Cutting the Gordian Knot of Theoretical Physics (The Unification of Gravitational and Maxwellian Fields)

Rolf Warnemünde

Rolf Warnemünde
E-Mail: rolf.warnemuende@t-online.de

Abstract

It concerns the unification of Maxwell-Field and Gravitational-Field without compromise consisting of:

1. *A derivation of the general equations of continuously differentiable fluctuating 3-dimensional vector fields turning out to be generalized Maxwell-Equations*
2. *Identifying the Einstein-Space as the result of deforming an Euclidean Space,*
3. *Identifying the fluctuating hypersurface of the Einstein-Space as gravitational wave propagating with the velocity of light seen from an observer space or rather coordinate space,*

This leads to

1. *the quantitative unification of Maxwell-Field and Gravitational-Field,*
2. *the facilitation of quantizing gravitational fields,*
3. *considerations of general gravitational waves from a new perspective.*

With the described unification electromagnetism is directly led back to the most fundamental terms of physics, space and time.**Last but not least the importance of the Einstein-Equations for microphysics is proved**

Contents

1 Introduction

In this treatise gravitational and electrodynamic field are unified not forces as is wished from an elementary particle physics perspective.
The unification of physically different fields requires a uniform mathematical description. This is for the gravitational and the electromagnetic field not evident. The gravitational field is seen as the consequence of the curved Space-Time characterized by nonlinear differential geometric formulations. The electromagnetic field satisfies the requirements for the linear Maxwell-Equations. In physics it is rated as sure knowledge that nonlinear and linear fields are assessed totally differently. On the other hand electrodynamic fields are suggestive of beeing properties of Space-Time itself. How to cut this Gordian knot proceeds as follows:
At first, it is searched for fluctuation equations of general 3-dimendional sufficiently often continuously differentiable vector fields. This is achieved by finding out the connection of a stochastic ensemble consideration of an unlimited number of existent deterministic fluctuating continuum fields with the deterministic consideration of a single ensemble system resulting in quasi-linear generalized Maxwell Equations. Requiring constant propagation speed the linear vacuum Maxwell Equations are found. The mentioned Gordian knot is cut considering the movements of the Riemannian hypersurface of the Einstein-Space as deformation fluctuations of a suitable Euclidean observer space. As these fluctuations proceed with light velocity the fluctuations are decribed by the usual Maxwell Equations. So there are the following results:

1. *the quantitative unification of Maxwell-Field and Gravitational-Field,*
2. *the facilitation of quantizing gravitational fields,*
3. *considerations of general gravitational waves from a new perspective.*

With the described unification electromagnetism is directly led back to the most fundamental terms of physics, space and time.
Last but not least the importance of the Einstein-Equations for microphysics is proved.

2 Stochastic and deterministic general vector fields

$$f_{t_\epsilon}(\vec{\mathbf{x}},t,\vec{\mathbf{E}},\vec{\mathbf{B}}) = \int\limits_{\vec{\mathbf{B}}}\int\limits_{\vec{\mathbf{E}}} W_{t_\epsilon}(\vec{\mathbf{x}},t,\vec{\mathbf{E}},\vec{\mathbf{B}},\vec{\mathbf{E}}',\vec{\mathbf{B}}')\cdot f_{t_\epsilon}(\vec{\mathbf{x}}-\Delta\vec{\mathbf{x}},t-t_\epsilon,\vec{\mathbf{E}}',\vec{\mathbf{B}}')d\vec{\mathbf{E}}'d\vec{\mathbf{B}}'$$
$$\Updownarrow$$
$$\frac{\partial}{\partial t}\vec{\mathbf{B}}-\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{E}}=0$$
$$\frac{\partial}{\partial t}\left(\frac{B^2}{E^2}\cdot\vec{\mathbf{E}}\right)+\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{B}}=0$$

Subsequently continuum fluctuations of general 3 dimensional vector fields $\vec{\mathbf{A}}(\vec{\mathbf{x}},t)$ with $\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{A}}\neq\mathbf{0}$ are analysed. They have to be sufficiently often continuously differentiable. Defining the vector fields $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ by

$$\begin{aligned}\vec{\mathbf{E}} &= \partial\vec{\mathbf{A}}/\partial t \neq 0\\ \vec{\mathbf{B}} &= \vec{\boldsymbol{\nabla}}\times\vec{\mathbf{A}}\neq 0\end{aligned} \tag{2.1}$$

and owing to the exchangeability of the operators $\partial/\partial t$ und $\vec{\boldsymbol{\nabla}}\times$

$$\frac{\partial\vec{\mathbf{B}}}{\partial t}=\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{E}} \tag{2.2}$$

follows. This is a necessary consequence of the condition of the continuous differentiability of $\vec{\mathbf{A}}(\vec{\mathbf{x}},t)$. This relation is known according to the Maxwell Equations. The for this purpose dual equation is subsequently beeing looked for. A stochastic continuum process in the frame of an ensemble theory is formulated such that according to a deterministic theory the already known as well as the related dual equation arise with fluctuating quantities $\vec{\mathbf{E}}$ und $\vec{\mathbf{B}}$.

2.1 The Transition: stochastic theory $\longleftrightarrow$ deterministic theory

Every space-time-point$(\vec{\mathbf{x}}, t)$ a continuously differentiable distribution density $f_{\boldsymbol{t_\epsilon}}$ is assigned to the motion quantities $\vec{\mathbf{E}}_{t_\epsilon} = \partial\vec{\mathbf{A}}_{t_\epsilon}/\partial t$ and $\vec{\mathbf{B}}_{t_\epsilon} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{A}}_{t_\epsilon}$ with

$$f_{\boldsymbol{t_\epsilon}} = f_{\boldsymbol{t_\epsilon}}(\vec{\mathbf{x}}, t, \vec{\mathbf{E}}, \vec{\mathbf{B}}). \tag{2.3}$$

In the with $\boldsymbol{t_\epsilon}$ or $\boldsymbol{\epsilon}$ indexed functions $f_{\boldsymbol{t_\epsilon}}$ it is automatically assumed that the included motion quantities $(\vec{\mathbf{E}}, \vec{\mathbf{B}})$ are assigned to a $\boldsymbol{t_\epsilon}$-measurement accuracy. The indexing of the motion quantities may be omitted in functions appropriately indexed themselves.

After the execution of a $\lim \boldsymbol{t_\epsilon} \to 0$-process

$$\lim_{\boldsymbol{t_\epsilon} \to 0} f_{\boldsymbol{t_\epsilon}}(\vec{\mathbf{x}}, t, \vec{\mathbf{E}}, \vec{\mathbf{B}}) = f(\vec{\mathbf{x}}, t, \vec{\mathbf{E}}, \vec{\mathbf{B}}) \tag{2.4}$$

f and $(\vec{\boldsymbol{E}}, \vec{\boldsymbol{B}})$ are understood in the sense of an exact measurement process.

The stochastic transport of the fluctuation quantities

$$\left(\vec{\mathbf{E}}'_{t_\epsilon}(\vec{\boldsymbol{x}} - \varDelta\vec{\boldsymbol{x}}, t - t_\epsilon), \vec{\mathbf{B}}'_{t_\epsilon}(\vec{\boldsymbol{x}} - \varDelta\vec{\boldsymbol{x}}, t - t_\epsilon)\right) \longrightarrow \left(\vec{\mathbf{E}}_{t_\epsilon}(\vec{\boldsymbol{x}}, t), \vec{\mathbf{B}}_{t_\epsilon}(\vec{\boldsymbol{x}}, t)\right)$$

happens by the transition probability density $W_{t_\epsilon} = W_{t_\epsilon}(\vec{\boldsymbol{x}}, t, \vec{\mathbf{E}}, \vec{\mathbf{B}}, \vec{\mathbf{E}}', \vec{\mathbf{B}}')$ with

$$\boxed{\begin{aligned} \lim_{t_\epsilon \to 0} W_{t_\epsilon} &= \delta(\vec{\mathbf{E}}, \vec{\mathbf{B}}; \vec{\mathbf{E}}', \vec{\mathbf{B}}') \\ f_{t_\epsilon}(\vec{\mathbf{x}}, t, \vec{\mathbf{E}}, \vec{\mathbf{B}}) &= \int_{\vec{\mathbf{B}}'} \int_{\vec{\mathbf{E}}'} W_{t_\epsilon}(\vec{\mathbf{x}}, t, \vec{\mathbf{E}}, \vec{\mathbf{B}}, \vec{\mathbf{E}}', \vec{\mathbf{B}}') \cdot f_{t_\epsilon}(\vec{\mathbf{x}} - \Delta\vec{\mathbf{x}}, t - t_\epsilon, \vec{\mathbf{E}}', \vec{\mathbf{B}}') d\vec{\mathbf{E}}' d\vec{\mathbf{B}}' \\ \Delta\vec{\mathbf{x}} &= t_\epsilon \cdot \vec{\mathbf{E}}' \times \frac{\vec{\mathbf{B}}'}{B'^2} \text{ and } \vec{\mathbf{E}}' \times \frac{\vec{\mathbf{B}}'}{B'^2} = \text{velocity of propagation.} \end{aligned}} \tag{2.5}$$

These equations define stochastic continuum fluctuations of the quantities $\vec{\mathbf{E}}$ und $\vec{\mathbf{B}}$ in the sense of an ensemble-theory and represent a Markov Process of natural causality. The test-functions of distribution theory obtain by this formulation of a transition probability density W_{t_ϵ} an immediate physical meaning.

f_{t_ϵ} is developed until the 1st order about $(\vec{\mathbf{x}},t) \Longrightarrow$

$$\begin{gathered} f_{t_\epsilon}(t-t_\epsilon,\vec{\mathbf{x}}-\triangle\vec{\mathbf{x}},\vec{\mathbf{E}}',\vec{\mathbf{B}}') = f'_{t_\epsilon} - \frac{\partial f'_{t_\epsilon}}{\partial t}\cdot t_\epsilon - \triangle\vec{\mathbf{x}}\cdot\vec{\boldsymbol{\nabla}}f'_{t_\epsilon} + \boldsymbol{O}({t_\epsilon}^2) \\ f'_{t_\epsilon} = f_{t_\epsilon}(\vec{\mathbf{x}},t,\vec{\mathbf{E}}',\vec{\mathbf{B}}') \end{gathered} \tag{2.6}$$

und one gets

$$\int_{\vec{\mathbf{E}}}\int_{\vec{\mathbf{B}}} W_{t_\epsilon}\left[\frac{\partial f'_{t_\epsilon}}{\partial t}+\vec{\mathbf{E}}'\times\frac{\vec{\mathbf{B}}'}{B'^2}\cdot\vec{\boldsymbol{\nabla}}f'_{t_\epsilon}\right]d\vec{\mathbf{E}}'d\vec{\mathbf{B}}'+\boldsymbol{O}({t_\epsilon}^2) = \frac{\int_{\vec{\mathbf{B}}}\int_{\vec{\mathbf{E}}} W_{t_\epsilon}f'_{t_\epsilon}d\vec{\mathbf{E}}'d\vec{\mathbf{B}}' - f_{t_\epsilon}}{t_\epsilon}. \tag{2.7}$$

By the process $t_\epsilon \to 0$ W_{t_ϵ} degenerates to a δ-function:

$$\lim_{\boldsymbol{t_\epsilon\to 0}} W_{t_\epsilon} = \delta(\vec{\mathbf{E}},\vec{\mathbf{B}};\vec{\mathbf{E}}',\vec{\mathbf{B}}') \tag{2.8}$$

$\lim \boldsymbol{t_\epsilon} \to 0$ applied leads to

$$\frac{\partial f}{\partial t} + \vec{\mathbf{E}}\times\frac{\vec{\mathbf{B}}}{B^2}\cdot\vec{\boldsymbol{\nabla}}f = \lim_{t_\epsilon\to 0}\frac{\int_{\vec{\mathbf{E}}}\int_{\vec{\mathbf{B}}} W_{t_\epsilon}f'_{t_\epsilon}d\vec{\mathbf{E}}'d\vec{\mathbf{B}}' - f_{t_\epsilon}}{t_\epsilon}. \tag{2.9}$$

Recovering equation (2.2) after the transition to deterministic consideration the exchange term has to vanish, in this case.

$$\lim_{t_\epsilon\to 0}\frac{\int_{\vec{\mathbf{B}}}\int_{\vec{\mathbf{E}}} W_{t_\epsilon}f'_{t_\epsilon}d\vec{\mathbf{E}}'d\vec{\mathbf{B}}' - f_{t_\epsilon}}{t_\epsilon} = \mathbf{0}. \tag{2.10}$$

This link is an integral part of the considered stochastic process.

