Deriving the electromagnetic radiation:(1) photon, (2) anti-photon, (3) unsuccessful radiation through the mutual energy principle and self-energy principle

Shuang-ren Zhao

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Abstract

The solutions of Maxwell equations includes the restarted and the advanced waves. There are a few famous scientists supported the concept of advanced wave. Wheeler and Feynman have introduced the absorber theory in1945 which told us the absorbers can send the advanced wave. The absorber theory is based on the action-at-a-distance of Schwarzschild, Tetrode and Fokker, which told us the electric current sends a half retarded wave and a half advanced wave. John Cramer has introduced transactional interpretation in quantum mechanics, which said the retarded wave and the advanced wave have a handshake. What is the advanced wave in electromagnetic field theory? In 1960, Welch introduced the reciprocity theorem in arbitrary time domain which involved the advanced wave. In 1963 V.H. Rumsey mentioned a method to transform the Lorentz reciprocity theorem to a new formula. In early of 1987 Shuang-ren Zhao (this author) introduced the mutual energy theorem in frequency domain. In the end of 1987 de Hoop introduced the time-domain cross-correlation reciprocity theorem. All these theories can be seen a same theorem in different domain: Fourier domain or in time domain. This theorem can be applied to prove the directivity diagram of a receiving antenna are equal to the directivity diagram when the receiving antenna are used as a transmitting antenna. According to this theory, the receiving antenna sends the advanced wave. As a reciprocity theorem the two fields in it do not need to be real for all. Hence, for the reciprocity theorem of Welch, Rumsey and Hoop do not need to claim that the advanced wave is a physical wave. However, when Shuang-ren Zhao said it is a energy theorem that means the two waves the retarded wave and the advanced wave in the theorem must be all real physical waves. After the mutual energy theorems has been published 30 years, Shuang-ren Zhao re-enter this field. First, the mutual energy flow theorem is derived. The mutual energy flow produced by the superposition of the retarded wave and the advanced wave. The mutual energy flow can carry the energy from

the transmitting antenna to the receiving antenna. Our textbook of electromagnetic field tell us the energy is carried by the energy flow of the Poynting vector which is the self-energy flow. Hence, there is a question that the energy of electromagnetic field is transferred by the mutual energy or self-energy or by both? This author found that only the former can offer a self-consistency theory. This author also proved that the energy is transferred by the self-energy or both all conflict with the energy conservation law and hence cannot be accept. If the energy is transferred by the mutual energy, the axioms of the electromagnetic field needs to be modified. Hence, the mutual energy principle is introduced to replace the Maxwell equations as axioms. The mutual energy principle can be derived from the Maxwell equations. The Maxwell equations can also be derived from the mutual energy principle. However, the mutual energy principle does not equivalent to the Maxwell's equations. Starting from the mutual energy principle, the solution needs to be two groups of Maxwell equations existing together. One group of the Maxwell equation is corresponding to the retarded wave, another is corresponding to the advanced wave. The two waves must be synchronized to produce the mutual energy flow. The conflict of Maxwell equations with the energy conservation law further suggest that there exist a time-reversal wave and self-energy principle. Self energy principle tells us that self-energy flow or the energy flow corresponding to Poynting vector does not carry or transfer the energy, because there exist 2 time-reversal waves corresponding to the retarded wave and the advanced wave. The energy flow of the time-reversal waves cancels the energy flows of the self-energy flows of the retarded wave and the advanced wave. This also tell us there are 4 waves for electromagnetic fields, the retarded wave, the advanced wave, the 2 time-reversal waves corresponding to the retarded wave and the advanced wave. The self-energy flow of these 4 waves are all canceled. However the mutual energy flow of the retarded wave and the advanced wave does not disappear. The energy of electromagnetic field is transferred by the mutual energy flow. Photons can be explained by the mutual energy flows. There is also the time-reversal mutual energy flow which can wipe out the half-photon or partial photon. Anti-particle can also be explained by the time-reversal mutual energy flow. This theory has been widen to the quantum mechanics. That means all particles for example electron is also consist of 4 waves and 6 energy flows. There is the mutual energy principle and self-energy principle corresponding to the Schrödinger equation. In this article 3 modes of radiation are introduced which are photon, anti-photon, ant unsuccessful radiation. Photon is consist of one mutual energy flow, self-energy flow for the retarded wave and advanced wave, self-energy flow of the time-reversal waves. All self-energy flows are canceled. Hence only the mutual energy flow survive. Anti-photon is consist of the time-reversal mutual energy flow; Time-reversal self-energy flow and self-energy flows. All the self-energy flow canceled. Only the time-reversal mutual energy survived. The last mode is the unsuccessful radiation. The retarded wave is sent out but it did not meat any advanced wave to handshake/synchronized. Hence, the energy is returned, the radiation is unsuccessful. Photon is transfer the radiation energy. Anti-photon is responsible to eliminate the half-photon or partial photon. The unsuccessful radiation is the necessary

result that the source and sink both send the waves, one is the retarded wave and the other is the advanced wave.

Keywords: Maxwell; Poynting; Schrödinger; Wheeler; Feynman; Mutual energy; Self-energy; Reciprocity theorem; Electromagnetic field; Photon; Advanced wave; Retarded wave; Causality; Energy conservation law; Photon; anti-photon;

1 Introduction

Maxwell equations have the retarded solution and the advanced solution. Many engineers and scientists do not accept the advanced wave based on the consideration of causality. However, there are scientists and engineers believe that the advanced wave is real. Wheeler and Feynman have introduced the absorber theory in 1945, which involved the advanced wave [1][2]. The absorber theory is based on the action-at-a-distance [21, 24, 8]. J. Cramer further worked on the absorber theory and introduced the transactional interpretation for quantum mechanics[5, 6] around 1980. Stephenson offered a good tutorial about the advanced wave [23].

In classical electromagnetic field theory the advanced wave is applied on the Welch's reciprocity theorem [25] in 1960, Rumsey's reciprocity theorem[20] in 1963, Zhao's (this author) mutual energy theorem [9, 27, 26] introduced in the first half of 1987. de Hoop's reciprocity theorem[7] introduced in the second half of 1987. This author found the above 4 theorems can be seen as one theorem in Fourier domain or in time domain. These theorems have the major difference comparing to the Lorentz reciprocity theorem[3, 4]. Lorentz reciprocity theorem is a mathematical theorem, Shuang-ren Zhao has noticed that these theorems are an energy theorems.

This author combined the absorber theory and the mutual energy theorem and further introduced the concept that the photon energy is transferred by the mutual energy flow[18, 17, 13, 22, 15, 14, 19, 16]. And this author further introduced the mutual energy principle[10] and the self-energy principle[11]. The mutual energy principle tell us that the electromagnetic field or the field for photon all should satisfy the formula of the mutual energy principle. The solution of the mutual energy principle is an retarded wave and an advanced wave. Both waves satisfy the Maxwell equations. The formula of the mutual energy principle require that the both waves must be synchronized. The mutual energy flow is the energy flow of the particle. The self-energy principle for photon tells us that the self-energy are canceled by the time-reversal waves. There are two time reversal waves, one corresponding to the retarded wave and another one is corresponding to the advanced wave. The energy flow of the two time-reversal waves offset the the self-energy flows. Hence, the self-energy flows do not contribute to any energy transfers of the particle. However, the retarded wave and the advanced wave combined together can build the mutual energy flow, which survived and which can transfer the energy from point **1** to point **2**. Here point **1** is a source of the energy flow, point **2** is a sink of the energy flow. It can be proven that the shape of the mutual energy flow is thin in the two ends **1** and 2, it is thick in the middle between 1 and **2**. Hence, the mutual energy flow looks like a wave in the middle between **1** and **2** and looks like a particle at the two ends **1** and **2**. Hence, photon is not wave or particle but the mutual energy flow.

In this article we simplified the whole theory and try to allow the reader more easily to understand the concept of the theory for the mutual energy and self-energy.

2 Why we need the mutual energy principle

2.1 Derive the mutual energy principle from the Maxwell equations

The Maxwell equations are,

$$\begin{cases} -\frac{\partial}{\partial t}(\epsilon \boldsymbol{E}) + \nabla \times \boldsymbol{H} = \boldsymbol{J} \\ -\nabla \times \boldsymbol{E} - \frac{\partial}{\partial t}(\mu \boldsymbol{H}) = 0 \end{cases} \tag{1}$$

where $\boldsymbol{E}$ is the electric field, $\boldsymbol{H}$ is the magnetic field, $\boldsymbol{J}$ is electric current intensity, ϵ is permittivity and μ is permeability.

In this article when we speak about the Maxwell equation, we mean this two equation, the other two Maxwell equation is not too important to the radiation waves and are omit here. Maxwell equations can be written as,

$$L\xi = \frac{1}{\epsilon}\tau \tag{2}$$

where $\xi = [\boldsymbol{E}, \boldsymbol{H}]$, $\tau = [\boldsymbol{J}, \boldsymbol{K}]$, $\boldsymbol{K} = 0$ is the magnetic current intensity. The operator L is defined as

$$L = \begin{bmatrix} -\frac{\partial}{\partial t} & \frac{1}{\epsilon}\nabla\times \\ -\frac{1}{\mu}\nabla\times & -\frac{\partial}{\partial t} \end{bmatrix} \tag{3}$$

We can define inner product,

$$(\xi_2, \xi_1)_V = \iiint\limits_V (\epsilon \boldsymbol{E} \cdot \boldsymbol{E} + \mu\, \boldsymbol{H} \cdot \boldsymbol{H}) dV \tag{4}$$

It can be obtained easily,

$$L\xi_1 = \begin{bmatrix} -\frac{\partial}{\partial t} & \frac{1}{\epsilon}\nabla\times \\ -\frac{1}{\mu}\nabla\times & -\frac{\partial}{\partial t} \end{bmatrix} \begin{bmatrix} \boldsymbol{E}_1 \\ \boldsymbol{H}_1 \end{bmatrix} = \begin{bmatrix} -\frac{\partial}{\partial t}\boldsymbol{E}_1 & +\frac{1}{\epsilon}\nabla \times \boldsymbol{H}_1 \\ -\frac{1}{\mu}\nabla \times \boldsymbol{E}_1 & -\frac{\partial}{\partial t}\boldsymbol{H}_1 \end{bmatrix} \tag{5}$$

and, hence,

$$(\xi_2, L\xi_1)_V = \iiint\limits_V \epsilon(-\boldsymbol{E}_2 \cdot \frac{\partial}{\partial t}\boldsymbol{E}_1 + \frac{1}{\epsilon}\boldsymbol{E}_2 \cdot \nabla \times \boldsymbol{H}_1) + \mu(-\frac{1}{\mu}\boldsymbol{H}_2 \cdot \nabla \times \boldsymbol{E}_1 - \boldsymbol{H}_2 \frac{\partial}{\partial t}\boldsymbol{H}_1) dV$$

$$= \iiint\limits_V (-\epsilon \boldsymbol{E}_2 \cdot \frac{\partial}{\partial t}\boldsymbol{E}_1 - \nabla \cdot (\boldsymbol{E}_2 \times \boldsymbol{H}_1) - \nabla \cdot (\boldsymbol{E}_1 \times \boldsymbol{H}_2) - \mu \boldsymbol{H}_2 \frac{\partial}{\partial t}\boldsymbol{H}_1) dV \quad (6)$$

