Mass interaction principle as an origin of quantum mechanics

Chu-Jun Gu*

Institute of Theoretical Physics, Grand Genius Group, 100085,

Beijing, P. R. China

January 8, 2023

Abstract

This paper proposes mass interaction principle (MIP) as: the particles will be subjected to the random frictionless quantum Brownian motion by the collision of space time particle (STP) ubiquitous in spacetime. The change in the amount of action of the particles during each collision is an integer multiple of the Planck constant h. The motion of particles under the action of STP is a quantum Markov process. Under this principle, we infer that the statistical inertial mass of a particle is a statistical property that characterizes the difficulty of particle diffusion in spacetime. Within the framework of MIP, all the essences of quantum mechanics are derived, which proves that MIP is the origin of quantum mechanics. Due to the random collisions between STP and the matter particles, matter particles are able to behave exactly as required by the supervisor and shepherd for all microscopic behaviors of matter particles. More importantly, we solve a world class puzzle about the anomalous magnetic moment of muon in the latest experiment, and give a self-consistent explanation to the lifetime discrepancy of muon between standard model prediction and experiments at the same time.

Keywords: *Mass Interaction Principle , Special Relativity, Schrödinger Equation , Quantum Measurement , Entanglement , Uncertainty Relation, Path Integral , Neutrino, Electromagnetism, Photon, Spin*

*E-mail: CJ_GU@grandgeniusgroup.com

1 Introduction

1.1 Spacetime Fluctuation, STP and MIP

We believe the energy fluctuations of spacetime are universal, which are defined as STP. In this picture, particles are classified into two groups: one is matter particles which interact with STP, another one is massless particles which freely move in spacetime. Matter particles change their states by all the collisions with STP. The underlying property of mass is a statistical property emerging from random impacts in spacetime. Different particles have different effects of impact by STP, which can be defined as some kind of inertia property of particles. This property corresponds to mass dimension (Following we will prove it happens to be the inertial mass from Schr$\ddot{o}$dinger's equation). Matter particles develop a Brownian motion due to random impacts from spacetime. We strongly suggest that all the probabilistic behaviours of quantum mechanics come from the Brownian motion, which is exactly the origin of quantum nature. In the framework of MIP, the photon represents itself as a Hopf link exicitation made of the 2+1-dim gauge field and its Hodge dual partner. On the other hand, under the MIP framework, photons not only exchange electromagnetic interactions, they also exchange spin information. It just explains that the annihilation condition of positive and negative electrons is not only the opposite of charge, but also the opposite of its spin. In modern physics, the spin and charge of matter particles are independent quantum properties. However, the spin has a magnetic moment and indicates that the spin and electromagnetic interactions are related. Under the MIP framework, this apparent contradiction can be self-consistently explained.

We believe the quantum behaviours of matter particle come from spacetime fluctuation. The energy fluctuation of spacetime is quantised. We call the quantised energy as spacetime particle. It is a massless and spinless scalar particle. The exchange of energy between particle and STP is not strictly random, which leads to a unique Brownian-like motion. Once the time interval of impact is fixed, the exchange of energy has to be quantised, which indeed is the quantum nature of particles. Therefore, all quantum nature of particles is a faithful representation of spacetime quantised fluctuation.

Matter particles will perform random fluctuation motion in spacetime because of stochastic interactions between STP and matter particles, within which the energy exchange can not be achieved instaneously. For free matter particles, we define the product of exchanged characteristic energy and the corresponding time interval as the change of action in the collision process(For more details, please refer to Appendix A).

1.2 Inertia Mass is a Statistical Property

Until now, our knowledge of mass, a fundamental concept of physics, mainly comes from Newton's laws of motion especially the first and second laws. The definition of mass in physics is a basic property of particles. The amount of matter contained in object is called the mass of object. The mass is related to the inertial nature of the object's original motion state.

The first law states that in an inertial reference frame, an object either remains at rest or continues to move at a constant speed, unless acted upon by a force. However according to the MIP, free particle has to do Brownian-like motions in spacetime, which is a Markov process. The mass of particle, in order to be sensed by spacetime, has to be collided randomly by STP. Mass cannot be well defined within the interval of two consecutive random collisions. In other words, mass is not a constant property belonging to the particle itself, but a discrete statistical property depending on dynamical collisions of spacetime. We will derive from MIP straightforwardly that mass must be a statistical term which has its own means and fluctuations.

Moreover, we prove the uncertainty relation asserting a fundamental limit to the precision regarding mass and diffusion coefficient. This implies that both mass and diffusion coefficient of any particle state can not simultaneously be exactly measured. Newton's Second law states that in an inertial reference frame, the vector sum of the forces F on an object is equal to the mass m of that object multiplied by the acceleration of the object. This connects the concept of mass and inertia and in principle defines a fundamental approach to measure the mass of any particle experimentally. However, according to the MIP, forces on a particle are changed constantly by the random impact of STP. Therefore, we are no longer able to take constant mass for granted. In conclusion, we believe that mass as a statistical property is much more natural within the framework of modern science, which completely overrules Newton's concept of mass based on Mathematical Principles of Natural Philosophy first published in 1687.

1.3 Realistic Interpretation of Quantum Mechanics

The main idea of Copenhagen interpretation is that the wave function does not have any real existence in addition to the abstract concept. In this article we do not deny the internal consistency of Copenhagen interpretation. We admit that Copenhagen′s quantum mechanics is a self-consistent theory. Einstein believed that for a complete physical theory, there must be such a requirement: a complete physical theory should include all of the physical reality, not merely its probable behaviour. From the materialistic point of view, the physical reality should be measured in principles , such as the position q and momentum p of particles. In the Copenhagen interpretation, the particle wave function $\Psi(q,t)$ or the momentum wave function $\Psi(p,t)$ is taken to be the only description of the physical system, which can not be called a complete physical theory, at most a phenomelogical effective theory. Therefore, in this paper, we propose a MIP where the coordinate and momentum of particles are objective reality irrespective of observations . With the postulation of MIP, quantum behaviour will emerge from a statistical description of spacetime random impacts on the experimental scale, including Schrödinger′s equation, Born rule, Heisenberg′s uncertainty principle and Feynman′s path integral formulation. Thus, we believe that non-relativistic quantum mechanics can be constructed under the MIP. Born rule and Heisenberg′s uncertainty relation are no longer fundamental within our framework.

1.4 Photon under the MIP framework

Within the framework of MIP, STP spread over spacetime, and its energy spectrum distribution is consistent with scalar particles. It can therefore be thought of as an excitation of a scalar field. The influence of material particles on its spacetime is local, so on the 2+1-dimensional time-space slice, the influence of material particles on spacetime can be regarded as a potential energy.

In modern quantum field theory, an important point is that microscopic energy can be non-conservative, and it can fluctuate to form pairs of virtual positive and negative particles. Within the framework of MIP, the fluctuation of spacetime energy is itself STP. The number of STP particles is not conserved locally, but globally, the energy of STP is conserved. So the picture of STP as a free particle is restored on a large scale. This just shows that STP has some local symmetry, which is broken at large scale. In essence, when the domain symmetry of the authority is $U(1)$, STP is the excitation of a complex scalar field.

On the other hand, the spacetime can be regarded as 2+1-dimensional around the spacetime in which the material particles are located. On this 2+1-dimensional spatiotemporal slice, STP is the excitation of the complex scalar field, which is accompanied by the excitation of the gauge field. The material particle produces a local non-perturbative potential energy in the surrounding space and time. The existence of this potential energy can cause the STP to spontaneously form a stable vortex solution. If the STP is not accompanied by a gauge field, then the vortex solution will cause the problem of energy divergence in the vortex center. The gauge field just eliminates the problem of local energy divergence.

The existence of a vortex solution also provides a possibility of duality, namely Hodge duality. The Hodge duality will extend the dynamics of the 2+1 dimensional gauge field to the 3+1 dimension. In the sense of Lagrangian, the 3+1-dimensional gauge field just describes the electromagnetic field theory. That is to say, the 3+1-dimensional equation of motion is Maxwell's equation. Therefore, we derive the classical electromagnetic theory from the vortex dynamics of STP.

In the MIP framework, the photon is essentially a topological excited state of two 2+1-dimensional gauge fields with their field strengths being Hodge's dual, and its topological configuration is a Hopf chain. Physically, photons transfer phase changes of material particles. Its equation of motion is the Maxwell equation.

On the other hand, the two topological circles of the photon, of which topological configuration Hopf link correspond to the topological subspace of the local spacetime. The Hopf links just represent the Lorentz representation of spin 1, which is a vector representation. Therefore, within the framework of MIP, the spin 1 of zero-mass photon is also self-consistently explained.

1.5 Fremion spin under the MIP framework

Within the framework of MIP, a careful observation of properties near the singularity at the center of the STP vortex, drive us to a new perspective of particle spin. We noticed there are not only energy divergence at the singularity on the center of the STP vortex, there also exists a disorientation property for a direction vector. To describe the disorientation, we introduce the torsion based on the cotangent vielbein field. The torsion tensor actually drives the cobordism topological phase transition between STP vortices on tangent space and its dual normal space. By the cobordism topological phase transition, we combined vortices on the 2+1 dimensional tangent space and normal space into a 3+1 dimensional instanton. The cost of this cobordism topological phase transition, is to calculate the corresponding topological order. By cohomological theory, we calculated the incomplete angle due to the cobordism topological phase transition, which concludes that the incomplete angle is an integer times π,this angle contributes to the STP vortex around matter particle a factor $e^{iN\pi}$. When rotating the particle a circle, the factor changed the signature of the wave function. This inveals the origination of particle spin is a topological phase transtion between STP vortices around the matter particle. Within the framework of MIP, particle spin describes the topological order of this cobordism phase trantion of STP vortices.

1.6 Muon anomalous magnetic moment under the MIP framework

On April 7, 2021, FermiLab performs a new muon anomalous magnetic moment experiment. The experimental value differs from the theoretical value predicted by the Standard Model with 4.2σ standard deviation. The probability of this deviation comes from statistical fluctuations is 1 in 40000, which implies possible physics beyond the Standard Model. The new massless scalar STP required by the MIP is a key step beyond the existing standard model. Introducing only one parameter, the interaction strength between STP and lepton, not only perfectly solves the world-class problem of the anomalous magnetic moment of muons in the latest experiment, but also explains the muon lifetime discrepancy between theory and experiment. It can be seen that this is a triumph for applications of MIP in modern particle physics.

Last and most importantly, we derived the generation for charged leptons. This is a completely new result and one can not derive this law in current quantum field theory framework. Within the MIP framework, by invoking the STP vortices, the generation is a direct inference.

2 Mass Interaction Principle

2.1 Proposing the MIP

Particles moving in spacetime interact with STP. The generation of STP itself should be regarded as a microscopic random excitation of local spacetime energy. We can assume the following two self-consistent ideal STP models. First, the spacetime itself is discrete, and each of the smallest spacetime units can act on the particle to change the particle's motion . However this spacetime unit acts as a random force on the particles, the motion of the particles in spacetime under the action of STP will also be random. Secondly, the energy distribution of STP is Gaussian, therefore, when they were scattering with matter particle, the force is random.

Furthermore, we propose in each interaction between matter particle and STP, the exchanging action should be nh, with n integer and h the Planck constant. According to this, we can define the MIP accurately. Suppose STP begin to collide with matter particle at time t_1 and end it at at time t_2 to exchange energy E. Without the collision of STP, the action of particle at the same interval will be

$$S = \int_{t_1}^{t_2} E_0 dt \tag{2.1}$$

With the collision of STP, the action of particle at the same interval will be

$$S' = \int_{t_1}^{t_2} E(t) dt \tag{2.2}$$

Therefore the change of action in Definition 1 is

$$\delta S = S' - S = \int_{t_1}^{t_2} [E(t) - E_0] dt \equiv \int_{t_1}^{t_2} f(t) dt \tag{2.3}$$

By definition, integral function $f(t)$ is a monistic increasing function $f(t)$ with following property

$$f(t_1) = 0, f(t_2) = E \tag{2.4}$$

According to Mean value theorems for integrals, there exists one point t^* at the interval satisfying

$$\int_{t_1}^{t_2} f(t) dt = f(t^*)(t_2 - t_1) \tag{2.5}$$

Setting exchange of energy be $E^* = f(t^*)$ at this point, we have $0 < E^* < E$. So the exact formula of the change of action is

$$\delta S = E^* \delta t \tag{2.6}$$

where $\delta t \equiv t_2 - t_1$. Therefore we are sure that, it is this characteristic exchange energy E^* not the energy of STP itself corresponding to the change of action. With MIP $\delta S = nh$, it's impossible to interact instantaneously, since the exchange energy E^* will blow up.
In our MIP framework, there are no instant interactions between matter particle and STP, in other words, the interaction takes time to transfer the energy. If the scattering STP has

an extremely low energy such that in Δt, the transfered action is less then h , we conclude that in Δt, the STP cannot collide the particle. We argue that such a collision is still in process, the particle as well as the STP are in a bound state, not a scattering state. This is similar to a completely inelastic collision in classic mechanics. While in such a process, the conservation of energy and momentum can not be satisfied simultaneously. Because of conservation of energy and momentum, the bound state actually is not a stable state. This observation leads to an important point: there exists a minimal energyE_{min} in Δt so that

$$E_{min}\Delta t = const. \tag{2.7}$$

In physics , the product of energy and time will have the dimension of action. It is natural to suggest such a constant with action dimension is the Planck constant, so we have

$$E_{min}\Delta t = nh, n \in Z. \tag{2.8}$$

At a certain moment, particle can be scattered by many STP with different momenta and energies. In Δt, we assume there are effectively N collisions. The state of the motion will depends on the net effect of the N times collision. This is a principle of superposition. We can use in total N vectors to superposite whole changes of the state of motion, which means if at time t the particle was at position $\vec{X}(t)$, with speed $\vec{V}_0$, then at the moment $t + \Delta t$, its position will be $\vec{x}(t + \Delta t) = \vec{X}(t) + \sum_{i=1}^{N} \Delta X_i$, and speed$\vec{V}_0 + \sum_{i=1}^{N} \Delta\vec{V}_i$. This simple analysis tells us in Δt, the ultimate state of motion of the particle can be separated as N different paths. This is the effect of separation of paths. While the weights of these paths, *aka* the probability distribution of universal diffusion, highly rely on the energy distribution of STP. Collisions by STP with different energies end up with different changes of the state of motion.

2.2 The Nature of Spacetime within the framework of MIP

At the beginning of the 20th century, the null result of the Michaelson-Morley experiment ended the ether theory. Within the framework of MIP, the concept of spacetime looks very similar to that of ether, but it is fundamentally different. To clarify this, let us first review the concept of ether. The ether is a gas medium filled in Newton's absolute static time and space. Its definition directly introduces a reference frame of God's perspective, which is Newton's static spacetime system. The earth and this frame of reference are relatively moving, so they will feel the ether wind blowing, which is the experimental basis of the Michaelson-Morley experiment. But spacetime is not a gaseous medium filled with absolute time and space. It is the fluctuation of time and space. From a large scale, the fluctuation of spacetime does not have significant effects. Spacetime seems to be smooth and differentiable, and the differential geometry theory of general relativity can effectively describe the physical properties of large-scale spacetime. However, on the microscopic scale, the fluctuation of spacetime indicates that spacetime itself does not have continuous property. There is no absolute static spacetime reference frame in the above discussion, so the STP within the framework of MIP is not etheric.

The null result of the Michaelson-Morley experiment actually promoted Einstein's most important hypothesis of the theory of relativity, which is the constant speed of light. In the theory of relativity, the constant speed of light is the only absolute assumption, and the relativity of all other speeds remains.

Within the framework of MIP, the energy fluctuation of spacetime forms STP. If you think of spacetime as a peaceful lake, then STP is the splash of water on the surface of the lake. When it falls on the surface of the lake, it will form ripples. Therefore, the emergence of STP is always accompanied by the spread of ripple. The propagation speed of ripple is the characteristic propagation speed in spacetime. Forming a STP means that fluctuation of spacetime will spread to a certain spatial distance within a certain period of time, so the spacetime around the STP is also changed. We now know that the smallest scale of time is the Planck scale, and the smallest scale of space is the Planck length. In the Planck time STP has to spread a Planck length of space, so the propagating speed of STP is the same as light speed.

From the spacetime view of MIP, any physical observable event in spacetime will inevitably accompany the fluctuation of spacetime energy, which will profoundly affect the spacetime after the event. Under such a view of spacetime, the current spacetime is actually the result of the joint influences of all events in the history.

2.3 Energy spectrum of STP

To consider the collision between STP and particle, it will be ambiguous if the energy spectrum of STP is not clear at first. In this subsection, we deal with this problem.

Let us consider a cubic with volume L^3, which we call a system. If there are in total N systems in spacetime, we can classify the N systems by states. We label a state by j so that there are N_j systems with energy E_j. The total energy of the ensemble(collection of N systems) is denoted as $\mathcal{E}$, we have

$$N = \sum_j N_j \tag{2.9}$$

$$\mathcal{E} = \sum_j N_j E_j, \tag{2.10}$$

for constant $\mathcal{E}$ and N, the possible total number of states in whole spacetime will be $\Omega = \frac{N!}{\prod_j N_j!}$. Physical reality is required by the maximum of Ω. There is a distribution $\{N_j\}$ maximizing Ω, so that

$$\ln\Omega = N\ln N - N - \sum_j N_j \ln N_j + \sum_j N_j \cdots \tag{2.11}$$

the question is under constraints (2.9,2.10), how to maximize $\ln\Omega$. With the method of Lagrangian multiplier,

$$\frac{\partial \ln\Omega}{\partial N_j} - \lambda_1 \frac{\partial \sum_j N_j}{\partial N_j} - \lambda_2 \frac{\partial\left(\sum_j N_j E_j\right)}{\partial N_j} = 0 \tag{2.12}$$

we can derive

$$\begin{aligned} -\ln N_j - \lambda_1 - \lambda_2 E_j &= 1 \Rightarrow \\ N_j &= e^{-1-\lambda_1-\lambda_2 E_j} \end{aligned} \tag{2.13}$$

hence the probability of being at state j

$$P_j = \frac{N_j}{N} = \frac{e^{-\lambda_1-\lambda_2 E_j}}{\sum_j e^{-\lambda_1-\lambda_2 E_j}} = \frac{e^{-\lambda_2 E_j}}{\sum_j e^{-\lambda_2 E_j}} \equiv \frac{e^{-\lambda_2 E_j}}{\mathcal{Z}} \tag{2.14}$$

and the average energy of the ensemble

$$E = \frac{\mathcal{E}}{N} = \sum_j E_j P_j = -\frac{\partial}{\partial \lambda_2} \ln \mathcal{Z} \tag{2.15}$$

In L^3 , suppose there are $n_{\vec{p}} = 0, 1, 2, \cdots$ STP have momentum $\vec{p}$, for giving distribution $\{n_{\vec{p}}\}$, the energy in L^3 is

$$E = \sum_{\{n_{\vec{p}}\}} n_{\vec{p}} E_{\vec{p}} \tag{2.16}$$

with $E_{\vec{p}} = c|\vec{p}| = cp$. Here STP are massless as proposed. We have

$$\begin{aligned} \mathcal{Z} &= \sum_{\{n_{\vec{p}}\}} e^{-\lambda_2 E} = \prod_{\vec{p}} \left(1 + e^{-c\lambda_2 p} + e^{-2c\lambda_2 p} + \cdots\right) \\ &= \prod_{\vec{p}} \frac{1}{1 - e^{-c\lambda_2 p}} \end{aligned} \tag{2.17}$$

and the average energy of a system is

$$\begin{aligned} E &= -\frac{\partial}{\partial \lambda_2} \ln \mathcal{Z} = \frac{\partial}{\partial \lambda_2} \sum_{\vec{p}} \ln\left(1 - e^{-c\lambda_2 p}\right) \\ &= \sum_{\vec{p}} \frac{pe^{-c\lambda_2 p}}{1 - e^{-c\lambda_2 p}} = \sum_{\vec{p}} \frac{cp}{e^{c\lambda_2 p} - 1} \end{aligned} \tag{2.18}$$

when $L \to \infty$, summation becomes integration as follow

$$\sum_{\vec{p}} \to \frac{L^3}{8\pi^3} \int d^3\vec{p}$$

from which we see

$$E = \frac{L^3}{2\pi^2} \int dp \frac{p^3}{e^{c\lambda_2 p} - 1} = \frac{\pi^2 L^3}{30\lambda_2^4} \tag{2.19}$$

so the density of STP will be

$$\epsilon_{ST} = \frac{\pi^2}{30\lambda_2^4} \tag{2.20}$$

Recover c and $\hbar$ in above equation, we obtain

$$\epsilon_{ST} = \frac{\pi^2}{30c^3\hbar^3\lambda_2^4}. \tag{2.21}$$

Now consider the physical meaning of λ_2, which determines the constraint that represents energy distribution of STP. While the multiplier λ_1 which determines the constraint represents the number distribution of STP has no affects on the dynamics of STP. This means we can classify STP arbitrarily, except to satisfy the total energy constraint. For example, the action of particle changed $kh, k \in \mathbb{Z}$ in a certain collision by STP. In physics we can not distinct one STP collision or many STP collision, since neither from energy spectrum of STP nor from the change of status of the particle can distinct them. From dimensional analysis and MIP, we have

$$\lambda_2 = \frac{g}{E_{ST}} \tag{2.22}$$

where g is a dimensionless coupling constant, and E_{ST} is the characteristic energy of STP. In the limit of extreme relativity, the colliding of STP can not be seen as perturbations, but strong disturbances.

3 Random Motion and Spacetime Diffusion Coefficient

Let m_{ST} be the statistical mass of the particle . We will prove the spacetime interaction coefficient of a m_{ST} mass particle will be universally given as

$$\Re = \frac{h}{2m_{ST}}. \tag{3.1}$$

Within the framework of random motion[1], or Wiener process in mathematics [2], this spacetime induced random motion is equivalent to the Markov process, moreover, the spacetime interaction coefficient is nothing but the diffusion coeffient [3]. In this section, we will start our journey from propability theory of random motion[3, 4], and then give a concrete proof that for the random motion induced by MIP, the spacetime interaction coefficient is given exactly by (3.1). The last two subsections discussed two spacetime models in order to investigate the origin of the spacetime interaction coefficient. From both we obtained the coefficient reading as $\Re = \frac{w\ell}{2}$, in which w is the average speed of the particle and ℓ the mean free path.

3.1 Langevin Equation

The space-time background can be seen as a fluctuation environment, and the particles move in this fluctuation environment. This is a Markov process. The position of the particle $\vec{q}$ is a random quantity. From a strict mathematical point of view, it can be decomposed into a super random part and a superimposable function

$$\vec{q}(t) = \vec{q}_0(t) + \vec{\omega}(t) \tag{3.2}$$

where $\vec{q}_0(t)$ is the differential part of position and $\vec{\omega}(t)$ represents random motions of particles. The whole motion of particle can be described by Langevin equation as

$$\frac{\delta q_i(t)}{\delta t} = \frac{dq_{0,i}(t)}{dt} + \frac{\delta \omega_i(t)}{\delta t} = U_i(\mathbf{q}(t)) + \nu_i(t) \tag{3.3}$$

In spacetime, particles are subjected to the impact of STP. But if some of the impact is relatively weak, then the change of the state of motion can only be regarded as a perturbation. Under perturbation, the velocity of the particles changes which can be seen as smoothly and continuously. The non-perturbative impacts of STP on the particles instantaneously change the motion state of the particles, leading to the completely random motion. Each impact should be treated as a sum of a differential impact and a random impact. A microscopic impact does not change the classic trajectory of the particle, but it will cause the trajectory to be superimposed on the motion of an envelope. This is precisely the "differentiable velocity function" $\mathbf{U}(\mathbf{q}(t))$ expressed by the first term in the three velocities decomposition of the Langevin's equation. Therefore, the true velocity of the particle $\mathbf{V}(t)$ should contain three contributions, which is

$$\mathbf{V}(t) = \mathbf{v}(t) + \mathbf{u}(\mathbf{q}(t)) + \vec{\nu}(t) \tag{3.4}$$

Where $\mathbf{v}(t)$ is the classic statistical velocity, $\mathbf{u}(\mathbf{q}(t))$ is the quantum envelope velocity of the particle, and $\vec{\nu}(t)$ is the diffusion velocity representing random motion. $\mathbf{U}(\mathbf{q}(t))$ denotes the union of the first and the second term in eq.(3.4)

$$\mathbf{U}(\mathbf{q}(t)) = \mathbf{v}(t) + \mathbf{u}(\mathbf{q}(t)) \tag{3.5}$$

.

For a Markov process, the average contribution of white noise vanishes. However, because of its Gaussian nature, its variation is not zero. We have

$$\langle \nu_i \rangle_\nu = 0, \ \langle \nu_i(t)\nu_j(t') \rangle_\nu = \Omega \delta_{i,j} \delta(t-t'), \ t \geq t' \tag{3.6}$$

here the $\delta_{i,j}$ in the later equation can be obtained from the spacetime homogeneous property, while $\delta(t-t')$ is determined from the Markov property. For a Markov process, only conditions at the very moment determine the dynamics of the system, and all information from future or past are irrelevant. We can write down the basic correlation function by introducing a probability measure $[d\rho(\nu)]$,which is given as

$$[d\rho(\nu)] := \left(\sqrt{\frac{1}{2\pi\Omega\delta(t-t')}} \right)^D [d\nu] \exp\left(-\frac{1}{2\Omega} \int dt \sum_i \nu_i^2 \right) \tag{3.7}$$

It is easy to see that

$$\begin{aligned} \nu_i(t)\rangle_\nu \equiv \int \nu_i(t)[d\rho(\nu)] &= 0 & (3.8) \\ \nu_i(t)\nu_j(t')\rangle_\nu \equiv \nu_i(t)\nu_j(t')[d\rho[\nu]] &= \Omega\delta_{i,j}\delta(t-t') & (3.9) \end{aligned}$$

Here Ω describes the strength of spacetime interaction on the particle. Notice $\delta(t-t')$ has the inverse dimension of time t , as

$$\int_0^\infty \delta(t-t')dt = 1.$$

However, from the definition of measure (3.7), we can see, ν_i have the unit of m/s, so Ω will have the unit of m^2/s. From previous analysis, each collision leads to a change of an action

h. h has the unit of angular momentum, $kg \cdot m^2/s$. From this we can define a quantity with mass unit, it is

$$m_{ST} \equiv \frac{h}{\Omega}. \tag{3.10}$$

The mass m_{ST} has the meaning such that it is the mass collided by STP and is a statistical property. Accordingly, the collision parameter $\Omega = \frac{h}{m_{ST}}$ reflects a physical realistic viewpoint: an object in our real nature, the larger its mass means the smaller its quantum effect.

Langevin equation generates a timedependent probability such that

$$\mathbf{P}[\mathbf{q}, t;\ \mathbf{q}', t'] = \langle \prod_{i=1}^{D} \delta[q_i(t) - q_i'(t')] \rangle_\nu,\ t \geq t' \tag{3.11}$$

which means for an operator $\mathcal{O}[\mathbf{q}]$, its average value at time t will be:

$$\langle \mathcal{O}[\mathbf{q}(t)] \rangle_\nu \equiv \int \mathbf{P}[\mathbf{q}, t;\ \mathbf{q}', t'] \mathcal{O}[\mathbf{q}] d\mathbf{q} \tag{3.12}$$

Using the probability distribution (3.11), one can immediately verify equation (3.12). Actually, the distribution (3.11) can be seen as an evolution process, which says

$$\mathbf{P}[\mathbf{q}, t;\ \mathbf{q}', t'] = \iint q(t) e^{-(t-t')H(p,q)} q'(t') d^D p \tag{3.13}$$

here the evolution Hamiltonian is the famous Fokk-Planck Hamiltonian, as we will derive its formalism in next subsection.

3.2 Fokker-Planck Equation

Given the Langevin equation (3.3), we can derive the corresponding Fokk-Planck equation, as well as the Fokk-Planck Hamiltonian [3].