Limiting ourselves to one system of the ensemble the function $f(\vec{\mathbf{x}},t,\vec{\mathbf{E}},\vec{\mathbf{B}})$ in the space-time-point $(\vec{\boldsymbol{x}},t)$ degenerates to a $\delta-$function

$$f(\vec{\mathbf{x}},t,\vec{\mathbf{E}},\vec{\mathbf{B}}) \longrightarrow \delta(\vec{\mathbf{E}}_{(\vec{\mathbf{x}},t)},\vec{\mathbf{B}}_{(\vec{\mathbf{x}},t)};\vec{\mathbf{E}},\vec{\mathbf{B}})\text{-function.} \tag{2.11}$$

From equation (2.9) arises the key-equation

$$\boxed{\frac{\partial}{\partial t}\delta + \vec{\mathbf{E}}_{(\vec{x},t)} \times \frac{\vec{\mathbf{B}}_{(\vec{x},t)}}{B^2_{(\vec{x},t)}} \cdot \vec{\boldsymbol{\nabla}}\delta = \mathbf{0}}. \tag{2.12}$$

The $\Xi[...]$-operator is inserted as follows

$$\begin{aligned}
\Xi\left[\int_{\vec{\mathbf{E}}}\int_{\vec{\mathbf{B}}} \delta(\vec{\mathbf{B}}_{(\vec{x},t)}, \vec{\mathbf{E}}_{(\vec{x},t)}; \vec{\mathbf{B}}, \vec{\mathbf{E}})\vec{\mathbf{B}} d\vec{\mathbf{B}} d\vec{\mathbf{E}}\right] &= \Xi[\vec{\mathbf{B}}_{(\vec{x},t)}] = \vec{\mathbf{B}}(\vec{x}, t) \\
\Xi\left[\int_{\vec{\mathbf{E}}}\int_{\vec{\mathbf{B}}} \delta(\vec{\mathbf{B}}_{(\vec{x},t)}, \vec{\mathbf{E}}_{(\vec{x},t)}; \vec{\mathbf{B}}, \vec{\mathbf{E}})\vec{\mathbf{E}} d\vec{\mathbf{B}} d\vec{\mathbf{E}}\right] &= \Xi\left[\vec{\mathbf{E}}_{(\vec{x},t)}\right] = \vec{\mathbf{E}}(\vec{x}, t)
\end{aligned} \tag{2.13}$$

or

$$\Xi\left[\int_{\vec{\mathbf{E}}}\int_{\vec{\mathbf{B}}} \delta(\vec{\mathbf{B}}_{(\vec{x},t)}, \vec{\mathbf{E}}_{(\vec{x},t)}; \vec{\mathbf{b}}, \vec{\mathbf{E}})\left(\frac{B^2}{E^2}\cdot\vec{\mathbf{E}}\right) d\vec{\mathbf{B}} d\vec{\mathbf{E}}\right] = \Xi\left[\frac{B^2_{(\vec{x},t)}}{E^2_{(\vec{x},t)}}\cdot\vec{\mathbf{E}}_{(\vec{x},t)}\right] = \frac{B^2(\vec{x},t)}{E^2(\vec{x},t)}\cdot\vec{\mathbf{E}}(\vec{x},t), \tag{2.14}$$

developing the deterministic equations from the key equation.

2.2 The deterministic fluctuation-equations

The key-equation (2.12) represents the interface for the transition of stochastic to deterministic consideration. From the perspective of statistics over the states of movement of the parallelly assumed deterministic processes in the respective point $(\vec{\mathbf{x}}, t)$ one is confined to a single system and such to a single state of motion $(\vec{\mathbf{E}}_{(\vec{\mathbf{x}},t)}, \vec{\mathbf{B}}_{(\vec{\mathbf{x}},t)})$. In this situation the vectors of the motion quantities may be pushed before and behind the differential operators

$$\begin{aligned}
\vec{\mathbf{E}}_{(\vec{x},t)} \times \frac{\vec{\mathbf{B}}_{(\vec{x},t)}}{B^2_{(\vec{x},t)}} \cdot \vec{\boldsymbol{\nabla}}\delta &= -\frac{\vec{\mathbf{B}}_{(\vec{x},t)}}{B^2_{(\vec{x},t)}} \times \vec{\mathbf{E}}_{(\vec{x},t)} \cdot \vec{\boldsymbol{\nabla}}\delta \\
&= -\frac{\vec{\mathbf{B}}_{(\vec{x},t)}}{B^2_{(\vec{x},t)}} \cdot \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}_{(\vec{x},t)}\delta
\end{aligned}$$

Further more there is

$$
\begin{aligned}
&\frac{\partial}{\partial t}\Big(\frac{\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t)} \cdot \vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t)}}{B^2_{(\vec{\boldsymbol{x}},t)}}\delta\Big) - \frac{\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t)}}{B^2_{(\vec{\boldsymbol{x}},t)}} \cdot \vec{\boldsymbol{\nabla}} \times (\vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t)}\delta) = 0 \\
\Longrightarrow\ &\frac{\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t)}}{B^2_{(\vec{\boldsymbol{x}},t)}} \cdot \Big[\frac{\partial}{\partial t}(\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t)}\delta) - \vec{\boldsymbol{\nabla}} \times (\vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t)}\delta)\Big] = 0 \\
\Longrightarrow\ &\frac{\partial}{\partial t}(\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t)}\delta) - \vec{\boldsymbol{\nabla}} \times (\vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t)}\delta) = 0.
\end{aligned}
\tag{2.15}
$$

Now the vector fields of the motion quantities $(\vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t)}, \vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t)})$ of the one determinstic system are created about the point $(\vec{\boldsymbol{x}}, t)$ and such the transition to the deterministic equations of the one system has succeeded.

One obtains

$$
\boldsymbol{\Xi}\left[\int_{\vec{\mathbf{B}}}\int_{\vec{\mathbf{E}}}\left[\frac{\partial}{\partial t}(\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t)}\delta) - \vec{\boldsymbol{\nabla}} \times (\vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t)}\delta) = 0\right] d\vec{\mathbf{E}} d\vec{\mathbf{B}}\right]. \tag{2.16}
$$

As integration and differentiation are exchangeable $\Longrightarrow$

$$
\frac{\partial}{\partial t}\boldsymbol{\Xi}[\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t)}] - \vec{\boldsymbol{\nabla}} \times \boldsymbol{\Xi}[\vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t)}] = 0 \tag{2.17}
$$

and it results in the 1.st of the dual fluctuation equations

$$
\frac{\partial}{\partial t}\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = 0. \tag{2.18}
$$

Hereby the stochastic-deterministic connection is established.

Back to the key-equation (2.12)

$$
\frac{\partial}{\partial t}\delta + \vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t)} \times \frac{\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t)}}{B^2_{(\vec{\boldsymbol{x}},t)}} \cdot \vec{\boldsymbol{\nabla}}\delta = \mathbf{0}
$$

one obtains by simple conversion

$$
\begin{aligned}
\frac{\partial}{\partial t}\left(\vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t)} \cdot \frac{\vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t)}}{E^2_{(\vec{\boldsymbol{x}},t)}}\delta\right) + \vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t)} \cdot \vec{\boldsymbol{\nabla}} \times \left(\frac{\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t)}}{B^2_{(\vec{\boldsymbol{x}},t)}}\delta\right) = 0 \\
\frac{\partial}{\partial t}\left(\frac{B^2_{(\vec{\boldsymbol{x}},t)}}{E^2_{(\vec{\boldsymbol{x}},t)}} \cdot \vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t)}\delta\right) + \vec{\boldsymbol{\nabla}} \times (\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t)}\delta) = 0
\end{aligned}
\tag{2.19}
$$

and

$$\boldsymbol{\Xi}\left[\int_{\vec{\mathbf{B}}}\int_{\vec{\mathbf{E}}}\left[\frac{\partial}{\partial t}\left(\frac{B^2_{(\vec{\boldsymbol{x}},t)}}{E^2_{(\vec{\boldsymbol{x}},t)}}\cdot\vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t)}\delta\right)+\vec{\boldsymbol{\nabla}}\times(\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t)}\delta)=0\right]d\vec{\mathbf{E}}d\vec{\mathbf{B}}\right] \tag{2.20}$$

respectively

$$\frac{\partial}{\partial t}\boldsymbol{\Xi}\left[\frac{B^2_{(\vec{\boldsymbol{x}},t)}}{E^2_{(\vec{\boldsymbol{x}},t)}}\cdot\vec{\mathbf{E}}_{(\vec{\boldsymbol{x}},t)}\right]+\vec{\boldsymbol{\nabla}}\times\boldsymbol{\Xi}[\vec{\mathbf{B}}_{(\vec{\boldsymbol{x}},t)}]=0. \tag{2.21}$$

So we have the second of the two dual equations

$$\frac{\partial}{\partial t}(\frac{B^2}{E^2}\cdot\vec{\mathbf{E}})+\vec{\boldsymbol{\nabla}}\times(\vec{\mathbf{B}})=0. \tag{2.22}$$

The result is recapitulated by the following equation system:

$$\boxed{\begin{array}{l}\frac{\partial}{\partial t}\vec{\mathbf{B}}-\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{E}}=0\\ \frac{\partial}{\partial t}\left(\frac{B^2}{E^2}\cdot\vec{\mathbf{E}}\right)+\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{B}}=0\\ \vec{\mathbf{E}}\times\frac{\vec{\mathbf{B}}}{B^2}=\text{propagation speed}\end{array}} \tag{2.23}$$

with $|\vec{\mathbf{E}}\times\frac{\vec{\mathbf{B}}}{B^2}|\leq|\vec{\mathbf{E}}|\cdot|\frac{\vec{\mathbf{B}}}{B^2}|$. I.e. $\frac{E^2}{B^2}$ is not the quadratic propagation speed. Interestingly, this only becomes clear after the involvement of the stochastic ensemble theory.

The equation system (2.23) is in such general terms that the physical significance depends on the interpretation of the starting field $\vec{\mathbf{A}}$, the boundary conditions as well as on the initial conditions. Hereunder, a deformation vector field, the velocity vector field of turbulence motions or the fluctuations of any other continuously differentiable vector field may be understood. These equations possess with boundary- and suitable initial conditions exactly one solution after the theorem of Cauchy-Kowalewskaja [2]. This statement is at first restricted to the calculation of the fields $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$. Calculating the field $\vec{\mathbf{A}}$ with the mere knowledge of

$$\frac{\partial\vec{\mathbf{A}}}{\partial\boldsymbol{t}}=\vec{\mathbf{E}} \tag{2.24}$$

is not possible in all cases. A negative example is the calculation of $\vec{\mathbf{v}}$ with the knowledge of $\frac{\partial \vec{\mathbf{v}}}{\partial t}$ related to turbulent velocity fluctuations.

2.2.1 The vacuum Maxwell Equations

The propagation speed having the constant amount of light velocity one obtains the known equations of vacuum-electrodynamics in the coordinate system of the observer:

$$\boxed{\begin{array}{l} \frac{\partial}{\partial t}\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = 0 \\ \frac{1}{c^2}\frac{\partial}{\partial t}\vec{\mathbf{E}} + \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} = 0 \qquad \text{mit} \qquad \vec{\mathbf{E}} \perp \vec{\mathbf{B}} \\ \vec{\mathbf{E}} \times \frac{\vec{\mathbf{B}}}{B^2} = \vec{\mathbf{c}} = \text{propagation speed of light.} \end{array}} \tag{2.25}$$

So the electrodynamic equations of vacuum are generally derived, too. Usually, they are seen in the above equations with $-\vec{\mathbf{E}}$. It is more than pure supposition, that they describe properties of space-time without a unification of General Relativity and electromagnetic field in vacuum having succeeded, though many physicists not least Einstein [3], Jordan [5] and many others having endeavoured.

There is still the explanation of the associated initial field $\vec{\mathbf{A}}$ it generally happening in the frame of vector potential considerations, without recognizing $\vec{\mathbf{A}}$ as definite physical object. Only by a direct comprehension of the vector potential the electromagnetic field may be explained without means of mechanical quantities.[1]

2.3 Surfacelike deformation-fluctuations in 3-dimensional space

Let $\vec{\mathbf{d}}$ be a continuously differentiable deformation vector field defining an area and $\vec{\mathbf{b}}$ und $\vec{\mathbf{e}}$ the derived fields

$$\vec{\mathbf{e}} = \frac{\partial}{\partial t}\vec{\mathbf{d}}, \qquad \vec{\mathbf{b}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{d}} \tag{2.26}$$

[1]Electrodynamics is introduced in physics via mechanical effects.

with

$$\begin{aligned}\vec{\mathbf{d}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) &= \Big(\mathrm{d_x}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}), \mathrm{d_y}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}), \mathrm{d_z}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})\Big)\\ \vec{\mathbf{e}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) &= \Big(\mathrm{e_x}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}), \mathrm{e_y}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}), \mathrm{e_z}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})\Big)\\ \vec{\mathbf{b}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}) &= \Big(\mathrm{b_x}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}), \mathrm{b_y}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t}), \mathrm{b_z}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t})\Big).\end{aligned} \tag{2.27}$$

Then the deformation is without loss of generality seen as deformation of the $\mathbf{x}-\mathbf{y}$-area. The equations of motion formally equal the equations of 3-dimensional fluctuations

$$\begin{aligned}&\frac{\partial}{\partial t}\vec{\mathbf{b}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{e}} = 0\\ &\frac{\partial}{\partial t}\left(\frac{b^2}{e^2}\cdot\vec{\mathbf{e}}\right) + \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{b}} = 0\\ &\vec{\mathbf{e}} \times \frac{\vec{\mathbf{b}}}{b^2} = \text{propagation speed,}\end{aligned} \tag{2.28}$$

only, the operator $\vec{\boldsymbol{\nabla}}\times$ corresponds to

$$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{d}} = \begin{pmatrix} \partial d_z/\partial y \\ -\partial d_z/\partial x \\ \partial d_y/\partial x - \partial d_x \partial y \end{pmatrix}. \tag{2.29}$$

The solution uniquely succeeds by the initial conditions $\vec{\mathbf{b}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t_0})$ and $\vec{\mathbf{e}}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{t_0})$ according to the theorem of Cauchy-Kowalewskaya [2]. The solution for this area corresponds to a partial solution of a 3-dimensional complete solution. Physical material properties are not explicitly included in these equations. They have to be implicitly considered by initial and boundary conditions. Sole precondition is that the appropriate materials act continuously. It also means that the physical process has to be clarified enabling the corresponding initial and border conditions.