We can also obtain,

$$L\xi_2 = \begin{bmatrix} -\frac{\partial}{\partial t} & \frac{1}{\epsilon}\nabla\times \\ -\frac{1}{\mu}\nabla\times & -\frac{\partial}{\partial t} \end{bmatrix} \begin{bmatrix} \boldsymbol{E}_2 \\ \boldsymbol{H}_2 \end{bmatrix} = \begin{bmatrix} -\frac{\partial}{\partial t}\boldsymbol{E}_2 & +\frac{1}{\epsilon}\nabla \times \boldsymbol{H}_2 \\ -\frac{1}{\mu}\nabla \times \boldsymbol{E}_2 & -\frac{\partial}{\partial t}\boldsymbol{H}_2 \end{bmatrix} \quad (7)$$

$$(L\xi_2, \xi_1)_V = \iiint\limits_V (\epsilon(-\boldsymbol{E}_1 \cdot \frac{\partial}{\partial t}\boldsymbol{E}_2 + \frac{1}{\epsilon}\boldsymbol{E}_1 \cdot \nabla \times \boldsymbol{H}_2) + \mu(-\frac{1}{\mu}\boldsymbol{H}_1 \cdot \nabla \times \boldsymbol{E}_2 - \boldsymbol{H}_1 \cdot \frac{\partial}{\partial t}\boldsymbol{H}_2)) dV$$

$$= \iiint\limits_V (-\epsilon \boldsymbol{E}_1 \cdot \frac{\partial}{\partial t}\boldsymbol{E}_2 - \nabla \cdot (\boldsymbol{E}_1 \times \boldsymbol{H}_2) - \nabla \cdot (\boldsymbol{E}_2 \times \boldsymbol{H}_1) - \mu \boldsymbol{H}_1 \cdot \frac{\partial}{\partial t}\boldsymbol{H}_2) dV \quad (8)$$

Hence, we can obtained the mathematical formula,

$$\int\limits_{-\infty}^{\infty} ((\xi_2, L\xi_1)_V + (L\xi_2, \xi_1)_V) dt$$

$$= -\int\limits_{-\infty}^{\infty} \iiint\limits_V (\epsilon \boldsymbol{E}_1 \frac{\partial}{\partial t}\boldsymbol{E}_2 + \epsilon \boldsymbol{E}_2 \frac{\partial}{\partial t}\boldsymbol{E}_1 + \mu \boldsymbol{H}_1 \frac{\partial}{\partial t}\boldsymbol{H}_2 + \mu \boldsymbol{H}_2 \frac{\partial}{\partial t}\boldsymbol{H}_1) dV$$

$$- \oiint\limits_\Gamma (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma \quad (9)$$

where V is a volume, Γ is the boundary surface of the volume V. $\hat{n}$ is the unit normal vector of the surface Γ. $\xi_1 = [\boldsymbol{E}_1, \boldsymbol{H}_1]$. $\xi_2 = [\boldsymbol{E}_2, \boldsymbol{H}_2]$. In this article, we assume $\xi = [\boldsymbol{E}, \boldsymbol{H}]$ and $\xi^T = \begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{bmatrix}$ are same vector. Hence, we do not need to write the Matrix transfer symbol T.

The above formula is not a physical formula, it is only a mathematical formula according to our definition of the Maxwell operator Eq.(3).

Substitute the Maxwell equations Eq.(2) to the above formula Eq.(9) we obtained,

$$\int\limits_{-\infty}^{\infty} ((\xi_2, \frac{1}{\epsilon}\tau_1)_V + (\frac{1}{\epsilon}\tau_2, \xi_1)_V) dt$$

$$= -\int\limits_{-\infty}^{\infty} \iiint\limits_V (\epsilon \boldsymbol{E}_1 \frac{\partial}{\partial t}\boldsymbol{E}_2 + \epsilon \boldsymbol{E}_2 \frac{\partial}{\partial t}\boldsymbol{E}_1 + \mu \boldsymbol{H}_1 \frac{\partial}{\partial t}\boldsymbol{H}_2 + \mu \boldsymbol{H}_2 \frac{\partial}{\partial t}\boldsymbol{H}_1) dV dt$$

$$-\int_{-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma dt \tag{10}$$

or

$$-\int_{-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n} d\Gamma dt$$

$$= \int_{-\infty}^{\infty} \iiint_{V} (\epsilon \boldsymbol{E}_1 \frac{\partial}{\partial t} \boldsymbol{E}_2 + \epsilon \boldsymbol{E}_2 \frac{\partial}{\partial t} \boldsymbol{E}_1 + \mu \boldsymbol{H}_1 \frac{\partial}{\partial t} \boldsymbol{H}_2 + \mu \boldsymbol{H}_2 \frac{\partial}{\partial t} \boldsymbol{H}_1) dV dt$$

$$+ \int_{-\infty}^{\infty} \iiint_{V} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2 + \boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV dt \tag{11}$$

The above formula is referred as the mutual energy principle. It is a physical formula because Maxwell equations are applied. Hence, we have proved the formula of the mutual energy principle from the Maxwell equations.

The mutual energy principle can be rewritten as,

$$-\sum_{i=1}^{2} \sum_{i=1, i\neq j}^{2} \int_{-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot \hat{n} d\Gamma dt$$

$$= \sum_{i=1}^{2} \sum_{i=1, i\neq j}^{2} \int_{-\infty}^{\infty} \iiint_{V} (\epsilon \boldsymbol{E}_i \frac{\partial}{\partial t} \boldsymbol{E}_j + \mu \boldsymbol{H}_i \frac{\partial}{\partial t} \boldsymbol{H}_j) dV dt$$

$$+ \sum_{i=1}^{2} \sum_{i=1, i\neq j}^{2} \int_{-\infty}^{\infty} \iiint_{V} (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV dt \tag{12}$$

If there are N charges instead of 2 charges, the mutual energy principle can be widen to

$$-\sum_{i=1}^{N} \sum_{i=1, i\neq j}^{N} \int_{-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot \hat{n} d\Gamma dt$$

$$= \sum_{i=1}^{N} \sum_{i=1, i\neq j}^{N} \int_{-\infty}^{\infty} \iiint_{V} (\epsilon \boldsymbol{E}_i \frac{\partial}{\partial t} \boldsymbol{E}_j + \mu \boldsymbol{H}_i \frac{\partial}{\partial t} \boldsymbol{H}_j) dV dt$$

$$+ \sum_{i=1}^{N} \sum_{i=1, i\neq j}^{N} \int_{-\infty}^{\infty} \iiint_{V} (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV dt \tag{13}$$

2.2 Derive the Maxwell equations from the mutual energy principle

If we know the mutual energy principle, we would like to found the solution of mutual energy principle, we compare the mutual energy principle Eq.(11) the mathematical formula Eq.(9), we can obtained 2 groups of Maxwell equations,

$$L\xi_1 = \frac{1}{\epsilon}\tau_1 \tag{14}$$

$$L\xi_2 = \frac{1}{\epsilon}\tau_2 \tag{15}$$

Two groups of the Maxwell equations must exist together. Where L is defined in Eq.(3).

It should be noticed that, $L\xi_1 = \frac{1}{\epsilon}\tau_1$ alone is not a solution of the mutual energy principle. Since if ξ_2=0, $\tau_2 = 0$, substitute this to the mutual energy principle we obtain $||\xi_1|| < \infty$ which is not an accepted solution for ξ_1.

Similarly $L\xi_2 = \frac{1}{\epsilon}\tau_2$ alone is also not a solution of the mutual energy principle. The two groups of the Maxwell equations must be satisfied the Maxwell equations simultaneously.

If the mutual energy principle used as axioms of the electromagnetic field theory, it requires that the two groups of the Maxwell equations exist simultaneously. If the Maxwell equations used as an axioms of the electromagnetic field theory, only one group of Maxwell equations is OK. This is the major difference about which should be used as axioms of the electromagnetic field theory, Maxwell equations or the mutual energy principle.

2.3 The energy conservation law

Assume there are N charges. the i-th charge's electric current is $\boldsymbol{J}_i$. The power of $\boldsymbol{J}_i$ to the j-th charge is $\iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV$. Since if i-th charge offer some energy to j-th charge, j-th charge's energy is increased, the energy of i-th charge will decrease, the total energy will not increase or decrease and hence we have,

$$\sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \int_{t=-\infty}^{\infty} \iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV dt = 0 \tag{16}$$

This is the energy conservation law of the electromagnetic charges system. This system have only N charges otherwise is nothing. This formula should be seen as self-explanatory.

Now we have two different axioms for the electromagnetic field theory. Which is correct? The energy conservation law will be applied as touchstone. Only the one agrees with the energy conservation law should be survived.

2.4 Derive the energy conservation law from the mutual energy principle

Compare the mutual energy principle Eq.(13) and the energy conservation law we know that if we can prove the two following terms as 0, we can derive the energy conservation law from the mutual energy principle, i.e.,

$$\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_j+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_j)dVdt=0 \tag{17}$$

$$\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma dt=0 \tag{18}$$

We will prove these only in the situation where $N=2$, i.e.,

$$\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_1\frac{\partial}{\partial t}\boldsymbol{E}_2+\epsilon\boldsymbol{E}_2\frac{\partial}{\partial t}\boldsymbol{E}_1+\mu\boldsymbol{H}_1\frac{\partial}{\partial t}\boldsymbol{H}_2+\mu\boldsymbol{H}_2\frac{\partial}{\partial t}\boldsymbol{H}_1)dVdt=0 \tag{19}$$

$$\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma dt=0 \tag{20}$$

Assume the volume V is infinite big sphere. Γ is the boundary of the surface. $\tau_1=[\boldsymbol{J}_1,0]$ and $\tau_2=[\boldsymbol{J}_2,0]$ are two electric current of the two charges. τ_1 and τ_2 are inside the volume$V$. If this two charges all send the retarded wave or all send advanced wave it is clear the surface integral cannot be 0 at the surface Γ. However if one sends the retarded wave and another sends the advanced wave, the two waves reach the surface Γ one is at a future time and another is at a past time. Hence, the two waves cannot no zero at the same time on the sphere surface Γ. Hence, the surface integral on Γ is 0, i.e,

$$\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma=0 \tag{21}$$

hence, we have

$$\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma dt=0 \tag{22}$$

In other hand,

$$\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_1\frac{\partial}{\partial t}\boldsymbol{E}_2+\epsilon\boldsymbol{E}_2\frac{\partial}{\partial t}\boldsymbol{E}_1+\mu\boldsymbol{H}_1\frac{\partial}{\partial t}\boldsymbol{H}_2+\mu\boldsymbol{H}_2\frac{\partial}{\partial t}\boldsymbol{H}_1)dVdt$$

$$= \int_{-\infty}^{\infty} dU = U(\infty) - U(-\infty)$$

$$= 0 \tag{23}$$

where

$$U = \iiint_V (\epsilon \boldsymbol{E}_1 \boldsymbol{E}_2 + \mu \boldsymbol{H}_1 \boldsymbol{H}_2) dV \tag{24}$$

is the space energy of the electromagnetic field, this energy should be equal at the time $t = -\infty$ and $t = \infty$. We assume there is an energy from move from the source 1 to the sink 2. The energy in the space is changed when the energy move from 1 to 2. However, in the time $t = -\infty$ the moving process has not started and at $t = \infty$ the energy moving process has finished. The energy in the space for this two particular time are same. Hence, we have $U(\infty) = U(-\infty)$.

