On Mass Interaction Principle

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Abstract

This paper proposes mass interaction principle (MIP) as: the particles will be subjected to the random frictionless quantum Brownian motion by the collision of space time particle (STP) prevalent in spacetime. The change in the amount of action of the particles during each collision is an integer multiple of the Planck constant h. The motion of particles under the action of STP is a quantum Markov process. Under this principle, we infer that the statistical inertial mass of a particle is a statistical property that characterizes the difficulty of particle diffusion in spacetime. Within the framework of MIP, there are three novel and important works in this paper: 1. We prove that the number of generations of lepton can not be over three. 2. We prove the principle of entropy increasing for noninteracting systems, and clarify the physical origin of entropy at absolute zero. 3. We solve a world class puzzles about the anomalous magnetic moment of muon, and give a self-consistent explanation to the lifetime discrepancy of muon at the same time.

Under the MIP framework of interaction between STP and matter particles, the relativistic, quantum mechanic, electromagnetic, spin, thermodynamics and gravitation properties are all interpreted self-consistently, which shows that they all have the common origin.

Keywords: *Mass Interaction Principle , Special Relativity, Schrödinger Equation , Quantum Measurement , Entanglement , Uncertainty Relation, Path Integral , Neutrino, Electromagnetism, Photon, Spin*

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1 Introduction

1.1 Spacetime Fluctuation, STP and MIP

We believe the energy fluctuations of spacetime are universal, which are defined as STP. In this picture, particles are classified into two groups: one is matter particles which interact with STP, another one is massless particles which freely move in spacetime. Matter particles change their states by all the collisions with STP. The underlying property of mass is a statistical property emerging from random impacts in spacetime. Different particles have different effects of impact by STP, which can be defined as some kind of inertia property of particles. This property corresponds to mass dimension (Following we will prove it happens to be the inertial mass from Schrödinger's equation). Matter particles develop a Brownian motion due to random impacts from spacetime. We strongly suggest that all the probabilistic behaviours of quantum mechanics come from the Brownian motion, which is exactly the origin of quantum nature. In the framework of MIP, the photon represents itself as a Hopf link exicitation made of the 2+1-dim gauge field and its Hodge dual partner. On the other hand, under the MIP framework, photons not only exchange electromagnetic interactions, they also exchange spin information. It just explains that the annihilation condition of positive and negative electrons is not only the opposite of charge, but also the opposite of its spin. In modern physics, the spin and charge of matter particles are independent quantum properties. However, the spin has a magnetic moment and indicates that the spin and electromagnetic interactions are related. Under the MIP framework, this apparent contradiction can be self-consistently explained.

We believe the quantum behaviours of matter particle come from spacetime fluctuation. The energy fluctuation of spacetime is quantised. We call the quantised energy as space-time particle. It is a massless and spinless scalar particle. The exchange of energy between particle and STP is not strictly random, which leads to a unique Brownian-like motion. Once the time interval of impact is fixed, the exchange of energy has to be quantised, which indeed is the quantum nature of particles. Therefore, all quantum nature of particles is a faithful representation of spacetime quantised fluctuation.

Definition 1:

Matter particles will perform random fluctuation motion in spacetime because of stochastic interactions between STP and matter particles, within which the energy exchange can not be achieved instaneously. For free matter particles, we define the product of exchanged characteristic energy and the corresponding time interval as the change of action in the collision process.

According to the above two fundamental propositions: 1. spacetime fluctuations are universal; 2. spacetime fluctuations are quantised, we propose a MIP: Any particle with mass m will involve Brownian-like motions without frictions, due to random impacts from spacetime. Each impact changes the amount nh (n is any integer)for an action of the particle. The motion of a particle under the action of STP is a random motion of a quantum Markov process (quantum Brownian motion).

The MIP is absolutely essential to mass, spin, all quantum properties as well as relativity properties of matter particles. We will prove two important results. At first, within the framework of MIP, fundamental results of special and general relativity are natural inferences. Secondly, many important principles of modern quantum mechanics can be derived from MIP. Within this framework, MIP plays the role of the zeroth interaction, which dictates all quantum behaviours. Moreover, it will be shown that modern quantum field theory is compatible with MIP in the sense of quantum statistical partition functions.

1.2 Inertia Mass is a Statistical Property

Until now, our knowledge of mass, a fundamental concept of physics, mainly comes from Newton's laws of motion especially the first and second laws. The definition of mass in physics is a basic property of particles. The amount of matter contained in object is called the mass of object. The mass is related to the inertial nature of the object's original motion state.

The first law states that in an inertial reference frame, an object either remains at rest or continues to move at a constant speed, unless acted upon by a force. However according to the MIP, free particle has to do Brownian-like motions in spacetime, which is a Markov process. The mass of particle, in order to be sensed by spacetime, has to be collided randomly by STP. Mass cannot be well defined within the interval of two consecutive random collisions. In other words, mass is not a constant property belonging to the particle itself, but a discrete statistical property depending on dynamical collisions of spacetime. We will derive from MIP straightforwardly that mass must be a statistical term which has its own means and fluctuations.

Moreover, we prove the uncertainty relation asserting a fundamental limit to the precision regarding mass and diffusion coefficient. This implies that both mass and diffusion coefficient of any particle state can not simultaneously be exactly measured. Newton's Second law states that in an inertial reference frame, the vector sum of the forces F on an object is equal to the mass m of that object multiplied by the acceleration of the object. This connects the concept of mass and inertia and in principle defines a fundamental approach to measure the mass of any particle experimentally. However, according to the MIP, forces on a particle are changed constantly by the random impact of STP. Therefore, we are no longer able to take constant mass for granted. In conclusion, we believe that mass as a statistical property is much more natural within the framework of modern science, which completely overrules Newton's concept of mass based on Mathematical Principles of Natural Philosophy first published in 1687.

1.3 Realistic Interpretation of Quantum Mechanics

The main idea of Copenhagen interpretation is that the wave function does not have any real existence in addition to the abstract concept. In this article we do not deny the internal consistency of Copenhagen interpretation. We admit that Copenhagen′s quantum mechanics is a self-consistent theory. Einstein believed that for a complete physical theory, there must be such a requirement: a complete physical theory should include all of the physical reality, not merely its probable behaviour. From the materialistic point of view, the physical reality should be measured in principles , such as the position q and momentum p of particles. In the Copenhagen interpretation, the particle wave function $\Psi(q,t)$ or the momentum wave function $\Psi(p,t)$ is taken to be the only description of the physical system, which can not be called a complete physical theory, at most a phenomelogical effective theory. Therefore, in this paper, we propose a MIP where the coordinate and momentum of particles are objective reality irrespective of observations . With the postulation of MIP, quantum behaviour will emerge from a statistical description of spacetime random impacts on the experimental scale, including Schrödinger′s equation, Born rule, Heisenberg′s uncertainty principle and Feynman′s path integral formulation. Thus, we believe that non-relativistic quantum mechanics can be constructed under the MIP. Born rule and Heisenberg′s uncertainty relation are no longer fundamental within our framework.

1.4 MIP and Statistical Properties of Spin

In modern quantum field theory, the spin properties of particles reflect the transform properties of particles under relativistic Lorentz transformation. The spin is a representation of the Lorentz group. The algebraic representation theory, simplifies the mathematical definition of spin, however it hides the fundamental physical properties of spin. Within the framework of MIP, particle spin has a complete new origin. It is a topological order, which describes a topological phase transition between the two STP vortices living on Hodge dual 2+1 dimensional space-time, respectively.

Within the framework of MIP, a careful observation of properties near the singularity at the center of the STP vortex, drive us to a new perspective of particle spin. We noticed there are not only energy divergence at the singularity on the center of the STP vortex, there also exists a disorientation property for a direction vector. To describe the disorientation, we introduce the torsion based on the cotangent vielbein field. The torsion tensor actually drives the cobordism topological phase transition between STP vortices on tangent space and its dual normal space. By the cobordism topological phase transition, we combined vortices on the 2+1 dimensional tangent space and normal space into a 3+1 dimensional instanton. The cost of this cobordism topological phase transition, is to calculate the corresponding topological order. By cohomological theory, we calculated the incomplete angle due to the cobordism topological phase transition, which concludes that the incomplete angle is an integer times π,this angle contributes to the STP vortex around matter particle a factor $e^{iN\pi}$. When rotating the particle a circle, the factor changed the signature of the wave function. This inveals the origination of particle spin is a topological phase transtion between STP vortices around the matter particle. Within the framework of MIP, particle spin describes the topological order of this cobordism phase trantion of STP vortices.

1.5 MIP and Electromagnetic Theory

Within the framework of MIP, STP spread over spacetime, and its energy spectrum distribution is consistent with scalar particles. It can therefore be thought of as an excitation of a scalar field. The influence of material particles on its spacetime is local, so on the 2+1-dimensional time-space slice, the influence of material particles on spacetime can be regarded as a potential energy.

In modern quantum field theory, an important point is that microscopic energy can be non-conservative, and it can fluctuate to form pairs of virtual positive and negative particles. Within the framework of MIP, the fluctuation of spacetime energy is itself STP. The number of STP particles is not conserved locally, but globally, the energy of STP is conserved. So the picture of STP as a free particle is restored on a large scale. This just shows that STP has some local symmetry, which is broken at large scale. In essence, when the domain symmetry of the authority is $U(1)$, STP is the excitation of a complex scalar field.

On the other hand, the spacetime can be regarded as 2+1-dimensional around the spacetime in which the material particles are located. On this 2+1-dimensional spatiotemporal slice, STP is the excitation of the complex scalar field, which is accompanied by the excitation of the gauge field. The material particle produces a local non-perturbative potential energy in the surrounding space and time. The existence of this potential energy can cause the STP to spontaneously form a stable vortex solution. If the STP is not accompanied by a gauge field, then the vortex solution will cause the problem of energy divergence in the vortex center. The gauge field just eliminates the problem of local energy divergence.

The existence of a vortex solution also provides a possibility of duality, namely Hodge duality. The Hodge duality will extend the dynamics of the 2+1 dimensional gauge field to the 3+1 dimension. In the sense of Lagrangian, the 3+1-dimensional gauge field just describes the electromagnetic field theory. That is to say, the 3+1-dimensional equation of motion is Maxwell's equation. Therefore, we derive the classical electromagnetic theory from the vortex dynamics of STP.

In the MIP framework, the photon is essentially a topological excited state of two 2+1-dimensional gauge fields with their field strengths being Hodge's dual, and its topological configuration is a Hopf chain. Physically, photons transfer phase changes of material particles. Its equation of motion is the Maxwell equation.

On the other hand, the two topological circles of the photon, of which topological configuration Hopf link correspond to the topological subspace of the local spacetime. The Hopf links just represent the Lorentz representation of spin 1, which is a vector representation. Therefore, within the framework of MIP, the spin 1 of zero-mass photon is also self-consistently explained.

1.6 Muon anomalous magnetic moment under the MIP framework

On April 7, 2021, FermiLab performs a new muon anomalous magnetic moment experiment. The experimental value differs from the theoretical value predicted by the Standard Model with 4.2 σ standard deviation. The probability of this deviation comes from statistical fluctuations is 1 in 40000, which implies possible physics beyond the Standard Model. The new massless scalar STP required by the MIP is a key step beyond the existing standard model. Introducing only one parameter, the interaction strength between STP and lepton, not only perfectly solves the world-class problem of the anomalous magnetic moment of muons in the latest experiment, but also explains the muon lifetime discrepancy between theory and experiment. It can be seen that this is a triumph for applications of MIP in modern particle physics.

Last and most importantly, we derived the generation for charged leptons. This is a completely new result and one can not derive this law in current quantum field theory framework. Within the MIP framework, by invoking the STP vortices, the generation is a direct inference.

1.7 MIP and Special Relativity

From the MIP point of view, the state change of free material particles can only be achieved by the impact of STP. We can say that when the microscopic properties of a particle of matter (such as its phase or spin eigenstate) change, the particle that propagates the information is a gauge particle. From MIP, we have obtained the classical electromagnetic theory in section 9 of this paper. It is a theory that is invariant under the transformation of the inertial reference system. In particular, the speed of light as a constant does not change under the transformation of inertial reference frame. Therefore, the assumption that the speed of light does not change is no longer a hypothesis, but a basic law.

On the other hand, the interaction of STP on particles causes the particles to perform random fluctuations. The speed of this kind of fluctuation movement is very different from the classic speed. It is essentially a relative speed that is constant under a time reversal. This random Markov fluctuation is not related to the classical motion and is therefore invariant under the transformation of the inertial reference frame. Therefore, the equivalence between inertial reference systems is no longer an assumption, but a natural inference under the MIP framework. We can naturally derive some basic results from the special theory of relativity. Under the framework of MIP, the effects of ”mass enhancement”, ”time dilation” and ”length contraction” all have new physical meanings.

1.8 MIP and Newton’s Universal Gravity

In the 3+1 dimension Minkowski spacetime, STP is a real scalar field. We consider the interaction between STP and matter particles. With the tree diagram approximation of quantum field theory, the interaction among matter particles induced by STP is finally embodied as the inverse square law and proportional to the product of mass of matter particles. We show that Newton’s theory of gravity is an effective theory within the framework of MIP.

Furthermore, we can judge from the overall perspective of modern physics that the inertia mass of fermions must be equal to the gravitational mass. We have obtained the equivalent principle. Both inertial mass and gravitational mass are no longer the basic physical quantities. The two are indeed equivalent, which come from the statistical mass of STP collisions.

1.9 The principle of entropy in MIP framework

Starting from MIP and recalling the mathematical property of Markov random collision, we obtained the principle of entropy of non-interacting particle naturally. It is important to emphrase that in modern physics, the principle of entropy is still a empirical law. It does not have an explanation from the first principle. What we explained in this article, reveals the deep meaning of the principle of entropy. Most importantly, within the MIP framework, the principle of entropy originates from the statistical effect of random collision between matter particle and STPs. The collision naturally leads to the increasing entropy of matter particles. This principle is a cornerstone of modern physics, which is also irreplaceable.

1.10 Outline

In summary, MIP provides quantum mechanics, special and general relativity, electromagnetic theory, spin, electric charge, generation of leptons and entropy principle a materialistic basis, where an intuitive physical picture can be constructed. In this picture, mass is a statistical property which emerges when a large number of STP collisions occur. The spin represents the statistical properties of the interaction of the particle with the STP around it. Charges interact with each other through the topological excited state of gauge field in 2+1-dim and its Hodge dual partner, i.e. photon. Within the framework of MIP, Born's probability interpretation, Heisenberg's uncertainty principle no longer are basic principles, but only the natural consequences of MIP. The main results and conclusions of this paper are as follows:
In Section 2, we propose MIP and the fundamental definition 1, and obtain the energy spectrum distribution of the STP.

In Section 3, based on MIP from the general random motion process, we derive the universal spacetime diffusion coefficients of particles.

In Section 4, within the framework of MIP, we derive the spacetime diffusion coefficient and introduce a non-relativistic mass, which does not only describe the inertia, but also a statistical property. It is especially important that statistical inertial mass is a measure of how easily particles diffuse in spacetime, which is therefore a statistical property. It can be inferred that there is an uncertainty relationship between statistical inertial mass and spacetime diffusion coefficient, The most fundamental coordinate-momentum uncertainty relationship of quantum mechanics can be derived from this uncertainty relationship. Therefore, the wave-particle duality and the Heisenberg's uncertainty relationship are the characteristics of the STP colliding particles within the framework of MIP.

In Section 5, we point out that the motion of particles in spacetime is a Markov process, which will emerge as a quantum wave, which satisfies the Schr*ö*dinger equation. Within the framework of MIP, we reinterpret the Born's rule.

In Section 6, we reinterpret the Feynman path integral and construct a system of non-relativistic quantum mechanics from materialist epistemology. We demonstrate the principle of quantum mechanics and the compatibility of path integrals. Based on the path integral and MIP, we derive the path integral of the free particle, wave function in the potential field and the steady state wave function.

Section 7 of this paper provides an explanation of the quantum measurement within the framework of MIP. The effects of the measurement leading to wavepacket collapse can be well understood. EPR can also be well explained within the framework of MIP.

In Section 8, we consider the 2+1-dimensional dynamics of a complex scalar field. The existence of matter particles causes the STP to form a vortex structure solution around the material particles. The existence of this solution extends the 2+1 dimensional gauge field to the 3+1 dimension and derives Maxwell's equations. The photon appears as a Hopf chain solution of two three-dimensional selfdual gauge fields, which is a topological invariant configuration. We show that the existence of a spacetime ANOZ vortex solution can explain the origin of the charge and electromagnetic interaction. A most important result in this chapter is that we also derived the generation for leptons, which says in 3+1 dimensional space-time, there are at most three kinds of flavor for all leptons. This firstly resolves the flavor in modern physics is a natural result in MIP.

In Section 9, by introducing STP scalar field, we considered the 1-loop corrections by interaction between STPs and leptons in two important physics: one is the muon anomalous magnetic moment, the other is muon decay. The interaction between STPs and muon(electron) not only explains the difference between FNAL experiment and theory on muon anomalous magnetic moment , but also preicts a more precise muon lifetime.

In Section 10, by investigating the topological phase transition of two vortices on dual 2+1 dimensional spaces, we found the origination of particle spin within the framework of MIP. Particle spin is the topological order for the cobordism topologcial phase transition which concatenating the two 2+1 dimensional vortices into a 3+1 dimensional instanton.

In Section 11, we derive three important results of the special theory of relativity from MIP, which are mass enhancement, time dilation and length contraction. These effects can be self-consistently explained within the framework of MIP.

In Section 12 , we derive Newtonian gravitational forces between two matter particles from MIP.

In Section 13, we derive the principle of entropy of non-interacting particle from MIP.

Finally, in Section 14 we summarize and explore the future of research directions.

2 Mass Interaction Principle

2.1 Proposing the MIP

Particles moving in spacetime interact with STP. The generation of STP itself should be regarded as a microscopic random excitation of local spacetime energy. We can assume the following two self-consistent ideal STP models. First, the spacetime itself is discrete, and each of the smallest spacetime units can act on the particle to change the particle's motion . However this spacetime unit acts as a random force on the particles, the motion of the particles in spacetime under the action of STP will also be random. Secondly, the energy distribution of STP is Gaussian, therefore, when they were scattering with matter particle, the force is random.

Furthermore, we propose in each interaction between matter particle and STP, the exchanging action should be nh, with n integer and h the Planck constant. According to this, we can define the MIP accurately. Suppose STP begin to collide with matter particle at time t_1 and end it at at time t_2 to exchange energy E. Without the collision of STP, the action of particle at the same interval will be

$$S = \int_{t_1}^{t_2} E_0 dt \tag{2.1}$$

With the collision of STP, the action of particle at the same interval will be

$$S' = \int_{t_1}^{t_2} E(t) dt \tag{2.2}$$

Therefore the change of action in Definition 1 is

$$\delta S = S' - S = \int_{t_1}^{t_2} [E(t) - E_0] dt \equiv \int_{t_1}^{t_2} f(t) dt \tag{2.3}$$

By definition, integral function $f(t)$ is a monistic increasing function $f(t)$ with following property

$$f(t_1) = 0, f(t_2) = E \tag{2.4}$$

According to Mean value theorems for integrals, there exists one point t^* at the interval satisfying

$$\int_{t_1}^{t_2} f(t) dt = f(t^*)(t_2 - t_1) \tag{2.5}$$

Setting exchange of energy be $E^* = f(t^*)$ at this point, we have $0 < E^* < E$. So the exact formula of the change of action is

$$\delta S = E^* \delta t \tag{2.6}$$

where $\delta t \equiv t_2 - t_1$. Therefore we are sure that, it is this characteristic exchange energy E^* not the energy of STP itself corresponding to the change of action. With MIP $\delta S = nh$, it's impossible to interact instantaneously, since the exchange energy E^* will blow up.
In our MIP framework, there are no instant interactions between matter particle and STP, in other words, the interaction takes time to transfer the energy. If the scattering STP has an extremely low energy such that in Δt, the transfered action is less then h , we conclude that in Δt, the STP cannot collide the particle. We argue that such a collision is still in process, the particle as well as the STP are in a bound state, not a scattering state. This is similar to a completely inelastic collision in classic mechanics. While in such a process, the conservation of energy and momentum can not be satisfied simultaneously. Because of conservation of energy and momentum, the bound state actually is not a stable state. This observation leads to an important point: there exists a minimal energyE_{min} in Δt so that

$$E_{min}\Delta t = const. \tag{2.7}$$

In physics , the product of energy and time will have the dimension of action. It is natural to suggest such a constant with action dimension is the Planck constant, so we have

$$E_{min}\Delta t = nh, n \in Z. \tag{2.8}$$

At a certain moment, particle can be scattered by many STP with different momenta and energies. In Δt, we assume there are effectively N collisions. The state of the motion will depends on the net effect of the N times collision. This is a principle of superposition. We can use in total N vectors to superposite whole changes of the state of motion, which means if at time t the particle was at position $\vec{X}(t)$, with speed $\vec{V}_0$, then at the moment $t + \Delta t$, its position will be $\vec{x}(t + \Delta t) = \vec{X}(t) + \sum_{i=1}^{N} \Delta X_i$, and speed$\vec{V}_0 + \sum_{i=1}^{N} \Delta\vec{V}_i$. This simple analysis tells us in Δt, the ultimate state of motion of the particle can be separated as N different paths. This is the effect of separation of paths. While the weights of these paths, *aka* the probability distribution of universal diffusion, highly rely on the energy distribution of STP. Collisions by STP with different energies end up with different changes of the state of motion.

2.2 The Nature of Spacetime within the framework of MIP

At the beginning of the 20th century, the null result of the Michaelson-Morley experiment ended the ether theory. Within the framework of MIP, the concept of spacetime looks very similar to that of ether, but it is fundamentally different. To clarify this, let us first review the concept of ether. The ether is a gas medium filled in Newton's absolute static time and space. Its definition directly introduces a reference frame of God's perspective, which is Newton's static spacetime system. The earth and this frame of reference are relatively moving, so they will feel the ether wind blowing, which is the experimental basis of the Michaelson-Morley experiment. But spacetime is not a gaseous medium filled with absolute time and space. It is the fluctuation of time and space. From a large scale, the fluctuation of spacetime does not have significant effects. Spacetime seems to be smooth and differentiable, and the differential geometry theory of general relativity can effectively describe the physical properties of large-scale spacetime. However, on the microscopic scale, the fluctuation of spacetime indicates that spacetime itself does not have continuous property. There is no absolute static spacetime reference frame in the above discussion, so the STP within the framework of MIP is not etheric.

The null result of the Michaelson-Morley experiment actually promoted Einstein's most important hypothesis of the theory of relativity, which is the constant speed of light. In the theory of relativity, the constant speed of light is the only absolute assumption, and the relativity of all other speeds remains.

Within the framework of MIP, the energy fluctuation of spacetime forms STP. If you think of spacetime as a peaceful lake, then STP is the splash of water on the surface of the lake. When it falls on the surface of the lake, it will form ripples. Therefore, the emergence of STP is always accompanied by the spread of ripple. The propagation speed of ripple is the characteristic propagation speed in spacetime. Forming a STP means that fluctuation of spacetime will spread to a certain spatial distance within a certain period of time, so the spacetime around the STP is also changed. We now know that the smallest scale of time is the Planck scale, and the smallest scale of space is the Planck length. In the Planck time STP has to spread a Planck length of space, so the propagating speed of STP is the same as light speed.

From the spacetime view of MIP, any physical observable event in spacetime will inevitably accompany the fluctuation of spacetime energy, which will profoundly affect the spacetime after the event. Under such a view of spacetime, the current spacetime is actually the result of the joint influences of all events in the history.

2.3 Energy spectrum of STP

To consider the collision between STP and particle, it will be ambiguous if the energy spectrum of STP is not clear at first. In this subsection, we deal with this problem.

Let us consider a cubic with volume L^3, which we call a system. If there are in total N systems in spacetime, we can classify the N systems by states. We label a state by j so that there are N_j systems with energy E_j. The total energy of the ensemble(collection of N systems) is denoted as $\mathcal{E}$, we have

$$\begin{aligned} N &= \sum_j N_j & (2.9) \\ \mathcal{E} &= \sum_j N_j E_j, & (2.10) \end{aligned}$$

for constant $\mathcal{E}$ and N, the possible total number of states in whole spacetime will be $\Omega = \frac{N!}{\prod_j N_j!}$. Physical reality is required by the maximum of Ω. There is a distribution $\{N_j\}$ maximizing Ω, so that

$$\ln \Omega = N \ln N - N - \sum_j N_j \ln N_j + \sum_j N_j \cdots \tag{2.11}$$

the question is under constraints (2.9,2.10), how to maximize $\ln \Omega$. With the method of Lagrangian multiplier,

$$\frac{\partial \ln \Omega}{\partial N_j} - \lambda_1 \frac{\partial \sum_j N_j}{\partial N_j} - \lambda_2 \frac{\partial \left(\sum_j N_j E_j\right)}{\partial N_j} = 0 \tag{2.12}$$

we can derive

$$\begin{aligned} -\ln N_j - \lambda_1 - \lambda_2 E_j &= 1 \Rightarrow \\ N_j &= e^{-1-\lambda_1-\lambda_2 E_j} & (2.13) \end{aligned}$$

hence the probability of being at state j

$$P_j = \frac{N_j}{N} = \frac{e^{-\lambda_1-\lambda_2 E_j}}{\sum_j e^{-\lambda_1-\lambda_2 E_j}} = \frac{e^{-\lambda_2 E_j}}{\sum_j e^{-\lambda_2 E_j}} \equiv \frac{e^{-\lambda_2 E_j}}{\mathcal{Z}} \tag{2.14}$$

and the average energy of the ensemble

$$E = \frac{\mathcal{E}}{N} = \sum_j E_j P_j = -\frac{\partial}{\partial \lambda_2} \ln \mathcal{Z} \tag{2.15}$$

In L^3 , suppose there are $n_{\vec{p}} = 0, 1, 2, \cdots$ STP have momentum $\vec{p}$, for giving distribution $\{n_{\vec{p}}\}$, the energy in L^3 is

$$E = \sum_{\{n_{\vec{p}}\}} n_{\vec{p}} E_{\vec{p}} \tag{2.16}$$

with $E_{\vec{p}} = c|\vec{p}| = cp$. Here STP are massless as proposed. We have

$$\begin{aligned} \mathcal{Z} &= \sum_{\{n_{\vec{p}}\}} e^{-\lambda_2 E} = \prod_{\vec{p}} \left(1 + e^{-c\lambda_2 p} + e^{-2c\lambda_2 p} + \cdots\right) \\ &= \prod_{\vec{p}} \frac{1}{1 - e^{-c\lambda_2 p}} & (2.17) \end{aligned}$$

and the average energy of a system is

$$\begin{aligned} E &= -\frac{\partial}{\partial\lambda_2}\ln\mathcal{Z} = \frac{\partial}{\partial\lambda_2}\sum_{\vec{p}}\ln\left(1-e^{-c\lambda_2 p}\right) \\ &= \sum_{\vec{p}}\frac{pe^{-c\lambda_2 p}}{1-e^{-c\lambda_2 p}} = \sum_{\vec{p}}\frac{cp}{e^{c\lambda_2 p}-1} \end{aligned} \tag{2.18}$$

when $L\to\infty$, summation becomes integration as follow

$$\sum_{\vec{p}} \to \frac{L^3}{8\pi^3}\int d^3\vec{p}$$

from which we see

$$E = \frac{L^3}{2\pi^2}\int dp\frac{p^3}{e^{c\lambda_2 p}-1} = \frac{\pi^2 L^3}{30\lambda_2^4} \tag{2.19}$$

so the density of STP will be

$$\epsilon_{ST} = \frac{\pi^2}{30\lambda_2^4} \tag{2.20}$$

Recover c and $\hbar$ in above equation, we obtain

$$\epsilon_{ST} = \frac{\pi^2}{30c^3\hbar^3\lambda_2^4}. \tag{2.21}$$

Now consider the physical meaning of λ_2, which determines the constraint that represents energy distribution of STP. While the multiplier λ_1 which determines the constraint represents the number distribution of STP has no affects on the dynamics of STP. This means we can classify STP arbitrarily, except to satisfy the total energy constraint. For example, the action of particle changed $kh, k\in\mathbb{Z}$ in a certain collision by STP. In physics we can not distinct one STP collision or many STP collision, since neither from energy spectrum of STP nor from the change of status of the particle can distinct them. From dimensional analysis and MIP, we have

$$\lambda_2 = \frac{g}{E_{ST}} \tag{2.22}$$

where g is a dimensionless coupling constant, and E_{ST} is the characteristic energy of STP. In the limit of extreme relativity, the colliding of STP can not be seen as perturbations, but strong disturbances.

3 Random Motion and Spacetime Diffusion Coefficient

Let m_{ST} be the statistical mass of the particle . We will prove the spacetime interaction coefficient of a m_{ST} mass particle will be universally given as

$$\Re = \frac{h}{2m_{ST}}. \tag{3.1}$$

Within the framework of random motion[1], or Wiener process in mathematics [2], this spacetime induced random motion is equivalent to the Markov process, moreover, the spacetime interaction coefficient is nothing but the diffusion coeffient [3]. In this section, we will start our journey from propability theory of random motion[3, 4], and then give a concrete proof that for the random motion induced by MIP, the spacetime interaction coefficient is given exactly by (3.1). The last two subsections discussed two spacetime models in order to investigate the origin of the spacetime interaction coefficient. From both we obtained the coefficient reading as $\Re = \frac{w\ell}{2}$, in which w is the average speed of the particle and ℓ the mean free path.

3.1 Langevin Equation

The space-time background can be seen as a fluctuation environment, and the particles move in this fluctuation environment. This is a Markov process. The position of the particle $\vec{q}$ is a random quantity. From a strict mathematical point of view, it can be decomposed into a super random part and a superimposable function

$$\vec{q}(t) = \vec{q}_0(t) + \vec{\omega}(t) \tag{3.2}$$

where $\vec{q}_0(t)$ is the differential part of position and $\vec{\omega}(t)$ represents random motions of particles. The whole motion of particle can be described by Langevin equation as

$$\frac{\delta q_i(t)}{\delta t} = \frac{dq_{0,i}(t)}{dt} + \frac{\delta \omega_i(t)}{\delta t} = U_i(\mathbf{q}(t)) + \nu_i(t) \tag{3.3}$$

In spacetime, particles are subjected to the impact of STP. But if some of the impact is relatively weak, then the change of the state of motion can only be regarded as a perturbation. Under perturbation, the velocity of the particles changes which can be seen as smoothly and continuously. The non-perturbative impacts of STP on the particles instantaneously change the motion state of the particles, leading to the completely random motion. Each impact should be treated as a sum of a differential impact and a random impact. A microscopic impact does not change the classic trajectory of the particle, but it will cause the trajectory to be superimposed on the motion of an envelope. This is precisely the "differentiable velocity function" $\mathbf{U}(\mathbf{q}(t))$ expressed by the first term in the three velocities decomposition of the Langevin's equation. Therefore, the true velocity of the particle $\mathbf{V}(t)$ should contain three contributions, which is

$$\mathbf{V}(t) = \mathbf{v}(t) + \mathbf{u}(\mathbf{q}(t)) + \vec{\nu}(t) \tag{3.4}$$

Where $\mathbf{v}(t)$ is the classic statistical velocity, $\mathbf{u}(\mathbf{q}(t))$ is the quantum envelope velocity of the particle, and $\vec{\nu}(t)$ is the diffusion velocity representing random motion. $\mathbf{U}(\mathbf{q}(t))$ denotes the union of the first and the second term in eq.(3.4)

$$\mathbf{U}(\mathbf{q}(t)) = \mathbf{v}(t) + \mathbf{u}(\mathbf{q}(t)) \tag{3.5}$$

.

For a Markov process, the average contribution of white noise vanishes. However, because of its Gaussian nature, its variation is not zero. We have

$$\langle \nu_i \rangle_\nu = 0, \ \langle \nu_i(t)\nu_j(t')\rangle_\nu = \Omega\delta_{i,j}\delta(t-t'), \ t \geq t' \tag{3.6}$$

here the $\delta_{i,j}$ in the later equation can be obtained from the spacetime homogeneous property, while $\delta(t-t')$ is determined from the Markov property. For a Markov process, only conditions at the very moment determine the dynamics of the system, and all information from future or past are irrelevant. We can write down the basic correlation function by introducing a probability measure $[d\rho(\nu)]$,which is given as

$$[d\rho(\nu)] := \left(\sqrt{\frac{1}{2\pi\Omega\delta(t-t')}}\right)^D [d\nu] \exp\left(-\frac{1}{2\Omega}\int dt \sum_i \nu_i^2\right) \tag{3.7}$$

It is easy to see that

$$\langle \nu_i(t)\rangle_\nu \equiv \int \nu_i(t)[d\rho(\nu)] \quad = \quad 0 \tag{3.8}$$

$$\langle \nu_i(t)\nu_j(t')\rangle_\nu \equiv \int \nu_i(t)\nu_j(t')[d\rho[\nu]] \quad = \quad \Omega\delta_{i,j}\delta(t-t') \tag{3.9}$$

Here Ω describes the strength of spacetime interaction on the particle. Notice $\delta(t-t')$ has the inverse dimension of time t , as

$$\int_0^\infty \delta(t-t')dt = 1.$$

However, from the definition of measure (3.7), we can see, ν_i have the unit of m/s, so Ω will have the unit of m^2/s. From previous analysis, each collision leads to a change of an action h. h has the unit of angular momentum, $kg \cdot m^2/s$. From this we can define a quantity with mass unit, it is

$$m_{ST} \equiv \frac{h}{\Omega}. \tag{3.10}$$

The mass m_{ST} has the meaning such that it is the mass collided by STP and is a statistical property. Accordingly, the collision parameter $\Omega = \frac{h}{m_{ST}}$ reflects a physical realistic viewpoint: an object in our real nature, the larger its mass means the smaller its quantum effect.

Langevin equation generates a timedependent probability such that

$$\mathbf{P}[\mathbf{q}, t;\ \mathbf{q}', t'] = \langle \prod_{i=1}^{D} \delta[q_i(t) - q_i'(t')]\rangle_\nu, \ t \geq t' \tag{3.11}$$

which means for an operator $\mathcal{O}[\mathbf{q}]$, its average value at time t will be:

$$\langle \mathcal{O}[\mathbf{q}(t)]\rangle_\nu \equiv \int \mathbf{P}[\mathbf{q}, t;\ \mathbf{q}', t']\mathcal{O}[\mathbf{q}]d\mathbf{q} \tag{3.12}$$

Using the probability distribution (3.11), one can immediately verify equation (3.12). Actually, the distribution (3.11) can be seen as an evolution process, which says

$$\mathbf{P}[\mathbf{q}, t;\ \mathbf{q}', t'] = \iint q(t) e^{-(t-t')H(p,q)} q'(t') d^D p \tag{3.13}$$

here the evolution Hamiltonian is the famous Fokk-Planck Hamiltonian, as we will derive its formalism in next subsection.

3.2 Fokk-Planck Equation

Given the Langevin equation (3.3), we can derive the corresponding Fokk-Planck equation, as well as the Fokk-Planck Hamiltonian [3].