2.4 1-dimensional deformation-fluctuations in 3-dimensional space

Let $\vec{\mathbf{d}}$ be a continuously differentiable deformation vector field defining a trajectory and $\vec{\mathbf{b}}$ und $\vec{\mathbf{e}}$ the derived fields

$$\vec{\mathbf{e}} = \frac{\partial}{\partial t}\vec{\mathbf{d}}, \qquad \vec{\mathbf{b}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{d}} \tag{2.30}$$

with

$$\begin{aligned} \vec{\mathbf{d}}(\boldsymbol{x},\boldsymbol{t}) =&(\mathbf{d_x}(\boldsymbol{x},\boldsymbol{t}), \mathbf{d_y}(\boldsymbol{x},\boldsymbol{t}), \mathbf{d_z}(\boldsymbol{x},\boldsymbol{t}))\\ \vec{\mathbf{e}}(\boldsymbol{x},\boldsymbol{t}) =&(\mathbf{e_x}(\boldsymbol{x},\boldsymbol{t}), \mathbf{e_y}(\boldsymbol{x},\boldsymbol{t}), \mathbf{e_z}(\boldsymbol{x},\boldsymbol{t}))\\ \vec{\mathbf{b}}(\boldsymbol{x},\boldsymbol{t}) =&(\mathbf{b_x}(\boldsymbol{x},\boldsymbol{t}), \mathbf{b_y}(\boldsymbol{x},\boldsymbol{t}), \mathbf{b_z}(\boldsymbol{x},\boldsymbol{t})). \end{aligned} \tag{2.31}$$

Then the deformation is without loss of generality seen as deformation of the **x**-coordinate. The equations of motion formally equal the equations of 3-dimensional fluctuations

$$\begin{aligned} &\frac{\partial}{\partial t}\vec{\mathbf{b}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{e}} = 0\\ &\frac{\partial}{\partial t}\left(\frac{b^2}{e^2} \cdot \vec{\mathbf{e}}\right) + \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{b}} = 0\\ &\vec{\mathbf{e}} \times \frac{\vec{\mathbf{b}}}{b^2} = \text{propagation speed,} \end{aligned} \tag{2.32}$$

only, the operator $\vec{\boldsymbol{\nabla}}\times$ corresponds to

$$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{d}} = \begin{pmatrix} & 0\\ - & \partial d_z/\partial x\\ & \partial d_y/\partial x \end{pmatrix}. \tag{2.33}$$

This leads to the component equations

$$\begin{aligned} \partial b_y/\partial t =& - \partial e_z/\partial x\\ \partial b_z/\partial x =&\partial e_y/\partial x\\ \\ \partial[(b^2/e^2) \cdot e_y)]\partial t =& - \partial b_z/\partial x\\ \partial[(b^2/e^2) \cdot e_z)]\partial t =&\partial b_y/\partial x\\ \\ \vec{\mathbf{e}} \times \vec{\mathbf{b}}/b^2 =&\text{propagation speed.} \end{aligned} \tag{2.34}$$

The x-component remains constant. The solution uniquely results from the initial conditions $\vec{\mathbf{b}}(\boldsymbol{x}, \boldsymbol{t_0})$ and $\vec{\mathbf{e}}(\boldsymbol{x}, \boldsymbol{t_0})$ according to the theorem of Cauchy-Kowalewskaya [2]. The solution for this 1-dimensional trajectory corresponds to a partial solution of a 3-dimensional complete solution. Physical material properties are not explicitly included in these equations. They have to be implicitly considered by initial and boundary conditions. Sole precondition is that the appropriate materials act continuously. It also means that the physical process has to be clarified enabling the corresponding initial and border conditions.

3 Space-Time-fluctuations in General Relativity

$$\boxed{\mathbf{R}_{\mu\nu} = 8\pi \cdot G_N \left(\mathbf{T}_{\mu\nu} - \frac{1}{2}\mathbf{g}_{\mu\nu}\mathbf{T} \right)}$$

Electrodynamics with its Maxwell Equations is the only field theory of classical physics students of physics are generally faced with in the frame of theoretical physics (at least in Germany). The Maxwell Equations above are shown formally beeing a limiting case of classical continuum physics. Because of the constant velocity of light they were the reason for setting up the Einsteinian Special Relativity. The adjustment of the electrodynamic field to Space-Time caused many physicists including Albert Einstein to try an identification of these fields with Space-Time fluctuations. Obviously, electromagnetic fluctuations are properties of Space-Time itself, though a prove is missing.

In chapter 2 continuum fluctuations of general vector fields are discussed. Now we consider deformation vector fields $\vec{\mathbf{d}}(\vec{\mathbf{x}}, t)$ with $\vec{\mathbf{\nabla}} \times \vec{\mathbf{d}} \neq \mathbf{0}$. They are sufficiently often continuously differentiable. Defining $\vec{\mathbf{e}}$ und $\vec{\mathbf{b}}$ by

$$\begin{aligned} \vec{\mathbf{e}} &= \partial \vec{\mathbf{d}} / \partial t \neq 0 \\ \vec{\mathbf{b}} &= \vec{\mathbf{\nabla}} \times \vec{\mathbf{d}} \neq 0 \end{aligned} \tag{3.1}$$

and interchanging the sequence of the operators $\partial/\partial t$ and $\vec{\mathbf{\nabla}}\times$

$$\frac{\partial \vec{\mathbf{b}}}{\partial t} = \vec{\mathbf{\nabla}} \times \vec{\mathbf{e}} \tag{3.2}$$

directly follows. So this equation is a necessary consequence of the continuous differentiability of $\vec{\mathbf{d}}(\vec{\mathbf{x}}, t)$. The hereto dual equation is found according to chapter 2 with

$$\begin{aligned}
&\frac{\partial}{\partial t}\vec{\mathbf{b}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{e}} = 0 \\
&\frac{\partial}{\partial t}\left(\frac{b^2}{e^2} \cdot \vec{\mathbf{e}}\right) + \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{b}} = 0 \\
&\vec{\mathbf{e}} \times \frac{\vec{\mathbf{b}}}{b^2} = \text{propagation speed}
\end{aligned} \tag{3.3}$$

Assuming the constant speed of light the Maxwell Equations of vacuum[1]are obtained:

$$\boxed{\begin{aligned}
&\frac{\partial}{\partial t}\vec{\mathbf{b}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{e}} = 0 \\
&\frac{1}{c^2}\frac{\partial}{\partial t}\vec{\mathbf{e}} + \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{b}} = 0 \\
&\vec{\mathbf{e}} \times \frac{\vec{\mathbf{b}}}{b^2} = \vec{\mathbf{c}} = \text{propagation speed of light.}
\end{aligned}} \tag{3.4}$$

3.1 Space-Time of General Relativity and its Riemannian hypersurface

First, the Riemannian hypersurface of Space-Time is considered as deformation of an Euclidian space. For a precise mathematical definition of the Riemannian space [7] is noted.

The Riemannian space is generally defined by a manifold, which consists of a point set, charts or coordinate systems and a symmetrical metric tensor field. Riemannian space and a suitable Euclidian space are one to one linked by the coordinate system. The according mapping is in mathematics not explicitly used as all considerations are abstractly concerned with the connections of the Riemannian space itself not interesting what kind of picture succeeds in the observational coordinate space. The metric tensor arises in the point $P(\vec{\boldsymbol{x}}) \in \boldsymbol{M}$ with $\vec{\boldsymbol{x}} \in \boldsymbol{E}$ (Euclidian space) by scalar products of the tangential vectors $\vec{\boldsymbol{g}}_{\boldsymbol{i}}$.

$$\boldsymbol{g_{ij}}(P(\vec{\boldsymbol{x}})) = \vec{\boldsymbol{g}}_{\boldsymbol{i}}(P(\vec{\boldsymbol{x}})) \cdot \vec{\boldsymbol{g}}_{\boldsymbol{j}}(P(\vec{\boldsymbol{x}})) \tag{3.5}$$

By free choice of the coordinate system $\boldsymbol{g_{ij}}(P(\vec{\boldsymbol{x}}))$ may be determined in one point $(P(\vec{\boldsymbol{x}}))$. But this does not simultaneously hold for the neighborhood of this point.

[1]The Maxwell Equations are usually presented by $\vec{\mathbf{e}} \to -\vec{\mathbf{e}}$

The isomorphic mapping from Euclidian space into the Riemannian hypersurface is brought to physical life when interpreted as deformation of the Euclidian space, both spaces, Euclidian and Riemannian space, tangentially merging in one point. Here the deformation vector field $\vec{\boldsymbol{d}} = \vec{\boldsymbol{d}}(\vec{\boldsymbol{x}}, t)$ vanishes. These time dependent mappings can be interpreted as gravitational waves. The Riemannian hypersurface arises from

$$\vec{\boldsymbol{y}}(\vec{\boldsymbol{x}}, t) = \vec{\boldsymbol{d}}(\vec{\boldsymbol{x}}, t) + \vec{\boldsymbol{x}} \quad . \tag{3.6}$$

The gradient on the deformed field is described by

$$\vec{\boldsymbol{\nabla}}\vec{\boldsymbol{y}} = \begin{pmatrix} \boldsymbol{\partial}_i \boldsymbol{y}_j \end{pmatrix} \tag{3.7}$$

and detailed

$$\begin{pmatrix} \boldsymbol{\partial}_i \boldsymbol{y}_j \end{pmatrix} = \begin{pmatrix} \boldsymbol{\partial}_1 \boldsymbol{y}_1 & \boldsymbol{\partial}_1 \boldsymbol{y}_2 & \boldsymbol{\partial}_1 \boldsymbol{y}_3 \\ \boldsymbol{\partial}_2 \boldsymbol{y}_1 & \boldsymbol{\partial}_2 \boldsymbol{y}_2 & \boldsymbol{\partial}_2 \boldsymbol{y}_3 \\ \boldsymbol{\partial}_3 \boldsymbol{y}_1 & \boldsymbol{\partial}_3 \boldsymbol{y}_2 & \boldsymbol{\partial}_3 \boldsymbol{y}_3 \end{pmatrix} \qquad i, j = 1, 2, 3. \tag{3.8}$$

Defining the spatially tangential vector $\vec{\boldsymbol{t}}_i$ with

$$\vec{\boldsymbol{t}}_i = \boldsymbol{\partial}_i \vec{\boldsymbol{y}} = (\boldsymbol{\partial}_i \boldsymbol{y}_1, \boldsymbol{\partial}_i \boldsymbol{y}_2, \boldsymbol{\partial}_i \boldsymbol{y}_3)\,, \tag{3.9}$$

one obtains the spatial metric tensor $\boldsymbol{t}_{ij} = \vec{\boldsymbol{t}}_i \cdot \vec{\boldsymbol{t}}_j$ by

$$\begin{pmatrix} \boldsymbol{t_{ij}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\partial}_i \boldsymbol{y}_j \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{\partial}_i \boldsymbol{y}_j \end{pmatrix}^{\boldsymbol{T}} \tag{3.10}$$

and

$$\boldsymbol{t_{ij}} = \boldsymbol{\partial}_i \boldsymbol{y}_1 \cdot \boldsymbol{\partial}_j \boldsymbol{y}_1 + \boldsymbol{\partial}_i \boldsymbol{y}_2 \cdot \boldsymbol{\partial}_j \boldsymbol{y}_2 + \boldsymbol{\partial}_i \boldsymbol{y}_3 \cdot \boldsymbol{\partial}_j \boldsymbol{y}_3 \tag{3.11}$$

as part of the metric tensors of Space-Time

$$\begin{pmatrix} \mathbf{g}_{\mu\nu} \end{pmatrix} = \begin{pmatrix} \mathbf{g_{00}} & \mathbf{g_{01}} & \mathbf{g_{02}} & \mathbf{g_{03}} \\ \mathbf{g_{10}} & \mathbf{t_{11}} & \mathbf{t_{12}} & \mathbf{t_{13}} \\ \mathbf{g_{20}} & \mathbf{t_{21}} & \mathbf{t_{22}} & \mathbf{t_{23}} \\ \mathbf{g_{30}} & \mathbf{t_{31}} & \mathbf{t_{32}} & \mathbf{t_{33}} \end{pmatrix} \qquad \mu, \nu = 0, 1, 2, 3\ . \tag{3.12}$$

The metric-tensor elements $\boldsymbol{t}_{ij}$ of the spatial hypersurface are components of the metric-tensor element set $\boldsymbol{g}_{\mu\nu}$ of Space-Time. The corresponding statement does not hold for the Ricci Curvature Tensor. The Ricci Tensor elements $\boldsymbol{r}_{ij}$ of the Riemannian hypersurface as subspace of Space-Time are not part of the Ricci Tensor element set $\boldsymbol{R}_{\mu\nu}$ of the overall space.