We consider the time segment from t to $t+\epsilon$, $\epsilon \to 0$, and have the Langevin equation as:

$$q_i(t+\epsilon) - q_i(t) = \epsilon U_i(\mathbf{q}(t)) + \int_t^{t+\epsilon} \nu_i(\tau) d\tau + O(\epsilon^2) \tag{3.14}$$

its related propability distribution is

$$\mathbf{P}[\mathbf{q}, t+\epsilon;\ \mathbf{q}', t] = \langle \delta(\mathbf{q} - \mathbf{q}(t+\epsilon)) \rangle_\nu \tag{3.15}$$

According MIP, everytime the STP collided with the particle, the action of particle will change nh, $n \in \mathbb{Z}$. To obtain the Fokk-Planck equation, we define following discreterization

$$\bar{\nu_i} \equiv \frac{1}{\sqrt{\epsilon}} \int_t^{t+\epsilon} \nu_i(\tau) d\tau \tag{3.16}$$

so that the discrete Langevin equation is

$$q_i(t+\epsilon) - q_i(t) = -\frac{1}{2}\epsilon f_i(\mathbf{q}(t)) + \sqrt{\epsilon}\bar{\nu_i} + O(\epsilon^2) \tag{3.17}$$

Notice here the time has been discreterized as

$$(t - t')/\epsilon \in \mathbb{Z}^+.$$

Now the Gaussian distribution and the property of Markov progcess determins the average value of discrete white noises ν_i, and we have

$$\langle \bar{\nu_i} \rangle_\nu = 0, \ \langle \bar{\nu_i}(t) \bar{\nu_j}(t') \rangle_\nu = \frac{\hbar}{m_{ST}} \delta_{i,j} \delta_{t,t'} \tag{3.18}$$

When $\epsilon \to 0$, the Fourier transformation of the probability distribution (3.15) is

$$\begin{aligned}
\tilde{\mathbf{P}}[\mathbf{p}, t; \ \mathbf{q}', t']|_{t=t'+\epsilon} &= \int e^{-i\mathbf{p}\cdot\mathbf{q}} \mathbf{P}[\mathbf{q}, t; \mathbf{q}', t'] d^D \mathbf{q}|_{t=t'+\epsilon} \\
&= \langle e^{-i\mathbf{p}\cdot\mathbf{q}'(\mathbf{t}-\epsilon)} \rangle_\nu \\
&= \langle e^{-i\mathbf{p}\cdot(\mathbf{q}'(\mathbf{t}) - \epsilon \frac{\delta \mathbf{q}'(\mathbf{t})}{\delta \mathbf{t}} - \mathbf{O}(\epsilon^2))} \rangle_\nu \\
&= \langle \exp(-i\mathbf{p} \cdot (\mathbf{q}'(\mathbf{t}) - \epsilon \mathbf{U}(\mathbf{q}'))) \rangle_\nu \\
&\quad \times \left\langle \exp\left[+i\mathbf{p} \centerdot \int_{t-\epsilon}^{t} \nu(\tau) d\tau \right] \right\rangle_\nu \times \left\langle \exp\left(O(\epsilon^2) \right) \right\rangle_\nu \\
&= \exp\left[-i\mathbf{p} \centerdot (\mathbf{q}' - \epsilon \mathbf{U}(\mathbf{q}')) \right] \\
&\quad \times \left\langle \exp\left[+i\mathbf{p} \centerdot \int_{t-\epsilon}^{t} \nu(\tau) d\tau \right] \right\rangle_\nu
\end{aligned} \tag{3.19}$$

Notice that the last average value can be evaluated out by Gaussian integration, which reads,

$$\begin{aligned}
&\left(\sqrt{\frac{h}{2\pi}} \right)^D \int [d\nu] \exp\left(-\frac{m_{ST}}{2h} \int dt \sum_i^D \nu_i^2 \right) \exp\left[+i\mathbf{p} \centerdot \int_{t-\epsilon}^{t} \nu(\tau) d\tau \right] \\
&= \left(\sqrt{\frac{h}{2\pi}} \right)^D \int [d\nu] \exp\left(-\frac{m_{ST}}{2h} \int dt \sum_i \nu_i^2 + i\mathbf{p} \centerdot \int_{t-\epsilon}^{t} \nu(\tau) d\tau \right) \\
&= \left(\sqrt{\frac{h}{2\pi}} \right)^D \int [d\nu] \exp\left(-\frac{m_{ST}}{2h} \int dt \sum_i \nu_i^2 + i\sqrt{\epsilon} \mathbf{p} \centerdot \bar{\nu} \right) \\
&\quad \times \exp\left(-\epsilon \frac{h}{2m_{ST}} \mathbf{p} \centerdot \mathbf{p} + \epsilon \frac{h}{2m_{ST}} \mathbf{p} \centerdot \mathbf{p} \right) \\
&= \left(\sqrt{\frac{h}{2\pi}} \right)^D \int [d^D \left(-\nu_i - \frac{ih}{2m_{ST}} \sqrt{\epsilon} p_i \right)] \\
&\quad \times \exp\left(-\frac{m_{ST}}{2h} \int dt \sum_{i=1}^{D} \left(\nu_i + \sqrt{\epsilon} \frac{ih}{2m_{ST}} p_i \right)^2 - \epsilon \frac{h}{2m_{ST}} \mathbf{p} \centerdot \mathbf{p} \right) \\
&= \exp\left(-\epsilon h \mathbf{p} \centerdot \mathbf{p} / (2m_{ST}) \right)
\end{aligned} \tag{3.20}$$

here we obtain the probability distribution under Fourier transformation ,

$$\tilde{\mathbf{P}}[\mathbf{p}, t+\epsilon; \ \mathbf{q}', t] = e^{-\epsilon h/2m_{ST} \mathbf{p}\cdot\mathbf{p} + i\epsilon \mathbf{p} \centerdot f(\mathbf{q}')/2 - i\mathbf{p}\cdot\mathbf{q}'} \tag{3.21}$$

for $\epsilon \to 0$, expanding (3.21) will end up with

$$\tilde{\mathbf{P}}[\mathbf{p}, t+\epsilon; \ \mathbf{q}', t] = e^{-i\mathbf{p}\cdot\mathbf{q}'} (1 - \epsilon H_{FP}(\mathbf{p}, \mathbf{q}') + O(\epsilon^2)).$$

Here we obtained the Fokk-Planck Hamiltonian

$$H_{FP}(\mathbf{p},\mathbf{q}) = -\frac{h}{2m_{ST}}\mathbf{p}\cdot\mathbf{p} - i\mathbf{p}\cdot f(\mathbf{q})/2 \tag{3.22}$$

From which we can read off the diffusion coefficient induced by collisions between STP and the particle, is exactly $\Re = h/2m_{ST}$. Later we will see in deriving the Schrödinger equation of free particle in spacetime, the spacetime mass $m_{ST} = 2\pi m$ will be identified as the inertial mass, in the framework of non-relativistic quantum mechanics.

3.3 From spacetime scattering to spacetime diffusion coefficient

3.3.1 From spacetime scattering to spacetime diffusion coefficient

Beginning with MIP, we want to investigate the origin of spacetime interaction coefficient. Within the framework of discrete spacetime, spacetime diffusion coefficient $\Re = \frac{h}{2m_{ST}}$ should be derived in terms of parameters of discrete spacetime. Let us consider the simplest discrete model (see Fig.1), where the length union of discrete space is ℓ. $P(j,t)$ is the probability of a particle at lattice site j at time t.

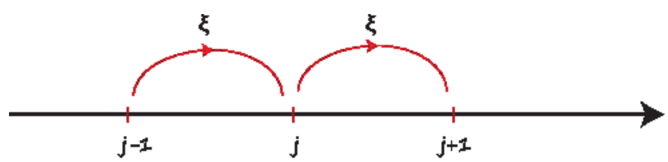

Figure 1: Random jumping model on one dimensional lattice

Because of the discrete nature of the space, all jumpings can only happen between nearest pair of positions. Given the rate of jumping between the nearest neighbour ζ and the isotropy of frictionless space, the evolution of probability should be

$$\partial_t P(j,t) = \zeta(\frac{1}{2}P(j-1,t) + \frac{1}{2}P(j+1,t) - P(j,t)) \tag{3.23}$$

the first two terms of RHS of (3.23) describe the fact that jumping forward and backward from neighbors $j-1$ and $j+1$ positions respectively, have the same probability, which is 1/2, the third term remarks the probability from j position to neighbors. Introducing the fundamental spacing of the lattice ℓ, the eq.(3.23) goes to

$$\partial_t P(j,t) = \frac{\zeta\ell^2}{2}(\frac{\frac{P(j+1,t)-P(j,t)}{\ell} - \frac{P(j,t)-P(j-1,t)}{\ell}}{\ell}) \tag{3.24}$$

In the continum limit of spacetime, which says $\ell \to 0$, and $\zeta \to \infty$, but keeping the quantity $\zeta\ell^2$ unchanged, the probability $P(j,t)$ now becomes the probability density $\rho(x,t)$, and the RHS of (3.23) becomes the definition of second derivative. Thus we have

$$\partial_t \rho(x,t) = \frac{\zeta\ell^2}{2}\partial_x^2\rho(x,t). \tag{3.25}$$

It is straightforward to generalise above equation to three dimension case, we have,

$$\partial_t \rho(\vec{r}, t) = \frac{\zeta \ell^2}{2} \nabla^2 \rho(\vec{r}, t) \tag{3.26}$$

Comparing with diffusion equation in Einstein's paper[6]

$$\partial_t \rho(\vec{r}, t) = \Re \nabla^2 \rho(\vec{r}, t) \tag{3.27}$$

the microscopic origin of spacetime diffusion coefficient will be

$$\Re = \frac{\zeta \ell^2}{2} \tag{3.28}$$

Furthermore, we can also discrete time with union $\tau = \frac{\ell}{w}$, where w is the average speed of particle. With $\zeta = \frac{1}{\tau}$, we obtain

$$\Re = \frac{w\ell}{2} \tag{3.29}$$

Combining the microscopic structure of discrete spacetime with the MIP, we have

$$\Re = \frac{w\ell}{2} = \frac{h}{2m_{ST}} \tag{3.30}$$

3.3.2 From Spacetime Scattering to the Spacetime Diffusion Coefficient

Particles will be scattered randomly from the STP with the speed of light, which leads to the probability distribution of speed $f(\vec{v})$, the number of partials within $v \to v + dv$ is $f(v)d^3\vec{v}$. Therefore, all the particles cross the section area dA during time dt will be inside the cylinder (see Fig.2).

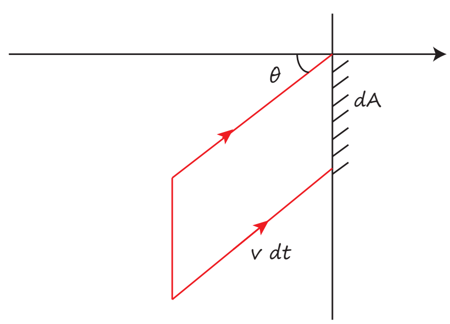

Figure 2: Probability distribution of spacetime scattering

The volume of this cylinder is

$$V = vdt \cos\theta dA \tag{3.31}$$

in which the number of particles is

$$N = f(\vec{v})d^3\vec{v}vdt \cos\theta dA \tag{3.32}$$

Because of the isotropy of space, we have $f(\vec{v}) = f(v)$. From left to right, the number of particle cross the unit area per unit time is

$$\begin{aligned}
\Phi &= \int_{v_z>0} \frac{N}{dAdt} \\
&= \int_0^{\frac{\pi}{2}} d\theta \cos\theta \sin\theta \int_0^{2\pi} d\phi \int_0^{+\infty} f(v) v^3 dv \\
&= \pi \int_0^{+\infty} f(v) v^3 dv
\end{aligned} \tag{3.33}$$

where $v_z > 0$ means $0 < \theta < \frac{\pi}{2}$. The average speed reads

$$w = \frac{\int_0^{+\infty} f(v) v d^3v}{\int_0^{+\infty} f(v) d^3v} = \frac{4\pi}{\rho} \int_0^{+\infty} f(v) v^3 dv \tag{3.34}$$

where the density of particle number is $\rho = \int_0^{+\infty} f(v) d^3v$. Correspondingly, the number of particle cross the unit area per unit time will be

$$\Phi = \frac{1}{4} \rho w \tag{3.35}$$

Let mean free path of particles be ℓ, i.e. the average distance traveled by the particle between successive impacts from spacetime. The net flux J_z through the z plane will be (see Fig.3)

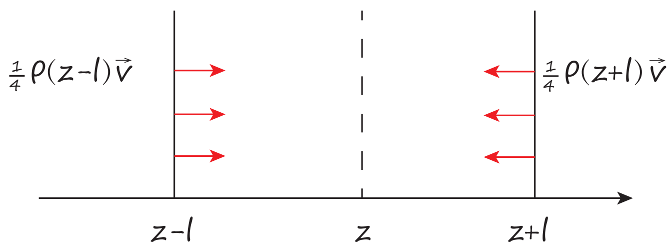

Figure 3: mean free path and scattering flux

$$J_z = \frac{1}{4} \rho(z-\ell) w - \frac{1}{4} \rho(z+\ell) w = -\frac{1}{2} \ell w \partial_z \rho \tag{3.36}$$

With the equation of continuity

$$\partial_t \rho + \nabla \cdot \vec{J} = 0 \tag{3.37}$$

and the isotropy of space, we have

$$\partial_t \rho = \frac{1}{2} \ell w \nabla^2 \rho \tag{3.38}$$

Combining the kinetics of spacetime scattering with quantum nature induced by STP, we obtain

$$\Re = \frac{w\ell}{2} = \frac{h}{2m_{ST}} \tag{3.39}$$

which is consistent with eq.(3.30).

3.4 Statistical mass of fundamental particles

Let's consider the electron at first. The mass of an electron is $m_e = 9.104 \times 10^{-31} kg$. So its static energy is

$$E_e = m_e c^2 = 9.104 \times 10^{-31} \times 9 \times 10^{18} J = 8.1936 \times 10^{-12} J$$

This energy, according to MIP, comes from "effective" collisions between STP and the electron. In our MIP theory, the electron is not a point-like particle. It is finite size, statistically. Because of symmetry, its shape is a ball with a sphere boundary. The effective collisions are considered as the number of STP which coming into and going out cross the sphere. Assume every effective collision gives energy, which numerically equals to Planck constant. Hence the times of effective collisons (TEC) can be calculated as follow

$$N_e = E_e/h = 1.2347 \times 10^{20} [s^{-1}]$$

The statistical mass of electron can be written in form of TEC

$$m_e = \frac{h}{c^2} N_e \tag{3.40}$$

The ratio of mass and TEC is

$$k_{st} \equiv \frac{h}{c^2} = 7.37 \times 10^{-51} kg \cdot s \tag{3.41}$$

It has the unit of $[mass] \cdot [time]$. The fluctuation of the density of STP, around the electron, denoted as $\Delta\rho^e_{st}$, can be written as

$$\Delta\rho^e_{st} \equiv \rho^e - \rho_0 = \frac{m_e c^2}{\frac{4}{3}\pi r^3 h} \tag{3.42}$$

For proton, it is easy to calculate exactly the same as the electron, we have

$$N_p = \frac{m_p}{k_{st}} = 1.6726 \times 10^{-27} / 7.37 \times 10^{-51} \simeq 2.227 \times 10^{23} [s^{-1}] \tag{3.43}$$

The radius of proton is

$$r_p \simeq 8.735 \times 10^{-16} m \tag{3.44}$$

from which we obtain the mean free path of a proton in the STP sea around it.

$$l_{st} = \sqrt[3]{\frac{4}{3}\pi r_p^3 / N_p} \simeq 2.3 \times 10^{-23} m$$

3.5 Momentum and energy within the framework of MIP

The time scale of physics spans many orders of magnitude. Cosmology studies the age of the universe at about 4×10^{17} seconds. Newtonian mechanics studies the low-velocity motion of macroscopic objects, and the time scale is usually on the order of seconds. The basic system of quantum mechanics is a hydrogen atom. When the electrons outside the hydrogen nucleus are in the ground state, the electrons move around the nucleus for about 1.5×10^{-15} seconds. The first excited state of the hydrogen atom transitions to the ground state emitting light with a wavelength of 121 nm, corresponding to a time period of 4×10^{-16} seconds. Modern physics believes that considering the principles of general relativity, special relativity and quantum mechanics, the smallest physical time scale is Planck time about 5×10^{-44} seconds, which is the smallest measurable time interval. According to academic consensus today, any changes during this time interval cannot be measured or detected.

Under the MIP framework, the average number of STP hitting electrons within one second is 10^{20}. That is to say, the theory derived from MIP in this paper has a typical time scale of 10^{-20} seconds. For electron, this time scale is 10,000 times shorter than quantum mechanics[1]. Therefore, energy conservation and momentum conservation in quantum mechanics are not constant conservation laws, but statistical average conservation under the MIP framework. The momentum and energy we define below are the results of statistically averaging the random effects of STP.

In the time interval of 10^{-20} seconds, we call the momentum of particle [2] as instant momentum. According to MIP, instant momentum is defined as

$$\vec{P_i} = m_i \vec{V} \tag{3.45}$$

Where m_i is the mass of the particles in the time interval of 10^{-20}seconds, which we call as instant mass. $\vec{V}$ is the true velocity of the particle

$$\vec{V} = \vec{u} + \vec{v} + \vec{\nu} \tag{3.46}$$

Similarly, we define the instant kinetic energy of the particle as

$$E_i = \frac{1}{2} m_i V^2 \tag{3.47}$$

The mass observed in modern physical experiments is the statistical mass of the particles, which is the inertial property at intervals greater than $\times 10^{-16}$ seconds. The momentum observed in modern physical experiments is the momentum predicted by quantum mechanics. Quantum mechanical momentum is the statistical average of instant momentum, which we call statistical momentum:

$$\vec{P_s} =< \vec{P_i} >= \frac{M_{st}}{2\pi} < \vec{v} + \vec{u} > \tag{3.48}$$

From this we relate the instant momentum at small time scales to the quantum mechanical momentum at large time scales. There is an important observation which we have proved in Chapter 5. The classical statistical velocity of any stationary state (the ground state is the lowest energy stationary state) is $\vec{v} = 0$, and the quantum envelop velocity of the ground state electrons of hydrogen atoms is

$$\vec{u} = -c\alpha\hat{r} \tag{3.49}$$

Where α is the Fine structure constant. Comparing the results of quantum mechanics: the momentum of the ground state electrons of a hydrogen atom must be zero, satisfying the

[1] In the field of particle physics, short lifetime such as the Higgs boson is about 1.5×10^{-22} seconds. For the Higgs boson, the average number of STP hitting a Higgs particle in a second is 10^{25} times. Its typical time scale is a thousand times smaller than quantum field theory.

[2] In the discussion below, the particles are all specific to electrons and represent the particles of matter.

isotropic wave function. Subtlely, the quantum envelope velocity does not contribute to the momentum of the ground state electrons because isotropic offsets each other by $< \vec{u} >= 0$. Because quantum mechanics is the combined result of statistical averaging three velocities and instant mass on large time scales, $\vec{P_s}$ is consistent with the momentum calculated by quantum mechanics.

The kinetic energy observed in modern physical experiments is the kinetic energy predicted by quantum mechanics theory. Quantum mechanical kinetic energy is the statistical average of instant kinetic energy, which we call statistical kinetic energy.

$$E_s =< E_i >= \frac{M_{st}}{4\pi} < V^2 > \tag{3.50}$$

The quantum envelop velocity contributes to the kinetic energy of the ground state electrons (always positive so cannot cancel out). Therefore, the energy of the ground state electron has two parts (the classical statistical velocity is always 0, and does not contribute to the ground state kinetic energy):

ground state energy = quantum envelop energy + coulomb potential

The calculated result is exactly -13.6 ev, which is also consistent with the energy calculated by quantum mechanics. The quantum envelop kinetic energy is defined as

$$E_e = \frac{1}{4\pi} M_{st} u^2 \tag{3.51}$$

Substituting the value of the electron energy of the ground state of a hydrogen atom

$$E = \frac{M_{st}}{4\pi} < (c\alpha)^2 > + < -\frac{e^2}{4\pi\epsilon_0} a >= -13.6ev \tag{3.52}$$

Where a is the Bohr radius of the hydrogen atom and ϵ_0 is the vacuum permittivity. Thus, we obtain the definitions of momentum and kinetic energy that are consistent with quantum mechanics.

More generally, the equivalence between statistical momentum and quantum mechanical momentum in any quantum state are proved as follows. According to the Ehrenfest theorem of quantum mechanics, the average value of particle positions evolves with time as

$$\frac{d}{dt}\langle \vec{x} \rangle = \frac{1}{i\hbar}\langle [\vec{x},\ H] \rangle = \frac{1}{i2m\hbar}\langle [\vec{x},\ p^2] \rangle = \frac{1}{i2m\hbar}\langle \vec{x}pp - pp\vec{x} \rangle \tag{3.53}$$

Combining with $\vec{x}pp - pp\vec{x} = i2\hbar\vec{p}$, we have

$$\frac{d}{dt}\langle \vec{x} \rangle = \frac{1}{m}\langle \vec{p} \rangle \tag{3.54}$$

This is a very important result, indicating how the momentum average of quantum mechanics is related to the mean value of the coordinates. In the MIP framework, the derivative of coordinates versus time is defined as

$$\frac{d}{dt}\vec{x} = \vec{u} + \vec{v} \tag{3.55}$$

Once two sides of the equation are averaged, the momentum average of quantum mechanics corresponds to the statistical momentum of the MIP as

$$\vec{P}_s = < \vec{P}_i > = \frac{M_{st}}{2\pi} < \vec{v} + \vec{u} > \tag{3.56}$$

which proves that the microscopic theoretical basis of quantum mechanics is exactly MIP.

4 Mass-Diffusion Uncertainty relation

We now consider the motion status of particle under impacts of STP collisions. The most important proposition of Copenhagen interperitation of quantum mechanics is the wave-particle duality. This allows one using the superposition rule of plane waves to describe the state of a particle. The kernel of the wave transformation from frequency space to time space will be the factor $\exp(ipx/\hbar)$. In fact it introduces the quantized operator formalism $\vec{p} = -i\hbar\vec{\nabla}$. Because of the duality, physical quantities of the particle can also be derived from wave, which implies some quantities can be described in phase space as eigenvalues of special operators. However, under the framework of MIP, we need to emphrase again that the wave-like property of the particle is an emergent property due to collision of STP, therefore it is not intrinsic. We can not borrow the quantization hypothesis directly. We consider the action of the particle

$$\begin{aligned} & S[\phi(t,x), \partial\phi(t,x), \bar{\nu}(t,x)] \\ & \quad = S_0[\phi(t,x), \partial\phi(t,x)] + \sum_{I=1}^{\infty} S_I[\bar{\nu}(t,x)] \end{aligned} \tag{4.1}$$

where $\phi(t,x)$ describing the classical trajectory of the particle, and S_0 is the related classical action. $S_I[\bar{\nu}(t,x)]$ is the contribution of $I - th$ collision between STP and the particle. It does not depend on the classical trajectory at all, which only depends on the fluctuation of STP. The MIP said this term should contribute integer number of h , that is $S_I = nh$.

The partition function of the particle now is

$$Z = \int [d\phi(t,x)] \exp(-\frac{i}{\hbar} S[\phi(t,x), \partial\phi(t,x), \bar{\nu}(t,x)]) \tag{4.2}$$

hence

$$\exp\left(-\frac{i}{\hbar} S_I[\bar{\nu}]\right) = \exp\left(-\frac{i}{\hbar} nh\right) = e^{-i2\pi n} = 1 \tag{4.3}$$

from which we see the introducing of MIP does not change the classical partition function, therefore physical quantity derived from classical action will not be affected.

4.1 Mass-Diffusion Uncertainty

We have claimed and proven that particle mass is a statistical property describing the diffusion ability of the particle in spacetime, ,which shows that mass and diffusion coefficient are indeed statistical properties, under continuous interaction of STP. However, MIP itself describes a special Markov process, which possesses the intrinsic characteristic property of being quantized.

Firstly, we will proof that within framework of MIP, the particle mass and the diffusion coeffient in spacetime are not only statistical conjuation to each other, but also satisfying the minimum uncertainty relation:

$$\Delta m \Delta \Re = h/2 \tag{4.4}$$

4.2 Instantaneous statistical inertia mass

In this article, mass reflects the statistical propterty of the motion of matter particle, which is drived by collisions of STPs with the particle. As a statistical physical quantity, its instantaneous value does not have an explicit meaning in physics. We do not know how to measure the collision of a single collision between one STP and the particle exactly. In the other way, when we consider the relation between collision and the spectrum of STPs, we had already proven the number of STPs can not be determinate accurately. Hence even for a single collision between STP and the particle, the mass of the particle is also a statistical property. With this point of view, the statistical mass can be defined instanteously. In Minkowski spacetime, the distribution of STPs is uniform and isotropic. The instantaneous mass of matter particle will be changed according to the speed of particle. Though the instantaneous mass of particle $\hat{m}$, varying every moment, when taking the mean of speeds of the particle, will regress to the statistical inertia mass m_{ST}.[3]

Because the exchanged action relating to every single collision is not the same, neither the energy of the STP in this collision. The time interval that accomplishing the exchanging of action, is also different in every collision. We know, as a reflection of the collision between STP and matter particle, the motion of particle will deviate from its classical velocity. The noise part $\vec{\nu}$ describes the deviation cause by the collision between STP and the particle. The bigger the noise is, the smaller the statistical inertia mass m_{ST} is. In another way, a bigger deviation means the particle can diffuse in spacetime easier, thus it corresponds to a bigger spacetime diffusion coefficient $\Re$. In the moment of measurement, because of the existence of noise, the instantaneous mass of the particle will not be exact as m_{ST}. We know

$$\Delta m = \hat{m} - m_{ST}$$

The instantaneous mass corresponds to every measurement does not have any real physical meaning. The standard deviation of many times of measurement results is what we care about, it is

$$\sigma(m) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{m}_i - m_{ST}\right)^2} \tag{4.5}$$

[3]It possible that collisions of STP give matter particles the statistical property of mass, while the Higgs particle produces the average mass of matter particles.

With the same reason, we only care about the standard deviation of spacetime diffusion coefficients of every measurement

$$\sigma(\Re) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\Re}_i - \Re\right)^2} \tag{4.6}$$

The relative difference of this two statistical quantity can be represented as the covariance, as

$$cov(m, \Re) = \frac{\sum_{i=1}^{N}\left(\hat{m}_i - m_{ST}\right)\left(\hat{\Re}_i - \Re\right)}{N\sigma(m)\sigma(\Re)} \tag{4.7}$$

Since the noise of STP is a white noise, its standard deviation is a constant, so we can normalize its magnitude as 1.

Notice that when $N \to \infty$,

$$\begin{aligned} cov(m, \Re) &= \lim_{N\to\infty}\frac{\sum_{i=1}^{N}\left(\hat{m}_i - m_{ST}\right)\left(\hat{\Re}_i - \Re\right)}{N} \\ &\equiv \langle \Delta m \Delta \Re \rangle \end{aligned} \tag{4.8}$$

which is the LHS of the uncertainty relation expression as we claimed in (4.4). The following task is to calculate its explicit value.