We consider the time segment from t to $t+\epsilon$, $\epsilon \to 0$, and have the Langevin equation as:

$$q_i(t+\epsilon) - q_i(t) = \epsilon U_i(\mathbf{q}(t)) + \int_t^{t+\epsilon} \nu_i(\tau) d\tau + O(\epsilon^2) \tag{3.14}$$

its related propability distribution is

$$\mathbf{P}[\mathbf{q}, t+\epsilon;\ \mathbf{q}', t] = \langle \delta(\mathbf{q} - \mathbf{q}(t+\epsilon)) \rangle_\nu \tag{3.15}$$

According MIP, everytime the STP collided with the particle, the action of particle will change nh, $n \in \mathbb{Z}$. To obtain the Fokk-Planck equation, we define following discreterization

$$\bar{\nu_i} \equiv \frac{1}{\sqrt{\epsilon}} \int_t^{t+\epsilon} \nu_i(\tau) d\tau \tag{3.16}$$

so that the discrete Langevin equation is

$$q_i(t+\epsilon) - q_i(t) = -\frac{1}{2}\epsilon f_i(\mathbf{q}(t)) + \sqrt{\epsilon}\bar{\nu_i} + O(\epsilon^2) \tag{3.17}$$

Notice here the time has been discreterized as

$$(t-t')/\epsilon \in \mathbb{Z}^+.$$

Now the Gaussian distribution and the property of Markov progcess determins the average value of discrete white noises ν_i, and we have

$$\langle \bar{\nu_i} \rangle_\nu = 0,\ \langle \bar{\nu_i}(t) \bar{\nu_j}(t') \rangle_\nu = \frac{\hbar}{m_{ST}} \delta_{i,j} \delta_{t,t'} \tag{3.18}$$

When $\epsilon \to 0$, the Fourier transformation of the probability distribution (3.15) is

$$\begin{aligned}
\tilde{\mathbf{P}}[\mathbf{p},t;\ \mathbf{q}',t']|_{t=t'+\epsilon} &= \int e^{-i\mathbf{p}\cdot\mathbf{q}}\mathbf{P}[\mathbf{q},t;\mathbf{q}',t']d^D\mathbf{q}|_{t=t'+\epsilon} \\
&= \langle e^{-i\mathbf{p}\cdot\mathbf{q}'(\mathbf{t}-\epsilon)}\rangle_\nu \\
&= \langle e^{-i\mathbf{p}\cdot(\mathbf{q}'(\mathbf{t})-\epsilon\frac{\delta\mathbf{q}'(\mathbf{t})}{\delta\mathbf{t}}-\mathbf{O}(\epsilon^2))}\rangle_\nu \\
&= \langle \exp(-i\mathbf{p}\cdot(\mathbf{q}'(\mathbf{t})-\epsilon\mathbf{U}(\mathbf{q}')))\rangle_\nu \\
&\quad \times \left\langle \exp\left[+i\mathbf{p}\,\boldsymbol{.}\int\limits_{t-\epsilon}^{t}\nu(\tau)d\tau\right]\right\rangle_\nu \times \left\langle \exp\left(O(\epsilon^2)\right)\right\rangle_\nu \\
&= \exp\left[-i\mathbf{p}\,\boldsymbol{.}\,(\mathbf{q}'-\epsilon\mathbf{U}(\mathbf{q}'))\right] \\
&\quad \times \left\langle \exp\left[+i\mathbf{p}\,\boldsymbol{.}\int\limits_{t-\epsilon}^{t}\nu(\tau)d\tau\right]\right\rangle_\nu
\end{aligned} \tag{3.19}$$

Notice that the last average value can be evaluated out by Gaussian integration, which reads,

$$\begin{aligned}
&\left(\sqrt{\frac{h}{2\pi}}\right)^D \int[d\nu]\exp\left(-\frac{m_{ST}}{2h}\int dt\sum_i^D\nu_i^2\right)\exp\left[+i\mathbf{p}\,\boldsymbol{.}\int\limits_{t-\epsilon}^{t}\nu(\tau)d\tau\right] \\
&= \left(\sqrt{\frac{h}{2\pi}}\right)^D \int[d\nu]\exp\left(-\frac{m_{ST}}{2h}\int dt\sum_i\nu_i^2+i\mathbf{p}\,\boldsymbol{.}\int\limits_{t-\epsilon}^{t}\nu(\tau)d\tau\right) \\
&= \left(\sqrt{\frac{h}{2\pi}}\right)^D \int[d\nu]\exp\left(-\frac{m_{ST}}{2h}\int dt\sum_i\nu_i^2+i\sqrt{\epsilon}\mathbf{p}\,\boldsymbol{.}\,\bar{\nu}\right) \\
&\qquad \times\exp\left(-\epsilon\frac{h}{2m_{ST}}\mathbf{p}\,\boldsymbol{.}\,\mathbf{p}+\epsilon\frac{h}{2m_{ST}}\mathbf{p}\,\boldsymbol{.}\,\mathbf{p}\right) \\
&= \left(\sqrt{\frac{h}{2\pi}}\right)^D \int[d^D\left(-\nu_i-\frac{ih}{2m_{ST}}\sqrt{\epsilon}p_i\right)] \\
&\times\exp\left(-\frac{m_{ST}}{2h}\int dt\sum_{i=1}^D\left(\nu_i+\sqrt{\epsilon}\frac{ih}{2m_{ST}}p_i\right)^2-\epsilon\frac{h}{2m_{ST}}\mathbf{p}\,\boldsymbol{.}\,\mathbf{p}\right) \\
&= \exp\left(-\epsilon h\mathbf{p}\,\boldsymbol{.}\,\mathbf{p}/(2m_{ST})\right)
\end{aligned} \tag{3.20}$$

here we obtain the probability distribution under Fourier transformation ,

$$\tilde{\mathbf{P}}[\mathbf{p},t+\epsilon;\ \mathbf{q}',t] = e^{-\epsilon h/2m_{ST}\mathbf{p}\cdot\mathbf{p}+i\epsilon\mathbf{p}.f(\mathbf{q}')/2-i\mathbf{p}\cdot\mathbf{q}'} \tag{3.21}$$

for $\epsilon \to 0$, expanding (3.21) will end up with

$$\tilde{\mathbf{P}}[\mathbf{p},t+\epsilon;\ \mathbf{q}',t] = e^{-i\mathbf{p}\cdot\mathbf{q}'}(1-\epsilon H_{FP}(\mathbf{p},\mathbf{q}')+O(\epsilon^2)).$$

Here we obtained the Fokk-Planck Hamiltonian

$$H_{FP}(\mathbf{p},\mathbf{q}) = -\frac{h}{2m_{ST}}\mathbf{p}\cdot\mathbf{p} - i\mathbf{p}\cdot f(\mathbf{q})/2 \tag{3.22}$$

From which we can read off the diffusion coefficient induced by collisions between STP and the particle, is exactly $\Re = h/2m_{ST}$. Later we will see in deriving the Schrödinger equation of free particle in spacetime, the spacetime mass $m_{ST} = 2\pi m$ will be identified as the inertial mass, in the framework of non-relativistic quantum mechanics.

3.3 From spacetime scattering to spacetime diffusion coefficient

3.3.1 From Discrete Spacetime to the Spacetime Diffusion Coefficient

Beginning with MIP, we want to investigate the origin of spacetime interaction coefficient. Within the framework of discrete spacetime, spacetime diffusion coefficient $\Re = \frac{h}{2m_{ST}}$ should be derived in terms of parameters of discrete spacetime. Let us consider the simplest discrete model (see Fig.3.1), where the length union of discrete space is ℓ. $P(j,t)$ is the probability of a particle at lattice site j at time t.

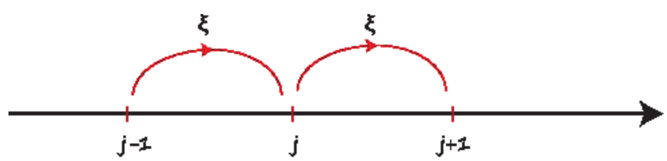

Fig. 3.1: Random jumping model on one dimensional lattice

Because of the discrete nature of the space, all jumpings can only happen between nearest pair of positions. Given the rate of jumping between the nearest neighbour ζ and the isotropy of frictionless space, the evolution of probability should be

$$\partial_t P(j,t) = \zeta(\frac{1}{2}P(j-1,t) + \frac{1}{2}P(j+1,t) - P(j,t)) \tag{3.23}$$

the first two terms of RHS of (3.23) describe the fact that jumping forward and backward from neighbors $j-1$ and $j+1$ positions respectively, have the same probability, which is 1/2, the third term remarks the probability from j position to neighbors. Introducing the fundamental spacing of the lattice ℓ, the eq.(3.23) goes to

$$\partial_t P(j,t) = \frac{\zeta\ell^2}{2}(\frac{\frac{P(j+1,t)-P(j,t)}{\ell} - \frac{P(j,t)-P(j-1,t)}{\ell}}{\ell}) \tag{3.24}$$

In the continum limit of spacetime, which says $\ell \to 0$, and $\zeta \to \infty$, but keeping the quantity $\zeta\ell^2$ unchanged, the probability $P(j,t)$ now becomes the probability density $\rho(x,t)$, and the RHS of (3.23) becomes the definition of second derivative. Thus we have

$$\partial_t \rho(x,t) = \frac{\zeta\ell^2}{2}\partial_x^2 \rho(x,t). \tag{3.25}$$

It is straightforward to generalise above equation to three dimension case, we have,

$$\partial_t \rho(\vec{r},t) = \frac{\zeta\ell^2}{2}\nabla^2 \rho(\vec{r},t) \tag{3.26}$$

Comparing with diffusion equation in Einstein's paper[6]

$$\partial_t \rho(\vec{r},t) = \Re\nabla^2 \rho(\vec{r},t) \tag{3.27}$$

the microscopic origin of spacetime diffusion coefficient will be

$$\Re = \frac{\zeta\ell^2}{2} \tag{3.28}$$

Furthermore, we can also discrete time with union $\tau = \frac{\ell}{w}$, where w is the average speed of particle. With $\zeta = \frac{1}{\tau}$, we obtain

$$\Re = \frac{w\ell}{2} \tag{3.29}$$

Combining the microscopic structure of discrete spacetime with the MIP, we have

$$\Re = \frac{w\ell}{2} = \frac{h}{2m_{ST}} \tag{3.30}$$

3.3.2 From Spacetime Scattering to the Spacetime Diffusion Coefficient

Particles will be scattered randomly from the STP with the speed of light, which leads to the probability distribution of speed $f(\vec{v})$, the number of partials within $v \to v + dv$ is $f(v)d^3\vec{v}$. Therefore, all the particles cross the section area dA during time dt will be inside the cylinder (see Fig.3.2).

The volume of this cylinder is

$$V = vdt\cos\theta dA \tag{3.31}$$

in which the number of particles is

$$N = f(\vec{v})d^3\vec{v}vdt\cos\theta dA \tag{3.32}$$

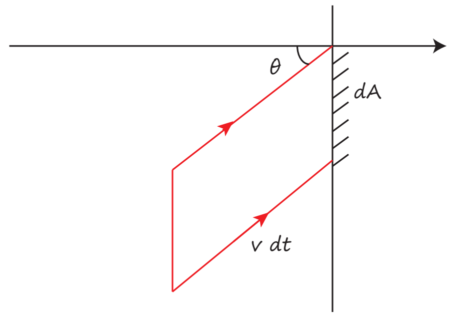

Fig. 3.2: Probability distribution of spacetime scattering

Because of the isotropy of space, we have $f(\vec{v}) = f(v)$. From left to right, the number of particle cross the unit area per unit time is

$$\begin{aligned}\Phi &= \int\limits_{v_z>0} \frac{N}{dAdt} \\ &= \int\limits_0^{\frac{\pi}{2}} d\theta \cos\theta \sin\theta \int\limits_0^{2\pi} d\varphi \int\limits_0^{+\infty} f(v)v^3 dv \\ &= \pi \int\limits_0^{+\infty} f(v)v^3 dv \end{aligned} \tag{3.33}$$

where $v_z > 0$ means $0 < \theta < \frac{\pi}{2}$. The average speed reads

$$w = \frac{\int_0^{+\infty} f(v) v d^3 v}{\int_0^{+\infty} f(v) d^3 v} = \frac{4\pi}{\rho} \int\limits_0^{+\infty} f(v)v^3 dv \tag{3.34}$$

where the density of particle number is $\rho = \int_0^{+\infty} f(v)d^3v$. Correspondingly, the number of particle cross the unit area per unit time will be

$$\Phi = \frac{1}{4}\rho w \tag{3.35}$$

Let mean free path of particles be ℓ, i.e. the average distance traveled by the particle between successive impacts from spacetime. The net flux J_z through the z plane will be (see Fig.3.3)

$$J_z = \frac{1}{4}\rho(z-\ell)w - \frac{1}{4}\rho(z+\ell)w = -\frac{1}{2}\ell w \partial_z \rho \tag{3.36}$$

With the equation of continuity

$$\partial_t \rho + \nabla \cdot \vec{J} = 0 \tag{3.37}$$

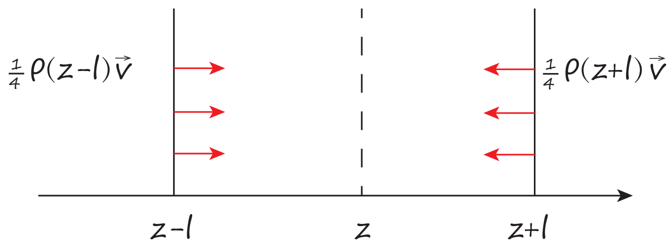

Fig. 3.3: mean free path and scattering flux

and the isotropy of space, we have

$$\partial_t \rho = \frac{1}{2}\ell w \nabla^2 \rho \tag{3.38}$$

Combining the kinetics of spacetime scattering with quantum nature induced by STP, we obtain

$$\Re = \frac{w\ell}{2} = \frac{h}{2m_{ST}} \tag{3.39}$$

which is consistent with eq.(3.30).

3.4 Statistical mass of fundamental particles

Let's consider the electron at first. The mass of an electron is $m_e = 9.104 \times 10^{-31} kg$. So its static energy is

$$E_e = m_e c^2 = 9.104 \times 10^{-31} \times 9 \times 10^{18} J = 8.1936 \times 10^{-12} J$$

This energy, according to MIP, comes from "effective" collisions between STP and the electron. In our MIP theory, the electron is not a point-like particle. It is finite size, statistically. Because of symmetry, its shape is a ball with a sphere boundary. The effective collisions are considered as the number of STP which coming into and going out cross the sphere. Assume every effective collision gives energy, which numerically equals to Planck constant. Hence the times of effective collisons (TEC) can be calculated as follow

$$N_e = E_e/h = 1.2347 \times 10^{20}[s^{-1}]$$

The statistical mass of electron can be written in form of TEC

$$m_e = \frac{h}{c^2} N_e \tag{3.40}$$

The ratio of mass and TEC is

$$k_{st} \equiv \frac{h}{c^2} = 7.37 \times 10^{-51} kg \cdot s \tag{3.41}$$

It has the unit of $[mass] \cdot [time]$. The fluctuation of the density of STP, around the electron, denoted as $\Delta\rho^e_{st}$, can be written as

$$\Delta\rho^e_{st} \equiv \rho^e - \rho_0 = \frac{m_e c^2}{\frac{4}{3}\pi r^3 h} \tag{3.42}$$

For proton, it is easy to calculate exactly the same as the electron, we have

$$N_p = \frac{m_p}{k_{st}} = 1.6726 \times 10^{-27}/7.37 \times 10^{-51} \simeq 2.227 \times 10^{23}[s^{-1}] \tag{3.43}$$

The radius of proton is

$$r_p \simeq 8.735 \times 10^{-16} m \tag{3.44}$$

from which we obtain the mean free path of a proton in the STP sea around it.

$$l_{st} = \sqrt[3]{\frac{4}{3}\pi r_p^3 / N_p} \simeq 2.3 \times 10^{-23} m$$

3.5 Momentum and energy within the framework of MIP

The time scale of physics spans many orders of magnitude. Cosmology studies the age of the universe at about 4×10^{17} seconds. Newtonian mechanics studies the low-velocity motion of macroscopic objects, and the time scale is usually on the order of seconds. The basic system of quantum mechanics is a hydrogen atom. When the electrons outside the hydrogen nucleus are in the ground state, the electrons move around the nucleus for about 1.5×10^{-15} seconds. The first excited state of the hydrogen atom transitions to the ground state emitting light with a wavelength of 121 nm, corresponding to a time period of 4×10^{-16} seconds. Modern physics believes that considering the principles of general relativity, special relativity and quantum mechanics, the smallest physical time scale is Planck time about 5×10^{-44} seconds, which is the smallest measurable time interval. According to academic consensus today, any changes during this time interval cannot be measured or detected.

Under the MIP framework, the average number of STP hitting electrons within one second is 10^{20}. That is to say, the theory derived from MIP in this paper has a typical time scale of 10^{-20} seconds. For electron, this time scale is 10,000 times shorter than quantum mechanics[1]. Therefore, energy conservation and momentum conservation in quantum mechanics are not constant conservation laws, but statistical average conservation under the MIP framework. The momentum and energy we define below are the results of statistically averaging the random effects of STP.

[1] In the field of particle physics, short lifetime such as the Higgs boson is about 1.5×10^{-22} seconds. For the Higgs boson, the average number of STP hitting a Higgs particle in a second is 10^{25} times. Its typical time scale is a thousand times smaller than quantum field theory.

In the time interval of 10^{-20} seconds, we call the momentum of particle [2] as instant momentum. According to MIP, instant momentum is defined as

$$\vec{P_i} = m_i \vec{V} \tag{3.45}$$

Where m_i is the mass of the particles in the time interval of 10^{-20}seconds, which we call as instant mass. $\vec{V}$ is the true velocity of the particle

$$\vec{V} = \vec{u} + \vec{v} + \vec{\nu} \tag{3.46}$$

Similarly, we define the instant kinetic energy of the particle as

$$E_i = \frac{1}{2} m_i V^2 \tag{3.47}$$

The mass observed in modern physical experiments is the statistical mass of the particles, which is the inertial property at intervals greater than $\times 10^{-16}$ seconds. The momentum observed in modern physical experiments is the momentum predicted by quantum mechanics. Quantum mechanical momentum is the statistical average of instant momentum, which we call statistical momentum:

$$\vec{P_s} =< \vec{P_i} >= \frac{M_{st}}{2\pi} < \vec{v} + \vec{u} > \tag{3.48}$$

From this we relate the instant momentum at small time scales to the quantum mechanical momentum at large time scales. There is an important observation which we have proved in Chapter 5. The classical statistical velocity of any stationary state (the ground state is the lowest energy stationary state) is $\vec{v} = 0$, and the quantum envelop velocity of the ground state electrons of hydrogen atoms is

$$\vec{u} = -\frac{c}{137} \hat{r} \tag{3.49}$$

Comparing the results of quantum mechanics: the momentum of the ground state electrons of a hydrogen atom must be zero, satisfying the isotropic wave function. Subtlely, the quantum envelope velocity does not contribute to the momentum of the ground state electrons because isotropic offsets each other by $< \vec{u} >= 0$. Because quantum mechanics is the combined result of statistical averaging three velocities and instant mass on large time scales, $\vec{P_s}$ is consistent with the momentum calculated by quantum mechanics.

The kinetic energy observed in modern physical experiments is the kinetic energy predicted by quantum mechanics theory. Quantum mechanical kinetic energy is the statistical average of instant kinetic energy, which we call statistical kinetic energy.

$$E_s =< E_i >= \frac{M_{st}}{4\pi} < V^2 > \tag{3.50}$$

[2] In the discussion below, the particles are all specific to electrons and represent the particles of matter.

The quantum envelop velocity contributes to the kinetic energy of the ground state electrons (always positive so cannot cancel out). Therefore, the energy of the ground state electron has two parts (the classical statistical velocity is always 0, and does not contribute to the ground state kinetic energy):

ground state energy = quantum envelop energy + coulomb potential

The calculated result is exactly -13.6 ev, which is also consistent with the energy calculated by quantum mechanics. The quantum envelop kinetic energy is defined as

$$E_e = \frac{1}{4\pi} M_{st} u^2 \tag{3.51}$$

Substituting the value of the electron energy of the ground state of a hydrogen atom

$$E = \frac{M_{st}}{4\pi} < (\frac{c}{137})^2 > + < -\frac{e^2}{4\pi\epsilon_0} a >= -13.6ev \tag{3.52}$$

Where a is the Bohr radius of the hydrogen atom and ϵ_0 is the vacuum permittivity. Thus, we obtain the definitions of momentum and kinetic energy that are consistent with quantum mechanics.

More generally, the equivalence between statistical momentum and quantum mechanical momentum in any quantum state are proved as follows. According to the Ehrenfest theorem of quantum mechanics, the average value of particle positions evolves with time as

$$\frac{d}{dt}\langle\vec{x}\rangle = \frac{1}{i\hbar}\langle[\vec{x},\ H]\rangle = \frac{1}{i2m\hbar}\langle[\vec{x},\ p^2]\rangle = \frac{1}{i2m\hbar}\langle\vec{x}pp - pp\vec{x}\rangle \tag{3.53}$$

Combining with $\vec{x}pp - pp\vec{x} = i2\hbar\vec{p}$, we have

$$\frac{d}{dt}\langle\vec{x}\rangle = \frac{1}{m}\langle\vec{p}\rangle \tag{3.54}$$

This is a very important result, indicating how the momentum average of quantum mechanics is related to the mean value of the coordinates. In the MIP framework, the derivative of coordinates versus time is defined as

$$\frac{d}{dt}\vec{x} = \vec{u} + \vec{v} \tag{3.55}$$

Once two sides of the equation are averaged, the momentum average of quantum mechanics corresponds to the statistical momentum of the MIP as

$$\vec{P}_s =< \vec{P}_i >= \frac{M_{st}}{2\pi} < \vec{v} + \vec{u} > \tag{3.56}$$

which proves that the microscopic theoretical basis of quantum mechanics is exactly MIP.

4 Mass-Diffusion Uncertainty relation

We now consider the motion status of particle under impacts of STP collisions. The most important proposition of Copenhagen interperitation of quantum mechanics is the wave-particle duality. This allows one using the superposition rule of plane waves to describe the state of a particle. The kernel of the wave transformation from frequency space to time space will be the factor $\exp(ipx/\hbar)$. In fact it introduces the quantized operator formalism $\vec{p} = -i\hbar\vec{\nabla}$. Because of the duality, physical quantities of the particle can also be derived from wave, which implies some quantities can be described in phase space as eigenvalues of special operators. However, under the framework of MIP, we need to emphrase again that the wave-like property of the particle is an emergent property due to collision of STP, therefore it is not intrinsic. We can not borrow the quantization hypothesis directly. We consider the action of the particle

$$\begin{aligned} &S[\phi(t,x), \partial\phi(t,x), \bar{\nu}(t,x)] \\ &\qquad = S_0[\phi(t,x), \partial\phi(t,x)] + \sum_{I=1}^{\infty} S_I[\bar{\nu}(t,x)] \end{aligned} \tag{4.1}$$

where $\phi(t,x)$ describing the classical trajectory of the particle, and S_0 is the related classical action. $S_I[\bar{\nu}(t,x)]$ is the contribution of $I-th$ collision between STP and the particle. It does not depend on the classical trajectory at all, which only depends on the fluctuation of STP. The MIP said this term should contribute integer number of h , that is $S_I = nh$.

The partition function of the particle now is

$$Z = \int [d\phi(t,x)] \exp(-\frac{i}{\hbar} S[\phi(t,x), \partial\phi(t,x), \bar{\nu}(t,x)]) \tag{4.2}$$

hence

$$\exp\left(-\frac{i}{\hbar} S_I[\bar{\nu}]\right) = \exp\left(-\frac{i}{\hbar} nh\right) = e^{-i2\pi n} = 1 \tag{4.3}$$

from which we see the introducing of MIP does not change the classical partition function, therefore physical quantity derived from classical action will not be affected.

4.1 Mass-Diffusion Uncertainty

We have claimed and proven that particle mass is a statistical property describing the diffusion ability of the particle in spacetime, ,which shows that mass and diffusion coefficient are indeed statistical properties, under continuous interaction of STP. However, MIP itself describes a special Markov process, which possesses the intrinsic characteristic property of being quantized.

Firstly, we will proof that within framework of MIP, the particle mass and the diffusion coeffient in spacetime are not only statistical conjuation to each other, but also satisfying the minimum uncertainty relation:

$$\Delta m \Delta \Re = h/2 \tag{4.4}$$

4.2 Instantaneous statistical inertia mass

In this article, mass reflects the statistical propterty of the motion of matter particle, which is drived by collisions of STPs with the particle. As a statistical physical quantity, its instantaneous value does not have an explicit meaning in physics. We do not know how to measure the collision of a single collision between one STP and the particle exactly. In the other way, when we consider the relation between collision and the spectrum of STPs, we had already proven the number of STPs can not be determinate accurately. Hence even for a single collision between STP and the particle, the mass of the particle is also a statistical property. With this point of view, the statistical mass can be defined instanteously. In Minkowski spacetime, the distribution of STPs is uniform and isotropic. The instantaneous mass of matter particle will be changed according to the speed of particle. Though the instantaneous mass of particle $\hat{m}$, varying every moment, when taking the mean of speeds of the particle, will regress to the statistical inertia mass m_{ST}.

Because the exchanged action relating to every single collision is not the same, neither the energy of the STP in this collision. The time interval that accomplishing the exchanging of action, is also different in every collision. We know, as a reflection of the collision between STP and matter particle, the motion of particle will deviate from its classical velocity. The noise part $\vec{\nu}$ describes the deviation cause by the collision between STP and the particle. The bigger the noise is, the smaller the statistical inertia mass m_{ST} is. In another way, a bigger deviation means the particle can diffuse in spacetime easier, thus it corresponds to a bigger spacetime diffusion coefficient $\Re$. In the moment of measurement, because of the existence of noise, the instantaneous mass of the particle will not be exact as m_{ST}. We know

$$\Delta m = \hat{m} - m_{ST}$$

The instantaneous mass corresponds to every measurement does not have any real physical meaning. The standard deviation of many times of measurement results is what we care about, it is

$$\sigma(m) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{m}_i - m_{ST}\right)^2} \tag{4.5}$$

With the same reason, we only care about the standard deviation of spacetime diffusion coefficients of every measurement

$$\sigma(\Re) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\Re}_i - \Re\right)^2} \tag{4.6}$$

The relative difference of this two statistical quantity can be represented as the covariance, as

$$cov(m,\Re) = \frac{\sum_{i=1}^{N}\left(\hat{m}_i - m_{ST}\right)\left(\hat{\Re}_i - \Re\right)}{N\sigma(m)\sigma(\Re)} \tag{4.7}$$

Since the noise of STP is a white noise, its standard deviation is a constant, so we can normalize its magnitude as 1.

Notice that when $N \to \infty$,

$$\begin{aligned} cov(m,\Re) &= \lim_{N\to\infty}\frac{\sum_{i=1}^{N}\left(\hat{m}_i - m_{ST}\right)\left(\hat{\Re}_i - \Re\right)}{N} \\ &\equiv \langle \Delta m \Delta \Re \rangle \end{aligned} \tag{4.8}$$

which is the LHS of the uncertainty relation expression as we claimed in (4.4). The following task is to calculate its explicit value.

We now cut the time into slides along the classical velocity of the particle. On each time slide, we only need to consider the collision of STPs parallel to the time slide. Defining the time interval for the cutting as $\delta\tau$. the instantaneous mass at the moment i could be defined as follows: from the moment $i-1$ to i, the action changing causing by STP collisions is $\Delta S_i = S_i - S_{i-1}$; Meanwhile the diffusion area is $\hat{\Re}_i$. The instantaneous mass is

$$\hat{m}_i \equiv \frac{\Delta S_i}{\hat{\Re}_i} \tag{4.9}$$

To varifying the (4.9) matches the statistical definition as in previous chapter, we need to reform the changing of action as the changing of motion status of the particle, it is

$$\Delta S_i = \frac{1}{4\pi} m_{ST}(V_i^2 - V_{i-1}^2)\delta\tau \tag{4.10}$$

hereV_i and V_{i-1}represent real velocities at moment i and $i-1$. Because there is no changing of classical velocity from moment $i-1$ to moment i , meanwhile the differentiable part of the collision, aka the quantum envelope velocity is also a slow varying quantity, so it could be seen as unchanged in this time interval. Thus all changing of the velocity is contributed from the STP noise. In classical situation, the previous equation could be written as

$$\begin{aligned}\Delta S_i &= \frac{1}{2}m(V_i^2 - V_{i-1}^2)\delta\tau \\ &= \frac{1}{2}m\left((V_{i-1}+\nu_i)^2 - V_{i-1}^2\right)\delta\tau \\ &= \frac{1}{2}m\left(\nu_i^2 + 2V_{i-1}\nu_i\right)\delta\tau \end{aligned} \tag{4.11}$$

Taking the mean value of this equation, we obtain

$$\begin{aligned}\langle\sum_i \Delta S_i\rangle_\nu &= \langle\int \frac{1}{2}m\left(\nu_i^2 + 2V_{i-1}\nu_i\right)dt\rangle_\nu \\ &= \hbar/4\end{aligned} \tag{4.12}$$

However, it is notable that the changing caused by STP collisions is not a classical kinetic variation, we need to consider the special relativety effect as well. In rest frame of classical velocity, the particle energy is

$$E = mc^2$$

In static observer frame, its energy is

$$E_0 = \frac{m_0c^2}{\sqrt{1 - V^2/c^2}} \tag{4.13}$$

Therefore we obtain

$$\begin{aligned}\Delta S_i &= \left[\frac{m_0c^2}{\sqrt{1-\frac{V_i^2}{c^2}}} - \frac{m_0c^2}{\sqrt{1-\frac{V_{i-1}^2}{c^2}}}\right]\frac{\delta\tau_0}{\sqrt{1-\frac{V_{i-1}^2}{c^2}}} \\ &= \frac{m_0c^2\delta\tau_0}{\sqrt{\left(1-\frac{(V_{i-1}+\nu_i)^2}{c^2}\right)\left(1-\frac{V_{i-1}^2}{c^2}\right)}} - \frac{m_0c^2\delta\tau_0}{\left(1-\frac{V_{i-1}^2}{c^2}\right)} \\ &= \frac{m_0c^2\delta\tau_0\left(\sqrt{\left(1-\frac{V_{i-1}^2}{c^2}\right)} - \sqrt{\left(1-\frac{(V_{i-1}+\nu_i)^2}{c^2}\right)}\right)}{\left(1-\frac{V_{i-1}^2}{c^2}\right)\sqrt{\left(1-\frac{(V_{i-1}+\nu_i)^2}{c^2}\right)}}\end{aligned} \tag{4.14}$$

especially, in above equation, we used the special relativity transformation that

$$m_i = \frac{m_0}{\sqrt{1-\frac{V_i^2}{c^2}}} \tag{4.15}$$

Because the changing of action from $i-1-th$ to $i-th$ time slide is a Lorentz scalar. We can take the $i-1-th$ slide as the rest frame with mass m_{i-1}, the

$i-th$ slide represents the frame with velocity ν_i. Therefore, we change the equation (4.14) as

$$\Delta S_i = \left(\frac{m_{i-1}c^2}{\sqrt{1-\nu_i^2/c^2}} - m_{i-1}c^2\right)\delta\tau_i \tag{4.16}$$
$$= \left(\frac{1}{2}m_{i-1}\nu_i^2 + \frac{3}{8}(\nu_i^2/c^2)^2 c^2 m_{i-1} + \cdots\right)\delta\tau_i$$

Taking mean value of the above, we obtain

$$\langle\left(\frac{1}{2}m_{i-1}\nu_i^2 + \frac{3}{8}(\nu_i^2/c^2)^2 c^2 m_{i-1} + \cdots\right)\delta\tau_i\rangle_\nu$$
$$= \frac{\hbar}{4} + \frac{3\hbar^2}{32c^2 m_{i-1}\delta\tau_i} + \frac{5\hbar^3}{256c^4 m_{i-1}^2\delta\tau_i^2}\cdots \tag{4.17}$$

When the cutting interval goes to the classical limit, say, $\delta\tau_i \gg 0$, and the number $\hbar/c$ is very small, we have:

$$\langle\hat{m}_i\hat{\Re}_i\rangle_\nu \simeq \frac{\hbar}{4} \tag{4.18}$$

It means at arbitrary time slide, the mean value of the product of instantaneous mass and diffusion coefficient is $\frac{\hbar}{4}$.

From the definition of statistical inertia mass $m_{ST}$and diffusion coefficient $\Re$, we have:

$$\Re \equiv \sum_{i=1}^{N}\hat{\Re}_i/N \tag{4.19}$$

$$m_{ST} \equiv 2\pi\sum_{i=1}^{N}\hat{m}_i/N \tag{4.20}$$

It will not change the essence of the relation

$$\langle m_{ST}\Re\rangle_\nu = \frac{h}{2}$$

This is because

$$\langle m_{ST}\Re\rangle_\nu = 2\pi\langle\sum_{i=1}^{N}\hat{m}_i/N\sum_{j=1}^{N}\hat{\Re}_j/N\rangle_\nu$$
$$= 2\pi\left[\sum_{i=j}^{N}\frac{\langle\hat{m}_i\hat{\Re}_i\rangle_\nu}{N^2} + \sum_{i\neq j}^{N}\frac{\langle\hat{m}_i\hat{\Re}_j\rangle_\nu}{N^2}\right]$$
$$= \frac{h}{4N} + 2\pi\frac{\sum_{i=1}^{N}\langle\hat{m}_i\rangle\sum_{j\neq i}^{N}\langle\hat{\Re}_j\rangle}{N^2} + \mathcal{O}(\frac{h^2}{c^2N})$$
$$= \frac{h}{4N} + \frac{N-1}{N}\frac{h}{2} + \mathcal{O}(\frac{h^2}{c^2N}) = \frac{h}{2} - \frac{h}{4N} - \mathcal{O}(\frac{h^2}{c^2N}) \tag{4.21}$$

when $N \to \infty$, $\langle m_{ST}\Re\rangle_\nu = \frac{h}{2}$. Therefore we know the time cutting definition and the statistical definition is coincident with each other.

Now we can calculate the covariance as following

$$cov(m, \Re) = \lim_{N\to\infty} \frac{\sum_{i=1}^N \Delta S_i - m_{ST}\sum_{i=1}^N \hat{\Re}_i - \Re \sum_{i=1}^N \hat{m}_i}{N} + h/2 \tag{4.22}$$

and we obtain:

$$cov(m, \Re) = \lim_{N\to\infty} \frac{\sum_{i=1}^N \Delta S_i}{N} - h/2 \tag{4.23}$$

From MIP, the changing of action caused by STP collsion is N times of Planck constant, where N is an arbitrary integer, when the number of collisions goes to infinity, it is obvious that

$$\lim_{N\to\infty} \frac{\sum_{i=1}^N \Delta S_i}{N} = \lim_{N\to\infty} \frac{\hbar/4}{N} = 0 \tag{4.24}$$

at last we obtain

$$\langle \Delta m \Delta \Re \rangle = h/2 \tag{4.25}$$

and the proof is closed.

4.3 Position-Momentum Uncertainty Relation

Extending the definition of commutation relation, and recall $m = m_{ST}/2\pi$, we consider the position-momentum commutator

$$\begin{aligned}[x,p] &= \lim_{\epsilon\to 0}\left(\frac{1}{2\pi}x(t+i\epsilon)m_{ST}\frac{\delta x(t)}{\delta t} - \frac{1}{2\pi}m_{ST}\frac{\delta x(t+i\epsilon)}{\delta t}x(t)\right) \\ &= \lim_{\epsilon\to 0}\left(i\frac{\epsilon}{2\pi}\left[m_{ST}\left(\frac{\delta x(t)}{\delta t}\right)^2 - m_{ST}\frac{\delta^2 x(t)}{\delta t^2}x(t)\right]\right)\end{aligned} \tag{4.26}$$

Here we didn't take the statistical inertia mass as a variable, because when considering the changing of the particle's position caused by STP collisions, its statistical property is unchanged. Noticed that in our derivation, the momentum and position both have its instantaneous value. However, the two measurements are not isochronous in priori. Our isochrony is essencially different from what in quantum mechanism. Here since there exist collisions between STPs and matter particle, any two measurements can not be exactly isochronous. We let the time interval ϵ goes to zero to achieve an isochronous commutation relation in posteriori.

Define

$$a_{ST}(t) := \frac{\partial^2 x(t)}{\partial t^2} \tag{4.27}$$

It is the instantaneous accelaration induced by the collison between STP and the particle. From which we can define the instantaneous "spacetime" force as

$$F_{ST}(t) = ma_{ST}(t) = m\frac{\delta^2 x(t)}{\delta t^2} \tag{4.28}$$

The statistical average of eq.(4.26) is

$$[x, p] = \lim_{\epsilon \to 0} \left(m\langle V(t)^2 \rangle_\nu i\epsilon - \langle F_{ST}(t)x(t) \rangle_\nu i\epsilon \right) \tag{4.29}$$

Its second term has an explicit meaning in physics. It is the the mean work done by STP acting on the particle. Obviously, this mean work is zero.

