \frac{q^3}{(1-q^2)(1-q^3)} + \frac{q^5}{(1-q^3)(1-q^4)(1-q^5)} + \dots \right\}
 \end{aligned}$$

$$F(q) = 1 + \frac{q^2}{1-q} + \frac{q^8}{(1-q)(1-q^3)} + \dots$$

$$\phi(-q) + \chi(q) = 2F(q).$$

$$f(-q) + 2F(q^2) - 2 = \phi(-q^2) + \psi(-q)$$

$$\begin{aligned}
 &= 2\phi(-q^2) - f(q) = \frac{1 - 2q + 2q^4 - 2q^9 + \dots}{(1-q)(1-q^4)(1-q^6)(1-q^9) \dots} \\
 \psi(q) - F(q^2) + 1 &= q \frac{1 + q^2 + q^6 + q^{12} + \dots}{(1-q^8)(1-q^{12})(1-q^{28}) \dots}
 \end{aligned}$$

Mock  $\vartheta$ -functions (of 5th order)

$$\begin{aligned}
 f(q) &= 1 + \frac{q^2}{1+q} + \frac{q^6}{(1+q)(1+q^2)} + \frac{q^{12}}{(1+q)(1+q^2)(1+q^3)} + \dots \\
 \phi(q) &= q + q^4(1+q) + q^9(1+q)(1+q^3) + \dots \\
 \psi(q) &= 1 + q(1+q) + q^3(1+q)(1+q^2) + q^6(1+q)(1+q^2)(1+q^3) + \dots
 \end{aligned}$$

$$\begin{aligned} \chi(q) &= \frac{1}{1-q} + \frac{q}{(1-q^2)(1-q^3)} + \frac{q^2}{(1-q^3)(1-q^4)(1-q^5)} \\ &\quad + \frac{q^3}{(1-q^4)(1-q^5)(1-q^6)(1-q^7)} + \dots \\ F(q) &= \frac{1}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^{12}}{(1-q)(1-q^3)(1-q^5)} + \dots \end{aligned}$$

have got similar relations as above.

Mock  $\vartheta$ -functions (of 7th order)

$$\begin{aligned} \text{(i)} \quad & 1 + \frac{q}{1-q^2} + \frac{q^4}{(1-q^3)(1-q^4)} + \frac{q^9}{(1-q^4)(1-q^5)(1-q^6)} + \dots \\ \text{(ii)} \quad & \frac{q}{1-q} + \frac{q^4}{(1-q^2)(1-q^3)} + \frac{q^9}{(1-q^3)(1-q^4)(1-q^5)} + \dots \\ \text{(iii)} \quad & \frac{1}{1-q} + \frac{q^2}{(1-q^2)(1-q^3)} + \frac{q^6}{(1-q^3)(1-q^4)(1-q^5)} + \dots \end{aligned}$$

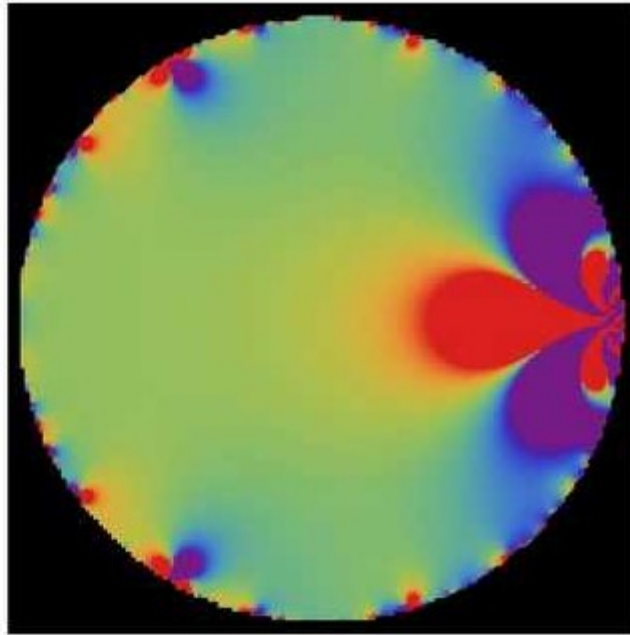
These are not related to each other.

Ever yours sincerely  
S.Ramanujan

## Results of the Ramanujan Mock Theta Functions



Ramanujan's  
Paradise:  
Mock Theta  
Functions



From:

**S. Ramanujan to G.H. Hardy**  
12 January 1920 - University of Madras

(From the original letter of S. Ramanujan)



I am extremely sorry for not writing you a single letter up to now ... I discovered very interesting functions recently which I call "Mock"  $\vartheta$ -functions. Unlike the "False"  $\vartheta$ -functions (studied partially by Prof. Rogers in his interesting paper) they enter into mathematics as beautifully as the ordinary  $\vartheta$ -function. I am sending you with this letter some examples ...