We have proved when $N = 2$ the two terms Eq.(19, 20) are indeed as 0. This can be easily widen to the situation where charge number $N > 2$. Hence, we have derived the energy conservation law from the mutual energy principle.

2.5 The conflict of the Poynting theorem with energy conservation law

Assume the Maxwell equations Eq.(1) is satisfied. From Maxwell equations the Poynting theorem can be derived which is,

$$-\oiint_\Gamma (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma = \iiint_V (\epsilon \boldsymbol{E} \frac{\partial}{\partial t} \boldsymbol{E} + \mu \boldsymbol{H} \frac{\partial}{\partial t} \boldsymbol{H}) dV + \iiint_V (\boldsymbol{E} \cdot \boldsymbol{J}) dV \tag{25}$$

The superposition principle for electromagnetic field can be written as,

$$\boldsymbol{E} = \sum_{i=1}^{N} \boldsymbol{E}_i, \quad \boldsymbol{H} = \sum_{i=1}^{N} \boldsymbol{H}_i, \quad \boldsymbol{J} = \sum_{i=1}^{N} \boldsymbol{J}_i \tag{26}$$

Substitute the superposition principle to the Poynting theorem we obtain,

$$-\sum_{i=1}^{N} \sum_{i=1}^{N} \int_{-\infty}^{\infty} \oiint_\Gamma (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot \hat{n} d\Gamma dt$$

$$= \sum_{i=1}^{N} \sum_{i=1}^{N} \int_{-\infty}^{\infty} \iiint_V (\epsilon \boldsymbol{E}_i \frac{\partial}{\partial t} \boldsymbol{E}_j + \mu \boldsymbol{H}_i \frac{\partial}{\partial t} \boldsymbol{H}_j) dV dt$$

$$+ \sum_{i=1}^{N} \sum_{i=1}^{N} \int_{-\infty}^{\infty} \iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV dt \tag{27}$$

This can be divided as two formula,

$$
\begin{aligned}
&-\sum_{i=1}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma dt\\
&=\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_i+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_i)dVdt\\
&+\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_i)dVdt
\end{aligned}
\tag{28}
$$

and

$$
\begin{aligned}
&-\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma dt\\
&=\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_j+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_j)dVdt\\
&+\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_j)dVdt
\end{aligned}
\tag{29}
$$

The Eq.(28) is the self-energy formula or Poynting theorem for all N charges. The Eq.(29) is the mutual energy principle Eq.(13). If each terms in Eq.(28) are 0, we obtained the mutual energy principle, from the derivation of last few subsections we can prove the energy conservation law Eq.(16). Every thing is fine. However, we cannot prove each terms of Eq.(28) is 0. Since one term of the Eq.(28) is Poynting theorem for i-th charge. If i-th term of the formula Eq.(28) is 0 then the electromagnetic field of i-th charge is 0. We know that i is arbitrary and, hence, this means that all the electromagnetic fields are 0. This cannot be accepted. We know the electromagnetic field is not 0. If there are the terms in Eq.(28) not 0, we cannot prove the energy conservation law from the Maxwell equations with the superposition principle. This means the Maxwell equations and the superposition principle are conflict with the energy conservation law.

Actually from Maxwell equations we have proven that the formula Eq.(16) is an energy theorem, we can call it as the mutual energy theorem. In order to prove it is an energy conservation law we have to prove in the space there is no any other kind of energy transfer. That is the reason we need all terms of Eq.(28) as 0.

Since the Maxwell equations can be derived from the mutual energy principle, this also means that the mutual energy principle has also the problem. However, the problem of the mutual energy principle is less than the Maxwell equations, because the problem of the mutual energy principle is only through the the Maxwell equations indirectly. The problem of the Maxwell equation is direct.

If we can solve the conflict between the Maxwell equations together with the superposition principle and the energy conservation law, we have solved all the problems. The mutual energy principle will also not conflict with the energy conservation law.

2.6 Introduce the self-energy principle

This problem cannot be solved inside the frame of the Maxwell's theory. Hence, we need to found a solution outside the theory of the Maxwell. We have heard the concept of wave function collapse. Perhaps wave function collapse can offer a solution, however, there are two reason we do not use the concept of wave function collapse: 1) it cannot offer any mathematical formula to describe it, 2) now we have the mutual energy which can transfer the energy in the space, we do not need the wave to transfer the energy. However, we apply a very similar concept which is wave function collapse back. The wave function collapse means the wave goes to its target. The wave function collapse back means the wave goes to its source. Hence, for a retarded wave if it collapses, it goes to the sink (absorber). If a retarded wave collapses back it goes back to the source (emitter). For an advanced wave if it collapses it goes to the source (emitter) and if it collapse back it goes back to the sink (absorber).

The wave function collapse back can be described by a time-reversal process which is obtained by the transform $t \to -t = \tau$ to the formula of the electromagnetic field, for example, Maxwell equations. Considering,

$$\boldsymbol{E}(t) \to \boldsymbol{E}(-t) = \boldsymbol{E}(\tau) \tag{30}$$

$$\frac{\partial}{\partial t}\boldsymbol{E}(t) \to \frac{\partial}{\partial t}\boldsymbol{E}(-t) = -\frac{\partial}{\partial(-t)}\boldsymbol{E}(-t) = -\frac{\partial}{\partial(\tau)}\boldsymbol{E}(\tau) \tag{31}$$

$$\frac{\partial}{\partial t}\boldsymbol{H}(t) \to \frac{\partial}{\partial t}\boldsymbol{H}(-t) = -\frac{\partial}{\partial(-t)}\boldsymbol{H}(-t) = -\frac{\partial}{\partial(\tau)}\boldsymbol{H}(\tau) \tag{32}$$

$$\boldsymbol{J} = \rho \boldsymbol{v} = \boldsymbol{\rho(t)\frac{\partial x(t)}{\partial t}}$$

$$\to \rho(-t)\frac{\partial x(-t)}{\partial t} = \rho(-t)\frac{\partial x(-t)}{-\partial(-t)} = -\rho(\tau)\frac{\partial x(\tau)}{\partial(\tau)} = -\rho\boldsymbol{v} = -\boldsymbol{J} \tag{33}$$

Hence substitute Eq.(31, 32, 33) to the Maxwell equations Eq.(1) we can obtain the time-reversal Maxwell equations which are,

$$\begin{array}{l} \frac{\partial}{\partial t}(\epsilon \boldsymbol{e}) + \nabla \times \boldsymbol{h} = -\boldsymbol{j} \\ -\nabla \times \boldsymbol{e} + \frac{\partial}{\partial t}(\mu \boldsymbol{h}) = 0 \end{array} \tag{34}$$

The corresponding time-reversal Poynting theorem can be derived from the time-reversal Maxwell equations which are,

$$-\oiint_{\Gamma}(\boldsymbol{e}\times\boldsymbol{h})\cdot\hat{n}d\Gamma=-\iiint_{V}(\epsilon\boldsymbol{e}\frac{\partial}{\partial t}\boldsymbol{e}+\mu\boldsymbol{h}\frac{\partial}{\partial t}\boldsymbol{h})dV-\iiint_{V}(\boldsymbol{e}\cdot\boldsymbol{j})dV \tag{35}$$

The above formula is obtained by substitute Eq.(31-33)) to the Poynting theorem Eq.(25). This is the time-reversal Poynting theorem, it is corresponding the time-reversal self-energy flow.

Actually we do not need all terms in the Eq.(28) are 0. What we need is the self-energy terms do not carry and transfer energy. This author noticed, if there are a time-reversal electromagnetic field which satisfy the time-reversal Maxwell equations, the time-reversal energy flow can cancel the normal energy flow, hence, we can have,

$$-\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma dt+\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h_i})\cdot\hat{n}d\Gamma dt=0 \tag{36}$$

$$\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_i+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_i)dVdt+\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{e}_i\frac{\partial}{\partial t}\boldsymbol{e}_i+\mu\boldsymbol{h}_i\frac{\partial}{\partial t}\boldsymbol{h}_i)dVdt=0 \tag{37}$$

$$\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_i)dVdt+\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{e}_i\cdot\boldsymbol{j}_i)dVdt=0 \tag{38}$$

There exist the time-reversal wave which satisfy the time-reversal Maxwell equations and can cancel the normal self-energy terms. These are referred as the self-energy principle. After the introduction of the self-energy principle is introduced, Eq.(28) do not carry any energy in the space. Hence, the Maxwell equations together with the self-energy principle Eq.(15,16,17) can also derive the energy conservation law. The conflict between the (Maxwell equations, the superposition principle) and the energy conservation law is solved.

The above formula means the source for the time-reversal wave just negative value of the normal current (assume the source is in very short time for example $\delta(t)$). The above formula can also be seen as the self-energy principle. The self-energy principle Eq.(15,16,17) can be derived from the time-reversal Maxwell equation, if we know that,

$$\boldsymbol{J}=-\boldsymbol{j} \tag{39}$$

When $\boldsymbol{E}_i$ increase, $\boldsymbol{e}_i$ decrease, $\frac{\partial}{\partial t}\boldsymbol{e}_i$=-$\frac{\partial}{\partial t}\boldsymbol{E}_i$, $\boldsymbol{H}_i$ increase, $\boldsymbol{h}_i$ decrease, $\frac{\partial}{\partial t}\boldsymbol{h}_i$=-$\frac{\partial}{\partial t}\boldsymbol{H}_i$. The $\boldsymbol{j}$ is a implicit current.

3 The mutual energy theorem and the mutual energy flow theorem

3.1 The mutual energy theorem

We have derived the energy conservation law from the mutual energy principle and self-energy principle. When the $N = 2$, we have,

$$\sum_{i=1}^{2} \sum_{j=1, j \neq i}^{2} \int_{t=-\infty}^{\infty} \iiint_{V} (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV dt = 0 \tag{40}$$

Or

$$\int_{t=-\infty}^{\infty} \iiint_{V} (\boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) + \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t)) dV dt = 0 \tag{41}$$

Or

$$\int_{t=-\infty}^{\infty} \iiint_{V_2} (\boldsymbol{E}_1(t) \cdot \boldsymbol{J}_2(t) = - \int_{t=-\infty}^{\infty} \iiint_{V_1} \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t)) dV dt \tag{42}$$

This is W.J. Welch's reciprocity theorem[25]. This can be widen to,

$$\int_{t=-\infty}^{\infty} \iiint_{V_2} (\boldsymbol{E}_1(t+\tau) \cdot \boldsymbol{J}_2(t) dV dt = - \int_{t=-\infty}^{\infty} \iiint_{V_1} \boldsymbol{E}_2(t) \cdot \boldsymbol{J}_1(t+\tau)) dV dt \tag{43}$$

This is the Hoop's reciprocity theorem[7]. After make the Fourier transform from Hoop's reciprocity theorem, it become,

$$\iiint_{V_2} (\boldsymbol{E}_1(\omega) \cdot \boldsymbol{J}_2^*(\omega) dV = - \iiint_{V_1} \boldsymbol{E}_2^*(\omega) \cdot \boldsymbol{J}_1(\omega)) dV \tag{44}$$

This is Rumsey's reciprocity theorem and this is also the mutual energy reciprocity theorem of the Shuang-ren Zhao [9]. Hence, in case there are only two charges in the empty space, these theorems actually are the energy conservation law.

