# Explicit Analysis of Spin-$\frac{1}{2}$ System, Young's Double-slit Experiment and Hanbury Brown-Twiss Effect Using the Non-Dualistic Interpretation of Quantum Mechanics

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The main ideas of the wave-particle non-dualistic interpretation of quantum mechanics are elucidated using two well-known examples, viz., (i) a spin-$\frac{1}{2}$ system in the Stern-Gerlach experiment and (ii) Young's double-slit experiment, representing the cases of observables with discrete and continuous eigenvalues, respectively. It's proved that only Born's rule can arise from quantum formalism as a limiting case of the relative frequency of detection. Finally, non-duality is used to unambiguously explain Hanbury Brown-Twiss effect, at the level of individual quanta, for the two-particle coincidence detection.

## I. INTRODUCTION

According to Prof. Feynman, the central mystery of quantum mechanics is contained in the Young's double-slit experiment which is about the wave-particle duality of a single quantum [1]. Not only light, but all material particles like electrons, protons, atoms, molecules etc., are known to exhibit wave-particle duality [2–7].

There are various interpretations of quantum formalism, like, the mainstream Copenhagen interpretation [8–11], de Broglie-Bohm theory [12, 13], 'many-worlds' interpretation [14–16], spontaneous collapse theories [17, 18], modal interpretation [19], relational interpretations [20], consistent histories [21], transactional interpretation [22–24], QBism [25] etc. Though, each one of them is interesting by itself, but, none of them achieves the derivation of Born's rule as a limiting case of relative frequency of detection (RFD) by making use of the single-quantum events as it's done by wave-particle non-dualistic interpretation of quantum mechanics [26–29].

Hanbury Brown-Twiss (HBT) effect was initially invented to estimate the size of stars by observing an interference pattern in the intensity correlations of incoherent light [30–37].

This effect can be explained by using classical electromagnetic theory [30–33] as well as by the quantum formalism [34, 37]. Later, the same effect is also observed in matter waves [38], ultra-cold quantum gas [39], bosons and fermions [40], interacting photons [41], twisted light [42] etc. There are various applications of the HBT effect ranging from condensed matter and fluid dynamics to nuclear and particle physics [43–46].

The main ideas of the non-dualistic interpretation of quantum mechanics, like the physical meaning of Schrödinger's wave function, derivation of Born's rule as a limiting case of the RFD using individual quantum events, etc., are presented in the Section-II. The overall phase associate with state vector is so far overlooked and unused in the entire literature of quantum mechanics. The non-duality proposes a relation between this phase and the experimental outcome of a particular eigenstate in a deterministic way; this relation is explicitly presented in Section-III for the case of an observable with discrete eigenstates by using an example of a coin tossed in 3D-Euclidean space (3DES), which is later analyzed using a complex vector space (CVS) and finally mapped into a spin-$\frac{1}{2}$ system in the Stern-Gerlach experiment (SG) [1, 47]. The same as in the Section-III is studied in Section-IV but for the case of an observable with continuous eigenstates by using Young's double-slit (YDS) experiment as an example. In the Section-V, an explanation for HBT effect is provided at the level of two individual quanta and also what's weird with the transactional interpretation [22–24] in the same context is discussed. Conclusions and discussions are presented in the Section-VI.

When the article given in the reference [29] was submitted to a research journal for publication, I received the reviewer's report as attached in the Appendix. The report shows how biased and prejudiced the reviewer is, who completely failed to understand the main idea of non-duality. I did not respond to the questions and doubts raised by the reviewer because, I felt it's useless. My reply is also attached in the same appendix.

## II. THE NON-DUALISTIC INTERPRETATION OF QUANTUM MECHANICS

Before presenting the main features of the non-duality, a couple of classical prejudices being entertained in the quantum domain, which are the root cause for believing quantum mechanics (QM) as strange, weird and counter-intuitive, are given below:

1. Treating wave and particle natures to be complementary to each other which is actually the case of classical mechanics (CM), instead of combining them into a single

entity as demanded by the quantum formalism and also as seen in all the quantum experiments. This does not mean that the position and momentum of a quantum are not complementary to each other.

2. Though the absolute space can be felt intuitively as nothingness, its true nature is unavailable to experimental observation independent of the material phenomena happening in it. Visualizing the quantum phenomena in 3DES rather than in the CVS as demanded by the heart of the quantum formalism viz., the canonical commutation relations (CCR). The CCR clearly indicate that the quantum systems live in CVS. As it's well-known, the QM is, in principle, applicable to all physical systems ranging from microscopic to macroscopic ones and hence, obviously it is much more fundamental than the CM. Therefore, it's unavoidable to accept the CVS as fundamental physical space which anyhow yields the 3DES for macroscopic objects - this aspect can be shown within the quantum formalism [27, 29] and implies that the QM is just a CM but, in CVS.

**The Wave-Particle Non-Dualistic Interpretation of the Quantum Formalism**

1. **Instantaneous Resonant Spacial Mode (IRSM)**: The physical reality of Schrödinger's wave function is shown to be an IRSM [27, 29]. For example, consider the eigenvalue equation for a free particle:

$$\hat{H}|\psi >= E|\psi >, \tag{1}$$

where, $\hat{H}$ is the free particle Hamiltonian operator, $|\psi >$ is the eigenstate and $E$, the energy eigenvalue carried by the particle. The moment a particle appears (like a photon from a light source or an electron from a metal surface), its eigenstate $|\psi >$ also appears at the same moment. Obviously, the eigenstate also disappears when the particle gets absorbed. Therefore, the particle (identified by the eigenvalue) and its eigenstate are in resonance with each other and the same can be said to the position representation, $< r|\psi >$, as well; where, $|r >$ is the eigenstate of the position operator, $\hat{r}$, with eigenvalue $r$. In other words, the moment a particle appears, instantaneously its wave function, $< r|\psi >$, appears at all the eigenvalues of $\hat{r}$. The set of all eigenvalues of $\hat{r}$, $\{r\}$, indeed spans the 3DES and therefore, $r$ ($\in \{r\}$) in $< r|\psi >$ need not be identified with the particle coordinate, since the particle is a localized entity. Whenever the particle is observed, it appears at some position eigenvalue, say $r_p \in \{r\}$, with energy eigenvalue, $E$. Therefore, the picture of wave-particle existing within the quantum formalism is that a quantum is confined to its IRSM, but, free to move akin to the case of a test particle in the curved space-time of general relativity [48]. The particle and wave natures always coexist resonantly together as a single physical entity, which is termed as wave-particle non-duality. Note that, throughout the present article, Schrödinger's wave function and IRSM are used synonymously.

2. **Inner-Product as an Interaction**: The intensity of a classical wave is proportional to the square of its amplitude. But, such an intensity can't be claimed for IRSM, because, it is unlike a propagating classical wave. If the particle is going to end up on a detector screen, then a dual vector gets excited in the same screen and interacts according to the inner-product which can be found within the quantum formalism. Let the IRSM, say $|\psi >$, gets scattered into some other state, say $|\psi' >$, at the screen. This process can be described by associating an operator, $\hat{O}_{DS} = |\psi' >< \psi|$, to the detector screen:

$$\hat{O}_{DS}|\psi >=< \psi|\psi > |\psi' > \tag{2}$$

Note that, the dual-vector $< \psi|$ is confined only to the detector screen which is analogues to an image formed in a mirror. It's unlike and having nothing to do with the 'advanced wave' advocated in transactional interpretation [22–24] as it can be easily seen from Eq. (2). Also, it's easy to check that, the unitary time evolution operator until the particle interacts with the screen and after that are different though it itself is continuous in time; its time derivative will be discontinuous at the moment of interaction.

Therefore, if the scattered state is discarded or it's a null-state, then the particle must have interacted or got absorbed at some location in the region of inner-product, $< \psi|\psi >$, respectively. Therefore, the present non-dualistic interpretation will not support the many worlds interpretation [14–16], for that matter any other interpretations [12, 13, 17–25] which directly make use of Born's probability - with an exception of the mainstream Copenhagen interpretation [8–11].