We now cut the time into slides along the classical velocity of the particle. On each time slide, we only need to consider the collision of STPs parallel to the time slide. Defining the time interval for the cutting as $\delta\tau$. the instantaneous mass at the moment i could be defined as follows: from the moment $i-1$ to i, the action changing causing by STP collisions is $\Delta S_i = S_i - S_{i-1}$; Meanwhile the diffusion area is $\hat{\Re}_i$. The instantaneous mass is

$$\hat{m}_i \equiv \frac{\Delta S_i}{\hat{\Re}_i} \tag{4.9}$$

To varifying the (4.9) matches the statistical definition as in previous chapter, we need to reform the changing of action as the changing of motion status of the particle, it is

$$\Delta S_i = \frac{1}{4\pi} m_{ST}(V_i^2 - V_{i-1}^2)\delta\tau \tag{4.10}$$

hereV_i and V_{i-1}represent real velocities at moment i and $i-1$. Because there is no changing of classical velocity from moment $i-1$ to moment i , meanwhile the differentiable part of the collision, aka the quantum envelope velocity is also a slow varying quantity, so it could be seen as unchanged in this time interval. Thus all changing of the velocity is contributed from the STP noise. In classical situation, the previous equation could be written as

$$\begin{aligned} \Delta S_i &= \frac{1}{2} m(V_i^2 - V_{i-1}^2)\delta\tau \\ &= \frac{1}{2} m\left((V_{i-1} + \nu_i)^2 - V_{i-1}^2\right)\delta\tau \\ &= \frac{1}{2} m\left(\nu_i^2 + 2V_{i-1}\nu_i\right)\delta\tau \end{aligned} \tag{4.11}$$

Taking the mean value of this equation, we obtain

$$\begin{aligned}\langle\sum_i \Delta S_i\rangle_\nu &= \langle\int \frac{1}{2} m\left(\nu_i^2 + 2V_{i-1}\nu_i\right) dt\rangle_\nu \\ &= \hbar/4 \end{aligned} \tag{4.12}$$

However, it is notable that the changing caused by STP collisions is not a classical kinetic variation, we need to consider the special relativety effect as well. In rest frame of classical velocity, the particle energy is

$$E = mc^2$$

In static observer frame, its energy is

$$E_0 = \frac{m_0 c^2}{\sqrt{1 - V^2/c^2}} \tag{4.13}$$

Therefore we obtain

$$\begin{aligned}\Delta S_i &= \left[\frac{m_0 c^2}{\sqrt{1-\frac{V_i^2}{c^2}}} - \frac{m_0 c^2}{\sqrt{1-\frac{V_{i-1}^2}{c^2}}}\right]\frac{\delta\tau_0}{\sqrt{1-\frac{V_{i-1}^2}{c^2}}} \\ &= \frac{m_0 c^2 \delta\tau_0}{\sqrt{\left(1-\frac{(V_{i-1}+\nu_i)^2}{c^2}\right)\left(1-\frac{V_{i-1}^2}{c^2}\right)}} - \frac{m_0 c^2 \delta\tau_0}{\left(1-\frac{V_{i-1}^2}{c^2}\right)} \\ &= \frac{m_0 c^2 \delta\tau_0\left(\sqrt{\left(1-\frac{V_{i-1}^2}{c^2}\right)} - \sqrt{\left(1-\frac{(V_{i-1}+\nu_i)^2}{c^2}\right)}\right)}{\left(1-\frac{V_{i-1}^2}{c^2}\right)\sqrt{\left(1-\frac{(V_{i-1}+\nu_i)^2}{c^2}\right)}}\end{aligned} \tag{4.14}$$

especially, in above equation, we used the special relativity transformation that

$$m_i = \frac{m_0}{\sqrt{1-\frac{V_i^2}{c^2}}} \tag{4.15}$$

Because the changing of action from $i-1-th$ to $i-th$ time slide is a Lorentz scalar. We can take the $i-1-th$ slide as the rest frame with mass m_{i-1}, the $i-th$ slide represents the frame with velocity ν_i. Therefore, we change the equation (4.14) as

$$\begin{aligned}\Delta S_i &= \left(\frac{m_{i-1}c^2}{\sqrt{1-\nu_i^2/c^2}} - m_{i-1}c^2\right)\delta\tau_i \\ &= \left(\frac{1}{2} m_{i-1}\nu_i^2 + \frac{3}{8}(\nu_i^2/c^2)^2 c^2 m_{i-1} + \cdots\right)\delta\tau_i\end{aligned} \tag{4.16}$$

Taking mean value of the above, we obtain

$$\begin{aligned}&\langle\left(\frac{1}{2} m_{i-1}\nu_i^2 + \frac{3}{8}(\nu_i^2/c^2)^2 c^2 m_{i-1} + \cdots\right)\delta\tau_i\rangle_\nu \\ &= \frac{\hbar}{4} + \frac{3\hbar^2}{32c^2 m_{i-1}\delta\tau_i} + \frac{5\hbar^3}{256c^4 m_{i-1}^2\delta\tau_i^2}\cdots\end{aligned} \tag{4.17}$$

When the cutting interval goes to the classical limit, say, $\delta\tau_i \gg 0$, and the number $\hbar/c$ is very small, we have:

$$\langle\hat{m}_i\hat{\Re}_i\rangle_\nu \simeq \frac{\hbar}{4} \tag{4.18}$$

It means at arbitrary time slide, the mean value of the product of instantaneous mass and diffusion coefficient is $\frac{\hbar}{4}$.

From the definition of statistical inertia mass $m_{ST}$and diffusion coefficient $\Re$, we have:

$$\Re \equiv \sum_{i=1}^{N} \hat{\Re}_i / N \tag{4.19}$$

$$m_{ST} \equiv 2\pi \sum_{i=1}^{N} \hat{m}_i / N \tag{4.20}$$

It will not change the essence of the relation

$$\langle m_{ST} \Re \rangle_\nu = \frac{h}{2}$$

This is because

$$\begin{aligned}
\langle m_{ST} \Re \rangle_\nu &= 2\pi \langle \sum_{i=1}^{N} \hat{m}_i / N \sum_{j=1}^{N} \hat{\Re}_j / N \rangle_\nu \\
&= 2\pi \left[\sum_{i=j}^{N} \frac{\langle \hat{m}_i \hat{\Re}_i \rangle_\nu}{N^2} + \sum_{i \neq j}^{N} \frac{\langle \hat{m}_i \hat{\Re}_j \rangle_\nu}{N^2} \right] \\
&= \frac{h}{4N} + 2\pi \frac{\sum_{i=1}^{N} \langle \hat{m}_i \rangle \sum_{j \neq i}^{N} \langle \hat{\Re}_j \rangle}{N^2} + \mathcal{O}(\frac{h^2}{c^2 N}) \\
&= \frac{h}{4N} + \frac{N-1}{N} \frac{h}{2} + \mathcal{O}(\frac{h^2}{c^2 N}) = \frac{h}{2} - \frac{h}{4N} - \mathcal{O}(\frac{h^2}{c^2 N})
\end{aligned} \tag{4.21}$$

when $N \to \infty$, $\langle m_{ST} \Re \rangle_\nu = \frac{h}{2}$. Therefore we know the time cutting definition and the statistical definition is coincident with each other.

Now we can calculate the covariance as following

$$cov(m, \Re) = \lim_{N \to \infty} \frac{\sum_{i=1}^{N} \Delta S_i - m_{ST} \sum_{i=1}^{N} \hat{\Re}_i - \Re \sum_{i=1}^{N} \hat{m}_i}{N} + h/2 \tag{4.22}$$

and we obtain:

$$cov(m, \Re) = \lim_{N \to \infty} \frac{\sum_{i=1}^{N} \Delta S_i}{N} - h/2 \tag{4.23}$$

From MIP, the changing of action caused by STP collsion is N times of Planck constant, where N is an arbitrary integer, when the number of collisions goes to infinity, it is obvious that

$$\lim_{N \to \infty} \frac{\sum_{i=1}^{N} \Delta S_i}{N} = \lim_{N \to \infty} \frac{\hbar/4}{N} = 0 \tag{4.24}$$

at last we obtain

$$\langle \Delta m \Delta \Re \rangle = h/2 \tag{4.25}$$

and the proof is closed.

4.3 Position-Momentum Uncertainty Relation

Extending the definition of commutation relation, and recall $m = m_{ST}/2\pi$, we consider the position-momentum commutator

$$
\begin{aligned}
[x,p] = \lim_{\epsilon\to 0}\left(\frac{1}{2\pi}x(t+i\epsilon)m_{ST}\frac{\delta x(t)}{\delta t} - \frac{1}{2\pi}m_{ST}\frac{\delta x(t+i\epsilon)}{\delta t}x(t)\right) \\
= \lim_{\epsilon\to 0}\left(i\frac{\epsilon}{2\pi}\left[m_{ST}\left(\frac{\delta x(t)}{\delta t}\right)^2 - m_{ST}\frac{\delta^2 x(t)}{\delta t^2}x(t)\right]\right) \quad (4.26)
\end{aligned}
$$

Here we didn't take the statistical inertia mass as a variable, because when considering the changing of the particle's position caused by STP collisions, its statistical property is unchanged. Noticed that in our derivation, the momentum and position both have its instantaneous value. However, the two measurements are not isochronous in priori. Our isochrony is essencially different from what in quantum mechanism. Here since there exist collisions between STPs and matter particle, any two measurements can not be exactly isochronous. We let the time interval ϵ goes to zero to achieve an isochronous commutation relation in posteriori.

Define

$$
a_{ST}(t) := \frac{\partial^2 x(t)}{\partial t^2} \quad (4.27)
$$

It is the instantaneous accelaration induced by the collison between STP and the particle. From which we can define the instantaneous "spacetime" force as

$$
F_{ST}(t) = m a_{ST}(t) = m\frac{\delta^2 x(t)}{\delta t^2} \quad (4.28)
$$

The statistical average of eq.(4.26) is

$$
[x,p] = \lim_{\epsilon\to 0}\left(m\langle V(t)^2\rangle_\nu i\epsilon - \langle F_{ST}(t)x(t)\rangle_\nu i\epsilon\right) \quad (4.29)
$$

Its second term has an explicit meaning in physics. It is the the mean work done by STP acting on the particle. Obviously, this mean work is zero.

Now we consider the contribution from the first term of eq.(4.29) Under discretization of the fluctuation, the average speed is

$$
\int_t^{t+\epsilon} \nu(\tau)d\tau/\epsilon = \bar{\nu}/\sqrt{\epsilon}
$$

therefore

$$
\langle \nu^2\rangle_\nu = \langle \bar{\nu}^2\rangle_\nu/\epsilon = \frac{h}{m_{ST}\epsilon} \quad (4.30)
$$

Substitude this into the first term of Eq. (4.29), we obtain

$$
\begin{aligned}
[x,\ p] = \lim_{\epsilon\to 0}\left(i\epsilon m\langle \nu^2\rangle_\nu + i\epsilon\langle U^2\rangle_\nu\right) \\
= \lim_{\epsilon\to 0} i\epsilon m\frac{h}{m_{ST}\epsilon} + 0 = i\hbar \quad (4.31)
\end{aligned}
$$

which is the most fundamental hypothesis of quantum mechanism, the position-momentum uncertainty relation.

4.4 Energy-Time Uncertainty Relation

Within the framwork of non-relativity quantum mechanism, the position-momentum uncertainty relation does not imply the energy-time uncertainty. This means we can not derive one kind of uncertainty relation from the other. Notice, position, momentum, energy are all dynamical variables. They are functions of time t, say, the time t is a self-variable. Experimentally, because in non-relativity quantum mechanism, time t is an independent variable and does not rely on particle status, we can measure the position, momentum, energy of a matter particle.

Now we define the Δt in energy-time uncertainty relation as: the characteric time describing a significant variation in the system study at hand. To describe the variation, we have to introduce a time-varying physical quantity Q. The 'significant' variation is defined as the time interval in which the Q changing by one standard deviation σ_Q. Mathematically, it is expressed as:

$$\sigma_Q = |\frac{d}{dt}\langle Q\rangle_\nu| \times \Delta t \tag{4.32}$$

Meanwhile, we can define the ΔE in energy-time uncertainty relation as the uncertainty of Hamiltonian of the system σ_H. The average evolution equation of Q along with the time is

$$\frac{d}{dt}\langle Q\rangle_\nu = \frac{i}{\hbar}\langle [H, Q]\rangle_\nu \tag{4.33}$$

combine with the Schwarz inequality in mathematics, we have

$$\sigma_H^2 \sigma_Q^2 \geq \left[\frac{1}{2i}\langle [H, Q]\rangle_\nu\right]^2 \tag{4.34}$$

and then substitude into the definition of ΔE andΔt, we arrive:

$$\Delta E \Delta t \geq \frac{\hbar}{2} \tag{4.35}$$

If any physical quantity in this system varies fast, say Δt is very small, then its energy uncertainty will be very large. If ΔE is very small, then the Δt is very large, it means all observables in this system are varying slow.

5 Random Motion of Free Particle under MIP

5.1 Decompositions of the Real Velocity

In modern quantum mechanics, particles do not have trajectories of motions, so their velocities are not well defined. Within the framework of MIP, the real velocity of the particles must be discussed in detail. Under the impact of STP, the velocity of the particle not only contains the classical velocity, but also the results of random mechanical interactions. It is especially important that the particles are subjected to the impact of the STP, and the change of action is quantized. Therefore, the real velocity of the particles should reflect the classical, random and quantum properties.

Within the framework of MIP, the motion of particles is a frictionless quantum Brownian motion. However, it should be noted that the impact of STP is not completely random. The exchanged action that each particle is subjected to STP is an integer multiple of the Planck constant h. Therefore, the movement of particles in spacetime cannot be a problem of random mechanics completely. It is the quantization of randomized motions. The corresponding theoretical system is a quantum Markov process. If there is no STP and other external forces, the motion of the free particles satisfies Newtonian mechanics. Its velocity is the classic velocity.
Within the framework of MIP, for the real velocity of motion of free particles $\vec{V}(\vec{x},t)$, we can first isolate the classical statistical velocity of the particle $\vec{v}(\vec{x},t)$. In the context of space-time, it is a simple mean of the statistics of the impact of STP as Gaussian noise. Since the simple mean contribution of Gaussian noise is zero, the classical statistical velocity of the particle and the classical velocity under Newtonian mechanics are exactly equal. Second, after separating the classical statistical velocity $\vec{v}(\vec{x},t)$, we will consider a random motion. This random motion is driven by the impact of STP, and we note it with the random motion velocity $\vec{W}(\vec{x},t)$. In Appendix B of this paper, we prove that any random function can be decomposed into a random function and a superposition of differentiable functions. Random motion under the framework of MIP also follows this important principle. Therefore, in general, we can decompose the random motion velocity $\vec{W}(\vec{x},t)$ as follow

$$\vec{W}(\vec{x},t) = \vec{u}(\vec{x},t) + \vec{\nu}(t) \tag{5.1}$$

Where $\vec{u}(\vec{x},t)$ is defined as the quantum envelope velocity of the particle. For free particles, $< \vec{u}(\vec{x},t) >_\nu = 0$. It corresponds to the perturbation part of the random motion. It reflects the physical fact that the impact of STP is random, but it is a small perturbation to the current motion of the particle. These impacts are "differential impacts" of STP on the particles. Under the action of the pertubation of space-time, the motion of particles is not an unpredictable random motion. It allows the motion state of particles to be described by a differentiable function and describes the corresponding motion state . The equation is a non-random partial differential equation. And $\vec{\nu}(t)$ represents the non-microscopic impact of the particle by STP, which is a non-perturbative effect on the velocity of the particle motion. We define it as the velocity of fluctuation. Because of the existence of such random impact, the state function that we finally describe the equation of motion of the particle will not be an accurate description. It can only be a probabilistic description on the background of this fluctuation.

We will see that in the framework of MIP, quantum envelope motion reflects the wave-particle duality of particles. Considering the impact between STP and particle, the amount of exchange action is nh. For particles with a statistical mass of m_0, the characteristic time of this collision is

$$t_c = \frac{nh}{m_0 c^2} \tag{5.2}$$

The so-called quantum envelope motion is essentially the differentiable part of the fluctuation motion.

The above discussion is based on the classification of particles by the impact of STP. From the above analysis we can see that there is actually another mathematical classification for the velocity of the particles, and we decompose the velocity of the particle into a differentiable part and a non-differentiable part. The differentiable part of the real motion velocity of a particle can be defined as:

$$\vec{U}(\vec{x},t) = \vec{v}(\vec{x},t) + \vec{u}(\vec{x},t) \tag{5.3}$$

It is a superposition of classic statistical velocity $\vec{v}(\vec{x},t)$ and quantum envelope velocity $\vec{u}(\vec{x},t)$. We call this differentiable velocity "statistical average velocity". Although mathematically it is a differentiable function, it is quite different from the classical velocity. Because there is a quantum envelope velocity $\vec{u}(\vec{x},t)$, it is a representation of the Markov process formed by the impact of STP. Therefore, the decomposition of the velocity of the particles caused by the collision of STP can be written in three parts in principle[4]:

$$\vec{V}(\vec{x},t) = \vec{u}(\vec{x},t) + \vec{v}(\vec{x},t) + \vec{\nu}(t) \tag{5.4}$$

Since a Markov process will still be a Markov process under time reversal[9], the quantum envelope velocity $\vec{u}(\vec{x},t)$ is invariant under time reversal as

$$T : \vec{u}(\vec{x},t) \to \tilde{\vec{u}}(\vec{x},t) = \vec{u}(\vec{x},t) \tag{5.5}$$

However, the classical statistical velocity $\vec{v}(\vec{x},t)$ is changed by the time reversal, that is,

$$T : \vec{v}(\vec{x},t) \to \tilde{\vec{u}}(\vec{x},t) = -\vec{v}(\vec{x},t) \tag{5.6}$$

With above properties of time reversal, we can have a well defined limit $\vec{u} = 0$ as Newtonian mechanics with

$$\vec{v} = \frac{1}{2}(\vec{U} - \tilde{\vec{U}}) \tag{5.7}$$

$$\vec{u} = \frac{1}{2}(\vec{U} + \tilde{\vec{U}}) \tag{5.8}$$

Where $\tilde{\vec{U}}$is the time reversal of the statistical average velocity $\vec{U}$. In the following, the physical quantities with time reversal are marked with tilde.

The non-differentiable part is the fluctuation velocity $\vec{\nu}(t)$ for the random "non-differentiable impact"of the particle. It causes the particle's velocity to deviate from the classical statistical mean, so it will be physically reflected as a random diffusion behavior of the particle in spacetime. Based on this, we named it the "diffusion velocity" of particles in space and time.

In the following subsections, we will see that the decomposition of the above two velocities is a very important theoretical basis for deriving the equation of motion of particles, that is, the Schrödinger equation in quantum mechanics and an in-depth understanding of its physical meaning.

[4]After we finished our manuscript, we found that this three-velocity decomposition is in fact consistent with Wold's decomposition theorem of the stochastic process in.

5.2 From MIP to Schr$\ddot{o}$dinger Equation

Without the interaction of spacetime, the velocity of particle $\vec{v}$ has to be the derivative $\vec{v} = \frac{d\vec{x}}{dt}$. Contrasting from usual Markov process, spacetime random motion is frictionless, otherwise the quantum effect of a particle will decay as time going, which is obviously not the case. According to the MIP, the coordinate of a free particle is a stochastic process $\vec{x}(t)$, in which the velocity $\vec{V}$ can not be expressed in terms of $\frac{d\vec{x}}{dt}$. The velocity $\vec{V}$ should be a statistical average corresponding to a distribution $\delta\vec{x} = \vec{x}(t+\frac{1}{\omega}) - \vec{x}(t)$, at the limit of spacetime collision frequency ω going to infinity. In Einstein's theory on Brownian motion, $\delta\vec{x}$ is a Gaussian distribution with zero mean and variance proportional to $\frac{1}{\omega}$[6]. However, Einstein's theory cannot be correct at the limit of spacetime collision frequency ω going to infinity[10, 11]. Therefore, we will construct the operator D as following, which plays the same role as $\frac{d}{dt}$ in Newtonian Mechanics. For any physical function $f(\vec{x}, t)$, we have

$$\begin{aligned}
&\omega(f(\vec{x}(t+\frac{1}{\omega}), t+\frac{1}{\omega}) - f(\vec{x}(t), t)) \\
&= [\partial_t + \sum_i \omega(x_i(t+\frac{1}{\omega}) - x_i(t))\partial_i \\
&\quad + \sum_{ij} \frac{\omega}{2}(x_i(t+\frac{1}{\omega}) - x_i(t))(x_j(t+\frac{1}{\omega}) - x_j(t))\partial_i\partial_j \\
&\quad + \sum_i (x_i(t+\frac{1}{\omega}) - x_i(t))\partial_i\partial_t + \frac{1}{2\omega}\partial_t^2] f(\vec{x}(t), t) \qquad (5.9)
\end{aligned}$$

At the limit of spacetime collision frequency ω going to infinity, in terms of statistical average $< ... >$ for δx, we can define the operator D as

$$\begin{aligned}
Df(x(t), t) &= \lim_{\omega\to+\infty} \omega\langle f(\vec{x}(t+\frac{1}{\omega}), t+\frac{1}{\omega}) - f(\vec{x}(t), t)\rangle_\nu \qquad (5.10) \\
&= (\partial_t + \sum_i U_i\partial_i + \sum_{ij} \Re_{ij}\partial_i\partial_j) f(\vec{x}(t), t) \qquad (5.11)
\end{aligned}$$

where we used

$$\vec{U} = \lim_{\omega\to+\infty} \omega\langle\delta\vec{x}\rangle_\nu \qquad (5.12)$$

it relates to the descreterization of Lagevin equation

$$x_i(t+\epsilon) - x_i(t) = \epsilon U_i(\mathbf{x}(t)) + \sqrt{\epsilon}\bar{\nu}_i + O(\epsilon^2) \qquad (5.13)$$

here

$$\epsilon = \frac{1}{\omega} \qquad (5.14)$$

In eq.(5.10) , we used the following deduced result

$$\begin{aligned}
\lim_{\omega\to+\infty} \frac{\omega\langle\delta x_i\delta x_j\rangle_\nu}{2} &= \lim_{\epsilon\to0^+} \frac{1}{2\epsilon}\langle(x_i(t+\epsilon) - x_i(t))(x_j(t+\epsilon) - x_j(t))\rangle_\nu \\
&= \lim_{\epsilon\to0^+} \frac{1}{2\epsilon}\left[\langle\epsilon^2 U_i(\mathbf{x}(\mathbf{t}))U_j(\mathbf{x}(t))\rangle_\nu + \epsilon\langle\bar{\nu}_i\bar{\nu}_j\rangle_\nu + \epsilon^{\frac{3}{2}}\langle(U_i\bar{\nu}_j + U_j\bar{\nu}_i)\rangle_\nu\right] \\
&= \frac{h}{2m_{ST}}\delta_{i,j} \qquad (5.15)
\end{aligned}$$

Because of the isotropy of space, the MIP coefficient will be

$$\Re_{ij} = \frac{\hbar}{2m_{ij}} = \Re\delta_{ij} \tag{5.16}$$

which is consistent with Eq.3.30 and 3.39. The operator D and its time reversal $\tilde{D}$ are

$$D = \partial_t + \vec{U} \cdot \nabla + \Re\nabla^2 \tag{5.17}$$

$$\tilde{D} = -\partial_t + \tilde{\vec{U}} \cdot \nabla + \Re\nabla^2 \tag{5.18}$$

Therefore, the statistical average velocity of particle $\vec{V}$ can be written as

$$\vec{U} = D\vec{x} \tag{5.19}$$

$$\tilde{\vec{U}} = \tilde{D}\vec{x} \tag{5.20}$$

Correspondingly, its classical statistical velocity and quantum envelope velocity are

$$\vec{v} = D^-\vec{x} \tag{5.21}$$

$$\vec{u} = D^+\vec{x} \tag{5.22}$$

with

$$D^- = \frac{1}{2}(D - \tilde{D}) \tag{5.23}$$

$$D^+ = \frac{1}{2}(D + \tilde{D}) \tag{5.24}$$

We define the statical average acceleration of particles as

$$\begin{aligned} \vec{a} &= D\vec{U} = (D^+ + D^-)(\vec{v} + \vec{u}) \\ &= D^+\vec{u} + D^-\vec{v} + D^-\vec{u} + D^+\vec{v} \end{aligned} \tag{5.25}$$

Under time reversal, it acts as

$$\begin{aligned} \tilde{\vec{a}} &= \tilde{D}\tilde{\vec{U}} = (D^+ - D^-)(-\vec{v} + \vec{u}) \\ &= D^+\vec{u} + D^-\vec{v} - D^-\vec{u} - D^+\vec{v} \end{aligned} \tag{5.26}$$

Define the classical average acceleration as

$$\vec{a}_c = \frac{1}{2}(\vec{a} + \tilde{\vec{a}}) = D^+\vec{u} + D^-\vec{v}, \tag{5.27}$$

obviously it is invariant under time reversal. The average acceleration of a free particle must be zero, which can be written as

$$D^+\vec{v} + D^-\vec{u} = 0. \tag{5.28}$$

However, the average acceleration of quantum envelope motion can not simply be zero,

$$D^+\vec{u} + D^-v \neq 0 \tag{5.29}$$

At classical and low speed case, the average acceleration of quantum envelope motion does not relate to classical statistical velocity, therefore we can have

$$D^-\vec{v} - D^+\vec{u} = 0. \tag{5.30}$$

These conditions are equivalent to the coupled non-linear partial differential equations as following

$$\frac{\partial \vec{u}}{\partial t} = -\Re\nabla^2\vec{v} - \nabla(\vec{u}\cdot\vec{v}) \tag{5.31}$$

$$\frac{\partial \vec{v}}{\partial t} = -(\vec{v}\cdot\nabla)\vec{v} + (\vec{u}\cdot\nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{5.32}$$

Random motions of free particles due to the random impacts of STP satisfy the Markov property, one can make predictions for the future of the process based solely on its present state just as well as one could know the process's full history. This is the simplest situation for random motions, the free particle does not involve any external potential. Now, we have an initial value problem, which is to solve $\vec{u}(\vec{x},t)$ and $\vec{v}(\vec{x},t)$given $\vec{u}(\vec{x},0) = \vec{u}_0(\vec{x})$, $\vec{v}(\vec{x},0) = \vec{v}_0(\vec{x})$. In order to solve the coupled non-linear partial differential equations, we have to linearise it firstly[12, 13, 14]. Let

$$\Psi = e^{R+iI}, \tag{5.33}$$

where

$$\nabla R = \frac{1}{2\Re}\vec{u} \tag{5.34}$$

$$\nabla I = \frac{1}{2\Re}\vec{v} \tag{5.35}$$

We can obtain

$$\frac{\partial \Psi}{\partial t} = i\Re\nabla^2\Psi \tag{5.36}$$

According to the MIP, the universal spacetime diffusion coefficient is the MIP coefficient $\Re = \frac{\hbar}{2m_{ST}}$. Substituting to the last equation, we will get the equation of motion of free particles as

$$i\frac{\partial \Psi}{\partial t} = -\frac{\hbar\nabla^2}{2m_{ST}}\Psi \tag{5.37}$$

which is the Schrödinger equation essentially.

According to the continuity equation

$$\partial_t\rho(\vec{r},t) + \nabla\cdot\vec{J} = 0 \tag{5.38}$$

The definition of particle current is density multiplied by velocity. In the framework of MIP, the velocity in this definition corresponds to the classical statistical velocity. (See Appendix C)We can naturally derive the Born's interpretation as follows:

$$\vec{J} = \rho\vec{v} \tag{5.39}$$

among them

$$\vec{v} = 2\Re\nabla I \tag{5.40}$$

Substitute (5.33) in Schrödinger equation

$$\partial_t\Psi = i\Re\nabla^2\Psi \tag{5.41}$$

Let the real and imaginary parts be equal respectively, there are

$$\partial_t R + \Re(2\nabla R\cdot\nabla I + \nabla^2 I) = 0 \tag{5.42}$$

and

$$\partial_t \rho(\vec{r}, t) + \nabla \cdot (\rho \vec{v}) = 0 \tag{5.43}$$

which can be solved as

$$\rho = e^{2R} \tag{5.44}$$

Therefore, we show that the distribution of the particle number density is exactly the wave function modulo square. Further considering the ensemble of many identical particles, the particle number density is interpreted as the probability density, which is exactly the Born's interpretation.

The Born rule is a law of quantum mechanics which gives the probability that a measurement on a quantum system will yield a given result, which became a fundamental ingredient of Copenhagen interpretation[15]. In this paper, we attempt to suggest an interpretation of Born rule according to the MIP, which can provide a realistic point of view for wave function. Emerging from random impacts of spacetime, it's absolutely necessary that wave function is complex. If wave function were a real sine or cosine function[16], according to $\rho = |\Psi|^2$, the probabilistic density of a free particle with definite momentum would oscillate periodically which violates the isotropy of physical space. Under the framework of this paper, we can prove the 'Uncertain principle' directly(For more details, see Appendix D).