We assume that the charge 1 is a source which sends the retarded wave. We assume the source 2 is a sink which sends the advanced wave, these theorems tell us that the energy sucked by the advanced wave $\boldsymbol{E}_2(\omega)$ on the source current $\boldsymbol{J}_1(\omega)$ is equal to the retarded wave $\boldsymbol{E}_1(\omega)$ applied the energy on the sink current $\boldsymbol{J}_2(\omega)$.

This theorem apply to the antenna, that means the energy received by the receiving antenna is equal to the energy sends out by the transmitting antenna, if we assume that there are only these two antennas in a otherwise empty space.

For sure normally there are other antennas and also the environment which can also receive the energy sent from the transmitting antenna. However, we can assume an ideal situation, there are only a transmitting antenna and a receiving antenna in an otherwise empty space, these theorem become the energy conservation law for the antenna system with one transmitting antenna and one receiving antenna.

3.2 The mutual energy flow theorem

In the derivation of the energy conservation law from the mutual energy principle, we assume the surface Γ is a infinite big sphere surface. If we assume this surface is not at infinite big sphere, in general we cannot obtained the surface integral is 0, hence, generally we have,

$$\begin{aligned} & -\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma dt \\ & =\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_j)dVdt \end{aligned} \tag{45}$$

Or if $N=2$, we have,

$$\begin{aligned} & -\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma dt \\ & =\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_1\cdot\boldsymbol{J}_2+\boldsymbol{E}_2\cdot\boldsymbol{J}_1)dVdt \end{aligned} \tag{46}$$

Here the volume V is arbitrary. Inside the volume there are two currents $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$. We assume $\boldsymbol{J}_1$ is the source which sends the retarded wave $\boldsymbol{E}_1, \boldsymbol{H}_1$ and $\boldsymbol{J}_2$ is the sink which sends the advanced wave $\boldsymbol{E}_2, \boldsymbol{H}_2$. See Figure 1 for details.

If we take the V as V_1, $\boldsymbol{J}_1$ is inside V_1, and Γ become Γ_1 which is the boundary of V_1, we can obtained,

$$\begin{aligned} & -\int_{-\infty}^{\infty}\oiint_{\Gamma_1}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}_{12}d\Gamma dt \\ & \qquad =\int_{-\infty}^{\infty}\iiint_{V_1}(\boldsymbol{E}_2\cdot\boldsymbol{J}_1)dVdt \end{aligned} \tag{47}$$

In the above formula, we have considered $\boldsymbol{J}_2$ is at outside of V_1. That $\hat{n}_{12}$ is the norm unit vector of the surface Γ_1, which is direct to the outside of V_1, that is from volume 1 direct to volume 2. Hence, the subscript of 12 is applied for the norm vector $\hat{n}$. Details please see Figure 2

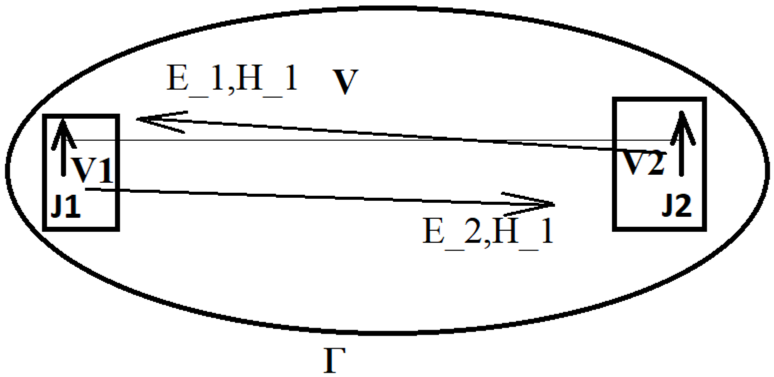

Figure 1: In the volume V, there are source $\boldsymbol{J}_1$, sink $\boldsymbol{J}_2$, $\boldsymbol{J}_1$ sends the retarded wave $[\boldsymbol{E}_1, \boldsymbol{H}_1]$. $\boldsymbol{J}_2$ sends the advanced wave $[\boldsymbol{E}_2, \boldsymbol{H}_2]$.

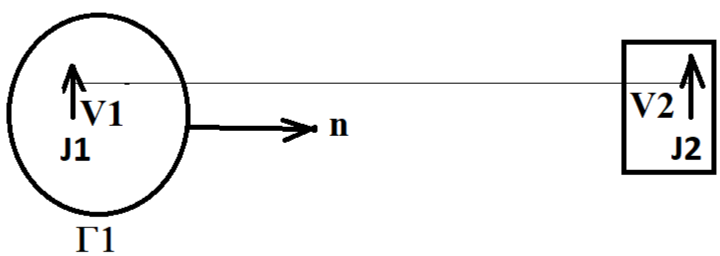

Figure 2: In the volume V_1, there are only the source $\boldsymbol{J}_1$, the sink $\boldsymbol{J}_2$ is at the outside of the volume V_1, $\boldsymbol{J}_1$ sends the retarded wave $[\boldsymbol{E}_1, \boldsymbol{H}_1]$. $\boldsymbol{J}_2$ sends the advanced wave $[\boldsymbol{E}_2, \boldsymbol{H}_2]$.

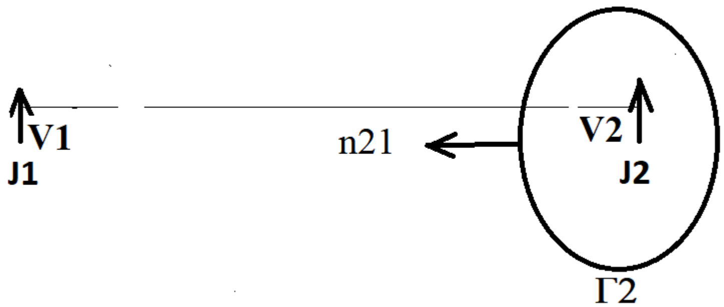

Figure 3: In the volume V_2, there are only the source $\boldsymbol{J}_2$, the source $\boldsymbol{J}_1$ is at the outside of the volume V_2, $\boldsymbol{J}_1$ sends the retarded wave $[\boldsymbol{E}_1, \boldsymbol{H}_1]$. $\boldsymbol{J}_2$ sends the advanced wave $[\boldsymbol{E}_2, \boldsymbol{H}_2]$. The norm vector of the surface is $\hat{n}_{12}$.

Similarly we have,

$$-\int_{-\infty}^{\infty} \oiint_{\Gamma_2} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n}_{21} d\Gamma dt$$

$$= \int_{-\infty}^{\infty} \iiint_{V_2} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2) dV dt \tag{48}$$

In equation Eq.(47) and (48) the direction of the norm vector are different. We can adjust $\hat{n}_{21}$ to $\hat{n}_{12}$ which is at the direction to the inside of the volume V_2. The details can be seen in Figure 3.

Change the direction of the norm vector of the surface $\hat{n}_{21}$ to $\hat{n}_{12}$ the above formula become,

$$\int_{-\infty}^{\infty} \oiint_{\Gamma_2} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n}_{12} d\Gamma dt$$

$$= \int_{-\infty}^{\infty} \iiint_{V_2} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2) dV dt \tag{49}$$

Substituting the above two formula Eq.(47, 49) to the mutual energy theorem Eq.(42), we obtained,

$$-\int_{-\infty}^{\infty} \iiint_{V_2} (\boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV dt$$

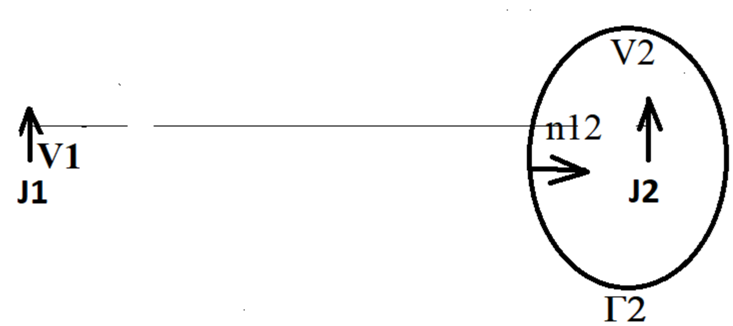

Figure 4: In the volume V_1, there are only the source $\boldsymbol{J}_1$, the sink $\boldsymbol{J}_2$ is at the outside of the volume V_1, $\boldsymbol{J}_1$ sends the retarded wave $[\boldsymbol{E}_1, \boldsymbol{H}_1]$. $\boldsymbol{J}_2$ sends the advanced wave $[\boldsymbol{E}_2, \boldsymbol{H}_2]$. The norm vector of the surface is $\hat{n}_{12}$

$$= \int_{-\infty}^{\infty} \oiint_{\Gamma_1} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n}_{12} d\Gamma dt$$

$$= \int_{-\infty}^{\infty} \oiint_{\Gamma_2} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n}_{12} d\Gamma dt$$

$$= \int_{-\infty}^{\infty} \iiint_{V_2} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2) dV dt \tag{50}$$

In the above formula there are two surface Γ_1 and Γ_2. This is the mutual energy flow theorem. See Figure 5 for details.