$$\left(\mathbf{R}_{\mu\nu}\right) = \begin{pmatrix} \mathbf{R_{00}} & \mathbf{R_{01}} & \mathbf{R_{02}} & \mathbf{R_{03}} \\ \mathbf{R_{10}} & \mathbf{R_{11}} & \mathbf{R_{12}} & \mathbf{R_{13}} \\ \mathbf{R_{20}} & \mathbf{R_{21}} & \mathbf{R_{22}} & \mathbf{R_{23}} \\ \mathbf{R_{30}} & \mathbf{R_{31}} & \mathbf{R_{32}} & \mathbf{R_{33}} \end{pmatrix} \neq \begin{pmatrix} \mathbf{R_{00}} & \mathbf{R_{01}} & \mathbf{R_{02}} & \mathbf{R_{03}} \\ \mathbf{R_{10}} & \mathbf{r_{11}} & \mathbf{r_{12}} & \mathbf{r_{13}} \\ \mathbf{R_{20}} & \mathbf{r_{21}} & \mathbf{r_{22}} & \mathbf{r_{23}} \\ \mathbf{R_{30}} & \mathbf{r_{31}} & \mathbf{r_{32}} & \mathbf{r_{33}} \end{pmatrix} \tag{3.13}$$

$$\text{i.e.} \qquad \boldsymbol{r}_{ij} \neq \boldsymbol{R}_{ij} \qquad i,j = 1,2,3$$

Initially, it is the plan to express the Ricci Tensor of Space Time by the Ricci Tensor of the spatial hypersurface and its time dependent metric tensor

$$\boldsymbol{R}_{ij} = \boldsymbol{R}_{ij}(\boldsymbol{r}_{ij}, \boldsymbol{t}_{ij}) \qquad i,j = 1,2,3. \tag{3.14}$$

Formulating the energy momentum tensor of the right side of the Einstein equations

$$\mathbf{R}_{\boldsymbol{\mu\nu}} - \frac{1}{2}\mathbf{g}_{\boldsymbol{\mu\nu}}\mathbf{R} = 8\pi \cdot G_N \mathbf{T}_{\boldsymbol{\mu\nu}} \qquad \mu,\nu = 0,1,2,3$$

by the related deformation fluctuations using its electromagnetic interpretation the unification of gravitational and electromagnetic field is outlined in the following chapter.

Originating from the Einstein equations

$$\mathbf{R}_{\boldsymbol{\mu\nu}} - \frac{1}{2}\mathbf{g}_{\boldsymbol{\mu\nu}}\mathbf{R} = 8\pi \cdot G_N \mathbf{T}_{\boldsymbol{\mu\nu}} \tag{3.15}$$

one obtains by contraction

$$\mathbf{trace}\left(\mathbf{R}_{\boldsymbol{\mu\nu}} - \frac{1}{2}\mathbf{g}_{\boldsymbol{\mu\nu}}\mathbf{R}\right) = \mathbf{g}^{\boldsymbol{\mu\mu}}\left(\mathbf{R}_{\boldsymbol{\mu\mu}} - \frac{1}{2}\mathbf{g}_{\boldsymbol{\mu\mu}}\mathbf{R}\right) = -\mathbf{R} = 8\pi \cdot G_N \mathbf{T}^{\mu}_{\mu} = 8\pi \cdot G_N \mathbf{T} \tag{3.16}$$

an alternative form of the Einstein Equations

$$\mathbf{R}_{\boldsymbol{\mu\nu}} = 8\pi \cdot G_N\left(\mathbf{T}_{\boldsymbol{\mu\nu}} - \frac{1}{2}\mathbf{g}_{\boldsymbol{\mu\nu}}\mathbf{T}\right). \tag{3.17}$$

3.2 The Ricci Tensor in the origin of a local inertial-system

The Riemannian curvature tensor $\mathbf{R}^{\mu}_{.\nu\alpha\beta}$ is described in any coordinate system by the Christoffel symbols

$$\boldsymbol{\Gamma}^{\boldsymbol{\mu}}_{\boldsymbol{\nu\alpha}} = \left\{ \begin{matrix} \mu \\ \nu\ \alpha \end{matrix} \right\} = \frac{1}{2}\mathbf{g}^{\mu\lambda}\Big[\partial_\nu \mathbf{g}_{\alpha\lambda} + \partial_\alpha \mathbf{g}_{\lambda\nu} - \partial_\lambda \mathbf{g}_{\nu\alpha}\Big] \tag{3.18}$$

$$\mathbf{R}^{\mu}_{.\nu\alpha\beta} = \frac{\partial \boldsymbol{\Gamma}^{\mu}_{\nu\beta}}{\partial \mathbf{x}^{\alpha}} - \frac{\partial \boldsymbol{\Gamma}^{\mu}_{\nu\alpha}}{\partial \mathbf{x}^{\beta}} + \boldsymbol{\Gamma}^{\mu}_{\rho\alpha}\boldsymbol{\Gamma}^{\rho}_{\nu\beta} - \boldsymbol{\Gamma}^{\mu}_{\rho\beta}\boldsymbol{\Gamma}^{\rho}_{\nu\alpha}. \tag{3.19}$$

In the origin $\vec{\mathbf{x}_0}$ of a local inertial system [1] the partial derivatives with respect to coordinates of the metric tensor $\mathbf{g}_{\lambda\nu}$ vanish such that

$$\boldsymbol{\Gamma}^{\boldsymbol{\mu}}_{\boldsymbol{\nu\alpha}}(\vec{\mathbf{x}_0}) = \mathbf{0} \tag{3.20}$$

and

$$\mathbf{R}^{\mu}_{.\nu\alpha\beta}(\vec{\mathbf{x}_0}) = \frac{\partial \boldsymbol{\Gamma}^{\mu}_{\nu\beta}}{\partial \mathbf{x}^{\alpha}} - \frac{\partial \boldsymbol{\Gamma}^{\mu}_{\nu\alpha}}{\partial \mathbf{x}^{\beta}}. \tag{3.21}$$

In the origin of the coordinate system the metric tensor itself equals the Minkowski tensor.

$$\mathbf{g}_{\mu\nu}(\vec{\mathbf{x}_0}) = \eta_{\mu\nu} = \begin{pmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix} \tag{3.22}$$

Written out one obtains

$$\mathbf{R}^{\mu}_{.\nu\alpha\beta}(\vec{\mathbf{x}_0}) = \frac{1}{2}\eta^{\mu\lambda}\frac{\partial}{\partial x^{\alpha}}\Big[\partial_\nu \mathbf{g}_{\beta\lambda} + \partial_\beta \mathbf{g}_{\lambda\nu} - \partial_\lambda \mathbf{g}_{\nu\beta}\Big] - \frac{1}{2}\eta^{\mu\lambda}\frac{\partial}{\partial x^{\beta}}\Big[\partial_\nu \mathbf{g}_{\alpha\lambda} + \partial_\alpha \mathbf{g}_{\lambda\nu} - \partial_\lambda \mathbf{g}_{\nu\alpha}\Big] \tag{3.23}$$

$\Longrightarrow$

$$\mathbf{R}^{\mu}_{.\nu\alpha\beta}(\vec{\mathbf{x}_0}) = \frac{1}{2}\eta^{\mu\lambda}\Big[\partial_\alpha\partial_\nu \mathbf{g}_{\beta\lambda} + \partial_\alpha\partial_\beta \mathbf{g}_{\lambda\nu} - \partial_\alpha\partial_\lambda \mathbf{g}_{\nu\beta}\Big] - \frac{1}{2}\eta^{\mu\lambda}\Big[\partial_\beta\partial_\nu \mathbf{g}_{\alpha\lambda} + \partial_\beta\partial_\alpha \mathbf{g}_{\lambda\nu} - \partial_\beta\partial_\lambda \mathbf{g}_{\nu\alpha}\Big] \tag{3.24}$$

$\Longrightarrow$

$$\mathbf{R}^{\mu}_{.\nu\alpha\beta}(\vec{\mathbf{x_0}}) = \frac{1}{2}\eta^{\mu\lambda}\Big[\partial_\alpha\partial_\nu\mathbf{g}_{\beta\lambda} + \partial_\beta\partial_\lambda\mathbf{g}_{\nu\alpha} - \partial_\alpha\partial_\lambda\mathbf{g}_{\nu\beta} - \partial_\beta\partial_\nu\mathbf{g}_{\alpha\lambda}\Big] \tag{3.25}$$

$\Longrightarrow$

$$\mathbf{R}_{\mu\nu\alpha\beta}(\vec{\mathbf{x_0}}) = \frac{1}{2}\Big[\partial_\alpha\partial_\nu\mathbf{g}_{\beta\lambda} + \partial_\beta\partial_\lambda\mathbf{g}_{\nu\alpha} - \partial_\alpha\partial_\lambda\mathbf{g}_{\nu\beta} - \partial_\beta\partial_\nu\mathbf{g}_{\alpha\lambda}\Big]. \tag{3.26}$$

After contraction there is the associated Ricci Tensor

$$\mathbf{R}_{\mu\nu}(\vec{\mathbf{x_0}}) = \frac{1}{2}\Big[\partial_\mu\partial_\alpha\mathbf{g}^{\alpha}_{\nu} + \partial_\nu\partial^\alpha\mathbf{g}_{\mu\alpha} - \partial_\alpha\partial^\alpha\mathbf{g}_{\mu\nu} - \partial_\nu\partial_\mu\mathbf{g}^{\alpha}_{\alpha}\Big] \tag{3.27}$$

and as $\partial_\alpha\partial^\alpha = \Box$ means the D'Alembert-Operator $\Longrightarrow$

$$\boxed{\mathbf{R}_{\mu\nu}(\vec{\mathbf{x_0}}) = \frac{1}{2}\Big[\partial_\mu\partial_\alpha\mathbf{g}^{\alpha}_{\nu} + \partial_\nu\partial_\alpha\mathbf{g}^{\alpha}_{\mu} - \Box\mathbf{g}_{\mu\nu} - \partial_\nu\partial_\mu\mathbf{g}\Big]}\,. \tag{3.28}$$

This result may be obtained by linearization of the Riemannian curvature tensor, too. Choosing point $(\vec{\mathbf{x_0}})$ as the origin of a local inertial system, linearization is not necessary.

3.3 The Ricci Tensor of the Einstein Space in dependence of temporal fluctuations of its Riemannian hypersurface

The following relations correspond to [4] Landau Lifschitz volume 2 page.308-309. A time orthogonal coordinate system is always possible. In contrary to [4], we do not equate the velocity of light with 1.

$$\text{Def:} \qquad \varkappa_{\mathbf{ij}} = \frac{\partial\mathbf{g_{ij}}}{\partial(c\boldsymbol{t})} \tag{3.29}$$

$\mathbf{r_{ij}}$ means the Ricci Tensor of the Riemannian hypersurface.
$\Longrightarrow$

$$\begin{aligned}
\mathbf{R_{00}} =& -\frac{1}{2}\frac{\partial \varkappa_i^i}{\partial(c\boldsymbol{t})} - \frac{1}{4}\varkappa_i^j \varkappa_j^i \\
\mathbf{R_{0i}} =& \frac{1}{2}\left(\varkappa_{\mathrm{i};j}^j - \varkappa_{\mathrm{j};i}^j\right) \\
\mathbf{R_{ij}} =& \mathbf{r_{ij}} + \frac{1}{2}\frac{\partial \varkappa_{ij}}{\partial(c\boldsymbol{t})} + \frac{1}{4}\left(\varkappa_{\mathrm{ij}}\varkappa_{\mathrm{k}}^k - 2\varkappa_i^k \varkappa_{jk}\right)
\end{aligned} \tag{3.30}$$

i, j, k pass through $1, 2, 3$. ";" means partial derivation, here.

Thus the geometry of Space-Time may be opened up from geometrodynamics of space. Gravitational waves existing the energy momentum tensor $\mathbf{T}_{\mu\nu} \neq \mathbf{0}$ is given in the considered Space-Time area even if there is no matter. [2]

[2]in contrary to Penrose [6] page 467 "The energy-momentum tensor in empty space is zero."