Now we consider the contribution from the first term of eq.(4.29) Under discretization of the fluctuation, the average speed is

$$\int_t^{t+\epsilon} \nu(\tau) d\tau / \epsilon = \bar{\nu}/\sqrt{\epsilon}$$

therefore

$$\langle \nu^2 \rangle_\nu = \langle \bar{\nu}^2 \rangle_\nu / \epsilon = \frac{h}{m_{ST}\epsilon} \tag{4.30}$$

Substitude this into the first term ofEq. (4.29), we obtain

$$\begin{aligned} [x,\ p] &= \lim_{\epsilon \to 0} \left(i\epsilon m \langle \nu^2 \rangle_\nu + i\epsilon \langle U^2 \rangle_\nu \right) \\ &= \lim_{\epsilon \to 0} i\epsilon m \frac{h}{m_{ST}\epsilon} + 0 = i\hbar \end{aligned} \tag{4.31}$$

which is the most fundamental hypothesis of quantum mechanism, the position-momentum uncertainty relation.

4.4 Energy-Time Uncertainty Relation

Within the framwork of non-relativity quantum mechanism, the position-momentum uncertainty relation does not imply the energy-time uncertainty. This means we can not derive one kind of uncertainty relation from the other. Notice, position, momentum, energy are all dynamical variables. They are functions of time t, say, the time t is a self-variable. Experimentally, because in non-relativity quantum mechanism, time t is an independent variable and does not rely on particle status, we can measure the position, momentum, energy of a matter particle.

Now we define the Δt in energy-time uncertainty relation as: the characteric time describing a significant variation in the system study at hand. To describe the variation, we have to introduce a time-varying physical quantity Q. The 'significant' variation is defined as the time interval in which the Q changing by one standard deviation σ_Q. Mathematically, it is expressed as:

$$\sigma_Q = |\frac{d}{dt}\langle Q\rangle_\nu| \times \Delta t \tag{4.32}$$

Meanwhile, we can define the ΔE in energy-time uncertainty relation as the uncertainty of Hamiltonian of the system σ_H. The average evolution equation of Q along with the time is

$$\frac{d}{dt}\langle Q\rangle_\nu = \frac{i}{\hbar}\langle[H,Q]\rangle_\nu \tag{4.33}$$

combine with the Schwarz inequality in mathematics, we have

$$\sigma_H^2\sigma_Q^2 \geq \left[\frac{1}{2i}\langle[H,Q]\rangle_\nu\right]^2 \tag{4.34}$$

and then substitude into the definition of ΔE andΔt, we arrive:

$$\Delta E\Delta t \geq \frac{\hbar}{2} \tag{4.35}$$

If any physical quantity in this system varies fast, say Δt is very small, then its energy uncertainty will be very large. If ΔE is very small, then the Δt is very large, it means all observables in this system are varying slow.

4.5 Neutrino mass and the neutrino diffusion experiment

In previous subsections, we derived the mass-diffusion uncertainty relation. We now discuss a possible important application of this .

In mordern physics, neutrino oscillation is provided as a longstanding puzzle for high energy physics. The current explaination is that neutrinos have a very strange property that they can not be eigenstate of mass and flavor simultaneously. However, in the progress of nuclear reaction, neutrinos are all considered as a flavor eigenstate, which means they have definitive flavors. This leads to a strange result that we can not detect the mass of neutrinos.

Within framework of MIP, we study the statistical mass of neutrino. By definition, statistical mass is an emergent mass resulting from random collisions of STPs and the particle. The statistical mass of a free neutrino should be smaller than the summation of masses of three kinds of neutrinos with different flavours. According to the most recent experiment of cosmology, for the three kinds of neutrinos with different flavours, their mass summation is much more smaller than a single neutrino with certain flavour, it implies the statistical mass of

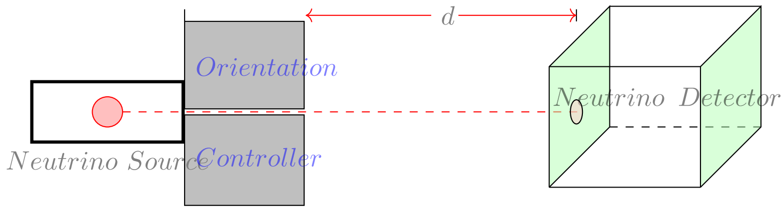

Fig. 4.1: Diffusion experiment of neutrino

neutrino maybe zero. We now use the neutrino emitted from nuclear reactor to design a crucial experiment to investigate whether the statistical mass of a neutrino is exact zero.

If the statistical mass of neutrino is exact zero, the uncertainty of its mass also vanished, hence we can not observe the following estimated diffusion effect. We will reach to a new important conclusion: the neutrino moves in speed of light, because the motion speed of a free particle is determined by its statistical mass. This conclusion coincides with the most recent experiment. If the statistical mass of neutrino is not zero, we will study its mass uncertainty and the diffusion caused by STP collisions. In this situation, we can conclude that the neutrino oscillation itself reflects the statistical inertia mass is a statistical property.

Experimently, physcists now can measure the mass square differences relating to neutrino oscillation indirectly. The mass square differences are between $2.6 \times 10^{-3} eV^2$ and $7.58 \times 10^{-5} eV^2$. This gives a good estimation for the mass-diffusion uncertainty. It is clear that the mass uncertainty of neutrino is in interval $0.0087 \sim 0.05 eV$.

From the mass-diffusion uncertainty relation, we have

$$\Delta m \Delta \Re = \hbar/2 \tag{4.36}$$

the diffusion coefficient reads

$$\begin{aligned} \Delta \Re = \hbar/2\Delta m &= \frac{6.626 \times 10^{-34} \times 9 \times 10^{16}}{4 \times 3.1416 \times 0.05 \times 1.6 \times 10^{-19}} \\ &= 1186.4[m^2/s] \end{aligned} \tag{4.37}$$

The physical meaning of this calculation is significant. Every second the neutrino propagates with a growing diffusion cone, with the bottom of the cone, increasing its area to $1186.4m^2$. If a neutrino goes from sun to earth, its diffusion radius will be about 307 meters.

Since sun cannot be seen as point-like source for neutrino ejection, the diffusion effect of neutrino can not be measured accurately. We could use the neutrino emitted from nuclear reactor to design a crucial experiment in labratory, as shown in Fig.4.1

Electron neutrinos came from reactor and were screened by screening matter, except those moving strictly toward x-direction. According to MIP and due to diffusion of neutrino, after propagating distance d, dectectors at distance d will detect neutrinos in a disk region with equal probability. The disk area can be calculated as

$$\delta r \simeq \sqrt{\Delta R \frac{d}{c\pi}} \tag{4.38}$$

If $d \simeq 100km$, $\delta r \simeq 0.3548m$, the disk is macro significant detectable. From the result of this crucial experiment, we can make a claim on whether the statistical mass of neutrino is zero, and also can deduce the speed, oscillation and other properties of neutrino.

5 Random Motion of Free Particle under MIP

5.1 Decompositions of the Real Velocity

In modern quantum mechanics, particles do not have trajectories of motions, so their velocities are not well defined. Within the framework of MIP, the real velocity of the particles must be discussed in detail. Under the impact of STP, the velocity of the particle not only contains the classical velocity, but also the results of random mechanical interactions. It is especially important that the particles are subjected to the impact of the STP, and the change of action is quantized. Therefore, the real velocity of the particles should reflect the classical, random and quantum properties.

Within the framework of MIP, the motion of particles is a frictionless quantum Brownian motion. However, it should be noted that the impact of STP is not completely random. The exchanged action that each particle is subjected to STP is an integer multiple of the Planck constant h. Therefore, the movement of particles in spacetime cannot be a problem of random mechanics completely. It is the quantization of randomized motions. The corresponding theoretical system is a quantum Markov process. If there is no STP and other external forces, the motion of the free particles satisfies Newtonian mechanics. Its velocity is the classic velocity.

Within the framework of MIP, for the real velocity of motion of free particles $\vec{V}(\vec{x},t)$, we can first isolate the classical statistical velocity of the particle $\vec{v}(\vec{x},t)$. In the context of spacetime, it is a simple mean of the statistics of the impact of STP as Gaussian noise. Since the simple mean contribution of Gaussian noise is zero, the classical statistical velocity of the particle and the classical velocity under Newtonian mechanics are exactly equal. Second, after separating the classical statistical velocity $\vec{v}(\vec{x},t)$, we will consider a random motion. This random motion is driven by the impact of STP, and we note it with the random motion velocity $\vec{W}(\vec{x},t)$. In Appendix B of this paper, we prove that any random function can be decomposed into a random function and a superposition of differentiable functions. Random motion under the framework of MIP also follows this important principle. Therefore, in general, we can decompose the random motion velocity $\vec{W}(\vec{x},t)$ as follow

$$\vec{W}(\vec{x},t) = \vec{u}(\vec{x},t) + \vec{\nu}(t) \tag{5.1}$$

Where $\vec{u}(\vec{x},t)$ is defined as the quantum envelope velocity of the particle. For free particles, $< \vec{u}(\vec{x},t) >_\nu = 0$. It corresponds to the perturbation part of the random motion. It reflects the physical fact that the impact of STP is random, but it is a small perturbation to the current motion of the particle. These impacts are "differential impacts" of STP on the particles. Under the action of the pertubation of space-time, the motion of particles is not an unpredictable random motion. It allows the motion state of particles to be described by a differentiable function and describes the corresponding motion state . The equation is a non-random partial differential equation. And $\vec{\nu}(t)$ represents the non-microscopic impact of the particle by STP, which is a non-perturbative effect on the velocity of the particle motion. We define it as the velocity of fluctuation. Because of the existence of such random impact, the state function that we finally describe the equation of motion of the particle will not be an accurate description. It can only be a probabilistic description on the background of this fluctuation.

We will see that in the framework of MIP, quantum envelope motion reflects the wave-particle duality of particles. Considering the impact between STP and particle, the amount of exchange action is nh. For particles with a statistical mass of m_0, the characteristic time of this collision is

$$t_c = \frac{nh}{m_0 c^2} \tag{5.2}$$

The so-called quantum envelope motion is essentially the differentiable part of the fluctuation motion.

The above discussion is based on the classification of particles by the impact of STP. From the above analysis we can see that there is actually another mathematical classification for the velocity of the particles, and we decompose the velocity of the particle into a differentiable part and a non-differentiable part. The differentiable part of the real motion velocity of a particle can be defined as:

$$\vec{U}(\vec{x},t) = \vec{v}(\vec{x},t) + \vec{u}(\vec{x},t) \tag{5.3}$$

It is a superposition of classic statistical velocity $\vec{v}(\vec{x},t)$ and quantum envelope velocity $\vec{u}(\vec{x},t)$. We call this differentiable velocity “statistical average velocity”. Although mathematically it is a differentiable function, it is quite different from the classical velocity. Because there is a quantum envelope velocity $\vec{u}(\vec{x},t)$, it is a representation of the Markov process formed by the impact of STP. Therefore, the decomposition of the velocity of the particles caused by the collision of STP can be written in three parts in principle[3]:

$$\vec{V}(\vec{x},t) = \vec{u}(\vec{x},t) + \vec{v}(\vec{x},t) + \vec{\nu}(t) \tag{5.4}$$

Since a Markov process will still be a Markov process under time reversal, the quantum envelope velocity $\vec{u}(\vec{x},t)$ is invariant under time reversal as

$$T : \vec{u}(\vec{x},t) \to \tilde{\vec{u}}(\vec{x},t) = \vec{u}(\vec{x},t) \tag{5.5}$$

However, the classical statistical velocity $\vec{v}(\vec{x},t)$ is changed by the time reversal, that is,

$$T : \vec{v}(\vec{x},t) \to \tilde{\vec{u}}(\vec{x},t) = -\vec{v}(\vec{x},t) \tag{5.6}$$

With above properties of time reversal, we can have a well defined limit $\vec{u} = 0$ as Newtonian mechanics with

$$\vec{v} = \frac{1}{2}(\vec{U} - \tilde{\vec{U}}) \tag{5.7}$$

$$\vec{u} = \frac{1}{2}(\vec{U} + \tilde{\vec{U}}) \tag{5.8}$$

Where $\tilde{\vec{U}}$is the time reversal of the statistical average velocity $\vec{U}$. In the following, the physical quantities with time reversal are marked with tilde.

The non-differentiable part is the fluctuation velocity $\vec{\nu}(t)$ for the random "non-differentiable impact"of the particle. It causes the particle's velocity to deviate from the classical statistical mean, so it will be physically reflected as a random diffusion behavior of the particle in spacetime. Based on this, we named it the "diffusion velocity" of particles in space and time.

In the following subsections, we will see that the decomposition of the above two velocities is a very important theoretical basis for deriving the equation of motion of particles, that is, the Schrödinger equation in quantum mechanics and an in-depth understanding of its physical meaning.

5.2 From MIP to Schrödinger Equation

Without the interaction of spacetime, the velocity of particle $\vec{v}$ has to be the derivative $\vec{v} = \frac{d\vec{x}}{dt}$. Contrasting from usual Markov process, spacetime random motion is frictionless, otherwise the quantum effect of a particle will decay as time going, which is obviously not the case. According to the MIP, the coordinate of a free particle is a stochastic process $\vec{x}(t)$, in which the velocity $\vec{V}$ can not be expressed in terms of $\frac{d\vec{x}}{dt}$. The velocity $\vec{V}$ should be a statistical average corresponding to a distribution $\delta\vec{x} = \vec{x}(t + \frac{1}{\omega}) - \vec{x}(t)$, at the limit of spacetime

[3] After we finished our manuscript, we found that this three-velocity decomposition is in fact consistent with Wold's decomposition theorem of the stochastic process in [20].

collision frequency ω going to infinity. In Einstein's theory on Brownian motion, $\delta\vec{x}$ is a Gaussian distribution with zero mean and variance proportional to $\frac{1}{\omega}$[6]. However, Einstein's theory cannot be correct at the limit of spacetime collision frequency ω going to infinity[21, 22]. Therefore, we will construct the operator D as following, which plays the same role as $\frac{d}{dt}$ in Newtonian Mechanics. For any physical function $f(\vec{x}, t)$, we have

$$\begin{aligned}
&\omega(f(\vec{x}(t+\frac{1}{\omega}), t+\frac{1}{\omega}) - f(\vec{x}(t), t)) \\
&= [\partial_t + \sum_i \omega(x_i(t+\frac{1}{\omega}) - x_i(t))\partial_i \\
&\quad + \sum_{ij} \frac{\omega}{2}(x_i(t+\frac{1}{\omega}) - x_i(t))(x_j(t+\frac{1}{\omega}) - x_j(t))\partial_i\partial_j \\
&\quad + \sum_i (x_i(t+\frac{1}{\omega}) - x_i(t))\partial_i\partial_t + \frac{1}{2\omega}\partial_t^2] f(\vec{x}(t), t) \qquad (5.9)
\end{aligned}$$

At the limit of spacetime collision frequency ω going to infinity, in terms of statistical average $< ... >$ for δx, we can define the operator D as

$$\begin{aligned}
Df(x(t), t) &= \lim_{\omega\to+\infty} \omega\langle f(\vec{x}(t+\frac{1}{\omega}), t+\frac{1}{\omega}) - f(\vec{x}(t), t)\rangle_\nu \qquad (5.10) \\
&= (\partial_t + \sum_i U_i\partial_i + \sum_{ij} \Re_{ij}\partial_i\partial_j) f(\vec{x}(t), t) \qquad (5.11)
\end{aligned}$$

where we used

$$\vec{U} = \lim_{\omega\to+\infty} \omega\langle\delta\vec{x}\rangle_\nu \qquad (5.12)$$

it relates to the descreterization of Lagevin equation

$$x_i(t+\epsilon) - x_i(t) = \epsilon U_i(\mathbf{x}(t)) + \sqrt{\epsilon}\bar{\nu}_i + O(\epsilon^2) \qquad (5.13)$$

here

$$\epsilon = \frac{1}{\omega} \qquad (5.14)$$

In eq.(5.10) , we used the following deduced result

$$\begin{aligned}
\lim_{\omega\to+\infty} \frac{\omega\langle\delta x_i\delta x_j\rangle_\nu}{2} &= \lim_{\epsilon\to0^+} \frac{1}{2\epsilon}\langle(x_i(t+\epsilon) - x_i(t))(x_j(t+\epsilon) - x_j(t))\rangle_\nu \\
&= \lim_{\epsilon\to0^+} \frac{1}{2\epsilon}\left[\langle\epsilon^2 U_i(\mathbf{x}(\mathbf{t}))U_j(\mathbf{x}(t))\rangle_\nu + \epsilon\langle\bar{\nu}_i\bar{\nu}_j\rangle_\nu + \epsilon^{\frac{3}{2}}\langle(U_i\bar{\nu}_j + U_j\bar{\nu}_i)\rangle_\nu\right] \\
&= \frac{h}{2m_{ST}}\delta_{i,j} \qquad (5.15)
\end{aligned}$$

Because of the isotropy of space, the MIP coefficient will be

$$\Re_{ij} = \frac{\hbar}{2m_{ij}} = \Re\delta_{ij} \qquad (5.16)$$

which is consistent with Eq.3.30 and 3.39. The operator D and its time reversal $\tilde{D}$ are

$$D = \partial_t + \vec{U} \cdot \nabla + \Re\nabla^2 \tag{5.17}$$

$$\tilde{D} = -\partial_t + \tilde{\vec{U}} \cdot \nabla + \Re\nabla^2 \tag{5.18}$$

Therefore, the statistical average velocity of particle $\vec{V}$ can be written as

$$\vec{U} = D\vec{x} \tag{5.19}$$

$$\tilde{\vec{U}} = \tilde{D}\vec{x} \tag{5.20}$$

Correspondingly, its classical statistical velocity and quantum envelope velocity are

$$\vec{v} = D^-\vec{x} \tag{5.21}$$

$$\vec{u} = D^+\vec{x} \tag{5.22}$$

with

$$D^- = \frac{1}{2}(D - \tilde{D}) \tag{5.23}$$

$$D^+ = \frac{1}{2}(D + \tilde{D}) \tag{5.24}$$

We define the statical average acceleration of particles as

$$\begin{aligned} \vec{a} &= D\vec{U} = (D^+ + D^-)(\vec{v} + \vec{u}) \\ &= D^+\vec{u} + D^-\vec{v} + D^-\vec{u} + D^+\vec{v} \end{aligned} \tag{5.25}$$

Under time reversal, it acts as

$$\begin{aligned} \tilde{\vec{a}} &= \tilde{D}\tilde{\vec{U}} = (D^+ - D^-)(-\vec{v} + \vec{u}) \\ &= D^+\vec{u} + D^-\vec{v} - D^-\vec{u} - D^+\vec{v} \end{aligned} \tag{5.26}$$

Define the classical average acceleration as

$$\vec{a}_c = \frac{1}{2}(\vec{a} + \tilde{\vec{a}}) = D^+\vec{u} + D^-\vec{v}, \tag{5.27}$$

obviously it is invariant under time reversal. The average acceleration of a free particle must be zero, which can be written as

$$D^+\vec{v} + D^-\vec{u} = 0. \tag{5.28}$$

However, the average acceleration of quantum envelope motion can not simply be zero,

$$D^+\vec{u} + D^-v \neq 0 \tag{5.29}$$

At classical and low speed case, the average acceleration of quantum envelope motion does not relate to classical statistical velocity, therefore we can have

$$D^{-}\vec{v} - D^{+}\vec{u} = 0. \tag{5.30}$$

These conditions are equivalent to the coupled non-linear partial differential equations as following

$$\frac{\partial \vec{u}}{\partial t} = -\Re\nabla^2\vec{v} - \nabla(\vec{u}\cdot\vec{v}) \tag{5.31}$$

$$\frac{\partial \vec{v}}{\partial t} = -(\vec{v}\cdot\nabla)\vec{v} + (\vec{u}\cdot\nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{5.32}$$

Random motions of free particles due to the random impacts of STP satisfy the Markov property, one can make predictions for the future of the process based solely on its present state just as well as one could know the process's full history. This is the simplest situation for random motions, the free particle does not involve any external potential. Now, we have an initial value problem, which is to solve $\vec{u}(\vec{x},t)$ and $\vec{v}(\vec{x},t)$given $\vec{u}(\vec{x},0) = \vec{u}_0(\vec{x})$, $\vec{v}(\vec{x},0) = \vec{v}_0(\vec{x})$. In order to solve the coupled non-linear partial differential equations, we have to linearise it firstly. Let

$$\Psi = e^{R+iI}, \tag{5.33}$$

where

$$\nabla R = \frac{1}{2\Re}\vec{u} \tag{5.34}$$

$$\nabla I = \frac{1}{2\Re}\vec{v} \tag{5.35}$$

We can obtain

$$\frac{\partial \Psi}{\partial t} = i\Re\nabla^2\Psi \tag{5.36}$$

According to the MIP, the universal spacetime diffusion coefficient is the MIP coefficient $\Re = \frac{\hbar}{2m_{ST}}$. Substituting to the last equation, we will get the equation of motion of free particles as

$$i\frac{\partial \Psi}{\partial t} = -\frac{\hbar\nabla^2}{2m_{ST}}\Psi \tag{5.37}$$

which is the Schrödinger equation essentially.

According to the continuity equation

$$\partial_t\rho(\vec{r},t) + \nabla\cdot\vec{J} = 0 \tag{5.38}$$

The definition of particle current is density multiplied by velocity. In the framework of MIP, the velocity in this definition corresponds to the classical statistical velocity. We can naturally derive the Born's interpretation as follows:

$$\vec{J} = \rho\vec{v} \tag{5.39}$$

among them

$$\vec{v} = 2\Re\nabla I \tag{5.40}$$

Substitute (5.33) in Schrödinger equation

$$\partial_t \Psi = i\Re\nabla^2\Psi \tag{5.41}$$

Let the real and imaginary parts be equal respectively, there are

$$\partial_t R + \Re(2\nabla R\cdot\nabla I + \nabla^2 I) = 0 \tag{5.42}$$

and

$$\partial_t \rho(\vec{r}, t) + \nabla\cdot(\rho\vec{v}) = 0 \tag{5.43}$$

which can be solved as

$$\rho = e^{2R} \tag{5.44}$$

Therefore, we show that the distribution of the particle number density is exactly the wave function modulo square. Further considering the ensemble of many identical particles, the particle number density is interpreted as the probability density, which is exactly the Born's interpretation.

The Born rule is a law of quantum mechanics which gives the probability that a measurement on a quantum system will yield a given result, which became a fundamental ingredient of Copenhagen interpretation. In this paper, we attempt to suggest an interpretation of Born rule according to the MIP, which can provide a realistic point of view for wave function. Emerging from random impacts of spacetime, it's absolutely necessary that wave function is complex. If wave function were a real sine or cosine function[27], according to $\rho = |\Psi|^2$, the probabilistic density of a free particle with definite momentum would oscillate periodically which violates the isotropy of physical space.

5.3 Physical Meanings of Potential Functions R and I

Substituting $\Psi = e^{R+iI}$ into $\frac{\partial\Psi}{\partial t} = i\Re\nabla^2\Psi$, we equalise the real and imaginary part separately as

$$\partial_t R = -\Re(2\nabla R\cdot\nabla I + \nabla^2 I) \tag{5.45}$$

$$\partial_t I = \Re[(\nabla R)^2 - (\nabla I)^2 + \nabla^2 R] \tag{5.46}$$

Combining with previous result $\rho = |\Psi|^2 = e^{2R}$, we have

$$\partial_t \rho = 2\rho\partial_t R \tag{5.47}$$

$$\nabla\rho = 2\rho\nabla R \tag{5.48}$$

The differential equation of potential R can be turned into

$$\partial_t \rho = -2\Re\nabla\cdot(\rho\nabla I) \tag{5.49}$$

With $\nabla I = \frac{1}{2\Re}\vec{v}$, the differential equation of potential R is equivalent to the equation of continuity

$$\partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0 \tag{5.50}$$

Noticing that the classical momentum of particle is $m\vec{v} = \hbar \nabla I$, we find that the differential equation of potential I goes to

$$\partial_t(\hbar I) + \frac{(\nabla(\hbar I))^2}{2m} - \hbar\Re[(\nabla R)^2 + \nabla^2 R] = 0 \tag{5.51}$$

Comparing with the Hamilton-Jacobi equation from classical mechanics [28, 29] as

$$\partial_t S + \frac{(\nabla S)^2}{2m} + V(x) = 0 \tag{5.52}$$

which is particularly useful in identifying conserved quantities for mechanical systems. There are two crucial remarks: Firstly, potential function I is proportional to the Hamilton-Jacobi function S as $S = \hbar I$. Secondly, for a free particle, the influence of spacetime can be summed up to the spacetime potential

$$V_{ST} = -\hbar\Re[(\nabla R)^2 + \nabla^2 R] \tag{5.53}$$

where the spacetime potential V_{ST} will play the same role of potential V in the Hamilton-Jacobi equation. The spacetime potential V_{ST} vanishes in the classical limit $\hbar = 0$, which is equivalent to $V = 0$ for free particles in classical mechanics. The quantum effect, which corresponding to nonzero $\hbar$, now is the natural result of the existence of the spacetime potential V_{ST}, induced by MIP. In principal, the moving of free particle can be described precisely by the spacetime potential V_{ST} as

$$m\frac{d^2\vec{x}}{dt^2} = -\nabla V_{ST} = \hbar\Re\nabla[(\nabla R)^2 + \nabla^2 R] \tag{5.54}$$

This equation indicates that free particle moves not along straight line within interactions of STP. It is affected by a space-time potential V_{ST}. The interactions between STP and particle give the statistcal mass to particle.

5.4 Space-time Random Motion of Charged Particles in Electromagnetic Field

According to the MIP, in case of low speed, electromagnetic field only serves as an external potential, which itself is not affected by random impacts of spacetime. In a electromagnetic field $(\vec{E}, \vec{B})$, the charged particle will experience a Lorentz force $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$. Therefore, the average acceleration of charged particles will be

$$\vec{a} = e(\vec{E} + \vec{v} \times \vec{B})/m \tag{5.55}$$

where m is the inertial mass of charged particle and e is the charge. Based on the spacetime principle, we are able to derive the equation of motion of charged particle in electromagnetic field, which is finally shown to be Schrödinger equation in electromegnatic field, which is

$$i\hbar\partial_t\Psi = \frac{1}{2m}(-i\hbar\nabla - \frac{e}{c}\vec{A})^2\Psi + e\phi\Psi \tag{5.56}$$

where the electromagnetic potential and the electromagnetic field are connected by

$$\vec{B} = \nabla\times\vec{A}, \vec{E} = -\partial_t\vec{A} - \nabla\phi. \tag{5.57}$$

We do not have average acceleration in absence of electromagnetic field. However, this is not the case when the particle have non-zero electric charge, moving in external electromagnetic field. Identifying the velocity in the Lorentz force as the classical velocity of random motion of particle in spacetime, we have

$$\partial_t\vec{v} = e(\vec{E} + \vec{v}\times\vec{B})/m - (\vec{v}\cdot\nabla)\vec{v} + (\vec{u}\cdot\nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{5.58}$$

In the electromagnetic field, the equation of motion of charged particle becomes coupled non-linear partial differential equations as following

$$\frac{\partial\vec{u}}{\partial t} = -\Re\nabla(\nabla\cdot\vec{v}) - \nabla(\vec{u}\cdot\vec{v}) \tag{5.59}$$

$$\frac{\partial\vec{v}}{\partial t} = e(\vec{E} + \vec{v}\times\vec{B})/m - (\vec{v}\cdot\nabla)\vec{v} + (\vec{u}\cdot\nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{5.60}$$

In order to solve the coupled non-linear partial differential equations, we have to linearise it firstly. Let $\Psi = e^{R+iI}$ and notice that the canonical momentum of charged particle [30] is $\vec{p} = m\vec{v} + e\vec{A}/c$, we suppose

$$\nabla R = \frac{1}{2\Re}\vec{u} \tag{5.61}$$

$$\nabla I = \frac{1}{2\Re}(\vec{v} + \frac{e\vec{A}}{mc}) \tag{5.62}$$

In order to prove Eq.(5.56), we expand the first term of right side of Eq.(5.56) as

$$\frac{1}{2m}(-i\hbar\nabla - \frac{e}{c}\vec{A})^2\Psi = -\frac{\hbar^2\nabla^2}{2m}\Psi + \frac{e^2A^2}{2mc^2}\Psi + \frac{i\hbar e}{2mc}(\nabla\cdot\vec{A})\Psi + \frac{i\hbar e}{mc}\vec{A}\cdot(\nabla\Psi) \tag{5.63}$$

Substituting $\Psi = e^{R+iI}$, it leads to

$$-\frac{\hbar^2}{2m}[\nabla^2 R + i\nabla^2 I + (\nabla R + i\nabla I)^2]\Psi + \frac{e^2A^2}{2mc^2}\Psi + \frac{i\hbar e}{2mc}(\nabla\cdot\vec{A})\Psi + \frac{i\hbar e}{mc}(\vec{A}\cdot(\nabla R + i\nabla I))\Psi \tag{5.64}$$

With vector formulas

$$\nabla(\vec{A}\cdot\vec{B}) = \vec{A}\times(\nabla\times\vec{B}) + \vec{B}\times(\nabla\times\vec{A}) + (\vec{A}\cdot\nabla)\vec{B} + (\vec{B}\cdot\nabla)\vec{A} \tag{5.65}$$

$$\nabla(\nabla\cdot\vec{A}) = \nabla\times(\nabla\times\vec{A}) + \nabla^2\vec{A} \tag{5.66}$$

and Eq.(5.61), we will obtain

$$\nabla\times\vec{u} = 0 \tag{5.67}$$

$$\nabla\times(\vec{v} + \frac{e\vec{A}}{mc}) = 0 \tag{5.68}$$

Straightforwardly, we have

$$i\hbar(\partial_t R + i\partial_t I) = -\frac{\hbar^2}{2m}[\nabla^2 R + i\nabla^2 I + (\nabla R + i\nabla I)^2] + \frac{e^2A^2}{2mc^2} + \frac{i\hbar e}{2mc}(\nabla\cdot\vec{A}) + \frac{i\hbar e}{mc}(\vec{A}\cdot(\nabla R + i\nabla I)) + e\phi \tag{5.69}$$

Now, let's prove that the real and imaginary parts are separately equaled as

$$\partial_t I = \frac{\hbar}{2m}(\nabla^2 R + (\nabla R)^2 - (\nabla I)^2) - \frac{e^2\vec{A}^2}{2mc^2} + \frac{e}{mc}(\vec{A}\cdot(\nabla I)) - \frac{e\phi}{\hbar} \tag{5.70}$$

$$\partial_t R = -\frac{\hbar}{2m}(\nabla^2 I + 2(\nabla R)\cdot(\nabla I)) + \frac{e}{2mc}(\nabla\cdot\vec{A}) + \frac{e}{mc}\vec{A}\cdot(\nabla R) \tag{5.71}$$

Taking the gradient from both sides and the definitions $\vec{B} = \nabla\times\vec{A}, \vec{E} = -\partial_t\vec{A} - \nabla\phi$, we have reproduced the Eq.(5.59). Therefore, we have proved that both sides of Eq.(5.59) are at most different from a zero gradient function. It's important to notice that the choices of electromagnetic potentials are not completely determined. It allows a gauge transformation [31]

$$\vec{A}' = \vec{A} + \nabla\Lambda \tag{5.72}$$

$$\phi' = \phi - \partial_t\Lambda \tag{5.73}$$

For any function $\Lambda(\vec{x}, t)$, the electromagnetic field is invariant. Therefore, the corresponding wave function cannot change essentially, at most changing a local phase factor. Given $\psi' = \psi e^{\frac{ie\Lambda}{\hbar c}}$, Schrödinger equation of charged particle in electromagnetic field is invariant, i.e., $U(1)$ gauge symmetry. By choosing the function $\Lambda(\vec{x}, t)$ properly, we are able to eliminate the redundant zero gradient function. So we have proved Eq.(5.56) at the end.

5.5 Stationary Schr$\ddot{o}$dinger Equation from MIP

Compare to the definition of classical statistical velocity as in eq.(5.35), it is easy to see that for the ground state, the classical statistical velocity is zero. Moreover, we can prove for all stationary states, their classical statistical velocities are zero. For a stationary state has exact energy E, the Schr$\ddot{o}$dinger equation is

$$[-\frac{\hbar^2\nabla^2}{2m}+V_c(\vec{x})]\Psi = E\Psi \tag{5.74}$$

its conjugation reads

$$[-\frac{\hbar^2\nabla^2}{2m}+V_c(\vec{x})]\Psi^* = E\Psi^* \tag{5.75}$$

here $V_c(\vec{x})$ is classical external potential. Add the above two equations, the new real wave function has to satisfy the Schr$\ddot{o}$dinger equation with same eigenenergy E.

Corresponding to the classical velocity from Eq.(5.35), it is easy to show that the classical velocity of particles must be zero in stationary states. Within the framework of MIP, we should interpret the stationary states from quantum mechanics as a spacetime random motion with zero classical velocity. Once we have all the stationary states, we will get the general solution by linear superposition. Therefore, we are going to derive stationary Schr$\ddot{o}$dinger equation from classical velocity $\vec{v} = 0$, which can provide a clear physical picture of MIP. Moreover, when $|\vec{v}|$ is large and close to velocity of light c, the generalisation of this framework is clear and will be explained in our further work.
The trajectory of random motion of particle can be understood as the superposition of classical path and fluctuated path. During time interval $\triangle t$, there are two contributions to the trajectory as

$$\delta\vec{x} = \vec{u}(\vec{x},t)\triangle t + \triangle\vec{x} \tag{5.76}$$

of which distribution satisfies $\varphi(\triangle\vec{x}) = \varphi(-\triangle\vec{x})$ and

$$\int \varphi(\triangle\vec{x})d(\triangle\vec{x}) = 1$$

. The spacetime coefficient reads

$$\Re = \frac{1}{2\triangle t}\int(\triangle\vec{x})^2\varphi(\triangle\vec{x})d(\triangle\vec{x}) \tag{5.77}$$

The probabilistic density $\rho(x,t)$ evolves as

$$\rho(\vec{x},t+\triangle t) = \int \rho(x-\delta\vec{x},t)\varphi(\triangle\vec{x}))d(\triangle\vec{x}) \tag{5.78}$$

Expanding Taylor series of both sides, we have

$$\partial_t \rho = -\nabla \cdot (\rho \vec{u}) + \Re \nabla^2 \rho \tag{5.79}$$

which is consistent with Fokker-Planck equation. In any external potential $V(\vec{x})$, there are two contributions to the changing of average velocity. One is from random impacts of spacetime, another one is from acceleration provided by external potential. Therefore, the average velocity will evolve during time interval $\triangle t$ as

$$\vec{u}(\vec{x}, t + \triangle t) = \frac{\int (\vec{u}(\vec{x} - \delta\vec{x}, t) - \frac{\triangle t \nabla V(\vec{x} - \delta\vec{x})}{m}) \rho(x - \delta\vec{x}, t) \varphi(\triangle\vec{x}) d(\triangle\vec{x})}{\int \rho(x - \delta\vec{x}, t) \varphi(\triangle\vec{x})) d(\triangle\vec{x})} \tag{5.80}$$

the denominator of eq. 5.80 is the normalisation factor of the probability distribution. Expanding Taylor series of both sides, we obtain

$$m\frac{d\vec{u}}{dt} = -\nabla V + \Re m(\frac{\nabla^2(\rho\vec{u})}{\rho} - \vec{u}\frac{\nabla^2 \rho}{\rho}) \tag{5.81}$$

From this we can see the acceleration of the quantum envelope velocity $\vec{u}$, whose dynamics are rooted in the joint contribution of the classical potential and the quantum potential. For the physical state with certain energy, the three-velocity decomposition $\vec{V}(\vec{x}, t) = \vec{u}(\vec{x}, t) + \vec{v}(\vec{x}, t) + \vec{\nu}(t)$ has clear physical meaning. The quantum envelope velocity $\vec{u}(\vec{x}, t)$ and the classical statistical velocity $\vec{v}(\vec{x}, t)$ are both velocity fields, which are functions of space-time coordinates. The classical statistical velocity field of a physical state with certain energy is zero, which can be used as a new interpretation of the steady state of quantum mechanics. The dynamic mechanism of the quantum envelope velocity field $\vec{u}(\vec{x}, t)$ has two contributions, the classical external potential field where the particle is located and the quantum potential field generated by the random collision of time-space. The diffusion velocity $\vec{\nu}(\vec{x}, t)$ is the background of space-time fluctuations, evenly distributed in space, and satisfies the properties of Brownian motion in time, which is the intrinsic property of space-time. The sum of these three velocities is the real velocity of the objective reality of the particles required by materialism. See appendix B where we proved these.With the condition of stationary state $\partial_t \rho = 0$, it goes to

$$\vec{u} = \Re \frac{\nabla \rho}{\rho} \tag{5.82}$$

$$\partial_t \vec{u} = 0 \tag{5.83}$$

It's important to notice that

$$\frac{d\vec{u}}{dt} = \partial_t \vec{u} + (\vec{u} \cdot \nabla)\vec{u} \tag{5.84}$$

The average velocity $\vec{u}$ is not zero in the stationary state, which exactly cancle out its fluctuation velocity. Therefore, given the condition of stationary state, we are able to get

$$-2m\Re^2\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}+V(x)=Const. \tag{5.85}$$

We can prove this constant is exactly the average energy of particle

$$E=\int\rho(\frac{1}{2}mu^2+V)d^3x \tag{5.86}$$

Now, we have derived

$$-2m\Re^2\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}+V(x) \quad = \quad E \tag{5.87}$$

$$\psi=\sqrt{\rho}e^{-iEt/\hbar} \tag{5.88}$$

Let $\Re=\frac{\hbar}{2m}$ once again, we arrive at the stationary Schrödinger equation

$$-\frac{\hbar^2\nabla^2}{2m}\psi+V\psi=E\psi \tag{5.89}$$

5.6 Ground States of Hydrogen Atoms in MIP

In the hydrogen atom system, we can take $\vec{A}=0$ and $\phi=-\frac{e}{4\pi\epsilon_0 r}$. The stationary solution of the equation (5.56) satisfies

$$E\Psi=\frac{1}{2m}(-i\hbar\nabla)^2\Psi-\frac{e^2}{4\pi\epsilon_0 r}\Psi \tag{5.90}$$

The lowest energy stationary state solution (ground state wave function) is $\Psi(r,\theta,\varphi)=\frac{1}{\sqrt{\pi a^3}}e^{-r/a}$, where $a=5\times10^{-11}m$ is the Bohr radius of the hydrogen atom. Using the wave function of the ground state of a hydrogen atom, we can get its quantum envelope velocity as

$$\vec{u}=2\Re\nabla R=-\frac{\hbar}{ma}\hat{r}=-\frac{c}{137}\hat{r} \tag{5.91}$$

Where c is the velocity of light in vacuum, $\hat{r}$ is the unit vector $\hat{r}=\frac{\vec{r}}{r}$. Similarly we can get its classic average velocity

$$\vec{v}=2\Re\nabla I=0 \tag{5.92}$$

Its spacetime fluctuation rate is satisfied

$$<\nu_i>=0,<\nu_i(t)\nu_j(t')>=\Re\delta_{ij}\delta(tt') \tag{5.93}$$

Then the electron in the ground state of the hydrogen atom has its coordinate $\vec{X}(t)$ as a random variable, and its real velocity $\vec{V}$ satisfies the following microscopic dynamic equations.

$$\frac{d\vec{X}(t)}{dt}=\vec{V}(t)=\vec{u}+\vec{v}+\vec{\nu}=-\frac{c}{137}\hat{r}+\vec{\nu}(t) \tag{5.94}$$

This is the real equation of motion of the ground state electrons of a hydrogen atom in the context of MIP. The quantum envelope velocity always points to the center of hydrogen atom. The closer to the center, the greater the repulsive force generated by the spacetime potential. Because this envelope velocity is balanced out by the combination of the classical Coulomb potential and the spacetime potential, the hydrogen atom can be stabilized on the ground state.