5.3 Physical Meanings of Potential Functions R and I

Substituting $\Psi = e^{R+iI}$ into $\frac{\partial \Psi}{\partial t} = i\Re\nabla^2\Psi$, we equalise the real and imaginary part separately as

$$\partial_t R = -\Re(2\nabla R \cdot \nabla I + \nabla^2 I) \tag{5.45}$$

$$\partial_t I = \Re[(\nabla R)^2 - (\nabla I)^2 + \nabla^2 R] \tag{5.46}$$

Combining with previous result $\rho = |\Psi|^2 = e^{2R}$, we have

$$\partial_t \rho = 2\rho \partial_t R \tag{5.47}$$

$$\nabla \rho = 2\rho \nabla R \tag{5.48}$$

The differential equation of potential R can be turned into

$$\partial_t \rho = -2\Re\nabla \cdot (\rho \nabla I) \tag{5.49}$$

With $\nabla I = \frac{1}{2\Re}\vec{v}$, the differential equation of potential R is equivalent to the equation of continuity

$$\partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0 \tag{5.50}$$

Noticing that the classical momentum of particle is $m\vec{v} = \hbar\nabla I$, we find that the differential equation of potential I goes to

$$\partial_t(\hbar I) + \frac{(\nabla(\hbar I))^2}{2m} - \hbar\Re[(\nabla R)^2 + \nabla^2 R] = 0 \tag{5.51}$$

Comparing with the Hamilton-Jacobi equation from classical mechanics [17, 18] as

$$\partial_t S + \frac{(\nabla S)^2}{2m} + V(x) = 0 \tag{5.52}$$

which is particularly useful in identifying conserved quantities for mechanical systems. There are two crucial remarks: Firstly, potential function I is proportional to the Hamilton-Jacobi function S as $S = \hbar I$. Secondly, for a free particle, the influence of spacetime can be summed up to the spacetime potential

$$V_{ST} = -\hbar\Re[(\nabla R)^2 + \nabla^2 R] \tag{5.53}$$

where the spacetime potential V_{ST} will play the same role of potential V in the Hamilton-Jacobi equation. The spacetime potential V_{ST} vanishes in the classical limit $\hbar = 0$, which is equivalent to $V = 0$ for free particles in classical mechanics. The quantum effect, which corresponding to nonzero $\hbar$, now is the natural result of the existence of the spacetime potential V_{ST}, induced by MIP. In principal, the moving of free particle can be described precisely by the spacetime potential V_{ST} as

$$m\frac{d^2\vec{x}}{dt^2} = -\nabla V_{ST} = \hbar\Re\nabla[(\nabla R)^2 + \nabla^2 R] \tag{5.54}$$

This equation indicates that free particle moves not along straight line within interactions of STP. It is affected by a space-time potential V_{ST}. The interactions between STP and particle give the statistcal mass to particle.

5.4 Space-time Random Motion of Charged Particles in Electromagnetic Field

According to the MIP, in case of low speed, electromagnetic field only serves as an external potential, which itself is not affected by random impacts of spacetime. In a electromagnetic field $(\vec{E}, \vec{B})$, the charged particle will experience a Lorentz force $\vec{F} = e(\vec{E}+\vec{v}\times\vec{B})$. Therefore, the average acceleration [19] of charged particles will be

$$\vec{a} = e(\vec{E} + \vec{v} \times \vec{B})/m \tag{5.55}$$

where m is the inertial mass of charged particle and e is the charge. Based on the spacetime principle, we are able to derive the equation of motion of charged particle in electromagnetic field, which is finally shown to be Schrödinger equation in electromegnatic field, which is

$$i\hbar\partial_t\Psi = \frac{1}{2m}(-i\hbar\nabla - \frac{e}{c}\vec{A})^2\Psi + e\phi\Psi \tag{5.56}$$

where the electromagnetic potential and the electromagnetic field are connected by

$$\vec{B} = \nabla \times \vec{A}, \vec{E} = -\partial_t\vec{A} - \nabla\phi. \tag{5.57}$$

We do not have average acceleration in absence of electromagnetic field. However, this is not the case when the particle have non-zero electric charge, moving in external electromagnetic field. Identifying the velocity in the Lorentz force as the classical velocity of random motion of particle in spacetime, we have

$$\partial_t\vec{v} = e(\vec{E} + \vec{v} \times \vec{B})/m - (\vec{v} \cdot \nabla)\vec{v} + (\vec{u} \cdot \nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{5.58}$$

In the electromagnetic field, the equation of motion of charged particle becomes coupled non-linear partial differential equations as following

$$\frac{\partial \vec{u}}{\partial t} = -\Re\nabla(\nabla\cdot\vec{v}) - \nabla(\vec{u}\cdot\vec{v}) \tag{5.59}$$

$$\frac{\partial \vec{v}}{\partial t} = e(\vec{E} + \vec{v}\times\vec{B})/m - (\vec{v}\cdot\nabla)\vec{v} + (\vec{u}\cdot\nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{5.60}$$

In order to solve the coupled non-linear partial differential equations, we have to linearise it firstly. Let $\Psi = e^{R+iI}$ and notice that the canonical momentum of charged particle [20] is $\vec{p} = m\vec{v} + e\vec{A}/c$, we suppose

$$\nabla R = \frac{1}{2\Re}\vec{u} \tag{5.61}$$

$$\nabla I = \frac{1}{2\Re}(\vec{v} + \frac{e\vec{A}}{mc}) \tag{5.62}$$

In order to prove Eq.(5.56), we expand the first term of right side of Eq.(5.56) as

$$\frac{1}{2m}(-i\hbar\nabla - \frac{e}{c}\vec{A})^2\Psi = -\frac{\hbar^2\nabla^2}{2m}\Psi + \frac{e^2A^2}{2mc^2}\Psi + \frac{i\hbar e}{2mc}(\nabla\cdot\vec{A})\Psi + \frac{i\hbar e}{mc}\vec{A}\cdot(\nabla\Psi) \tag{5.63}$$

Substituting $\Psi = e^{R+iI}$, it leads to

$$-\frac{\hbar^2}{2m}[\nabla^2R + i\nabla^2I + (\nabla R + i\nabla I)^2]\Psi + \frac{e^2A^2}{2mc^2}\Psi + \frac{i\hbar e}{2mc}(\nabla\cdot\vec{A})\Psi + \frac{i\hbar e}{mc}(\vec{A}\cdot(\nabla R + i\nabla I))\Psi \tag{5.64}$$

With vector formulas

$$\nabla(\vec{A}\cdot\vec{B}) = \vec{A}\times(\nabla\times\vec{B}) + \vec{B}\times(\nabla\times\vec{A}) + (\vec{A}\cdot\nabla)\vec{B} + (\vec{B}\cdot\nabla)\vec{A} \tag{5.65}$$

$$\nabla(\nabla\cdot\vec{A}) = \nabla\times(\nabla\times\vec{A}) + \nabla^2\vec{A} \tag{5.66}$$

and Eq.(5.61), we will obtain

$$\nabla\times\vec{u} = 0 \tag{5.67}$$

$$\nabla\times(\vec{v} + \frac{e\vec{A}}{mc}) = 0 \tag{5.68}$$

Straightforwardly, we have

$$i\hbar(\partial_t R + i\partial_t I) = -\frac{\hbar^2}{2m}[\nabla^2R + i\nabla^2I + (\nabla R + i\nabla I)^2] + \frac{e^2A^2}{2mc^2} \tag{5.69}$$
$$+\frac{i\hbar e}{2mc}(\nabla\cdot\vec{A}) + \frac{i\hbar e}{mc}(\vec{A}\cdot(\nabla R + i\nabla I)) + e\phi$$

Now, let's prove that the real and imaginary parts are separately equaled as

$$\partial_t I = \frac{\hbar}{2m}(\nabla^2 R + (\nabla R)^2 - (\nabla I)^2) - \frac{e^2\vec{A}^2}{2mc^2} + \frac{e}{mc}(\vec{A}\cdot(\nabla I)) - \frac{e\phi}{\hbar} \tag{5.70}$$

$$\partial_t R = -\frac{\hbar}{2m}(\nabla^2 I + 2(\nabla R)\cdot(\nabla I)) + \frac{e}{2mc}(\nabla\cdot\vec{A}) + \frac{e}{mc}\vec{A}\cdot(\nabla R) \tag{5.71}$$

Taking the gradient from both sides and the definitions $\vec{B} = \nabla\times\vec{A}, \vec{E} = -\partial_t\vec{A} - \nabla\phi$, we have reproduced the Eq.(5.59). Therefore, we have proved that both sides of Eq.(5.59) are at most different from a zero gradient function. It's important to notice that the choices of electromagnetic potentials are not completely determined. It allows a gauge transformation [20]

$$\vec{A}' = \vec{A} + \nabla\Lambda \tag{5.72}$$

$$\phi' = \phi - \partial_t\Lambda \tag{5.73}$$

For any function $\Lambda(\vec{x}, t)$, the electromagnetic field is invariant. Therefore, the corresponding wave function cannot change essentially, at most changing a local phase factor. Given $\psi' = \psi e^{\frac{ie\Lambda}{\hbar c}}$, Schrödinger equation of charged particle in electromagnetic field is invariant, i.e., $U(1)$ gauge symmetry. By choosing the function $\Lambda(\vec{x}, t)$ properly, we are able to eliminate the redundant zero gradient function. So we have proved Eq.(5.56) at the end.

5.5 Stationary Schrödinger Equation from MIP

Compare to the definition of classical statistical velocity as in eq.(5.35), it is easy to see that for the ground state, the classical statistical velocity is zero. Moreover, we can prove for all stationary states, their classical statistical velocities are zero. For a stationary state has exact energy E, the Schrödinger equation is

$$[-\frac{\hbar^2\nabla^2}{2m} + V_c(\vec{x})]\Psi = E\Psi \tag{5.74}$$

its conjugation reads

$$[-\frac{\hbar^2\nabla^2}{2m} + V_c(\vec{x})]\Psi^* = E\Psi^* \tag{5.75}$$

here $V_c(\vec{x})$ is classical external potential. Add the above two equations, the new real wave function has to satisfy the Schrödinger equation with same eigen-energy E.

Corresponding to the classical velocity from Eq.(5.35), it is easy to show that the classical velocity of particles must be zero in stationary states. Within the framework of MIP, we should interpret the stationary states from quantum mechanics as a spacetime random motion with zero classical velocity. Once we have all the stationary states, we will get the general solution by linear superposition. Therefore, we are going to derive stationary

Schrödinger equation from classical velocity $\vec{v}=0$, which can provide a clear physical picture of MIP. Moreover, when $|\vec{v}|$ is large and close to velocity of light c, the generalisation of this framework is clear and will be explained in our further work.
The trajectory of random motion of particle can be understood as the superposition of classical path and fluctuated path. During time interval $\triangle t$, there are two contributions to the trajectory as

$$\delta\vec{x} = \vec{u}(\vec{x},t)\triangle t + \triangle\vec{x} \tag{5.76}$$

of which distribution satisfies $\varphi(\triangle\vec{x}) = \varphi(-\triangle\vec{x})$ and

$$\int \varphi(\triangle\vec{x})d(\triangle\vec{x}) = 1$$

. The spacetime coefficient reads

$$\Re = \frac{1}{2\triangle t}\int(\triangle\vec{x})^2\varphi(\triangle\vec{x})d(\triangle\vec{x}) \tag{5.77}$$

The probabilistic density $\rho(x,t)$ evolves [21, 22, 23] as

$$\rho(\vec{x},t+\triangle t) = \int \rho(x-\delta\vec{x},t)\varphi(\triangle\vec{x}))d(\triangle\vec{x}) \tag{5.78}$$

Expanding Taylor series of both sides, we have

$$\partial_t\rho = -\nabla\cdot(\rho\vec{u}) + \Re\nabla^2\rho \tag{5.79}$$

which is consistent with Fokker-Planck equation. In any external potential $V(\vec{x})$, there are two contributions to the changing of average velocity. One is from random impacts of spacetime, another one is from acceleration provided by external potential. Therefore, the average velocity will evolve during time interval $\triangle t$ as

$$\begin{aligned}&\vec{u}(\vec{x},t+\triangle t) = \\ &\frac{\int(\vec{u}(\vec{x}-\delta\vec{x},t) - \frac{\triangle t\nabla V(\vec{x}-\delta\vec{x})}{m})\rho(x-\delta\vec{x},t)\varphi(\triangle\vec{x}))d(\triangle\vec{x})}{\int\rho(x-\delta\vec{x},t)\varphi(\triangle\vec{x}))d(\triangle\vec{x})}\end{aligned} \tag{5.80}$$

the denominator of eq. 5.80 is the normalisation factor of the probability distribution. Expanding Taylor series of both sides, we obtain

$$m\frac{d\vec{u}}{dt} = -\nabla V + \Re m(\frac{\nabla^2(\rho\vec{u})}{\rho} - \vec{u}\frac{\nabla^2\rho}{\rho}) \tag{5.81}$$

From this we can see the acceleration of the quantum envelope velocity $\vec{u}$, whose dynamics are rooted in the joint contribution of the classical potential and the quantum potential. For the physical state with certain energy, the three-velocity decomposition $\vec{V}(\vec{x},t) = \vec{u}(\vec{x},t) + \vec{v}(\vec{x},t) + \vec{\nu}(t)$ has clear physical meaning. The quantum envelope velocity $\vec{u}(\vec{x},t)$ and the classical statistical velocity $\vec{v}(\vec{x},t)$ are both velocity fields, which are functions of spacetime coordinates. The classical statistical velocity field of a physical state with certain energy is zero, which can be used as a new interpretation of the steady state of quantum mechanics. The dynamic mechanism of the quantum envelope velocity field $\vec{u}(\vec{x},t)$ has two contributions, the classical external potential field where the particle is located and the quantum potential field generated by the random collision of time-space. The diffusion velocity $\vec{\nu}(\vec{x},t)$ is the background of space-time fluctuations, evenly distributed in space, and satisfies the properties of Brownian motion in time, which is the intrinsic property of space-time. The sum of these three velocities is the real velocity of the objective reality of the particles required by materialism. See appendix B where we proved these.With the condition of stationary state $\partial_t \rho = 0$, it goes to

$$\vec{u} = \Re \frac{\nabla \rho}{\rho} \tag{5.82}$$

$$\partial_t \vec{u} = 0 \tag{5.83}$$

It's important to notice that

$$\frac{d\vec{u}}{dt} = \partial_t \vec{u} + (\vec{u} \cdot \nabla)\vec{u} \tag{5.84}$$

The average velocity $\vec{u}$ is not zero in the stationary state, which exactly cancle out its fluctuation velocity. Therefore, given the condition of stationary state, we are able to get

$$-2m\Re^2 \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} + V(x) = Const. \tag{5.85}$$

We can prove this constant is exactly the average energy of particle

$$E = \int \rho(\frac{1}{2}mu^2 + V)d^3x \tag{5.86}$$

Now, we have derived

$$-2m\Re^2 \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} + V(x) \quad = \quad E \tag{5.87}$$

$$\psi = \sqrt{\rho} e^{-iEt/\hbar} \tag{5.88}$$

Let $\Re = \frac{\hbar}{2m}$ once again, we arrive at the stationary Schrödinger equation

$$-\frac{\hbar^2 \nabla^2}{2m}\psi + V\psi = E\psi \tag{5.89}$$

5.6 Ground States of Hydrogen Atoms in MIP

In the hydrogen atom system, we can take $\vec{A} = 0$ and $\phi = -\frac{e}{4\pi\epsilon_0 r}$. The stationary solution of the equation (5.56) satisfies

$$E\Psi = \frac{1}{2m}(-i\hbar\nabla)^2\Psi - \frac{e^2}{4\pi\epsilon_0 r}\Psi \tag{5.90}$$

The lowest energy stationary state solution (ground state wave function) is $\Psi(r,\theta,\varphi) = \frac{1}{\sqrt{\pi a^3}}e^{-r/a}$, where $a = 5\times 10^{-11}m$ is the Bohr radius of the hydrogen atom. Using the wave function of the ground state of a hydrogen atom, we can get its quantum envelope velocity as

$$\vec{u} = 2\Re\nabla R = -\frac{\hbar}{ma}\hat{r} = -c\alpha\hat{r} \tag{5.91}$$

Where c is the velocity of light in vacuum, $\hat{r}$ is the unit vector $\hat{r} = \frac{\vec{r}}{r}$. Similarly we can get its classic average velocity

$$\vec{v} = 2\Re\nabla I = 0 \tag{5.92}$$

Its spacetime fluctuation rate is satisfied

$$< \nu_i >= 0, < \nu_i(t)\nu_j(t') >= \Re\delta_{ij}\delta(tt') \tag{5.93}$$

Then the electron in the ground state of the hydrogen atom has its coordinate $\vec{X}(t)$ as a random variable, and its real velocity $\vec{V}$ satisfies the following microscopic dynamic equations.

$$\frac{d\vec{X}(t)}{dt} = \vec{V}(t) = \vec{u} + \vec{v} + \vec{\nu} = -c\alpha\hat{r} + \vec{\nu}(t) \tag{5.94}$$

This is the real equation of motion of the ground state electrons of a hydrogen atom in the context of MIP. The quantum envelope velocity always points to the center of hydrogen atom. The closer to the center, the greater the repulsive force generated by the spacetime potential. Because this envelope velocity is balanced out by the combination of the classical Coulomb potential and the spacetime potential, the hydrogen atom can be stabilized on the ground state.

According to MIP, the real motion of electrons in the ground state of hydrogen atoms, we can calculate the average kinetic energy of electrons as

$$< K >= \frac{m}{2} < \vec{V}(t)^2 >= \frac{m}{2}(c\alpha)^2 + \frac{m}{2} < \vec{\nu}(t)^2 > \tag{5.95}$$

The average of the square of the spacetime fluctuation is

$$< \vec{\nu}(t)^2 >= \Re/T \tag{5.96}$$

Where T is the cumulative interaction time of the electrons. The ground state of a hydrogen atom can exist forever, that is, T tends to infinity, and thus we can obtain the average kinetic energy of the ground state electron as

$$< K >= \frac{m}{2} < \vec{V}(t)^2 >= \frac{m}{2}(c\alpha)^2 \tag{5.97}$$

We can calculate the average potential energy of the electron as

$$< U(r) >=< -\frac{e^2}{4\pi\epsilon_0 r} >=< -\frac{e^2}{4\pi\epsilon_0 a} > \tag{5.98}$$

Where a is the Bohr radius and ϵ_0 is the vacuum permittivity. The average energy of the ground state electrons is the sum of the average kinetic energy and the average potential energy. Substituting the standard values of physical constants, we can get the numerical result of the average energy of the ground state electrons as

$$E =< K > + < U >= -13.6ev \tag{5.99}$$

We have reached the same conclusion as quantum mechanics through the microscopic equation of motion of MIP. It can be seen that quantum mechanics only reflects the statistical average nature of the real motion process and does not reflect all the physics under the framework of MIP.

5.6.1 Deriving the amount of elementary charge from MIP

According to MIP, the interaction between particles and STP (the basic definition of the action is the product of momentum and displacement)

$$Nh = \oint pdq \tag{5.100}$$

For example, the simplest uniform circular motion is

$$\oint pdq = 2\pi mvr \tag{5.101}$$

Consider the electrons inside the hydrogen atom. STP collisions provide random Brownian motion, and attraction from proton provides centripetal force with equilibrium conditions

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \tag{5.102}$$

The amount of charge can be solved as

$$e = nh\sqrt{\frac{\epsilon_0}{m\pi r}} \tag{5.103}$$

The exact value of the electronic charge can be accurately obtained. We know that in MIP, the exchange action is nh, where n can be any integer.

We only need to make a hypothesis that the orbit of the electron is determined by the quantum number n of STP interaction. The proof of this hypothesis is shown in the next section. That is, when $n = 1$, the electron falls on the Bohr's orbit ($r = 0.53 \times 10^{-10}m$). When $n = 2$, the electrons fall on the second orbit (by analogy). You can get important results (all values below are with international units)

$$h = 6.62 \times 10^{-34}, m = 9.11 \times 10^{-31}, \epsilon_0 = 8.85 \times 10^{-12}$$

After substituting, we obtain the amount of charge as

$$e = 1.6 \times 10^{-19}C \tag{5.104}$$

5.6.2 Quantum number n of STP determining the orbit of hydrogen atoms

What we want to prove is that when the electrons are in Bohr's orbit ($r = a$), the amount of exchange action of STP is just a Planck constant, ie

$$h = 2\pi mva \tag{5.105}$$

Using the ground state wave function of the hydrogen atom derived above

$$\psi = \frac{1}{\sqrt{\pi a^3}} e^{-r/a} \tag{5.106}$$

The average value of the momentum can be found as

$$Mv = p = |\int \psi^*(-i\hbar\nabla)\psi d\tau| = \frac{\hbar}{a} \tag{5.107}$$

The integral volume element is $d\tau = r^2 sin\theta d\theta d\varphi dr$ and $h = 2\pi mva$.

5.6.3 Generalisation to Hydrogen-like atoms

The exchanged action between particles and STP

$$nh = \oint pdq \tag{5.108}$$

In uniform circular motion

$$\oint pdq = 2\pi mvr \tag{5.109}$$

An electron in a hydrogen-like atom with a positively charged nucleus. STP collisions provide random Brownian motion, and the attraction of the nucleus provides centripetal force with equilibrium conditions

$$\frac{Ze^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \tag{5.110}$$

The amount of charge can be solved as

$$e = nh\sqrt{\frac{\epsilon_0}{Zm\pi r}} \tag{5.111}$$

The Bohr-like orbital electron corresponding to $n = 1$ has a Bohr radius of $r = a/Z$, from which the elementary charge can be derived as

$$e = 1.6 \times 10^{-19}C \tag{5.112}$$

Starting from MIP, we have made a thorough study of free matter particles and obtained the most important conclusions of quantum mechanics. Furthermore, the most fundamental cause of atomic stability is explained by MIP, and from the first principle we calculate the basic physical quantity of electron charge unit. It can be seen that the random collision of STP does not only provide chaotic background noise , but also the stability of all matter in a seemingly chaotic background. At the most profound level, materialism interpret the physical world and the contradictions are unified.

6 Quantum Measurement in MIP

6.1 General Principle

There are fundamental distinctions on quantum measurement between MIP and Copenhagen interpretation. Within the framework of MIP, since matter particle is collided randomly by STP. Any measurement related to position and momentum can not be done in a time interval between two collisions, therefore any this kind of measurement cannot lead to precise result, which means we cannot make errors as small as possible in principle. Therefore, incommutable observables can not only be measured precisely at the same time, but also cannot be measured precisely separately. Theoretically, all measure values means statistical average, which include intrinsic uncertainty from spacetime besides normal measurement errors. For examples, the momentum uncertainty from MIP is due to the statistical properties of fluctuated mass. As a statistical mass, the minimum fluctuation is $\triangle m_{st}$, which roughly is one part per million of electron mass. The position intrinsic uncertainty $\triangle X_{st}$ from MIP is the mean free path between two consecutive collision by STP.

When the spacetime sensible mass is equivalent to the statistical inertial mass, the equation of motion will be determined by Schrödinger equation. In other words, moving matter particle and propagational wave are unified in spacetime. If we want to measure a matter particle, we need apparatus to interact with particle somehow. However, every such measurement has to interrupt the random motion of particle. Therefore, measurement means the end of a Markov process. When the measurement is finished, a new Markov process will begin. For the moving matter particle, the phases of wave functions before and after measurements is completely irrelevant, which cannot interfere each other. Under this framework, it's unnecessary to introduce hypothesises of wave function collapse or multi universe.

6.2 EPR Paradox in MIP

In a 1935 paper[24], Einstein with Podolsky and Rosen considered an experiment in which two particles that move along the x-axis with coordinates x_1 and x_2 and momenta p_1 and p_2 were somehow produced in an eigenstate of the observables $X = x_1 - x_2$ and $P = p_1 + p_2$ (these two observables commute $[X, P] = 0$).It's easy to understand that the measurement of the position of particle 1 can interfere with its momentum, so that after the second measurement the momentum of particle 1 no longer has a definite value. However two particles are far apart, how can the second measurement interfere with the momentum of particle 2? And if it does not, then after both measurements particle 2 must have both definite position and momentum, contradicting the quantum uncertainty principle. If it does, there exist some "spooky" interaction between two far apart particles, contradicting the locality principle in the special theory of relativity. The orthodox interpretation of quantum mechanics suppose that the second measurement which gives particle 1 a definite position, prevents particle 2 from having a definite momentum, even though the two particles are far apart. The states of the two particles are so call quantum entanglement.

Let's investigate the experimental process in detailed and estimate every uncertainty relations. Suppose two particles that are originally bound in some sort of unstable molecule at rest fly apart freely in opposite directions, with equal and opposite momenta until their separation becomes macroscopically large. Their separation will evolve as

$$x_1 - x_2 = x_{10} - x_{20} + (p_1 - p_2)t/m \tag{6.1}$$

where x_{10}, x_{20} are initial positions of two particles. It's noticed that under MIP, every massive particle is collided randomly by STP, the initial separation of two particle cannot be measured precisely. There exists intrinsic uncertainty $\triangle X_{st} = \triangle|x_{10} - x_{20}|$ as the mean free path between two consecutive collision by STP. According to the uncertainty relation derived from MIP, the momentum difference at least has intrinsic uncertainty as $\triangle P_{st} = \triangle|p_1 - p_2| \geq \frac{\hbar}{\triangle X_{st}}$, because of the commutation $[x_1 - x_2, p_1 - p_2] = 2i\hbar$. Therefore the uncertainty of separation will be

$$\triangle|x_1 - x_2| = \triangle X_{st} + \frac{\hbar t}{\triangle X_{st} m} \tag{6.2}$$

Its minimum is at $\triangle X_{st} = \sqrt{\frac{\hbar t}{m}}$, leading to

$$\triangle|x_1 - x_2| \geq 2\sqrt{\frac{\hbar t}{m}} \tag{6.3}$$

Similarly, the total momentum P is not strictly zero under MIP, which includes at least the intrinsic uncertainty due to

$$\triangle P = \triangle m_{st} v \tag{6.4}$$

where $\triangle m_{st}$is the fluctuation of statistical mass, according to MIP, roughly as one part per million of electron mass. Perform EPR experiment after the second measurement of particle 1, the uncertainty of particle 2 at least will be

$$\triangle p_2 \triangle x_2 = 2\sqrt{\frac{\hbar t}{m}} \triangle m_{st} v \tag{6.5}$$

More importantly , does the intrinsic uncertainty of particle 2 given by MIP contradict the uncertainty relation given by quantum mechanics? If

$$\triangle p_2 \triangle x_2 \leq \frac{\hbar}{2} \tag{6.6}$$

it still contradicts uncertainty relation of quantum mechanics, which means that we will observe the quantum entanglement experimentally, because we have to suppose the "spooky" interaction between two far apart particles to satisfy uncertainty relation. Therefore, we obtain the key criterion of quantum entanglement (momentum-position type) as

$$\frac{\triangle m_{st}^2}{m^2} \leq \frac{\pi \lambda_d}{8L} \tag{6.7}$$

where $\lambda_d = \frac{h}{mv}$ is de Broglie′s wavelength and L is the separation of two particles. So we can conclude that there is a characteristic separation of quantum entanglement as

$$L^* = \frac{\pi \lambda_d}{8} (\frac{m}{\triangle m_{st}})^2 \tag{6.8}$$

When the separation of two particles is larger than L^*, the inequality of (8) cannot be satisfied which means we are no longer able to determine the existence of quantum entanglement from experimental results. The reason is that the intrinsic uncertainty of particle 2 given by MIP has already satisfy uncertainty relation of quantum mechanics automatically. We cannot deduce the existence of 'spooky' interaction in this scenario. For two electrons moving at the speed of $0.01c$, the corresponding characteristic separation will be $L^* \approx 40m$. For two atoms moving at the speed of $0.01c$, the corresponding characteristic separation will be $L^* \approx 4 \times 10^7 m$.

7 From MIP to Path Integral

The path integral representation of quantum mechanics is a generalization and formulation method for quantum physics, which extends from the principle of action in classical mechanics. It replaces a single path in classical mechanics with a quantum amplitude that includes the sum or functional integral of all paths between two points. The path integral expression was theoretically published by theoretical physicist Richard Feynman in 1948 [25]. Prior to this, Dirac's 1933 paper[26], had major ideas and some early results. The main advantages of the path integral expression is that it treats spacetime equally, so it is easy to generalize to the theory of relativity, which is widely used in modern quantum field theory. However, the basic assumptions of MIP tell us that the effect of each STP colliding on particles can be seen as an independent path. The weight of each independent path is related to the distribution of energy. This is essentially a process of path integration. To understand this concept more clearly, we consider a simple process as follows. Assuming that the effect of random motion of particles over time Δt is from point A to point B. According to MIP, in this process, the change of the action can only be h, 2h, 3h, ..., but the paths are different corresponding to each specific action change. For example, the smallest amount of action change is one h, corresponding to a linear motion from A to B, and the 2h change corresponds to the movement of the polyline, during which the particle is struck twice by STP, and so on. In this picture, the so-called infrared effect is naturally ruled out, that is, the process of less than one h in Δt. The effect of infinity is also ruled out because the instantaneous velocity has certain upper bound which is the speed of light. This suggests that such a path integral effect is a finite summation rather than an infinite, so there is no need to introduce a so-called renormalization procedure. We see that under the framework of MIP, the quantum properties of particles are rooted in nature as the statistical description of their random motion.