4 Unification of Maxwell Field and gravitational field

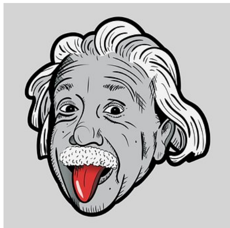

Figure 4.1: Maybe, Einstein would have had fun at this theory

4.1 Gravitational waves corresponding to electromagnetic Fluctuations

The deformation fluctuations of space and its as electromagnetic fluctuations noticed phenomena are subsequently faced to each other in a limited volume area as fourier developments . The considerations are performed based on treatments of natural vibrations of the electomagnetic field in vacuum in accordance to [4]. The usual electric field $\vec{\mathbf{E}}$ is replaced by $-\vec{\mathbf{E}}$, without loss of generality. An explicit dependency of the viewed overall volume in the canonical variables and such in the resulting energy density and the electromagnetic fields is avoided by modified normalisation of the canonical variables, in contrast to [4].
In pure field theories energy densities and accellerations should occur as **primary** quantities not energies and forces. The energy in one point $(\vec{\mathbf{x}}, t)$ is always zero but not the energy density. Analogically, the same is true for the relation of accelleration and force.

deformation fluctuations

From
$\vec{\mathbf{d}}=$ deformation vectorfield
$\frac{\partial}{\partial t}\vec{\mathbf{b}}-\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{e}}=0$
$\frac{1}{c^2}\frac{\partial}{\partial t}\vec{\mathbf{e}}+\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{b}}=0$

and

$\vec{\mathbf{e}}=\partial\vec{\mathbf{d}}/\partial t\neq 0$
$\vec{\mathbf{b}}=\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{d}}\neq 0$

one obtains

$\frac{1}{\boldsymbol{c^2}}\frac{\partial^2\vec{\mathbf{d}}}{\partial t^2}=\boldsymbol{\Delta}\vec{\mathbf{d}}$

electromagnetic fluctuations

$\vec{\mathbf{A}}=$ vector potential
$\frac{\partial}{\partial t}\vec{\mathbf{B}}-\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{E}}=0$
$\frac{1}{c^2}\frac{\partial}{\partial t}\vec{\mathbf{E}}+\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{B}}=0$

$\vec{\mathbf{E}}=\partial\vec{\mathbf{A}}/\partial t\neq 0$
$\vec{\mathbf{B}}=\vec{\boldsymbol{\nabla}}\times\vec{\mathbf{A}}\neq 0$

$\frac{1}{\boldsymbol{c^2}}\frac{\partial^2\vec{\mathbf{A}}}{\partial t^2}=\boldsymbol{\Delta}\vec{\mathbf{A}}$

Deformation field and according vector potential field are formally described by

$\vec{\mathbf{d}}=\sum_{\vec{\mathbf{k}}}\vec{\mathbf{d}}_{\vec{\mathbf{k}}}=\sum_{\vec{\mathbf{k}}}\vec{\mathbf{a}}_{\vec{\mathbf{k}}}\boldsymbol{e}^{i\vec{\mathbf{k}}\vec{\mathbf{r}}}+\vec{\mathbf{a}^*}_{\vec{\mathbf{k}}}\boldsymbol{e}^{-i\vec{\mathbf{k}}\vec{\mathbf{r}}}$ $\qquad$ $\vec{\mathbf{A}}=\sum_{\vec{\mathbf{k}}}\vec{\mathbf{A}}_{\vec{\mathbf{k}}}=\sum_{\vec{\mathbf{k}}}\vec{\mathfrak{A}}_{\vec{\mathbf{k}}}\boldsymbol{e}^{i\vec{\mathbf{k}}\vec{\mathbf{r}}}+\vec{\mathfrak{A}}^*_{\vec{\mathbf{k}}}\boldsymbol{e}^{-i\vec{\mathbf{k}}\vec{\mathbf{r}}}$
and it follows

$\ddot{\vec{\mathbf{d}}}_{\vec{\mathbf{k}}}+c^2k^2\vec{\mathbf{d}}_{\vec{\mathbf{k}}}=\mathbf{0}$ $\qquad$ $\ddot{\vec{\mathbf{A}}}_{\vec{\mathbf{k}}}+c^2k^2\vec{\mathbf{A}}_{\vec{\mathbf{k}}}=\mathbf{0}$

with

$\vec{\boldsymbol{e}}=\dot{\vec{\boldsymbol{d}}}=\sum_{\vec{\mathbf{k}}}\dot{\vec{\mathbf{d}}}_{\vec{\mathbf{k}}}=\sum_{\vec{\mathbf{k}}}\left(\dot{\vec{\mathbf{a}}}_{\vec{\mathbf{k}}}\boldsymbol{e}^{i\vec{\mathbf{k}}\vec{\mathbf{r}}}+\dot{\vec{\mathbf{a}}}^*_{\vec{\mathbf{k}}}\boldsymbol{e}^{-i\vec{\mathbf{k}}\vec{\mathbf{r}}}\right)$ $\qquad$ $\vec{\boldsymbol{E}}=\dot{\vec{\boldsymbol{A}}}=\sum_{\vec{\mathbf{k}}}\dot{\vec{\mathbf{A}}}_{\vec{\mathbf{k}}}=\sum_{\vec{\mathbf{k}}}\left(\dot{\vec{\mathfrak{A}}}_{\vec{\mathbf{k}}}\boldsymbol{e}^{i\vec{\mathbf{k}}\vec{\mathbf{r}}}+\dot{\vec{\mathfrak{A}}}^*_{\vec{\mathbf{k}}}\boldsymbol{e}^{-i\vec{\mathbf{k}}\vec{\mathbf{r}}}\right)$

and

$\vec{\boldsymbol{b}}=-\boldsymbol{i}\sum_{\vec{\mathbf{k}}}\vec{\mathbf{k}}\times\left(\vec{\mathbf{a}}_{\vec{\mathbf{k}}}\boldsymbol{e}^{i\vec{\mathbf{k}}\vec{\mathbf{r}}}+\vec{\mathbf{a}^*}_{\vec{\mathbf{k}}}\boldsymbol{e}^{-i\vec{\mathbf{k}}\vec{\mathbf{r}}}\right)$ $\qquad$ $\vec{\boldsymbol{B}}=-\boldsymbol{i}\sum_{\vec{\mathbf{k}}}\vec{\mathbf{k}}\times\left(\vec{\mathfrak{A}}_{\vec{\mathbf{k}}}\boldsymbol{e}^{i\vec{\mathbf{k}}\vec{\mathbf{r}}}+\vec{\mathfrak{A}}^*_{\vec{\mathbf{k}}}\boldsymbol{e}^{-i\vec{\mathbf{k}}\vec{\mathbf{r}}}\right)$
$\mathbf{k_1}=\frac{2\pi\cdot n_x}{L_x},\ \mathbf{k_2}=\frac{2\pi\cdot n_y}{L_y},\ \mathbf{k_3}=\frac{2\pi\cdot n_z}{L_z};$
$\vec{\mathbf{k}}=(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3})$

$\mathbf{a}_{\mathbf{k}_i}\sim\boldsymbol{e}^{-i\omega_{\mathbf{k}_i}t},\qquad\omega_{\mathbf{k}_i}=ck_i$ $\qquad$ $\mathfrak{A}_{\vec{\mathbf{k}}_i}\sim\boldsymbol{e}^{-i\omega_{\mathbf{k}_i}t},\qquad\omega_{\mathbf{k}_i}=ck_i$

The wave vectors are calculated in a sufficiently great volume $\boldsymbol{V}=\boldsymbol{L_x}\cdot\boldsymbol{L_y}\cdot\boldsymbol{L_z}$.
$\mathcal{E}=\frac{1}{8\pi}\int_{V_0}(\boldsymbol{E^2}/\boldsymbol{c^2}+\boldsymbol{B^2})dV$ means the energy of the field in volume V_0.

The energy density of the field is $\mathfrak{E}=\frac{1}{8\pi}\sum_{\vec{\mathbf{k}}}(\boldsymbol{E}^2_{\vec{\mathbf{k}}}/\boldsymbol{c^2}+\boldsymbol{B}^2_{\vec{\mathbf{k}}})$

deformation fluctuations | **electromagnetic fluctuations**

Now, the following vectorial quantities (canonical variables) are defined:

$$\vec{\mathbf{q}}_{\vec{\mathbf{k}}} = \sqrt{\tfrac{1}{4\pi c^2}}(\vec{\mathbf{a}}_{\vec{\mathbf{k}}} + \vec{\mathbf{a}}^*_{\vec{\mathbf{k}}}) \qquad \vec{\mathbf{Q}}_{\vec{\mathbf{k}}} = \sqrt{\tfrac{1}{4\pi c^2}}(\vec{\mathfrak{A}}_{\vec{\mathbf{k}}} + \vec{\mathfrak{A}}^*_{\vec{\mathbf{k}}})$$

$$\vec{\mathbf{p}}_{\vec{\mathbf{k}}} = -i\omega_{\vec{\mathbf{k}}}\sqrt{\tfrac{1}{4\pi c^2}}(\vec{\mathbf{a}}_{\vec{\mathbf{k}}} - \vec{\mathbf{a}}^*_{\vec{\mathbf{k}}}) = \dot{\vec{\mathbf{q}}}_{\vec{\mathbf{k}}} \qquad \vec{\mathbf{P}}_{\vec{\mathbf{k}}} = -i\omega_{\vec{\mathbf{k}}}\sqrt{\tfrac{1}{4\pi c^2}}(\vec{\mathfrak{A}}_{\vec{\mathbf{k}}} - \vec{\mathfrak{A}}^*_{\vec{\mathbf{k}}}) = \dot{\vec{\mathbf{Q}}}_{\vec{\mathbf{k}}}$$

$$\vec{\mathbf{q}}_{\mathbf{k}_i} \sim \boldsymbol{cos}(\omega_{\mathbf{k}_i}t), \qquad \vec{\mathbf{p}}_{\mathbf{k}_i} \sim \boldsymbol{sin}(\omega_{\mathbf{k}_i}t) \qquad \vec{\mathbf{Q}}_{\mathbf{k}_i} \sim \boldsymbol{cos}(\omega_{\mathbf{k}_i}t), \qquad \vec{\boldsymbol{P}}_{\mathbf{k}_i} \sim \boldsymbol{sin}(\omega_{\boldsymbol{k}_i}t)$$

Obviously, they are real and resolved according to complex quantities they give

$$\vec{\mathbf{a}}_{\mathbf{k_j}} = \tfrac{i}{k_j}\sqrt{\pi}(\vec{\mathbf{p}}_{\mathbf{k_j}} - i\omega_{\vec{\mathbf{k_j}}}\vec{\mathbf{q}}_{\mathbf{k_j}}) \qquad \vec{\mathfrak{A}}_{\mathbf{k_j}} = \tfrac{i}{k_j}\sqrt{\pi}(\vec{\mathbf{P}}_{\mathbf{k_j}} - i\omega_{\mathbf{k_j}}\vec{\mathbf{Q}}_{\mathbf{k_j}})$$
$$\vec{\mathbf{a}}^*_{\mathbf{k_j}} = -\tfrac{i}{k_j}\sqrt{\pi}(\vec{\mathbf{p}}_{\mathbf{k_j}} + i\omega_{\mathbf{k_j}}\vec{\mathbf{q}}_{\mathbf{k_j}}) \qquad \vec{\mathfrak{A}}^*_{\mathbf{k_j}} = -\tfrac{i}{k_j}\sqrt{\pi}(\vec{\mathbf{P}}_{\mathbf{k_j}} + i\omega_{\mathbf{k_j}}\vec{\mathbf{Q}}_{\mathbf{k_j}}).$$

Thus one obtains as expansion by characteristic vibrations (in concise presentation):

$$\vec{\mathbf{d}} = \sqrt{4\pi}\textstyle\sum_{\vec{\mathbf{k}}}\frac{1}{k}\left(ck\vec{\mathbf{q}}_{\vec{\mathbf{k}}}cos(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}) - \vec{\mathbf{p}}_{\vec{\mathbf{k}}}sin(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}})\right) \qquad \vec{\mathbf{A}} = \sqrt{4\pi}\textstyle\sum_{\vec{\mathbf{k}}}\frac{1}{k}\left(ck\vec{\mathbf{Q}}_{\vec{\mathbf{k}}}cos(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}) - \vec{\mathbf{P}}_{\vec{\mathbf{k}}}sin(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}})\right)$$
$$\vec{\mathbf{e}} = \sqrt{4\pi}\textstyle\sum_{\vec{\mathbf{k}}}\boldsymbol{c}\left(ck\vec{\mathbf{q}}_{\vec{\mathbf{k}}}sin(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}) + \vec{\mathbf{p}}_{\vec{\mathbf{k}}}cos(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}})\right) \qquad \vec{\mathbf{E}} = \sqrt{4\pi}\textstyle\sum_{\vec{\mathbf{k}}}\boldsymbol{c}\left(ck\vec{\mathbf{Q}}_{\vec{\mathbf{k}}}sin(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}) + \vec{\mathbf{P}}_{\vec{\mathbf{k}}}cos(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}})\right)$$
$$\vec{\mathbf{b}} = -\sqrt{4\pi}\textstyle\sum_{\vec{\mathbf{k}}}\frac{1}{k}\vec{\mathbf{k}}\times[ck\vec{\mathbf{q}}_{\vec{\mathbf{k}}}sin(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}) + \vec{\mathbf{p}}_{\vec{\mathbf{k}}}cos(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}})] \qquad \vec{\mathbf{B}} = -\sqrt{4\pi}\textstyle\sum_{\vec{\mathbf{k}}}\frac{1}{k}\vec{\mathbf{k}}\times[ck\vec{\mathbf{Q}}_{\vec{\mathbf{k}}}sin(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}) + \vec{\mathbf{P}}_{\vec{\mathbf{k}}}cos(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}})]$$

respectively noted for the single modes:

