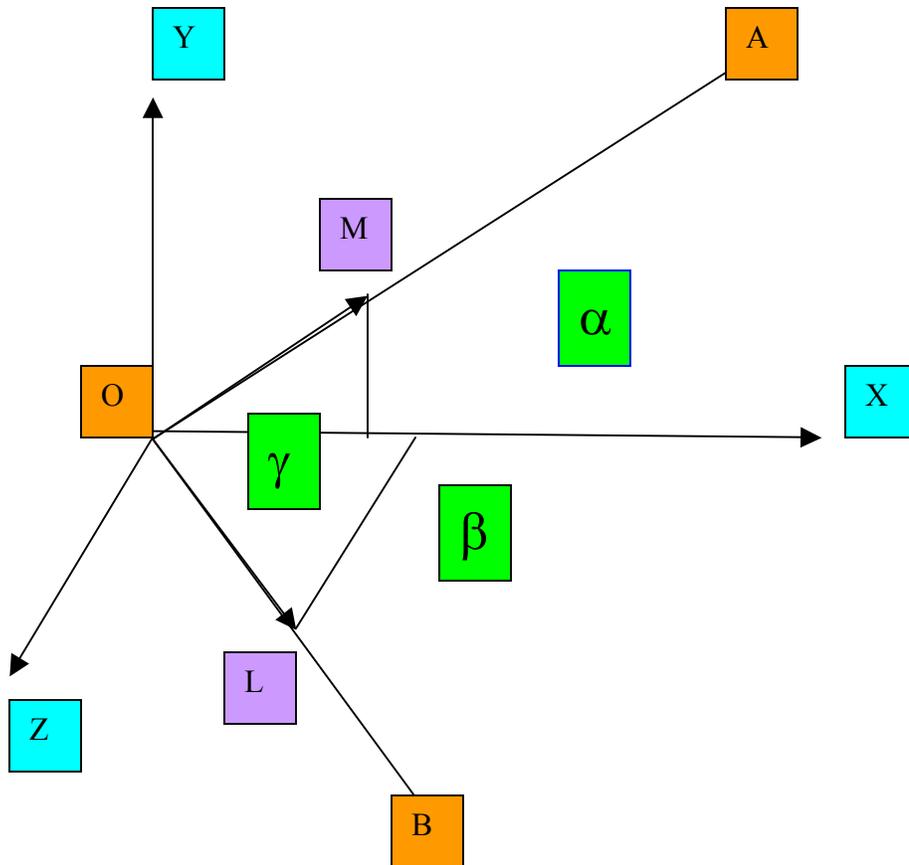


Triple Cosines Theorem

Yuly Shipilevsky

yulysh2000@yahoo.ca

Toronto, Ontario, Canada



In a Cartesian coordinate system for a three-dimensional space of an ordered triplet of axes: \mathbf{OX} , \mathbf{OY} , \mathbf{OZ} that go through the origin \mathbf{O} , let the angle $\mathbf{AOX} = \alpha$, the angle $\mathbf{XOB} = \beta$.

Let us find the angle $\mathbf{AOB} = \gamma$.

Solution. Let \mathbf{OM} be a unit vector in the direction of \mathbf{OA} , let \mathbf{OL} be a unit vector in the direction of \mathbf{OB} .

$$\mathbf{OM} = (\cos \alpha, \sin \alpha, 0), \mathbf{OL} = (\cos \beta, 0, \sin \beta)$$

Since the dot product of vectors \mathbf{OM} and \mathbf{OL} is:

$$\mathbf{OM} * \mathbf{OL} = |\mathbf{OM}| |\mathbf{OL}| \cos \gamma = \cos \gamma,$$

finally: $\cos \gamma = \cos \alpha \cos \beta$.