According to MIP, the real motion of electrons in the ground state of hydrogen atoms, we can calculate the average kinetic energy of electrons as

$$< K >= \frac{m}{2} < \vec{V}(t)^2 >= \frac{m}{2}(\frac{c}{137})^2 + \frac{m}{2} < \vec{\nu}(t)^2 > \tag{5.95}$$

The average of the square of the spacetime fluctuation is

$$< \vec{\nu}(t)^2 >= \Re/T \tag{5.96}$$

Where T is the cumulative interaction time of the electrons. The ground state of a hydrogen atom can exist forever, that is, T tends to infinity, and thus we can obtain the average kinetic energy of the ground state electron as

$$< K >= \frac{m}{2} < \vec{V}(t)^2 >= \frac{m}{2}(\frac{c}{137})^2 \tag{5.97}$$

We can calculate the average potential energy of the electron as

$$< U(r) >=< -\frac{e^2}{4\pi\epsilon_0 r} >=< -\frac{e^2}{4\pi\epsilon_0 a} > \tag{5.98}$$

Where a is the Bohr radius and ϵ_0 is the vacuum permittivity. The average energy of the ground state electrons is the sum of the average kinetic energy and the average potential energy. Substituting the standard values of physical constants, we can get the numerical result of the average energy of the ground state electrons as

$$E =< K > + < U >= -13.6ev \tag{5.99}$$

We have reached the same conclusion as quantum mechanics through the microscopic equation of motion of MIP. It can be seen that quantum mechanics only reflects the statistical average nature of the real motion process and does not reflect all the physics under the framework of MIP.

5.6.1 Deriving the amount of elementary charge from MIP

According to MIP, the interaction between particles and STP (the basic definition of the action is the product of momentum and displacement)

$$Nh = \oint pdq \tag{5.100}$$

For example, the simplest uniform circular motion is

$$\oint pdq = 2\pi mvr \tag{5.101}$$

Consider the electrons inside the hydrogen atom. STP collisions provide random Brownian motion, and attraction from proton provides centripetal force with equilibrium conditions

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \tag{5.102}$$

The amount of charge can be solved as

$$e = nh\sqrt{\frac{\epsilon_0}{m\pi r}} \tag{5.103}$$

The exact value of the electronic charge can be accurately obtained. We know that in MIP, the exchange action is nh, where n can be any integer.
We only need to make a hypothesis that the orbit of the electron is determined by the quantum number n of STP interaction. The proof of this hypothesis is shown in the next section. That is, when $n = 1$, the electron falls on the Bohr's orbit ($r = 0.53 \times 10^{-10} m$). When $n = 2$, the electrons fall on the second orbit (by analogy). You can get important results (all values below are with international units)

$$h = 6.62 \times 10^{-34}, m = 9.11 \times 10^{-31}, \epsilon_0 = 8.85 \times 10^{-12}$$

After substituting, we obtain the amount of charge as

$$e = 1.6 \times 10^{-19} C \tag{5.104}$$

5.6.2 Quantum number n of STP determining the orbit of hydrogen atoms

What we want to prove is that when the electrons are in Bohr's orbit ($r = a$), the amount of exchange action of STP is just a Planck constant, ie

$$h = 2\pi mva \tag{5.105}$$

Using the ground state wave function of the hydrogen atom derived above

$$\psi = \frac{1}{\sqrt{\pi a^3}} e^{-r/a} \tag{5.106}$$

The average value of the momentum can be found as

$$Mv = p = |\int \psi^*(-i\hbar\nabla)\psi d\tau| = \frac{\hbar}{a} \tag{5.107}$$

The integral volume element is $d\tau = r^2 sin\theta d\theta d\varphi dr$ and $h = 2\pi mva$.

5.6.3 Generalisation to Hydrogen-like atoms

The exchanged action between particles and STP

$$nh = \oint pdq \tag{5.108}$$

In uniform circular motion

$$\oint pdq = 2\pi mvr \tag{5.109}$$

An electron in a hydrogen-like atom with a positively charged nucleus. STP collisions provide random Brownian motion, and the attraction of the nucleus provides centripetal force with equilibrium conditions

$$\frac{Ze^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \tag{5.110}$$

The amount of charge can be solved as

$$e = nh\sqrt{\frac{\epsilon_0}{Zm\pi r}} \tag{5.111}$$

The Bohr-like orbital electron corresponding to $n = 1$ has a Bohr radius of $r = a/Z$, from which the elementary charge can be derived as

$$e = 1.6 \times 10^{-19} C \tag{5.112}$$

Starting from MIP, we have made a thorough study of free matter particles and obtained the most important conclusions of quantum mechanics. Furthermore, the most fundamental cause of atomic stability is explained by MIP, and from the first principle we calculate the basic physical quantity of electron charge unit. It can be seen that the random collision of STP does not only provide chaotic background noise , but also the stability of all matter in a seemingly chaotic background. At the most profound level, materialism interpret the physical world and the contradictions are unified.

6 Quantum Measurement in MIP

6.1 General Principle

There are fundamental distinctions on quantum measurement between MIP and Copenhagen interpretation. Within the framework of MIP, since matter particle is collided randomly by STP. Any measurement related to position and momentum can not be done in a time interval between two collisions, therefore any this kind of measurement cannot lead to precise result, which means we cannot make errors as small as possible in principle. Therefore, incommutable observables can not only be measured precisely at the same time, but also cannot be measured precisely separately. Theoretically, all measure values means statistical average, which include intrinsic uncertainty from spacetime besides normal measurement errors. For examples, the momentum uncertainty from MIP is due to the statistical properties of fluctuated mass. As a statistical mass, the minimum fluctuation is $\triangle m_{st}$, which roughly is one part per million of electron mass. The position intrinsic uncertainty $\triangle X_{st}$ from MIP is the mean free path between two consecutive collision by STP.

When the spacetime sensible mass is equivalent to the statistical inertial mass, the equation of motion will be determined by Schrödinger equation. In other words, moving matter particle and propagational wave are unified in spacetime. If we want to measure a matter particle, we need apparatus to interact with particle somehow. However, every such measurement has to interrupt the random motion of particle. Therefore, measurement means the end of a Markov process. When the measurement is finished, a new Markov process will begin. For the moving matter particle, the phases of wave functions before and after measurements is completely irrelevant, which cannot interfere each other. Under this framework, it's unnecessary to introduce hypothesises of wave function collapse or multi universe.

6.2 EPR Paradox in MIP

In a 1935 paper[45], Einstein with Podolsky and Rosen considered an experiment in which two particles that move along the x-axis with coordinates x_1 and x_2 and momenta p_1 and p_2 were somehow produced in an eigenstate of the observables $X = x_1 - x_2$ and $P = p_1 + p_2$ (these two observables commute $[X, P] = 0$).It's easy to understand that the measurement of the position of particle 1 can interfere with its momentum, so that after the second measurement the momentum of particle 1 no longer has a definite value. However two particles are far apart, how can the second measurement interfere with the momentum of particle 2? And if it does not, then after both measurements particle 2 must have both definite position and momentum, contradicting the quantum uncertainty principle. If it does, there exist some "spooky" interaction between two far apart particles, contradicting the locality principle in the special theory of relativity. The orthodox interpretation of quantum mechanics suppose that the second measurement which gives particle 1 a definite position, prevents particle 2 from having a definite momentum, even though the two particles are far apart. The states of the two particles are so call quantum entanglement.

Let's investigate the experimental process in detailed and estimate every uncertainty relations. Suppose two particles that are originally bound in some sort of unstable molecule at rest fly apart freely in opposite directions, with equal and opposite momenta until their separation becomes macroscopically large. Their separation will evolve as

$$x_1 - x_2 = x_{10} - x_{20} + (p_1 - p_2)t/m \tag{6.1}$$

where x_{10}, x_{20} are initial positions of two particles. It's noticed that under MIP, every massive particle is collided randomly by STP, the initial separation of two particle cannot be measured precisely. There exists intrinsic uncertainty $\triangle X_{st} = \triangle|x_{10} - x_{20}|$ as the mean free path between two consecutive collision by STP. According to the uncertainty relation derived from MIP, the momentum difference at least has intrinsic uncertainty as $\triangle P_{st} = \triangle|p_1 - p_2| \geq \frac{\hbar}{\triangle X_{st}}$, because of the commutation $[x_1 - x_2, p_1 - p_2] = 2i\hbar$. Therefore the uncertainty of separation will be

$$\triangle|x_1 - x_2| = \triangle X_{st} + \frac{\hbar t}{\triangle X_{st} m} \tag{6.2}$$

Its minimum is at $\triangle X_{st} = \sqrt{\frac{\hbar t}{m}}$, leading to

$$\triangle|x_1 - x_2| \geq 2\sqrt{\frac{\hbar t}{m}} \tag{6.3}$$

Similarly, the total momentum P is not strictly zero under MIP, which includes at least the intrinsic uncertainty due to

$$\triangle P = \triangle m_{st} v \tag{6.4}$$

where $\triangle m_{st}$is the fluctuation of statistical mass, according to MIP, roughly as one part per million of electron mass. Perform EPR experiment after the second measurement of particle 1, the uncertainty of particle 2 at least will be

$$\triangle p_2 \triangle x_2 = 2\sqrt{\frac{\hbar t}{m}} \triangle m_{st} v \tag{6.5}$$

More importantly , does the intrinsic uncertainty of particle 2 given by MIP contradict the uncertainty relation given by quantum mechanics? If

$$\triangle p_2 \triangle x_2 \leq \frac{\hbar}{2} \tag{6.6}$$

it still contradicts uncertainty relation of quantum mechanics, which means that we will observe the quantum entanglement experimentally, because we have to suppose the "spooky" interaction between two far apart particles to satisfy uncertainty relation. Therefore, we obtain the key criterion of quantum entanglement (momentum-position type) as

$$\frac{\triangle m_{st}^2}{m^2} \leq \frac{\lambda_d}{16\pi L} \tag{6.7}$$

where $\lambda_d = \frac{h}{mv}$ is de Broglie′s wavelength and L is the separation of two particles. So we can conclude that there is a characteristic separation of quantum entanglement as

$$L^* = \frac{\lambda_d}{16\pi}(\frac{m}{\triangle m_{st}})^2 \tag{6.8}$$

When the separation of two particles is larger than L^*, the inequality of (8) cannot be satisfied which means we are no longer able to determine the existence of quantum entanglement from experimental results. The reason is that the intrinsic uncertainty of particle 2 given by MIP has already satisfy uncertainty relation of quantum mechanics automatically. We cannot deduce the existence of 'spooky' interaction in this scenario. For two electrons moving at the speed of $0.01c$, the corresponding characteristic separation will be $L^* \approx 1m$. For two atoms moving at the speed of $0.01c$, the corresponding characteristic separation will be $L^* \approx 10^6 m$.

7 From MIP to Path Integral

The path integral representation of quantum mechanics is a generalization and formulation method for quantum physics, which extends from the principle of action in classical mechanics. It replaces a single path in classical mechanics with a quantum amplitude that includes the sum or functional integral of all paths between two points. The path integral expression was theoretically published by theoretical physicist Richard Feynman in 1948 [49]. Prior to this, Dirac's 1933 paper[50], had major ideas and some early results. The main advantages of the path integral expression is that it treats spacetime equally, so it is easy to generalize to the theory of relativity, which is widely used in modern quantum field theory. However, the basic assumptions of MIP tell us that the effect of each STP colliding on particles can be seen as an independent path. The weight of each independent path is related to the distribution of energy. This is essentially a process of path integration. To understand this concept more clearly, we consider a simple process as follows. Assuming that the effect of random motion of particles over time Δt is from point A to point B. According to MIP, in this process, the change of the action can only be h, 2h, 3h, ..., but the paths are different corresponding to each specific action change. For example, the smallest amount of action change is one h, corresponding to a linear motion from A to B, and the 2h change corresponds to the movement of the polyline, during which the particle is struck twice by STP, and so on. In this picture, the so-called infrared effect is naturally ruled out, that is, the process of less than one h in Δt. The effect of infinity is also ruled out because the instantaneous velocity has certain upper bound which is the speed of light. This suggests that such a path integral effect is a finite summation rather than an infinite, so there is no need to introduce a so-called renormalization procedure. We see that under the framework of MIP, the quantum properties of particles are rooted in nature as the statistical description of their random motion.

7.1 Path Integral of Free Particle and Spacetime Interaction Coefficient

We had argued the real velocity of free particle in space-time satisfies the decomposition as

$$\vec{V}(\vec{x},t) = \vec{v}(\vec{x},t) + \vec{u}(\vec{x},t) + \vec{\nu}(t) \tag{7.1}$$

in which there are two kinetic arguments, they are classical statistical velocity $\vec{v}$ and quantum envelope velocity $\vec{u}$.

There are two kinetic variables with random motion particle in spacetime, which are classical speed $\vec{v}$ and fluctuated speed $\vec{u}$. The corresponding kinetic equations are

$$\frac{\partial \vec{u}}{\partial t} = -\Re\nabla(\nabla \cdot \vec{v}) - \nabla(\vec{u}\cdot\vec{v}) \tag{7.2}$$

$$\frac{\partial \vec{v}}{\partial t} = -(\vec{v}\cdot\nabla)\vec{v} + (\vec{u}\cdot\nabla)\vec{u} + \Re\nabla^2\vec{u} \tag{7.3}$$

Setting $\Psi = e^{R+iI}$, we are able to linearise as

$$\nabla R = \frac{1}{2\Re}\vec{u} \tag{7.4}$$

$$\nabla I = \frac{1}{2\Re}\vec{v} \tag{7.5}$$

which leads to

$$\frac{\partial \Psi}{\partial t} = i\Re\nabla^2\Psi \tag{7.6}$$

During an infinite small time interval ϵ, the solution can be written in terms of integrals as

$$\Psi(x, t+\epsilon) = \int G(x,y,\epsilon)\Psi(y,t)dy \tag{7.7}$$

which represents the superposition of all the possible paths from y to x. The critical observation of Feynman is the weight factor $G(x,y,\epsilon)$ will be proportional to $e^{iS(x,y,\epsilon)/\hbar}$, where $S(x,y,\epsilon)$ is the classical action of particle as

$$S(x,y,\epsilon) = \int L(x,y,\epsilon)dt = \int (K-U)dt = (\bar{K}-\bar{U})\epsilon \tag{7.8}$$

$\bar{K}$ and $\bar{U}$ are average kinetic energy and potential energy separately. In order to show the equivalence between path integral formulation and the spacetime interacting picture, we should derive our basic kinetic equations from the postulation of path integral $G(x,y,\epsilon) = Ae^{iS(x,y,\epsilon)/\hbar}$. For a free particle in spacetime, one has $\bar{U} = 0$,$\bar{L} = \frac{m}{2}(\frac{x-y}{\epsilon})^2$ and $S = \frac{m(x-y)^2}{2\epsilon}$, which leads to

$$\Psi(x, t+\epsilon) = A\int e^{\frac{im(x-y)^2}{2\hbar\epsilon}}\Psi(y,t)dy \tag{7.9}$$

Setting $y - x = \xi$ and $\alpha = -\frac{im}{2\hbar\epsilon}$, it can be written in terms of

$$\begin{aligned}\Psi(x, t+\epsilon) &= A\int e^{-\alpha\xi^2}\Psi(x+\xi, t)d\xi \qquad (7.10)\\ &= A\int e^{-\alpha\xi^2}(\Psi(x,t) + \xi\frac{\partial\Psi}{\partial x} + \frac{1}{2}\xi^2\frac{\partial^2\Psi}{\partial x^2} + \mathcal{O}(\xi^4))d\xi\end{aligned}$$

With the properties of Gaussian integral

$$\int e^{-\alpha\xi^2}d\xi = \sqrt{\frac{\pi}{\alpha}} \qquad (7.11)$$

$$\int e^{-\alpha\xi^2}\xi d\xi = 0 \qquad (7.12)$$

$$\int e^{-\alpha\xi^2}\xi^2 d\xi = \frac{1}{2\alpha}\sqrt{\frac{\pi}{\alpha}} \qquad (7.13)$$

we can obtain

$$\Psi(x, t+\epsilon) = A(\sqrt{\frac{\pi}{\alpha}}\Psi(x,t) + \frac{1}{4\alpha}\sqrt{\frac{\pi}{\alpha}}\frac{\partial^2\Psi}{\partial x^2} + \mathcal{O}(\alpha^{-\frac{5}{2}})) \qquad (7.14)$$

Setting $A = \sqrt{\frac{\alpha}{\pi}}$, we have

$$\Psi(x, t+\epsilon) - \Psi(x,t) = \epsilon\partial_t\Psi(x,t) = \frac{1}{4\alpha}\frac{\partial^2\Psi}{\partial x^2} \qquad (7.15)$$

From this integral, We observed that the most important contribution comes from $y - x = \xi \propto \sqrt{\epsilon}$, where the speed of particle is $\frac{y-x}{\epsilon} \propto \sqrt{\frac{\hbar}{m\epsilon}}$, we see here when $\epsilon \to 0$, the speed divergent in order $\sqrt{1/\epsilon}$. The paths involved are, therefore continuous but possess no derivative, which are of a type familiar from study of stochastic process. With the isotropy of space, we have

$$\partial_t\Psi(\vec{x}, t) = \frac{1}{4\alpha\epsilon}\nabla^2\Psi(\vec{x}, t) \qquad (7.16)$$

Corresponding to the Eq. (7.6), if one requires the equivalence between path integral formulation and MIP, there must be

$$i\Re = \frac{1}{4\alpha\epsilon} \qquad (7.17)$$

$$\Re = \frac{1}{4i\alpha\epsilon} = \frac{1}{4i(-\frac{im}{2\hbar\epsilon})\epsilon} = \frac{\hbar}{2m} \qquad (7.18)$$

Notice that $\Re$ is only an arbitrary parameter in the Eq.(5.31). The consistency between path integral and MIP requires $\Re = \frac{\hbar}{2m}$. An arbitrary finite time interval Δt, can be cut into infinitely many slides of infnitesimal time interval ϵ. And in each ϵ, the collisions leads to many different paths, one can pick one path and consectively another along the time direction, this will end up a path in Δt, sum over all possible paths in Δt gives an integration over path space, which is the celebrated historical summation or path integral. The method here can be straightforwardly generalised to the particle in the external potential as in following section.

7.2 Path Integral of Particle in an External Potential and Spacetime Interaction Coefficient

In an external potential U, one has $\bar{U} = U(\frac{x+y}{2})$ and $\bar{L} = \frac{m}{2}(\frac{x-y}{\epsilon})^2$, which leads to the action

$$S = \frac{m(x-y)^2}{2\epsilon} - U(\frac{x+y}{2})\epsilon \tag{7.19}$$

According to the path integral formulation, it must satisfy

$$\begin{aligned} \Psi(x, t+\epsilon) &= A \int e^{\frac{im(x-y)^2}{2\hbar\epsilon} - \frac{iU(\frac{x+y}{2})\epsilon}{\hbar}} \Psi(y,t) dy \\ &= A \int e^{\frac{im(x-y)^2}{2\hbar\epsilon}} (1 - \frac{iU(\frac{x+y}{2})\epsilon}{\hbar}) \Psi(y,t) dy \end{aligned} \tag{7.20}$$

To the lowest order of ϵ, it shows

$$U(\frac{x+y}{2})\epsilon = U(x + \frac{\xi}{2})\epsilon = U(x)\epsilon \tag{7.21}$$

$$\Psi(x, t+\epsilon) = A \int e^{-\alpha\xi^2} (1 - \frac{iU(x)\epsilon}{\hbar}) \Psi(x+\xi, t) d\xi \tag{7.22}$$

From the properties of Gaussian integral in the previous section, we obtain

$$\Psi(x, t+\epsilon) = A(1 - \frac{iU(x)\epsilon}{\hbar}) \sqrt{\frac{\pi}{\alpha}} \Psi(x,t) + A \frac{1}{4\alpha} \sqrt{\frac{\pi}{\alpha}} \frac{\partial^2 \Psi}{\partial x^2} \tag{7.23}$$

Setting $A = \sqrt{\frac{\alpha}{\pi}}$, $\epsilon \to 0$, we have

$$\partial_t \Psi(\vec{x}, t) = \frac{1}{4\alpha\epsilon} \nabla^2 \Psi(\vec{x}, t) + \frac{1}{i\hbar} U \Psi(\vec{x}, t) \tag{7.24}$$

To be consistent with the case of free particle, let's take $\Re = \frac{\hbar}{2m}$ which leads to

$$\partial_t \Psi(\vec{x}, t) = i\Re \nabla^2 \Psi(\vec{x}, t) + \frac{1}{i\hbar} U \Psi(\vec{x}, t) \tag{7.25}$$

Therefore we have derived the equation of motion from MIP.

8 Electromagnetism: An MIP Approach

8.1 Essential Properties of Electronic Charge In Modern Physics

In framework of modern physic, fundamental matter particles are all electric charged. The fundamental electric charge is defined as the amount of charge of an electron or a positron.

For electric charge [4], there are five fundamental properties. Firstly, there are only two kinds of charges, as known as the positive and negative charges. The characteristic quantum numbers of positron and electron are 1 and -1. Secondly, same charges repel each other, different charges attract each other.Thirdly, the interaction between charges is known as the Coulomb force, obeys the inverse square law. Electron and positron can annihilation each other, emit photons.Forthly, in an isolated system, the algebraic amount of charges are conserved. Finally, the amount of fundamental charge is $1.6 \times 10^{-19}C$.

Dynamics of STP revisited

Since there are no interactions between STP, the differential dynamics of STP is discribed by a massless free scale field theory, its Lagrangian is:

$$\mathcal{L}_{ST} = \partial_\mu \phi \partial^\mu \phi \,. \tag{8.1}$$

The dynamic equation is the 3+1 dimentional Klein-Gordon equation,

$$\partial_\mu \partial^\mu \phi = 0, \tag{8.2}$$

the solution of above equation is a wave solution, it can be written as follow

$$\phi(\vec{x}, t) = \sum_{E^2 = \sum_{i=1}^{3} p_i^2} f(E, \vec{p}) \exp(iEt - i\vec{p} \cdot \vec{x}), \tag{8.3}$$

in which $f(E, \vec{p})$ is an analytic function in momentum space.

Now let us consider putting a particle into space-time. The impact of introducing the matter particle into space-time scalar field, is somehow like dropping a cobble into the water surface of a peaceful lake, leads to the ripple effect. Compare to the fluctuation of space-time, the matter particle introduces a non-perturbative effect, which will bring into the space-time a strong potential. The reason that the matter particle results a strong potential is as follows Any perturbative disturbance will be get drowned out by the fluctuation of microscopic space-time energy fluctuation. In general, strong perturbation will lead to non-linear effects, especially non-perturbative soliton effect. The soliton effect is steady and relatively large than STP. We know STP are local excitation of space-time energy, obviously, a cluster of STP describes a “huge” excitation of space-time energy. So it is nature to introduce solitons into space-time field since a local non-perturbative energy disturbance leads a local space-time soliton, discribing a cluster effect of STP.

[4] We will use charge instead of electric charge in this section, for simplicity.

8.2 2+1-dim Complex Scalar Space-time field

In modern quantum field theory, the microscopic energy can be non-conserved locally, which is saying the vacuum can excit any pair of virtual conjugated particles. In framework of MIP, the fluctuations of space-time energy are STP. The non-conservation nature of local space-time energy is saying the number of STP are locally non-conserved. However, in a global viewpoint, the energy of STP are conserved.

In framework of MIP, we introduce a local companion for STP field, which is a local field that can interact with STP. Howerver, in global, the companion field will decouple with the STP field. The existence of the local companion field also implies in local there is a kind of local symmetry, which is broken in global. In fact, when the local symmetry is $U(1)$, STP are excitations of a complex scalar field.

In framework of MIP, matter particle experiences quantum Brownian motion, which essentially is a Markov progress. This implies the past and future of the matter particle are causual irrelated. So at an arbitrary point of time, one can cut the slice vertical to the direction of the velocity of the matter particle, as known as the normal slice. The dynamics of matter particle on normal slice is a 2+1-dim dynamics. The whole 3+1-dim dynamics could be extented from the dynamics on slices. Notice there are two kinds of dynamics on the 2+1-dim normal slice, one for matter particle, the other for STP, respectively.

We now consider the 2+1 dimensional dynamics of STP. As is stated above, the matter particle drops a cobble into the STP lake and results a period potential. We denote the potential as $V(\phi,\phi^*)$, thus the Lagrangian of complex STP field now becomes

$$\mathcal{L}_{ST} = -\frac{1}{2}\partial_j\phi\partial^j\phi^* + V(\phi,\phi^*), \quad j = 0,1,2. \tag{8.4}$$

8.3 Abrikosov-Nielsen-Olesen-Zumino Vortex

In 2+1 dimension, the famous non-pertubative solution for a complex scalar field is the Abrikosov-Nielsen-Olesen-Zumino(ANOZ) vortex solution [54][55]. The Lagrangian supports the ANOZ vortex is

$$\mathcal{L} = -\partial_j\phi^*\partial^j\phi - \frac{\lambda}{2}\left(\phi^*\phi - F^2\right)^2 \tag{8.5}$$

The minimum of the potential is obvious, it is

$$\phi = F\cdot e^{i\varphi}$$

which is a cycle with radius F. Notice this configuration is compatible with the "ripple" effect of matter particle acting on STP field. It also introduces a symmetry $U(1)$. Since this $U(1)$ now is a local symmetry, it implies there should be a gauge field companion with the STP field. The soliton solution is obtained when introducing the boundary condition at infinity, that is

$$|x| \to \infty : \quad \vec{\phi} \to F\frac{\vec{x}}{|x|}, \quad \phi \to Fe^{i\varphi}. \tag{8.6}$$

However, the soliton solition suffers an energy divergence because

$$E = \int d^2x \left(\vec{\partial}\phi^* \vec{\partial}\phi + V(\phi, \phi^*)\right) \tag{8.7}$$

goes to infinity. One can check this as follows

$$\begin{aligned} |x| \quad &\to \quad \infty : \quad \partial_i\phi_j \to \frac{F}{|x|}\left(\delta_{ij} - \frac{x_i x_j}{|x|^2}\right) \\ \sum_{i,j=1}^{2} (\partial_i\phi_j)^2 \quad &\to \quad \frac{F^2}{|x|^2} \\ \int d^2x \vec{\partial}\phi^* \vec{\partial}\phi \quad &\to \quad 2\pi \int_0^\infty d|x| \frac{F^2}{|x|} : \quad Log \ \ divergent. \end{aligned} \tag{8.8}$$

We saw the energy of the vortex is divergent at spatial infinity, this is unphysical since it implies there is an infinity energy source at spatial infinity. To avoid this divergence, the way out is to introduce a gauge vector field to smear the infinity energy on whole 2+1-dim normal slice. In fact, the local non-conservation of space-time energy implies we need a companion field for STP field in the first place. Here it is clear that the field is a gauge field. To do so, we need introduce the covariant derivative for STP field, instead of original derivative, as well as a kinetic term for the gauge field. Now the Lagrangian is

$$\mathcal{L} = -\frac{1}{2}D_\mu\phi^* D^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi, \phi^*) \tag{8.9}$$

$$D_\mu\phi = \partial_\mu - igA_\mu \tag{8.10}$$

The complex STP field degenerates into a real scalar field. This is because the energy non-conservation is recovered in global. The complexity of the STP field reflects the local property of STP. At spatial infinity,

$$\phi \to Fe^{i\varphi}|_{\varphi=0} = F \tag{8.11}$$

the gradient of STP field is

$$\vec{\partial}\phi = (\partial_r\phi\vec{e}_r + \partial_\varphi\phi\vec{e}_\varphi)_{\varphi=0} = iF/r \tag{8.12}$$

and the gauge field becomes pure gauge field (with vanishing field strength), that is

$$\vec{A} \to \frac{1}{ig}\phi^{-1}\vec{\partial}\phi \tag{8.13}$$

In form of polar coordinates,

$$A_r = 0, \quad A_\varphi = \frac{1}{gr} \tag{8.14}$$

In general, we can not let a complex scalar field directly equals to a real scalar field at an arbitrary spatial point. However, we can let them equals to each other up to a gauge transformation, say

$$\phi \to \Omega F, \quad \Omega(\vec{x}) = e^{i\varphi(\vec{x})} \tag{8.15}$$

and then we have

$$\vec{A} \to -\frac{1}{ig}\Omega\vec{\partial}\Omega^{-1} \tag{8.16}$$

Actually, under this general configuration, the divergence of energy will be strictly vanished, as

$$\vec{D}\phi \to \left(\vec{\partial}\Omega + \Omega(\vec{\partial}\Omega^{-1})\Omega\right) F = \Omega\vec{\partial}(\Omega^{-1}\Omega)F = 0 \tag{8.17}$$

In terms of component, the gauge field reads

$$A_i = -\frac{1}{g}\epsilon_{ij}\frac{x_j}{r^2} \tag{8.18}$$

From the Stokes theorem, we have

$$\Phi \equiv \oint_{C=n\cdot\partial\Sigma} \vec{A}d\vec{x} = \int_\Sigma \vec{B}d\vec{\sigma} = \frac{2\pi n}{g} \equiv g_m \tag{8.19}$$

here we recognize the famous Dirac quantization condition [53] for electronic charges, say

$$g \cdot g_m = 2\pi n, \quad n \in \mathbb{Z} \tag{8.20}$$

This implies if there was an ANOZ vortex solution, the electronic charge is quantized. When n is a negative integer, it describes an opposite spinning vortex solution and also describes a negative charge. In modern physics, there should be a Dirc monople to support the Dirac quantization condition of charges. In framework of MIP, the only origin of quantized charge is the STP field.

8.4 From 2+1-d to 3+1-d

In 3+1 Minkowski space-time, the local space-time symmetry is Lorentz symmetry, denoted by $SO(3,1)$. In Lie group theory, $SO(3,1)$ is algebraic isomorphism to $SU(2) \times SU(2)$, that is

$$so(3,1) \cong su(2) \times su(2) \cong so(3) \times so(3). \tag{8.21}$$

In fact, this isomorphism reveals locally, the 3+1-dim space-time equals to cross extension of two 2+1-dim space-time.

Now we consider how this local extension of dimension can be done from Lie algebra. Notice the six generator of Lorentz group can be written explicityly as

$$K_i \equiv L_{0i} = t\partial_i - x_i\partial_t \quad i,j,k \in [1,2,3] \tag{8.22}$$

$$R_k = \epsilon^{ij}{}_k L_{ij} = \epsilon^{ij}{}_k x_i\partial_j \tag{8.23}$$

The two algebra $su(2)$ are isomorphic to $so(3)$, in terms of derivative, they are

$$S_a = \epsilon^{abc} r_a \partial_{r_b} \tag{8.24}$$

$$\tilde{S}_a = \epsilon^{abc} l_a \partial_{l_b} \tag{8.25}$$

in which there are six degrees of freedom, in the meaning of linear space, they are

$$r_1, r_2, r_3, \quad l_1, l_2, l_3$$

Though the Lie algebras of $SO(3,1)$ and $SU(2) \times SU(2)$ is isomorphic to each other, from the viewpoint of degree of freedom, they are not the same. Notice there is a hidden duality, which maps 2-dim surface to 1+1-dim surface and vice versa, as follows

$$\begin{aligned} \star : e_0 \otimes e_i \to \epsilon_{0i}^{\;\;jk} e_j \otimes e_k \\ \star : e_j \otimes e_k \to \epsilon^{0i}_{\;\;jk} e_0 \otimes e_i \end{aligned} \tag{8.26}$$

This duality is actually the Hodge duality in differential geometry. It implies extension rules should be followed when extending a theory from 2+1-dim to 3+1-dim.

In conclude, we know the rule guiding the extension from 2+1-dim to 3+1-dim is Hodge duality. In the vortex situation considered at hand, the Hodge duality actually corresponds to a resolving of singularity. The vortex has a singular tube which shrinks to a point when goes to its center. If one wants to resolving the singularity, the general way in differential topology is to introduce a finite size sphere instead of the singularity. The resolving operation can be done by two steps: cut the vortex tube at a finite size, which will be a circle, then rotate the circle into a sphere. This rotation was been done in 3+1-dim and is the physical saying of the Hodge duality.

8.5 The Origin of Photon from ANOZ Vortex

In discussion of ANOZ Vortex, we obtained the gauge constraint and the quantization condition of electric charge, however, we didn't obtain the dynamics of the vortex. Because vortex is not a fundamental excitation, its dynamics can not be analytically achieved from fundamental STPs. So in order to obtain the vortex dynamics. We need to introduce the Lagrangian for vortices.