3. **Derivation of the Born's Rule Using Individual Quantum Events**:

(a) **Minimum Phase and Quantum Jump:**

Instead of $\hat{O}_{DS}$, if the IRSM encounters a CVS spanned by discrete orthogonal eigenstates, $|a_i>; i = 1, 2, 3, \cdots$, of an operator, $\hat{A}$:

$$|\psi>= \sum_i |a_i>< a_i|\psi>, \tag{3}$$

then, the particle enters into one of the eigenstate, say $|a_p>$, which makes the minimum phase with $|\psi>$, i.e., phase$\{< a_p|\psi>\} <$ phase$\{< a_i|\psi>\}$, for all $i \neq p$. It's well-known that any physical system tries to be in a minimum energy state. When $|\psi>$ undergoes the decomposition as given in Eq. (3), the particle, which can't split, enters $|a_p>$ as a whole from $|\psi>$. Only this quantum jump requires a minimum energy when compared to all other eigenstates, $|a_i>; i \neq p$. This minimum energy will be either absorbed from the device of observation or emitted out depending upon which process really makes the minimum phase. During the observation, IRSM interacts with its excited dual, $<\psi|$, in the detector as mentioned in Eq. (2):

$$<\psi|\psi>= \sum_i <\psi|a_i>< a_i|\psi> \xrightarrow{\text{observation}} |< a_p|\psi>|^2, \tag{4}$$

and the particle will be naturally found in $|a_p>$ with an eigenvalue $a_p$, because, the remaining orthogonal empty states have nothing to contribute. Note that, since $|\psi>$ obeys the eigenvalue equation, any change in its eigenvalue due to observation will assume a new eigenstate with new eigenvalue. Now, the boundary conditions for this new eigenstate has obviously changed. The initial one being at the point of interaction and final one depends on where the particle will be ending up. In other words, the initial eigenstate, $|\psi>$, disappears and a new eigenstate, $|\psi'>$, appears.

The particle itself contributes a point to the function $|< a_p|\psi>|^2$. Note that, this physical mechanism is indeed in one-to-one correspondence with the 'wave function collapse' advocated in the Copenhagen interpretation [8–10]. In other words, the eigenvalue equation along with the boundary conditions and the inner-product interaction are sufficient to provide the underlying physical mechanism behind 'wave function collapse'. Repeating the detection procedure on several identical particle states, all with different initial phases, yields various eigenvalues, because, different initial phases make the particles to enter into different eigenstates. Note that, in the case of discrete eigenstates, there will be certain range for a given initial phase such that all the phases of particle states lying in that range will be detected in a particular eigenstate, $|a_p>$. By normalizing the number of particles found in $|a_p>$ with respect to the total number of particles yields the relative frequency of detection (RFD). As it can be easily seen from Eq. (4), in the limit of infinite number of particles, the RFD coincides with $|<a_p|\psi>|^2$. Therefore,

$$<\psi|\psi>=\sum_i <\psi|a_i><a_i|\psi>=\sum_i |<a_i|\psi>|^2 = 1, \tag{5}$$

which is the well-known Born's rule. Actually, how this mechanism of minimum phase in Eq. (4) works is explicitly explained in Section-III, where, a spin-$\frac{1}{2}$ system in SG experiment is considered as an example.

(b) **Zero-Phase and No Quantum Jump:** In the case of an observable with continuous eigenvalues, there will be always an eigenstate making a phase with $|\psi>$ which will be exactly the same as the initial phase of $|\psi>$. Instead of $\hat{A}$ in the Eq. (3), consider the position operator, $\hat{\mathbf{r}}$, with orthogonal eigenstates, $|\mathbf{r}>$ and continuous eigenvalues, $\mathbf{r}(=\{x,y,z\})$, as an example:

$$|\psi>=\int d\mathbf{r}|\mathbf{r}><\mathbf{r}|\psi>. \tag{6}$$

The particle naturally enters into a position eigenstate, say $|\mathbf{r_p}><\mathbf{r_p}|\psi>$, without any quantum jump such that its absolute phase is same as that of $|\psi>$, i.e., phase$\{<\mathbf{r_p}|\psi>\}$ = phase$\{|\psi>\}$. Therefore, the interaction of IRSM with its excited dual, $<\psi|$, in an apparatus is,

$$<\psi|\psi>=\int d\mathbf{r}<\psi|\mathbf{r}><\mathbf{r}|\psi>\xrightarrow{\text{observation}}|<\mathbf{r_p}|\psi>|^2, \tag{7}$$

because, except the state $|\mathbf{r_p}><\mathbf{r_p}|\psi>$, the remaining orthogonal ones, $|\mathbf{r}><\mathbf{r}|\psi>$, are empty. Therefore, quantum mechanics is not about probabilities. It can be described at a single quantum level which, anyhow, statistically yields Born's rule and *only* Born's rule for a large number of identical particles.

Therefore, without introducing any additional hidden variables (for example, like in the Refs. [12, 13]), the unavoidable initial phase associated with any state vector along with the inner-product interaction available within the quantum formalism naturally allows one to derive Born's rule and hence, the Copenhagen interpretation is completely contained within the present non-dualistic interpretation. Note that, though a single quantum phenomenon can be deterministically described, the unavailability of the information about the absolute phase of IRSM due to inner-product forces experiments to observe only RFD. These aspects are explained in detail in Section-IV, where the classic Young's double-slit experiment is considered as an example.

4. **Bohr's Complementarity at a Single-Quantum Level:** Suppose that, instead of $\hat{A}$, the same IRSM, $|\psi>$, encounters a different observable, $\hat{B}$, whose CVS is spanned by the eigenstates, say $|b_i>$:

$$|\psi> = \sum_i |b_i><b_i|\psi>, \tag{8}$$

and the particle will be present in some eigenstate, $|b_p>$, which makes a minimum phase with $|\psi>$. The inner-product interaction at the detector is,

$$<\psi|\psi> = \sum_i <\psi|b_i><b_i|\psi> \xrightarrow[\text{at B}]{\text{Detection}} |<b_p|\psi>|^2,$$

yielding the eigenvalue $b_p$ and the particle itself contributes a point to $|<b_p|\psi>|^2$. Therefore, it's the measuring device, either A or B where the inner-product interaction occurs, decides which property, either $a_p$ or $b_p$, of the quantum to be observed. This is actually Bohr's principle of complementarity [49–51], but, at a single-quantum level. However, notice that, *the non-dualistic picture of a particle flying in its own IRSM is further irreducible and is independent of any measuring device.*

## III. SPIN-$\frac{1}{2}$ SYSTEM IN THE STERN-GERLACH EXPERIMENT: AN EXAMPLE FOR THE OBSERVABLE WITH DISCRETE EIGENVALUES

This section is completely devoted to explain the concept of minimum phase, occurrence of RFD and the derivation of Born's rule using a spin-$\frac{1}{2}$ system as an explicit example.

In this regard, a generalized representation for the $SU(2)$ algebra is constructed using the quantum formalism as demanded by the non-dualistic interpretation.

Consider tossing of a coin in 3DES as shown in Fig. (1), which will be later addressed using the CVS and finally mapped into the case of an electron in SG apparatus [1, 47].