7.1 Path Integral of Free Particle and Spacetime Interaction Coefficient

We had argued the real velocity of free particle in space-time satisfies the decomposition as

$$\vec{V}(\vec{x},t) = \vec{v}(\vec{x},t) + \vec{u}(\vec{x},t) + \vec{\nu}(t) \tag{7.1}$$

in which there are two kinetic arguments, they are classical statistical velocity $\vec{v}$ and quantum envelope velocity $\vec{u}$.

There are two kinetic variables with random motion particle in spacetime, which are classical speed $\vec{v}$ and fluctuated speed $\vec{u}$. The corresponding kinetic equations are

$$\frac{\partial \vec{u}}{\partial t} = -\Re\nabla(\nabla\cdot\vec{v}) - \nabla(\vec{u}\cdot\vec{v}) \tag{7.2}$$

$$\frac{\partial \vec{v}}{\partial t} = -(\vec{v}\cdot\nabla)\vec{v} + (\vec{u}\cdot\nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{7.3}$$

Setting $\Psi = e^{R+iI}$, we are able to linearise as

$$\nabla R = \frac{1}{2\Re}\vec{u} \tag{7.4}$$

$$\nabla I = \frac{1}{2\Re}\vec{v} \tag{7.5}$$

which leads to

$$\frac{\partial \Psi}{\partial t} = i\Re\nabla^2\Psi \tag{7.6}$$

During an infinite small time interval ϵ, the solution can be written in terms of integrals as

$$\Psi(x, t+\epsilon) = \int G(x, y, \epsilon)\Psi(y, t)dy \tag{7.7}$$

which represents the superposition of all the possible paths from y to x. The critical observation of Feynman is the weight factor $G(x, y, \epsilon)$ will be proportional to $e^{iS(x,y,\epsilon)/\hbar}$, where $S(x, y, \epsilon)$ is the classical action of particle as

$$S(x, y, \epsilon) = \int L(x, y, \epsilon)dt = \int (K - U)dt = (\bar{K} - \bar{U})\epsilon \tag{7.8}$$

$\bar{K}$ and $\bar{U}$ are average kinetic energy and potential energy separately. In order to show the equivalence between path integral formulation and the spacetime interacting picture, we should derive our basic kinetic equations from the postulation of path integral $G(x, y, \epsilon) = Ae^{iS(x,y,\epsilon)/\hbar}$. For a free particle in spacetime, one has $\bar{U} = 0$,$\bar{L} = \frac{m}{2}(\frac{x-y}{\epsilon})^2$ and $S = \frac{m(x-y)^2}{2\epsilon}$, which leads to

$$\Psi(x, t+\epsilon) = A\int e^{\frac{im(x-y)^2}{2\hbar\epsilon}}\Psi(y, t)dy \tag{7.9}$$

Setting $y - x = \xi$ and $\alpha = -\frac{im}{2\hbar\epsilon}$, it can be written in terms of

$$\begin{aligned}\Psi(x, t+\epsilon) &= A\int e^{-\alpha\xi^2}\Psi(x+\xi, t)d\xi \\ &= A\int e^{-\alpha\xi^2}(\Psi(x, t) + \xi\frac{\partial\Psi}{\partial x} + \frac{1}{2}\xi^2\frac{\partial^2\Psi}{\partial x^2} + \mathcal{O}(\xi^4))d\xi\end{aligned} \tag{7.10}$$

With the properties of Gaussian integral

$$\int e^{-\alpha\xi^2}d\xi = \sqrt{\frac{\pi}{\alpha}} \tag{7.11}$$

$$\int e^{-\alpha\xi^2}\xi d\xi = 0 \tag{7.12}$$

$$\int e^{-\alpha\xi^2}\xi^2 d\xi = \frac{1}{2\alpha}\sqrt{\frac{\pi}{\alpha}} \tag{7.13}$$

we can obtain

$$\Psi(x, t+\epsilon) = A(\sqrt{\frac{\pi}{\alpha}}\Psi(x, t) + \frac{1}{4\alpha}\sqrt{\frac{\pi}{\alpha}}\frac{\partial^2\Psi}{\partial x^2} + \mathcal{O}(\alpha^{-\frac{5}{2}})) \tag{7.14}$$

Setting $A = \sqrt{\frac{\alpha}{\pi}}$, we have

$$\Psi(x, t+\epsilon) - \Psi(x, t) = \epsilon\partial_t\Psi(x, t) = \frac{1}{4\alpha}\frac{\partial^2\Psi}{\partial x^2} \tag{7.15}$$

From this integral, We observed that the most important contribution comes from $y - x = \xi \propto \sqrt{\epsilon}$, where the speed of particle is $\frac{y-x}{\epsilon} \propto \sqrt{\frac{\hbar}{m\epsilon}}$, we see here when $\epsilon \to 0$, the speed divergent in order $\sqrt{1/\epsilon}$. The paths involved are, therefore continuous but possess no derivative, which are of a type familiar from study of stochastic process. With the isotropy of space, we have

$$\partial_t \Psi(\vec{x}, t) = \frac{1}{4\alpha\epsilon} \nabla^2 \Psi(\vec{x}, t) \tag{7.16}$$

Corresponding to the Eq. (7.6), if one requires the equivalence between path integral formulation and MIP, there must be

$$i\Re = \frac{1}{4\alpha\epsilon} \tag{7.17}$$

$$\Re = \frac{1}{4i\alpha\epsilon} = \frac{1}{4i(-\frac{im}{2\hbar\epsilon})\epsilon} = \frac{\hbar}{2m} \tag{7.18}$$

Notice that $\Re$ is only an arbitrary parameter in the Eq.(5.31). The consistency between path integral and MIP requires $\Re = \frac{\hbar}{2m}$. An arbitrary finite time interval Δt, can be cut into infinitely many slides of infnitesimal time interval ϵ. And in each ϵ, the collisions leads to many different paths, one can pick one path and consectively another along the time direction, this will end up a path in Δt, sum over all possible paths in Δt gives an integration over path space, which is the celebrated historical summation or path integral. The method here can be straightforwardly generalised to the particle in the external potential as in following section.

7.2 Path Integral of Particle in an External Potential and Spacetime Interaction Coefficient

In an external potential U, one has $\bar{U} = U(\frac{x+y}{2})$ and $\bar{L} = \frac{m}{2}(\frac{x-y}{\epsilon})^2$, which leads to the action

$$S = \frac{m(x-y)^2}{2\epsilon} - U(\frac{x+y}{2})\epsilon \tag{7.19}$$

According to the path integral formulation, it must satisfy

$$\begin{aligned} \Psi(x, t+\epsilon) &= A \int e^{\frac{im(x-y)^2}{2\hbar\epsilon} - \frac{iU(\frac{x+y}{2})\epsilon}{\hbar}} \Psi(y, t) dy \\ &= A \int e^{\frac{im(x-y)^2}{2\hbar\epsilon}} (1 - \frac{iU(\frac{x+y}{2})\epsilon}{\hbar}) \Psi(y, t) dy \end{aligned} \tag{7.20}$$

To the lowest order of ϵ, it shows

$$U(\frac{x+y}{2})\epsilon = U(x + \frac{\xi}{2})\epsilon = U(x)\epsilon \tag{7.21}$$

$$\Psi(x, t+\epsilon) = A \int e^{-\alpha\xi^2} (1 - \frac{iU(x)\epsilon}{\hbar}) \Psi(x+\xi, t) d\xi \tag{7.22}$$

From the properties of Gaussian integral in the previous section, we obtain

$$\Psi(x, t+\epsilon) = A(1 - \frac{iU(x)\epsilon}{\hbar}) \sqrt{\frac{\pi}{\alpha}} \Psi(x, t) + A \frac{1}{4\alpha} \sqrt{\frac{\pi}{\alpha}} \frac{\partial^2 \Psi}{\partial x^2} \tag{7.23}$$

Setting $A = \sqrt{\frac{\alpha}{\pi}}$, $\epsilon \to 0$, we have

$$\partial_t \Psi(\vec{x}, t) = \frac{1}{4\alpha\epsilon} \nabla^2 \Psi(\vec{x}, t) + \frac{1}{i\hbar} U \Psi(\vec{x}, t) \tag{7.24}$$

To be consistent with the case of free particle, let's take $\Re = \frac{\hbar}{2m}$ which leads to

$$\partial_t \Psi(\vec{x}, t) = i\Re\nabla^2\Psi(\vec{x}, t) + \frac{1}{i\hbar}U\Psi(\vec{x}, t) \tag{7.25}$$

Therefore we have derived the equation of motion from MIP.

8 STP Vortices as an origin of photon

8.1 Essential Properties of Electronic Charge In Modern Physics

In framework of modern physic, fundamental matter particles are all electric charged. The fundamental electric charge is defined as the amount of charge of an electron or a positron.

For electric charge [5], there are five fundamental properties. Firstly, there are only two kinds of charges, as known as the positive and negative charges. The characteristic quantum numbers of positron and electron are 1 and -1. Secondly, same charges repel each other, different charges attract each other.Thirdly, the interaction between charges is known as the Coulomb force, obeys the inverse square law. Electron and positron can annihilation each other, emit photons.Forthly, in an isolated system, the algebraic amount of charges are conserved. Finally, the amount of fundamental charge is $1.6 \times 10^{-19}C$.

Since there are no interactions between STP, the differential dynamics of STP is discribed by a massless free scale field theory, its Lagrangian is:

$$\mathcal{L}_{ST} = \partial_\mu\phi\partial^\mu\phi\,. \tag{8.1}$$

The dynamic equation is the 3+1 dimentional Klein-Gordon equation,

$$\partial_\mu\partial^\mu\phi = 0, \tag{8.2}$$

the solution of above equation is a wave solution, it can be written as follow

$$\phi(\vec{x}, t) = \sum_{E^2=\sum_{i=1}^3 p_i^2} f(E, \vec{p}) \exp(iEt - i\vec{p}\cdot\vec{x}), \tag{8.3}$$

in which $f(E, \vec{p})$ is an analytic function in momentum space.

Now let us consider putting a particle into space-time. The impact of introducing the matter particle into space-time scalar field, is somehow like dropping a cobble into the water surface of a peaceful lake, leads to the ripple effect. Compare to the fluctuation of space-time, the matter particle introduces a non-perturbative effect, which will bring into the space-time a strong potential. The reason that the matter particle results a strong potential is as follows Any perturbative disturbance will be get drowned out by the fluctuation of microscopic space-time energy fluctuation. In general, strong perturbation will lead to nonlinear effects, especially non-perturbative soliton effect. The soliton effect is steady and relatively large than STP. We know STP are local excitation of space-time energy, obviously, a cluster

[5] We will use charge instead of electric charge in this section, for simplicity.

of STP describes a “huge” excitation of space-time energy. So it is nature to introduce solitons into space-time field since a local non-perturbative energy disturbance leads a local space-time soliton, discribing a cluster effect of STP.

8.2 2+1-dim Complex Scalar Space-time field

In modern quantum field theory, the microscopic energy can be non-conserved locally, which is saying the vacuum can excit any pair of virtual conjugated particles. In framework of MIP, the fluctuations of space-time energy are STP. The non-conservation nature of local space-time energy is saying the number of STP are locally non-conserved. However, in a global viewpoint, the energy of STP are conserved.

In framework of MIP, we introduce a local companion for STP field, which is a local field that can interact with STP. However, in global, the companion field will decouple with the STP field. The existence of the local companion field also implies in local there is a kind of local symmetry, which is broken in global. In fact, when the local symmetry is $U(1)$, STP are excitations of a complex scalar field.

In framework of MIP, matter particle experiences quantum Brownian motion, which essentially is a Markov progress. This implies the past and future of the matter particle are causual irrelated. So at an arbitrary point of time, one can cut the slice vertical to the direction of the velocity of the matter particle, as known as the normal slice. The dynamics of matter particle on normal slice is a 2+1-dim dynamics. The whole 3+1-dim dynamics could be extented from the dynamics on slices. Notice there are two kinds of dynamics on the 2+1-dim normal slice, one for matter particle, the other for STP, respectively.

We now consider the 2+1 dimensional dynamics of STP. As is stated above, the matter particle drops a cobble into the STP lake and results a period potential. We denote the potential as $V(\phi,\phi^*)$, thus the Lagrangian of complex STP field now becomes

$$\mathcal{L}_{ST} = -\frac{1}{2}\partial_j\phi\partial^j\phi^* + V(\phi,\phi^*), \quad j = 0,1,2. \tag{8.4}$$

8.3 Abrikosov-Nielsen-Olesen-Zumino Vortex

In 2+1 dimension, the famous non-pertubative solution for a complex scalar field is the Abrikosov-Nielsen-Olesen-Zumino(ANOZ) vortex solution. The Lagrangian supports the ANOZ vortex is

$$\mathcal{L} = -\partial_j\phi^*\partial^j\phi - \frac{\lambda}{2}\left(\phi^*\phi - F^2\right)^2 \tag{8.5}$$

The minimum of the potential is obvious, it is

$$\phi = F\cdot e^{i\varphi}$$

which is a cycle with radius F. Notice this configuration is compatible with the “ripple” effect of matter particle acting on STP field. It also introduces a symmetry $U(1)$. Since this $U(1)$ now is a local symmetry, it implies there should be a gauge field companion with the STP field. The soliton solution is obtained when introducing the boundary condition at infinity, that is

$$|x| \to \infty : \quad \vec{\phi} \to F\frac{\vec{x}}{|x|}, \quad \phi \to Fe^{i\varphi}. \tag{8.6}$$

However, the soliton solition suffers an energy divergence because

$$E = \int d^2x \left(\vec{\partial}\phi^* \vec{\partial}\phi + V(\phi, \phi^*)\right) \tag{8.7}$$

goes to infinity. One can check this as follows

$$\begin{aligned} |x| &\to \infty : \quad \partial_i \phi_j \to \frac{F}{|x|}\left(\delta_{ij} - \frac{x_i x_j}{|x|^2}\right) \\ \sum_{i,j=1}^{2} (\partial_i \phi_j)^2 &\to \frac{F^2}{|x|^2} \\ \int d^2x \vec{\partial}\phi^* \vec{\partial}\phi &\to 2\pi \int_0^\infty d|x| \frac{F^2}{|x|} : \quad Log \;\; divergent. \end{aligned} \tag{8.8}$$

We saw the energy of the vortex is divergent at spatial infinity, this is unphysical since it implies there is an infinity energy source at spatial infinity. To avoid this divergence, the way out is to introduce a gauge vector field to smear the infinity energy on whole 2+1-dim normal slice. In fact, the local non-conservation of space-time energy implies we need a companion field for STP field in the first place. Here it is clear that the field is a gauge field. To do so, we need introduce the covariant derivative for STP field, instead of original derivative, as well as a kinetic term for the gauge field. Now the Lagrangian is

$$\mathcal{L} = -\frac{1}{2} D_\mu \phi^* D^\mu \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\phi, \phi^*) \tag{8.9}$$

$$D_\mu \phi = \partial_\mu - igA_\mu \tag{8.10}$$

The complex STP field degenerates into a real scalar field. This is because the energy non-conservation is recovered in global. The complexity of the STP field reflects the local property of STP. At spatial infinity,

$$\phi \to Fe^{i\varphi}|_{\varphi=0} = F \tag{8.11}$$

the gradient of STP field is

$$\vec{\partial}\phi = (\partial_r \phi \vec{e}_r + \partial_\varphi \phi \vec{e}_\varphi)_{\varphi=0} = iF/r \tag{8.12}$$

and the gauge field becomes pure gauge field (with vanishing field strength), that is

$$\vec{A} \to \frac{1}{ig} \phi^{-1} \vec{\partial}\phi \tag{8.13}$$

In form of polar coordinates,

$$A_r = 0, \quad A_\varphi = \frac{1}{gr} \tag{8.14}$$

In general, we can not let a complex scalar field directly equals to a real scalar field at an arbitrary spatial point. However, we can let them equals to each other up to a gauge transformation, say

$$\phi \to \Omega F, \quad \Omega(\vec{x}) = e^{i\varphi(\vec{x})} \tag{8.15}$$

and then we have

$$\vec{A} \to -\frac{1}{ig}\Omega\vec{\partial}\Omega^{-1} \tag{8.16}$$

Actually, under this general configuration, the divergence of energy will be strictly vanished, as

$$\vec{D}\phi \to \left(\vec{\partial}\Omega + \Omega(\vec{\partial}\Omega^{-1})\Omega\right) F = \Omega\vec{\partial}(\Omega^{-1}\Omega)F = 0 \tag{8.17}$$

In terms of component, the gauge field reads

$$A_i = -\frac{1}{g}\epsilon_{ij}\frac{x_j}{r^2} \tag{8.18}$$

From the Stokes theorem, we have

$$\Phi \equiv \oint_{C=n\cdot\partial\Sigma} \vec{A}d\vec{x} = \int_{\Sigma} \vec{B}d\vec{\sigma} = \frac{2\pi n}{g} \equiv g_m \tag{8.19}$$

here we recognize the famous Dirac quantization condition [9] for electronic charges, say

$$g \cdot g_m = 2\pi n, \quad n \in \mathbb{Z} \tag{8.20}$$

This implies if there was an ANOZ vortex solution, the electronic charge is quantized. When n is a negative integer, it describes an opposite spinning vortex solution and also describes a negative charge. In modern physics, there should be a Dirc monople to support the Dirac quantization condition of charges. In framework of MIP, the only origin of quantized charge is the STP field.

8.4 From 2+1-d to 3+1-d

In 3+1 Minkowski space-time, the local space-time symmetry is Lorentz symmetry, denoted by $SO(3,1)$. In Lie group theory, $SO(3,1)$ is algebraic isomorphism to $SU(2)\times SU(2)$, that is

$$so(3,1) \cong su(2) \times su(2) \cong so(3) \times so(3). \tag{8.21}$$

In fact, this isomorphism reveals locally, the 3+1-dim space-time equals to cross extension of two 2+1-dim space-time.

Now we consider how this local extension of dimension can be done from Lie algebra. Notice the six generator of Lorentz group can be written explicityly as

$$K_i \equiv L_{0i} = t\partial_i - x_i\partial_t \quad i,j,k \in [1,2,3] \tag{8.22}$$

$$R_k = \epsilon^{ij}{}_k L_{ij} = \epsilon^{ij}{}_k x_i \partial_j \tag{8.23}$$

The two algebra $su(2)$ are isomorphic to $so(3)$, in terms of derivative, they are

$$S_a = \epsilon^{abc} r_a \partial_{r_b} \tag{8.24}$$

$$\tilde{S}_a = \epsilon^{abc} l_a \partial_{l_b} \tag{8.25}$$

in which there are six degrees of freedom, in the meaning of linear space, they are

$$r_1, r_2, r_3, \quad l_1, l_2, l_3$$

Though the Lie algebras of $SO(3,1)$ and $SU(2) \times SU(2)$ is isomorphic to each other, from the viewpoint of degree of freedom, they are not the same. Notice there is a hidden duality, which maps 2-dim surface to 1+1-dim surface and vice versa, as follows

$$\begin{aligned} \star : e_0 \otimes e_i \to \epsilon_{0i}^{\ \ jk} e_j \otimes e_k \\ \star : e_j \otimes e_k \to \epsilon^{0i}_{\ \ jk} e_0 \otimes e_i \end{aligned} \tag{8.26}$$

This duality is actually the Hodge duality in differential geometry. It implies extension rules should be followed when extending a theory from 2+1-dim to 3+1-dim.

In conclude, we know the rule guiding the extension from 2+1-dim to 3+1-dim is Hodge duality. In the vortex situation considered at hand, the Hodge duality actually corresponds to a resolving of singularity. The vortex has a singular tube which shrinks to a point when goes to its center. If one wants to resolving the singularity, the general way in differential topology is to introduce a finite size sphere instead of the singularity. The resolving operation can be done by two steps: cut the vortex tube at a finite size, which will be a circle, then rotate the circle into a sphere. This rotation was been done in 3+1-dim and is the physical saying of the Hodge duality.

8.5 The Origin of Photon from ANOZ Vortex

In discussion of ANOZ Vortex, we obtained the gauge constraint and the quantization condition of electric charge, however, we didn't obtain the dynamics of the vortex. Because vortex is not a fundamental excitation, its dynamics can not be analytically achieved from fundamental STPs. So in order to obtain the vortex dynamics. We need to introduce the Lagrangian for vortices.

8.5.1 Dynamics on normal slice

For the kinetic part of STPs field, say,

$$\mathcal{L}_\phi = \frac{1}{2} \vec{D}_i \phi^* \vec{D}^i \phi = \frac{1}{2} |(\partial_i - igA_i)\phi|^2 \tag{8.27}$$

in this subsection, $i, j, k, l, m, n = 0, 1, 2$ label indices on the 2+1-dim normal slice. We only consider the excitations nearby the vortex potential, which is $\phi = Fe^{i\varphi}$. The above STP field kinetic Lagrangian can be written as

$$\mathcal{L}_\phi = \frac{1}{2} F^2 \left(\partial_i \varphi - gA_i\right)^2 \tag{8.28}$$

After a simple square matching operation, we arrive a linear form

$$\mathcal{L}_\phi = -\frac{1}{2F^2} \xi^i \xi_i + \xi_i (\partial^i \varphi - gA^i) \tag{8.29}$$

here ξ^i is a static auxillary field. Notice that for vortex solution, the phase angle field φ is singular at the vortex center, we now separate the phase angle into two parts, one is smooth and the other is for vortex, say,

$$\varphi = \varphi_0 + \varphi_{vortex} \tag{8.30}$$

The smooth part does not have a significant effect on what we concerned, we integral it out and it results a constraint equation for the auxillary field,

$$\partial_i \xi^i = 0 \tag{8.31}$$

This reveals the auxillary field is a 2+1-dim sourceless field, and it can be rewritten as a pure curl as

$$\xi^i = \epsilon^{ijk}\partial_j a_k \tag{8.32}$$

On the other hand, the equation of motion of auxillary field ξ can also be obtained from Euler-Lagrange equation, it reads

$$\xi^i = F^2(\partial^i \varphi - gA^i) \tag{8.33}$$

The above two equations define a hidden duality as follow

$$F^2(\partial^i \varphi - gA^i) = \epsilon^{ijk}\partial_j a_k \tag{8.34}$$

Substitute it into equation (8.29), we obtain

$$\begin{aligned} \mathcal{L}_\phi &= \frac{1}{2F^2}\xi^i \xi_i = \frac{1}{2F^2}\epsilon^{ijk}\partial_j a_k \epsilon_{imn}\partial^m a^n \\ &= \frac{1}{2F^2} f^{jk} f_{jk} \end{aligned} \tag{8.35}$$

here

$$f_{jk} = \partial_j a_k - \partial_k a_j \tag{8.36}$$

is the field strength of a field. Here we saw the dynamics of the STP field on normal slice is fully equivalent to a vector field a . Recall the kinetic term of gauge field A , we obtain a effective Lagrangian on normal slice

$$\mathcal{L}_{total} = \mathcal{L}_A + \mathcal{L}_\phi = -\frac{1}{4g^2}F^{jk}F_{jk} + \frac{1}{2F^2} f^{jk} f_{jk} \tag{8.37}$$

8.5.2 The Hodge duality

Notice in the dynamics of 2+1-dim vortex, the singularity of the phase angle is essential, which results that the corresponding gauge field A is also singular at the center of the vortex. This singularity could be resolved in higher dimension, for example, in 3+1-dim space-time, we can extend the 2+1-dim Hodge duality (8.34) to 3+1-dim. This 3+1-dim Hodge duality reflects the local duality of 3+1-dim Lorentz group, as revealed in last subsection. In 3+1-dim, the complex STPs field becomes real because the phase angle is fixed to zero and has no dynamics at all, leads a free STP scalar field in 3+1-dim. Actually, in 3+1-dim, we can define the Hodge duality of a field as:

$$F'^{\alpha\beta} = \sqrt{2} g F i \epsilon^{\alpha\beta}_{\ \ ij} f^{ij} \tag{8.38}$$

from which we has defined a gauge field A' , its field strength is

$$F'^{\alpha\beta} = \partial^\alpha A'^\beta - \partial^\beta A'^\alpha \tag{8.39}$$

It is an extension of a field in 3+1-dim and on any 2+1-dim sub-manifold of the 3+1-dim space-time, its dynamics is equivalent to field a . In total, we know

$$\mathcal{L}_{total} = -\frac{1}{4g^2} F^{jk} F_{jk} - \frac{1}{4g^2} F'^{\alpha\beta} F'_{\alpha\beta} \tag{8.40}$$

Actually, in 3+1-dim, the two parts of above Lagrangian can be written as a single term when defined a new field $\tilde{A}$ satisfying

$$\frac{1}{g}\tilde{F}_{ij} = F_{ij}, \quad \frac{1}{g}\tilde{F}_{\alpha\beta} = F'_{\alpha\beta} \tag{8.41}$$

Notice the above equations are six equations, which are

$$\partial_0 \tilde{A}_1 - \partial_1 \tilde{A}_0 = g(\partial_0 A_1 - \partial_1 A_0) \tag{8.42}$$
$$\partial_0 \tilde{A}_2 - \partial_2 \tilde{A}_0 = g(\partial_0 A_2 - \partial_2 A_0) \tag{8.43}$$
$$\partial_1 \tilde{A}_2 - \partial_2 \tilde{A}_1 = g(\partial_1 A_2 - \partial_2 A_1) \tag{8.44}$$
$$\partial_0 \tilde{A}_3 - \partial_3 \tilde{A}_0 = g(\partial_0 A'_3 - \partial_3 A'_0) \tag{8.45}$$
$$\partial_1 \tilde{A}_3 - \partial_3 \tilde{A}_1 = g(\partial_1 A'_3 - \partial_3 A'_1) \tag{8.46}$$
$$\partial_2 \tilde{A}_3 - \partial_3 \tilde{A}_2 = g(\partial_2 A'_3 - \partial_3 A'_2) \tag{8.47}$$

On 0-1-2 normal slice, we can assume

$$\tilde{A}_0|_{\Sigma=(t,x_1,x_2)} = gA_0, \quad \tilde{A}_1|_{\Sigma=(t,x_1,x_2)} = gA_1, \quad \tilde{A}_2|_{\Sigma=(t,x_1,x_2)} = gA_2 \tag{8.48}$$

here $\tilde{A}_i|_{\Sigma=(t,x_1,x_2)}$ denotes the reduced field of the four dimensional gauge field $\tilde{A}$ onto normal slice $\Sigma = (t, x_1, x_2, 0)$. Hence from eq.(8.45-8.47) we see, the constraint equations require that on x_3 direction, $\tilde{A}_0, \tilde{A}_1, \tilde{A}_2$ should coincide with A'_0, A'_1, A'_2 ,

$$A_i(0,0,0,x_3) = A'_i(0,0,0,x_3), \quad i = 0,1,2 \tag{8.49}$$

then we obtain

$$\tilde{A}_3(t, x_1, x_2, x_3) = gA'_3(t, x_1, x_2, x_3) \tag{8.50}$$

Actually, the A'_3 is a new component of the gauge field results from the Hodge duality, it is unique up to a pure gauge with vanishing field strength. Now we see how to extend the gauge field on 2+1-dim to 3+1-dim guiding by the Hodge duality. A simple extension is

$$\tilde{A}_i(t, x_1, x_2, x_3) = g(A_i(t, x_1, x_2, 0) + A'_i(0,0,0,x_3)), \quad i = 0,1,2 \tag{8.51}$$
$$\tilde{A}_i(t, x_1, x_2, x_3) = gA'_3(t, x_1, x_2, x_3) \tag{8.52}$$

Under this extension, we arrive a simple Lagrangian

$$\mathcal{L}^{eff}_{3+1d} = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}, \quad \mu,\nu = 0,1,2,3 \tag{8.53}$$

it is the famous Lagrangian for 3+1-dim gauge field, the field strength is the same as Maxwell field strength. In three dimensional form, the field strength can be written as electric and magnetic field strengths as

$$E_i = \tilde{F}_{0i}, \quad B_i = \epsilon_{ijk}\tilde{F}^{jk}, \quad i,j,k = 1,2,3 \tag{8.54}$$

In above derivation, we saw that the dynamic effects of STP ANOZ vortex and 3+1-dim electromagnetic field are completely equivalent. This reveals an important assertion: photons are companion particles of STP vortices. In 3+1-dim space-time, Maxwell field strength is a derived result because of vanishing of the ANOZ vortex singularity.

In conclusion, when introducing the third spatial dimension, the singularity of ANOZ vortex is vanished. Meanwhile the equation of motion for ANOZ vortices is equivalent to 3+1-dim Maxwell equations, they are

$$\vec{\nabla} \cdot \vec{E} = 0 \tag{8.55}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{8.56}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{8.57}$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} \tag{8.58}$$

Here what we obtained is the source-free Maxwell equations because we didn't consider the effect of matter particles, which will couple to gauge field as will considered in next subsection.