8.5.1 Dynamics on normal slice

For the kinetic part of STPs field, say,

$$\mathcal{L}_\phi = \frac{1}{2}\vec{D}_i\phi^*\vec{D}^i\phi = \frac{1}{2}|(\partial_i - igA_i)\phi|^2 \tag{8.27}$$

in this subsection, $i, j, k, l, m, n = 0, 1, 2$ label indices on the 2+1-dim normal slice. We only consider the excitations nearby the vortex potential, which is $\phi = Fe^{i\varphi}$. The above STP field kinetic Lagrangian can be written as

$$\mathcal{L}_\phi = \frac{1}{2}F^2\left(\partial_i\varphi - gA_i\right)^2 \tag{8.28}$$

After a simple square matching operation, we arrive a linear form

$$\mathcal{L}_\phi = -\frac{1}{2F^2}\xi^i\xi_i + \xi_i(\partial^i\varphi - gA^i) \tag{8.29}$$

here ξ^i is a static auxillary field. Notice that for vortex solution, the phase angle field φ is singular at the vortex center, we now separate the phase angle into two parts, one is smooth and the other is for vortex, say,

$$\varphi = \varphi_0 + \varphi_{vortex} \tag{8.30}$$

The smooth part does not have a significant effect on what we concerned, we integral it out and it results a constraint equation for the auxillary field,

$$\partial_i\xi^i = 0 \tag{8.31}$$

This reveals the auxillary field is a 2+1-dim sourceless field, and it can be rewritten as a pure curl as

$$\xi^i = \epsilon^{ijk}\partial_j a_k \tag{8.32}$$

On the other hand, the equation of motion of auxillary field ξ can also be obtained from Euler-Lagrange equation, it reads

$$\xi^i = F^2(\partial^i\varphi - gA^i) \tag{8.33}$$

The above two equations define a hidden duality as follow

$$F^2(\partial^i\varphi - gA^i) = \epsilon^{ijk}\partial_j a_k \tag{8.34}$$

Substitute it into equation (8.29), we obtain

$$\begin{aligned}\mathcal{L}_\phi &= \frac{1}{2F^2}\xi^i\xi_i = \frac{1}{2F^2}\epsilon^{ijk}\partial_j a_k\epsilon_{imn}\partial^m a^n \\ &= \frac{1}{2F^2}f^{jk}f_{jk}\end{aligned} \tag{8.35}$$

here

$$f_{jk} = \partial_j a_k - \partial_k a_j \tag{8.36}$$

is the field strength of a field. Here we saw the dynamics of the STP field on normal slice is fully equivalent to a vector field a . Recall the kinetic term of gauge field A , we obtain a effective Lagrangian on normal slice

$$\mathcal{L}_{total} = \mathcal{L}_A + \mathcal{L}_\phi = -\frac{1}{4g^2}F^{jk}F_{jk} + \frac{1}{2F^2}f^{jk}f_{jk} \tag{8.37}$$

8.5.2 The Hodge duality

Notice in the dynamics of 2+1-dim vortex, the singularity of the phase angle is essential, which results that the corresponding gauge field A is also singular at the center of the vortex. This singularity could be resolved in higher dimension, for example, in 3+1-dim space-time, we can extend the 2+1-dim Hodge duality (8.34) to 3+1-dim. This 3+1-dim Hodge duality reflects the local duality of 3+1-dim Lorentz group, as revealed in last subsection. In 3+1-dim, the complex STPs field becomes real because the phase angle is fixed to zero and has no dynamics at all, leads a free STP scalar field in 3+1-dim. Actually, in 3+1-dim, we can define the Hodge duality of a field as:

$$F'^{\alpha\beta} = \sqrt{2}gFi\epsilon^{\alpha\beta}_{\ \ ij}f^{ij} \tag{8.38}$$

from which we has defined a gauge field A' , its field strength is

$$F'^{\alpha\beta} = \partial^\alpha A'^\beta - \partial^\beta A'^\alpha \tag{8.39}$$

It is an extension of a field in 3+1-dim and on any 2+1-dim sub-manifold of the 3+1-dim space-time, its dynamics is equivalent to field a . In total, we know

$$\mathcal{L}_{total} = -\frac{1}{4g^2}F^{jk}F_{jk} - \frac{1}{4g^2}F'^{\alpha\beta}F'_{\alpha\beta} \tag{8.40}$$

Actually, in 3+1-dim, the two parts of above Lagrangian can be written as a single term when defined a new field $\tilde{A}$ satisfying

$$\frac{1}{g}\tilde{F}_{ij} = F_{ij}, \quad \frac{1}{g}\tilde{F}_{\alpha\beta} = F'_{\alpha\beta} \tag{8.41}$$

Notice the above equations are six equations, which are

$$\partial_0\tilde{A}_1 - \partial_1\tilde{A}_0 = g(\partial_0 A_1 - \partial_1 A_0) \tag{8.42}$$
$$\partial_0\tilde{A}_2 - \partial_2\tilde{A}_0 = g(\partial_0 A_2 - \partial_2 A_0) \tag{8.43}$$
$$\partial_1\tilde{A}_2 - \partial_2\tilde{A}_1 = g(\partial_1 A_2 - \partial_2 A_1) \tag{8.44}$$
$$\partial_0\tilde{A}_3 - \partial_3\tilde{A}_0 = g(\partial_0 A'_3 - \partial_3 A'_0) \tag{8.45}$$
$$\partial_1\tilde{A}_3 - \partial_3\tilde{A}_1 = g(\partial_1 A'_3 - \partial_3 A'_1) \tag{8.46}$$
$$\partial_2\tilde{A}_3 - \partial_3\tilde{A}_2 = g(\partial_2 A'_3 - \partial_3 A'_2) \tag{8.47}$$

On 0-1-2 normal slice, we can assume

$$\tilde{A}_0|_{\Sigma=(t,x_1,x_2)} = gA_0, \quad \tilde{A}_1|_{\Sigma=(t,x_1,x_2)} = gA_1, \quad \tilde{A}_2|_{\Sigma=(t,x_1,x_2)} = gA_2 \tag{8.48}$$

here $\tilde{A}_i|_{\Sigma=(t,x_1,x_2)}$ denotes the reduced field of the four dimensional gauge field $\tilde{A}$ onto normal slice $\Sigma = (t, x_1, x_2, 0)$. Hence from eq.(8.45-8.47) we see, the constraint equations require that on x_3 direction, $\tilde{A}_0, \tilde{A}_1, \tilde{A}_2$ should coincide with A'_0, A'_1, A'_2 ,

$$A_i(0,0,0,x_3) = A'_i(0,0,0,x_3), \quad i = 0,1,2 \tag{8.49}$$

then we obtain

$$\tilde{A}_3(t,x_1,x_2,x_3) = gA'_3(t,x_1,x_2,x_3) \tag{8.50}$$

Actually, the A'_3 is a new component of the gauge field results from the Hodge duality, it is unique up to a pure gauge with vanishing field strength. Now we see how to extend the gauge field on 2+1-dim to 3+1-dim guiding by the Hodge duality. A simple extension is

$$\tilde{A}_i(t,x_1,x_2,x_3) = g(A_i(t,x_1,x_2,0) + A'_i(0,0,0,x_3)), \quad i = 0,1,2 \tag{8.51}$$

$$\tilde{A}_i(t,x_1,x_2,x_3) = gA'_3(t,x_1,x_2,x_3) \tag{8.52}$$

Under this extension, we arrive a simple Lagrangian

$$\mathcal{L}^{eff}_{3+1d} = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}, \quad \mu,\nu = 0,1,2,3 \tag{8.53}$$

it is the famous Lagrangian for 3+1-dim gauge field, the field strength is the same as Maxwell field strength. In three dimensional form, the field strength can be written as electric and magnetic field strengths as

$$E_i = \tilde{F}_{0i}, \quad B_i = \epsilon_{ijk}\tilde{F}^{jk}, \quad i,j,k = 1,2,3 \tag{8.54}$$

In above derivation, we saw that the dynamic effects of STP ANOZ vortex and 3+1-dim electromagnetic field are completely equivalent. This reveals an important assertion: photons are companion particles of STP vortices. In 3+1-dim space-time, Maxwell field strength is a derived result because of vanishing of the ANOZ vortex singularity.

In conclusion, when introducing the third spatial dimension, the singularity of ANOZ vortex is vanished. Meanwhile the equation of motion for ANOZ vortices is equivalent to 3+1-dim Maxwell equations, they are

$$\vec{\nabla} \cdot \vec{E} = 0 \tag{8.55}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{8.56}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{8.57}$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} \tag{8.58}$$

Here what we obtained is the source-free Maxwell equations because we didn't consider the effect of matter particles, which will couple to gauge field as will considered in next subsection.

8.6 The Coulomb Force

We now consider the force between two matter particles. In hydrodynamics, two vortices will repel each other if their handing of spins are the same, and will attract each other if their handing of spins are different. This is a nature derivation from Bernoulli principle. There are only two kinds of charity for 2+1-dim vortices, left and right, respectively.

More than two decades ago, people had already found the correspondence between equations of motions of hydrodynamics and Maxwell eletromagenetism [56]. This correspondence was supported by [58] with a detailed derivation. The correspondence between hydrodynamics and eletromagnetism is much more like a coincidence in previous researches. However, in framework of the STP vortex, the fluid-eletromagnetism correspondence now has a concrete theoretic origin.

In previous subsections, we only considered dynamics of STP and gauge fields, leaving the matter particle as a source of potential. It is nature to consider the interaction between matter field and gauge field as well. To do so, we introduce the matter field in Lagrangian as follow

$$\mathcal{L}_{total} = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - i\bar{\psi}\gamma^{\mu}\tilde{D}_{\mu}\psi + m\bar{\psi}\psi \tag{8.59}$$

$$\tilde{D}_{\mu} \equiv \partial_{\mu} + ie\tilde{A}_{\mu} \tag{8.60}$$

This interaction can be understood as an effective representation of the collision between matter particle and STP vortices, though their are no terms representing vortices in the Lagrangian. This is because the dynamics of vortices now is equivalent to gauge field in 3+1-dim. Other collisions between matter particle and STP are not considered in this section, as we will see, they also play important roles in deriving gravity between matter particles.

In global, the STP and gauge field are decoupled, hence all local dynamics have been reduced to gauge field dynamics in 3+1-dim space-time. Notice the Lagrangian we obtained above is the same as that in famous QED [17]. Under standard calculation, the interaction between matter particles will be the Coulomb interaction. However, in framework of MIP, the gauge field is not originated from matter field, but from STP vortices. This is an essential difference between modern quantum field theory and the MIP proposed in this article.

Define the four dimensional current as

$$j^{\mu} \equiv i\bar{\psi}\gamma^{\mu}\psi \tag{8.61}$$

we can explicitly see the minimal couple between gauge field $\tilde{A}$ and the electronic current j. The equation of motions now becomes the famous sourced Maxwell equation, as known as

$$\vec{\nabla} \cdot \vec{E} = j_0 \tag{8.62}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{8.63}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{8.64}$$

$$\vec{\nabla} \times \vec{B} = \vec{j} + \frac{\partial \vec{E}}{\partial t} \tag{8.65}$$

8.7 Another Derivation of EoM of Photons

In framework of MIP, we had obtained four properties of charges, they are: 1. There are only two kinds of charges corresponding to left and right chiralities of STP vortices. Same charges repel each other while different ones attract each other. 2. The charges are quantized guiding by the Dirac quantization condition derived from STP vortex. 3. Force between charges are mediated by photons. 4. The force between charges is the Coulomb force.

Based on calculations in previous subsections and discussion of the Hodge duality, we know some properties of phtons in frame work of MIP. At first, it companies with the non-pertubative solition solution, as known as the vortex solution. Secondly, it is a gauge field in 2+1-dim normal slice on which another effective auxillary gauge field lives as well. Thirdly, the 3+1-dim Hodge duality acts on the effective auxillary gauge field does not only resolve the phase singularity of the STP vortex, it introduces the dual part of 2+1-dim gauge field. So the 2+1-dim gauge field and its Hodge dual merged into a 3+1-dim gauge field, which is the photon field, which means on 3+1-dim space-time, the photon field can be understood as toplogical excitations of 2+1-dim gauge field, the topological configuration is known as the Hopf link excitation. We now clarify the conclusion in detail since it is very important to understand the spin of photon, which has a zero mass.

In framework of STP vortex, the vortex tube is made of two fields, one is the STP field ϕ , whose gradient defines the flow direction of the vortex, the other is 2+1-dim gauge field A whose field strength characterizes the spinning direction of the vortex. So in this picture, A describes the rotation and ϕ the flowing. Under the Hodge duality, the dynamics of the soliton part of STP field is equivalent to another gauge field A' , which is Hodge dual to A. Topologically, the vortex tube reprensents a Wilson loop, its Hodge duality is t'Hooft loop. Put them together forms a famous topological object, the Hopf link, as shown in Fig.8.1. The Hopf link is obvious a non-local object. The topological stability of the Hopf link protects it from pertubative destruction, so it can propagate in space-time without dissipation unless it meets another vortex. This is very similar to what happens in electromegnatic interaction, where photons propagates the

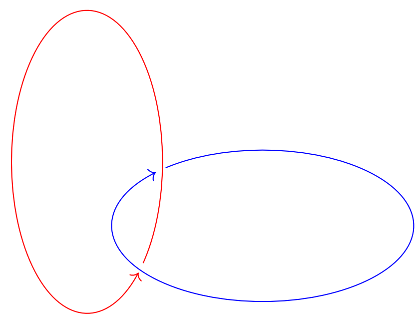

Fig. 8.1: Photon as a topolotical excitation: a Hopf link

interaction between charges. We had seen the equation of motion of the $\tilde{A}$, aka the joint representation of A and A' , is nothing but the Maxwell equations. The $\tilde{A}$ field is an effective representation of the Hopf link.

There are two circles in a Hopf link, they wind the topological subgroup (mathematically, the minimal torus) of Lorentz group separately. As we knew in previous section, they are left and right hand topological circles, each corresponds to a spinor fiber. However, in physics, there are no purely topological objects. So we need to consider the dynamics of the Hopf link, say, the effect resulted from deformation of either circle.

Consider an arbitrary deformation on one of the two circles, it will affect the whole Hopf link and defines a self isomorphism as follow

$$A : \Lambda_L \otimes \Lambda_R \to \Lambda_L \otimes \Lambda_R \tag{8.66}$$

here A denotes the self isomorphism on $\Lambda_L \otimes \Lambda_R$, Λ_L and Λ_R are left and right spinor firbers respectively. In appendix E, we proved that such a self isomorphism should be a vector map. Relatively, all derivatives should be changed into covariant derivatives, as

$$\partial_\mu \to D_\mu = \partial_\mu + igA_\mu \tag{8.67}$$

This leads to non-trivial local transmition that

$$[D_\mu, D_\nu] = D_\mu D_\nu - D_\nu D_\mu = ig(\partial_\mu A_\nu - \partial_\nu A_\mu). \tag{8.68}$$

This reflects the local homomorphism deformation. The strength of the deformation is described by the coefficient g, which relates to charge of matter particle. So we could propose an assertion: the amount of electric charge reflects the strength of local deformation of local space-time. The RHS of above equation is nothing but a field strength of four dimensional gauge field

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{8.69}$$

Since

$$\begin{aligned} D^\mu[D_\mu, D_\nu] &= ig\partial^\mu(\partial_\mu A_\nu - \partial_\nu A_\mu) - g^2 A^\mu(\partial_\mu A_\nu - \partial_\nu A_\mu) \\ &= -ig\Box A_\nu + ig\partial_\nu(\partial^\mu A_\mu) - g^2\partial_\mu(A^\mu A_\nu) + g^2(\partial_\mu A^\mu)A_\nu \\ &= \frac{1}{2}g^2\partial_\nu(A_\mu A^\mu) \end{aligned} \quad (8.70)$$

under Lorentz gauge $\partial_\mu A^\mu = 0$,the above equation only have pure derivation contributions, with vanishing contributions for no-boundary free field. So this equation can be simply written as

$$D^\mu F_{\mu\nu} = 0 \quad (8.71)$$

In three dimensional form, it can be written as

$$\vec{\nabla} \cdot \vec{E} = 0 \quad (8.72)$$

$$\partial_t E - \vec{\nabla} \times \vec{B} = 0 \quad (8.73)$$

In another way, because the Hopf link configuration is unchanged under left-right flop symmetry, this leads to a electromagnetic duality for field strength $F_{\mu\nu}$. The left-right flop symmetry actually means a flop between pair of indices $(0, i) \leftrightarrow (j, k)$, this can be achieved by introducing the Levi-Cevita connection

$$\epsilon^{0ijk} : (0, i) \rightarrow (j, k) \quad (8.74)$$

thus for the field strength $F_{\mu\nu}$, we have the following dual relation

$$\tilde{F}_{\alpha\beta} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu} \quad (8.75)$$

The Levi-Cevita connection flip electric and magnetic fields in three dimensions, and the above dual relation reads

$$\vec{E} \rightarrow \vec{B}, \quad \vec{B} \rightarrow -\vec{E} \quad (8.76)$$

The dual equation in four dimensional is written as

$$D_\mu \tilde{F}^{\mu\nu} = 0 \quad (8.77)$$

In three dimension, it becomes two equations

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (8.78)$$

$$\partial_t \vec{B} - \vec{\nabla} \times \vec{E} = 0 \quad (8.79)$$

Equations (8.72,8.73,8.78,8.79) are Maxwell equations for source-free electromagnetic fields, which proves in 3+1-dim, the Hopf link transforms the local deformation just the same as photon propagates in space-time.

The figure fig.8.2 shows how a deformation propagates from an electron to a positron, where red upper arrows denote left topological circles and blue downer arrows denote right topological circles.

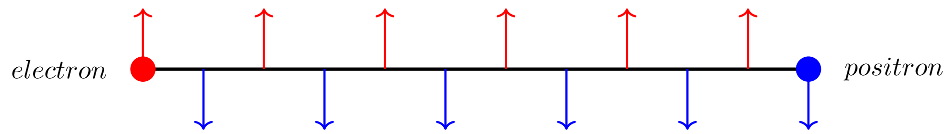

Fig. 8.2: Photons deliver the interaction between electron and positron

8.8 Photon and vortex tube

We had already known that in framework of MIP, the spins of matter particles are originated from collisions between them and STP along topological circles in local space-time. Now we knew the photon could be represented as a Hopf link, which also is winding topological circles in 3+1-dim local space-time. So it is possible the spin of photons are also originated from STP.

In case of matter particles, for examples, electron and positron, their spins are sourced from local winding along left and right topological circles $U(1)_L$ and $U(1)_R$ in local space-time, respectively. At arbitrary moment, electron or positron has a phase angle φ_L or φ_R. These two phase angles are undetermined. It means electron or positron has a local phase angle symmetry, which is $U(1)$ symmetry. Because it is deduced from local space-time symmetry, it is a gauge symmetry.

Let us choose the phase angle be θ . The identical principle for fundamental particles requests the following equqtions

$$\psi \to \psi e^{-i\varphi_L} \equiv \psi e^{-i\theta}, \quad \bar{\psi} \to \bar{\psi} e^{-i\varphi_R} \equiv \psi e^{i\theta} \tag{8.80}$$

from which we know

$$\varphi_L = -\varphi_R = \theta \tag{8.81}$$

It means the gauge group $U(1)$ is the diagonal subgroup of $U(1)_L \times U(1)_R$, with transition matrix be -1. This perspective could be extend to higher dimensional transition matrices, which will leads to non-Abelian gauge groups, for example, $SU(2)$ or $SU(3)$.

In this picture, photon is represented as a Hopf link of 2+1-dim gauge fields, it is massless. However, it carries the information of collisions between matter particle and STP vortices. So it will also record the motion of the matter particle, as well as its spin. Since it is a $(1,1)$ representation of the topological subgroup of Lorentz group. Therefore, from the Hopf link proposition, we obtained photon has spin 1, and massless, and satisfies Maxwell equations. It actually explains how a massless photon has non-zero spin.

8.9 The generation of charged leptons in MIP

In the frame of MIP, there are no more than 3 generations of charged leptons.

According to MIP, the mass of matter particles is a statistical mass deriving from collision of STP. This collision effects of STP can be described by an effective potential $V(x)$,which reflects the strength of the interactions between STP and matter particles and will vary with the statistical mass: the bigger the statistical mass, the stronger effective potential you will get. If the particle is massless, there is no collision. Therefore the space is homogeneous and isotropic so that we can write $V(x) = 0$. On the other hand the previous discussion has shown us that the 3+1-dimensional electromagnetic field is born in vortex solution in 2+1-dimensional spacetime.

In the following we will make a study of the number of generations of charged leptons in the Standard Model, which is still an open question. Crossing any point O in 3-dimensional space there are 3 independent orthogonal 2-dimensional planes. Take O as the origin and choose rectangular coordinate system with the coordinates (x^0, x^1, x^2, x^3) . The Lagrangian equipped with vortex solution in the 2+1-dimensional subspaces can take the following forms

$$L_a^{2+1} = \partial_\mu \phi^* \partial^\mu \phi - \frac{\lambda_a}{2} \left(\phi\phi^* - F^2\right)^2, \tag{8.82}$$

with $a = 1, 2, 3$ respectively corresponding to 3-dimensional spacetime $(x^0, x^2, x^3),\ (x^0, x^1, x^3),\ (x^0, x^1, x^2)$; λ_a is the coupling constant which reflects the strength of the effective potential and is closely related to the statistical mass of the particle. If $\lambda_a = 0$, that is $V(x) = 0$, indicating the particle is massless ,there is neither collision nor vortex solution. So the statistical mass is an essential prerequisite for a particle to get charge. Following the steps in the previous section, bring in the gauge field $\vec{A}$ and investigate the excited states near the lowest point of the potential. From (8.27), we get

$$L_a^{2+1} = L_{\bar{A}} + L_\phi = -\frac{1}{4} F_{ij} F^{ij} + \frac{1}{4} f_{ij} f^{ij}, \tag{8.83}$$

with (i, j) taking values in the corresponding subspace. For the sake of simplicity, we have chosen the coupling constants of the gauge fields to be 1. Now the Lagrangians do not obviously involve λ_a any more and therefore have nothing to do with the statistical mass of the particle. Take Hodge $*$ duality, and lift the 2+1-dimensional theory to 3+1-dimensional spacetime. We take the notation $F_{\alpha\beta} = i\epsilon^{ij}_{\alpha\beta} f_{ij}$. For L_1^{2+1},

$$L_1^{2+1} = -\frac{1}{4} F_{ij} F^{ij} + \frac{1}{4} f_{ij} f^{ij} \tag{8.84}$$

Here the indexes i, j come from the subspace (x^0, x^2, x^3), with $i, j = 0, 2, 3$. The independent components of the field strength are $F_{02}, F_{03}, F_{23}, f_{02}, f_{03}, f_{23}$. and

$$\begin{cases} F_{\alpha\beta} = i\epsilon^{02}_{\alpha\beta} f_{02} \Rightarrow F_{13} = -if_{02} \\ F_{\alpha\beta} = i\epsilon^{03}_{\alpha\beta} f_{03} \Rightarrow F_{12} = if_{03} \\ F_{\alpha\beta} = i\epsilon^{23}_{\alpha\beta} f_{23} \Rightarrow F_{01} = -if_{23} \end{cases} \tag{8.85}$$

Here we take the usual notations as $g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$, $\epsilon^{0123} = 1$. So that for L_1^{2+1} in the 3+1-dimensional spacetime, we have

$$\begin{aligned} L_1^{3+1} &= -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} = -\frac{1}{4}\tilde{F}_{ij}\tilde{F}^{ij} + \frac{1}{4}\tilde{F}_{\alpha\beta}\tilde{F}^{\alpha\beta} \\ &= -\frac{1}{4}\tilde{F}_{ij}\tilde{F}^{ij} - \frac{1}{4}\tilde{f}_{ij}\tilde{f}^{ij} \end{aligned} \tag{8.86}$$

with

$$\tilde{F}_{\mu\nu} = \begin{pmatrix} 0 & -i\tilde{f}_{23} & \tilde{F}_{02} & \tilde{F}_{03} \\ i\tilde{f}_{23} & 0 & i\tilde{f}_{03} & -i\tilde{f}_{02} \\ -\tilde{F}_{02} & -i\tilde{f}_{03} & 0 & \tilde{F}_{23} \\ -\tilde{F}_{03} & i\tilde{f}_{02} & -\tilde{F}_{23} & 0 \end{pmatrix} \tag{8.87}$$

Following the same way, for L_2^{2+1} , we get

$$F_{\alpha\beta} = i\epsilon^{ij}_{\alpha\beta} f_{ij} \Rightarrow \begin{cases} F_{\alpha\beta} = i\epsilon^{01}_{\alpha\beta} f_{01} \Rightarrow F_{23} = if_{01} \\ F_{\alpha\beta} = i\epsilon^{03}_{\alpha\beta} f_{03} \Rightarrow F_{12} = if_{03} \\ F_{\alpha\beta} = i\epsilon^{13}_{\alpha\beta} f_{13} \Rightarrow F_{02} = if_{13} \end{cases} \tag{8.88}$$

$$\begin{aligned} L_2^{3+1} &= -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} = -\frac{1}{4}\tilde{F}_{ij}\tilde{F}^{ij} + \frac{1}{4}\tilde{F}_{\alpha\beta}\tilde{F}^{\alpha\beta} \\ &= -\frac{1}{4}\tilde{F}_{ij}\tilde{F}^{ij} - \frac{1}{4}\tilde{f}_{ij}\tilde{f}^{ij} \end{aligned} \tag{8.89}$$

with

$$\tilde{F}_{\mu\nu} = \begin{pmatrix} 0 & \tilde{F}_{01} & i\tilde{f}_{13} & \tilde{F}_{03} \\ -\tilde{F}_{01} & 0 & i\tilde{f}_{03} & \tilde{F}_{13} \\ -i\tilde{f}_{13} & -i\tilde{f}_{03} & 0 & i\tilde{f}_{01} \\ -\tilde{F}_{03} & -\tilde{F}_{13} & -i\tilde{f}_{01} & 0 \end{pmatrix}. \tag{8.90}$$

For L_3^{2+1} , we can obtain

$$F_{\alpha\beta} = i\epsilon^{ij}_{\alpha\beta} f_{ij} \Rightarrow \begin{cases} F_{\alpha\beta} = i\epsilon^{01}_{\alpha\beta} f_{01} \Rightarrow F_{23} = if_{01} \\ F_{\alpha\beta} = i\epsilon^{02}_{\alpha\beta} f_{02} \Rightarrow F_{13} = -if_{02} \\ F_{\alpha\beta} = i\epsilon^{12}_{\alpha\beta} f_{12} \Rightarrow F_{03} = -if_{12} \end{cases} \tag{8.91}$$

$$\begin{aligned} L_3^{3+1} &= -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} = -\frac{1}{4}\tilde{F}_{ij}\tilde{F}^{ij} + \frac{1}{4}\tilde{F}_{\alpha\beta}\tilde{F}^{\alpha\beta} \\ &= -\frac{1}{4}\tilde{F}_{ij}\tilde{F}^{ij} - \frac{1}{4}\tilde{f}_{ij}\tilde{f}^{ij} \end{aligned} \tag{8.92}$$

with

$$\tilde{F}_{\mu\nu} = \begin{pmatrix} 0 & \tilde{F}_{01} & \tilde{F}_{02} & -i\tilde{f}_{12} \\ -\tilde{F}_{01} & 0 & \tilde{F}_{12} & -i\tilde{f}_{02} \\ -\tilde{F}_{02} & -\tilde{F}_{12} & 0 & i\tilde{f}_{01} \\ i\tilde{f}_{12} & i\tilde{f}_{02} & -i\tilde{f}_{01} & 0 \end{pmatrix} . \quad (8.93)$$

According to the above, starting with 3 different 2+1-dimensional Lagrangians L_a^{2+1}, we end up with the 3+1-dimensional Lagrangians which have the uniform description as

$$\begin{aligned} L_a^{3+1} &= -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} = -\frac{1}{4}\tilde{F}_{ij}\tilde{F}^{ij} + \frac{1}{4}\tilde{F}_{\alpha\beta}\tilde{F}^{\alpha\beta} \\ &= -\frac{1}{4}\tilde{F}_{ij}\tilde{F}^{ij} - \frac{1}{4}\tilde{f}_{ij}\tilde{f}^{ij} \ . \end{aligned} \quad (8.94)$$

In fact they are the same one since they can be converted to each other by rotating the proper coordinate axis as follows

$$L_1^{3+1} \leftarrow (\hat{e}_1 \leftrightarrow \hat{e}_2) \rightarrow L_2^{3+1} \leftarrow (\hat{e}_2 \leftrightarrow \hat{e}_3) \rightarrow L_3^{3+1}, \quad (8.95)$$

which is equivalent to internal rotations of the gauge fields $\vec{A}, \vec{a}$. For the electromagnetic field arising from the lepton with fundamental charge in 3+1-dimensional spacetime, when we trace back to its birth in 2+1-dimensional subspace, we will find out we have 3 degrees of freedom described by $\lambda_a,\ a = 1, 2, 3$, and just corresponding to the 3 different subspaces. Therefore the type of charged leptons is no more than 3. Actually the modern science has told us there are 3 generations of charged leptons in our real world, which is just in accordance with $\lambda_1 \neq \lambda_2 \neq \lambda_3 \neq 0$ in our framework and from the aspect of STP the local isotropy of spacetime is broken. In conclusion, in the frame of MIP, there are no more than 3 generations of charged leptons, which is firmly rooted in the fact that we live in a 3+1dimensional spacetime.

8.10 Conclusion of the section

In this section, from the MIP picture, we explained the origin of electromagnetic interaction in detail. In framework of MIP, the 3+1-dim electromagnetic field represents itself as a Hopf link exicitation made of 2+1-dim gauge field and its Hodge dual partner. It is a topological state. From this topological configuration, we obtained the Maxwell equations in two different ways, also from which, we explained why massless photons have spin 1. In this section, we studied four properties of electric charges, say, positive and negative, quantization, repelling and attracting , Coulomb inverse square law ,and equations of motions of photons, which propagates the Coulomb interaction between charged particles. In addition, together with the charge amount calculated in section 5, we obtained all five properties of the electric charge.

There is one additional expression for the STP vortex configuration.In this section, we only considered the non-pertubative potential came from matter particle. However, a non-pertubative disturbance of space-time energy does not only have such a single origin in our universe. In early universe, the disturbance is very large and STP vortices could also be generated as well as its partner field, the photon field. It implies in early time, the universe was dominated by radiation, which coincides with observations in cosmology. Another example for non-pertubative potential is black holes, near the horizon of a black hole, the space-time energy disturbance is quite large, and it will also generate electromagnetic radiation. This kind of radiation has a completely different origin comparing with Hawking radiation. This may offer quite a lot of new perspectives on black hole and cosmology researches.

Last and most importantly, we derived the generation for charged leptons. This is a completely new result and one can not derive this law in current quantum field theory framework. Within the MIP framework, by invoking the STP vortices, the generation is a direct inference.

9 Muon physics and MIP

9.1 Theoretical framework

Under the framework of MIP, STPs collide with material particles. In quantum field theory, this is equivalent to introducing a massless scalar field into the theory and its interaction with material particles. Therefore, the Standard Model of particle physics needs to be revised as:

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{ST} + \mathcal{L}_{int} \tag{9.1}$$

In the above formula, $\mathcal{L}_{SM}$ is the Lagrangian of the standard model of particle physics; $\mathcal{L}_{ST}$ is the kinetic energy term of the STPs scalar field, which can be expressed as for:

$$\mathcal{L}_{ST} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi \tag{9.2}$$

Since the strength of the collision between STPs and material particles is proportional to the mass of the particles, the interaction term between STPs and material particles is expressed as:

$$\mathcal{L}_{int} = \lambda \sum_{i \in \text{all matter fields}} m_i \phi \bar{\psi}_i \psi \ . \tag{9.3}$$

Where ψ_i represents the material particles in the Standard Model, that is, leptons and quarks. m_i is the mass of the corresponding material particles.

Obviously, for material particles, the mass itself already reflects the information of the collision and interaction between STPs and material particles. So at the

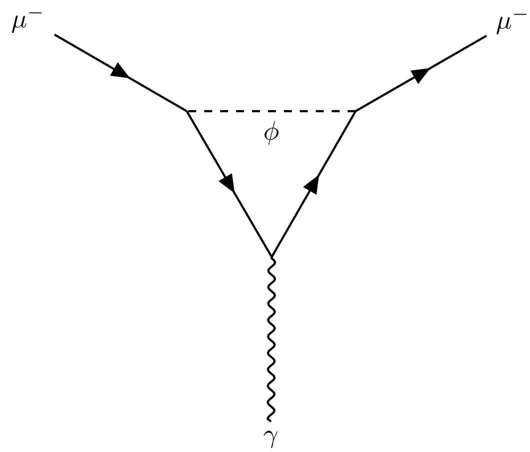

Fig. 9.1: Feynman diagram of the contribution of STPs to the anomalous magnetic moment of muon

tree level, the interaction (9.3) does not change any physics. But at the order of loop diagrams, the interaction of the above equation is ignored by the Standard Model of particle physics.

In this chapter, we will consider the modification of muon physics caused by the interaction of STPs with muons, which includes two aspects. One is the correction of muon anomalous magnetic moment. The second is the lifetime of muon. Muon physics is considered because muons are two hundred times more massive relative to electrons. This means that at the loop diagrams, STPs are about 10^4 times larger than electrons for the correction of muon physics. On the other hand, electrons do not decay, and the effect of STPs cannot be verified in experiments.

9.2 muon anomalous magnetic moment

The anomalous magnetic moment of the muon is contributed by a triangular Feynman diagram. The single loop contribution of the STPs scalar field to the muon anomalous magnetic moment can be represented by a Feynman diagram 9.1. As early as 1972, Jackiw and Weinberg have calculated the contribution of this graph [37], and its contribution to the muon anomalous magnetic moment is:

$$\Delta g_\mu = \frac{3\lambda^2 m_\mu^2}{8\pi^2} \ . \tag{9.4}$$

Jackiw and Weinberg call this contribution in their paper the "virtual scalar field" contribution. Since this "virtual scalar field" does not exist in the Standard Model, the contribution of this scalar field is not considered in subsequent experimental verifications. But in MIP, this scalar field exists undoubtedly , and it refers to the scalar field of STPs. Therefore we need to consider its contribution to the anomalous magnetic moment of muon.

As early as 2006, Brookhaven National Laboratory in the United States discovered experimentally that there is a 3.3σ difference between the anomalous magnetic moment of muon and the prediction of the Standard Model[38], that is,

$$a_\mu(\text{BNL}) = 116592080(63) \times 10^{-11}(0.54\text{ppm}).$$

Where $a_\mu = \frac{g_\mu - 2}{2}$ is the difference value of muon anomalous magnetic moment. In 2021, the Fermi National Laboratory in the United States accurately measured the difference value of the muon anomalous magnetic moment[39], and the result was:

$$a_\mu(\text{FNAL}) = 116592040(54) \times 10^{-11}(0.46\text{ppm}).$$

Combining two experiments, the average of anomalous magnetic moment is:

$$a_\mu(\text{EXP}) = 116592061(59) \times 10^{-11}(0.35\text{ppm}).$$

From the standard model, the theoretical value of a_μ is:

$$a_\mu(\text{SM}) = 116591810(43) \times 10^{-11}(0.37\text{ppm}).$$

The deviation between experiment and theory is:

$$a_\mu(\text{EXP}) - a_\mu(\text{SM}) = 251 \pm 59 \times 10^{-11}.$$

This deviation reaches 4.2σ, so it is a very significant deviation. This means there is a high probability that the contribution of a certain particle is missing from the Standard Model. Under the MIP framework, we believe that this deviation comes entirely from the contribution of STPs. From this deviation, the coupling constant λ of STPs and material particles can be determined, Its value is given as follows:

$$\begin{aligned}\lambda^2 &= (a_\mu(\text{EXP}) - a_\mu(\text{SM}))\frac{16\pi^2}{3m_\mu^2} \\ &= 1.18349(\pm 0.27819) \times 10^{-11}\text{MeV}^{-2} \qquad (9.5)\\ \lambda &= 3.44019^{+0.38300}_{-0.43137} \times 10^{-6}\text{Mev}^{-1} \qquad (9.6)\end{aligned}$$

Therefore, the introduction of the interaction between STPs and muon can completely match the theoretical and experimental results of muon anomalous magnetic moment.