**(a). Tossing a Coin in the Euclidean Space**

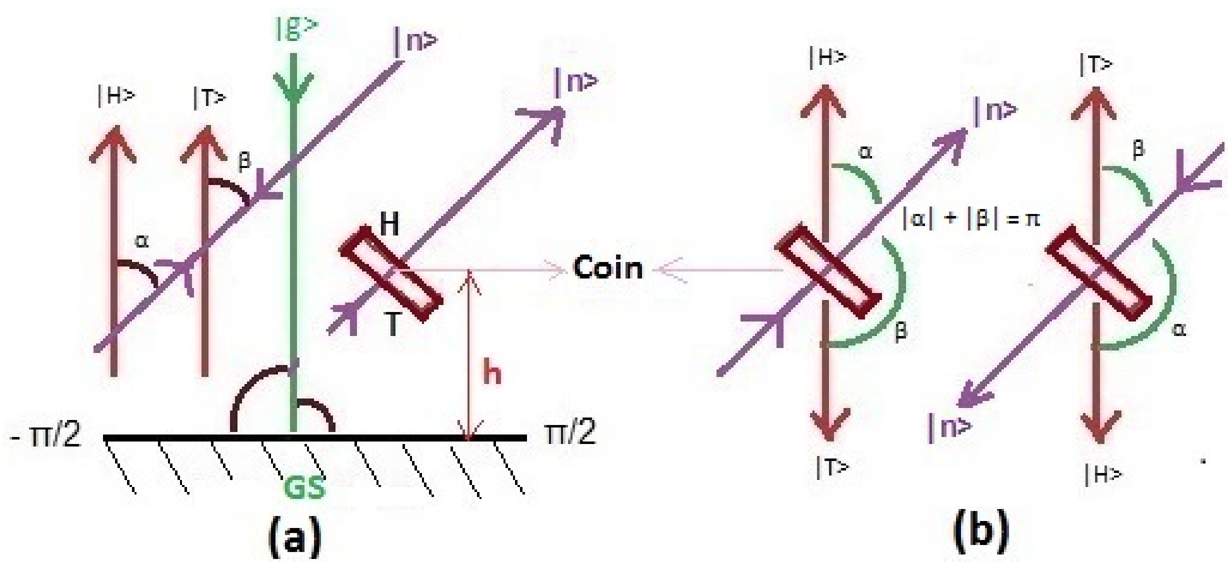


FIG. 1: **Schematic Diagram for the Tossing of a Classical Coin**: (a) $h$ is the height of the coin above the ground surface (GS) and is supposed to be less than the radius of the coin (which is not explicitly shown in the diagram). $|g>$ is a vector parallel to the gravitational field direction and perpendicular to the GS. $|n>$ is a vector normal to the head-surface passing through the center-of-mass of the coin. The outcomes, head and tail, are represented by the state vectors $|H>$ and $|T>$, respectively, which can be taken anti-parallel to $|g>$ but they are mutually exclusive with respect to the observation, i.e., $<T|H>= 0$ (in the space above the GS). (b) $\alpha$ and $\beta$ are the angles between $|H> \& |n>$ and $|T> \& |n>$, respectively; $|\alpha| + |\beta| = \pi$. If $|\alpha| < |\beta|$ ($|\beta| < |\alpha|$), then the coin enters into $|H>$ ($|T>$). The case of $|\alpha| = |\beta|$ is ruled out for an ideal classical coin.

Using the Newtonian mechanics, it's possible, in principle, to predict exactly whether head or tail of the coin will occur on a horizontal flat ground. If there is an ignorance about some parameters involved in the dynamics of the coin, then probability can be invoked for the final outcome - provided a large number of coins were tossed.

Let $|n>$ be a normal vector to the head-surface passing through coin's center-of-mass and $\alpha$ be an angle between $|n>$ and a vector, $|g>$, parallel to the gravitational field. Just before coin lands, consider its position at a height $h \leq r$ above the ground surface; here, $r$ is the radius of the coin. If $-\pi/2 < \alpha < \pi/2$, then head will be the out come. Otherwise, tail occurs for $\pi/2 < \alpha < 3\pi/2$. Depending on the value of $\alpha$, coin will jump into either head or tail state. Upon the outcome, $|n>$ will be pointing either parallel or anti-parallel to $|g>$. Note that, from the moment of toss to a point at $h$ above the ground, $|n>$ itself will be varying from given initial conditions, both in space and time, obeying Newton's equations of motion. The detailed dynamics of $|n>$ is immaterial for the probabilistic description, but only the value of $\alpha$ matters.

### (b). Analyzing the Toss of Coin Using Complex Vector Space

Since the coin system described earlier is aimed to map onto spin-$\frac{1}{2}$ system in SG apparatus, choose eigenvalues $+\frac{1}{2}$ and $-\frac{1}{2}$ for the outcomes of head and tail, respectively. Spin eigenvalues are intrinsic property of an electron but there are no such things for the coin. Still, one can associate two different numbers for the outcomes of head and tail, respectively. Also note that, all the vectors considered in this subsection belongs to a CVS.

Let $|H>$ and $|T>$ be the eigenstates for the head and tail, respectively. Upon the outcome, $|n>$ will be pointing either along $|H>$ or $|T>$ which can also be regarded as anti-parallel vectors to $|g>$. Since, head and tail are mutually exclusive with respect to observation, one needs $<T|H>=0$. Only the vector space above the ground is relevant and it can be taken as a direct sum of $|H>$ and $|T>$ as shown in Fig. (1a). Let $\alpha$ and $\beta$ be the angles made by $|n>$ with $|H>$ and $|T>$, respectively such that $|\alpha| + |\beta| = \pi$.

In any CVS of any dimensionality, one can always write $<a|b>=|<a|b>|.e^{i\theta}$ between any pair of vectors $|a>$ and $|b>$; where, $|<a|b>|$ is the absolute value of the complex number, $<a|b>$, and $\theta$ is the phase angle between the vectors. Hence, one has,

$$<H|n>=|<H|n>|.e^{i\alpha} \; ;<T|n>=|<T|n>|.e^{i\beta} \; ; \; |\alpha| + |\beta| = \pi \; . \qquad (9)$$

Let $\hat{C}$ be an operator associated with the observables of the coin, which can be expressed as,

$$\hat{C} = \frac{1}{2}(|H><H| - |T><T|) \; ; \; \hat{C}|H>=\frac{1}{2}|H> \; ; \; \hat{C}|T>=-\frac{1}{2}|T>, \qquad (10)$$

where, $< H|H >=< T|T >= 1$. By making use of the unit operator, $\hat{I} = |H >< H| + |T >< T|$, the state, $|n >$, when subjected to the observation, can be written as,

$$\begin{aligned} |n > &= |H >< H|n > + |T >< T|n > \\ &= |H > .| < H|n > |.e^{i\alpha} + |T > .| < T|n > |.e^{i\beta}. \end{aligned} \tag{11}$$

According to the criterion of the minimum phase, if $|\alpha| < |\beta|$, then the coin enters into $|H >$ and if $|\alpha| > |\beta|$, then it enters into $|T >$. Note that, either $\alpha$ or $\beta$ will be minimum at a time because $|\alpha| + |\beta| = \pi$. As an explicit example, consider the case $|\alpha| < |\beta|$; then, upon observation,

$$< n|n > \xrightarrow{\text{observation}} | < H|n > |^2 \ ; \left(\text{observation of the eigenvalue } + \frac{1}{2}\right). \tag{12}$$

Consider another tossed coin represented by a state vector $|\tilde{n} >$ which is related to the previous coin as,

$$|\tilde{n} >= e^{i\phi}.|n >, \tag{13}$$

where, $\phi$ is the overall phase by which the second coin differs from the first one. Now, one has,

$$|\tilde{n} >= |H > .| < H|n > |.e^{i(\phi+\alpha)} + |T > .| < T|n > |.e^{i(\phi+\beta)}. \tag{14}$$

Depending upon whether $|(\phi + \alpha)| < |(\phi + \beta)|$ or $|(\phi + \alpha)| > |(\phi + \beta)|$, the coin will enter into either $|H >$ or $|T >$, respectively.

Note in the Eq. (12) that, the absolute length of $|n >$ and hence the value of $| < H|n > |$ is immaterial in the case of single observation except for the eigenvalue. However, for an infinitely large number of tosses, the RFD, $\frac{|<H|n>|^2}{<n|n>}$, must coincide with the probability of occurrence for heads i.e., $\frac{1}{2}$ which fixes $| < H|n > | = \frac{1}{\sqrt{2}}$.