8.6 The Coulomb Force

We now consider the force between two matter particles. In hydrodynamics, two vortices will repel each other if their handing of spins are the same, and will attract each other if their handing of spins are different. This is a nature derivation from Bernoulli principle. There are only two kinds of charity for 2+1-dim vortices, left and right, respectively.

More than two decades ago, people had already found the correspondence between equations of motions of hydrodynamics and Maxwell eletromagenetism [27]. This correspondence was supported by [28] with a detailed derivation. The correspondence between hydrodynamics and eletromagnetism is much more like a coincidence in previous researches. However, in framework of the STP vortex, the fluid-eletromagnetism correspondence now has a concrete theoretic origin.

In previous subsections, we only considered dynamics of STP and gauge fields, leaving the matter particle as a source of potential. It is nature to consider the interaction between matter field and gauge field as well. To do so, we introduce the matter field in Lagrangian as follow

$$\mathcal{L}_{total} = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - i\bar{\psi}\gamma^{\mu}\tilde{D}_{\mu}\psi + m\bar{\psi}\psi \tag{8.59}$$

$$\tilde{D}_{\mu} \equiv \partial_{\mu} + ie\tilde{A}_{\mu} \tag{8.60}$$

This interaction can be understood as an effective representation of the collision between matter particle and STP vortices, though their are no terms representing vortices in the Lagrangian. This is because the dynamics of vortices now is equivalent to gauge field in 3+1-dim. Other collisions between matter particle and STP are not considered in this section, as we will see, they also play important roles in deriving gravity between matter particles.

In global, the STP and gauge field are decoupled, hence all local dynamics have been reduced to gauge field dynamics in 3+1-dim space-time. Notice the Lagrangian we obtained above is the same as that in famous QED [29]. Under standard calculation, the interaction between matter particles will be the Coulomb interaction. However, in framework of MIP, the gauge field is not originated from matter field, but from STP vortices. This is an essential difference between modern quantum field theory and the MIP proposed in this article.

Define the four dimensional current as

$$j^\mu \equiv i\bar{\psi}\gamma^\mu\psi \tag{8.61}$$

we can explicitly see the minimal couple between gauge field $\tilde{A}$ and the electronic current j. The equation of motions now becomes the famous sourced Maxwell equation, as known as

$$\vec{\nabla}\cdot\vec{E} = j_0 \tag{8.62}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \tag{8.63}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \tag{8.64}$$

$$\vec{\nabla}\times\vec{B} = \vec{j} + \frac{\partial\vec{E}}{\partial t} \tag{8.65}$$

8.7 Another Derivation of EoM of Photons

In framework of MIP, we had obtained four properties of charges, they are: 1. There are only two kinds of charges corresponding to left and right chiralities of STP vortices. Same charges repel each other while different ones attract each other. 2. The charges are quantized guiding by the Dirac quantization condition derived from STP vortex. 3. Force between charges are mediated by photons. 4. The force between charges is the Coulomb force.

Based on calculations in previous subsections and discussion of the Hodge duality, we know some properties of phtons in frame work of MIP. At first, it companies with the non-pertubative solition solution, as known as the vortex solution. Secondly, it is a gauge field in 2+1-dim normal slice on which another effective auxillary gauge field lives as well. Thirdly, the 3+1-dim Hodge duality acts on the effective auxillary gauge field does not only resolve the phase singularity of the STP vortex, it introduces the dual part of 2+1-dim gauge field. So the 2+1-dim gauge field and its Hodge dual merged into a 3+1-dim gauge field, which is the photon field, which means on 3+1-dim space-time, the photon field can be understood as toplogical excitations of 2+1-dim gauge field, the topological configuration is known as the Hopf link excitation. We now clarify the conclusion in detail since it is very important to understand the spin of photon, which has a zero mass.

In framework of STP vortex, the vortex tube is made of two fields, one is the STP field ϕ , whose gradient defines the flow direction of the vortex, the other is 2+1-dim gauge field A whose field strength characterizes the spinning direction of the vortex. So in this picture,

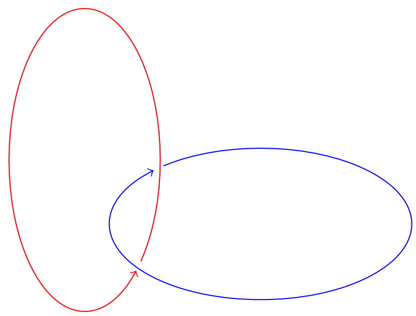

Figure 4: Photon as a topolotical excitation: a Hopf link

A describes the rotation and ϕ the flowing. Under the Hodge duality, the dynamics of the soliton part of STP field is equivalent to another gauge field A' , which is Hodge dual to A. Topologically, the vortex tube reprensents a Wilson loop, its Hodge duality is t'Hooft loop. Put them together forms a famous topological object, the Hopf link, as shown in Fig.4. The Hopf link is obvious a non-local object. The topological stability of the Hopf link protects it from pertubative destruction, so it can propagate in space-time without dissipation unless it meets another vortex. This is very similar to what happens in electromegnatic interaction, where photons propagates the interaction between charges. We had seen the equation of motion of the $\tilde{A}$, aka the joint representation of A and A' , is nothing but the Maxwell equations. The $\tilde{A}$ field is an effective representation of the Hopf link.

There are two circles in a Hopf link, they wind the topological subgroup (mathematically, the minimal torus) of Lorentz group separately. As we knew in previous section, they are left and right hand topological circles, each corresponds to a spinor fiber. However, in physics, there are no purely topological objects. So we need to consider the dynamics of the Hopf link, say, the effect resulted from deformation of either circle.

Consider an arbitrary deformation on one of the two circles, it will affect the whole Hopf link and defines a self isomorphism as follow

$$A : \Lambda_L \otimes \Lambda_R \to \Lambda_L \otimes \Lambda_R \tag{8.66}$$

here A denotes the self isomorphism on $\Lambda_L \otimes \Lambda_R$, Λ_L and Λ_R are left and right spinor firbers respectively. In appendix E, we proved that such a self isomorphism should be a vector map. Relatively, all derivatives should be changed into covariant derivatives, as

$$\partial_\mu \to D_\mu = \partial_\mu + igA_\mu \tag{8.67}$$

This leads to non-trivial local transmition that

$$[D_\mu, D_\nu] = D_\mu D_\nu - D_\nu D_\mu = ig(\partial_\mu A_\nu - \partial_\nu A_\mu). \tag{8.68}$$

This reflects the local homomorphism deformation. The strength of the deformation is described by the coefficient g, which relates to charge of matter particle. So we could propose an assertion: the amount of electric charge reflects the strength of local deformation of local space-time. The RHS of above equation is nothing but a field strength of four dimensional gauge field

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{8.69}$$

Since

$$\begin{aligned} D^\mu[D_\mu, D_\nu] &= ig\partial^\mu(\partial_\mu A_\nu - \partial_\nu A_\mu) - g^2 A^\mu(\partial_\mu A_\nu - \partial_\nu A_\mu) \\ &= -ig\Box A_\nu + ig\partial_\nu(\partial^\mu A_\mu) - g^2\partial_\mu(A^\mu A_\nu) + g^2(\partial_\mu A^\mu)A_\nu \\ &= \frac{1}{2}g^2\partial_\nu(A_\mu A^\mu) \end{aligned} \tag{8.70}$$

under Lorentz gauge $\partial_\mu A^\mu = 0$,the above equation only have pure derivation contributions, with vanishing contributions for no-boundary free field. So this equation can be simply written as

$$D^\mu F_{\mu\nu} = 0 \tag{8.71}$$

In three dimensional form, it can be written as

$$\vec{\nabla} \cdot \vec{E} = 0 \tag{8.72}$$

$$\partial_t E - \vec{\nabla} \times \vec{B} = 0 \tag{8.73}$$

In another way, because the Hopf link configuration is unchanged under left-right flop symmetry, this leads to a electromagnetic duality for field strength $F_{\mu\nu}$. The left-right flop symmetry actually means a flop between pair of indices $(0, i) \leftrightarrow (j, k)$, this can be achieved by introducing the Levi-Cevita connection

$$\epsilon^{0ijk} : (0, i) \to (j, k) \tag{8.74}$$

thus for the field strength $F_{\mu\nu}$, we have the following dual relation

$$\tilde{F}_{\alpha\beta} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu} \tag{8.75}$$

The Levi-Cevita connection flip electric and magnetic fields in three dimensions, and the above dual relation reads

$$\vec{E} \to \vec{B}, \quad \vec{B} \to -\vec{E} \tag{8.76}$$

The dual equation in four dimensional is written as

$$D_\mu \tilde{F}^{\mu\nu} = 0 \tag{8.77}$$

In three dimension, it becomes two equations

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{8.78}$$

$$\partial_t \vec{B} - \vec{\nabla} \times \vec{E} = 0 \tag{8.79}$$

Equations (8.72,8.73,8.78,8.79) are Maxwell equations for source-free electromagnetic fields, which proves in 3+1-dim, the Hopf link transforms the local deformation just the same as photon propagates in space-time.

The figure fig.5 shows how a deformation propagates from an electron to a positron, where red upper arrows denote left topological circles and blue downer arrows denote right topological circles.

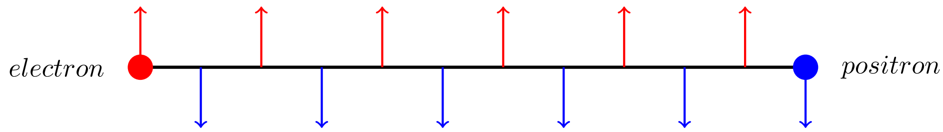

Figure 5: Photons deliver the interaction between electron and positron

8.8 Photon and vortex tube

We had already known that in framework of MIP, the spins of matter particles are originated from collisions between them and STP along topological circles in local space-time. Now we knew the photon could be represented as a Hopf link, which also is winding topological circles in 3+1-dim local space-time. So it is possible the spin of photons are also originated from STP.

In case of matter particles, for examples, electron and positron, their spins are sourced from local winding along left and right topological circles $U(1)_L$ and $U(1)_R$ in local space-time, respectively. At arbitrary moment, electron or positron has a phase angle φ_L or φ_R. These two phase angles are undetermined. It means electron or positron has a local phase angle symmetry, which is $U(1)$ symmetry. Because it is deduced from local space-time symmetry, it is a gauge symmetry.

Let us choose the phase angle be θ . The identical principle for fundamental particles requests the following equations

$$\psi \to \psi e^{-i\varphi_L} \equiv \psi e^{-i\theta}, \quad \bar{\psi} \to \bar{\psi} e^{-i\varphi_R} \equiv \psi e^{i\theta} \tag{8.80}$$

from which we know

$$\varphi_L = -\varphi_R = \theta \tag{8.81}$$

It means the gauge group $U(1)$ is the diagonal subgroup of $U(1)_L \times U(1)_R$, with transition matrix be -1. This perspective could be extend to higher dimensional transition matrices, which will leads to non-Abelian gauge groups, for example, $SU(2)$ or $SU(3)$.

In this picture, photon is represented as a Hopf link of 2+1-dim gauge fields, it is massless. However, it carries the information of collisions between matter particle and STP vortices. So it will also record the motion of the matter particle, as well as its spin. Since it is a $(1,1)$ representation of the topological subgroup of Lorentz group. Therefore, from the Hopf link proposition, we obtained photon has spin 1, and massless, and satisfies Maxwell equations. It actually explains how a massless photon has non-zero spin.

8.9 The generation of charged leptons in MIP

In the frame of MIP, there are no more than 3 generations of charged leptons.

According to MIP, the mass of matter particles is a statistical mass deriving from collision of STP. This collision effects of STP can be described by an effective potential $V(x)$,which reflects the strength of the interactions between STP and matter particles and will vary with the statistical mass: the bigger the statistical mass, the stronger effective potential you will get. If the particle is massless, there is no collision. Therefore the space is homogeneous and isotropic so that we can write $V(x) = 0$. On the other hand the previous discussion has shown us that the 3+1-dimensional electromagnetic field is born in vortex solution in 2+1-dimensional spacetime.

In the following we will make a study of the number of generations of charged leptons in the Standard Model, which is still an open question. Crossing any point O in 3-dimensional space there are 3 independent orthogonal 2-dimensional planes. Take O as the origin and choose rectangular coordinate system with the coordinates (x^0, x^1, x^2, x^3) . The Lagrangian equipped with vortex solution in the 2+1-dimensional subspaces can take the following forms

$$L_a^{2+1} = \partial_\mu \phi^* \partial^\mu \phi - \frac{\lambda_a}{2} \left(\phi\phi^* - F^2\right)^2, \tag{8.82}$$

with $a = 1, 2, 3$ respectively corresponding to 3-dimensional spacetime:

$$(x^0, x^2, x^3),\ (x^0, x^1, x^3),\ (x^0, x^1, x^2),$$

λ_a is the coupling constant which reflects the strength of the effective potential and is closely related to the statistical mass of the particle. If $\lambda_a = 0$, that is $V(x) = 0$, indicating the particle is massless ,there is neither collision nor vortex solution. So the statistical mass is an essential prerequisite for a particle to get charge. Following the steps in the previous section, bring in the gauge field $\vec{A}$ and investigate the excited states near the lowest point of the potential. From (8.27), we get

$$L_a^{2+1} = L_{\bar{A}} + L_\phi = -\frac{1}{4} F_{ij} F^{ij} + \frac{1}{4} f_{ij} f^{ij}, \tag{8.83}$$

with (i, j) taking values in the corresponding subspace. For the sake of simplicity, we have chosen the coupling constants of the gauge fields to be 1. Now the Lagrangians do not obviously involve λ_a any more and therefore have nothing to do with the statistical mass of the particle. Take Hodge $*$ duality, and lift the 2+1-dimensional theory to 3+1-dimensional spacetime. We take the notation $F_{\alpha\beta} = i\epsilon^{ij}_{\alpha\beta} f_{ij}$. For L_1^{2+1},

$$L_1^{2+1} = -\frac{1}{4} F_{ij} F^{ij} + \frac{1}{4} f_{ij} f^{ij} \tag{8.84}$$

Here the indexes i, j come from the subspace (x^0, x^2, x^3), with $i, j = 0, 2, 3$. The independent components of the field strength are $F_{02}, F_{03}, F_{23}, f_{02}, f_{03}, f_{23}$. and

$$\begin{cases} F_{\alpha\beta} = i\epsilon^{02}_{\alpha\beta} f_{02} \Rightarrow F_{13} = -i f_{02} \\ F_{\alpha\beta} = i\epsilon^{03}_{\alpha\beta} f_{03} \Rightarrow F_{12} = i f_{03} \\ F_{\alpha\beta} = i\epsilon^{23}_{\alpha\beta} f_{23} \Rightarrow F_{01} = -i f_{23} \end{cases} \tag{8.85}$$

Here we take the usual notations as $g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$, $\epsilon^{0123} = 1$. So that for L_1^{2+1} in the 3+1-dimensional spacetime, we have

$$\begin{aligned} L_1^{3+1} &= -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} = -\frac{1}{4} \tilde{F}_{ij} \tilde{F}^{ij} + \frac{1}{4} \tilde{F}_{\alpha\beta} \tilde{F}^{\alpha\beta} \\ &= -\frac{1}{4} \tilde{F}_{ij} \tilde{F}^{ij} - \frac{1}{4} \tilde{f}_{ij} \tilde{f}^{ij} \end{aligned} \tag{8.86}$$

with

$$\tilde{F}_{\mu\nu} = \begin{pmatrix} 0 & -i\tilde{f}_{23} & \tilde{F}_{02} & \tilde{F}_{03} \\ i\tilde{f}_{23} & 0 & i\tilde{f}_{03} & -i\tilde{f}_{02} \\ -\tilde{F}_{02} & -i\tilde{f}_{03} & 0 & \tilde{F}_{23} \\ -\tilde{F}_{03} & i\tilde{f}_{02} & -\tilde{F}_{23} & 0 \end{pmatrix} \tag{8.87}$$

Following the same way, for L_2^{2+1} , we get

$$F_{\alpha\beta} = i\epsilon_{\alpha\beta}^{ij} f_{ij} \Rightarrow \begin{cases} F_{\alpha\beta} = i\epsilon_{\alpha\beta}^{01} f_{01} \Rightarrow F_{23} = if_{01} \\ F_{\alpha\beta} = i\epsilon_{\alpha\beta}^{03} f_{03} \Rightarrow F_{12} = if_{03} \\ F_{\alpha\beta} = i\epsilon_{\alpha\beta}^{13} f_{13} \Rightarrow F_{02} = if_{13} \end{cases} \tag{8.88}$$

$$\begin{aligned} L_2^{3+1} &= -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} = -\frac{1}{4}\tilde{F}_{ij}\tilde{F}^{ij} + \frac{1}{4}\tilde{F}_{\alpha\beta}\tilde{F}^{\alpha\beta} \\ &= -\frac{1}{4}\tilde{F}_{ij}\tilde{F}^{ij} - \frac{1}{4}\tilde{f}_{ij}\tilde{f}^{ij} \end{aligned} \tag{8.89}$$

with

$$\tilde{F}_{\mu\nu} = \begin{pmatrix} 0 & \tilde{F}_{01} & i\tilde{f}_{13} & \tilde{F}_{03} \\ -\tilde{F}_{01} & 0 & i\tilde{f}_{03} & \tilde{F}_{13} \\ -i\tilde{f}_{13} & -i\tilde{f}_{03} & 0 & i\tilde{f}_{01} \\ -\tilde{F}_{03} & -\tilde{F}_{13} & -i\tilde{f}_{01} & 0 \end{pmatrix}. \tag{8.90}$$

For L_3^{2+1} , we can obtain

$$F_{\alpha\beta} = i\epsilon_{\alpha\beta}^{ij} f_{ij} \Rightarrow \begin{cases} F_{\alpha\beta} = i\epsilon_{\alpha\beta}^{01} f_{01} \Rightarrow F_{23} = if_{01} \\ F_{\alpha\beta} = i\epsilon_{\alpha\beta}^{02} f_{02} \Rightarrow F_{13} = -if_{02} \\ F_{\alpha\beta} = i\epsilon_{\alpha\beta}^{12} f_{12} \Rightarrow F_{03} = -if_{12} \end{cases} \tag{8.91}$$

$$\begin{aligned} L_3^{3+1} &= -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} = -\frac{1}{4}\tilde{F}_{ij}\tilde{F}^{ij} + \frac{1}{4}\tilde{F}_{\alpha\beta}\tilde{F}^{\alpha\beta} \\ &= -\frac{1}{4}\tilde{F}_{ij}\tilde{F}^{ij} - \frac{1}{4}\tilde{f}_{ij}\tilde{f}^{ij} \end{aligned} \tag{8.92}$$

with

$$\tilde{F}_{\mu\nu} = \begin{pmatrix} 0 & \tilde{F}_{01} & \tilde{F}_{02} & -i\tilde{f}_{12} \\ -\tilde{F}_{01} & 0 & \tilde{F}_{12} & -i\tilde{f}_{02} \\ -\tilde{F}_{02} & -\tilde{F}_{12} & 0 & i\tilde{f}_{01} \\ i\tilde{f}_{12} & i\tilde{f}_{02} & -i\tilde{f}_{01} & 0 \end{pmatrix}. \tag{8.93}$$

According to the above, starting with 3 different 2+1-dimensional Lagrangians L_a^{2+1}, we end up with the 3+1-dimensional Lagrangians which have the uniform description as

$$\begin{aligned} L_a^{3+1} &= -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} = -\frac{1}{4}\tilde{F}_{ij}\tilde{F}^{ij} + \frac{1}{4}\tilde{F}_{\alpha\beta}\tilde{F}^{\alpha\beta} \\ &= -\frac{1}{4}\tilde{F}_{ij}\tilde{F}^{ij} - \frac{1}{4}\tilde{f}_{ij}\tilde{f}^{ij} \ . \end{aligned} \tag{8.94}$$

In fact they are the same one since they can be converted to each other by rotating the proper coordinate axis as follows

$$L_1^{3+1} \leftarrow (\hat{e}_1 \leftrightarrow \hat{e}_2) \rightarrow L_2^{3+1} \leftarrow (\hat{e}_2 \leftrightarrow \hat{e}_3) \rightarrow L_3^{3+1}, \tag{8.95}$$

which is equivalent to internal rotations of the gauge fields $\vec{A}, \vec{a}$. For the electromagnetic field arising from the lepton with fundamental charge in 3+1-dimensional spacetime, when we trace back to its birth in 2+1-dimensional subspace, we will find out we have 3 degrees of freedom described by $\lambda_a,\ a = 1, 2, 3$, and just corresponding to the 3 different subspaces. Therefore the type of charged leptons is no more than 3. Actually the modern science has told us there are 3 generations of charged leptons in our real world, which is just in accordance with $\lambda_1 \neq \lambda_2 \neq \lambda_3 \neq 0$ in our framework and from the aspect of STP the local isotropy of spacetime is broken. In conclusion, in the frame of MIP, there are no more than 3 generations of charged leptons, which is firmly rooted in the fact that we live in a 3+1dimensional spacetime.

8.10 Conclusion of the section

In this section, from the MIP picture, we explained the origin of electromagnetic interaction in detail. In framework of MIP, the 3+1-dim electromagnetic field represents itself as a Hopf link exicitation made of 2+1-dim gauge field and its Hodge dual partner. It is a topological state. From this topological configuration, we obtained the Maxwell equations in two different ways, also from which, we explained why massless photons have spin 1. In this section, we studied four properties of electric charges, say, positive and negative, quantization, repelling and attracting , Coulomb inverse square law ,and equations of motions of photons, which propagates the Coulomb interaction between charged particles. In addition, together with the charge amount calculated in section 5, we obtained all five properties of the electric charge.

There is one additional expression for the STP vortex configuration.In this section, we only considered the non-pertubative potential came from matter particle. However, a non-pertubative disturbance of space-time energy does not only have such a single origin in our universe. In early universe, the disturbance is very large and STP vortices could also be generated as well as its partner field, the photon field. It implies in early time, the universe was dominated by radiation, which coincides with observations in cosmology. Another example for non-pertubative potential is black holes, near the horizon of a black hole, the space-time energy disturbance is quite large, and it will also generate electromagnetic radiation. This kind of radiation has a completely different origin comparing with Hawking radiation. This may offer quite a lot of new perspectives on black hole and cosmology researches.

Last and most importantly, we derived the generation for charged leptons. This is a completely new result and one can not derive this law in current quantum field theory framework. Within the MIP framework, by invoking the STP vortices, the generation is a direct inference.

9 STP Vortices as an origin of fermion spin

In this chapter, we will discuss the essence of spin from the topological structure of STP vortex.

While introducing into the gauge field in the 2+1 dimensional normal space, the singularity at the center of votex was resolved as a S^1. On the differential geometry point of view, this S^1 can be seen as the spatial edge of the vortex. Because of Hodge duality, we can obtain the dual S^1 which will be denoted as $S^{1\star}$. Hence in the 3+1 dimensional space-time, the simplest topological structure involving S^1 and $S^{1\star}$ is a Hopf link, which is a direct intersection of these two circles. As known in knots theory, there are more fundamental connect way for S^1 and $S^{1\star}$. The fundamental stone of topological intersection is the famous skein relation, which can be explicit as in the Fig.6

A single Hopf link actually have two twisted points, each of them is the mirror image of the other one. Mathematically, the two twisted points Hopf link is not the most fundamental topological structure. The most fundamental one is the single twisted point connection, which is shown in Fig.7

Within the STP vortex configuration, we could have the following algebra-knot correspondence: the fundamental representation of the Lorentz group corresponds to the single twisted point connection of two cirles, which are edges of two dual vortices. The two twisted points connection corresponds to the adjoint representation of the Lorentz group.Under this framework, the algebraic representation of Lorentz group and the topological knot representation has a deep and explicit corelation.

Even in mathematics, this correspondence is a new conjecture, we do not have a direct proof at this stage. However, the indirect way to proof the conjecture is worth to study. For example, connect the affine representation to each other, that is saying, finding an integrable correlation between Schur polynomial and Jones polynomial.

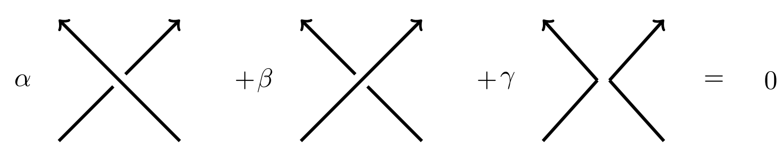

Figure 6: Skein relation

9.1 Topological phase transition of STP vortices

There are vortices on the tangent space and the normal space, since from the point view of isotropic STPs, there are no differences between these two spaces. Actually, in previous chapter, what we solved on the normal space has its Hodge dual on the tangent space. Therefore, in 3+1 dimensional space-time, we need to understand the interaction theory of two vortices living on dual spaces.

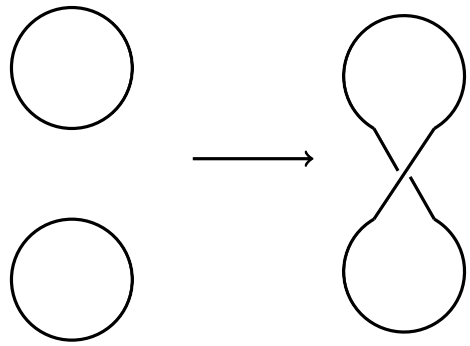

Figure 7: Topological phase transition of STP vortices

The interaction between two vortices can make centers of them fuse or intersect to each other. As we had known in previous chapter, because of the existence of gauge field, the singular center of the vortex had been resolved into a S^1. If there are no interactions between S^1 and its dual $S^{1\star}$, the dynamics on tangent space and normal space will completely decoupled. If this is the case, the dynamic of STPs around the matter particle will be un-isotropic and un-uniform. This obviously violates the physical fact. In other words, if the dynamics on tangent space and the one on normal space do not couple to each other, the space-time will be choked as slides. Hence the naturally way to couple these two dynamics of STPs leads to a phase transition.

The simplest topological phase transition is as shown in Fig.7. Notice that Edward Witten had used the skein relation developed by John Conway in 1969 to study knot invariant. It is amazing that the topological phase transition shown in7 is the same as John Conway's skein relation.

Therefore, we have already known the two vortices on tangent space and normal space respectively can form a topological twisted point through topological transition as well as the skein relation. For current double vortex case, because we could related the two vortices to each other by a single Lorentz rotation. This means the double vortex sysem has an internal symmetry. A careful study reveals the group is a double cover of $SO(3)$, respect to the Z_2 symmetry of the double vortices. this is because the center of STP vortex is what the matter particle sit on, hence in 3+1 dimensional spacetime, the two vortices have the same center. We splitted these two vortice by hand is a convenient way to explictly reprensent them. Therefore, the rotation subgroup of Lorentz group is the double cover of $SO(3)$, that is, $SU(2)$. This concludes the internal consistence between topological twisted point and spin.

9.2 The isotropic vortex

In previous chapter, we introduced into the 2+1 dimensional gauge field for vanishing the energy singularity at the center of STP vortex. The resolving of the singularity as an S^1 is the same as to introducing a $U(1)$ principal bundle structure in mathematics. The 2+1 dimensional gauge field is nothing but the connection on this principal bundle. However, the resolving operation blows up the singularity on the center of STP vortex does not reconfiguration all properties the singularity. As the center of STP vortex, the singularity is isotropic, but the circle S^1 is orientable. This means we covered the un-orientability of the singularity by the resolving operation. Now it is clear that we need to recover the un-orientability on the circleS^1.

In 1976, T. Martin [30] noticed that there is a correspondence in mathematics as follows. The rotation and translation effects can be separated geometrically. Hence there are two connections correspond to rotation and translation, respectively. The rotation connection corresponds to the torsion tensor, which has the similar meaning as curvature to translation effects.

We now consider the 2+1 dimensional STP vortex, it is nothing but a microscopic space-time. In this space-time, the torsion can not be negligible. The existence of microscopic torsion has no influence to the general relativity, since the geodesic line is unrelying on the torsion at all.

As saying in MIP, the matter particle obtains the mass property by collision of STPs and itself. In this picture, without STPs, the matter particle generated a space-time potential. The potential leads to a curved space-time around the matter particle. Microscopically, the metric around the matter is curved.