Actually the surface can be any surface Γ between V_1 and V_2. Hence, we have,

$$-\int_{-\infty}^{\infty} \iiint_{V_2} (\boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV dt$$

$$= \int_{-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n}_{12} d\Gamma dt$$

$$= \int_{-\infty}^{\infty} \iiint_{V_2} (\boldsymbol{E}_1 \cdot \boldsymbol{J}_2) dV dt \tag{51}$$

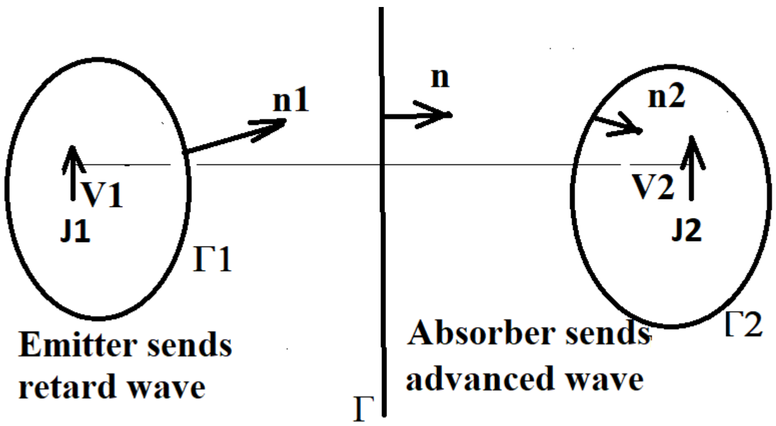

Figure 5: In the volume V_1, there are only the source $\boldsymbol{J}_1$, in the volume V_2 there is the sink $\boldsymbol{J}_2$, $\boldsymbol{J}_1$ sends the retarded wave $[\boldsymbol{E}_1, \boldsymbol{H}_1]$. $\boldsymbol{J}_2$ sends the advanced wave $[\boldsymbol{E}_2, \boldsymbol{H}_2]$. Γ_1 is the boundary of volume V_1. Γ_2 is the boundary of volume V_2. Γ is surface between V_1 and V_2. The norm vector of the surface are all in the direction from 1 to 2, hence is written as $\hat{n}_{12}$

The above is the mutual energy flow theorem. Γ can be taken at any place between volume V_1 and V_2. Γ can be a close surface for example a sphere or a infinite plane. The Mutual energy flow theorem gave more details of the explanation of the mutual energy theorem. Mutual energy theorem tell us there are energy from source move to sink. The sink received energy is equal to the energy radiated out by the source. The mutual energy flow theorem further tells us the energy that goes in the space is accounting to the mutual energy flow, which is define as,

$$Q = \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n}_{12} d\Gamma$$

The total energy go through the surface Γ from time $t = -\infty$ to $t = \infty$ is

$$Energy = \int_{-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1) \cdot \hat{n}_{12} d\Gamma dt$$

The mutual energy flow intensity is corresponding to the mixed Poynting vector:

$$\boldsymbol{S}_{12} = \boldsymbol{E}_1 \times \boldsymbol{H}_2 + \boldsymbol{E}_2 \times \boldsymbol{H}_1 \quad (52)$$

which is different comparing to the Poynting vector,

$$\boldsymbol{S}_1 = \boldsymbol{E}_1 \times \boldsymbol{H}_1 \quad (53)$$

There is also the Poynting vector corresponding the advanced wave,

$$S_2 = E_2 \times H_2 \tag{54}$$

In the theory of Poynting theorem, only the source can radiate the energy, the radiation is nothing to do with the sink. In the theory of mutual energy of this author, the energy flow is depended only on the mutual energy flow which is corresponding to the mixed Poynting vector $\boldsymbol{S}_{12}$ instead of the Poynting vector which is $\boldsymbol{S}_1$.

In this author's mutual energy theory, $\boldsymbol{S}_1$ is corresponding to the self-energy flow. This part of energy flow does not transfer and carry energy because there are time-reversal waves. The energy flow of self-energy terms is canceled by the energy flow of the corresponding time-reversal wave. This is also true for $\boldsymbol{S}_2$ which is the energy flow of the advanced wave. The energy is transferred only by the mutual energy flow. Hence, the mutual energy flow is the energy flow. The word "mutual" can be wipe off. The mutual energy principle is the energy principle. However, we still call it as mutual energy principle for history reason.

3.3 The relation of Maxwell equations vs the mutual energy principle and the self-energy principle

$$solution\ of\ Maxwell\ equation$$

$$\supset solution\ of\ mutual\ energy\ principle$$

The solution of Maxwell equations include the retarded wave solution and advanced wave solution.

The solution of Mutual energy principle has the solution only when the retarded wave and the advanced wave are synchronized.

$$solution\ of\ (Maxwell\ equation{+}self\ energy\ principle{+}timereversed\ Maxwell\ equation)$$

$$= solution\ of\ (mutual\ energy\ principle{+}self\ energy\ principle{+}timereversed\ Maxwell\ equation)$$

The solution of Maxwell equations are bigger concept set than the solution of the mutual energy principle. The solution of Maxwell equations has some kind of solution we can call it as weak solution in physics, that is the retarded wave and the advanced wave alone without synchronization. There is the strong solution in physics, which are the solutions satisfy the mutual energy principle that is the solution the retarded wave and the advanced wave can be synchronized hence there is the mutual energy flow produced. self-energy principle together with the self-energy principle is the new restrict condition. With this new restrict condition the weak solution in physics has been role out from the Maxwell equations, the rest are the strong solution which satisfy the mutual energy principle.

This also means that the really new principle are the self-energy principle and the time-reversal Maxwell equations. The mutual energy principle and Maxwell equations can be exchangeable. However, since the mutual energy principle is only one formula, the Maxwell equation is two formulas, the mutual energy principle is simpler than Maxwell equations. Maxwell equations cannot work without superposition principle. The mutual energy principle do not need the support of the superposition because the superposition can be derived from the mutual energy principle. Another advantage of the mutual energy principle is the mutual energy flow theorem and the mutual energy theorem (the conservation law) can be derived from the mutual energy principle easily. All these are usually difficult to be obtained from the Maxwell equations or at least confused people, the reason is most scientist work at Maxwell equations do not accept the advanced wave, since the advanced wave doesn't obey the traditional causality consideration. If we apply the mutual energy principle as the axioms, the advanced wave have to be accept.

4 Possible modes of the electromagnetic field radiations

In this section the possible modes of the radiation is discussed. We often head photons. However consider the physics law should be symmetric, it should be also the anti-photon. In the author's assumption the source randomly sends the retarded wave, the sink also randomly sends the advanced wave. It is possible the retarded wave cannot matches any advanced wave from many absorbers. The advanced wave send from an absorber is also possible cannot found a retarded wave to match. In this case the radiation process cannot be done.

4.1 Mode 1) invalid radiation

In this situation, the retarded wave is sent out by the emitter, but it doesn't find an advanced wave to synchronize. Hence, the retarded wave is returned by the corresponding time-reversal wave.

The advanced wave is sent out by the absorber but it doesn't find a retarded wave to synchronize. Hence, the advanced wave is returned by the corresponding time-reversal wave. In the above both situations, there is no any mutual energy flow is produced. The retarded wave and the advanced wave both satisfy the Maxwell equations, which further can derive the Poynting theorems,

$$
\begin{aligned}
&-\oiint_{\Gamma}\!\!\int (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma dt \\
&\iiint_V\!\!\int (\epsilon \boldsymbol{E}_i \frac{\partial}{\partial t} \boldsymbol{E}_i + \mu \boldsymbol{H}_i \frac{\partial}{\partial t} \boldsymbol{H}_i) dV dt \\
&+ \iiint_V\!\!\int (\boldsymbol{E}_i \cdot \boldsymbol{J}_i) dV dt
\end{aligned}
\tag{55}
$$

and the time-reversal Maxwell equations can derive the time-reversal Poynting theorem,

$$\oiint_{\Gamma} (\boldsymbol{e}_i \times \boldsymbol{h}_i) \cdot \hat{n} d\Gamma dt$$

$$\iiint_V (\epsilon \boldsymbol{e}_i \frac{\partial}{\partial t} \boldsymbol{e}_i + \mu \boldsymbol{h}_i \frac{\partial}{\partial t} \boldsymbol{h}_i) dV dt$$

$$+ \iiint_V (\boldsymbol{e}_i \cdot \boldsymbol{j}_i) dV dt \tag{56}$$

In this situation the self-energy principle is,

$$-\int_{-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma dt + \int_{-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{e}_i \times \boldsymbol{h}_i) \cdot \hat{n} d\Gamma dt = 0 \tag{57}$$

$$\int_{-\infty}^{\infty} \iiint_V (\epsilon \boldsymbol{E}_i \frac{\partial}{\partial t} \boldsymbol{E}_i + \mu \boldsymbol{H}_i \frac{\partial}{\partial t} \boldsymbol{H}_i) dV dt + \int_{-\infty}^{\infty} \iiint_V (\epsilon \boldsymbol{e}_i \frac{\partial}{\partial t} \boldsymbol{e}_i + \mu \boldsymbol{h}_i \frac{\partial}{\partial t} \boldsymbol{h}_i) dV dt = 0 \tag{58}$$

$$\int_{-\infty}^{\infty} \iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_i) dV dt + \iiint_V (\boldsymbol{e}_i \cdot \boldsymbol{j}_i) dV dt = 0 \tag{59}$$

It is notice we have use Poynting theorem replaced the Maxwell equations and use the time-reversal Poynting theorem replace the time-reversal Maxwell equations. This replacement will reduce the number of formula. Because the Poynting theorem is the necessary and sufficient condition of the Maxwell equations.

In the above situation all self-energy terms are canceled. Hence, the wave did not carry the energy. Or we can say the energy is collapse back. We assume the current $\boldsymbol{J}_i$ randomly changed and hence, the wave is randomly sent out. In this situation there is no mutual energy flow produced and, hence, there is no energy which have been sent out. Even, the self-energy items sends out the energy, the energy is collapse back through the time-reversal wave. Since, the collapsed-back wave is the time-reversal wave, all self-energy flow are canceled. Hence, it looks like that no any energy has been sent out ever.

In this situation $\boldsymbol{J}_i = -\boldsymbol{j}_i$. We assume $\boldsymbol{J}_i$ is happened at a very short time for example as $\delta(t)$, this time interval close to 0. Hence, we can have,

$$\boldsymbol{J}_i + \boldsymbol{j}_i = 0 \tag{60}$$

This means the current is also recovered to the situation before the radiation. We call this unsuccessful radiation. The current $\boldsymbol{J}_i$ has tried to make a radiation, the wave is sent to the space, however, since it doesn't find an advanced wave to match, the wave sends to the end of our universe and then it automatically returned or collapses back through the time-reversal wave to the source. In the place of source a negative current is produced. The negative current element cancel the original current element which tries to produce the radiation. Finally this radiation is not successfully sent out. The current element come back to the original state.

The sink (absorber) all can also sends the self-energy wave out. This wave is advanced wave. This waves are also canceled by the corresponding time-reversal waves.

We assume the emitter randomly sends the energy out, if this wave cannot find a advanced wave to match it, it will be returned by the time-reversal waves.

In this situation we have 3 formulas, 2 Poynting theorems Eq.(55,56) and one self-energy principle Eq.(42-44). The 3 formulas can fully describe this phenomena. In this situation the mutual energy principle is not involved.

4.2 The photon situation

In the photon situation, the mutual energy flow is produced, which is decided by the mutual energy principle,

$$-\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma dt$$

$$\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_j+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_j)dVdt$$

$$+\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_j)dVdt \tag{61}$$

In this situation, the emitter (an electron) has spring done from the higher level to lower level. The the electron in the absorber from lower level spring to the higher level, the current $\boldsymbol{J}_i$ and $\boldsymbol{J}_j$ are not 0. The self-energy flow satisfies the Poynting theorem,

$$-\sum_{i=1}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma dt$$

$$\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_i+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_i)dVdt$$

$$+\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_i)dVdt \tag{62}$$

This author assume that there is the time-reversal wave which satisfy the time-reversal Poynting theorem,

$$-\sum_{i=1}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h}_i)\cdot\hat{n}d\Gamma dt$$

$$=-\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{e}_i\frac{\partial}{\partial t}\boldsymbol{e}_i+\mu\boldsymbol{h}_i\frac{\partial}{\partial t}\boldsymbol{h}_i)dVdt$$

$$-\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{e}_i\cdot\boldsymbol{j}_i)dVdt \tag{63}$$

And the time-reversal self-energy flow can cancel the normal self-energy flow through the self-energy principle, i.e.,

$$+\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma dt-\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h}_i)\cdot\hat{n}d\Gamma dt=0 \tag{64}$$

$$-\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_i+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_i)dVdt-\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{e}_i\frac{\partial}{\partial t}\boldsymbol{e}_i+\mu\boldsymbol{h}_i\frac{\partial}{\partial t}\boldsymbol{h}_i)dVdt=0 \tag{65}$$

$$-\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_i)dVdt-\iiint_{V}(\boldsymbol{e}_i\cdot\boldsymbol{j}_i)dVdt=0 \tag{66}$$

$\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma dt$ is the self-energy flow go outside the volume V. $-\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h}_i)\cdot\hat{n}d\Gamma dt$ is the energy flow go inside the volume $-\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h}_i)\cdot\hat{n}d\Gamma dt$. This two energy flow has the same value and different direction and hence canceled.