### Note in Italian:

Funzioni di finto theta

Sono estremamente dispiaciuto per non averti scritto una sola lettera fino ad ora. . . Recentemente ho scoperto delle funzioni molto interessanti che chiamo "Finte" funzioni  $\vartheta$ . A differenza delle "False" funzioni  $\vartheta$  (studiate in parte dal Prof. Rogers nel suo interessante documento), esse si inseriscono in matematica magnificamente come la normale funzione  $\vartheta$ . Ti invio con questa lettera alcuni esempi ...

We have the following values concerning the Ramanujan mock theta functions

### MOCK THETA ORDER 7

Mock  $\vartheta$ -functions (of 7th order)

$$\begin{aligned}
 \text{(i)} \quad & 1 + \frac{q}{1-q^2} + \frac{q^4}{(1-q^3)(1-q^4)} + \frac{q^9}{(1-q^4)(1-q^5)(1-q^6)} + \dots \\
 \text{(ii)} \quad & \frac{q}{1-q} + \frac{q^4}{(1-q^2)(1-q^3)} + \frac{q^9}{(1-q^3)(1-q^4)(1-q^5)} + \dots \\
 \text{(iii)} \quad & \frac{1}{1-q} + \frac{q^2}{(1-q^2)(1-q^3)} + \frac{q^6}{(1-q^3)(1-q^4)(1-q^5)} + \dots
 \end{aligned}$$

From the (iii), we have:

$$\begin{aligned} & -0.081849047367565973116419938674252971482398018961922 \\ & 0.0004357345630640457140757853070834281049705616972466 \\ & -1.8762261787851325482986508127679968797519452065 \times 10^{-7} \\ & -0.081849047367565973116419938674252971482398018961922 + \\ & 0.0004357345630640457140757853070834281049705616972466 - \\ & 1.8762261787851325482986508127679968797519452065 \times 10^{-7} \\ & -0.08141350042711980591559898323225082017711543245919605 \end{aligned}$$

The result is:

$$-0.08141350042711980591559898323225082017711543245919605$$

**-0.0814135**

From the (ii), we have:

$$\begin{aligned} & -1.081849047367565973116419938674252971482398018961922 + \\ & 0.0761251367814440464022202749466671971676215118725857 \\ & -0.000433255719961759072744149660169833646052283127278 \end{aligned}$$

Input interpretation:

$$\begin{aligned} & -1.081849047367565973116419938674252971482398018961922 + \\ & 0.0761251367814440464022202749466671971676215118725857 - \\ & 0.000433255719961759072744149660169833646052283127278 \end{aligned}$$

Open code

Result:

$$-1.0061571663060836857869438133877556079608287902166143$$

The result is -1.0061571663...

$$-1.0061571663060836857869438133877556079608287902166143$$

**Partial result 0.07612513678...**

**Final result -1.0061571663...**

The sum of the two mock theta functions (ii) and (iii) is:

$$-0,0814135 - 1,00615716 = -1,08757066$$

And

$$-1 - 0.0814135 = -1.0814135$$

From the (i), we have:

$$0.9239078 + 0.000433255 + (-1.8754140254243246404383299476354805043847163776 \times 10^{-7})$$

Input interpretation:

$$0.9239078 + 0.000433255 - 1.8754140254243246404383299476354805043847163776 \times 10^{-7}$$

[Open code](#)

Enlarge Data Customize A Printout Interactive

Result

$$0.92434086745859745756753595616700523645194956152836224$$

[Open code](#)

The result is

$$0.92434086745859745756753595616700523645194956152836224$$

**0.9243408674589...**

We have also that:

$$0,9243408 - 1,00615716 = -0,08181636; \text{ and}$$

$$-0,08181636 - 1,00615716 = -1,08797352$$

$$-1 - 0.08181636 = -1.08181636$$

MOCK THETA ORDER 3

Mock  $\vartheta$ -functions

$$\begin{aligned}\phi(q) &= 1 + \frac{q}{1+q^2} + \frac{q^4}{(1+q^2)(1+q^4)} + \dots \\ \psi(q) &= \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^9}{(1-q)(1-q^3)(1-q^5)} + \dots \\ \chi(q) &= 1 + \frac{q}{1-q+q^2} + \frac{q^4}{(1-q+q^2)(1-q^2+q^4)} + \dots\end{aligned}$$