### (c). Spin-$\frac{1}{2}$ System

Consider $SG_x$, $SG_y$ and $SG_z$ apparatuses [1, 47], where magnetic field directions are along X, Y and Z axises, respectively. By taking the gravitational and magnetic field directions to be same and along Z-axis, the states of the tossed coin discussed above can be mapped into that of an electron's spin in $SG_z$ apparatus as follows:

$$|H >\rightarrow |S_z; \uparrow> \ ; |T >\rightarrow |S_z; \downarrow>, \tag{15}$$

$$\hat{C} \rightarrow \hat{S}_z = \frac{1}{2}(|S_z;\uparrow><S_z;\uparrow| - |S_z;\downarrow><S_z;\downarrow|), \tag{16}$$

$$\hat{I} \rightarrow \hat{I}_z = |S_z;\uparrow><S_z;\uparrow| + |S_z;\downarrow><S_z;\downarrow|, \tag{17}$$

where, $\hat{S}_z$ is the Z-component of total spin operator, $\hat{S}$, with eigenstates $|S_z;\uparrow>$ and $|S_z;\downarrow>$ corresponding to spin-up and spin-down states, respectively, and having $\hat{I}_z$ as the unit operator in its CVS. Consider an initial spin state 'up along Y', $|S_y;\uparrow>$, subjected to $SG_z$: By making use of $\hat{I}_z$, one has,

$$|n> \rightarrow |S_y;\uparrow> = |S_z;\uparrow><S_z;\uparrow|S_y;\uparrow> + |S_z;\downarrow><S_z;\downarrow|S_y;\uparrow> . \tag{18}$$

Akin to the case of Eq. (11), the above equation can be written:

$$\begin{aligned} |S_y;\uparrow> &= |S_z;\uparrow> .|<S_z;\uparrow|S_y;\uparrow>|.e^{i\alpha} + |S_z;\downarrow> .|<S_z;\downarrow|S_y;\uparrow>|.e^{\beta} \\ &= |S_z;\uparrow> .R.e^{i\alpha} + |S_z;\downarrow> .R.e^{\beta}, \end{aligned} \tag{19}$$

where, $|<S_z;\uparrow|S_y;\uparrow>| = |<S_z;\downarrow|S_y;\uparrow>| = R$, $<S_z;\uparrow|S_y;\uparrow> = Re^{i\alpha}$ and $<S_z;\downarrow|S_y;\uparrow> = Re^{\beta}$; here, $R$ is a positive real number. Depending on whether $|\alpha| < |\beta|$ or $|\alpha| > |\beta|$, the electron enters into either $|S_z;\uparrow>$ or $|S_z;\downarrow>$, respectively. Let's suppose that $|\alpha| < |\beta|$. Then the electron will enter into the state $|S_z;\uparrow>$ and $|S_z;\downarrow>$ will remain as an ontological empty state. Therefore, upon observation,

$$<S_y;\uparrow|S_y;\uparrow> \xrightarrow{\text{observation}} |<S_z;\uparrow|S_y;\uparrow>|^2 = R^2 \; ; \left(\text{observation of eigenvalue } +\frac{1}{2}\right) \tag{20}$$

because, $|S_z;\downarrow>$ has no electron to contribute.

Consider another spin state prepared 'up along Y' and represented by a state vector $|\tilde{S}_y;\uparrow>$, which differs from the previous one by an overall phase as,

$$|\tilde{S}_y;\uparrow> = e^{i\phi}.|S_y;\uparrow> . \tag{21}$$

The $SG_z$ feels $|\tilde{S}_y;\uparrow>$ as,

$$|\tilde{S}_y;\uparrow> = |S_z;\uparrow> .R.e^{i(\alpha+\phi)} + |S_z;\downarrow> .R.e^{(\beta+\phi)}. \tag{22}$$

Depending on whether $|(\alpha+\phi)| < |(\beta+\phi)|$ or $|(\alpha+\phi)| > |(\beta+\phi)|$, the electron enters into either $|S_z;\uparrow>$ or $|S_z;\downarrow>$, respectively. Therefore, it's sufficient to note that in Eq. (19), the values of $\alpha$ and $\beta$ will be different for different 'up along Y' spin states of electrons.

Quantum formalism avoids the possibility $|\alpha| = |\beta|$, because, in such situations, it's sufficient if the electron stays with the detector for a brief moment. The phases evolving due to time will drive the spin into any one of the eigenstates (further, the actual state vector of the electron has contributions from other eigenstates, like space, apart from the spin, whose overall phases also should be taken into account).

It's needless to show that Born's rule emerges out as a limiting case of RFD akin to the case of the coin described in the previous subsection.

Similar to Eq. (19), let's write

$$|S_y;\downarrow> \; = \; |S_z;\uparrow> .R.e^{i\alpha'} + |S_z;\downarrow> .R.e^{i\beta'}, \tag{23}$$

$$|S_x;\uparrow> \; = \; |S_z;\uparrow> .R.e^{i\gamma} + |S_z;\downarrow> .R.e^{\delta} \tag{24}$$

$$\text{and} \quad |S_x;\downarrow> \; = \; |S_z;\uparrow> .R.e^{i\gamma'} + |S_z;\downarrow> .R.e^{i\delta'}. \tag{25}$$

Now, block the $|S_z;\downarrow>$ in Eq. (19) and subject the $|S_z;\uparrow>$ component to $SG_x$ having the unit operator $\hat{I}_x = |S_x;\uparrow>< S_x;\uparrow| + |S_x;\downarrow>< S_x;\downarrow|$:

$$\begin{aligned} R.e^{i\alpha}.|S_z;\uparrow> \; &= \; R.e^{i\alpha}.|S_x;\uparrow>< S_x;\uparrow|S_z;\uparrow> + R.e^{i\alpha}.|S_x;\downarrow>< S_x;\downarrow|S_z;\uparrow> \\ &= \; R^2.e^{i(\alpha-\gamma)}.|S_x;\uparrow> + R^2.e^{i(\alpha-\delta)}.|S_x;\downarrow> . \end{aligned} \tag{26}$$

Therefore, depending on whether $|(\alpha - \gamma)|$ or $|(\alpha - \delta)|$ is minimum, which in turn depends on $\alpha$, electron will enter into either $|S_x;\uparrow>$ or $|S_x;\downarrow>$, respectively.

According to the requirement of the non-duality to describe a single-quantum behavior, a generalized representation for the $SU(2)$ algebra respecting the Eqs. (19), (23), (24) and (25) is explicitly worked out below:

Writing down the other operators,

$$\begin{aligned} \hat{S}_x \; &= \; \frac{1}{2}(|S_x;\uparrow>< S_x;\uparrow| - |S_x;\downarrow>< S_x;\downarrow|) \\ &= \; \frac{R^2}{2}(C_x|S_z;\uparrow>< S_z;\downarrow| + C_x^*|S_z;\downarrow>< S_z;\uparrow|), \end{aligned} \tag{27}$$

$$\begin{aligned} \hat{S}_y \; &= \; \frac{1}{2}(|S_y;\uparrow>< S_y;\uparrow| - |S_y;\downarrow>< S_y;\downarrow|) \\ &= \; \frac{R^2}{2}(C_y|S_z;\uparrow>< S_z;\downarrow| + C_y^*|S_z;\downarrow>< S_z;\uparrow|), \end{aligned} \tag{28}$$

where, $C_x = e^{i(\gamma-\delta)} - e^{i(\gamma'-\delta')}$ and $C_y = e^{i(\alpha-\beta)} - e^{i(\alpha'-\beta')}$ and $| < S_z;\uparrow |S_x;\uparrow> | = | < S_z;\downarrow |S_x;\uparrow> | = | < S_z;\uparrow |S_x;\downarrow> | = | < S_z;\downarrow |S_x;\downarrow> | = R$. It can be shown that,

$$< S_x;\downarrow |S_x;\uparrow> \ = \ 0 \implies (\gamma - \gamma') - (\delta - \delta') = \pm\pi \tag{29}$$

$$< S_y;\downarrow |S_y;\uparrow> \ = \ 0 \implies (\alpha - \alpha') - (\beta - \beta') = \pm\pi \tag{30}$$