9.3 Muon decay problem

Furthermore, to demonstrate the self-consistency of the scalar field introduced into STPs, we also need to consider the corresponding physics of the single loop interactions between STPs and material particles. In other words, if the introduction of the STPs scalar field and its coupling strength λ results in a contradiction between the theory of a certain physical process and the corresponding experimental results, it is proved that the STPs scalar field is not the source of the deviation of the muon anomalous magnetic moment. Therefore, we consider the single loop process in the muon decay problem. With the participation of STPs, the corresponding Feynman diagram is shown in Figure 9.2:

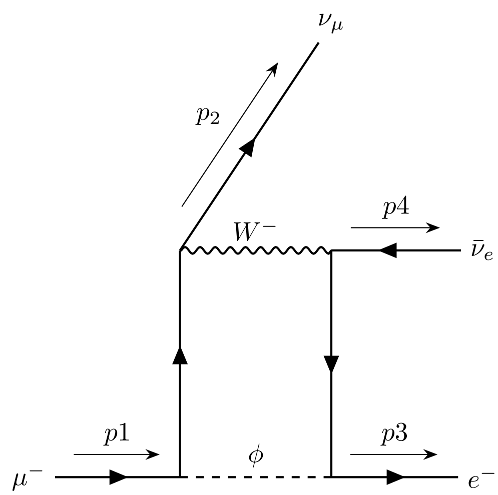

Fig. 9.2: Feynman diagram of the single loop contribution of STPs to the muon decay

The contribution of this box Feynman diagram is the scattering amplitude $\mathcal{M}_\square$ is:

$$\begin{aligned} i\mathcal{M}_\square &= -\frac{g_w^2\lambda^2 m_\mu m_e}{8}\int\frac{d^4k}{(2\pi)^4}\frac{\mathcal{D}(k,p,m)}{\mathcal{N}(k,p,m)} \qquad (9.7)\\ \mathcal{D}(k,p,m) &= \bar{u}(p_2)\gamma^\mu(1+\gamma^5)u(p_1)\bar{u}(p_2)(\not{k}-\not{p}_2-\not{p}_4+m_e)\gamma_\mu(1-\gamma^5)v(p_4)\\ \mathcal{N}(k,p,m) &= \left[(k^2-m_\mu^2+i\epsilon)\right]\left[((k-p_2)^2-m_W^2+i\epsilon)\right]\\ &\quad\times\left[(k-p_2-p_4)^2-m_e^2+i\epsilon\right]\left[(k-p_1)^2+i\epsilon\right] \end{aligned}$$

Without introduction of the STP scalar field, the scattering amplitude of the muon decay can be labeled as follows:

$$\mathcal{M}_{ST} = \mathcal{M}_{tree} + \mathcal{M}_{1-loop} + \mathcal{M}_{2-loop} + \cdots$$

After introducing the STP scalar field, the absolute square of the scattering amplitude can be written as:

$$\begin{aligned} |\mathcal{M}|^2 &= (\mathcal{M}_{ST} + \mathcal{M}_{\Box})\left(\mathcal{M}^*_{ST} + \mathcal{M}^*_{\Box}\right) \\ &= |\mathcal{M}_{ST}|^2 + 2\mathrm{Re}\left[\sum_{\text{all spins}} \mathcal{M}^*_{tree}\mathcal{M}_{\Box}\right] + \text{higher order terms} \end{aligned} \tag{9.8}$$

Therefore, we only need to calculate Re $\left[\sum_{\text{all spins}} \mathcal{M}^*_{tree}\mathcal{M}_{\Box}\right]$ to get the correction of the scattering amplitude.

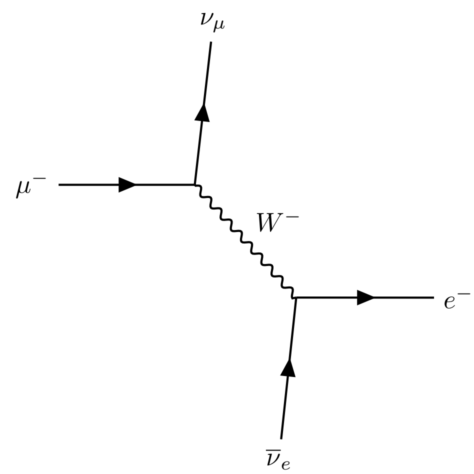

Fig. 9.3: muon decay tree diagram

$\mathcal{M}_{tree}$ represents the contribution of figure 9.3, and its expression is as follows:

$$\mathcal{M^*}_{tree} = -\frac{g_w^2}{8m_W^2c^2}\bar{u}(p_1)\gamma^\mu\left(1-\gamma^5\right)u(p_2)\bar{v}(p_4)\left(1+\gamma^5\right)\gamma_\mu u(p_3) \tag{9.9}$$

Condensing all Dirac matrices using Casimir’ s trick, we finally get:

$$\sum_{\text{all spins}} \mathcal{M}^*_{tree}\mathcal{M}_{\Box} = i\frac{4g_w^4\lambda^2 m_\mu m_e}{m_W^2c^2}\int\frac{d^4k}{(2\pi^4)}\frac{[(k+p_1)\cdot p_4]\,[(k+p_1-2p_4)\cdot p_2]}{\mathcal{N}(k,p,m)} \tag{9.10}$$

We compute this integral using the Mellin – Barnes (MB) representation [40, 41, 42, 43, 44] developed by V. A. Smirnov et al. For the integral kernel in

(9.10), we can do the substitution $k + p_1 \rightarrow k$, and then use the Mellin – Barnes representation to express it as factor multiple form of the Γ function, and finally the MB integral is used to do the appropriate contour integration. Since there are multiple Γ function poles that overlap, the order of the contour integration needs to be evaluated at multiple singular points. We denote the result of the integration of k as $\mathcal{F}(s,t,m)$, where $s = (p_1 - p_2)^2, t = (p_1 - p_3)^2$ is the Mandelstam variable. In muon's stationary reference frame, where $p_1 = (m_\mu c^2, 0, 0, 0)$, the decay rate of muon is

$$\begin{aligned} \mathrm{d}\Gamma = & \frac{\langle |\mathcal{M}|^2 \rangle}{2\hbar m_\mu} \left(\frac{\mathrm{d}^3 \mathbf{p}_2}{(2\pi)^3 2|\mathbf{p}_2|} \right) \left(\frac{\mathrm{d}^3 \mathbf{p}_3}{(2\pi)^3 2|\mathbf{p}_3|} \right) \left(\frac{\mathrm{d}\ {}^3 \mathbf{p}_4}{(2\pi)^3 2|\mathbf{p}_4|} \right) \\ & \times (2\pi)^4 \delta^4 (p_1 - p_2 - p_3 - p_4) \end{aligned} \tag{9.11}$$

The momentum of the electron, anti-electron neutrino and muon neutrino are also clearly written down, which are:

$$p_2 = (|\mathbf{p}_2|c, \mathbf{p}_2),\ p_3 = (\sqrt{|\mathbf{p}_3|^2 c^2 + m_e^2 c^4}, \mathbf{p}_3),\ p_4 = (|\mathbf{p}_4|c, \mathbf{p}_4) \tag{9.12}$$

Substituting the above formula and the momentum of muon p_1 into $\mathcal{F}(s,t,m)$, it becomes $\mathcal{F}(|\mathbf{p}_2|, |\mathbf{p}_3|, m_e, m_\mu, m_W)$ Then the change of decay rate caused by STP is:

$$\Delta\Gamma_{ST} = -\frac{g_w^4 \lambda^2 m_e}{8\pi^3 m_W^2 c^2 \hbar} \int_0^{m_\mu c/2} \mathrm{d}|\mathbf{p}_2| \int_{m_\mu c/2 - |\mathbf{p}_2|}^{m_\mu c/2} \mathrm{d}|\mathbf{p}_3| \mathrm{Im}\left(\mathcal{F}(|\mathbf{p}_2|, |\mathbf{p}_3|, m_e, m_\mu, m_W)\right) \tag{9.13}$$

Substituting into the numerical calculation shows that:

$$\Delta\Gamma_{ST} = 1.2141 \pm (0.2854) \tag{9.14}$$

The muon decay rate calculated from the Standard Model is:

$$\Gamma_{SM} = 455169.311 \tag{9.15}$$

Therefore, the lifetime of muon under the action of STP is:

$$\tau_\mu = 1/(\Gamma_{SM} + \Delta\Gamma_{ST}) = 21969788(\pm 14) \times 10^{-13} s \tag{9.16}$$

Experimentally, the muon lifetime is

$$\tau_\mu(\mathrm{Exp}) = 21969811(\pm 22) \times 10^{-13} s \tag{9.17}$$

It can be seen that after adding the contribution of the STP scalar field, the theoretical lifetime of the muon perfectly matches the experimental observations.

9.4 Summary

In this chapter, we consider two modifications for muon physics due to STP. First, we consider the correction of the STP scalar field to the muon anomalous magnetic moment. The interaction strength λ between the STP scalar field and the matter particle is determined. Second, we calculate the correction of the STP scalar field for muon decay, which makes the theoretical predictions agree with the experimental observations perfectly. It can be seen that we only need to introduce one free parameter, the STP scalar field interaction strength, we achieved a great triumph in the area of muon physics.

10 STP Vortices as origin of spin

In this chapter, we will discuss the essence of spin from the topological structure of STP vortex.

While introducing into the gauge field in the 2+1 dimensional normal space, the singularity at the center of votex was resolved as a S^1. On the differential geometry point of view, this S^1 can be seen as the spatial edge of the vortex. Because of Hodge duality, we can obtain the dual S^1 which will be denoted as $S^{1\star}$. Hence in the 3+1 dimensional space-time, the simplest topological structure involving S^1 and $S^{1\star}$ is a Hopf link, which is a direct intersection of these two circles. As known in knots theory, there are more fundamental connect way for S^1 and $S^{1\star}$. The fundamental stone of topological intersection is the famous skein relation, which can be explicit as in the Fig.10.1

A single Hopf link actually have two twisted points, each of them is the mirror image of the other one. Mathematically, the two twisted points Hopf link is not the most fundamental topological structure. The most fundamental one is the single twisted point connection, which is shown in Fig.10.2

Within the STP vortex configuration, we could have the following algebra-knot correspondence: the fundamental representation of the Lorentz group corresponds to the single twisted point connection of two cirles, which are edges of two dual vortices. The two twisted points connection corresponds to the adjoint representation of the Lorentz group.Under this framework, the algebraic representation of Lorentz group and the topological knot representation has a deep and explicit corelation.

Even in mathematics, this correspondence is a new conjecture, we do not have a direct proof at this stage. However, the indirect way to proof the conjecture is worth to study. For example, connect the affine representation to each other, that is saying, finding an integrable correlation between Schur polynomial and Jones polynomial.

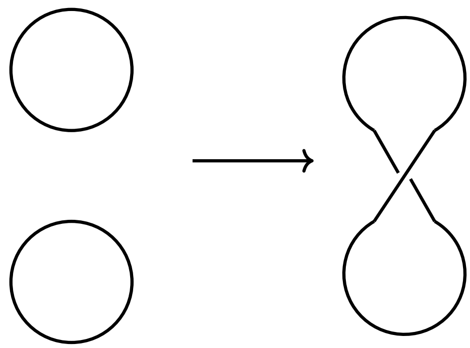

Fig. 10.2: Topological phase transition of STP vortices

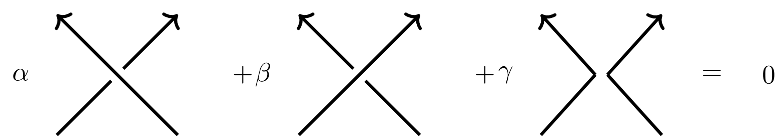

Fig. 10.1: Skein relation

10.1 Topological phase transition of STP vortices

There are vortices on the tangent space and the normal space, since from the point view of isotropic STPs, there are no differences between these two spaces. Actually, in previous chapter, what we solved on the normal space has its Hodge dual on the tangent space. Therefore, in 3+1 dimensional space-time, we need to understand the interaction theory of two vortices living on dual spaces.

The interaction between two vortices can make centers of them fuse or intersect to each other. As we had known in previous chapter, because of the existence of gauge field, the singular center of the vortex had been resolved into a S^1. If there are no interactions between S^1 and its dual $S^{1\star}$, the dynamics on tangent space and normal space will completely decoupled. If this is the case, the dynamic of STPs around the matter particle will be un-isotropic and un-uniform. This obviously violates the physical fact. In other words, if the dynamics on tangent space and the one on normal space do not couple to each other, the space-time will be choked as slides. Hence the naturally way to couple these two dynamics of STPs leads to a phase transition.

The simplest topological phase transition is as shown in Fig.10.2. Notice that Edward Witten had used the skein relation developed by John Conway in 1969 to study knot invariant. It is amazing that the topological phase transition shown in10.2 is the same as John Conway's skein relation.

Therefore, we have already known the two vortices on tangent space and normal space respectively can form a topological twisted point through topological transition as well as the skein relation. For current double vortex case, because we could related the two vortices to each other by a single Lorentz rotation. This means the double vortex sysem has an internal symmetry. A careful study reveals the group is a double cover of $SO(3)$, respect to the Z_2 symmetry of the double vortices. this is because the center of STP vortex is what the matter particle sit on, hence in 3+1 dimensional spacetime, the two vortices have the same center. We splitted these two vortice by hand is a convenient way to explictly reprensent them. Therefore, the rotation subgroup of Lorentz group is the double cover of $SO(3)$, that is, $SU(2)$. This concludes the internal consistence between topological twisted point and spin.

10.2 The isotropic vortex

In previous chapter, we introduced into the 2+1 dimensional gauge field for vanishing the energy singularity at the center of STP vortex. The resolving of the singularity as an S^1 is the same as to introducing a $U(1)$ principal bundle structure in mathematics. The 2+1 dimensional gauge field is nothing but the connection on this principal bundle. However, the resolving operation blows up the singularity on the center of STP vortex does not reconfiguration all properties the singularity. As the center of STP vortex, the singularity is isotropic, but the circle S^1 is orientable. This means we covered the un-orientability of the singularity by the resolving operation. Now it is clear that we need to recover the un-orientability on the circleS^1.

In 1976, T. Martin [57] noticed that there is a correspondence in mathematics as follows. The rotation and translation effects can be separated geometrically. Hence there are two connections correspond to rotation and translation, respectively. The rotation connection corresponds to the torsion tensor, which has the similar meaning as curvature to translation effects.

We now consider the 2+1 dimensional STP vortex, it is nothing but a microscopic space-time. In this space-time, the torsion can not be negligible. The existence of microscopic torsion has no influence to the general relativity, since the geodesic line is unrelying on the torsion at all.

As saying in MIP, the matter particle obtains the mass property by collision of STPs and itself. In this picture, without STPs, the matter particle generated a space-time potential. The potential leads to a curved space-time around the matter particle. Microscopically, the metric around the matter is curved.

Before introducing the torsion tensor, we need to introduce the everywhere othogonal tangent vielbein field $e_a(x)$ as following

$$e_a(x) = e_a^i \frac{\partial}{\partial x^i}, \quad a = 0, 1, 2 \tag{10.1}$$

it satisfies the relation as:

$$g^{ij} = \eta^{ab} e_a^i e_b^j, \quad \eta_{ab} = g_{ij} e_a^j e_b^i \ . \tag{10.2}$$

It is natural to define the dual cotangent vielbein field, as:

$$\theta^a(x) = \theta_i^a dx^i \tag{10.3}$$

they satisfies the normal condition

$$< \theta^a, e_b >= \delta_b^a \tag{10.4}$$

and

$$g_{ij} = \eta_{ab} \theta_i^a \theta_j^b, \quad \eta^{ab} = g^{ij} \theta_i^a \theta_j^b \tag{10.5}$$

now the differential interval

$$ds^2 = g_{ij} dx^i dx^j = \eta_{ab} \theta_i^a \theta_j^b dx^i dx^j = \eta_{ab} \theta^a \theta^b \tag{10.6}$$

the spin connection can be defined by covariant differential on tangent vielbein field, as:

$$\omega_{ia}^b e_b = D_i e_a, \quad \omega_{ia}^b =< D_i e_a, \theta^b > \tag{10.7}$$

where $\omega_{ia}^b(x)$is the spin connection coefficient, and

$$\omega_a^b(x) = \omega_{ia}^b(x) dx^i \tag{10.8}$$

is the spin connection 1-form field. The covariant differential now is defined as following:

$$D' = \partial + \omega \tag{10.9}$$

when acting on a vector field $\xi^a(x)$,

$$D_i' \xi^a = \frac{\partial \xi^a}{\partial x^i} + \omega_{ib}^a \xi^b \tag{10.10}$$

Now we can discuss the coupling between spinor field and space-time under local Lorentz symmetry. If there is a spinor field $\psi(x)$, aka a spin representation of local Lorentz group, then on dynamical point of view, the momentum term of this spinor field can be written as:

$$D_i' \psi = \partial_i \psi + \frac{1}{2} \omega_i^{ab} \Sigma_{ab} \psi \tag{10.11}$$

here Σ_{ab} is the spin representation of Lorentz algebra,

$$[\Sigma_{ab}, \Sigma_{cd}] = \eta_{bc} \Sigma_{ad} + \eta_{ad} \Sigma_{bc} - \eta_{ac} \Sigma_{bd} - \eta_{bd} \Sigma_{ac} \tag{10.12}$$

Introducing the spin connection ω_i^{ab}, the parallel transition of cotangent field $\theta(x)$ defines the torsion of this manifold

$$\begin{aligned}\tau_{ik}^a &= D_i'\theta_k^a - D_k'\theta_i^a \\ &= \frac{\partial\theta_k^a}{\partial x^i} - \frac{\partial\theta_i^a}{\partial x^k} + \omega_{ib}^a\theta_k^b - \omega_{kb}^a\theta_i^b\end{aligned} \tag{10.13}$$

it is the field strength of the cotangent vielbein. When it is not zero, the manifold is not torsion-free and hence intrinsic twisted.The un-vanishing of the field strength of cotangent vielbein implies there is a multi-value property when we joint two 2+1 dimensional theories into a single 3+1 dimesnional theory. We know there exists a sigularity at the center of STP vortex, meanwhile the vielbein rounds the singularity, the vielbein will generate a monodromy matrix at the singularity. To incomplete the contribution of this 2×2 monodromy matrix, we need to consider the following action:

$$I = \int d^3xTr[\epsilon^{ijk}\theta_i^a\tau_{jk}^a] + \int d^3x^\star Tr[\epsilon^{ijk}\theta_i^{a\star}\tau_{jk}^{a\star}] \tag{10.14}$$

here the Tr means summation on vielbein indices. The $\star$ indices means those torsion related variables are defined on dual 2+1 dimensional space-time. As we saw, (10.14) actually is a simple split joint of two 2+1 dimensional Chern-Simons theory defined on different boundary of the 3+1 dimensional space-time. Therefore, we need to introduce the joint constraint, which is obvious the Hodge duality. It is easy to proof that within the following constraint, the first term and the second term in (10.14) Hodge dual to each other. The constraint is :

$$\epsilon^{ijk}\theta_i^a = \epsilon^{ijkl}\tau_{il}^{a\star}, \quad \epsilon^{ijkl}\tau_{jk}^a = \epsilon^{ijl}\theta_j^{a\star} \tag{10.15}$$

Now the two 2+1dimensional Chern-Simons theory is fused into a 3+1 dimensional instanton interaction:

$$I = 2\int d^4x\epsilon^{ijkl}Tr\left(\tau_{il}^{a\star}\tau_{jk}^a\right) \tag{10.16}$$

We see, under the fused situation, the contribution of cotangent vielbein is completely equivalent to a topological instanton contribution of a gauge field. The instanton contribution is nothing but a constant, so now the task is to calculate this constant factor.

Written (10.16) as the differential form, it can be recognized as a characteric number in 3+1dimensional space-time. Notice when accomplish with the cotangent vielbein, on the 2+1 dimensional space-time, the boundary of the vortex could be seen as an S^2. We now joint two S^2 into a boundary of 3+1 dimensional space-time. If the concatenation is trivial, then the 3+1 dimensional spacetime has a boundary with topology $S^2 \times I$. However, the 3+1 dimensional space-time is $R^{3,1}$, when there exists no particles, the boundary is a null set. The boundary can be seen as an S^3 within the matter particle. So it means when we transform the two 2+1 dimensional vortices, the concatenation of their boundaries (S^2) is non-trivial. The final result of this concatenation is to generate an S^3. In fact, this is the way how the two 2+1 dimensional vortices become a microscopic stable configuration around the matter particle in 3+1 dimensional space-time.

Now consider the cobordism characteristic number of (10.16), it discribes the phase angle changing from $S^2 \times I$ to S^3. The phase angle difference describes the charactersitic number, we obtain:

$$I = 2 \times \frac{vol(S^3)}{vol(S^2)} \times N = 2 \times \frac{2\pi^2}{4\pi} \times N = \pi N \tag{10.17}$$

Here N is the topological number according to torsion τ, aka the winding number. It describes the multiplicity of the mapping from $S^2 \times I$ to S^3 . In physics, it is the *theta* contribution.

When considering the wave function of matter particle, we do not see the contribution of the characteristic number. Therefore the topological phase transition just contributes the signature of the wave function, as:

$$\Psi^{[N]} = \psi(x,t)\exp(iI) = (-)^N \psi(x,t) \tag{10.18}$$

when the particle rotate around some fixed axe one whole circle, the corresponding 2+1 dimensional STP vortex also rotated one times around the S^3 , the result is the topological winding number changes by 1, now

$$\Psi^{[N]} \to \Psi^{[N+1]} \ or \ \Psi^{[N-1]} \tag{10.19}$$

as

$$\Psi \to -\Psi \tag{10.20}$$

so we have proved the spin of matter particle should be 1/2, as known as the Fermionic property.

From which we observed above, we obtain an important conclusion. The spin statistical property of matter particle is originate from the un-orintable of singularity sitting on the center of STP vortex around matter particle. This singularity is double covered, there are two 2+1 dimensional vortices around it. The two vortices reconstruct the singularity by manifold cobordism and thus incomplete the isotropic property of the singularity. The spin property of matter particle corresponds to the topological phase transition at the cobordism. In general, in the frame of MIP, the spin of matter particle describes the topological order that corresponding to topological phase transition of STP vortices around the matter particle.

10.3 Pauli exclusion principle

We now use s to label the topological order according to the topological phase transition of STP vortices. For union definition convenience, we let the topological order as a quantum evolution operator, that is:

$$e^{\frac{i}{\hbar}\hat{s}\theta}|\Psi\rangle = e^{i\theta/2}|\Psi\rangle \tag{10.21}$$

From this definition we could take this topological order as an operartor that has eigenvalue $\frac{\hbar}{2}$, for example, $\langle\hat{s}\rangle = \frac{\hbar}{2}$. The parameter of rotation one circle is $\theta = 2\pi$, subsitute this parameter into previous equation, one obtains the Fermionic statistical property immediately.

Now let us consider a permutation of two coincident particles. Suppose particle 1 is on the state $|\Psi_{x_1}(p)\rangle$ and particle 2 is on the state $|\Psi_{x_2}(p)\rangle$. Then the direct product system of these two particle is on the state $|\Psi_{x_1}(p)\rangle \otimes |\Psi_{x_2}(p)\rangle$. We could rotate the $|\Psi_{x_1}(p)\rangle$ as well as the$|\Psi_{x_2}(p)\rangle$ half a cirlce around the center between x_1, x_2 . Because the vortices around these two particles also rotated two half a circles, hence

$$\begin{aligned} T_{x_1,x_2}e^{i\pi\hat{s}}|\Psi_{x_1}(p)\rangle \otimes |\Psi_{x_2}(p)\rangle &= e^{i\frac{\pi}{2}}|\Psi_{x_2}(p)\rangle \otimes e^{i\frac{\pi}{2}}|\Psi_{x_1}(p)\rangle \\ &= -|\Psi_{x_2}(p)\rangle \otimes |\Psi_{x_1}(p)\rangle \end{aligned} \tag{10.22}$$

here T_{x_1,x_2} exchanges x_1, x_2. Therefore if there are two coincident matter particles, on the same state, and sit on a same position, then it is easy to see a direct result from (10.22):

$$|\Psi_x(p)\rangle \otimes |\Psi_x(p)\rangle = -|\Psi_x(p)\rangle \otimes |\Psi_x(p)\rangle \tag{10.23}$$

when and only when $|\Psi_x(p)\rangle \otimes |\Psi_x(p)\rangle = 0$ the previous result can be the case. however, $|\Psi_x(p)\rangle \otimes |\Psi_x(p)\rangle = 0$ means the state actually does not exist! So the Pauli exclusive principle is a natural result in the frame of MIP.

11 MIP and Special Relativity

Under the framework of MIP, STP itself has no self-interaction. The speed of STP is constant at v_{st}. The effect of STP on particles is a stochastic dynamics problem, which makes the particle's time derivative $d\vec{x}/dt$ not well defined. Under this framework, the classical speed of particles only has clear meaning under statistical average. Because any experimental results can't isolate the effect of STP which are generated at the time that cannot be accurately known, it can't be determined from the beginning that the initial velocity of the particles is the classic speed in textbooks. This is actually just an ideal concept, and there is actually no such so-called classic speed. In the framework of MIP, the so-called "classic speed" only has a statistical meaning, which actually represents the statistical average speed of particles in spacetime. In the following, when we say classical speed, the actual meaning refers to the statistical average speed of the particles.

11.1 Equivalence between Inertial Reference Systems

Under the MIP framework, the real moving speed of a material particle is

$$\vec{V} = \vec{v} + \vec{u} + \vec{\nu} \tag{11.1}$$

The quantum envelope velocity of free material particles can be obtained by combining the wave function corresponding to the particle of matter, $\psi = e^{i(\vec{p}\cdot\vec{x}-Et)/\hbar}$

$$\vec{u} = 2\Re\nabla R = 0 \tag{11.2}$$

Classic statistical speed of matter particles

$$\vec{v} = 2\Re\nabla I = \frac{\vec{p}}{m} \tag{11.3}$$

From the three-speed decomposition process, we know that the classic statistical speed $\vec{v}$ and the fluctuation speed $\vec{\nu}$ are independent, so the fluctuation speed of $\vec{\nu}$ is completely caused by STP collision as

$$\vec{\nu} = f(\{\vec{v}_{st}\}) \tag{11.4}$$

Where $\{\vec{v}_{st}\}$ is the speed of all STP and satisfies $f(0) = 0$. Regardless of the classical speed of free material particles , the principle of MIP holds true in all inertial systems. Therefore equivalence between the inertial reference systems is no longer a postulate, but a basic law. In the following, we will study the transformation law between inertial systems and prove that if and only if the speed of light and the speed of STP are equal, the Lorentz transformation of special relativity can be derived naturally.

We also notice that the interaction of STP on particles causes the particles to perform random fluctuations. The speed of fluctuational movement is very different from that of the classic speed. It is essentially a relative speed that is constant under time reversal. In the Appendix B, we prove that this random Markov fluctuation is not related to the classical motion, so it is also invariant under the transformation of the inertial reference system. From this perspective, the equivalence between inertial reference systems is a natural inference under the MIP framework.

11.2 STP Collision and Particle Mass

In a random process in which particles are collided by STP, the instantaneous velocity of the particle is equal to its classical statistical speed $\vec{v}$, the fluctuation velocity is $\vec{\nu}$, and the quantum envelope velocity is $\vec{u}$ overlay, ie the eq.(11.1) . We know that under time reversal T ($T: (t, \vec{x}) \rightarrow (-t, \vec{x})$), there is

$$T: \vec{v} \Rightarrow -\vec{v}, \quad T: \vec{u} \Rightarrow \vec{u}, \quad T: \vec{\nu} \Rightarrow \vec{\nu} \tag{11.5}$$

This is because for a continuous time Markov process under time reversal, it is still a Markov process. The quantum envelope velocity $\vec{u}$ and the fluctuation velocity caused by the STP collision $\vec{\nu}$ are actually the consequences of the STP collision on the particles. Both the quantum envelope velocity and the fluctuation velocity are independent of the classical statistical velocity under the time inversion transform. Therefore, we can always consider the scenario where the particles with zero classic velocity are collided by STP. At this point, the relative speed of STP to particles is v_{st} and will always be v_{st}.

Because of STP, absolutely free particles do not exist. We refer to particles whose classical statistical speed is constant as "free particles." For free particles, the quantum envelope velocity is $\vec{u} = 0$, and $\vec{\nu}$ is a random variable. As can be seen from the probability distribution of $\vec{\nu}$, $\vec{\nu}$ has nothing to do with the classic statistical speed $\vec{v}$. The true speed of such free particles is $\vec{V}$ which consists of two parts that are independent of each other, namely

$$\vec{V} = \vec{v} + \vec{\nu} \tag{11.6}$$

When the particle's classic speed $\vec{v}$ changes, $\vec{\nu}$ does not change. It shows that the STP background does not change with the change of the classical speed of the particles themselves. This is actually the equivalence of the inertial reference system.

On the other hand, we already know that the statistical mass of a particle actually induced by impact of STP. The more particles are hit by STP per unit time, the greater the statistical mass of the particles.

First let's consider the static mass of the particles. Particles in spacetime are always subject to random collisions of STP. We know that the mass of a particle reflects the statistical properties of the motion which manifests in STP impacts. This does not mean that there is no statistical mass in the reference frame where the particle's classical velocity is zero. Because in the stationary reference frame, the motion of the particle is still a random quantum Brownian motion, except that its statistical position is at the origin. Therefore, when considering the motion of particles, we should first separate the mass in the stationary reference frame and then consider the change in the number of relative collisions due to motion. If the stationary particles are subjected to N_0 in the z direction in the z direction unit time, and the moving particles are subjected to N collisions in z direction per unit time. . At this point, the number of impacts on the particle in the same time period on the $x - y$ plane still appears to be N_0, so the mass is not a scalar property. But in fact, if we want to guarantee the principle of relativity is right, then the number of collisions in any direction will increase relatively. Here comes the mass observed by the laboratory observer:

$$m = m_0 N/N_0 \tag{11.7}$$

Where N is the number of times which the particle is hit by STP per unit time observed by the laboratory observer, and m_0 is the static mass of the particle.

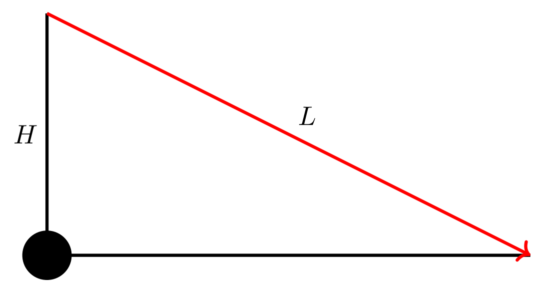

Fig. 11.1: Collision between STP with matter particle.

11.3 Time Dilation Effect

From the relative speed constant assumption, the distribution of STP under the reference system transformation will not change. If the distribution of the STP is uniform and isotropic in the rest frame , then because the speed of STP relative to the particle does not change, in the frame of relative velocity $\vec{v}$. The distribution of STP is still uniform and isotropicl. It should be noted, however, that the time costs for the same collision process in different reference frames are different. This can be explained by the following explanation.

In the rest frame, the STP is at a constant speed v_{st} , moving toward the particle from the distance H . After time t , it will collide with the particle, so the time

$$t = H/v_{st}. \tag{11.8}$$

However, in the moving frame with constant velocity v, after time t'. The distance between the time and space will be L. And the distance from the particle is$\sqrt{L^2 - H^2}$, Then $t' = L/v_{st}$, the following formula pops

$$\frac{L}{v_{st}} = \frac{\sqrt{L^2 - H^2}}{v} \tag{11.9}$$

hence

$$t' = \sqrt{t'^2 - t^2}\frac{v_{st}}{v} \Rightarrow T = t'\sqrt{1 - \frac{v^2}{v_{st}^2}} \tag{11.10}$$

As long as the speed of STP v_{st} equal to the speed of light c , the above equation returns to the relativity of simultaneousity in special relativity. Because of zero static mass of photon , the STP and photons have no interaction. On the other hand, due to the isotropy of STP, there can always exist STP moving parallel to the photon, so there are no relative movement between such STP and the photon. Hence the speed of light should be essentially equal to the speed of STP, that is $c = v_{st}$, which is a rigorous conclusion. and is the physical origin of the axiom of invariance of speed of light in special relativity.

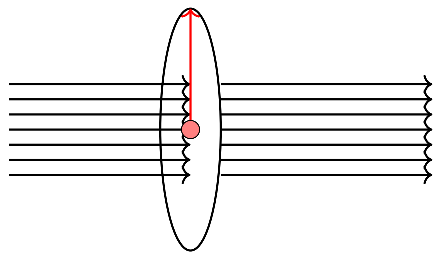

Fig. 11.2: Flux of STP cross a disk.

11.4 Relativistic Mass Effect

Now we consider the expression of the particle mass under the frame transformation. Due to the homogeneity and isotropy of STP distribution, we can assume the density of the STP is ρ_0, and in moving frame with constant velocity v, the particle moves along the horizontal direction .

Then in the rest frame, the number of STP passing through the disc on the vertical direction is

$$N_0 = \rho_0 \pi r^2 v_{st} \Delta t \tag{11.11}$$

The mass of the particle is

$$m_{st} = k_{st} \rho_0 \pi r^2 v_{st} \Delta t \tag{11.12}$$

where k_{st} is the propotional coefficient. In the moving frame with constant velocity v, the number of STP passing through the same disc is

$$N = \rho_0 \pi r^2 v_{st} \Delta t' \tag{11.13}$$

Notice that in moving frame, the time interval $\Delta t'$ is coincident with that in rest frame. The time dilation is universal along all directions, not only along the moving direction. Therefore, numbers of collisions on three spatial directions are uniformal N.

Since

$$\Delta t' = \Delta t / \sqrt{1 - v^2/v_{st}^2} \tag{11.14}$$

we have

$$m'_{st} = k_{st} N = k_{st} \rho_0 \pi r^2 v_{st} \Delta t' = m_{st} / \sqrt{1 - v^2/v_{st}^2} \tag{11.15}$$

When $v_{st} = c$, it is the same expression of relativistic mass as in special relativity.

11.5 Length Contraction Effect

Next, we consider the relativity of spatial distance, which is the length contraction effect in the special theory of relativity. In the framework MIP, we have two independent methods to derive this effect.

The first derivation is a natural inference of time dilation and mass enhancment effects, which we have derived in the above two sections. In each direction, STP and the particle's unit cross-section have the same collision number N. The time dilation is independent of the spatial direction (that is, the orthogonality is guaranteed). Based on these two points, in the parallel direction and the vertical direction of the particle motion, each of the collisions of a rectangular cross section is considered. In the case where the particles are stationary, two rectangular sections are both a long , the width is also b.

For a rectangular section in the parallel direction of particle motion (which is perpendicular to the direction of particle motion), the increase in the number of collisions N is due to the dilation of time, see the equation (11.15) . In the view of a stationary observer, if the area of the rectangle is constant, then the rectangular section in the vertical direction of the particle motion (which is parallel to the direction of particle motion) will have more STP passes. Similar to calculation of time dilation, we can get it at $\Delta t'$ The number of STP passing the same area are:

$$\tilde{N} = \rho_0 abv_{st}\Delta t'/\sqrt{1 - v^2/v_{st}^2} = N/\sqrt{1 - v^2/v_{st}^2}. \tag{11.16}$$

Then this result will show that the mass is not isotropic, which clearly violates the definition of statistical mass. The only way to resolve this contradiction is to make the length in the particle's direction of motion contract, and its contraction ratio is exactly equal to $\sqrt{1 - v^2/v_{st}^2}$. This makes $a \to a' = a\sqrt{1 - v^2/v_{st}^2}, \quad \tilde{N} = N$. Therefore, from the inherent self-consistency of the theory, the moving ruler under the framework of MIP must be contracted.

The second derivation is discussed below.

We consider the relativity of the spatial distance, that is, the measure effect in special relativity. We first consider the rest reference system, the length of the ruler is

$$l_0 = x_B - x_A \tag{11.17}$$

When the ruler is moving along x- direction in speed of v , as shown in the following figure.