$-\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_i+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_i)dVdt$ is the energy decrease of volume V. When this decrease, $-\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_i+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_i)dVdt$ will increase the value.

$-\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_i)dVdt$ is the energy offered by the source $\boldsymbol{J}_i$. $-\iiint_{V}(\boldsymbol{e}_i\cdot\boldsymbol{j}_i)dVdt$ is energy is the energy received by the direction of $\boldsymbol{j}_i$. This two terms cancel each other.

Hence, all self-energy items together have no effect on the current $\boldsymbol{J}_i$. Hence, we can safely say that the self-energy flow has no any physical effect.

All physical effect comes from the mutual energy terms Eq.(61), this formula tell us that the electron in the emitter has spring from higher level to a lower level. For the absorber the electron has spring from a lower level to a higher level. The electron spring from level to another level can be explain as the current $\boldsymbol{J}_i$. The mutual energy flow bring the energy from emitter to the absorber. This energy is the energy of the photon. Hence, we can say that the photon is the mutual energy flow.

4.3 Anti-photon

Anti-photon can be described by the time-reversal mutual energy principle which satisfies,

$$\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h}_j)\cdot\hat{n}d\Gamma dt$$
$$=\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{e}_i\frac{\partial}{\partial t}\boldsymbol{e}_j+\mu\boldsymbol{h}_i\frac{\partial}{\partial t}\boldsymbol{h}_j)dVdt$$
$$+\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{e}_i\cdot\boldsymbol{j}_j)dVdt \quad (67)$$

For an anti-photon the time-reversal self-energy flow satisfies,

$$\sum_{i=1}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h}_i)\cdot\hat{n}d\Gamma dt$$
$$\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{e}_i\frac{\partial}{\partial t}\boldsymbol{e}_i+\mu\boldsymbol{h}_i\frac{\partial}{\partial t}\boldsymbol{h}_i)dVdt$$
$$\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{e}_i\cdot\boldsymbol{j}_i)dVdt \quad (68)$$

and the self-energy flow satisfies,

$$-\sum_{i=1}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma dt$$
$$\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_i+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_i)dVdt$$
$$+\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_i)dVdt \quad (69)$$

The time-reversal self-energy flow is canceled by the self-energy flow through the self-energy principle,

$$-\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma dt+\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h_i})\cdot\hat{n}d\Gamma dt=0 \quad (70)$$

$$\int_{-\infty}^{\infty} \iiint_V (\epsilon \boldsymbol{E}_i \frac{\partial}{\partial t} \boldsymbol{E}_i + \mu \boldsymbol{H}_i \frac{\partial}{\partial t} \boldsymbol{H}_i) dV dt + \int_{-\infty}^{\infty} \iiint_V (\epsilon \boldsymbol{e}_i \frac{\partial}{\partial t} \boldsymbol{e}_i + \mu \boldsymbol{h}_i \frac{\partial}{\partial t} \boldsymbol{h}_i) dV dt = 0 \tag{71}$$

$$\int_{-\infty}^{\infty} \iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_i) dV dt + \int_{-\infty}^{\infty} \iiint_V (\boldsymbol{e}_i \cdot \boldsymbol{j}_i) dV dt = 0 \tag{72}$$

For an anti-photon the energy is transferred by time-reversal mutual energy flow Eq.(67). We do not assume that the retarded wave $\boldsymbol{E}_i$, $\boldsymbol{H}_i$ and the advanced wave $\boldsymbol{E}_j$, $\boldsymbol{H}_j$ will produce any mutual energy flow. If it produces the mutual energy flow, this energy flow will cancel the time-reversal mutual energy flow. In this situation the anti-photon will not produced. We assume it is possible the anti-photon does not produced, but it is also possible an anti-photon is produced.

The anti-photon often cancel the partial photon or half photon. If two absorbers receive the energy of one photon (energy is sent from one emitter), each absorber can only get the energy of a half photon. The electron in the absorber can only spring to a half way from lower level to the higher level. In this situation the anti-photon will produced, the half photon is canceled by an anti-half photon. The electron spring up a half way to the higher level will spring back to the low level again. This way the anti-photon can eliminate all half photons or partial photons.

4.4 Summary

There 3 different radiation modes, 1) unsuccessful radiation. This radiation make the wave become a probability wave. The wave is happened but there is no any energy sending out. The wave looks like ghost wave or probability waves.

2) Photon situation, the radiation is successful. The retarded wave sent out has met an advanced wave. Hence, produced the mutual energy flow. The photon's energy is carried by the mutual energy flow. The mutual energy flow bring the energy from the emitter to the absorber. The self-energy terms are all canceled by the time-reversal waves. The time-reversal waves do not produce any time-reversal mutual energy flow. The mutual energy flow is produced by the synchronized of the retarded wave and the advanced wave. Since the retarded waves and the advanced wave are randomly sends out by the emitters and absorbers, The synchronization of the two waves is also a random events. Hence, the photon sends from the source can only be randomly received by many absorbers in the screen. Since the mutual energy flow is produced by the retarded wave and the advanced wave, if there are double slits, the mutual energy flow can have interference stripes.

3) Anti-photon. Anti-photon is just the mirror situation of the photon. Anti-photon can cancel the normal photon. However, normally the anti-photon happens when the energy is equal to a half-photon or a partial photon. Since the exist of the the anti-photon, all half-photons and partial photons are eliminated.

4) Since there is the anti-photon, the energy can oscillate between the light source and the light receive screen. Among these oscillates, perhaps there will some absorber received much much more energy, if received energy larger than a photon, the absorber can receive, this energy is a photon. The overplus of the energy is sent back through an anti-photons. In the photon- receiving screen, the place where can received a photon is propositional to the size of the area and the square of the amplitude of the retarded wave. This explains the reason of the probability in the quantum mechanics.

It should be noticed, the charge movement of the absorber is not because of the the electromagnetic field of the emitter. This movement randomly happens and is independent to the emitter. If the movement of the charge in absorber is because of the emitter, for example the emitter's electromagnetic field drive the charge to move, if the absorber become infinite far a way from the emitter, the field become infinite small, and there is no any possibility to obtained enough energy as large as a photon. However we know the absorber can always receive a whole photon even it is infinite far away from the emitter.

According to the traditional electromagnetic field theory, the charge movement is because of the field of the source. The energy flow is carried by the energy flow corresponding to the Poynting vector, which is the self-energy flow of the charge. When the absorber is far away from the emitter, the absorber can receive one whole photon means its effective scattering section area become infinite. A emitter can have a infinite scattering section area means the energy flow calculation based on the Poynting vector is wrong. Self-energy flow do not carry the energy, the energy flow is the mutual energy flow.

If we apply the theory of the mutual energy principle, the current of the absorber is independent to the current of the emitter. Both are randomly run, however there is a energy flow from emitter go to the absorber.

This can be widen to the tradition electromagnetic field theory, for example a current inside a receiving antenna or a secondary coil of a transformer, is not caused by the electromagnetic field of the transmitting antenna or primary coil of the transformer. The charge inside the receiving antenna or a secondary coil of a transformer runs randomly and spontaneously. The movement of the the charge inside the receiving antenna or a secondary coil can obtained energy from through the mutual energy flow built between the transmitting antenna and the receiving antenna or between the primary coil and the secondary coil. When this charge received the energy the movement is encouraged and confirmed, this will further increase the movement. Otherwise there is always a randomly happened movement which is at the opposite direction and will cancel the movement.

5 Different axiom systems

In this section we discuss which formula should be chosen as axioms. What is the advantage and disadvantage of these choices.

5.1 The mutual energy principle can be applied as axiom

What should be chosen as axioms for the radiation system. For example in the case of photon see subsection 4.2, the axioms can be chosen is the mutual energy principle, The Maxwell equations can be derived from the mutual energy principle, hence, the Maxwell equations do not need to be applied as axioms. The time-reversal Maxwell equations and the self-energy principle should also be applied as axioms. The time-reversal Maxwell equations has two equations which can be replaced by the corresponding Poynting theorem, which is only one formula. This axiom system has 3 formulas, i.e., one mutual energy principle, one self-energy principle and one time-reversal Poynting theorem. The self-energy principle which tell us that the self-energy flow is canceled by the time-reversal self-energy flow.

In this situation the superposition principle is not necessary. The superposition principle can be derived from the mutual energy principle. Assume in the system there are N charges.

Axiom 1) the mutual energy principle:

$$-\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\oint\!\!\!\oint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma dt$$

$$\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_j+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_j)dVdt$$

$$+\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_j)dVdt \tag{73}$$

Axiom 2) time-reversal Poynting theorem:

$$\int_{-\infty}^{\infty}\oint\!\!\!\oint_{\Gamma}(\boldsymbol{e}_i\times\boldsymbol{h}_i)\cdot\hat{n}d\Gamma dt$$

$$=\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{e}_i\frac{\partial}{\partial t}\boldsymbol{e}_i+\mu\boldsymbol{h}_i\frac{\partial}{\partial t}\boldsymbol{h}_i)dVdt$$

$$+\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{e}_i\cdot\boldsymbol{j}_i)dVdt \tag{74}$$

One Poynting theorem is equivalent to the two Maxwell equation formulas. This is also true to the time-reversal Poynting theorem.

Axiom 3) the self-energy principle:

$$-\int_{-\infty}^{\infty} \oint\!\!\!\oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma dt + \int_{-\infty}^{\infty} \oint\!\!\!\oint_{\Gamma} (\boldsymbol{e}_i \times \boldsymbol{h_i}) \cdot \hat{n} d\Gamma dt = 0 \tag{75}$$

$$\int_{-\infty}^{\infty} \iiint_V (\epsilon \boldsymbol{E}_i \frac{\partial}{\partial t} \boldsymbol{E}_i + \mu \boldsymbol{H}_i \frac{\partial}{\partial t} \boldsymbol{H}_i) dV dt + \int_{-\infty}^{\infty} \iiint_V (\epsilon \boldsymbol{e}_i \frac{\partial}{\partial t} \boldsymbol{e}_i + \mu \boldsymbol{h}_i \frac{\partial}{\partial t} \boldsymbol{h}_i) dV dt = 0 \tag{76}$$

$$\int_{-\infty}^{\infty} \iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_i) dV dt + \iiint_V (\boldsymbol{e}_i \cdot \boldsymbol{j}_i) dV dt = 0 \tag{77}$$

This 3 formulas can be summed together and hence put in one formula. Hence, we only count them as one formula.

In this situation Maxwell equations can be derived from the mutual energy principle. This has been done in subsection 2.2.