Before introducing the torsion tensor, we need to introduce the everywhere othogonal tangent vielbein field $e_a(x)$ as following

$$e_a(x) = e_a^i \frac{\partial}{\partial x^i}, \quad a = 0, 1, 2 \tag{9.1}$$

it satisfies the relation as:

$$g^{ij} = \eta^{ab} e_a^i e_b^j, \quad \eta_{ab} = g_{ij} e_a^j e_b^i \ . \tag{9.2}$$

It is natural to define the dual cotangent vielbein field, as:

$$\theta^a(x) = \theta_i^a dx^i \tag{9.3}$$

they satisfies the normal condition

$$< \theta^a, e_b >= \delta_b^a \tag{9.4}$$

and

$$g_{ij} = \eta_{ab} \theta_i^a \theta_j^b, \quad \eta^{ab} = g^{ij} \theta_i^a \theta_j^b \tag{9.5}$$

now the differential interval

$$ds^2 = g_{ij} dx^i dx^j = \eta_{ab} \theta_i^a \theta_j^b dx^i dx^j = \eta_{ab} \theta^a \theta^b \tag{9.6}$$

the spin connection can be defined by covariant differential on tangent vielbein field, as:

$$\omega_{ia}^b e_b = D_i e_a, \quad \omega_{ia}^b =< D_i e_a, \theta^b > \tag{9.7}$$

where $\omega_{ia}^b(x)$is the spin connection coefficient, and

$$\omega_a^b(x) = \omega_{ia}^b(x)dx^i \tag{9.8}$$

is the spin connection 1-form field. The covariant differential now is defined as following:

$$D' = \partial + \omega \tag{9.9}$$

when acting on a vector field $\xi^a(x)$,

$$D_i'\xi^a = \frac{\partial \xi^a}{\partial x^i} + \omega_{ib}^a \xi^b \tag{9.10}$$

Now we can discuss the coupling between spinor field and space-time under local Lorentz symmetry. If there is a spinor field $\psi(x)$, aka a spin representation of local Lorentz group, then on dynamical point of view, the momentum term of this spinor field can be written as:

$$D_i'\psi = \partial_i \psi + \frac{1}{2}\omega_i^{ab}\Sigma_{ab}\psi \tag{9.11}$$

here Σ_{ab} is the spin representation of Lorentz algebra,

$$[\Sigma_{ab}, \Sigma_{cd}] = \eta_{bc}\Sigma_{ad} + \eta_{ad}\Sigma_{bc} - \eta_{ac}\Sigma_{bd} - \eta_{bd}\Sigma_{ac} \tag{9.12}$$

Introducing the spin connection ω_i^{ab}, the parallel transition of cotangent field $\theta(x)$ defines the torsion of this manifold

$$\begin{aligned} \tau_{ik}^a &= D_i'\theta_k^a - D_k'\theta_i^a \\ &= \frac{\partial \theta_k^a}{\partial x^i} - \frac{\partial \theta_i^a}{\partial x^k} + \omega_{ib}^a \theta_k^b - \omega_{kb}^a \theta_i^b \end{aligned} \tag{9.13}$$

it is the field strength of the cotangent vielbein. When it is not zero, the manifold is not torsion-free and hence intrinsic twisted.The un-vanishing of the field strength of cotangent vielbein implies there is a multi-value property when we joint two 2+1 dimensional theories into a single 3+1 dimesnional theory. We know there exists a sigularity at the center of STP vortex, meanwhile the vielbein rounds the singularity, the vielbein will generate a monodromy matrix at the singularity. To incomplete the contribution of this 2×2 monodromy matrix, we need to consider the following action:

$$I = \int d^3x Tr[\epsilon^{ijk}\theta_i^a \tau_{jk}^a] + \int d^3x^\star Tr[\epsilon^{ijk}\theta_i^{a\star}\tau_{jk}^{a\star}] \tag{9.14}$$

here the Tr means summation on vielbein indices. The $\star$ indices means those torsion related variables are defined on dual 2+1 dimensional space-time. As we saw, (9.14) actually is a simple split joint of two 2+1 dimensional Chern-Simons theory defined on different boundary of the 3+1 dimensional space-time. Therefore, we need to introduce the joint constraint, which is obvious the Hodge duality. It is easy to proof that within the following constraint, the first term and the second term in (9.14) Hodge dual to each other. The constraint is :

$$\epsilon^{ijk}\theta_i^a = \epsilon^{ijkl}\tau_{il}^{a\star}, \quad \epsilon^{ijkl}\tau_{jk}^a = \epsilon^{ijl}\theta_j^{a\star} \tag{9.15}$$

Now the two 2+1dimensional Chern-Simons theory is fused into a 3+1 dimensional instanton interaction:

$$I = 2\int d^4x \epsilon^{ijkl} Tr\left(\tau_{il}^{a\star}\tau_{jk}^a\right) \tag{9.16}$$

We see, under the fused situation, the contribution of cotangent vielbein is completely equivalent to a topological instanton contribution of a gauge field. The instanton contribution is nothing but a constant, so now the task is to calculate this constant factor.

Written (9.16) as the differential form, it can be recognized as a characteric number in 3+1dimensional space-time. Notice when accomplish with the cotangent vielbein, on the 2+1 dimensional space-time, the boundary of the vortex could be seen as an S^2. We now joint two S^2 into a boundary of 3+1 dimensional space-time. If the concatenation is trivial, then the 3+1 dimensional spacetime has a boundary with topology $S^2 \times I$. However, the 3+1 dimensional space-time is $R^{3,1}$, when there exists no particles, the boundary is a null set. The boundary can be seen as an S^3 within the matter particle. So it means when we transform the two 2+1 dimensional vortices, the concatenation of their boundaries (S^2) is non-trivial. The final result of this concatenation is to generate an S^3. In fact, this is the way how the two 2+1 dimensional vortices become a microscopic stable configuration around the matter particle in 3+1 dimensional space-time.

Now consider the cobordism characteristic number of (9.16), it discribes the phase angle changing from $S^2 \times I$ to S^3. The phase angle difference describes the charactersitic number, we obtain:

$$I = 2 \times \frac{vol(S^3)}{vol(S^2)} \times N = 2 \times \frac{2\pi^2}{4\pi} \times N = \pi N \tag{9.17}$$

Here N is the topological number according to torsion τ, also known as the winding number. It describes the multiplicity of the mapping from $S^2 \times I$ to S^3 . In physics, it is the *theta* contribution.

When considering the wave function of matter particle, we do not see the contribution of the characteristic number. Therefore the topological phase transition just contributes the signature of the wave function, as:

$$\Psi^{[N]} = \psi(x,t)\exp(iI) = (-)^N \psi(x,t) \tag{9.18}$$

when the particle rotate around some fixed axe one whole circle, the corresponding 2+1 dimensional STP vortex also rotated one times around the S^3 , the result is the topological winding number changes by 1, now

$$\Psi^{[N]} \to \Psi^{[N+1]} \; or \; \Psi^{[N-1]} \tag{9.19}$$

as

$$\Psi \to -\Psi \tag{9.20}$$

so we have proved the spin of matter particle should be 1/2, as known as the Fermionic property.

From which we observed above, we obtain an important conclusion. The spin statistical property of matter particle is originate from the un-orintable of singularity sitting on the center of STP vortex around matter particle. This singularity is double covered, there are two 2+1 dimensional vortices around it. The two vortices reconstruct the singularity by manifold cobordism and thus incomplete the isotropic property of the singularity. The spin property of matter particle corresponds to the topological phase transition at the cobordism. In general, in the frame of MIP, the spin of matter particle describes the topological order that corresponding to topological phase transition of STP vortices around the matter particle.

9.3 Pauli exclusion principle

We now use s to label the topological order according to the topological phase transition of STP vortices. For union definition convenience, we let the topological order as a quantum evolution operator, that is:

$$e^{\frac{i}{\hbar}\hat{s}\theta}|\Psi\rangle = e^{i\theta/2}|\Psi\rangle \tag{9.21}$$

From this definition we could take this topological order as an operartor that has eigenvalue $\frac{\hbar}{2}$, for example, $\langle\hat{s}\rangle = \frac{\hbar}{2}$. The parameter of rotation one circle is $\theta = 2\pi$, subsitute this parameter into previous equation, one obtains the Fermionic statistical property immediately.

Now let us consider a permutation of two coincident particles. Suppose particle 1 is on the state $|\Psi_{x_1}(p)\rangle$ and particle 2 is on the state $|\Psi_{x_2}(p)\rangle$. Then the direct product system of these two particle is on the state $|\Psi_{x_1}(p)\rangle \otimes |\Psi_{x_2}(p)\rangle$. We could rotate the $|\Psi_{x_1}(p)\rangle$ as well as the$|\Psi_{x_2}(p)\rangle$ half a cirlce around the center between x_1, x_2 . Because the vortices around these two particles also rotated two half a circles, hence

$$\begin{aligned} T_{x_1,x_2}e^{i\pi\hat{s}}|\Psi_{x_1}(p)\rangle \otimes |\Psi_{x_2}(p)\rangle &= e^{i\frac{\pi}{2}}|\Psi_{x_2}(p)\rangle \otimes e^{i\frac{\pi}{2}}|\Psi_{x_1}(p)\rangle \\ &= -|\Psi_{x_2}(p)\rangle \otimes |\Psi_{x_1}(p)\rangle \end{aligned} \tag{9.22}$$

here T_{x_1,x_2} exchanges x_1, x_2. Therefore if there are two coincident matter particles, on the same state, and sit on a same position, then it is easy to see a direct result from (9.22):

$$|\Psi_x(p)\rangle \otimes |\Psi_x(p)\rangle = -|\Psi_x(p)\rangle \otimes |\Psi_x(p)\rangle \tag{9.23}$$

when and only when $|\Psi_x(p)\rangle \otimes |\Psi_x(p)\rangle = 0$ the previous result can be the case. however, $|\Psi_x(p)\rangle \otimes |\Psi_x(p)\rangle = 0$ means the state actually does not exist! So the Pauli exclusive principle is a natural result in the frame of MIP.

9.4 STP as shepherd of matter particles

MIP explains all the essences of quantum mechanics both qualitatively and quantitatively without relying on any special hypothesis, which sweeps away the last traces of idealism in quantum mechanics from the Copenhagen interpretation. Fundamentally, MIP requires a massless and spinless scalar particle, i.e. STP. The existence of STP is an objective fact in physics, which is different from the wave function of quantum mechanics. Therefore, this paper can thoroughly solve a series of extremely important problems that cannot be answered by quantum mechanics alone:
How can a matter particle be sure that its momentum and position satisfy a certain uncertainty relationship? Why can a matter particle exhibit wave-particle duality? How does a matter particle know energy levels where it can go and where it absolutely cannot go, that

is , how can they satisfy the Pauli exclusion principle? Quantum mechanics only reles on a series of postulations to avoid the above problems. MIP not only solves the above problems by mathematical derivations, but also allows matter praticles behave exactly as required by quantum mechanical postulations.
At the level of objective reality, all the microscopic behaviors of matter particles are rooted in their commen shepherd– STP. The random collision of STP, which seems to be chaotic, is actually the supervisor of all microscopic behaviors of matter particles. The wonderful quantum world was born because of these ubiquitous supervisors.
Quantum mechanics, as one of the most successful physical sciences, ultimately cannot avoid the problem of its completeness. In an era where the fundamental question of how matter particles obey the postulations of quantum mechanics cannot be answered, the completness of quantums mechanics cannot be treated properly. When the theory of STP and MIP are discovered, in which STP is the supervisor and shepherd of all quantum behavious of matter particles, the question of quantum mechanical completeness are ready for profound investigations. MIP and its STP have taken a big step forward to finally slove the long lasting puzzle of quantum mechanical completeness.

10 Muon physics and MIP

10.1 Theoretical framework

Under the framework of MIP, STPs collide with material particles. In quantum field theory, this is equivalent to introduce a massless scalar field into the theory and its interaction with material particles. Therefore, the Standard Model of particle physics needs to be revised as:

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{ST} + \mathcal{L}_{int} \tag{10.1}$$

In the above formula, $\mathcal{L}_{SM}$ is the Lagrangian of the standard model of particle physics; $\mathcal{L}_{ST}$ is the kinetic energy term of the STPs scalar field, which can be expressed as for:

$$\mathcal{L}_{ST} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \tag{10.2}$$

Since the strength of the collision between STPs and material particles is proportional to the mass of the particles, the interaction term between STPs and material particles is expressed as:

$$\mathcal{L}_{int} = \lambda \sum_{i\in\text{all matter fields}} m_i\phi\bar{\psi}_i\psi_i\ . \tag{10.3}$$

Where ψ_i represents the material particles in the Standard Model, that is, leptons and quarks. m_i is the mass of the corresponding material particles.

Obviously, for material particles, the mass itself already reflects the information of the collision and interaction between STPs and material particles. So at the tree level, the interaction (10.3) does not change any physics. But at the order of loop diagrams, the interaction of the above equation is ignored by the Standard Model of particle physics.

In this chapter, we will consider the modification of muon physics caused by the interaction of STPs with muons, which includes two aspects. One is the correction of muon anomalous magnetic moment. The second is the lifetime of muon. Muon physics is considered because muons are two hundred times more massive relative to electrons. This means that at the loop diagrams, STPs are about 10^4 times larger than electrons for the correction of muon physics. On the other hand, electrons do not decay, and the effect of STPs cannot be verified in experiments.

10.2 muon anomalous magnetic moment

The anomalous magnetic moment of the muon is contributed by a triangular Feynman diagram. The single loop contribution of the STPs scalar field to the muon anomalous magnetic moment can be represented by a Feynman diagram 8. As early as 1972, Jackiw

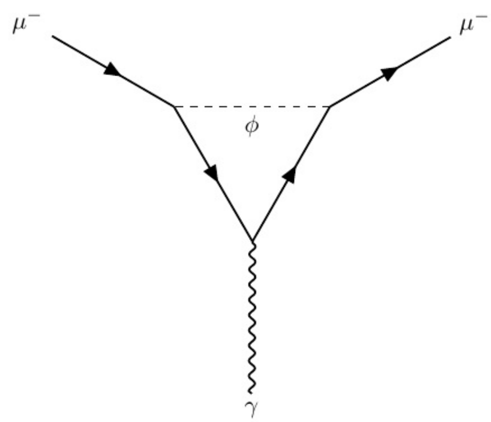

Figure 8: Feynman diagram of the contribution of STPs to the anomalous magnetic moment of muon

and Weinberg have calculated the contribution of this graph [31], and its contribution to the muon anomalous magnetic moment is:

$$\Delta g_\mu = \frac{3\lambda^2 m_\mu^2}{8\pi^2} \ . \tag{10.4}$$

Jackiw and Weinberg call this contribution in their paper the "virtual scalar field" contribution. Since this "virtual scalar field" does not exist in the Standard Model, the contribution of this scalar field is not considered in subsequent experimental verifications. But in MIP, this scalar field exists undoubtedly , and it refers to the scalar field of STPs. Therefore we need to consider its contribution to the anomalous magnetic moment of muon.

As early as 2006, Brookhaven National Laboratory in the United States discovered experimentally that there is a 3.3σ difference between the anomalous magnetic moment of muon and the prediction of the Standard Model[32], that is,

$$a_\mu(\text{BNL}) = 116592080(63) \times 10^{-11}(0.54\text{ppm}).$$

Where $a_\mu = \frac{g_\mu - 2}{2}$ is the difference value of muon anomalous magnetic moment. In 2021, the Fermi National Laboratory in the United States accurately measured the difference value of the muon anomalous magnetic moment[33], and the result was:

$$a_\mu(\text{FNAL}) = 116592040(54) \times 10^{-11}(0.46\text{ppm}).$$

Combining two experiments, the average of anomalous magnetic moment is:

$$a_\mu(\text{EXP}) = 116592061(59) \times 10^{-11}(0.35\text{ppm}).$$

From the standard model, the theoretical value of a_μ is:

$$a_\mu(\text{SM}) = 116591810(43) \times 10^{-11}(0.37\text{ppm}).$$

The deviation between experiment and theory is:

$$a_\mu(\text{EXP}) - a_\mu(\text{SM}) = 251 \pm 59 \times 10^{-11}.$$

This deviation reaches 4.2σ, so it is a very significant deviation. This means there is a high probability that the contribution of a certain particle is missing from the Standard Model. Under the MIP framework, we believe that this deviation comes entirely from the contribution of STPs. From this deviation, the coupling constant λ of STPs and material particles can be determined, Its value is given as follows:

$$\begin{aligned}\lambda^2 &= (a_\mu(\text{EXP}) - a_\mu(\text{SM}))\,\frac{16\pi^2}{3m_\mu^2}\\ &= 1.18349(\pm 0.27819) \times 10^{-11}\text{MeV}^{-2} \qquad (10.5)\\ \lambda &= 3.44019^{+0.38300}_{-0.43137} \times 10^{-6}\text{Mev}^{-1} \qquad (10.6)\end{aligned}$$

Therefore, the introduction of the interaction between STPs and muon can completely match the theoretical and experimental results of muon anomalous magnetic moment.

10.3 Muon decay problem

Furthermore, to demonstrate the self-consistency of the scalar field introduced into STPs, we also need to consider the corresponding physics of the single loop interactions between STPs and material particles. In other words, if the introduction of the STPs scalar field and its coupling strength λ results in a contradiction between the theory of a certain physical process and the corresponding experimental results, it is proved that the STPs scalar field is not the source of the deviation of the muon anomalous magnetic moment. Therefore, we consider the single loop process in the muon decay problem. With the participation of STPs, the corresponding Feynman diagram is shown in Figure 9:

The scattering amplitude $\mathcal{M}$ of this box Feynman diagram is:

$$\begin{aligned} i\mathcal{M} &= -\frac{g_w^2\lambda^2 m_\mu m_e}{8}\int \frac{d^4k}{(2\pi)^4}\frac{\mathcal{D}(k,p,m)}{\mathcal{N}(k,p,m)} \qquad (10.7)\\ \mathcal{D}(k,p,m) &= \bar{v}(p_2)\gamma^\mu(1+\gamma^5)(\not{k}+m_\mu)u(p_1)\times\\ &\quad \bar{u}(p_3)(\not{k}-\not{p}_3-\not{p}_4+m_e)\gamma_\mu(1-\gamma^5)v(p_4) \qquad (10.8)\\ \mathcal{N}(k,p,m) &= \left[(k^2-m_\mu^2+i\epsilon)\right]\left[((k-p_2)^2-m_W^2+i\epsilon)\right]\times\\ &\quad \left[(k-p_3-p_4)^2-m_e^2+i\epsilon\right]\left[(k-p_1)^2+i\epsilon\right] \qquad (10.9)\end{aligned}$$

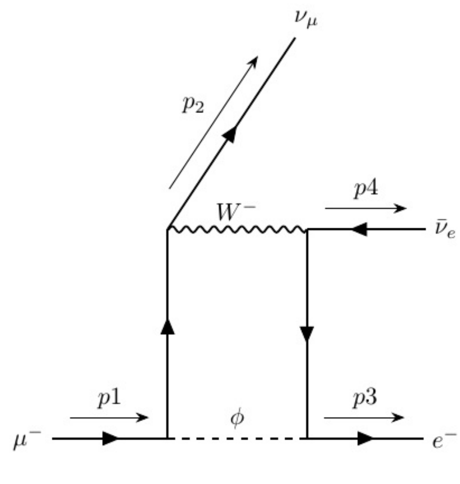

Figure 9: Feynman diagram of the single loop contribution of STPs to the muon decay

Without introduction of the STP scalar field, the scattering amplitude of the muon decay can be labeled as follows:

$$\mathcal{M}_{ST} = \mathcal{M}_{tree} + \mathcal{M}_{1-loop} + \mathcal{M}_{2-loop} + \cdots$$

After introducing the STP scalar field, the absolute square of the scattering amplitude can be written as:

$$\begin{aligned} |\mathcal{M}|^2 &= (\mathcal{M}_{ST} + \mathcal{M})\,(\mathcal{M}_{ST}^* + \mathcal{M}^*) \\ &= |\mathcal{M}_{ST}|^2 + 2\mathrm{Re}\left[\sum_{\text{all spins}} \mathcal{M}_{tree}^* \mathcal{M}\right] + \text{higher order terms} \end{aligned} \tag{10.10}$$

therefore, we only need to calculate $\mathrm{Re}\left[\sum_{\text{all spins}} \mathcal{M}_{tree}^* \mathcal{M}\right]$ to get the correction of the scattering amplitude.

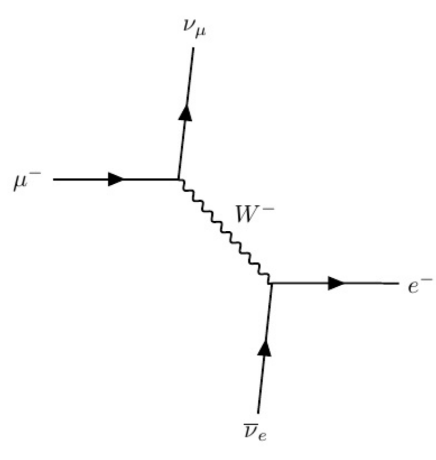

Figure 10: muon decay tree diagram

$\mathcal{M}_{tree}$ represents the contribution of figure 10, and its expression is as follows:

$$\mathcal{M^*}_{tree} = -\frac{g_w^2}{8m_W^2c^2}\bar{u}(p_1)\gamma^\mu\left(1-\gamma^5\right)u(p_2)\bar{v}(p_4)\left(1+\gamma^5\right)\gamma_\mu u(p_3) \tag{10.11}$$

Condensing all Dirac matrices and using Casimir's trick, we finally get:

$$\sum_{\text{all spins}} \mathcal{M}^*_{tree}\mathcal{M} = i\frac{4g_w^4\lambda^2 m_\mu m_e}{m_W^2c^2}\int\frac{d^4k}{(2\pi)^4}\frac{[(k+p_1)\cdot p_4]\,[(k+p_1-2p_4)\cdot p_2]}{\mathcal{N}(k,p,m)} \tag{10.12}$$

We compute this integral using the Mellin–Barnes (MB) representation [34, 35, 36, 37, 38] developed by V. A. Smirnov et al. (See Appendix F)For the integral kernel in (10.12), we can do the substitution $k+p_1 \to k$, and then use the Mellin–Barnes representation to express it as factor multiple form of the Γ function(See Appendix F), and finally the MB integral is used to do the appropriate contour integration. Since there are multiple Γ function poles that overlap, the order of the contour integration needs to be evaluated at multiple singular points. We denote the result of the integration of k as $\mathcal{F}(s,t,m)$, where $s=(p_1-p_2)^2, t=(p_1-p_3)^2$ is the Mandelstam variable. In muon's stationary reference frame, where $p_1=(m_\mu c^2,0,0,0)$, the decay rate of muon is

$$\begin{aligned} d\Gamma = {} & \frac{\langle|\mathcal{M}|^2\rangle}{2\hbar m_\mu}\left(\frac{d^3\mathbf{p}_2}{(2\pi)^3 2|\mathbf{p}_2|}\right)\left(\frac{d^3\mathbf{p}_3}{(2\pi)^3 2|\mathbf{p}_3|}\right)\left(\frac{d\ ^3\mathbf{p}_4}{(2\pi)^3 2|\mathbf{p}_4|}\right) \\ & \times (2\pi)^4\delta^4(p_1-p_2-p_3-p_4) \end{aligned} \tag{10.13}$$

The momentum of the electron, anti-electron neutrino and muon neutrino are also clearly written down, which are:

$$p_2 = (|\mathbf{p}_2|c, \mathbf{p}_2),\ p_3 = (\sqrt{|\mathbf{p}_3|^2c^2+m_e^2c^4}, \mathbf{p}_3),\ p_4 = (|\mathbf{p}_4|c, \mathbf{p}_4) \tag{10.14}$$

Substituting the above formula and the momentum of muon p_1 into $\mathcal{F}(s,t,m)$, it becomes $\mathcal{F}(|\mathbf{p}_2|,|\mathbf{p}_3|,m_e,m_\mu,m_W)$ Then the change of decay rate caused by STP is:

$$\Delta\Gamma_{ST} = -\frac{g_w^4\lambda^2 m_e}{8\pi^3 m_W^2c^2\hbar}\int_0^{m_\mu c/2} d|\mathbf{p}_2|\int_{m_\mu c/2-|\mathbf{p}_2|}^{m_\mu c/2} d|\mathbf{p}_3|\,\text{Im}\left(\mathcal{F}(|\mathbf{p}_2|,|\mathbf{p}_3|,m_e,m_\mu,m_W)\right) \tag{10.15}$$

Substituting into the numerical calculation shows that:

$$\Delta\Gamma_{ST} = 1.2141 \pm (0.2854)s^{-1} \tag{10.16}$$

The muon decay rate calculated from the Standard Model is:

$$\Gamma_{SM} = 455169.311s^{-1} \tag{10.17}$$

Therefore, the lifetime of muon under the action of STP is:

$$\tau_\mu = 1/(\Gamma_{SM}+\Delta\Gamma_{ST}) = 21969788(\pm 14)\times 10^{-13}s \tag{10.18}$$

Experimentally, the muon lifetime is

$$\tau_\mu(\text{Exp}) = 21969811(\pm 22)\times 10^{-13}s \tag{10.19}$$

It can be seen that after adding the contribution of the STP scalar field, the theoretical lifetime of the muon perfectly matches the experimental observations.

Just as we finished the writting work on this article, we noticed the breaking news about the mass of W boson. In this article [44], the mass of W boson is given as:

$$m_W = 80433.5(\pm 9.4)\ MeV/c^2\ . \tag{10.20}$$

Using this new W boson mass, we recalculated the STP scalar field contribution, and it shows:

$$\Delta\Gamma_{ST}^{new} = 0.9273 \pm (0.2180)s^{-1}, \tag{10.21}$$

hence the new lifetime of muon under the action of STP is:

$$\tau_\mu^{new} = 1/(\Gamma_{SM} + \Delta\Gamma_{ST}^{new}) = 21969802(\pm 10.5) \times 10^{-13}s\ . \tag{10.22}$$

This result is even better fitting the experiment result than the privious one, which provides strong support on our propose of STP. We also noticed the breaking news about the mass of W boson [44], which says that there is a significiant deviation between the standard model prediction and experiment. Using this new boson mass, we recalculated the STP scalar field contribution, which shous the result obtained in (10.12) is not sensitive to the new W boson mass. This is because in this article the Feynman diagrams we calculated for interaction between STP and W boson is not sensitive to the mass of W boson.

10.4 Lepton anomalous magnetic moment and MIP

We have considered the effects of MIP in muon physics. Introducing the STP scalar field, the anomalous magnetic moment of the muon and the decay lifetime can be well explained. Correspondingly, we can consider the deviations of other leptons after the introduction of STP.

10.4.1 Electron anomalous magnetic moment

The measurement of the electronic anomalous magnetic moment has been very accurate. The current experimentally determined electron anomalous magnetic moment is[39]:

$$a_e(\text{Exp}) = (1159652180.91 \pm 0.26) \times 10^{-12} \tag{10.23}$$

On the other side, in framework of standard model, the calculation of the anomalous magnetic moment of electron, strongly depends on the accurate value of fine structure constant α , which determined by experiment. At the level of 10^{-12}, the deviation of α is relative big. Therefore the theoretical calculation for anomalous magnetic moment of electron spands on a relative big range[40, 41, 42]. The results obtained by the theoretical calculation of the standard model are:

$$a_e^{\mathrm{SM}}(\mathrm{Rb}) = (1159652180.252 \pm 0.095) \times 10^{-12} \tag{10.24}$$

$$a_e^{\mathrm{SM}}(\mathrm{Cs}) = (1159652181.61 \pm 0.23) \times 10^{-12} \tag{10.25}$$

The differences between the theoretical calculation and experimental value are:

$$\Delta a_e(\mathrm{Rb}) = a_e^{\mathrm{SM}}(\mathrm{Rb}) - a_e(\mathrm{Exp}) = -(0.658 \pm 0.355) \times 10^{-12} \tag{10.26}$$

$$\Delta a_e(\mathrm{Cs}) = a_e^{\mathrm{SM}}(\mathrm{Cs}) - a_e(\mathrm{Exp}) = +(0.7 \pm 0.49) \times 10^{-12} \tag{10.27}$$

After introducing STP, the correction value of the electronic magnetic moment is:

$$\begin{aligned}\Delta a_e^{\mathrm{MIP}} &= \frac{3\lambda^2 m_e^2}{16\pi^2} = \frac{3 \times (1.18349 \pm 0.27819) \times (0.51099895)^2 \times 10^{-11}}{16 \times 3.1415926536^2} \\ &= (0.0587 \pm 0.0138) \times 10^{-12}\end{aligned} \tag{10.28}$$

According to above calculation, we know the correction due to STP scalar field is one level smaller than current theory-experment gap. However, the gap is mainly caused by the accuracy of fine structure constant, which is an experimental error. Therefore, under the current experiments, the electronic anomalous magnetic moment does not have a bigger deviation due to the existence of STP. The deviation due to STP field, is consistent with current experiments on the anomalous magnetic moment of electron.