The spacetime coordinates at both ends of the ruler are

$$(x'_A, t'_A), \quad (x'_B, t'_B),$$

as a rigid body, it requires $t'_A = t'_B$. In this coordinate system, the special relativistic transformation is

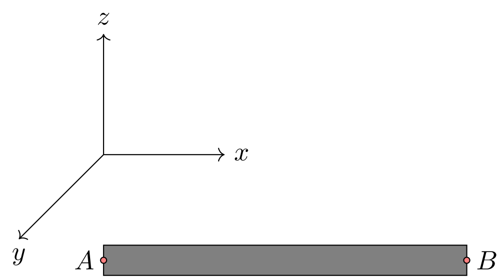

Fig. 11.3: Ruler in rest reference frame.

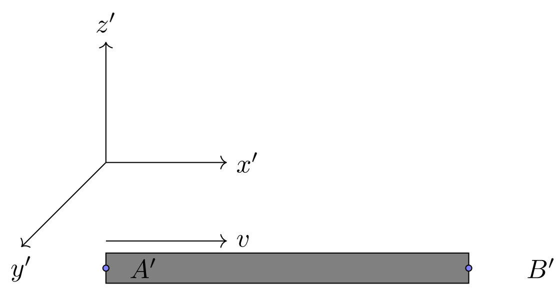

Fig. 11.4: Ruler in moving reference frame

$$X_A = \frac{x'_A + vt'_A}{\sqrt{1 - \frac{v^2}{v_{st}^2}}}, \qquad x_B = \frac{x'_B + vt'_B}{\sqrt{1 - \frac{v^2}{v_{st}^2}}} \tag{11.18}$$

Hence

$$x_B - x_A = \frac{x'_B - x'_A}{\sqrt{1 - \frac{v^2}{v_{st}^2}}} \tag{11.19}$$

and

$$l = x'_B - x'_A = l_0 \sqrt{1 - \frac{v^2}{v_{st}^2}} \tag{11.20}$$

In the framework of MIP, we consider the differential distance dx' The MIP requires $\delta(px) = nh = \delta(p'x'),\ n \in \mathbb{Z}$. In all inertial frames, each time the STP acting on matter particle , the changing of action is nh. The basic principle will remain the same regardless of inertial reference frames.

In the motion reference frame, we know that the mass $m' = m_0/\sqrt{1 - v^2/v_{st}^2}$, thus inducing $\delta p' = m'\delta v$. In the rest reference frame $\delta p = m\delta v_0$, we can easily seethat to ensure the MIP is independent of reference frame transformation, there must exists the relation

$$dx' = dx\sqrt{1 - v^2/v_{st}^2} \tag{11.21}$$

The length now is the integral of the above formula, and we have

$$l = \int_A^B dx' = l_0\sqrt{1 - v^2/v_{st}^2} \tag{11.22}$$

Thus we have derived the same result as in special relativity. However, its intrinsic meaning is not the same as in special relativity. Since we study within frameword of MIP, in which the STP' relative movement to the particle does not change under refrence frame transformation. Distinct frome macro length contraction effect of special relativity, the differntial distance is also constracted under MIP, which precisely reflects the universal applicability of the MIP.

11.6 An Alternative Method of Deriving Special Relativity

From the nature of time and space to study the transformation between inertial systems, we don't have to discuss the nature of light from beginning to end. We only need: the speed of STP is the maximum possible speed, and we can obtain special relativity. In Model 1 of Chapter 3 of this paper, we discuss examples of discrete space-time, where the maximum STP speed is not an assumption.

Consider two inertial systems K, K'. When $t = 0$ is set, the two origins of inertial systems coincide. K' relative K with speed ν . Simplifying the derivation of symbols, using $1+1$ dimension space-time. Most commonly, the time and space of K and K' are transformed as follow:

$$x' = X(x,t,v)\ , \quad t' = T(x,t,v), \tag{11.23}$$

where X and T are two universal functions. We will determine the form of X, T by the nature of space and time in the following.

Consider that the space is uniform. That is, a ruler is in K and the endpoints are in x_1 and x_2. It must be the same length as the endpoints in $x_1 + \Delta x$ and $x_2 + \Delta x\dot{W}$ith spatial invariance, also in the K' system:

$$X(x_2 + \Delta x) - X(x_2) = X(x_1 + \Delta x) - X(x_1). \tag{11.24}$$

Get $\Delta x \to 0$, you can get:

$$\frac{\partial X(x,t,v)}{\partial x}\ |_{x_1} = \frac{\partial X(x,t,v)}{\partial x}\ |_{x_2}\ . \tag{11.25}$$

With the arbitrariness of x_1 and x_2 , we know that the equation must be constant at both ends, i.e. X is a linear function of x . By the same token, a linear function of T is t can be obtained from the homogenous of time.

With $t = 0 = t'$, the origin coincides with $x = 0 = x'$, we can get

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} a_v & b_v \\ C_v & d_v \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \tag{11.26}$$

where a_v, b_v, c_v, d_v are functions of ν and the diagonal elements are dimensionless. When $x = vt$ $x' = 0$ gets $b_v = -va_v$.

Let's consider the spatial homogenous. In the 1+1 dimension spacetime, it can be understood as the inversion of the x axis. $x \to -x$, $v \to -v$, $x' \to -x'$, the spatio-temporal transformation is unchanged, that is, union:

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} a_v & b_v \\ C_v & d_v \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \tag{11.27}$$

as well as

$$\begin{pmatrix} -x' \\ t' \end{pmatrix} = \begin{pmatrix} a_{-v} & b_{-v} \\ C_{-v} & d_{-v} \end{pmatrix} \begin{pmatrix} -x \\ t \end{pmatrix} \tag{11.28}$$

Can get:

$$a_{-v} = a_v,\ b_{-v} = -b_v,\ c_{-v} = -c_v,\ d_{-v} = d_v. \tag{11.29}$$

By definition, the transformation of K to K' must be equivalent to the transformation of K' to K with a relative speed of $-\nu$, ie:

$$\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} a_{-v} & b_{-v} \\ C_{-v} & d_{-v} \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix} \tag{11.30}$$

Lianli available:

$$d_v = a_v,\ c_v = \frac{a_v^2 - 1}{b_v}. \tag{11.31}$$

which is,

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} a_v & -va_{-v} \\ -\frac{a_v^2-1}{va_v} & a_v \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \tag{11.32}$$

Very importantly, we generally determined that the diagonal elements of the spacetime transformation must be equal.
With the definition of the inertial system, there is a K'' inertial system relative to K' to w movement, then K'' relative K Must be an inertia transformation. This can be formulated as:

$$\begin{pmatrix} x'' \\ t'' \end{pmatrix} = a_w a_v \begin{pmatrix} 1 + w\frac{a_v^2-1}{\nu a_v^2} & -(w+v) \\ -\frac{a_w^2-1}{wa_w^2} - \frac{a_v^2-1}{va_v^2} & 1 + v\frac{a_w^2-1}{wa_w^2} \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \tag{11.33}$$

Combining the above two equations, we get an important relationship:

$$1 + w\frac{a_v^2 - 1}{va_v^2} = 1 + v\frac{a_w^2 - 1}{wa_w^2} \tag{11.34}$$

$$\frac{v^2 a_v^2}{a_v^2 - 1} = \frac{w^2 a_w^2}{a_w^2 - 1} = G. \tag{11.35}$$

Because w, v is any speed, this formula must be equal to both constants and set to G. The following proves that this formula must be no less than zero:

Available

$$a_v = \frac{1}{\sqrt{1 - \frac{v^2}{G}}}, \tag{11.36}$$

Since $v = 0$, $a_v = 1$ is equivalent to no transformation, so we can remove the negative root. It can be seen that for any speed v, there must be $a_v > 0$. Using (11.33), after two inertial transformations, it is still an inertial system transformation. So have

$$1 + w\frac{a_v^2 - 1}{\nu a_v^2} > 0 \tag{11.37}$$

If $a_v < 1$, when $w \gg v$, the above formula cannot be maintained. So we prove that $a_v \geq 1$ is equivalent to $G \geq 0$. From this we can set

$$\frac{v^2 a_v^2}{a_v^2 - 1} = \frac{w^2 a_w^2}{a_w^2 - 1} = \theta^2. \tag{11.38}$$

In summary, the final transformation can be obtained:

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \frac{1}{\sqrt{1 - \frac{v^2}{\theta^2}}} \begin{pmatrix} 1 & -v \\ \frac{-v}{\theta^2} & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \tag{11.39}$$

From this formula, you can see that $v < \theta$. (space-time coordinates cannot be imaginary). The joint transformation satisfies:

$$\omega = \frac{w + v}{1 + \frac{wv}{\theta^2}}. \tag{11.40}$$

Because $w < \theta$, $v < \theta$, no matter how we combine, we must have $\omega < \theta$. In summary, θ must be the maximum possible speed allowed for time and space. Combined with our discrete space-time model, there is a minimum length in time and space l and a minimum time τ, and STP speed is defined as the ratio of the minimum length and time.

Let's prove $\theta = \frac{l}{\tau} \equiv v_{st}$. as follows: If there was a speed $V > \theta$, it would move τ time interval, The distance must be greater than l, which is no problem and will not cause any contradiction. But it travels through the shortest distance l . It only takes $\frac{l}{V} < \tau$ time, which contradicts the definition of the shortest time.

Therefore we prove that the STP speed $\theta = v_{st}$ is the maximum possible speed in space and time. Time and space of this nature must have the properties derived above. When the STP speed is exactly equal to the propagation speed of light in a vacuum, our results are equivalent to the special relativity.

11.7 Lorentz invariant form of MIP

MIP is a principle of the change of action, so the definition of action is required. The basic definition of action is:

$$S = \int L dt \tag{11.41}$$

This formal definition does not give any specific content of the action, but it is very useful. Because it transfers the task of constructing the action S of a system to the Lagrangian L of this system. Because MIP is a general principle, the action must be invariant in all inertial systems. Therefore, S must be a Lorentz invariant .

Obviously, dt is not Lorentz invariant, because of time dilation effect of special relativity. Then, the Lagrangian L of the system cannot be a Lorentz invariant . Because a Lorentz invariant is multiplied by a non-Lorentz invariant and then summed (integral), the result cannot be a Lorentz invariant . To construct Lorentz invariant action, it is necessary to use physical quantities of material particle that have the Lorentz invariant properties.

In order to facilitate the construction of the Lorentz invariant action, we first use the clock fixed on the material particle, because proper time $d\tau$ is Lorentz invariant, which satisfies

$$d\tau = \sqrt{1 - \frac{v^2}{c^2}}dt \tag{11.42}$$

Substituting to get

$$S = \int_{t_1}^{t_2} Ldt = \int_{\tau_1}^{\tau_2} \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}d\tau \tag{11.43}$$

Because proper time $d\tau$ is a Lorentz invariant , our task becomes to construct an L such that $\frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$ is Lorentz invariant. In addition to the proper time of material particles, what other physical quantities are Lorentz invariant that can be used to construct L? For electrons, we can list all the physical quantities of Lorentz invariant:

$$m_0, c, \hbar, e \tag{11.44}$$

The quantities constructed from these original Lorentz invariant physical quantities will not be listed, such as the fine structure constant $\alpha = \frac{e^2}{\hbar c}$. Note that the velocity v of the particle of matter is not Lorentz invariant , to ensure that $\frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$ invariant, the most general Lagrangian L can only take the form

$$L = G(m_0, c, \hbar, e)\sqrt{1 - \frac{v^2}{c^2}} \tag{11.45}$$

where G is a unknown function. To make the Lagrangian of free material particles transition to the correct classic form when the speed of light approaches infinity, that is, the dependence on speed is $\frac{1}{2}m_0v^2$. Therefore the function G can be completely determined as

$$G(m_0, c, \hbar, e) = -m_0c^2 \tag{11.46}$$

because

$$\lim_{c\to\infty} -m_0c^2\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2}m_0v^2 - m_0c^2 \tag{11.47}$$

According to the definition of Lagrangian , the Lagrangian with a different constant corresponds to exactly the same physics. We get the final result

$$S = \int_{t_1}^{t_2} Ldt = -m_0c^2 \int_{t_1}^{t_2} \sqrt{1 - \frac{v^2}{c^2}}dt \tag{11.48}$$

If between t_1 and t_2, no STP collides with the material particles, so the material particles maintain a uniform linear motion. Then the effect of this interval is S_0

$$S_0 = -m_0c^2 \int_{t_1}^{t_2} \sqrt{1 - \frac{v^2}{c^2}}dt = -m_0c^2\sqrt{1 - \frac{v^2}{c^2}}(t_2 - t_1) \tag{11.49}$$

When STP colliding matter particles with initial time at t_1 and the end time at t_2, and action in this interval is S. This establishes the MIP of Lorentz invariant form :

$$S - S_0 = nh \tag{11.50}$$

Where n is an integer and h is Planck's constant.

12 MIP and General Relativity

Gravitation refers to the attraction of objects with mass, and together with electromagnetic force, weak interaction force and strong interaction force constitute the four fundamental interactions of nature. Among these four fundamental interactions, gravity is the weakest, but it is also a universal long-range attraction. Whenever there is an attraction between any two particles, the magnitude of the force is inversely proportional to the square of the distance, and proportional to the product of mass, we can conclude that this force must be gravitational. In the framework of the mass principle, we naturally derive the gravitation from the interaction between the massive fermion and STP.

12.1 Electron and STP

The electrons are described by the spinor field ψ, and the dynamics of free electrons are determined by the amount of Dirac Lagrangian:

$$\mathcal{L}_{\mathrm{D}}(\psi) = \bar{\psi}(i\partial\!\!\!/ - m)\psi \tag{12.1}$$

Where m is the electronic mass.
STP is described by the massless scalar field ϕ, and the dynamics of free STP is determined by Klein Gordon Lagrangian:

$$\mathcal{L}_{\mathrm{sp}}(\phi) = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi \tag{12.2}$$

The interaction between STP and electrons is determined by the Lagrangian:

$$\mathcal{L}_{\mathrm{int}}(\phi, \psi) = -\lambda\bar{\psi}\phi\psi \tag{12.3}$$

According to MIP, the interaction strength of STP and electrons λ must be positively correlated with the mass m of the electron and

$$\lim_{m\to 0} \lambda(m) = 0 \tag{12.4}$$

What needs to be studied here is the lowest order behavior of the interaction strength λ when the electron mass tends to zero. When we replace all the electronic masses in the system with $-m$, the interaction strength λ , all physical laws must remain the same. Therefore, the lowest order behavior is

$$\lambda(m) = A\sqrt{m^2} \tag{12.5}$$

At $m = 0$ is the branch point of the function, which corresponds to the singularity implied by MIP, that is the disappearance of interaction with STP. Comparing the description of photon frequencies and wavevectors by relativistic quantum mechanics $\omega = ck$, its correct interpretation should be:

$$\omega = c\sqrt{k^2} \tag{12.6}$$

This leads to the important conclusion that λ^2 (instead of λ) is an analytic function for m^2. The correct expansion must be of the form:

$$\lambda^2 = C_0 + C_2 m^2 + C_4 m^4 + ... \tag{12.7}$$

According to the MIP, there must be $C_0 = 0$. The electron mass is very small compared to the natural Planck mass. we can almost ignore the contribution of all high-order terms, leaving only the lowest-order behavior:

$$\lambda^2 = C_2 m^2 \tag{12.8}$$

In summary, the dynamics of the entire system are composed of STP and electrons, which are determined by the following action:

$$S[\phi, \psi] = \int d^4x \ [\mathcal{L}_{\text{sp}}(\phi) + \mathcal{L}_{\text{D}}(\psi) + \mathcal{L}_{\text{int}}(\phi, \psi)] \tag{12.9}$$

Substituting a concrete expression, the total amount of action of the system is:

$$S[\phi, \psi] = \int d^4x \left[\frac{1}{2}\partial^\mu \phi \partial_\mu \phi + \bar{\psi}(i\partial\!\!\!/ - m)\psi - \lambda\bar{\psi}\phi\psi\right] \tag{12.10}$$

According to path integration, the total partition function of this system is:

$$Z = \int D\phi D\psi e^{iS[\phi,\psi]} = \int D\psi e^{iS_{eff}[\psi]} \tag{12.11}$$

Let $J = -\lambda\bar{\psi}\psi$, the result for the STP field ϕ is

$$Z = \int D\phi \exp(\frac{1}{2}\partial^\mu \phi \partial_\mu \phi + J\phi) = e^{iW(J)} \tag{12.12}$$

When the fermion is at static limit, $J(\vec{x_i}) = \lambda\delta(\vec{x} - \vec{x_i})$, we have

$$W(J) = -\frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\frac{J^*(k)J(k)}{k^2} = \int dx^0 \int \frac{d^3\vec{k}}{(2\pi)^3}\frac{\lambda^2 e^{i\vec{k}\cdot\vec{r}}}{\vec{k}^2} \tag{12.13}$$

Using $Z = e^{iW(J)} = e^{-iV_{eff}T}$ and $T = \int dx^0$, we can get the effective interaction between electrons as:

$$V_{eff}(r) = -\int \frac{d^3\vec{k}}{(2\pi)^3} \frac{\lambda^2 e^{i\ Veck\cdot\vec{r}}}{\vec{k}^2} = -\frac{\lambda^2}{4\pi r} = -\frac{C_2 m^2}{4\pi r} \tag{12.14}$$

The effective interaction forces between the corresponding electrons are:

$$F_{eff}(r) = -\frac{\lambda^2}{4\pi r^2} = -\frac{C_2 m^2}{4\pi r^2} \tag{12.15}$$

Where $r = |\vec{r}|$ is the distance between the electrons, and the preceding negative sign indicates this is an attractive interaction. This universal attraction force is inversely proportional to the square of the distance and proportional to the product of mass. We can conclude that it can only be gravitational. So we can compare the gravitational formula and determine the scale factor $C_2 = 4\pi G$, that is:

$$F_{eff}(r) = -\frac{Gm^2}{r^2} \tag{12.16}$$

Where G is the gravitational constant, and the interaction force between the electrons induced by STP is universal gravitation.

12.2 Universal Gravity among Macroscopic Bodies

Let us consider a system with three components: free protons, free neutrons, STP. The nucleon is described by the spinor field ψ_i (subscript i=1 is denoted as proton, i=2 is denoted as neutron, repeating indicator is summed), and the dynamics of free nucleus is

$$\mathcal{L}_{\mathrm{N}}(\psi_i) = \bar{\psi}_i(i\partial\!\!\!/ - m_i)\psi_i \tag{12.17}$$

Where m_1 is the proton mass and m_2 is the neutron mass.

The description of the free STP section is as described in the previous section. The interaction between STP and nucleon is determined by the Lagrangian quantity $\mathcal{L}_{\mathrm{int}}$:

$$\mathcal{L}_{\mathrm{int}}(\phi, \psi) = -\lambda_{ij}\bar{\psi}_i\phi\psi_j \tag{12.18}$$

The repeated index are automatically summed, and there are four terms in this formula. According to MIP, the interaction strength of STP and electrons λ_{ij} must be positively related to the mass of the nucleus $m_i m_j$, and must have:

$$\lim_{m_1\to 0} \lambda_{11} = \lambda_{12} = \lambda_{21} = 0 \tag{12.19}$$

$$\lim_{m_2\to 0} \lambda_{22} = \lambda_{12} = \lambda_{21} = 0 \tag{12.20}$$

With the same reasoning of the previous section, according to the analytical nature of the function and the limitations of the MIP, the lowest order behavior must be:

$$\lambda_{ij}^2 = C_2 m_i m_j \tag{12.21}$$

In summary, the dynamics of the entire system are composed of STP and nucleon, which are determined by the following action:

$$S[\phi,\psi] = \int d^4x\ [\mathcal{L}_{\rm sp}(\phi) + \mathcal{L}_{\rm N}(\psi) + \mathcal{L}_{\rm int}(\phi,\psi)] \tag{12.22}$$

Substituting a concrete expression, the total amount of action of the system is:

$$S[\phi,\psi] = \int d^4x[\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \bar{\psi}_i(i\partial\!\!\!/ - m_i)\psi_i - \lambda_{ij}\bar{\psi}_i\phi\psi_j] \tag{12.23}$$

According to path integration, the total partition function of this system is:

$$Z = \int D\phi D\psi_1 D\psi_2 e^{iS[\phi,\psi_1,\psi_2]} = \int D\psi_1 D\psi_2 e^{iS_{eff}[\psi_1,\psi_2]} \tag{12.24}$$

The result of accumulating the STP field ϕ can further obtain the effective interaction potential between the nucleons:

$$V_{ij}(r) = -\frac{\lambda_{ij}^2}{4\pi r} = -\frac{C_2 m_i m_j}{4\pi r} \tag{12.25}$$

Same as the previous section, let the lowest order proportional coefficient $C_2 = 4\pi G$, where G is the gravitational constant. This leads to the important conclusion that the interaction between the nucleus induced by STP is universal gravitation:

$$F_{ij}(r) = -\frac{G m_i m_j}{r^2} \tag{12.26}$$

We obtained the correct expression of the gravitational potential between protons and protons, protons and neutrons, neutrons and neutrons. Considering that the gravitational potential is a scalar potential, a direct summation superposition can be performed, thereby obtaining the gravitational force between the macroscopic objects, and mathematically proves that the sum of the gravitational forces between the constituent elements of the two macroscopic objects is equivalent to mass center. This is the classic proof of Newton and will not be repeated here.

12.3 MIP and Equivalence Principle

The principle of equivalence plays a very important role in the general relativity. The principle of equivalence means that the observer cannot distinguish the inertial force generated by the acceleration and the gravitational force generated by the mass , which is derived from the fact that the gravitational mass and the inertial mass are strictly equal. In the theoretical framework of general relativity, the gravitational mass and the inertial mass are strictly equal, which is a postulation of the theory. Aparting from the support of empirical facts, there is no theoretical further explanation.

Within the framework of MIP, we have the microscopic origins of gravitation in the above two sections, which implies important physical results: in the universal gravitational formula induced by STP and fermion interaction, the mass of fermion from Dirac equation is inertial mass. From the overall perspective of modern physics, we can determine that this force must be gravitational, and it can be inferred that the inertial mass of the fermion must be equal to its gravitational mass. Of course, this conclusion cannot completely replace the principle of equivalence, and does not solve the problem of whether the mass of the boson in the natural world is equal to the mass of gravity. However, it is still an extremely important conclusion, because the principle of equivalence and general relativity are mainly applied to macroscopic objects in nature (such as various celestial bodies), and their mass components are completely derived from fermions. It can be said that when general relativity is applied in macroscopic fields such as cosmology and astrophysics, the equivalence principle is no longer a hypothesis, but a property that can be derived from microscopic STP dynamics.

Starting from the equivalent principle we derived, combined with Einstein's elevator thought experiment, we can demonstrate the inevitable bending of time and space with gravitational source. Describing the curve time and space , mathematically we have to use the metric field of Riemannian geometry. The core of general relativity is the physical equation that is satisfied in this Riemannian background. To get the Einstein field equation furthermore, we must add some constraints.

The important ones are as follows: 1.The Newton gravitational potential is the classical limit. The static gravitational field is determined by the mass density distribution of matter, which is a component of the momentum energy tensor. Extending to the general gravitational field and determining the momentum energy tensor of the gravitational field must correspond to this limit.

2. Referring to the Newtonian gravitational potential equation, we also require that the differential equations satisfied cannot have more than two orders of derivatives. This is combined with a theorem of Riemannian geometry [5] and the classical limit of Newton's gravitation, the form of the gravitational field equation can be determined. The coefficient of the cosmological constant term has not been completely determined. This item has no effects on this paper and will not be discussed here.

Furthermore, according to MIP, inertial mass and gravitational mass are no

[5] Weinberg, "Gravitation and Cosmology", the uniqueness of curvature tensor is a very important mathematical theorem in Riemannian geometry.

longer fundamental physical quantities. The two remain equal because both come from the statistical mass of STP collisions.

13 Entropy in MIP

Phase space is a delicate concept. Within the framework of MIP, the coordinates and momentum of particles can be completely independent, so the phase space has real physical meaning. Discussing the issue of entropy for non-interacting particle in the phase space will be more clear and insightful.

13.1 Entropy in phase space

Let us first consider a matter particle of mass m in a harmonic oscillator potential. The energy of a particle is the sum of its kinetic energy and potential energy

$$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 \tag{13.1}$$

Where k is the stiffness coefficient of the spring. According to Newton's second law

$$m\ddot{x} = F = -\frac{dV}{dx} = -kx \tag{13.2}$$

and

$$k = m\omega^2 \tag{13.3}$$

Where ω is the frequency of particle vibration.

The state of a particle is characterized by (x, p). In the (x, p) space, each point represents a state of the particle. This space is named phase space, and the motion of the particles constitutes the trajectory in the phase space.

For each fixed energy E, the particle's trajectory in phase space is an ellipse. According to the definition of ellipse, we can write

$$\frac{p^2}{2mE} + \frac{x^2}{\frac{2E}{m\omega^2}} = 1 \tag{13.4}$$

We can determine the two axis lengths of the ellipse

$$a = \sqrt{2mE} \tag{13.5}$$

$$b = \sqrt{\frac{2E}{m\omega^2}} \tag{13.6}$$

According to MIP, a particle moves along an ellipse in phase space for a period, and it must exchange with the STP an integer multiple of the Planck constant

$$\oint pdx = nh \tag{13.7}$$

It can be seen from the geometric meaning of the integral that the integer value corresponds exactly to the area of the ellipse

$$\oint pdx = \pi ab = \frac{2\pi E}{\omega} \tag{13.8}$$

From this we get important results

$$E = n\hbar\omega \tag{13.9}$$

This proves that every possible state occupies the same area in the phase space. This is the most important difference between quantum mechanics and classical mechanics: The energy levels are discrete, which means not all energy levels are allowed for real motions. In the phase space, only discrete ellipses are possible movements, corresponding to possible states.

With this important result, we can start to count the number of possible states to determine the entropy. Intuitively, for the elliptic family of phase space, the volume of the phase space occupied by each possible E (for the sake of intuition, we are talking about one-dimensional motion, the corresponding phase space is 2 dimensions, and therefore the area), which is exactly The area A surrounded by two adjacent ellipses. The most important thing is that this area A is a constant. Similarly we can calculate this constant as

$$A = \oint_{E=(n+1)\hbar\omega} pdx - \oint_{E=n\hbar\omega} pdx = h \tag{13.10}$$

If we further consider the net effect $1 + 1 + 1 + ... = \zeta(0) = -\frac{1}{2}$ of the infinite collisions of STP, we can get the complete result of quantum mechanics: The energy level of a simple harmonic oscillator is $E = (n + \frac{1}{2})\hbar\omega$.

13.2 Entropy at absolute zero

Let us first consider the entropy at absolute zero, and then discuss the entropy of thermodynamics. The entropy at absolute zero must be equal to zero, according to the definition of Boltzmann entropy, which means the entropy is the logarithm of all possible microscopic states at the same energy. It is equivalent to say that there can be only one state at absolute zero, so its entropy is 0.
If we look at the typical time scale of quantum mechanics, this conclusion is

correct. Due to STP colliding at the short time scale of MIP, the wave function formed on the quantum mechanical time scale is a pure state, and its entropy must be zero.
Free particles in quantum mechanics can be characterized by plane waves $e^{i\vec{p}\cdot\vec{x}/\hbar}$, where $\vec{p}\cdot\vec{x}/\hbar$ is called the phase factor of the wave function. In the time scale of MIP, we will generalize this key factor.
First, in this extremely short time scale, according to Section 3.5, we generalized the momentum of quantum mechanics $\vec{p}$ to instantaneous momentum $\vec{P}_i$. Second, the instantaneous momentum is not a conserved quantity. The original phase factor $\vec{p}\cdot\vec{x}$ must be generalized to $\int_\gamma \vec{P}_i \cdot d\vec{x}$. Third, for non-interacting particles, aka free particles, we can always choose an inertial frame of reference with zero classic statistical velocity. Furthermore, the integral of the random velocity through the path γ on the time scale of quantum mechanics is 0. Therefore, the contribution only comes from envelope velocity.
The number of all possible microscopic states of a particle can be characterized by its envelope velocity u. Within a short time scale, different envelope velocity can represent different possible states. We can construct entropy Within the framework of MIP, and then find the following way to pass to the long-term scale, the result of quantum mechanics about entropy.

Based on the above three points about particles traveling a path γ at the time scale of quantum mechanics, we can generalize the phase factor in quantum mechanics to

$$K_i = \frac{1}{\hbar}\int_\gamma \vec{P}_i \cdot d\vec{x} = \frac{m_{st}}{h}\int_\gamma \vec{u_i}\cdot d\vec{x} \tag{13.11}$$

All the possible state under the time scale of MIP are represented by different i and K_i is a dimensionless quantity. From the conclusion of Chapter 5, the envelope velocity is an irrotational field

$$\nabla \times \vec{u} = 0 \tag{13.12}$$

So K_i does not depend on the path γ, which is just a function of the endpoint. It must be noted that entropy is a variable of state, regardless of how to reach the state.
The probability of possible state i is defined as

$$p_i = \frac{1}{N}e^{2K_i} \tag{13.13}$$

Where the normalization constant

$$N = \sum_i e^{2K_i} \tag{13.14}$$

In order to guarantee that the probability sum of various possible states equals to 1, we have

$$\sum_i p_i = 1 \tag{13.15}$$

and the probability of all possible states are greater than 0.
Within the framework of MIP and the probability of possible states on a short time scale, we can define the corresponding entropy as

$$S = -\sum_i p_i \log p_i \tag{13.16}$$

By this definition, we claim that all results of quantum mechanical entropy can be derived.

The derivation is as follows:
Obtaining the gradient on both sides of Equation 11:

$$\nabla K_i = \frac{m_{st}}{h}\vec{u_i} \tag{13.17}$$

For each possible state i, the wave function ψ_i will emerge on the quantum mechanical time scale as

$$|\psi_i| = \frac{1}{\sqrt{N}}e^{K_i} \tag{13.18}$$

In every possible state, K_i corresponds exactly to the original potential function R. Thus our definition of entropy is equivalent to

$$S = -\sum_i 2|\psi_i|^2 \log|\psi_i| \tag{13.19}$$

This is completely equivalent to the definition of von Neumann entropy in quantum mechanics. Therefore, from the microscopic behavior of the envelope velocity in the short time scale of MIP, quantum mechanical entropy in the long time scale is derived.

We can summarize this section: at absolute zero and within the time scale of MIP, the entropy of matter particles is not zero, and its various microscopic states are characterized by different envelope velocities. According to the conclusions in Chapter 5, reaching the time scale of quantum mechanics after a long time of random collision, the material particles at absolute zero appear as a pure state wave function, and its evolution satisfies the Schrödinger equation. Then the probability of only one state i is 1, and the probability of other states is 0, which naturally leads to the conclusion of quantum mechanics: the entropy is 0 at absolute zero .

13.3 Entropy at finite temperature

When we consider not only the quantum behavior of single particle but also the thermodynamic properties of multiple particles without interactions, the work in the previous section needs to be further generalized. In the first step, we generalize to the case of two particles. The probability that one is in state i and the other is in state j is $p_i\tilde{p}_j$. If they are identical particles, two probability distribution functions are the same. According to the definition of entropy, the entropy of two particle systems is

$$\begin{aligned} S &= -\sum_{ij}(p_i\tilde{p}_j)\log(p_i\tilde{p}_j) = -\sum_{ij} p_i\tilde{p}_j \log p_i - \sum_{ij} p_i\tilde{p}_j \log \tilde{p}_j \\ &= -\sum_i p_i \log p_i \sum_j \tilde{p}_j - \sum_j \tilde{p}_j \log \tilde{p}_j \sum_i p_i \\ &= -\sum_i p_i \log p_i - \sum_j \tilde{p}_j \log \tilde{p}_j \\ &= S_1 + S_2 \end{aligned} \tag{13.20}$$

Additivity is obtained, which is the fundamental property of entropy. This can be directly extended to the entropy of any multi-particle system, which is equal to the sum of the entropy of each single particle. That is to say, the macroscopic thermodynamic entropy is the sum of the entropy of each part of the subsystem. And we treat each single particle as an independent subsystem, which is the smallest subsystem possibly. To be connected with thermodynamic entropy, we need to introduce temperature.

The second step is to define the temperature in the MIP framework as following. We have proved the additivity of entropy from MIP. Use this basic property to define the physical quantity of temperature. Assuming that the energy of two subsystems is E_1 and E_2, the total energy of the system $E = E_1 + E_2$ is a conserved quantity. By the additivity of entropy, we have

$$S(E) = S_1(E_1) + S_2(E_2) \tag{13.21}$$

The total system is a closed system. When in equilibrium, the derivative of both sides with respect to E_1 leads to

$$0 = \frac{dS_1}{dE_1} + \frac{dS_2}{dE_2}\frac{dE_2}{dE_1} = \frac{dS_1}{dE_1} - \frac{dS_2}{dE_2} \tag{13.22}$$

It can be seen that there is a physical quantity in equilibrium, which is possessed by all subsystems equally. We call this physical quantity the temperature T, which is defined as

$$\frac{dS}{dE} = \frac{1}{T} \tag{13.23}$$

That is

$$\frac{dS_1}{dE_1} = \frac{1}{T_1} = \frac{dS_2}{dE_2} = \frac{1}{T_2} \tag{13.24}$$

In the third step, we introduce temperature into the definition of entropy, in order to study the entropy of thermodynamics under the framework of MIP. Let us consider the microscopic collision process under the MIP framework. When a material particle collides with an STP, the material particle is in state a and the STP is in state b. After the collision, the state of the material particle changes to c, and the STP state changes to d. The probability of this process is proportional to $n_a n_b$, that is, in the initial state, there are n_a material particles in state a, and n_b material particles are in state b. Then we consider a reverse process whose probability is proportional to $n_c n_d$. According to MIP, the STP collision process has time-reversal symmetry and the material particles must reach an equilibrium state with the STP, that is, the average number of particles in each state does not change. Then we have

$$n_a n_b = n_c n_d \tag{13.25}$$

Conservation of energy of the collision process leads to

$$\epsilon_a + \epsilon_b = \epsilon_c + \epsilon_d \tag{13.26}$$

It can be proved that

$$n_i = Ce^{-\beta\epsilon_i} \tag{13.27}$$

Among them, the constant C is given by

$$\sum_i n_i = N \tag{13.28}$$

According to the basic definition of probability theory, the probability of being in the i state is

$$p_i = \frac{n_i}{N} = \frac{1}{\sum_i e^{-\beta\epsilon_i}} e^{-\beta\epsilon_i} \tag{13.29}$$

This p_i is the generalization of the probability distribution at a finite temperature. According to the definition of temperature, it can be proved that the coefficient β must be equal to $\frac{1}{T}$. From this we get the entropy at finite temperature

$$S = -\sum_i p_i \log p_i \tag{13.30}$$

We can directly substitute the specific expression of the probability distribution p_i to get the thermodynamic entropy as

$$S = -\sum_i \frac{1}{\sum_i e^{-\beta\epsilon_i}} e^{-\beta\epsilon_i} \log \frac{1}{\sum_i e^{-\beta\epsilon_i}} e^{-\beta\epsilon_i} = \frac{E-F}{T} \tag{13.31}$$

In this way, the general expression of thermodynamics is obtained, where the free energy of thermodynamics reads

$$F = -T \log \sum_i e^{-\epsilon_i/T} \tag{13.32}$$

And internal energy as

$$E = \frac{1}{\sum_i e^{-\epsilon_i/T}} \sum_i \epsilon_i e^{-\epsilon_i/T} \tag{13.33}$$

Within the framework of MIP, the energy of non-relativistic free material particles is expressed in terms of true velocity

$$\epsilon = \frac{1}{2}mV^2 \tag{13.34}$$

Substituting the distribution function of the true velocity of the material particles at a finite temperature

$$\Phi(V^2) = (\frac{m}{2\pi T})^{3/2}e^{-\frac{mV^2}{2T}} \tag{13.35}$$

Then we can further give the definition of classical statistical velocity in the decomposition of three velocities

$$v = \int V\Phi(V^2)d^3V = \sqrt{\frac{8T}{m\pi}} \tag{13.36}$$

which shows a deeper understanding of the physical meaning of the decomposition of three velocities. The entropy at absolute zero corresponds to the quantum envelope velocity of material particles, while the entropy at finite temperature includes the contributions from all three velocities. From finite temperature to absolute zero, the physical quantity describing the system has undergone a fundamental change. Therefore, thermodynamics cannot determine the value of entropy at absolute zero, which can be obtained naturally under the MIP framework.