$$\vec{\mathbf{d}}_{k_j} = \sqrt{4\pi}\tfrac{1}{k_j}\left(ck_j\vec{\mathbf{q}}_{\mathbf{k_j}}cos(\vec{\mathbf{k_j}}\cdot\vec{\mathbf{r}}) - \vec{\mathbf{p}}_{\mathbf{k_j}}sin(\vec{\mathbf{k_j}}\cdot\vec{\mathbf{r}})\right) \qquad \vec{\mathbf{A}}_{k_j} = \sqrt{4\pi}\tfrac{1}{k_j}\left(ck_j\vec{\mathbf{Q}}_{\mathbf{k_j}}cos(\vec{\mathbf{k_j}}\cdot\vec{\mathbf{r}}) - \vec{\mathbf{P}}_{\mathbf{k_j}}sin(\vec{\mathbf{k_j}}\cdot\vec{\mathbf{r}})\right)$$
$$\vec{\mathbf{e}}_{k_j} = \sqrt{4\pi}\boldsymbol{c}\left(ck_j\vec{\mathbf{q}}_{\mathbf{k_j}}sin(\vec{\mathbf{k_j}}\cdot\vec{\mathbf{r}}) + \vec{\mathbf{p}}_{\mathbf{k_j}}cos(\vec{\mathbf{k_j}}\cdot\vec{\mathbf{r}})\right) \qquad \vec{\mathbf{E}}_{k_j} = \sqrt{4\pi}\boldsymbol{c}\left(ck_j\vec{\mathbf{Q}}_{\mathbf{k_j}}sin(\vec{\mathbf{k_j}}\cdot\vec{\mathbf{r}}) + \vec{\mathbf{P}}_{\mathbf{k_j}}cos(\vec{\mathbf{k_j}}\cdot\vec{\mathbf{r}})\right)$$
$$\vec{\mathbf{b}}_{k_j} = -\sqrt{4\pi}\tfrac{1}{k_j}\vec{\mathbf{k_j}}\times[ck_j\vec{\mathbf{q}}_{\mathbf{k_j}}sin(\vec{\mathbf{k_j}}\cdot\vec{\mathbf{r}}) + \vec{\mathbf{p}}_{\mathbf{k_j}}cos(\vec{\mathbf{k_j}}\cdot\vec{\mathbf{r}})] \qquad \vec{\mathbf{B}}_{k_j} = -\sqrt{4\pi}\tfrac{1}{k_j}\vec{\mathbf{k_j}}\times[ck_j\vec{\mathbf{Q}}_{\mathbf{k_j}}sin(\vec{\mathbf{k_j}}\cdot\vec{\mathbf{r}}) + \vec{\mathbf{P}}_{\mathbf{k_j}}cos(\vec{\mathbf{k_j}}\cdot\vec{\mathbf{r}})]$$

with $\mathfrak{E} = \sum_{\vec{\mathbf{k}}}\mathfrak{E}_{\vec{\mathbf{k}}} = \frac{1}{2}\sum_{\vec{\mathbf{k}}}(\boldsymbol{E}^2_{\vec{\mathbf{k}}}/\boldsymbol{c^2} + \boldsymbol{B}^2_{\vec{\mathbf{k}}})$ and $\mathcal{E} = \sum_{\vec{\mathbf{k}}}\mathcal{E}_{\vec{\mathbf{k}}} = \frac{1}{2}\sum_{\vec{\mathbf{k}}}\int_{V_0}(\boldsymbol{E}^2_{\vec{\mathbf{k}}}/\boldsymbol{c^2} + \boldsymbol{B}^2_{\vec{\mathbf{k}}})dV.$
respectively $\mathfrak{E}_{\vec{\mathbf{k_j}}} = \frac{1}{2}(\boldsymbol{E}^2_{\vec{\mathbf{k_j}}}/\boldsymbol{c^2} + \boldsymbol{B}^2_{\vec{\mathbf{k_j}}})$ and $\mathcal{E}_{\vec{\mathbf{k_j}}} = \frac{1}{2}\int_{V_0}(\boldsymbol{E}^2_{\vec{\mathbf{k_j}}}/\boldsymbol{c^2} + \boldsymbol{B}^2_{\vec{\mathbf{k_j}}})dV.$

They may formally considered as running waves moving discrete quantities of harmonic oscillators with the Hamilton Functions

$$\mathbf{H} = \sum_{\vec{\mathbf{k}}}\mathbf{H}_{\vec{\mathbf{k}}} = \sum_{\vec{\mathbf{k}}}\frac{1}{2}(\mathbf{p}^2_{\vec{\mathbf{k}}} + \omega^2_{\vec{\mathbf{k}}}\mathbf{q}^2_{\vec{\mathbf{k}}}), \qquad \mathcal{H} = \sum_{\vec{\mathbf{k}}}\mathcal{H}_{\vec{\mathbf{k}}} = \sum_{\vec{\mathbf{k}}}\frac{1}{2}(\mathbf{P}^2_{\vec{\mathbf{k}}} + \omega^2_{\vec{\mathbf{k}}}\mathbf{Q}^2_{\vec{\mathbf{k}}}) \tag{4.1}$$

and the oscillator equations

$$\ddot{\vec{\mathbf{q}}}_{\vec{\mathbf{k}}} + \omega^{\mathbf{2}}_{\vec{\mathbf{k}}}\vec{\mathbf{q}}_{\vec{\mathbf{k}}} = \mathbf{0}, \qquad \ddot{\vec{\mathbf{Q}}}_{\vec{\mathbf{k}}} + \omega^{\mathbf{2}}_{\vec{\mathbf{k}}}\vec{\mathbf{Q}}_{\vec{\mathbf{k}}} = \mathbf{0} \tag{4.2}$$

.

$$\mathbf{H} = \sum_{\vec{\mathbf{k}}} \mathbf{H}_{\vec{\mathbf{k}}} \qquad \mathbf{H}_{\vec{\mathbf{k}}} = \frac{1}{2}(\mathbf{p}^2_{\vec{\mathbf{k}}} + \omega^2_{\vec{\mathbf{k}}}\mathbf{q}^2_{\vec{\mathbf{k}}}), \qquad \mathcal{H} = \sum_{\vec{\mathbf{k}}} \mathcal{H}_{\vec{\mathbf{k}}} \qquad \mathcal{H}_{\vec{\mathbf{k}}} = \frac{1}{2}(\mathbf{P}^2_{\vec{\mathbf{k}}} + \omega^2_{\vec{\mathbf{k}}}\mathbf{Q}^2_{\vec{\mathbf{k}}}) \tag{4.3}$$

4.2 The energy-momentum-tensor of the electromagnetic field

The energy momentum density tensor for the electromagnetic field (generally called Energy momentum tensor) in covariant components [8] is written with the choosen signature $(-1, 1, 1, 1)$

$$\mathbf{T}_{\mu\nu} = \frac{1}{4\pi}\left(\mathbf{F}^{\alpha}_{\mu}\mathbf{F}_{\alpha\nu} - \frac{1}{4}\mathbf{g}_{\mu\nu}\mathbf{F}_{\alpha\beta}\mathbf{F}^{\alpha\beta}\right) \tag{4.4}$$

It is symmetric: $\mathbf{T}_{\mu\nu} = \mathbf{T}_{\nu\mu}$.
One obtains the Faraday-tensor of the electromagnetic field from

$$\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu \qquad \mu, \nu = 0, 1, 2, 3 \tag{4.5}$$

and detailed (they are chosen respectively the form of the above Maxwell Equations)

$$\begin{aligned}
\mathbf{F}_{0i} &= \partial_0 \mathbf{A}_i - \partial_i \mathbf{A}_0 = \mathbf{E}_i/c, \qquad i = 1, 2, 3 \\
\mathbf{F}_{i0} &= \partial_i \mathbf{A}_0 - \partial_0 \mathbf{A}_i = -\mathbf{E}_i/c, \qquad i = 1, 2, 3 \\
\mathbf{F}_{12} &= \partial_1 \mathbf{A}_2 - \partial_2 \mathbf{A}_1 = \mathbf{B}_3 \\
\mathbf{F}_{13} &= \partial_1 \mathbf{A}_3 - \partial_3 \mathbf{A}_1 = -\mathbf{B}_2 \\
\mathbf{F}_{23} &= \partial_2 \mathbf{A}_3 - \partial_3 \mathbf{A}_2 = \mathbf{B}_1
\end{aligned} \tag{4.6}$$

$$\Longrightarrow \mathbf{F}_{\mu\nu} = -\mathbf{F}_{\nu\mu}$$

$$\partial_\rho \mathbf{F}_{\mu\nu} + \partial_\mu \mathbf{F}_{\nu\rho} + \partial_\nu \mathbf{F}_{\rho\mu} = 0$$

and in greater detail

$$\begin{aligned}\partial_1\mathbf{F_{23}} + \partial_3\mathbf{F_{12}} + \partial_2\mathbf{F_{31}} &= \mathbf{0}\\ \partial_2\mathbf{F_{30}} + \partial_0\mathbf{F_{23}} + \partial_3\mathbf{F_{02}} &= \mathbf{0}\\ \partial_3\mathbf{F_{01}} + \partial_1\mathbf{F_{30}} + \partial_0\mathbf{F_{13}} &= \mathbf{0}\\ \partial_0\mathbf{F_{12}} + \partial_2\mathbf{F_{01}} + \partial_1\mathbf{F_{20}} &= \mathbf{0}\end{aligned}.$$

The indices correspond to $0 \to ct$, $1 \to x$, $2 \to y$, $3 \to z$ complying with the following electrodynamic equations of vacuum[1]

$$\boldsymbol{div\vec{B} = 0} \qquad \text{and} \qquad \frac{\partial}{\partial t}\vec{\mathbf{B}} - \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = 0.$$

The expressions of the covariant and contravariant Faraday-tensors considering the minkowski tensor

$$\boldsymbol{\eta_{\mu\nu}} = \begin{pmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1}\end{pmatrix} \tag{4.7}$$

lead to

$$\mathbf{F}_{\mu\nu} = \begin{pmatrix} \mathbf{0} & \mathbf{E_1}/\boldsymbol{c} & \mathbf{E_2}/\boldsymbol{c} & \mathbf{E_3}/\boldsymbol{c}\\ -\mathbf{E_1}/\boldsymbol{c} & \mathbf{0} & \mathbf{B_3} & -\mathbf{B_2}\\ -\mathbf{E_2}/\boldsymbol{c} & -\mathbf{B_3} & \mathbf{0} & \mathbf{B_1}\\ -\mathbf{E_3}/\boldsymbol{c} & \mathbf{B_2} & -\mathbf{B_1} & \mathbf{0}\end{pmatrix} \qquad \mathbf{F}^{\mu\nu} = \begin{pmatrix} \mathbf{0} & -\mathbf{E_1}/\boldsymbol{c} & -\mathbf{E_2}/\boldsymbol{c} & -\mathbf{E_3}/\boldsymbol{c}\\ \mathbf{E_1}/\boldsymbol{c} & \mathbf{0} & \mathbf{B_3} & -\mathbf{B_2}\\ \mathbf{E_2}/\boldsymbol{c} & -\mathbf{B_3} & \mathbf{0} & \mathbf{B_1}\\ \mathbf{E_3}/\boldsymbol{c} & \mathbf{B_2} & -\mathbf{B_1} & \mathbf{0}\end{pmatrix} \tag{4.8}$$

$$\mathbf{F}^{\mu}_{\nu} = \begin{pmatrix} \mathbf{0} & -\mathbf{E_1}/\boldsymbol{c} & -\mathbf{E_2}/\boldsymbol{c} & -\mathbf{E_3}/\boldsymbol{c}\\ -\mathbf{E_1}/\boldsymbol{c} & \mathbf{0} & \mathbf{B_3} & -\mathbf{B_2}\\ -\mathbf{E_2}/\boldsymbol{c} & -\mathbf{B_3} & \mathbf{0} & \mathbf{B_1}\\ -\mathbf{E_3}/\boldsymbol{c} & \mathbf{B_2} & -\mathbf{B_1} & \mathbf{0}\end{pmatrix} \qquad . \tag{4.9}$$

Thus the covariant components of the electromagnetic energy momentum tensor are written

[1]the polarity reversal $\vec{\boldsymbol{E}} \longrightarrow -\vec{\boldsymbol{E}}$ recognised

$$\mathbf{T}_{\mu\nu} = \frac{1}{4\pi}\begin{pmatrix} \boldsymbol{Q} & (\frac{\vec{\boldsymbol{E}}}{\boldsymbol{c}} \times \vec{\boldsymbol{B}})_1 & (\frac{\vec{\boldsymbol{E}}}{\boldsymbol{c}} \times \vec{\boldsymbol{B}})_2 & (\frac{\vec{\boldsymbol{E}}}{\boldsymbol{c}} \times \vec{\boldsymbol{B}})_3 \\ (\frac{\vec{\boldsymbol{E}}}{\boldsymbol{c}} \times \vec{\boldsymbol{B}})_1 & -[\frac{\boldsymbol{E_1^2}}{\boldsymbol{c^2}} + \boldsymbol{B_1^2} - \boldsymbol{Q}] & -\frac{\mathbf{E_1E_2}}{\mathbf{c^2}} - \mathbf{B_1B_2} & -\frac{\mathbf{E_1E_3}}{\mathbf{c^2}} - \mathbf{B_1B_3} \\ (\frac{\vec{\boldsymbol{E}}}{\boldsymbol{c}} \times \vec{\boldsymbol{B}})_2 & -\frac{\mathbf{E_1E_2}}{\mathbf{c^2}} - \mathbf{B_1B_2} & -[\frac{\boldsymbol{E_2^2}}{\boldsymbol{c^2}} + \boldsymbol{B_2^2} - \boldsymbol{Q}] & -\frac{\mathbf{E_2E_3}}{\mathbf{c^2}} - \mathbf{B_2B_3} \\ (\frac{\vec{\boldsymbol{E}}}{\boldsymbol{c}} \times \vec{\boldsymbol{B}})_3 & -\frac{\mathbf{E_1E_3}}{\mathbf{c^2}} - \mathbf{B_1B_3} & -\frac{\mathbf{E_2E_3}}{\mathbf{c^2}} - \mathbf{B_2B_3} & -[\frac{\boldsymbol{E_3^2}}{\boldsymbol{c^2}} + \boldsymbol{B_3^2} - \boldsymbol{Q}] \end{pmatrix} \tag{4.10}$$

$$\text{with} \qquad \boldsymbol{Q} = \frac{\mathbf{1}}{\mathbf{2}}(\frac{\boldsymbol{E^2}}{\boldsymbol{c^2}} + \boldsymbol{B^2})$$