The sign ambiguity in the above equations is related to two possible ways of writing the commutation relations viz., $[\hat{S}_x \ , \ \hat{S}_y] = i\hat{S}_z$ or $[\hat{S}_y \ , \ \hat{S}_x] = i\hat{S}_z$ due to the rotational invariance about Z-axis, which can be fixed using the $SU(2)$ algebra:

$$[\hat{S}_x \ , \ \hat{S}_y] = \frac{R^4}{4}(A_{xy}|S_z;\uparrow>< S_z;\uparrow | + A^*_{xy}|S_z;\downarrow>< S_z;\downarrow |) = i\hat{S}_z, \tag{31}$$

$$[\hat{S}_z \ , \ \hat{S}_x] = \frac{R^2}{2}(C_x|S_z;\uparrow>< S_z;\downarrow | - C^*_x|S_z;\downarrow>< S_z;\uparrow |) = i\hat{S}_y, \tag{32}$$

$$[\hat{S}_y \ , \ \hat{S}_z] = \frac{R^2}{2}(C^*_y|S_z;\uparrow>< S_z;\downarrow | - C_y|S_z;\downarrow>< S_z;\uparrow |) = i\hat{S}_x, \tag{33}$$

where, $A_{xy} = C_x C^*_y - C^*_x C_y$. The above commutation relations yield $C_x = iC_y$ and $\frac{R^4}{4}A_{xy} = \frac{i}{2}$, which result in the following unique relations:

$$(\gamma - \delta) - (\alpha - \beta) = (\gamma' - \delta') - (\alpha' - \beta') = \frac{\pi}{2}, \tag{34}$$

$$\text{and} \qquad (\alpha - \alpha') - (\beta - \beta') = +\pi \ ; \ (\gamma - \gamma') - (\delta - \delta') = -\pi \ ; \ R = \frac{1}{\sqrt{2}}, \tag{35}$$

which are sufficient to satisfy the other aspects of $SU(2)$ algebra, viz.,

$$\{\hat{S}_i \ , \ \hat{S}_j\} = \frac{1}{2}\delta_{ij} \ ; \ \hat{S}^2 = \hat{S}^2_x + \hat{S}^2_y + \hat{S}^2_z = \frac{3}{4}\hat{I} \ ; \ [\hat{S}^2 \ , \ \hat{S}_i] = 0 \tag{36}$$

where, $i$ and $j$ run over $x$, $y$ and $z$ and $\{ \quad , \quad \}$ stands for anti-commutator.

It's straightforward to check the special case by setting $\alpha = \alpha' = \gamma = \gamma' = 0$ in Eqs. (34) and (35) which yields the well-known representation of $SU(2)$ algebra available in any text book of quantum mechanics [47]. This special case will not admit the concept of minimum phase and is good only for the probabilistic description i.e., over a large number of quantum events.

## IV. YOUNG'S DOUBLE-SLIT EXPERIMENT: AN EXAMPLE FOR THE OBSERVABLE WITH CONTINUOUS EIGENVALUES

In this section, YDS experiment (Fig. 2) is considered as an explicit example for the case of an observable with continuous eigenvalues to explain the concept of zero-phase, occurrence of RFD and the derivation of Born's rule.

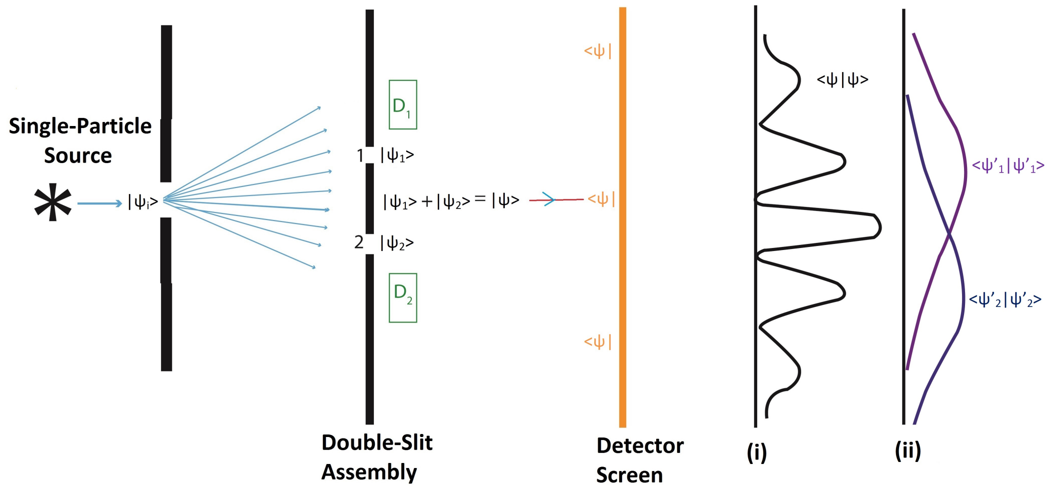


FIG. 2: **Schematic Sketch of the Young's Double-Slit Experiment**: A source shoots single-particles, one at a time, towards a double-slit assembly, where 1 and 2 represent two slits. $|\psi_i>$ is the initial state vector and the state vectors $|\psi_1>$ and $|\psi_2>$ from 1 and 2 get superposed as $|\psi>\equiv|\psi_1>+|\psi_2>$. The particle flying in the superposition gets detected by the screen where the dual $<\psi|$ gets excited and interacts as $<\psi|\psi>$. $D_1$ and $D_2$ are two detectors which can find out through which slit any particle is going through. After collecting a large number of quanta, the resulting particle distribution patterns at the screen, when $D_1$ and $D_2$ are turned off or on, are given in (i) and (ii), respectively. If the particles are detected by $D_1$ ($D_2$), then the resulting particle distribution on the screes is given by $<\psi_1'|\psi_1'>$ ($<\psi_2'|\psi_2'>$).

A source shoots monochromatic single-particles onto the screen through the YDS assembly. Let's suppose that every particle is shot only after the registration of the previous one. If particles are moving in 3DES, then they must leave a classically expected pattern of two strips on the screen, as some of them pass through slit-1 and the others through slit-2. But, according to non-duality, particles actually move in their own IRSMs obeying Schrödinger's wave equation, resulting in an interference pattern. Let $|\psi_1>$ and $|\psi_2>$ be the sate vectors due to the slits 1 and 2, respectively. Then, the superposed state,

$$|\psi >= |\psi_1 > +|\psi_2 >, \tag{37}$$

induces a dual-mode, $< \psi|$, in the screen and interacts as,

$$< \psi|\psi >=< \psi_1|\psi_1 > + < \psi_2|\psi_2 > + < \psi_1|\psi_2 > + < \psi_2|\psi_1 > . \tag{38}$$

As explained in Eq. (7), upon detection,

$$\begin{aligned} < \psi|\psi > &= \int d\mathbf{r} < \psi|\mathbf{r} >< \mathbf{r}|\psi > \xrightarrow{\text{observation}} | < \mathbf{r_p}|\psi > |^2, \\ \Longrightarrow \ | < \mathbf{r_p}|\psi > |^2 &= | < \mathbf{r_p}|\psi_1 > |^2 + | < \mathbf{r_p}|\psi_2 > |^2 \\ &\quad + < \psi_1|\mathbf{r_p} >< \mathbf{r_p}|\psi_2 > + < \psi_2|\mathbf{r_p} >< \mathbf{r_p}|\psi_1 >, \end{aligned} \tag{39}$$

where, $|\mathbf{r_p} >$ is the position eigenstate in which the particle is found at a position eigenvalue, $\mathbf{r_p}$, on the screen. Note that, the above inner-product interaction happens instantaneously the moment a particle appears at source, but, its effect remains unnoticed until the detection of particle.