The superposition principle can also be derived from the mutual energy principle. However, it is should be noticed, the superposition principle derived from the mutual energy principle is a reduced superposition principle, which says that all the retarded waves can be superposed or all advanced waves can be superposed. Or the superposition is that with a test charge. This problem is more complicated we have a separated paper to deal the problem [12].

It should be make clear about the current $\boldsymbol{J}_i$. $\boldsymbol{J}_i$ is current when an electron from higher energy level spring down to an energy lower level or an electron from a lower level spring up to a higher level. This current doing work to both mutual energy flow and self-energy flow. For simple we assume $N = 2$. Considering $\boldsymbol{J}_1$ is the source. The work is done for the mutual energy is

$$\int_{-\infty}^{\infty} \iiint_V (\boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV dt \tag{78}$$

The work is done for the self energy is,

$$\int_{-\infty}^{\infty} \iiint_V (\boldsymbol{E}_1 \cdot \boldsymbol{J}_1) dV dt \tag{79}$$

$\boldsymbol{J}_1$ can be seen as a current source, $\boldsymbol{E}_2$ and $\boldsymbol{E}_1$ can be seen as two loads to the source $\boldsymbol{J}_1$. The two loads are connected in series, See Figure 6.

Since the two loads are connected in series, it can be divided as two independent circuits Figure .

For $\boldsymbol{j}_1$ it connected also a load $\boldsymbol{e}_i$ see Figure 8.

The two works done by the current $\boldsymbol{J}_1$ and the return current $\boldsymbol{j}_1$ has canceled part, the results of cancellation is that only the work corresponding to the mutual energy is left, see Figure 9.

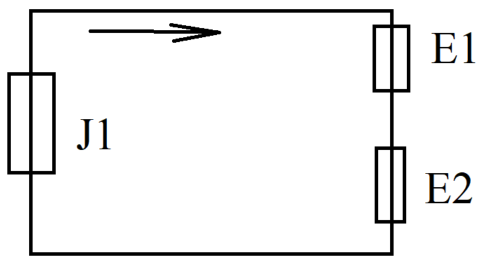

Figure 6: Two loads are connected as series.

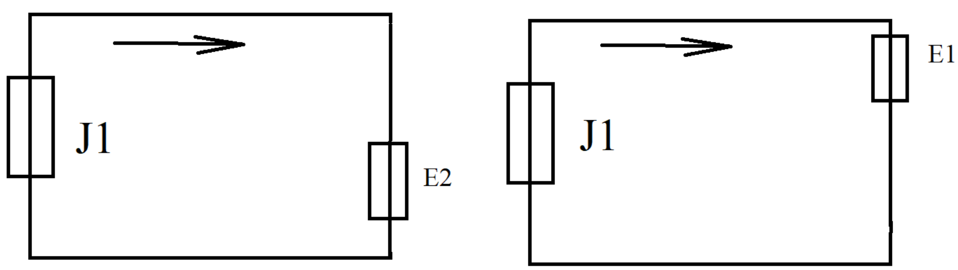

Figure 7: Two loads can be see as two separated circuits.

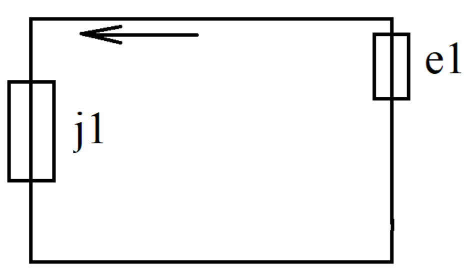

Figure 8: The work is done by the self-energy.

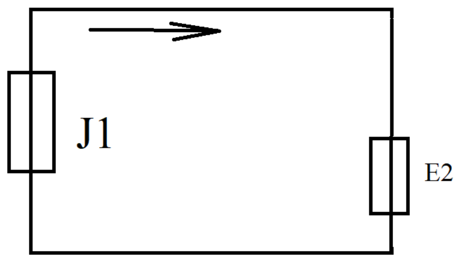

Figure 9: The work is done by the mutual energy is left.

That is the following part of work corresponding to the mutual energy is left.

$$\int_{-\infty}^{\infty} \iiint_V (\boldsymbol{E}_2 \cdot \boldsymbol{J}_1) dV dt \tag{80}$$

This means the self-energy principle allow the time-reversal Maxwell equations cancels the contribution of the Maxwell equations. After this, only the work corresponding to the mutual energy survived.

5.2 Maxwell equations for the superposed field and superposition principle as axiom

Axiom 1) the the Maxwell equations for the superposed field, which is, $\boldsymbol{E}, \boldsymbol{H}$. The Maxwell equation can be replaced by the Poynting theorems:

$$-\int_{-\infty}^{\infty} \oiint_\Gamma (\boldsymbol{E} \times \boldsymbol{H}) \cdot \hat{n} d\Gamma dt$$
$$= \int_{-\infty}^{\infty} \iiint_V (\epsilon \boldsymbol{E} \frac{\partial}{\partial t} \boldsymbol{E} + \mu \boldsymbol{H} \frac{\partial}{\partial t} \boldsymbol{H}) dV dt$$
$$+ \int_{-\infty}^{\infty} \iiint_V (\boldsymbol{E} \cdot \boldsymbol{J}) dV dt \tag{81}$$

Axiom 2) the superposition principle which are:

$$\begin{cases} \boldsymbol{E} = \sum_{i=1}^N \boldsymbol{E}_i \\ \boldsymbol{H} = \sum_{i=1}^N \boldsymbol{H}_i \\ \boldsymbol{J} = \sum_{i=1}^N \boldsymbol{J}_i \end{cases} \tag{82}$$

Axiom 3) time-reversal Poynting theorem:

$$\int_{-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{e}_i \times \boldsymbol{h}_i) \cdot \hat{n} d\Gamma dt$$
$$= \int_{-\infty}^{\infty} \iiint_{V} (\epsilon \boldsymbol{e}_i \frac{\partial}{\partial t} \boldsymbol{e}_i + \mu \boldsymbol{h}_i \frac{\partial}{\partial t} \boldsymbol{h}_i) dV dt$$
$$+ \int_{-\infty}^{\infty} \iiint_{V} (\boldsymbol{e}_i \cdot \boldsymbol{j}_i) dV dt \tag{83}$$

Axiom 4) the self-energy principle:

$$- \int_{-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma dt + \int_{-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{e}_i \times \boldsymbol{h}_i) \cdot \hat{n} d\Gamma dt = 0 \tag{84}$$

$$\int_{-\infty}^{\infty} \iiint_{V} (\epsilon \boldsymbol{E}_i \frac{\partial}{\partial t} \boldsymbol{E}_i + \mu \boldsymbol{H}_i \frac{\partial}{\partial t} \boldsymbol{H}_i) dV dt + \int_{-\infty}^{\infty} \iiint_{V} (\epsilon \boldsymbol{e}_i \frac{\partial}{\partial t} \boldsymbol{e}_i + \mu \boldsymbol{h}_i \frac{\partial}{\partial t} \boldsymbol{h}_i) dV dt = 0 \tag{85}$$

$$\int_{-\infty}^{\infty} \iiint_{V} (\boldsymbol{E}_i \cdot \boldsymbol{J}_i) dV dt + \iiint_{V} (\boldsymbol{e}_i \cdot \boldsymbol{j}_i) dV dt = 0 \tag{86}$$

In this situation, substitute axiom 2) to axiom 1) we can obtain,

$$- \sum_{i=1}^{N} \sum_{i=1}^{N} \int_{-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot \hat{n} d\Gamma dt$$
$$= \sum_{i=1}^{N} \sum_{i=1}^{N} \int_{-\infty}^{\infty} \iiint_{V} (\epsilon \boldsymbol{E}_i \frac{\partial}{\partial t} \boldsymbol{E}_j + \mu \boldsymbol{H}_i \frac{\partial}{\partial t} \boldsymbol{H}_j) dV dt$$
$$+ \sum_{i=1}^{N} \sum_{i=1}^{N} \int_{-\infty}^{\infty} \iiint_{V} (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV dt \tag{87}$$

consider the self-energy principle 4) the self-energy terms has no contribution to the energy transfer, it can taken away from the above formula that means all the terms

$$- \sum_{i=1}^{N} \int_{-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma dt$$

$$\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_i+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_i)dVdt$$

$$+\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_i)dVdt \tag{88}$$

can be taken away from Eq.(87). Hence we obtain,

$$-\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma dt$$

$$\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_j+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_j)dVdt$$

$$+\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_j)dVdt \tag{89}$$

This is the mutual energy principle. Hence, we have proved from the second axiom system to the first axiom system.

5.3 Maxwell equations for the single charge is applied as axioms

Maxwell equations for single charge can be written as,

$$L\xi=\frac{1}{\epsilon}\tau \tag{90}$$

where

$$L=\begin{bmatrix} -\frac{\partial}{\partial t} & \frac{1}{\epsilon}\nabla\times \\ -\frac{1}{\mu}\nabla\times & -\frac{\partial}{\partial t}\end{bmatrix} \tag{91}$$

From this we can derive the mutual energy principle, by using the Mathematical formula,

$$\int_{-\infty}^{\infty}((\xi_2,L\xi_1)_V+(L\xi_2,\xi_1)_V)dt$$

$$=-\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_1\frac{\partial}{\partial t}\boldsymbol{E}_2+\epsilon\boldsymbol{E}_2\frac{\partial}{\partial t}\boldsymbol{E}_1+\mu\boldsymbol{H}_1\frac{\partial}{\partial t}\boldsymbol{H}_2+\mu\boldsymbol{H}_2\frac{\partial}{\partial t}\boldsymbol{H}_1)dV$$

$$-\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma \tag{92}$$

Hence, we obtained the mutual energy principle,

$$\int_{-\infty}^{\infty}((\xi_2,\frac{1}{\epsilon}\tau_1)_V+(\frac{1}{\epsilon}\tau_2,\xi_1)_V)dt$$

$$=-\int_{-\infty}^{\infty}\iiint_V(\epsilon\boldsymbol{E}_1\frac{\partial}{\partial t}\boldsymbol{E}_2+\epsilon\boldsymbol{E}_2\frac{\partial}{\partial t}\boldsymbol{E}_1+\mu\boldsymbol{H}_1\frac{\partial}{\partial t}\boldsymbol{H}_2+\mu\boldsymbol{H}_2\frac{\partial}{\partial t}\boldsymbol{H}_1)dV$$

$$-\oiint_{\Gamma}(\boldsymbol{E}_1\times\boldsymbol{H}_2+\boldsymbol{E}_2\times\boldsymbol{H}_1)\cdot\hat{n}d\Gamma \tag{93}$$

Then we need also add the self-energy principle and time-reversal Maxwell equations.

5.4 Compare the two axiom systems

The advantage of the first axiom system is the formulas are less. The mutual energy principle is applied as axiom which can easily to derive the mutual energy theorem and mutual energy flow theorem. The Maxwell equation can be derived from the Mutual energy principle. Here, the important thing is there are two group Maxwell equations that are synchronized. These theorem can be applied to electromagnetic field calculation.

The advantage axiom of the second system is it more close to the current text book of the electromagnetic field theory. We only need to add the self-energy principle and the time-reversal Poynting theorem on the top of the original electromagnetic field theory of the current text book.