10.4.2 Tauon anomalous magnetic moment

Due to the relatively short lifetime of tauon, it is difficult to accurately measure its anomalous magnetic moment in such a short time. The relative experiment only can give a very rough region as follows [43]:

$$-0.052 < a_\tau(\mathrm{Exp}) < 0.013 \tag{10.29}$$

with confidential level 95%.

The tauon anomalous magnetic moment calculated by the current standard model is[43]:

$$a_\tau(\mathrm{SM}) = (117721 \pm 5) \times 10^{-8} \tag{10.30}$$

After the introduction of STP, the corrected value of tauon magnetic moment is:

$$\begin{aligned}\Delta a_\tau^{\mathrm{MIP}} &= \frac{3\lambda^2 m_\tau^2}{16\pi^2} = \frac{3 \times (1.18349 \pm 0.27819) \times (1776.86)^2 \times 10^{-11}}{16 \times 3.1415926536^2} \\ &= (7.0986 \pm 1.6686) \times 10^{-7}\end{aligned} \tag{10.31}$$

The ratio of this corrected value to the theoretical value is

$$\rho_\tau = \frac{\Delta a_\tau^{\mathrm{MIP}}}{a_\tau(\mathrm{SM})} \simeq 0.0006 \tag{10.32}$$

In fact, this correction ratio is the largest among the three generations of leptons. However, the experiment on tauon anomalous magnetic moment is quite difficult, the experiment uncertainty is huge comparing to the deviation due to STP field. Though we can predict the STP scalar field would bring in a small correction, the current tauon experiments are far away to the correction. However, the deviation due to STP field, is consistent with current experiments on the anomalous magnetic moment of tauon.

10.5 Summary

In this chapter, we consider two modifications for muon physics due to STP. First, we consider the correction of the STP scalar field to the muon anomalous magnetic moment. The interaction strength λ between the STP scalar field and the matter particle is determined. Second, we calculate the correction of the STP scalar field for muon decay, which makes the theoretical predictions agree with the experimental observations perfectly. It can be seen that we only need to introduce one free parameter, the STP scalar field interaction strength, we achieved a great triumph in the area of muon physics.

11 Summary

Starting from the fundamental concept innovation of statistical mass, this paper proposes MIP: material particles will be subjected to random collision of STP's which is ubiquitous in spacetime to make frictionless quantum Brownian motion. The change of the action of material particles in each collision is integer multiple of Planck constant h. From MIP, we can prove all the important results of quantum theory. The quantum theory obtained within the framework if MIP is fully compatible with the existing quantum theory. The advantage of this new framework is that it does not require the introduction of additional wave function assumptions, and is able to derive the Schrödinger equation directly. In particular, the concept of wave pack collapse is not required to be introduced under our MIP framework. The Heisenberg uncertainty principle no longer has a fundamental position but a natural inference under the MIP framework. From the statistical uncertainty between inertial mass and spacetime diffusion coefficient, the most basic coordinate momentum uncertainty relationship of quantum mechanics can be derived. Therefore, it is proved that the wave-particle duality is a property exhibited by the STP colliding particles under the MIP framework. Furthermore, we apply MIP to quantum measurement problems, and have a new breakthrough interpretation of the EPR paradox problem that has confused physics for nearly a century. The STP colliding matter particles is a zero-spin scalar particle without mass. According to MIP, the topological properties and dynamic properties of STP can explain the nature of photons, and thus naturally obtain the complete electromagnetic theory and all important properties of charge. Furthermore, from the vortex structure of spacetime, we obtain the origin of the spin and the relationship between spin and mass. Going back to the 2+1d vortex when we investigate the electromagnetic fields in 3+1d spacetime, we prove the strong constrain on the number of generations of charged leptons, at most three generations.

Due to the random collisions between STP and matter particles, matter particles are able to behave exactly as requierd by the postulations of quantum mechanics, which shows that STP is the supervisor and shepherd for all the microscopic behaviors of matter particles, MIP also creates the foundation for further investigation on the completeness problem of quantum mechanical descriptions.

Last but not least, MIP requires a novel massless scalar particle STP. The random collision between STPs and muons is the crucial step beyond standard model. Our extension of

standard model is minimal, which only introduce on free parameter decribing the interaction strength between STPs and muons, then we are able to explain two key experiments of muon simultaneously. By thorough calculations of corresponding Feyman's diagrams, the contributions from random collisions between STPs and muons explain the anomalous magnetic moment of muon and its lifetime excellently, which solve a world class puzzle about the anomalous magnetic moment of muon, and give a self-consistent explanation to the lifetime discrepancy of muon at the same time. Recent experomental results from FermiLab are the most precision verfication of MIP, which guarantee the correctness of MIP and the advantages over other alternative theories.
In summary, MIP is the origin of quantum mechanics. MIP is able to revise standard model at minimal cost to explain the anomalous magnetic moment of muon, which provides a whole new framework to research phenomena beyond standard model.

Appendix A: Brown Motion and Markov Process

When the displacement of the material particle $X(t)$ satisfies the following conditions, we call the material particle doing Brownian motion:

1.$X(0) = 0$.

2. On any finite disjoint interval set $(s_i, s_i + t_t)$, the displacement of the particle is $X(s_i + t_t) - X(s_i)$, which are random variables that are independent of each other.

3. For each $s \geqslant 0, t \geqslant 0$, $X(s+t) - X(s)$ obeys the normal distribution $N(0, t)$.

For each constant a, the process $X(t) + a$ is called the Brownian motion starting from a. For the Brownian motion that is physically free of friction, we call it the quantum Brownian motion in this paper.

Consider any past set of times $(\cdots, p_2, p_1)$, any "current time" s, and any "future time" t, all of which are within the range of X, if any

$$\cdots < p_2 < p_1 < s \tag{11.1}$$

Then the Markov property is established, and the process is a Markov process, but only if:

$$\begin{aligned} \Pr\big[X(t) &= x(t) \mid X(s) = x(s), X(p_1) = x(p_1), X(p_2) = x(p_2), \ldots\big] \\ &= \Pr\big[X(t) = x(t) \mid X(s) = x(s)\big] \end{aligned} \tag{11.2}$$

Set up for all time sets. Then calculate the conditional probability

$$\Pr\big[X(t) = x(t) \mid X(s) = x(s), X(p_1) = x(p_1), X(p_2) = x(p_2), \ldots\big] \tag{11.3}$$

Future state is independent of any historical state and is only relevant to the current state.

In summary, the quantum Brownian motion studied in this paper is a Markov process.

Appendix B: Decomposition of Random Variables

In the Langevin equation, the true velocity of particle motion $\vec{V}$ contains three parts: the classic statistical velocity $\vec{v}$, quantum envelope velocity $\vec{u}$ and Gaussian noise $\vec{\nu}$

We do not consider the impact of classic statistical velocity. Then the random motion of the particles will be determined by the quantum envelope motion and Gaussian noise. The fact that we need to prove is that we can distinguish the quantum envelope motion $\vec{u}$ in the strict mathematical differential sense. The quantum envelope motion corresponds to the smooth continuous part of the random motion, and the Gaussian noise corresponds to the continuous non-differentiable part of the random motion.

First, for any random variable $r(x,t)$, if a smooth function $f(x,t)$ is superimposed, the result is still a random variable. which is a random variable, as

$$w(x,t) = r(x,t) + f(x,t) \tag{11.4}$$

But if $r(x,t)$ or $w(x,t)$ has a finite order autocorrelation association, then theoretically we can strictly distinguish $w(x,t)$ and other two random variables of $r(x,t)$, which is:

$$\langle r(x_1,t_1)r(x_2,t_2)\cdots r(x_n,t_n)\rangle_r = \mathcal{F}_n(\vec{x},\vec{t}), \quad mod(n,N) \equiv 0 \tag{11.5}$$

$$\langle r(x_1,t_1)r(x_2,t_2)\cdots r(x_n,t_n)\rangle_r = 0, \quad mod(n,N) \neq 0 \tag{11.6}$$

Then there is

$$\langle w(x_1,t_1)w(x_2,t_2)\cdots w(x_N,t_N)\cdots w(x_n,t_n)\rangle_r \neq 0, \quad n > N \tag{11.7}$$

Therefore, it can be strictly distinguished mathematically. In the case we considered, Gaussian noise $\vec{\nu}$ has a second-order correlation

$$\langle \nu_i(t)\nu_j(t')\rangle = \Omega\delta_{i,j}\delta(t-t') \tag{11.8}$$

And all odd-order associations are zero

$$\langle \nu(t)\rangle_\nu = 0$$

So obviously

$$\vec{w}(t) = \vec{u}(t) + \vec{\nu}(t)$$

The odd-order correlation is not zero. So you can strictly distinguish between $\vec{w}(t)$ and $\vec{\nu}(t)$. Due to the MIP, there is only one kind of Gaussian noise, and there is no other noise source. So continuous functions other than noise are smooth and differentiable functions. So $\vec{u}$ is a smooth function.

Appendix C: Additional Physics Example with Three-speed Decomposition

The superposition of orbitals and the formation of chemical bonds, which are common in chemistry, involves quantum superposition states. In the simplest case, the ground state of the hydrogen atom and the first excited state are superimposed with equal probability as

$$\psi(r,t) = \psi_{100}e^{-iE_1 t} + \psi_{200}e^{-iE_2 t} \tag{11.9}$$

Where $E_1 = -13.6ev, E_2 = -13.6/4ev = -3.4ev$, the wave function of the ground state of the hydrogen atom and the first excited state are

$$\psi_{100} = \frac{1}{\sqrt{\pi a^3}}e^{-r/a} \tag{11.10}$$

$$\psi_{200} = \frac{1}{\sqrt{2a^3}}e^{-r/2a}(1 - \frac{r}{2a}) \tag{11.11}$$

Where a is the Bohr's radius $a = 0.529 \times 10^{-10}m$.

With the Euler formula, we can write the superimposed wave function as

$$\psi = [\psi_{100}cos(E_1 t) + \psi_{200}cos(E_2 t)] - i[[\psi_{100}sin(E_1 t) + \psi_{200}sin(E_2 t)] \tag{11.12}$$

From the real and imaginary part, the two potential functions R and I of the superposition wave function can be further determined. It is found by equation (5.34) and (5.35) that the electrons u and v are not zero in this state.

This physics example is not a special case, and has general physical meaning. When the quantum state has definite energy, its classical statistical velocity v must be zero. Generally speaking, the particle is in the superposition state of the energy eigenstate, and its three speeds are not zero which has clear physical meaning.

Appendix D: From MIP to the Uncertainty Principle

We believe that the uncertainty principle comes from the kinematic equation of stochastic spacetime motion, which is rooted in the non-differentiable motion path, i.e. the particle coordinate $\vec{x}(t)$ derivative of time $d\vec{x}/dt$ does not exist. Therefore, it must be noted that the particle's momentum $\vec{p} = md\vec{x}/dt$ cannot be well defined. The momentum is defined as follows

$$\vec{p} = mD\vec{x} = m\vec{v} + m\vec{u} \tag{11.13}$$

Kinematic equation

$$\vec{u} = \Re\frac{\nabla\rho}{\rho} \tag{11.14}$$

For the sake of simplicity, the following discussion uses only one component in the x direction, and all vector equations become equations of one component. For any random variable O, the statistical average is $< O >= \int O\rho(x)dx$. Multiplying both sides of the equation by ρ and integrate x, we can get the x and u_x covariance

$$\sigma(x, u_x) =< (x- < x >)(u_x- < u_x >) >= -\Re \tag{11.15}$$

The covariance represents the total error of two variables, which is different from the variance that only represents the error of one variable. If two variables change in the same directions, then the covariance between two variables is positive. If two variables change in opposite directions, the covariance between two variables is negative. For any two real random variables A and B, there is the Schwarz inequality $|\sigma(A, B)| \leqslant \sigma(A)\sigma(B)$, which leads to

$$\sigma(x)\sigma(u_x) \geqslant \Re = \hbar/2m \tag{11.16}$$

The statistical definition of uncertainty is

$$\sigma(x) = \sqrt{< x^2 > - < x >^2} \tag{11.17}$$

$$\sigma(u_x) = \sqrt{< u_x^2 > - < u_x >^2} \tag{11.18}$$

So far we have proved the uncertainty relationship between the position of random spacetime moving particles and the fluctuation speed. Further, if the uncertainty of momentum has two parts of contributions

$$\sigma^2(p) = m^2(\sigma^2(v) + \sigma^2(u)) \tag{11.19}$$

That is, $\sigma(p) \geqslant m\sigma(u)$, the uncertainty of the position and the fluctuation speed can be obtained.

$$\sigma(x)\sigma(p_x) \geqslant \hbar/2 \tag{11.20}$$

The proof of our paper interprets Heisenberg's uncertainty principle as the uncertainty relationship between random spacetime moving particle position and fluctuation speed.The random spacetime motion has no friction and no irreversible dissipation.

The uncertainty of the fluctuation speed is entirely from spacetime fluctuations. According to Heisenberg's original statement, the measured action inevitably interferes with the state of the particles being measured, thus creating uncertainty. Later that year, Kennard gave another statement. The following year, Herman also obtained this result independently. According to Kennard's statement, the uncertainty of position and the uncertainty of momentum are the nature of the particle, and cannot be suppressed below a certain limit, regardless of the measured action. Thus, for the principle of uncertainty, there are two completely different interpretations. Landau believes that the two interpretations are equivalent, so one expression can be derived from another expressions (Ref. quantum mechanics of Landau). However, in the latest experimental progress, Japanese scholars published on January 15, 2012, the empirical results of the Heisenberg uncertainty principle. They used two instruments to measure the spin angle of the neutron and obtained a smaller measurement than the Heisenberg limit, which proves the measurement interpretation by Heisenberg is wrong. However, the principle of uncertainty is still correct, because this is the quantum nature of the particle.

The derivation process of this paper has nothing to do with the measurement theory, and it has nothing to do with the internal properties of the particles. It is believed that the uncertainty principle is rooted in the fluctuation of spacetime. Under the non-relativistic framework, spacetime fluctuations are only related to the mass of the particles. The mass of a particle is the only perceptible property of the particle in spacetime.

Appendix E: Self Isomorphism on Direct Product Spin Clusters

We hope to prove the following conclusions in this appendix:
Theorem 2: Given any topological excited state deformation: $A : \Lambda_L \otimes \Lambda_R \mapsto \Lambda_L \otimes \Lambda_R$, where A For automorphism mapping, Λ_L, Λ_R represent the left-hand spin cluster and the right-hand spin cluster, respectively, and A is the vector map.

Proof: First of all, from the symmetry of the spin structure, it is not difficult to know that we only need to prove arbitrary automorphism: $A : \Lambda_L \mapsto \Lambda_L$ Both are vector maps. This is because if we can determine that A is a vector map, we can get it through conjugate expansion: $\widetilde{A} : \Lambda_L \otimes \Lambda_R \mapsto \Lambda_L \otimes \Lambda_R$ for vector mapping.

To prove that any automorphism: $A : \Lambda_L \mapsto \Lambda_L$ is a vector map, we need to consider the model on the left-handed spin sector, which is corresponding to the $Clifford$ algebra .Proposition 1.3.2 by [45] It can be seen that for the finite form $Clifford$ algebra, the following forms are isomorphic:

$$Cl_{r,s} \cong Cl_1 \ \widehat{\otimes} ... \ \widehat{\otimes} \ Cl_1 \widehat{\otimes} \ Cl_1^* ... \ \widehat{\otimes} \ Cl_1^*.$$

Among them, the number of Cl_1 corresponds to r, and the number of Cl_1^* corresponds to s.

From the theorem 1.5.4 of [45], all $Clifford$ $K-$ means that ρ can be decomposed into straight sums of irreducible algebra representations of the following form:

$$\rho = \rho_1 \oplus ... \oplus \rho_m.$$

The feature subspace W_i corresponding to ρ_i is the smallest subspace.

In additions, by the $Bott$ cycle law theorem [45], we can get the algebraic representation of all Cl_m, $(m = 1, ...8)$, and the representation follows the indicator m Repeated with a period of 8. That is: we can get the algebra of any Cl_m as follows:

$$\begin{gathered} Cl_1 = \mathbb{C},\ Cl_2 = \mathbb{H},\ Cl_3 = \mathbb{H} \oplus \mathbb{H},\ Cl_4 = \mathbb{H}(2), \\ Cl_5 = \mathbb{C}(4),\ Cl_6 = \mathbb{R}(8),\ Cl_7 = \mathbb{R}(8) \oplus \mathbb{R}(8),\ Cl_8 = \mathbb{R}(16). \end{gathered} \tag{11.21}$$

For any combination of the above forms, the straight and broken parts ρ_i Can be split into direct product form:

$$Cl_{r,s} \cong \ Cl_1 \ \widehat{\otimes} ... \ \widehat{\otimes} \ Cl_1 \ \widehat{\otimes} \ Cl_1^* ... \widehat{\otimes} \ Cl_1^*.$$

The automorphism mapping between any part of the above direct product form can be made by $Cl_1 = \mathbb{C}, ...Cl_8 = \mathbb{R}(16)$ Algebraic combination between parts. Since the above parts are all vector spaces, the automorphism must be a vector mapping, that is, the automorphism of ρ_i must correspond to the matrix form.

In addition, from the algebraic decomposition process described above, it is not difficult to know that the homomorphic mapping between all corresponding different sub-blocks is also a vector mapping. Finally, we will be $\rho_i,\ i = 1, ...8$ All of them are combined together in a straight form, and we can get the automorphism $A : \Lambda_L \mapsto \Lambda_L$ when $i = 1, ...8$ for vector mapping. When the indicator i is greater than 8, by the *Bott* cycle law, we can still get the automorphism mapping by the above process. A is the vector map. The conclusion is proved.

Appendix F: Field Theory Calculations for Fermionic Loop Integral

We consider the following Fermion loop momentum integrals

$$\int \frac{d^d k}{D_1^{n_1} D_2^{n_2}} = i\pi^{d/2}(-p^2)^{d/2-n_1-n_2} G(n_1, n_2)\,, \quad D_1 = -(k+p)^2\,, \quad D_2 = -k^2 \tag{11.22}$$

Noting that in the denominator, D_1, D_2 should actually have an infinitesimal analytic continuation $(-i0^+)$. But for the sake of simplicity, we don't explicitly write it out. After analysing the continuation, we need to consider the contribution of $p^2 < 0$, and the power contribution of $-p^2$ can be easily obtained from dimensional analysis. In fact, what needs to be calculated now is the dimensionless function $G(n_1, n_2)$; to simplify the calculation, we can let $-p^2 = 1$. When $n_1 \leq 0$ or $n_2 \leq 0$, the score can be strictly calculated and $G(n_1, n_2) = 0$ can be obtained.

Using Wick rotation and α parameterization, we can rewrite $G(n_1, n_2)$ as:

$$G(n_1, n_2) = \frac{\pi^{-d/2}}{\Gamma(n_1)\Gamma(n_2)} \int e^{-\alpha_1(k+p)^2 - \alpha_2 k^2} \alpha_1^{n_1-1} \alpha_2^{n_2-1} d\alpha_1\, d\alpha_2\, d^d k\,. \tag{11.23}$$

Let

$$k' = k + \frac{\alpha_1}{\alpha_1 + \alpha_2} p\,,$$

We can get

$$\begin{aligned} G(n_1, n_2) &= \frac{\pi^{-d/2}}{\Gamma(n_1)\Gamma(n_2)} \int \exp\left[-\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right] \alpha_1^{n_1-1}\alpha_2^{n_2-1} d\alpha_1\, d\alpha_2 \int e^{-(\alpha_1+\alpha_2)k^2} d^d k \\ &= \frac{1}{\Gamma(n_1)\Gamma(n_2)} \int \exp\left[-\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right] (\alpha_1+\alpha_2)^{-d/2} \alpha_1^{n_1-1}\alpha_2^{n_2-1} d\alpha_1\, d\alpha_2\,. \end{aligned} \tag{11.24}$$

Using the substitution $\alpha_1 = \eta x$, $\alpha_2 = \eta(1-x)$, the above formula can be rewritten as

$$\begin{aligned} G(n_1, n_2) &= \frac{1}{\Gamma(n_1)\Gamma(n_2)} \int_0^1 x^{n_1-1}(1-x)^{n_2-1} dx \int_0^\infty e^{-\eta x(1-x)} \eta^{-d/2+n_1+n_2-1} d\eta \\ &= \frac{\Gamma(-d/2+n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)} \int_0^1 x^{d/2-n_2-1}(1-x)^{d/2-n_1-1} dx\,. \end{aligned} \tag{11.25}$$

The integrand is an Euler B function, so we can get the final result

$$G(n_1, n_2) = \frac{\Gamma(-d/2+n_1+n_2)\Gamma(d/2-n_1)\Gamma(d/2-n_2)}{\Gamma(n_1)\Gamma(n_2)\Gamma(d-n_1-n_2)}\,. \tag{11.26}$$

For all positive integers $n_{1,2}$ they are proportional to

$$G_1 = G(1,1) = -\frac{2g_1}{(d-3)(d-4)}\,, \quad g_1 = \frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\,, \tag{11.27}$$

The scale factor is a rational function of d.

Noting that at $k \to \infty$, the denominator part of (11.22) behaves as $(k^2)^{n_1+n_2}$. Therefore, this integral is divergent when $d \geq 2(n_1+n_2)$.

Acknowledgement

Special thanks to Dr. Yu Xiaolu, Dr. He Yali and Dr. Wu Jianfeng for their patient discussions and critical suggestions. In addition, Dr. Song Fei has made important contributions to the certification of Appendix E of this paper. At the beginning of this research project, Dr. Zhang Peng, Dr. Ma Mingwei, Dr. Liu Jinyan, Dr. Tian Yuan, Cui Ying, and Hu Xiaohui also participated in the discussion and made many meaningful suggestions. I would like to thank them all.

References

[1] L. Erdos, Lecture Notes on Quantum Brownian Motion, arXiv: 1009.0843, (2010)

[2] N. Wiener, Differential space. J. Math and Phys. 58 , 131-174 (1923)

[3] J. Zinn-Justin, Quantum Field Theory and Critical Phenomena (4-th edition), Oxford University, (2002)

[4] S. Ross, A First Course in Probability (8th Edition), Pearson Prentice Hall, (2009)

[5] F. Reif, Fundamentals of Statistical and Thermal Physics, Waveland, (2009)

[6] A. Einstein, Investigations on the theory of the brownian movement. Dover Edition(1956).

[7] E. Nelson, Dynamical theories of Brownian motion, (Princeton University Press, Princeton, 1967).

[8] E. Nelson, Quantum Fluctuations, (Princeton University Press, Princeton, 1985).

[9] P.Dirac, Quantised Singularities in the Electromagnetic Field, Proc. Roy. Soc. A 133, 60 (1931)

[10] G. E. Uhlenbeck and L. S. Ornstein, Rev. Mod. Phys., 36,823(1930)

[11] M. C. Wang and G. E Uhlenbeck, Rev. Mod. Phys., 17,323(1945).

[12] L. de Broglie, C.R. Acad. Sci. 264B, 1041 (1967).

[13] M. Davidson, Lett. Math. Phys. 3, 271 (1979).

[14] M. Davidson, Lett. Math. Phys. 3, 367 (1979)

[15] Max Born, Zur Quantenmechanik der Sto vorg nge, Zeitschrift f'r Physik, 37, #12 (Dec. 1926), pp. 863-C867 (German); English translation, On the quantum mechanics of collisions, in Quantum theory and measurement, section I.2, J. A. Wheeler and W. H. Zurek, eds., Princeton, New Jersey: Princeton University Press, 1983, ISBN 0-691-08316-9.

[16] L. Kadanoff , Statistical Physics: statics, dynamics and renormalization. World Scientific Press, (2000).

[17] L. Landau and E. Lifshitz , Courses in theoretical physics, vol 1, Mechanics. Butterworth-Heinemann. (1976)

[18] H. Goldstein, Classical Mechanics (3rd Edition), Addison-Wesley, (2001).

[19] J. D. Jackson, Classical Electrodynamics (3-rd Edition), Wiley, (1998)

[20] L. Landau and E. Lifshitz , Courses in theoretical physics, vol 3, Quantum Mechanics. Pergamon Press, (1977)

[21] R.J. Glauber, Quantum optics and electronics, edited by C. De Witt (Gordon-Breach New York, 1965); H. Haken, in Encyclopedia of Physics (Springer, New York, 1976).

[22] G. Birkhoff and J. von Neumann, Ann. Math. 37, 823 (1932).

[23] H. Haken and W. Weidlich, Z. Phys. 205, 96 (1967).

[24] Einstein, A; B Podolsky; N Rosen. Physical Review. 47 (10): 777-780.

[25] R. P. Feynman, Ph.D thesis. Princeton Press (1942).

[26] P.A.M. Dirac, Physikalische Zeitschrift der Sowjetunion, Band 3, Heft 1 , pp. 64-72 (1933).

[27] L. Mario. "Hydrodynamic theory of electromagnetic fields in continuous media." Physical Review Letters 70.23(1993):3580-3583.

[28] Martins, Alexandre A , Pinheiro, et al. Fluidic electrodynamics: Approach to electromagnetic propulsion[C] Aip Conf Proc. American Institute of Physics, 2009.

[29] S. Weinberg, The Quantum Theory of Fields, vol 2, Cambridge University, (1995)

[30] Martin, T.. (1976). Torsion and the geometry of spin.

[31] Jackiw R , Weinberg S . Weak-Interaction Corrections to the Muon Magnetic Moment and to Muonic-Atom Energy Levels. Phys. Rev. D. 5, 2396 (1972).

[32] G. Bennett et al. (Muon g-2 Collaboration), Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL, Phys.Rev. D73, 072003 (2006).

[33] B. Abi, et al. Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm, Phys. Rev. Lett. 126 (14) (2021) 141801.

[34] V. A. Smirnov, Feynman Integral Calculus, (Springer Verlag, Berlin, 2006) DOI: https://doi.org/10.1007/3-540-30611-0.

[35] I. Dubovyka , J. Gluza and T. Riemann, Optimizing the Mellin-Barnes Approach to Numerical Multiloop Calculations. Acta Physica Polonica B, Vol. 50 (2019), 11, 1993.

[36] AMBRE webpage: http://prac.us.edu.pl/˜gluza/ambre.

[37] J. Gluza, K. Kajda, T. Riemann, AMBRE - a Mathematica package for the construction of Mellin-Barnes representations for Feynman integrals, Comput. Phys. Commun. 177 (2007) 879–893. arXiv:0704.2423, doi: 10.1016/j.cpc.2007.07.001.

[38] A. Smirnov, V. Smirnov, On the Resolution of Singularities of Multiple Mellin-Barnes Integrals, Eur. Phys. J. C62 (2009) 445–449. arXiv: 0901.0386, doi:10.1140/epjc/s10052-009-1039-6.

[39] P. A. Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 update

[40] Keshavarzi, Alex and Khaw, Kim Siang and Yoshiokam Tamaki. (2021). Muon $g-2$: current status.

[41] Aoyama, Tatsumi, et al. "Revised and Improved value of the QED Tenth-Order Electron Anomalous Magnetic Moment." Physical Review D, Vol. 97. no. 3, Feb. 2018. Crossref.

[42] R.H. Parker, C.Yu, W. Zhong, B. Estey, H. Muller, Measurment of the fine-structure constant as a test of the standard Model, Science 360 (2018) 191.

[43] Eidelman, M. Giacomini, F.V. Ignatov, M. Passera, The τ lepton anomalous magnetic moment, Nuclear Physics B - Proceedings Supplements, Volume 169, 2007, Pages 226-231, ISSN 0920-5632, https://doi.org/10.1016/j.nuclphysbps.2007.03.002. (https://www.sciencedirect.com/science/article/pii/S0920563207002496)

[44] CDF COLLABORATION, High-precision measurement of the W boson mass with the CDF II detector, SCIENCE, 376,(2022) 170

[45] H. B. Lawson. Spin Geometry [M]. Princeton University Press, New Jersey, 1994.