In this experiment, $< \mathbf{r}|\psi >$ is a free-particle solution of Schrödinger's wave equation,

$$(\nabla^2_{\mathbf{r}} + \mathbf{k}^2) < \mathbf{r}|\psi >= 0, \tag{40}$$

which can be found to be,

$$< \mathbf{r}|\psi >= |A|.e^{i(\epsilon+\mathbf{p}.\mathbf{r}/\hbar)}, \tag{41}$$

where, $|A|$ and $\epsilon$ are constants; $\mathbf{p}$ is the momentum of particle and $\hbar$ is the reduced Plank's constant. According to the criterion of zero-phase presented in Section-II, the particle will be found in $|\mathbf{r_p} >$, if

$$\begin{aligned} \text{phase}\{|\psi >\}|_{t=t_i} &= \text{phase}\{< \mathbf{r_p}|\psi >\}|_{t=t_i}, \\ \text{phase}\{|\psi >\}|_{t=t_f} &= \text{phase}\{< \mathbf{r_p}|\psi >\}|_{t=t_f} = \epsilon + \mathbf{p}.\mathbf{r_p}/\hbar, \end{aligned} \tag{42}$$

where, $t_i$ and $t_f$ are the initial time of appearance and final time of detection of the particle, respectively. It's shown in [27, 29] that phase$\{|\psi >\}|_{t=t_i}$ and phase$\{|\psi >\}|_{t=t_f}$ are related to each other by a phase arising due to the trajectory of the particle and hence, the set of eigenvalues, $\{\mathbf{r_p}(t)\}$, at various times between $t_i$ and $t_f$, lies on a classical trajectory connecting the initial and final locations. The phase$\{|\psi >\}|_{t=t_i}$ randomly changes for particles coming out of the source and hence, phase$\{|\psi >\}|_{t=t_f}$ also changes accordingly.

Now, a couple of possibilities can be seen from the above equation, viz.,
As the phase$\{|\psi>\}|_{t=t_f}$ randomly changes, then

1. $\epsilon$ can change for a fixed $\mathbf{r_p}$ which corresponds to the landing of many particles at same location $\mathbf{r_p}$ on the screen,

2. $\epsilon$ can remain unchanged, then the particles land at various values of $\mathbf{r_p}$ on the screen.

In general, both the above two aspects can occur simultaneously. Therefore, after collecting a large number of detection events, $|<\mathbf{r_p}|\psi>|^2$ in Eq. (39) can be regarded as a smooth function of position eigenvalues, $\mathbf{r_p}$, which will obviously obeys the following Schrödinger equation,

$$(\nabla^2_{\mathbf{r_p}} + \mathbf{k}^2) < \mathbf{r_p}|\psi >= 0 \tag{43}$$

Note that, both the Eqs. (40) and (43) describe the same physical situation, but, according to the non-duality, they have very different physical meaning. In the former case, $<\mathbf{r}|\psi>$ is an IRSM in which the particle flies and depending on its initial phase, the particle will land on a definite location $\mathbf{r_p}$ on the screen. The randomness in the landing of particles at various values of $\mathbf{r_p}$ is due the random phases of their state vectors generated due to the nature of source. In the later case, the initial phases have no role to play and $<\mathbf{r_p}|\psi>$ is regarded as same for all particles which demands an inference that any given particle will be simultaneously present at all locations until observed. Upon observation, the wave function collapses to some position eigenstate randomly. Moreover, this randomness has to be treated as an intrinsic property of the Nature.

Therefore, depending on the initial phase of IRSM, the particle will pass either through slit-1 or slit-2. If its momentum changes either due absorption or scattering at the screen, then the entire IRSM disappears in such way that the particle contributes a point to $<\psi|\psi>$. As already mentioned, the next particle appears at the source along with its IRSM whose absolute phase will be different from the previous one. However, its interaction region, $<\psi|\psi>$, is the same as all previous ones. Since, Born's rule is derived as a limiting case of RFD, the notion of probability for a single quantum's single event is not supported by non-duality. Therefore, Schrödinger's "cat paradox" [53] simply doesn't exist within quantum mechanics according to non-dualistic interpretation, though such a physical situation becomes observable over a large number of 'cat states' [54–65]. A further proof is provided in reference [28], where, the entanglement swapping, both in space and time, are analyzed at individual quantum level which is in perfect agreement with the experimental findings. Since, quantum formalism admits only linear vector spaces, higher order interference effects which can exist in generalized probabilistic theories [66–70], for a single quantum in YDS like experiments, are simply ruled out (see Eq. (2) for scattering case and the same for bound-state will be considered elsewhere) by non-duality.

As mentioned in Section-II, any momentum changing interaction of the particle with probes of detectors $D_1$ and $D_2$ will result in the disappearance of $|\psi >$, which had two origins, one at each slit. A new IRSM, either $|\psi_1' >$ or $|\psi_2' >$, of new momentum appears with a single origin where the interaction took place in the vicinity of the respective slit. Its interaction with the detector screen is either $< \psi_1'|\psi_1' >$ or $< \psi_2'|\psi_2' >$:

$$|\psi > \xrightarrow[\text{detectors, } D_1 \text{ \& } D_2]{\text{Interaction with}} |\psi' >= |\psi_1' > + |\psi_2' > \; ; \; < \psi_1'|\psi_2' >= 0 \tag{44}$$

$$\Longrightarrow < \psi'|\psi' >=< \psi_1'|\psi_1' > + < \psi_2'|\psi_2' > \tag{45}$$

Therefore, in the presence of detectors, clump patters occur and in their absence, the interference comes back. Also, when the source is emitting a large number of particles at a time, then the observed intensities on the screen will be that of several superimposed spherical waves whose origins lie at the regions where the detectors' probes interacted with the particles in the vicinity of the slits. This can be confirmed by observing the HBT effect [30, 31] at the location of the screen.

The existence of single quantum events, as considered by the non-dualistic interpretation, is clearly against the basic idea of 'many-worlds' [14–16] interpretation and there is no *measurement problem* in QM. This does not mean that there is no entanglement between the particle being detected and the measuring instrument. The full fledged explanation of YDS experiment, where the entanglement is taken into care, will be reported elsewhere. But, the present conclusions still hold as they are.

## V. THE HANBURY-BROWN-TWISS EFFECT

The outline sketch of HBT experimental arrangement is given in Fig. (3). Aim of this experiment is to estimate the distance of separation, $d$, between two incoherent particle sources, $a$ and $b$ [30–37]. Two independent particle detectors A and B separated by a distance $D$ register the particle intensities from $a$ and $b$. A coincidence detector, CD, measures the correlation between the intensities from A and B. The existence of an interference pattern in the intensity correlations when measured for various values of $D$ is known as the HBT effect.