The trace of the electromagnetic energy momentum tensors vanishes

$$\mathbf{T = 0} \tag{4.11}$$

and the Einstein Equations simplify to

$$\mathbf{R}_{\boldsymbol{ij}} = 8\pi \cdot G_N \mathbf{T}_{\boldsymbol{ij}}. \tag{4.12}$$

For further considerations the following eigenwave is choosen:

$$\mathbf{E_2 = E_3 = B_1 = B_3 = 0}, \qquad \mathbf{E_1 \neq 0}, \qquad \mathbf{B_2 \neq 0}. \tag{4.13}$$

$\Longrightarrow$

$$\mathbf{T_{00}} = \frac{1}{\mathbf{8\pi}}\left(\frac{\mathbf{E_1^2}}{\mathbf{c^2}} + \mathbf{B_2^2}\right), \qquad \mathbf{T_{01} = T_{02} = 0}, \qquad \mathbf{T_{03}} = \frac{1}{\mathbf{4\pi}}(\frac{\vec{\mathbf{E}}_\mathbf{1}}{\boldsymbol{c}} \times \vec{\mathbf{B}}_\mathbf{2}) \tag{4.14}$$

$$\mathbf{T_{ik} = 0} \qquad \text{für } \boldsymbol{i \neq k} \qquad \boldsymbol{i, k} = \mathbf{1, 2, 3} \tag{4.15}$$

$$\mathbf{T_{11}} = \frac{-1}{\mathbf{8\pi}}\left(\frac{\mathbf{E_1^2}}{\mathbf{c^2}} - \mathbf{B_2^2}\right), \qquad \mathbf{T_{22}} = \frac{1}{\mathbf{8\pi}}\left(\frac{\mathbf{E_1^2}}{\mathbf{c^2}} - \mathbf{B_2^2}\right) \tag{4.16}$$

$$\mathbf{T_{33}} = \frac{1}{\mathbf{8\pi}}\left(\frac{\mathbf{E_1^2}}{\mathbf{c^2}} + \mathbf{B_2^2}\right) \tag{4.17}$$

4.3 The quantitative relation of electromagnetic and gravitational waves

The quantitative connection is achieved via the Einstein Equations

$$\mathbf{R}_{\boldsymbol{\mu\nu}} = 8\pi \cdot G_N \mathbf{T}_{\boldsymbol{\mu\nu}}.$$

The description of a natural oscillation takes place using deformation interpretation by

$$\boxed{\begin{aligned}\vec{\boldsymbol{d}}_{k_i} =& \sqrt{4\pi}\frac{1}{k_i}\left(ck_i\vec{\mathbf{q}}_{\boldsymbol{k_i}} cos(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}}) - \vec{\boldsymbol{p}}_{\boldsymbol{k_i}} sin(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})\right)\\ \vec{\boldsymbol{e}}_{k_i} =& \sqrt{4\pi}\boldsymbol{c}\left(ck_i\vec{\boldsymbol{q}}_{\vec{\boldsymbol{k_i}}} sin(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}}) + \vec{\boldsymbol{p}}_{\vec{\boldsymbol{k_i}}} cos(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})\right)\\ \vec{\boldsymbol{b}}_{k_i} =& -\sqrt{4\pi}\frac{1}{k_i}\vec{\boldsymbol{k_i}}\times\left[ck_i\vec{\boldsymbol{q}}_{\vec{\boldsymbol{k_i}}} sin(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}}) + \vec{\boldsymbol{p}}_{\vec{\boldsymbol{k_i}}} cos(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})\right],\end{aligned}} \tag{4.18}$$

and using the electromagnetic field interpretation by

$$\boxed{\begin{aligned}\vec{\boldsymbol{A}}_{k_i} =& \sqrt{4\pi}\frac{1}{k}\left(ck_i\vec{\boldsymbol{Q}}_{\vec{\boldsymbol{k_i}}} cos(\vec{\boldsymbol{k_i}}\cdot\vec{)} - \vec{\boldsymbol{P}}_{\vec{\boldsymbol{k_i}}} sin(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})\right)\\ \vec{\boldsymbol{E}}_{k_i} =& \sqrt{4\pi}\boldsymbol{c}\left(ck_i\vec{\boldsymbol{Q}}_{\vec{\boldsymbol{k_i}}} sin(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}}) + \vec{\boldsymbol{P}}_{\vec{\boldsymbol{k_i}}} cos(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})\right)\\ \vec{\mathbf{B}}_{k_i} =& -\sqrt{4\pi}\frac{1}{k}\vec{\boldsymbol{k_i}}\times\left[ck_i\vec{\boldsymbol{Q}}_{\vec{\boldsymbol{k_i}}} sin(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}}) + \vec{\boldsymbol{P}}_{\vec{\boldsymbol{k_i}}} cos(\vec{\boldsymbol{k_i}}\cdot\vec{\boldsymbol{r}})\right]\end{aligned}} \tag{4.19}$$

with their corresponding energy density and energy in a volume surrounding the coordinate origin ($\vec{\mathbf{x}_0}$).

$$\boxed{\begin{aligned}\mathfrak{E}_{k_i} =& \frac{1}{2}\left(\frac{\boldsymbol{E}_{k_i}^2}{\boldsymbol{c^2}} + \boldsymbol{B}_{k_i}^2\right) && \text{Energiedichte}\\ \mathcal{E}_{k_i} =& \frac{1}{2}\int_{V_0}\left(\frac{\boldsymbol{E}_{k_i}^2}{\boldsymbol{c^2}} + \boldsymbol{B}_{k_i}^2\right)dV && \text{Energie}\end{aligned}} \tag{4.20}$$

.

The metric tensor of an elementary wave with $\vec{\mathbf{q}}_{\vec{\mathbf{k}}} \parallel \vec{\mathbf{e}}_x$, $\vec{\mathbf{p}}_{\vec{\mathbf{k}}} \parallel \vec{\mathbf{e}}_y$ and $\vec{\mathbf{k}} \parallel \vec{\mathbf{e}}_z$, $\vec{\mathbf{k}}\times\vec{\mathbf{q}}_{\vec{\mathbf{k}}} \parallel \vec{\mathbf{e}}_y$ is given by the tangential vectors:

$$\vec{\boldsymbol{t}}_i = \boldsymbol{\partial}_i\vec{\boldsymbol{y}} = (\boldsymbol{\partial}_i\boldsymbol{y}_1, \boldsymbol{\partial}_i\boldsymbol{y}_2, \boldsymbol{\partial}_i\boldsymbol{y}_3), \qquad \vec{\mathbf{y}} = \vec{\mathbf{d}} + \vec{\mathbf{x}}$$

$\Longrightarrow$

$$\vec{\boldsymbol{t}}_z = \boldsymbol{\partial_z \vec{y}} = \left(\boldsymbol{\partial}_z \boldsymbol{d}_x, \boldsymbol{0}, \boldsymbol{1}\right).$$

With $\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} = \boldsymbol{k} \cdot \boldsymbol{z} = \boldsymbol{\omega}_k / c \cdot \boldsymbol{z}$ one obtains

$$\vec{\boldsymbol{t}}_z = \Bigg(-\sqrt{4\pi}\omega_k \vec{\mathbf{q}}_{\vec{\mathbf{k}}} sin(\boldsymbol{\omega}_k/c \cdot \boldsymbol{z}), -\sqrt{4\pi}\vec{\mathbf{p}}_{\vec{\mathbf{k}}} cos(\boldsymbol{\omega}_k/c \cdot \boldsymbol{z}), 1\Bigg). \tag{4.21}$$

As searched spatial metric tensor element remains

$$\boldsymbol{t}_{zz} = 4\pi\Bigg(\omega_k^2 \mathbf{q}_{\vec{\mathbf{k}}}^2 sin^2(\boldsymbol{\omega}_k/c \cdot \boldsymbol{z}) + \mathbf{p}_{\vec{\mathbf{k}}}^2 cos^2(\boldsymbol{\omega}_k/c \cdot \boldsymbol{z})\Bigg) + \mathbf{1} \tag{4.22}$$

with

$$\mathbf{q_k} = \mathbf{u_k}\boldsymbol{cos}(\omega_\mathbf{k} t), \qquad \mathbf{p_k} = \mathbf{v_k}\boldsymbol{sin}(\omega_\mathbf{k} t). \tag{4.23}$$

The purpose is the evaluation of the equation

$$\mathbf{R}_{\boldsymbol{zz}} = 8\pi \cdot G_N \mathbf{T}_{\boldsymbol{zz}}. \tag{4.24}$$

It is appropriate to note, that

$$\mathbf{T}_{zz} = \frac{1}{\boldsymbol{8\pi}}\left(\frac{\mathbf{E_x^2}}{c^2} + \mathbf{B_y^2}\right) = \frac{\mathfrak{E}_\mathbf{k}}{4\pi}. \tag{4.25}$$

Starting from the Riemannian curvature tensor

$$\mathbf{R}^\sigma_{.\nu\alpha\beta} = \partial_\alpha \mathbf{\Gamma}^\sigma_{\nu\beta} - \partial_\beta \mathbf{\Gamma}^\sigma_{\nu\alpha} + \mathbf{\Gamma}^\sigma_{\rho\alpha}\mathbf{\Gamma}^\rho_{\nu\beta} - \mathbf{\Gamma}^\sigma_{\rho\beta}\mathbf{\Gamma}^\rho_{\nu\alpha}. \tag{4.26}$$

with

$$\mathbf{\Gamma}^{\boldsymbol{\mu}}_{\boldsymbol{\nu\alpha}} = \frac{1}{2}\mathbf{g}^{\mu\lambda}\Bigg[\partial_\nu \mathbf{g}_{\alpha\lambda} + \partial_\alpha \mathbf{g}_{\lambda\nu} - \partial_\lambda \mathbf{g}_{\nu\alpha}\Bigg] \tag{4.27}$$

leads by contraction to the Ricci tensor

$$\mathbf{R}_{\mu\nu} = \mathbf{R}^\sigma_{.\mu\nu\sigma} = \partial_\nu \mathbf{\Gamma}^\sigma_{\mu\sigma} - \partial_\sigma \mathbf{\Gamma}^\sigma_{\mu\nu} + \mathbf{\Gamma}^\sigma_{\rho\nu}\mathbf{\Gamma}^\rho_{\mu\sigma} - \mathbf{\Gamma}^\sigma_{\rho\sigma}\mathbf{\Gamma}^\rho_{\mu\nu}. \tag{4.28}$$

The metric tensor after the deformation by the above elementary wave is used in the time orthogonal coordinate system.

$$\eta_{\mu\nu} = \begin{pmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix} \tag{4.29}$$

$$\mathbf{g}_{\mu\nu}(\vec{\mathbf{x}_0}) = \begin{pmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{t_{zz}} \end{pmatrix} \qquad \mathbf{g}^{\mu\nu}(\vec{\mathbf{x}_0}) = \begin{pmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1/t_{zz}} \end{pmatrix} \tag{4.30}$$

$$\begin{gathered} \mathbf{g}_{\mu\nu} \approx \eta_{\mu\nu} + \mathbf{h}_{\mu\nu}, \qquad \mathbf{g}^{\mu\nu} \approx \eta^{\mu\nu} - \mathbf{h}^{\mu\nu} \\ |\mathbf{h}_{\mu\nu}|,|\mathbf{h}^{\mu\nu}| \ll \mathbf{1} \end{gathered} \tag{4.31}$$

The Ricci tensor is typically written in a linear and non-linear proportion with respect to the Christoffel symbols stripped down.

$$\mathbf{R}^{(1)}_{\mu\nu}(\vec{\mathbf{x}_0}) = \partial_\nu \mathbf{\Gamma}^\sigma_{\mu\sigma} - \partial_\sigma \mathbf{\Gamma}^\sigma_{\mu\nu}, \qquad \mathbf{R}^{(2)}_{\mu\nu}(\vec{\mathbf{x}_0}) = \mathbf{\Gamma}^\sigma_{\rho\nu}\mathbf{\Gamma}^\rho_{\mu\sigma} - \mathbf{\Gamma}^\sigma_{\rho\sigma}\mathbf{\Gamma}^\rho_{\mu\nu} \tag{4.32}$$