The advantage axiom of the third system is that it also works. The only things is if the Maxwell equations of single charge applied as axiom, when we derive the formula of mutual energy principle, it is difficult to realize that the mutual energy principle is the real physical formula, hence, we need to solve the mutual energy principle to obtain a pair of single charge. Hence, if the Maxwell equation for single charge used as axiom we should mention that the Maxwell equations should paired. One is for the retarded wave and the other is for advanced wave. However, this make things too complicated. The complicate things for example, it the retarded wave and the advanced wave should put in the simple formula and is derived later.

Hence, this author would like to apply the mutual energy principle as the axioms for electromagnetic field theory. The mutual energy principle should be derived out from the original electromagnetic field theory plus to consider the energy conservation law.

6 Compare the theory of the mutual energy principle with the traditional electromagnetic theory

In this section we summary what the difference is between the author's theory of the mutual energy principle and the traditional electromagnetic theory.

The author's theory can also started from Maxwell equations for single charge,

$$\begin{cases} -\frac{\partial}{\partial t}(\epsilon \boldsymbol{E}_i) + \nabla \times \boldsymbol{H}_i = \boldsymbol{J}_i \\ -\nabla \times \boldsymbol{E}_i - \frac{\partial}{\partial t}(\mu \boldsymbol{H}_i) = 0 \end{cases} \tag{94}$$

This author assume that the energy conservation law is correct:

$$\sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \int_{t=-\infty}^{\infty} \iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV dt = 0 \tag{95}$$

However, this author does not assume the superposed Maxwell equations together with superposition principle is correct,

$$\begin{cases} -\frac{\partial}{\partial t}(\epsilon \boldsymbol{E}) + \nabla \times \boldsymbol{H} = \boldsymbol{J} \\ -\nabla \times \boldsymbol{E} - \frac{\partial}{\partial t}(\mu \boldsymbol{H}) = 0 \end{cases} \tag{96}$$

$$\begin{cases} \boldsymbol{E} = \sum_{i=1}^{N} \boldsymbol{E}_i \\ \boldsymbol{H} = \sum_{i=1}^{N} \boldsymbol{H}_i \\ \boldsymbol{J} = \sum_{i=1}^{N} \boldsymbol{J}_i \end{cases} \tag{97}$$

From Maxwell equations Eq.(94) we can derived the mutual energy principle,

$$-\sum_{i=1}^{N} \sum_{i=1, i\neq j}^{N} \int_{-\infty}^{\infty} \oiint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_j) \cdot \hat{n} d\Gamma dt$$

$$\sum_{i=1}^{N} \sum_{i=1, i\neq j}^{N} \int_{-\infty}^{\infty} \iiint_V (\epsilon \boldsymbol{E}_i \frac{\partial}{\partial t} \boldsymbol{E}_j + \mu \boldsymbol{H}_i \frac{\partial}{\partial t} \boldsymbol{H}_j) dV dt$$

$$+\sum_{i=1}^{N} \sum_{i=1, i\neq j}^{N} \int_{-\infty}^{\infty} \iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV dt \tag{98}$$

From the mutual energy principle, we can derive the mutual energy theorem which is just the energy conservation law Eq.(95). This further tell us the mutual energy principle is correct. From mutual energy principle we can further derive that the Maxwell equations which must pair together, i.e. there are two groups Maxwell equations, one is for the retarded wave and another is for the advanced wave.

The next step is consider the superposition principle with the Maxwell equations Eq.(96). This Maxwell equations are not same as the Maxwell equations for N charges. Substitute the superposition principle to the Maxwell equations we obtain,

$$-\sum_{i=1}^{N}\sum_{i=1}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma dt$$

$$=\sum_{i=1}^{N}\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_j+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_j)dVdt$$

$$+\sum_{i=1}^{N}\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_j)dVdt \qquad (99)$$

This can be divided as the mutual energy part,

$$-\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_j)\cdot\hat{n}d\Gamma dt$$

$$=\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_j+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_j)dVdt$$

$$+\sum_{i=1}^{N}\sum_{i=1,i\neq j}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_j)dVdt \qquad (100)$$

and the self-energy part,

$$-\sum_{i=1}^{N}\int_{-\infty}^{\infty}\oiint_{\Gamma}(\boldsymbol{E}_i\times\boldsymbol{H}_i)\cdot\hat{n}d\Gamma dt$$

$$\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\epsilon\boldsymbol{E}_i\frac{\partial}{\partial t}\boldsymbol{E}_i+\mu\boldsymbol{H}_i\frac{\partial}{\partial t}\boldsymbol{H}_i)dVdt$$

$$+\sum_{i=1}^{N}\int_{-\infty}^{\infty}\iiint_{V}(\boldsymbol{E}_i\cdot\boldsymbol{J}_i)dVdt \qquad (101)$$

Eq.(100) is the mutual energy principle which we have accept it as correct, but we got another extra-formula Eq.(101). In order to solve the problem of the extra-formula we have to introduce the self-energy principle and the time-reversal Maxwell equations,

$$\begin{cases} -\frac{\partial}{\partial t}(\epsilon \boldsymbol{e}_i) - \nabla \times \boldsymbol{h}_i = \boldsymbol{j}_i \\ \nabla \times \boldsymbol{e}_i - \frac{\partial}{\partial t}(\mu \boldsymbol{h}_i) = 0 \end{cases} \tag{102}$$

and the self-energy principle,

$$-\int_{-\infty}^{\infty} \oint\!\!\!\oint_{\Gamma} (\boldsymbol{E}_i \times \boldsymbol{H}_i) \cdot \hat{n} d\Gamma dt + \int_{-\infty}^{\infty} \oint\!\!\!\oint_{\Gamma} (\boldsymbol{e}_i \times \boldsymbol{h}_{\boldsymbol{i}}) \cdot \hat{n} d\Gamma dt = 0 \tag{103}$$

$$\int_{-\infty}^{\infty} \iiint_V (\epsilon \boldsymbol{E}_i \frac{\partial}{\partial t} \boldsymbol{E}_i + \mu \boldsymbol{H}_i \frac{\partial}{\partial t} \boldsymbol{H}_i) dV dt + \int_{-\infty}^{\infty} \iiint_V (\epsilon \boldsymbol{e}_i \frac{\partial}{\partial t} \boldsymbol{e}_i + \mu \boldsymbol{h}_i \frac{\partial}{\partial t} \boldsymbol{h}_i) dV dt = 0 \tag{104}$$

$$\int_{-\infty}^{\infty} \iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_i) dV dt + \iiint_V (\boldsymbol{e}_i \cdot \boldsymbol{j}_i) dV dt = 0 \tag{105}$$

After we have assumed the above self-energy principle and time-reversal Maxwell equations, the superposition principle superposed Maxwell equations can also be applied. The conflict of the electromagnetic field theory is solved.

In the derivation of this section we can see that we still started from Maxwell equations for single charge.

For myself, we first meet the mutual energy theorem for two antenna,

$$\iiint_{V_2} (\boldsymbol{E}_1(\omega) \cdot \boldsymbol{J}_2^*(\omega) dV = -\iiint_{V_1} \boldsymbol{E}_2^*(\omega) \cdot \boldsymbol{J}_1(\omega)) dV \tag{106}$$

I did not realize it is an energy conservation law. However when I wrote the widened version for N charges,

$$\sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \int_{t=-\infty}^{\infty} \iiint_V (\boldsymbol{E}_i \cdot \boldsymbol{J}_j) dV dt = 0 \tag{107}$$

This author realize it is a energy conservation law. Eq.(106) can be easily understand as a energy theorem, but it is difficult to be realize as an energy conservation law for only two charges.

Hence, this author would just started from Maxwell equations, that is traditional electromagnetic field theory. On this base plus the energy conservation law. Pointer out the conflict. From this conflict, we obtained the formula of the mutual energy principle. Then we need point out the mutual energy principle now is the new axiom of the electromagnetic field theory.

Then started from the mutual energy principle, we can derive the pair of Maxwell equations, one is for the retarded wave, one is for the advanced wave. When the we obtained two Maxwell equations we can derive the Poynting theorems, the Poynting theorem will tell us there will have the self-energy flow, the self-energy flow offers additional energy on the top of the energy conservation law. In order to satisfy the energy conservation law, we must introduce the self-energy principle and the time-reversal Maxwell equations. After introduced the time-reversal Maxwell equations and self-energy principle, the self-energy flow do not have any contribution to the energy transfer. This further guarantees the energy conservation law is satisfied.

7 Conclusion

This article compared the two axioms system, one is Maxwell equations, another one is the mutual energy principle and self-energy principle introduced by this author. The two system are equivalent in the meaning they can be derived by each other. The mutual energy principle can be derived from the Maxwell equations, the Maxwell equations can also be derived from the mutual energy principle. However the two systems are not exactly same. That is because if the mutual energy principle is applied as axiom, it need two groups Maxwell equations together as a solution. One group of the Maxwell equations is corresponding the retarded wave, another is corresponding the advanced wave. A retarded wave alone or an advanced wave alone is not the solution of the mutual energy principle, but it is a solution of the Maxwell equations.

This author also found that the Poynting theorem and hence, Maxwell equations together with the superposition principle conflict with the energy conservation law. However, the mutual energy principle does not directly conflict with the energy conservation law. The mutual energy principle only conflict with the energy conservation law through the Maxwell equations. Hence, the author realized that the Maxwell equations need to be modified. Hence, the author introduced the self-energy principle. The self-energy principle tell us that there are another kind of the electromagnetic fields, which satisfies the time-reversal Maxwell equations. The energy flow of the time-reversal wave can cancel the energy flow of the normal electromagnetic fields. The energy flow of the time-reversal wave corresponding to the retarded wave can cancel the energy flow of the retarded wave. The energy flow of the time-reversal wave corresponding to the advanced wave can cancel the energy flow of the advanced wave. All self-energy flows are all canceled. Hence, the self-energy flow do not carry the energy. However, the mutual energy flows survived. There are two kinds of mutual energy flows: the mutual energy flow and the time-reversal mutual energy flow. The energy of photon is transferred by the mutual energy flow. Hence, we can say that photon is the mutual energy flow. The transferred energy from the transmitting antenna to the receiving antenna is also the mutual energy flow. The mutual energy theorem and the mutual energy flow theorem guarantees the energy is transferred by the mutual energy flow. The self-energy principle guarantees the self-energy flow do not transfer or carry any energy. The self-energy flow is the energy flow corresponding the Poynting vector. The time-reversal mutual energy flow can eliminate the half or partial photons which we have discussed before on another article the "mutual energy flow interpretation of quantum mechanics".

In this article we have also introduced 3 modes of radiation: (1) photons, (2) anti-photons, (3) unsuccessful radiation. Photon is the mutual energy flow. Anti-photon is the time-reversal mutual energy flow. Unsuccessful radiation is the situation the retarded wave is sent out but it did not found any advanced wave to synchronized and hence collapse back. The energy of a photon is transferred by the mutual energy flow. The self-energy principle guarantees the energy still travel in space will be cleared. The retarded wave and the advanced wave together build the the mutual energy flow, but the mutual energy flow is additional energy flow, the original energy flow of the retarded wave and the advanced wave still in space. It need to be cleared.

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