MOCK THETA ORDER 3

For  $\phi(q)$   $q = -e^{-t}$ ,  $t = 0.5$   $q^n = -21.79216 * -e^{-0.5}$ , we obtain:

$$\begin{aligned}\phi(q) &= 1 + \frac{q}{1+q^2} + \frac{q^4}{(1+q^2)(1+q^4)} + \dots \\ \psi(q) &= \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^9}{(1-q)(1-q^3)(1-q^5)} + \dots \\ \chi(q) &= 1 + \frac{q}{1-q+q^2} + \frac{q^4}{(1-q+q^2)(1-q^2+q^4)} + \dots\end{aligned}$$

$\phi(q) = 1.075226 + 0.00572374 = \mathbf{1.08094974}$

$\psi(q) = -1.08185 + 1.08232 - 1.08232 = \mathbf{-1.08185}$

$\chi(q) = 1.081345 + 0.00618954 = \mathbf{1.08753454}$

The sum of  $\phi(q) + \psi(q) + \chi(q) = \mathbf{1.08663428}$  very near to the value 1.08643 already calculated from Ramanujan. The mean is:

$\mathbf{1.08344476}$  ( Note that:  $1 - 1.08344476 = \mathbf{0.08344476}$ )

We have also that:

$1,08663428 + 1,0864055 = 2,17303978 \div 2 = \mathbf{1,08651989}$

12 Jan. 1920.

[This letter was written under difficulties and is in places very obscure. Ramanujan however makes it clear that what he means by a "mock  $\mathfrak{J}$ -function" is a function, defined by a  $q$ -series convergent for  $|q| < 1$ , for which we can calculate asymptotic formulæ, when  $q$  tends to a "rational point"  $e^{2\pi i/s}$ , of the same degree of precision as those furnished, for the ordinary  $\mathfrak{J}$ -functions, by the theory of linear transformation. Thus he asserts, for example, that if

$$f(q) = 1 + \frac{q}{(1+q)^2} + \frac{q^4}{(1+q)^2(1+q^2)^2} + \dots$$

and  $q = e^{-t} \rightarrow 1$  by positive values, then

$$f(q) + \sqrt{\left(\frac{\pi}{t}\right)} \exp\left(\frac{\pi^2}{24t^2} - \frac{t}{24}\right) \rightarrow 4.$$

$$1 + \frac{0.449329}{(1+0.449329)^2} + \frac{0.449329^4}{(1+0.449329)^2(1+0.449329^2)^2}$$

**Result:**

1.227343217712591575927923383010083014681378887610525818831...

$$f(q) = 1.22734321771259\dots$$

*Mock  $\mathfrak{J}$ -functions.*

$$\phi(q) = 1 + \frac{q}{1+q^2} + \frac{q^4}{(1+q^2)(1+q^4)} + \dots,$$

$$\psi(q) = \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^9}{(1-q)(1-q^3)(1-q^5)} + \dots$$

$$\chi(q) = 1 + \frac{q}{1-q+q^2} + \frac{q^4}{(1-q+q^2)(1-q^2+q^4)} + \dots,$$

$$\phi(q) = 1.40643658\dots$$

$$\psi(q) = 0.898893179095\dots$$

$$\chi(q) = 1.66162973306\dots$$

$$\phi(q) = 1.40643658... \quad \psi(q) = 0.898893179095.... \quad \chi(q) = 1.66162973306...$$

$$\text{Sum} = 3.966959492155 \quad (R_1)$$

*Mock 9-functions (of 5th order).*

$$\begin{aligned} f(q) &= 1 + \frac{q}{1+q} + \frac{q^4}{(1+q)(1+q^3)} + \dots, \\ \phi(q) &= 1 + q(1+q) + q^4(1+q)(1+q^3) + q^9(1+q)(1+q^3)(1+q^5) + \dots, \\ \psi(q) &= q + q^3(1+q) + q^6(1+q)(1+q^2) + q^{10}(1+q)(1+q^2)(1+q^3) + \dots, \\ \chi(q) &= 1 + \frac{q}{1-q^2} + \frac{q^2}{(1-q^3)(1-q^4)} + \frac{q^3}{(1-q^4)(1-q^6)(1-q^8)} + \dots \\ &= 1 + \frac{q}{1-q} + \frac{q^3}{(1-q^2)(1-q^3)} + \frac{q^5}{(1-q^3)(1-q^4)(1-q^5)} + \dots, \end{aligned}$$

$$f(q) = 1.333425959... \quad \phi(q) = 1.7168646644... \quad \psi(q) = 0.5957823226...$$