Detailed examination of the Christoffel symbols

$$\mathbf{\Gamma}^{\boldsymbol{\sigma}}_{\boldsymbol{\mu\sigma}} = \frac{1}{2}\sum_\sigma\sum_\rho g^{\sigma\rho}\partial_\sigma g_{\mu\rho} + \frac{1}{2}\sum_\sigma\sum_\rho g^{\sigma\rho}\partial_\mu g_{\mu\rho} - \frac{1}{2}\sum_\sigma\sum_\rho g^{\sigma\rho}\partial_\rho g_{\mu\rho} \tag{4.33}$$

$$\begin{aligned} \frac{1}{2}\sum_\sigma\sum_\rho g^{\sigma\rho}\partial_\sigma g_{z\rho} &= \frac{1}{2}\underbrace{g^{00}\partial_0 g_{z0}}_{=0} + \frac{1}{2}g^{zz}\partial_z g_{zz} \\ \frac{1}{2}\sum_\sigma\sum_\rho g^{\sigma\rho}\partial_z g_{\rho\sigma} &= \frac{1}{2}\underbrace{g^{00}\partial_z g_{00}}_{=0} + \frac{1}{2}g^{zz}\partial_z g_{zz} \\ \frac{1}{2}\sum_\sigma\sum_\rho g^{\sigma\rho}\partial_\rho g_{\sigma z} &= \frac{1}{2}\underbrace{g^{00}\partial_0 g_{00}}_{=0} + \frac{1}{2}g^{zz}\partial_z g_{zz} \end{aligned} \tag{4.34}$$

$$\partial_z \mathbf{\Gamma}^\sigma_{\mathbf{z}\sigma} = \frac{1}{2}\partial_z \mathbf{g}^{zz}\partial_z \mathbf{g}_{zz} \tag{4.35}$$

$$\begin{aligned} \partial_\sigma \mathbf{\Gamma}^\sigma_{\mathbf{zz}} =& \frac{1}{2}\partial_0 \mathbf{g}^{00}[\partial_z \underbrace{\mathbf{g}_{z0}}_{=0} + \partial_z \underbrace{\mathbf{g}_{0z}}_{=0} - \partial_0 \mathbf{g}_{zz}] + \frac{1}{2}\partial_z \mathbf{g}^{zz}[\partial_z \mathbf{g}_{zz} + \partial_z \mathbf{g}_{zz} - \partial_z \mathbf{g}_{zz}] \\ =& -\frac{1}{2}\partial_0^2 \mathbf{g}_{zz} + \frac{1}{2}\partial_z \mathbf{g}^{zz}\partial_z \mathbf{g}_{zz} \end{aligned} \tag{4.36}$$

lead for the linear part to

$$\mathbf{R}^{(1)}_{zz}(\vec{\mathbf{x}_0}) = \partial_z \mathbf{\Gamma}^{\sigma}_{z\sigma} - \partial_\sigma \mathbf{\Gamma}^{\sigma}_{zz} = +\frac{1}{2}\partial_0^2 \mathbf{g}_{zz}. \tag{4.37}$$

The nonlinear part is determined for the considered elementary wave by the following steps

$$\mathbf{R}^{(2)}_{zz}(\vec{\mathbf{x}_0}) = \mathbf{\Gamma}^{\sigma}_{\rho z}\mathbf{\Gamma}^{\rho}_{z\sigma} - \mathbf{\Gamma}^{\sigma}_{\rho\sigma}\mathbf{\Gamma}^{\rho}_{zz} \tag{4.38}$$

$$\boldsymbol{\Gamma^{\sigma}_{\rho z}} = \frac{1}{2}\mathbf{g}^{\sigma\sigma}\Big[\partial_\rho \mathbf{g}_{z\sigma} + \partial_z \mathbf{g}_{\sigma\rho} - \partial_\sigma \mathbf{g}_{\rho z}\Big] \tag{4.39}$$

$$\boldsymbol{\Gamma^{\rho}_{z\sigma}} = \frac{1}{2}\mathbf{g}^{\rho\rho}\Big[\partial_z \mathbf{g}_{\sigma\rho} + \partial_z \mathbf{g}_{\rho z} - \partial_\rho \mathbf{g}_{z\sigma}\Big] \tag{4.40}$$

$$\begin{aligned}\boldsymbol{\Gamma^{\sigma}_{\rho z}\Gamma^{\rho}_{z\sigma}} &= \frac{1}{4}\mathbf{g}^{\sigma\sigma}\mathbf{g}^{\rho\rho}\Big[\partial_\rho \mathbf{g}_{z\sigma} + \partial_z \mathbf{g}_{\sigma\rho} - \partial_\sigma \mathbf{g}_{\rho z}\Big]\Big[\partial_z \mathbf{g}_{\sigma\rho} + \partial_z \mathbf{g}_{\rho z} - \partial_\rho \mathbf{g}_{z\sigma}\Big] \\ &= \frac{1}{4}\mathbf{g}^{zz}\Big[\mathbf{g}^{zz}(\partial_z \mathbf{g}_{zz})^2 + (\partial_0 \mathbf{g}_{zz})(\partial_z \mathbf{g}_{zz}) + (\partial_0 \mathbf{g}_{zz})^2\Big]\end{aligned} \tag{4.41}$$

$$\begin{aligned}\boldsymbol{\Gamma^{\rho}_{\rho\sigma}} &= \frac{1}{2}\mathbf{g}^{\rho\alpha}\Big[\partial_\rho \mathbf{g}_{\sigma\alpha} + \partial_\sigma \mathbf{g}_{\alpha\rho} - \partial_\alpha \mathbf{g}_{\rho\sigma}\Big] \\ &= \frac{1}{2}\mathbf{g}^{\rho\rho}\Big[\partial_\rho \mathbf{g}_{\sigma\rho} + \partial_\sigma \mathbf{g}_{\rho\rho} - \partial_\rho \mathbf{g}_{\rho\sigma}\Big]\end{aligned} \tag{4.42}$$

$$\begin{aligned}\boldsymbol{\Gamma^{\sigma}_{\nu\mu}} &= \frac{1}{2}\mathbf{g}^{\sigma\alpha}\Big[\partial_\nu \mathbf{g}_{\mu\alpha} + \partial_\mu \mathbf{g}_{\alpha\nu} - \partial_\alpha \mathbf{g}_{\nu\mu}\Big] \\ &= \frac{1}{2}\mathbf{g}^{\sigma\sigma}\Big[\partial_\nu \mathbf{g}_{\mu\sigma} + \partial_\mu \mathbf{g}_{\sigma\nu} - \partial_\sigma \mathbf{g}_{\nu\mu}\Big]\end{aligned} \tag{4.43}$$

$$\begin{aligned}\boldsymbol{\Gamma^{\rho}_{\rho\sigma}\Gamma^{\sigma}_{zz}} &= \frac{1}{4}\mathbf{g}^{\rho\rho}\mathbf{g}^{\sigma\sigma}\Big[\partial_\rho \mathbf{g}_{\sigma\rho} + \partial_\sigma \mathbf{g}_{\rho\rho} - \partial_\rho \mathbf{g}_{\rho\sigma}\Big]\Big[\partial_z \mathbf{g}_{z\sigma} + \partial_z \mathbf{g}_{\sigma z} - \partial_\sigma \mathbf{g}_{zz}\Big] \\ &= \frac{1}{4}\mathbf{g}^{zz}\Big[\mathbf{g}^{zz}(\partial_z \mathbf{g}_{zz})^2 + (\partial_0 \mathbf{g}_{zz})^2\Big]\end{aligned} \tag{4.44}$$

$\Longrightarrow$

$$\begin{aligned}\mathbf{R}^{(2)}_{zz}(\vec{\mathbf{x}_0}) &= \mathbf{\Gamma}^{\sigma}_{\rho z}\mathbf{\Gamma}^{\rho}_{z\sigma} - \mathbf{\Gamma}^{\sigma}_{\rho\sigma}\mathbf{\Gamma}^{\rho}_{zz} \\ &= \frac{1}{4}\mathbf{g}^{zz}\Big[\mathbf{g}^{zz}(\partial_z\mathbf{g}_{zz})^2 + (\partial_0\mathbf{g}_{zz})(\partial_z\mathbf{g}_{zz}) + (\partial_0\mathbf{g}_{zz})^2\Big] - \frac{1}{4}\mathbf{g}^{zz}\Big[\mathbf{g}^{zz}(\partial_z\mathbf{g}_{zz})^2 + (\partial_0\mathbf{g}_{zz})^2\Big]\end{aligned} \tag{4.45}$$

$$\mathbf{R}^{(2)}_{zz}(\vec{\mathbf{x}_0}) = \mathbf{\Gamma}^{\sigma}_{\rho z}\mathbf{\Gamma}^{\rho}_{z\sigma} - \mathbf{\Gamma}^{\sigma}_{\rho\sigma}\mathbf{\Gamma}^{\rho}_{zz} = \frac{1}{4}\mathbf{g}^{zz}\Big[(\partial_0\mathbf{g}_{zz})(\partial_z\mathbf{g}_{zz})\Big] \tag{4.46}$$

The whole tensor element results in $\Longrightarrow$

$$\mathbf{R}_{zz}(\vec{\mathbf{x}_0}) = \mathbf{R}^{(1)}_{zz}(\vec{\mathbf{x}_0}) + \mathbf{R}^{(2)}_{zz}(\vec{\mathbf{x}_0}) = +\frac{1}{2}\partial_0^2\mathbf{g}_{zz} + \frac{1}{4}\mathbf{g}^{zz}\Big[(\partial_0\mathbf{g}_{zz})(\partial_z\mathbf{g}_{zz})\Big]. \tag{4.47}$$

For the point $\vec{\mathbf{x}_0} = \mathbf{(0,0,0,0)}$ this means

$$\partial_0\mathbf{g}_{zz}(\vec{\mathbf{x}_0}) = \partial_z\mathbf{g}_{zz}(\vec{\mathbf{x}_0}) = \mathbf{0} \qquad \longrightarrow \qquad \mathbf{R}^{(2)}_{zz}(\vec{\mathbf{x}_0}) = \mathbf{0}. \tag{4.48}$$

Thus one gets

$$\mathbf{R}_{zz}(\vec{\mathbf{x}_0}) = \mathbf{R}^{(1)}_{zz}(\vec{\mathbf{x}_0}) = \frac{1}{2}\partial_0^2\mathbf{g}_{zz}(\vec{\mathbf{x}_0}). \tag{4.49}$$

Now using

$$\mathbf{R}_{\boldsymbol{zz}} = 8\pi\cdot G_N\mathbf{T}_{\boldsymbol{zz}}.$$

the amplitude of the elementary gravitational wave (electromagnetic wave) gives the quantitative deformation of space by an electrodynamic elementary wave. Such the importance of the Einstein-Equations for microphysics is proved.

$$\boxed{\mathbf{d_k} = \frac{\mathbf{2}}{\omega_k^2}\sqrt{\pi\gamma\mathfrak{E}_\mathbf{k}}}. \tag{4.50}$$

with the constant of gravitation $\gamma = 6.67\cdot 10^{-11}m^3kg^{-1}s^{-2}$ and $\mathfrak{E}_\mathbf{k}$ = as energy density. In these considerations the light velocity c does not occur explicitly.

Setting $\mathfrak{E}_\mathbf{k} = 1Wsec/m^3$ and using $\omega_k^2 = (\mathbf{2}\pi\cdot\nu)^\mathbf{2}$ with $\nu = 50$ this results in $\mathbf{d_k} = 2.933\cdot 10^{-10}m$. In comparison, the measured atomic radius of H^1 is given by $\approx 2.5\cdot 10^{-11}m$. **Obviously, that effect has to be considered in practice.**

As Spin 1 is assigned to photons the same has to be assumed for the graviton. (A photon of giant wavelength from an other perspective, if it is existent.)

The Einstein Equations maybe achieve much more than describing cosmological processes!

4.4 Summary

Until now, electromagnetism is not directly understood. It is described with detours via mechanical effects though for physicists it has manifested in immediate clearness after more than a century of successful handling. With the described unification electromagnetism is directly led back to the most fundamental terms of physics, space and time. The usually discussed gauge transformations are chosen by the observation space respectivly the coordinate space. The vector potential achieves an absolute significance.
Establishing the facilitation of quantizing gravitational fields is a trivial statement concerning the Maxwell structure of the gravitational waves identified as deformation waves of an appropiate Euclidian space. This applies accepting the known canonical quantization for the electromagnetic field.

Bibliography

[1] Blau Mathias: Lecture Notes on General Relativity 28.12.2017

[2] Dieudonne J.: Grundzüge der modernen Analysis Band 4 1976 page 67

[3] Einstein Albert: The Meaning of relativity 1950

[4] Landau L. D., Lifshitz E. M.: The Classical Theory of Fields Volume 2 Fourth Revised English Edition

[5] Jordan Pascual: SCHWERKRAFT UND WELTALL 1952

[6] Penrose Roger: The Road to Reality Jonathan Cape 2004

[7] Rossmann Wulf: Lectures on Differential Geometry Oktober 2003

[8] Straumann Norbert: The General Theory of Relativity Springer 2004