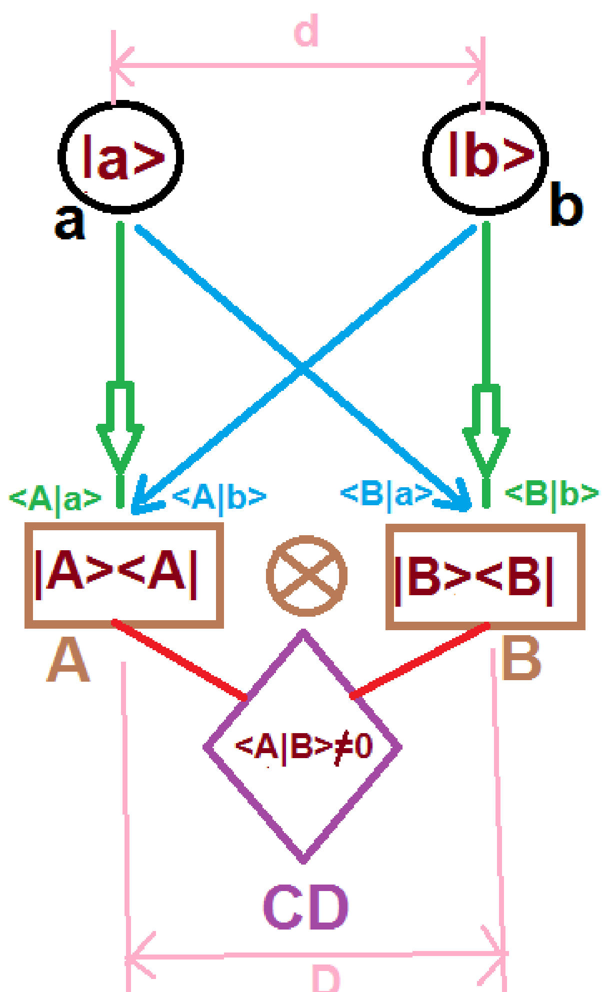


FIG. 3: **Schematic Sketch of the HBT Intensity Interferometer**: $a$ and $b$ are two particle sources separated by a distance $d$ and $|a>$ and $|b>$ are the corresponding state vectors associated with them, respectively. Two particle detctors A and B, separated by a distance $D$, independently register the particle intensity, because, $< A|B >= 0$. CD is a coincidence detector measuring the correlation between intensities from A and B and hence, with respect to the CD, $< A|B >\neq 0$, i.e., CD feels both the vector spaces of A and B as a superimposed vector space.

**(a). Explanation of the HBT Effect Using the Non-Dualistic Interpretation**

Let $\hat{H}_a|a>= E|a>$ and $\hat{H}_b|b>= E|b>$; where $\hat{H}_a$ and $\hat{H}_b$ are the Hamiltonian operators of the particles from $a$ and $b$ with $|a>$ and $|b>$ as their respective eigenstates and $E$ as energy eigenvalue. One needs two particle eigenstate of eigenvalue $2E$ for joint-detection to happen in the CD (notice that, $|a>$ and $|b>$ need not have the same eigenvalues). Such a state obeys the following eigenvalue equation,

$$(\hat{H}_a + \hat{H}_b)|a>|b>= 2E|a>|b>. \quad (46)$$

Let $\hat{P}_A = |A><A|$ and $\hat{P}_B = |B><B|$ be the operators associated with $A$ and $B$. Then CD can be described by an operator,

$$\hat{P}_{CD} \equiv \hat{P}_{AB} = |A><A| \otimes |B><B|. \quad (47)$$

Since, CD is insensitive to a particular particle from particular source and observes only energy, one can write down,

$$\begin{aligned} &|a>|b>= |\phi_+>> + |\phi_->>; \ |b>|a>= |\phi_+>> - |\phi_->>, \\ &\Longrightarrow |\phi_\pm>>= \frac{1}{2}(|a>|b> \pm |b>|a>), \end{aligned} \quad (48)$$

where, $|\phi_+>>$ and $|\phi_->>$ correspond to symmetric and anti-symmetric two-particle states. Since they are mutually orthogonal, both particles must be present either in $|\phi_+>>$ or $|\phi_->>$ for joint detection to happen. Therefore, the states which contribute to CD for the coincidence detection is,

$$|\tilde{\phi}_\pm>>= \hat{P}_{CD}|\phi_\pm>= \frac{1}{2}(<A|a><B|b>|A>|B> \pm <B|a><A|b>|B>|A>), \quad (49)$$

which interacts with its excited dual in CD as,

$$\begin{aligned} <<\tilde{\phi}_\pm|\tilde{\phi}_\pm>> &= \frac{1}{4}(|<A|a>|^2|<B|b>|^2 + |<B|a>|^2|<A|b>|^2).||A||^2.||B||^2 \\ &\pm \frac{1}{4}<a|B><b|A><A|a><B|b>|<A|B>|^2 \\ &\pm \frac{1}{4}<a|A><b|B><A|b><B|a>|<A|B>|^2. \end{aligned} \quad (50)$$

Notice that, if A and B are truly independent, then $<A|B>= 0$. But, with respect to the CD, $<A|B>\neq 0$. The above is the RFD at CD. Note that, each $|a>$ and $|b>$ have influences on both the detectors $A$ and $B$ simultaneously, even though the particle in $|a>$ or $|b>$ is detected by either $A$ or $B$.

**(b). Weirdness in the Transactional Interpretation in Explaining HBT Effect**

According to Copenhagen Interpretation (CI), a quantum entity propagates as a wave but behaves like a particle upon observation. Though no mechanism is provided for the wave collapsing to a particle, but at least, it's what seems to be happening in any given experiment. But, the transactional interpretation (TI) [22–24] demands much bigger mechanism than the wave function collapse demanded by the CI as pointed out below:

According to TI, the detectors $A$ and $B$ receive a half-photon from $a$ and another half-photon from $b$ and assemble them into one whole photon, one at each detector. When a photon from the source $a$ ($b$) divides into two halves, each piece travels different distance to reach $A$ and $B$. Therefore, which half of the which photon from $a$ is assembled with which half of the photon from $b$ is unclear. Further, why and how the detectors are doing to this assembling job just to fool the observers that the photons can only be detected as an integer multiple of a whole photon, albeit they are capable of traveling to various detectors as fractions? Certainly, the physical mechanism behind the process of 'assembling' seems to be much more weirder than the 'collapse of the wave function', because it's easy to explain the later by using quantum formalism as shown by the non-dualistic interpretation.

The TI contains and majorly depends on the so called 'offer wave' (OW), a retarded wave emitted by an emitter and 'confirmation wave' (CW), an advanced wave emitted in response to OW by an absorber. It's relativistically invariant, but, what it has to say about the propagation speeds of the OW and the CW is unclear, because, the process of 'transaction' is shown to be atemporal, even when the emitter and absorber are separated by an astronomical distance! - implying that the moment emitter emits OW, the CW is already received at the same moment from the absorber which will come into existence only after billions of years! Though TI is supposed to be 'time symmetric' formulation, it treats emitter and absorber asymmetrically with respect to the arrow of time; otherwise, causality can not be addressed. Therefore, the causality is just put in by hand into the TI. In a time symmetric formulation, the absorber can also produce OW and the emitter, a CW - resulting in a 'transaction' in such a way that the absorber first absorbs the photon which will be emitted by the emitter in the future of billions of years. It's not at all a simple picture when compared to Copenhagen interpretation and in particular, to non-dualistic interpretation.

Therefore, the TI itself is a big paradox. So, there is no wonder that it paradoxically explains all quantum paradoxes. Irrespective of its explanation, since it is constructed entirely based on reproducing Born's rule, it will somehow come in contact with experiments. Without accepting Born's probabilistic interpretation, TI has no meaning. But, Born's rule is a consequence of the quantum formalism according to the non-dualistic interpretation. Further, the non-duality decisively proves that only Born's rule but none other than Born's rule is possible within the quantum formalism, provided, the measurements are made on a large number of identical systems.

## VI. CONCLUSIONS AND DISCUSSIONS

By introducing the concept of minimum phase for the observables with discrete eigenvalues (zero-phase in the case of continuous eigenvalues), the pre-determinism in the experimental outcome of a particular eigenstate is shown to be related to the overall absolute phase of the state vector. The absolute phase itself is not experimentally observable due to the inner-product interaction and is like a hidden variable available within the quantum formalism. This forces experiments to observe only the relative frequency of detection which in turn, in the limit of infinite number of events, results in Born's rule. The physical reality of Schrödinger's wave function is shown to be an instantaneous resonant spatial mode in which a quantum flies akin to the case of a test particle in general theory of relativity. This notion, except for the concept of probability, is consistent with the Born's Probabilistic Interpretation [8]: "*The wave function determines only the probability that a particle - which brings with itself energy and momentum - takes a path; but no energy and no momentum pertains to the wave*". Finally, the *measurement problem* doesn't exist in the non-dualistic interpretation due to the principle of minimum phase and the inner-product interaction. There is no distinction between microscopic and macroscopic physical systems, because, the non-duality treats all of them on equal footing by recognizing them as represented by suitable complex vector spaces of the observables prescribed in the quantum formalism. As it can be shown by using quantum formalism, the complex vector space 'effectively' appears to be 3D-Euclidean for macroscopic objects.