$$\chi(q) = 1.962364415...$$

$$\text{Sum} = 5.608437361 \quad (R_2)$$

$$\text{Difference } R_1 \text{ and } R_2 = 5.608437361 - 3.966959492155$$

*Mock 9-functions (of 5th order).*

$$\begin{aligned} f(q) &= 1 + \frac{q^2}{1+q} + \frac{q^6}{(1+q)(1+q^2)} + \frac{q^{12}}{(1+q)(1+q^2)(1+q^3)} + \dots, \\ \phi(q) &= q + q^4(1+q) + q^9(1+q)(1+q^3) + \dots, \\ \psi(q) &= 1 + q(1+q) + q^3(1+q)(1+q^2) + q^6(1+q)(1+q^2)(1+q^3) + \dots, \\ \chi(q) &= \frac{1}{1-q} + \frac{q}{(1-q^2)(1-q^3)} + \frac{q^2}{(1-q^3)(1-q^4)(1-q^6)} + \dots, \\ F(q) &= \frac{1}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^{12}}{(1-q)(1-q^3)(1-q^5)} + \dots \end{aligned}$$

$$f(q) = 1.1424432422... \quad \chi(q) = 2.6709253774829... \quad F(q) = 1.897512108...$$

$$\phi(q) = 0.50970737445\dots \quad \psi(q) = 1.8236681145196\dots$$

$$\text{Sum} = 8.0442562166525 \text{ (R}_3\text{)}$$

$$\text{Sum} = 3.966959492155 \text{ (R}_1\text{)}$$

$$\text{Sum} = 5.608437361 \text{ (R}_2\text{)}$$

$$\text{Sum} = 8.0442562166525 \text{ (R}_3\text{)}$$

*Mock 9-functions (of 7th order).*

$$1 + \frac{q}{1-q^2} + \frac{q^4}{(1-q^3)(1-q^4)} + \frac{q^9}{(1-q^4)(1-q^6)(1-q^6)} + \dots,$$

$$\frac{q}{1-q} + \frac{q^4}{(1-q^2)(1-q^3)} + \frac{q^9}{(1-q^3)(1-q^4)(1-q^6)} + \dots,$$

$$\frac{1}{1-q} + \frac{q^2}{(1-q^2)(1-q^3)} + \frac{q^6}{(1-q^3)(1-q^4)(1-q^6)} + \dots$$

$$1.61052934557\dots \quad 0.8730077\dots \quad 2.103786766\dots$$

$$\text{Sum} = 4.58732381157$$

$$\text{Sum} = 3.966959492155 \text{ (R}_1\text{)}$$

$$\text{Sum} = 5.608437361 \text{ (R}_2\text{)}$$

$$\text{Sum} = 8.0442562166525 \text{ (R}_3\text{)}$$

$$\text{Sum} = 4.58732381157 \text{ (R}_4\text{)}$$

$$\text{Total result} = 22.2069768813775$$

$$F(q) = 1 + \frac{q^2}{1-q} + \frac{q^8}{(1-q)(1-q^3)} + \dots,$$

$$\phi(-q) + \chi(q) = 2F(q),$$

$$2\phi(-q^2) - f(q) = \frac{1 - 2q + 2q^4 - 2q^9 + \dots}{(1-q)(1-q^4)(1-q^6)(1-q^9)\dots},$$

$$\psi(q) - F(q^2) + 1 = q \frac{1 + q^2 + q^6 + q^{12} + \dots}{(1-q^8)(1-q^{12})(1-q^{28})\dots}$$

$$F(q) = 1.369955709\dots$$

$$2F(q) = 2.73991141808516\dots$$

$$2\phi(-q^2) - f(q) = 0.34647193607819\dots$$

$$\psi(q) - F(q^2) + 1 = 0.54471718545239\dots$$

$$1.369955709 + 2.73991141808516 + 0.34647193607819 + 0.54471718545239$$

$$\text{Sum} = 5.00105624861574$$

**Total of the 26 mock theta functions**

$$(3.966959492155 + 5.608437361 + 8.0442562166525 + 4.58732381157 + 5.00105624861574) = 27.20803312999324$$



Now, we have also these other results concerning the Ramanujan mock theta functions:

$$R = -1.08185; \quad R = 1.08753454; \quad R = 1.08094974; \quad \text{SUM} = R_a = 1.08663428$$

$$R_b = -4267.24; \quad R_c = 6.5960861587 * 10^{20}$$

$$R_d = -1.0058343895 * 10^{-12}; \quad R_e = -5.74968 * 10^{-40}; \quad R_f = -4.9290621621 * 10^6;$$

$$R_g = 4.04237000433962 * 10^{14}; \quad R_h = 3.0773505768788923 * 10^{13};$$

$$R_i = -0.0818160338; \quad R_l = -2498.279529; \quad R_m = -0.07609064; \quad R_n = 0.923910279;$$

$$R_o = 33021.1005; \quad R_p = -2122.1867; \quad R_q = 1.63161 * 10^{20}; \quad R_r = 9.39267 * 10^{17};$$

$$R_s = -0.0814135...; \quad R_t = -1.0061571663...; \quad R_u = 0.924340867458.$$

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