Each initial state prepared in an experiment, like spin-$\frac{1}{2}$ particles 'up along Y' in Stern-Gerlach apparatus with magnetic field along Z-axis or the monochromatic particle states in Young's double-slit experiment, will essentially differ from any other identical state by an overall phase which will never contribute to the inner-product interaction and hence to Born's rule but responsible for the different experimental outcomes. In other words, all initial conditions are different which result in different outcomes though they are all confined to Born's rule. This situation is in perfect agreement with a philosophical saying, "*It is necessary for the very existence of science that the same conditions always produce the same result*". '...outside the domain of science' - mentioned in one of the profound statements by Prof. Dirac [52]: "*Questions about what decides whether the photon is to go through or not and how it changes its direction of polarization when it does go through can not be investigated by experiment and should be regarded as outside the domain of science*", is brought inside the domain of science by non-duality. This was shown explicitly for spin-$\frac{1}{2}$ system and the will go through for photon's polarization as well. Towards the end, the Hanbury Brown-Twiss effect was unambiguously explained using non-duality. Also, how the transactional interpretation provides a weird explanation for the same effect was pointed out.

In fact, the better name for the non-dualistic interpretation may be *the quantum formalism as it is* interpretation.

With the tools of probabilities and uncertainties, we will be never able to dig deep into the fundamental secretes of Nature.

## ACKNOWLEDGMENT

I sincerely thank the anonymous reviewer for rejecting the article [29] for publication. It's only because of this rejection, I was motivated to write the present article.

## APPENDIX

**Review Report:**

*The author claims to have provided 'the only physical account of the Born Rule,' but what he gives is basically the same as the transactional interpretation (TI). In particular, the dual states entering in an ad hoc fashion here are already part of TI (where they are physically motivated advanced states arising in the direct-action theory).*

*The author is to be commended for his realist approach to the quantum level. But besides the fact that his ad hoc dual states are already part of TI, his notion of 'minimum phase' as* *the criterion for measurement outcomes, if I understand what he is proposing, does not work. For example, an initial spin state of 'up along y' could be subject to a z-axis measurement, in which case the amplitude for 'up along z' is* $1/\sqrt{2}$ *and for 'down along ' it is* $i/\sqrt{2}$*. According to this criterion, the result 'up along z' has the minimum phase, so it should always occur, which violates the Born Rule. In other cases with equal amplitudes for the outcomes, e.g. spin 'up along x' subjected to a z-spin measurement, there is no 'minimum phase'. Another (less serious) problem is the reliance on non-relativistic quantum theory, in particular wave functions, as fundamental. There is no position operator at the relativistic level, so no well-defined position eigenstates* $|x>$*. Nature is relativistic; so arguably, wave functions* $< x|\psi >$ *are not ontologically fundamental.*

**Reviewer**: "*The author claims to have provided 'the only physical account of the Born Rule,' but what he gives is basically the same as the transactional interpretation (TI)*".

**Author**: In my paper, I clearly stated, 'Although, each one of them is interesting by itself, none of them gives a derivation for the Born rule using the single-quantum events as it will be shown in the present article' (here, 'them' standing for various interpretations)-As it's very clear that the derivation given as in the present article, i.e., showing Born's rule as a limiting case of the relative frequency of detection, is not shown in any of the earlier interpretations. Anybody, who is familiar with the TI will know that my way of derivation is completely different from that of TI; in the presentation of entire TI, there are no such words like 'relative frequency of detection'. TI postulates retarded and advanced waves only to cook up the well-known Born's rule. Can it be considered as a derivation at all?

**Reviewer**: "*In particular, the dual states entering in an ad hoc fashion here are already part of TI (where they are physically motivated advanced states arising in the direct-action theory)*".

**Author**: The above statement of the reviewer is really too much. I am deriving the inner-product by introducing the dual vector as a consequence of quantum formalism for the case of a scattered particle at detector screen. The dual vector introduced by me is unlike the advanced wave of TI propagating in the space-time. It just gets excited in the detector screen and completely confined only to the screen. It's analogous to an image in a mirror. (The advanced states them self are unphysical; there seems to be physical motivation for this unphysical states! Mirror images are just images. They should not be mistaken to be existing in the physical space though they are caused by the objects living in the physical space).

**Reviewer**: "*The author is to be commended for his realist approach to the quantum level. But besides the fact that his ad hoc dual states are already part of TI, his notion of 'minimum phase' as the criterion for measurement outcomes, if I understand what he is proposing, does not work*".

**Author**: As I already described above, the dual vector is absolutely not a part of TI. Also I am 100% sure that the reviewer was unable to see why the natural notion of 'minimum phase' works because the report came within a day. Moreover, such a biased person towards a particular interpretation will naturally overlook the whole point in other interpretations. I even wonder whether TI makes any common sense at all (see Section-V (b), for an account of new kind of weirdness discovered by TI, though the QM itself is not weird.). See Sections III and IV where, it's shown by using explicit calculations for the well-known quantum systems, spin-$\frac{1}{2}$ system in Stern-Gerlach apparatus and Yong's double-slit experiment, that how the notion of 'minimum phase' actually works.

**Reviewer**: "*For example, an initial spin state of 'up along y' could be subject to a z-axis measurement, in which case the amplitude for 'up along z' is $1/\sqrt{2}$ and for 'down along ' it is $i/\sqrt{2}$. According to this criterion, the result 'up along z' has the minimum phase, so it should always occur, which violates the Born Rule. In other cases with equal amplitudes for the outcomes, e.g. spin 'up along x' subjected to a z-spin measurement, there is no 'minimum phase'.*"

**Author**: The reviewer failed to understand the fact that in quantum mechanics, all the initial state vectors of identical particles differ, in general, by overall phases, which are experimentally unobservable due to the nature of Born's rule. The reviewer is using a special case of a more general representation allowed by the quantum formalism to refute the concept of 'minimum phase'. The reviewer is reminded of the fact that, his/her special case is suitable only for the description of a quantum system probabilistically. It's of no use to say anything about a single quantum's single event where, probability is of no use. What's wrong with the reviewer's understanding in the above argument is explained in detail in Section-III and particularly in the subsection III-(c).

**Reviewer**: "*Another (less serious) problem is the reliance on non-relativistic quantum theory, in particular wave functions, as fundamental. There is no position operator at the relativistic level, so no well-defined position eigenstates $|x>$. Nature is relativistic; so* *arguably, wave functions $< x|\psi >$ are not ontologically fundamental*".

**Author**: In my paper, I clearly mentioned, "In the present article, only the time-independent non-relativistic quantum mechanics is considered, because, its interpretation naturally goes through time-dependent and relativistic cases". Many interpretation like Bohmian mechanics were initially at the non-relativistic level only. Moreover, my claim is that the same ideas can be carried over to the relativistic cases. Of course, when doing that, one has to look at the relativistic formalism to interpret in the same way as done for non-relativistic case. In the relativistic case, it doesn't matter whether the position eigenstates $|x>$ is well defined or not, because my interpretation crucially depends on recognizing the CVS of well-defined Hermitian operators as the fundamental.

**Reviewer**: "*Nature is relativistic; so arguably, wave functions $< x|\psi >$ are not ontologically fundamental*".

**Author**: By saying like the above, it's unfair to keep away the unbelievable success of Schrödinger's equation. Further, this statement - $< x|\psi >$ are not ontologically fundamental - actually belongs to Copenhagen interpretation. I am surprised to hear this from a TI person because TI is basically proposed against Copenhagen interpretation! Anyhow, whether the wave function is ontologically fundamental or not is the reviewer's personal opinion, but need not be a fact of Nature. But, according to me, since the observed physical effects, like the interference pattern in Young's double-slit experiment, are due to the wave function, then it must be an ontological entity. Further, how relativity disqualifies the ontological nature of Schrödinger's wave function is unclear.

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