13.4 Comparing between entropy at finite temperature and absolute zero

In modern information theory, entropy (Shannon entropy) is a measure of uncertainty. This basic concept is consistent with MIP. The study of the diffusion coefficient of material particles at finite temperature shows that, the uncertainty of the thermodynamic contribution of finite temperature is much smaller than the quantum contribution at absolute zero in MIP.

MIP shows that matter particles do Brownian motion under random collisions of STP, the most important property of this motion is

$$< X^2 >= 2\Re t \tag{13.37}$$

Where $\Re$ is the space-time diffusion coefficient $\Re = \frac{\hbar}{2m}$. Obviously, this is a result at absolute zero, has nothing to do with temperature, purely caused by Planck's constant.

In the framework of classical physics, the Planck constant is 0, so there is no such diffusion coefficient, and of course there is no such Brownian motion. However, there will still be Brownian motion caused by thermal motion. So the space-time diffusion coefficient has two parts, one is $\Re$ independent of the temperature T , and the other is related to the temperature T.

Our goal is the principle of entropy increasing, which is the thermodynamic properties of matter particles. How the diffusion coefficient depends on temperature T? Which part is more important?

The material particles do Brownian motion under the random collision of STP. According to the estimation in Section 3.4, the most important physical parameter is the average time interval between two collisions τ as

$$\tau \approx 10^{-20}s \tag{13.38}$$

The time scale of electrons in quantum mechanics is $\tau \approx 10^{-16}s$.Therefore, the electrons in hydrogen atoms are much larger than the time scale of MIP. We will explicitly construct the average time interval τ into the equation of motion:

$$m\frac{dV}{dt} = -\frac{mV}{\tau} + F(t) \tag{13.39}$$

We are able to get the answers to the above two questions at the same time as follows: Multiply both sides of the equation by X, using

$$\frac{d(XV)}{dt} = V^2 + X\frac{dV}{dt} \tag{13.40}$$

We can get

$$m\frac{d(XV)}{dt} = mV^2 - \frac{mXV}{\tau} + F(t)X \tag{13.41}$$

Taking the average of both sides of the equation, at a temperature of T the average kinetic energy of the particles is

$$\frac{1}{2}m < V^2 >= \frac{1}{2}kT \tag{13.42}$$

Substitute

$$m\frac{d(< XV >)}{dt} = kT - m\frac{< XV >}{\tau} \tag{13.43}$$

Combined with the initial condition $X(t = 0) = 0$, we solve this differential equation as

$$< XV >= \frac{kT\tau}{m}(1 - e^{-t/\tau}) \tag{13.44}$$

and

$$< XV >= \frac{1}{2}\frac{d < X^2 >}{dt} \tag{13.45}$$

Solve another differential equation to get

$$< X^2 >= \frac{2kT\tau}{m}(t - \tau(1 - e^{-t/\tau})) \tag{13.46}$$

This result is very important because it has both the properties wanted:

1. Under very short time scale $t \ll \tau$

$$< X^2 >= \frac{kT}{m} t^2 \tag{13.47}$$

At this time scale, the particles are moving at a uniform linear velocity, which comes from thermal motion $\sqrt{\frac{kT}{m}}$.

2. More importantly, under the time scale observed in the experiment, $t \gg \tau$

$$< X^2 >= \frac{2kT\tau}{m} t = 2\Re_T t \tag{13.48}$$

It shows that at this time scale, the particles are diffusive. Compared with equation (13.37), we can calculate the ratio of the diffusion caused by thermal motion to the diffusion at absolute zero. Assuming the system at room temperature 300K, the ratio will be

$$\frac{\Re_T}{\Re} \approx 10^{-6} \tag{13.49}$$

Therefore, the results we obtained without considering the temperature effect are very good approximations. The diffusion effect of material particles due to thermal motion can be ignored, and the diffusion coefficient at absolute zero based on MIP calculation is very accurate. In MIP, whenever considering quantum effects only, the entropy at finite temperature can be ignored, just as in the Schr*ö*dinger equation where is no need for a term directly related to temperature.

13.5 Proof of entropy increasing principle

Within the framework of MIP, we can use the definitions of entropy, combining with the general nature of Markov process, to prove the entropy increasing principle for non-interacting particle, both at finite temperature and absolute zero.

$$S = -\sum_i p_i \log p_i \tag{13.50}$$

Straightforwardly, proving the entropy increasing principle means

$$\frac{dS}{dt} \geqslant 0 \tag{13.51}$$

Use the definition of probability

$$\sum_i p_i = 1 \tag{13.52}$$

we have

$$\sum_i \frac{dp_i}{dt} = 0 \tag{13.53}$$

Then the definition of entropy goes to

$$\frac{dS}{dt} = -\sum_i \left(\frac{dp_i}{dt}\log p_i + \frac{dp_i}{dt}\right) = -\sum_i \frac{dp_i}{dt}\log p_i \tag{13.54}$$

If there is equal probability distribution, all p_i are equal to constant

$$p_i = \frac{1}{\Omega} \tag{13.55}$$

which is a very useful constraint. We will use it below.

The STP collision causes the transition between different states of material particles, which is a Markov process. For the Markov process, the following mathematical properties

$$\frac{dp_i}{dt} = \sum_j (p_j - p_i) g_{ij} \tag{13.56}$$

$$\frac{dp_j}{dt} = \sum_i (p_i - p_j) g_{ji} \tag{13.57}$$

This property has already been used in Section 3.3 equation (3.23), which is a special case of this mathematical property. If the probability distribution is equal, the probability no longer changes. It is an important step to prove that the collision of STP is invariant in time reversal, requiring the transfer matrix g to have

$$g_{ij} = g_{ji} \geqslant 0 \tag{13.58}$$

Therefore, the transition between various states is reversible on the time scale of STP collision, because the matter particle's Brownian motion in spacetime is frictionless. The proof of entropy increasing principle is irrelevant about the specific expression of entropy, whether or not including the temperature T. From this microscopic reversibility, it is possible to deduce the irreversibility of entropy on the macroscopic time scale, which is the essence point.

With this mathematical property, we get

$$\begin{aligned}\frac{dS}{dt} &= -\frac{1}{2}\left(\sum_i \frac{dp_i}{dt}\log p_i + \sum_j \frac{dp_j}{dt}\log p_j\right) \\ &= -\frac{1}{2}\left(\sum_{ij}(p_j - p_i)g_{ij}\log p_i + \sum_{ij}(p_i - p_j)g_{ji}\log p_j\right) \\ &= \frac{1}{2}\sum_{ij}(p_j - p_i)g_{ij}(\log p_j - \log p_i)\end{aligned} \tag{13.59}$$

If $p_j \geqslant p_i$, then $\log p_j \geqslant \log p_i$, which guarantees $\frac{dS}{dt} \geqslant 0$.

If $p_j \leqslant p_i$, then $\log p_j \leqslant \log p_i$, which also guarantees $\frac{dS}{dt} \geqslant 0$.

So the entropy increasing principle has been proved. This principle has profound significance in physics and other scientific fields, and can be used as a criterion for irreversibility and time flow. However, it must be emphasized that this principle is still an empirical law in modern physics and cannot be explained from the first principle. Therefore, our results are of great significance. In MIP, the random collision of STP can naturally generate the fundamental principle of increasing entropy. Within the framework of MIP, we unify the concept of entropy both at finite temperature and absolute zero and prove that both types of entropy are never decreasing with time.

14 Summary

Starting from the fundamental concept innovation of statistical mass, this paper proposes MIP: material particles will be subjected to random collision of STP's which is ubiquitous in space and time to make frictionless quantum Brownian motion. The change of the action of material particles in each collision is integer multiple of Planck constant h. From MIP, we can prove all the important results of the special theory of relativity. The speed of light in a vacuum no longer has a special physical meaning, but instead the speed of STP represents the upper limit of the speed of physical information propagation in spacetime. The constant speed of light is a natural consequence of MIP. The relative invariance of the speed of light actually reflects that the speed of STP relative to the particle of matter is always relatively constant. The quantum theory obtained under the framework of MIP is fully compatible with the existing quantum theory. The advantage of this new framework is that it does not require the introduction of additional wave function assumptions, which can directly derive the Schrödinger equation. In particular, the concept of wave pack collapse is not required to be introduced under our MIP framework. The Heisenberg uncertainty principle no longer has a fundamental position but a natural inference under the MIP framework. From the statistical uncertainty between inertial mass and space-time diffusion coefficient, the most basic coordinate momentum uncertainty relationship of quantum mechanics can be derived. Therefore, it is proved that the wave-particle duality is a property exhibited by the STP colliding particles under the MIP framework. Furthermore, we apply MIP to quantum measurement problems, and have a new breakthrough interpretation of the EPR paradox problem that has confused physics for nearly a century. The STP colliding matter particles is a zero-spin scalar particle without mass. According to MIP, the topological properties and dynamic properties of STP can explain the nature of photons, and thus naturally obtain the complete electromagnetic theory and all important properties of charge. Furthermore, from the vortex structure of spacetime, we obtain the origin of the spin and the relationship between spin and mass. Going back to the 2+1d vortex when we investigate the electromagnetic fields in 3+1d spacetime, we prove the strong constrain on the number of generations of charged leptons, at most three generations. From the microscopic behavior of a large number of STP, the macroscopic gravitational effect can be derived, and the Newton's universal gravitation formula are obtained. Inertia mass and gravitation mass are no longer basic physical quantities. The real root of the equivalence principle is that both come from the statistical mass of STP collisions. We define entropy of matter particle microscopically within the frame- work of MIP, and derive the principle of increasing entropy of free particle, which is a cornerstone in thermodynamics. We generalize the concept of entropy to absolute zero, where we also prove the principle of increasing entropy. Therefore, we unify the entropy at finite temperature and absolute zero. Last but not least, MIP requires a novel massless scalar particle STP. The random collisions between STPs and muons is the crucial step beyond standard model. Our extension of standard model is minimal, which only introduce one free parameter describing the interaction strength between STPs and muons, then we are able to explain two key experiments of muon simultaneously. By thorough calculations of corresponding Feynman's diagrams, the contributions from random collisions between STPs and muons explain the anomalous magnetic moment of muon and its lifetime excellently, which solve a world class puzzle about the anomalous magnetic moment of muon, and give a self-consistent explanation to the lifetime discrepancy of muon at the same time. Recent experimental results from FermiLab are the most precision verification of MIP, which not only guarantees the correctness of MIP, but also rejects other possible alternative model.

In summary, MIP may systematically solve all the basic problems of modern physics, which is the common origin of special relativity, general relativity, electromagnetic theory ,quantum nature and thermodynamics. Starting from the only one principle postulation, we may reconstruct the foundation of modern physics and unify all important areas of modern physics. Furthermore, proof of three generations of charged leptons create a theoretical framework of flavor problem of neutrino, which lead to a novel research direction of neutrino oscillation. The existence of STP vortex provide a research program for quantum gravity and grand unifying of four fundamental interactions.

Appendix A: Brown Motion and Markov Process

When the displacement of the material particle $X(t)$ satisfies the following conditions, we call the material particle doing Brownian motion:

1.$X(0) = 0$.

2. On any finite disjoint interval set $(s_i, s_i + t_t)$, the displacement of the particle

is $X(s_i + t_t) - X(s_i)$, which are random variables that are independent of each other.

3. For each $s \geqslant 0, t \geqslant 0$, $X(s+t) - X(s)$ obeys the normal distribution $N(0,t)$.

For each constant a, the process $X(t) + a$ is called the Brownian motion starting from a. For the Brownian motion that is physically free of friction, we call it the quantum Brownian motion in this paper.

Consider any past set of times $(\cdots, p_2, p_1)$, any "current time" s, and any "future time" t, all of which are within the range of X, if any

$$\cdots < p_2 < p_1 < s \tag{14.1}$$

Then the Markov property is established, and the process is a Markov process, but only if:

$$\begin{aligned} \Pr\big[X(t) &= x(t) \mid X(s) = x(s), X(p_1) = x(p_1), X(p_2) = x(p_2), \ldots\big] \\ &= \Pr\big[X(t) = x(t) \mid X(s) = x(s)\big] \end{aligned} \tag{14.2}$$

Set up for all time sets. Then calculate the conditional probability

$$\Pr\big[X(t) = x(t) \mid X(s) = x(s), X(p_1) = x(p_1), X(p_2) = x(p_2), \ldots\big] \tag{14.3}$$

Future state is independent of any historical state and is only relevant to the current state.

In summary, the quantum Brownian motion studied in this paper is a Markov process.

Appendix B: Decomposition of Random Variables

In the Langevin equation, the true velocity of particle motion $\vec{V}$ contains three parts: the classic statistical velocity $\vec{v}$, quantum envelope velocity $\vec{u}$ and Gaussian noise $\vec{\nu}$

We do not consider the impact of classic statistical velocity. Then the random motion of the particles will be determined by the quantum envelope motion and Gaussian noise. The fact that we need to prove is that we can distinguish the quantum envelope motion $\vec{u}$ in the strict mathematical differential sense. The quantum envelope motion corresponds to the smooth continuous part of the random motion, and the Gaussian noise corresponds to the continuous non-differentiable part of the random motion.

First, for any random variable $r(x,t)$, if a smooth function $f(x,t)$ is superimposed, the result is still a random variable. which is a random variable, as

$$w(x,t) = r(x,t) + f(x,t) \tag{14.4}$$

But if $r(x,t)$ or $w(x,t)$ has a finite order autocorrelation association, then theoretically we can strictly distinguish $w(x,t)$ and other two random variables of $r(x,t)$, which is:

$$\langle r(x_1,t_1)r(x_2,t_2)\cdots r(x_n,t_n)\rangle_r = \mathcal{F}_n(\vec{x},\vec{t}), \quad mod(n,N)\equiv 0 \quad (14.5)$$

$$\langle r(x_1,t_1)r(x_2,t_2)\cdots r(x_n,t_n)\rangle_r = 0, \quad mod(n,N)\neq 0 \quad (14.6)$$

Then there is

$$\langle w(x_1,t_1)w(x_2,t_2)\cdots w(x_N,t_N)\cdots w(x_n,t_n)\rangle_r \neq 0, \quad n>N \quad (14.7)$$

Therefore, it can be strictly distinguished mathematically. In the case we considered, Gaussian noise $\vec{\nu}$ has a second-order correlation

$$\langle \nu_i(t)\nu_j(t')\rangle = \Omega\delta_{i,j}\delta(t-t') \quad (14.8)$$

And all odd-order associations are zero

$$\langle \nu(t)\rangle_\nu = 0$$

So obviously

$$\vec{w}(t) = \vec{u}(t) + \vec{\nu}(t)$$

The odd-order correlation is not zero. So you can strictly distinguish between $\vec{w}(t)$ and $\vec{\nu}(t)$. Due to the MIP, there is only one kind of Gaussian noise, and there is no other noise source. So continuous functions other than noise are smooth and differentiable functions. So $\vec{u}$ is a smooth function.

Appendix C: From MIP to the Uncertainty Principle

We believe that the uncertainty principle comes from the kinematic equation of stochastic spacetime motion, which is rooted in the non-differentiable motion path, i.e. the particle coordinate $\vec{x}(t)$ derivative of time $d\vec{x}/dt$ does not exist. Therefore, it must be noted that the particle's momentum $\vec{p} = md\vec{x}/dt$ cannot be well defined. The momentum is defined as follows

$$\vec{p} = mD\vec{x} = m\vec{v} + m\vec{u} \quad (14.9)$$

Kinematic equation

$$\vec{u} = \Re\frac{\nabla\rho}{\rho} \quad (14.10)$$

For the sake of simplicity, the following discussion uses only one component in the x direction, and all vector equations become equations of one component. For any random variable O, the statistical average is $< O >= \int O\rho(x)dx$.

Multiplying both sides of the equation by ρ and integrate x, we can get the x and u_x covariance

$$\sigma(x, u_x) =< (x- < x >)(u_x- < u_x >) >= -\Re \tag{14.11}$$

The covariance represents the total error of two variables, which is different from the variance that only represents the error of one variable. If two variables change in the same directions, then the covariance between two variables is positive. If two variables change in opposite directions, the covariance between two variables is negative. For any two real random variables A and B, there is the Schwarz inequality $|\sigma(A, B)| \leqslant \sigma(A)\sigma(B)$, which leads to

$$\sigma(x)\sigma(u_x) \geqslant \Re = \hbar/2m \tag{14.12}$$

The statistical definition of uncertainty is

$$\sigma(x) = \sqrt{< x^2 > - < x >^2} \tag{14.13}$$

$$\sigma(u_x) = \sqrt{< u_x^2 > - < u_x >^2} \tag{14.14}$$

So far we have proved the uncertainty relationship between the position of random spacetime moving particles and the fluctuation speed. Further, if the uncertainty of momentum has two parts of contributions

$$\sigma^2(p) = m^2(\sigma^2(v) + \sigma^2(u)) \tag{14.15}$$

That is, $\sigma(p) \geqslant m\sigma(u)$, the uncertainty of the position and the fluctuation speed can be obtained.

$$\sigma(x)\sigma(p_x) \geqslant \hbar/2 \tag{14.16}$$

The proof of our paper interprets Heisenberg's uncertainty principle as the uncertainty relationship between random spacetime moving particle position and fluctuation speed.The random spacetime motion has no friction and no irreversible dissipation.

The uncertainty of the fluctuation speed is entirely from spacetime fluctuations. According to Heisenberg's original statement, the measured action inevitably interferes with the state of the particles being measured, thus creating uncertainty. Later that year, Kennard gave another statement. The following year, Herman also obtained this result independently. According to Kennard's statement, the uncertainty of position and the uncertainty of momentum are the nature of the particle, and cannot be suppressed below a certain limit, regardless of the measured action. Thus, for the principle of uncertainty, there are two completely different interpretations. Landau believes that the two interpretations are equivalent, so one expression can be derived from another expressions (Ref. quantum mechanics of Landau). However, in the latest experimental progress, Japanese scholars published on January 15, 2012, the empirical results of the Heisenberg uncertainty principle. They used two instruments to measure the spin angle of the neutron and obtained a smaller measurement than the Heisenberg limit, which proves the measurement interpretation by Heisenberg is wrong. However,

the principle of uncertainty is still correct, because this is the quantum nature of the particle.
The derivation process of this paper has nothing to do with the measurement theory, and it has nothing to do with the internal properties of the particles. It is believed that the uncertainty principle is rooted in the fluctuation of spacetime. Under the non-relativistic framework, spacetime fluctuations are only related to the mass of the particles. The mass of a particle is the only perceptible property of the particle in spacetime.

Appendix D: Additional Physics Example with Three-speed Decomposition

The superposition of orbitals and the formation of chemical bonds, which are common in chemistry, involves quantum superposition states. In the simplest case, the ground state of the hydrogen atom and the first excited state are superimposed with equal probability as

$$\psi(r,t) = \psi_{100}e^{-iE_1t} + \psi_{200}e^{-iE_2t} \tag{14.17}$$

Where $E_1 = -13.6ev, E_2 = -13.6/4ev = -3.4ev$, the wave function of the ground state of the hydrogen atom and the first excited state are

$$\psi_{100} = \frac{1}{\sqrt{\pi a^3}}e^{-r/a} \tag{14.18}$$

$$\psi_{200} = \frac{1}{\sqrt{2a^3}}e^{-r/2a}(1-\frac{r}{2a}) \tag{14.19}$$

Where a is the Bohr's radius $a = 0.529 \times 10^{-10}m$.

With the Euler formula, we can write the superimposed wave function as

$$\psi = [\psi_{100}cos(E_1t) + \psi_{200}cos(E_2t)] - i[[\psi_{100}sin(E_1t) + \psi_{200}sin(E_2t)] \tag{14.20}$$

From the real and imaginary part, the two potential functions R and I of the superposition wave function can be further determined. It is found by equation (5.34) and (5.35) that the electrons u and v are not zero in this state.

This physics example is not a special case, and has general physical meaning. When the quantum state has definite energy, its classical statistical velocity v must be zero. Generally speaking, the particle is in the superposition state of the energy eigenstate, and its three speeds are not zero which has clear physical meaning.

Appendix E: Self Isomorphism on Direct Product Spin Clusters

We hope to prove the following conclusions in this appendix:

Theorem 14.1. *Given any topological excited state deformation:* $A : \Lambda_L \otimes \Lambda_R \mapsto \Lambda_L \otimes \Lambda_R$, *where* A *For automorphism mapping,* Λ_L, Λ_R *represent the left-hand spin cluster and the right-hand spin cluster, respectively, and* A *is the vector map.*

Proof: First of all, from the symmetry of the spin structure, it is not difficult to know that we only need to prove arbitrary automorphism: $A : \Lambda_L \mapsto \Lambda_L$ Both are vector maps. This is because if we can determine that A is a vector map, we can get it through conjugate expansion: $\widetilde{A} : \Lambda_L \otimes \Lambda_R \mapsto \Lambda_L \otimes \Lambda_R$ for vector mapping.

To prove that any automorphism: $A : \Lambda_L \mapsto \Lambda_L$ is a vector map, we need to consider the model on the left-handed spin sector, which is corresponding to the $Clifford$ algebra .Proposition 1.3.2 by [59] It can be seen that for the finite form $Clifford$ algebra, the following forms are isomorphic:

$$Cl_{r,s} \cong Cl_1 \widehat{\otimes} ... \widehat{\otimes} Cl_1 \widehat{\otimes} Cl_1^* ... \widehat{\otimes} Cl_1^*.$$

Among them, the number of Cl_1 corresponds to r, and the number of Cl_1^* corresponds to s.

From the theorem 1.5.4 of [59], all $Clifford\ K-$ means that ρ can be decomposed into straight sums of irreducible algebra representations of the following form:

$$\rho = \rho_1 \oplus ... \oplus \rho_m.$$

The feature subspace W_i corresponding to ρ_i is the smallest subspace.

In additions, by the $Bott$ cycle law theorem [59], we can get the algebraic representation of all Cl_m, $(m = 1, ...8)$, and the representation follows the indicator m Repeated with a period of 8. That is: we can get the algebra of any Cl_m as follows:

$$\begin{gathered} Cl_1 = \mathbb{C},\ Cl_2 = \mathbb{H},\ Cl_3 = \mathbb{H} \oplus \mathbb{H},\ Cl_4 = \mathbb{H}(2), \\ Cl_5 = \mathbb{C}(4),\ Cl_6 = \mathbb{R}(8),\ Cl_7 = \mathbb{R}(8) \oplus \mathbb{R}(8),\ Cl_8 = \mathbb{R}(16). \end{gathered} \tag{14.21}$$

For any combination of the above forms, the straight and broken parts ρ_i Can be split into direct product form:

$$Cl_{r,s} \cong Cl_1 \widehat{\otimes} ... \widehat{\otimes} Cl_1 \widehat{\otimes} Cl_1^* ... \widehat{\otimes} Cl_1^*.$$

The automorphism mapping between any part of the above direct product form can be made by $Cl_1 = \mathbb{C}, ...Cl_8 = \mathbb{R}(16)$ Algebraic combination between parts. Since the above parts are all vector spaces, the automorphism must be a vector mapping, that is, the automorphism of ρ_i must correspond to the matrix form.

In addition, from the algebraic decomposition process described above, it is not difficult to know that the homomorphic mapping between all corresponding different sub-blocks is also a vector mapping. Finally, we will be $\rho_i,\ i = 1, ...8$ All of them are combined together in a straight form, and we can get the automorphism $A : \Lambda_L \mapsto \Lambda_L$ when $i = 1, ...8$ for vector mapping. When the indicator i is greater than 8, by the *Bott* cycle law, we can still get the automorphism mapping by the above process. A is the vector map. The conclusion is proved.

Appendix F: Field Theory Calculations for Fermionic Loop Integral

We consider the following Fermion loop momentum integrals

$$\int \frac{d^d k}{D_1^{n_1} D_2^{n_2}} = i\pi^{d/2}(-p^2)^{d/2-n_1-n_2} G(n_1, n_2)\,, \quad D_1 = -(k+p)^2\,, \quad D_2 = -k^2 \tag{14.22}$$

Noting that in the denominator, D_1, D_2 should actually have an infinitesimal analytic continuation $(-i0^+)$. But for the sake of simplicity, we don't explicitly write it out. After analysing the continuation, we need to consider the contribution of $p^2 < 0$, and the power contribution of $-p^2$ can be easily obtained from dimensional analysis. In fact, what needs to be calculated now is the dimensionless function $G(n_1, n_2)$; to simplify the calculation, we can let $-p^2 = 1$. When $n_1 \leq 0$ or $n_2 \leq 0$, the score can be strictly calculated and $G(n_1, n_2) = 0$ can be obtained.

Using Wick rotation and α parameterization, we can rewrite $G(n_1, n_2)$ as:

$$G(n_1, n_2) = \frac{\pi^{-d/2}}{\Gamma(n_1)\Gamma(n_2)} \int e^{-\alpha_1(\boldsymbol{k}+\boldsymbol{p})^2 - \alpha_2 \boldsymbol{k}^2} \alpha_1^{n_1-1} \alpha_2^{n_2-1} d\alpha_1\, d\alpha_2\, d^d\boldsymbol{k}\,. \tag{14.23}$$

Let

$$\boldsymbol{k}' = \boldsymbol{k} + \frac{\alpha_1}{\alpha_1+\alpha_2}\boldsymbol{p}\,,$$

We can get

$$\begin{aligned} G(n_1, n_2) &= \frac{\pi^{-d/2}}{\Gamma(n_1)\Gamma(n_2)} \int \exp\left[-\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right] \alpha_1^{n_1-1}\alpha_2^{n_2-1} d\alpha_1\, d\alpha_2 \int e^{-(\alpha_1+\alpha_2)\ Bmk^2} d^d\boldsymbol{k} \\ &= \frac{1}{\Gamma(n_1)\Gamma(n_2)} \int \exp\left[-\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\right] (\alpha_1+\alpha_2)^{-d/2} \alpha_1^{n_1-1}\alpha_2^{n_2-1} d\alpha_1\, d\alpha_2\,. \end{aligned} \tag{14.24}$$

Using the substitution $\alpha_1 = \eta x$, $\alpha_2 = \eta(1-x)$, the above formula can be rewritten as

$$\begin{aligned} G(n_1, n_2) &= \frac{1}{\Gamma(n_1)\Gamma(n_2)} \int_0^1 x^{n_1-1}(1-x)^{n_2-1} dx \int_0^\infty e^{-\eta x(1-x)} \eta^{-d/2+n_1+n_2-1} d\eta \\ &= \frac{\Gamma(-d/2+n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)} \int_0^1 x^{d/2-n_2-1}(1-x)^{d/2-n_1-1} dx\,. \end{aligned} \tag{14.25}$$

The integrand is an Euler B function, so we can get the final result

$$G(n_1,n_2) = \frac{\Gamma(-d/2+n_1+n_2)\Gamma(d/2-n_1)\Gamma(d/2-n_2)}{\Gamma(n_1)\Gamma(n_2)\Gamma(d-n_1-n_2)}\,. \tag{14.26}$$

For all positive integers $n_{1,2}$ they are proportional to

$$G_1 = G(1,1) = -\frac{2g_1}{(d-3)(d-4)}\,,\quad g_1 = \frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\,, \tag{14.27}$$

The scale factor is a rational function of d.

Noting that at $k \to \infty$, the denominator part of (14.22) behaves as $(k^2)^{n_1+n_2}$. Therefore, this integral is divergent when $d \geq 2(n_1+n_2)$.

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References

[1] L. Erdos, Lecture Notes on Quantum Brownian Motion, arXiv: 1009.0843, (2010)

[2] N. Wiener, Differential space. J. Math and Phys. 58 , 131-174 (1923)

[3] J. Zinn-Justin, Quantum Field Theory and Critical Phenomena (4-th edition), Oxford University, (2002)

[4] S. Ross, A First Course in Probability (8th Edition), Pearson Prentice Hall, (2009)

[5] F. Reif, Fundamentals of Statistical and Thermal Physics, Waveland, (2009)

[6] A. Einstein, Investigations on the theory of the brownian movement. Dover Edition(1956).

[7] E. Nelson, Dynamical theories of Brownian motion, (Princeton University Press, Princeton, 1967).

[8] E. Nelson, Quantum Fluctuations, (Princeton University Press, Princeton, 1985).

[9] S. Weinberg, Cosmology, Oxford University, (2008)

[10] F. W. Stecker “Constraining Superluminal Electron and Neutrino Velocities using the 2010 Crab Nebula Flare and the IceCube Pev Neutrino Events”. Astroparticle Physics 56: 16?C18. arXiv:1306.6095

[11] K. Kodama et al. (DONUT Collaboration; Kodama; Ushida; Andreopoulos; Saoulidou; Tzanakos; Yager; Baller; Boehnlein; Freeman; Lundberg; Morfin; Rameika; Yun; Song; Yoon; Chung; Berghaus; Kubantsev; Reay; Sidwell; Stanton; Yoshida; Aoki; Hara; Rhee; Ciampa; Erickson; Graham; et al. (2001). “Observation of tau neutrino interactions”. Physics Letters B 504 (3): 218. arXiv:hep-ex/0012035.

[12] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018) and 2019 update

[13] Adams, Scott; et, al (2013). "Observing the Next Galactic Supernova". Astrophysical Journal. 778 (2): 164.

[14] R. A. Battye and A. Moss, “Evidence for Massive Neutrinos from Cosmic Microwave Background and Lensing Observations”. Physical Review Letters 112 (5): 051303 (2014). arXiv:1308.5870v2.

[15] Planck Collaboration, P. A. R.; Ade, P. A. R.; Aghanim, N.; Armitage-Caplan, C.; Arnaud, M.; Ashdown, M.; Atrio-Barandela, F.; Aumont, J.; Baccigalupi, C.; Banday, A. J.; Barreiro, R. B.; Bartlett, J. G.; Battaner, E.; Benabed, K.; Benot, A.; Benoit-L’vy, A.; Bernard, J.-P.; Bersanelli, M.; Bielewicz, P.; Bobin, J.; Bock, J. J.; Bonaldi, A.; Bond, J. R.; Borrill, J.; Bouchet, F. R.; Bridges, M.; Bucher, M.; Burigana, C.; Butler, R. C.; et al. (2013). “Planck 2013 results. XVI. Cosmological parameters”. Astronomy & Astrophysics 1303: 5076. arXiv:1303.5076. Bibcode:2013arXiv1303.5076P. doi:10.1051/0004-6361/201321591.

[16] S. Weinberg, Lectures on Quantum Mechanics, Cambridge University, (2012)

[17] S. Weinberg, The Quantum Theory of Fields, vol 2, Cambridge University, (1995)

[18] S. Chandrashekhar, Rev. Mod. Phys., 15,1(1943).

[19] J. Doob, Stochastic processes. Wiley: New York, (1953).

[20] Wold, H. (1954) A Study in the Analysis of Stationary Time Series, Second revised edition, with an Appendix on "Recent Developments in Time Series Analysis" by Peter Whittle. Almqvist and Wiksell Book Co., Uppsala.

[21] G. E. Uhlenbeck and L. S. Ornstein, Rev. Mod. Phys., 36,823(1930)

[22] M. C. Wang and G. E Uhlenbeck, Rev. Mod. Phys., 17,323(1945).

[23] L. de Broglie, C.R. Acad. Sci. 264B, 1041 (1967).

[24] M. Davidson, Lett. Math. Phys. 3, 271 (1979).

[25] M. Davidson, Lett. Math. Phys. 3, 367 (1979)

[26] Max Born, Zur Quantenmechanik der Sto vorg nge, Zeitschrift f'r Physik, 37, #12 (Dec. 1926), pp. 863-C867 (German); English translation, On the quantum mechanics of collisions, in Quantum theory and measurement, section I.2, J. A. Wheeler and W. H. Zurek, eds., Princeton, New Jersey: Princeton University Press, 1983, ISBN 0-691-08316-9.

[27] L. Kadanoff , Statistical Physics: statics, dynamics and renormalization. World Scientific Press, (2000).

[28] L. Landau and E. Lifshitz , Courses in theoretical physics, vol 1, Mechanics. Butterworth-Heinemann. (1976)

[29] H. Goldstein, Classical Mechanics (3rd Edition), Addison-Wesley, (2001)

[30] J. D. Jackson, Classical Electrodynamics (3-rd Edition), Wiley, (1998)

[31] L. Landau and E. Lifshitz , Courses in theoretical physics, vol 3, Quantum Mechanics. Pergamon Press, (1977)

[32] R.J. Glauber, Quantum optics and electronics, edited by C. De Witt (Gordon-Breach New York, 1965); H. Haken, in Encyclopedia of Physics (Springer, New York, 1976).

[33] G. Birkhoff and J. von Neumann, Ann. Math. 37, 823 (1932).

[34] H. Haken and W. Weidlich, Z. Phys. 205, 96 (1967).

[35] E. H. Kennard, "Zur Quantenmechanik einfacher Bewegungstypen". Zeitschrift f'r Physik 44 (4-C5): 326 (1927)

[36] T.D.Lee, Particle physics and introduction to field theory,Harwood Academic Publishers , 1981.

[37] Jackiw R , Weinberg S . Weak-Interaction Corrections to the Muon Magnetic Moment and to Muonic-Atom Energy Levels. Phys. Rev. D. 5, 2396 (1972).

[38] G. Bennett et al. (Muon g-2 Collaboration), Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL, Phys.Rev. D73, 072003 (2006).

[39] B. Abi, et al. Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm, Phys. Rev. Lett. 126 (14) (2021) 141801.

[40] V. A. Smirnov, Feynman Integral Calculus, (Springer Verlag, Berlin, 2006) DOI: https://doi.org/10.1007/3-540-30611-0.

[41] I. Dubovyka , J. Gluza and T. Riemann, Optimizing the Mellin-Barnes Approach to Numerical Multiloop Calculations. Acta Physica Polonica B, Vol. 50 (2019), 11, 1993.

[42] AMBRE webpage: http://prac.us.edu.pl/~gluza/ambre.

[43] J. Gluza, K. Kajda, T. Riemann, AMBRE - a Mathematica package for the construction of Mellin-Barnes representations for Feynman integrals, Comput. Phys. Commun. 177 (2007) 879–893. arXiv:0704.2423, doi: 10.1016/j.cpc.2007.07.001.

[44] A. Smirnov, V. Smirnov, On the Resolution of Singularities of Multiple Mellin-Barnes Integrals, Eur. Phys. J. C62 (2009) 445–449. arXiv: 0901.0386, doi:10.1140/epjc/s10052-009-1039-6.

[45] Einstein, A; B Podolsky; N Rosen. Physical Review. 47 (10): 777-780.

[46] H. Weyl, The Theory of Groups and Quantum Mechanics, Dover Edition, (1950)

[47] L. A. Rozema, A. Darabi, D. H. Mahler, A. Hayat, Y. Soudagar, and A. M. Steinberg, Phys. Rev. Lett. 109, 100404 (2012)

[48] L. A. Rozema, A. Darabi, D. H. Mahler, A. Hayat, Y. Soudagar, and A. M. Steinberg, Phys. Rev. Lett. 109, 189902 (2012)

[49] R. P. Feynman, Ph.D thesis. Princeton Press (1942).

[50] P.A.M. Dirac, Physikalische Zeitschrift der Sowjetunion, Band 3, Heft 1 , pp. 64-72 (1933)

[51] P. Blanchard, S. Golin and M. Serva, Phys. Rev. D34, 3732 (1986).

[52] P. Blanchard and M. Serva, Phys. Rev. D51, 3132 (1995).

[53] P. A. M. Dirac, Quantised Singularities in the Electromagnetic Field, *Proc. Roy. Soc. A* **133**, 60 (1931)

[54] G. 't Hooft and F. Bruckmann, Monopoles, Instantons and Confinement. arXiv:hep-th/0010225

[55] N. K. Nielsen and P. Olesen, Vortex-Line Models for Dual Strings, *Nucl. Phys. B* **61**, 45 (1973)

[56] L. Mario. "Hydrodynamic theory of electromagnetic fields in continuous media." Physical Review Letters 70.23(1993):3580-3583.

[57] Martin, T.. (1976). Torsion and the geometry of spin.

[58] Martins, Alexandre A , Pinheiro, et al. Fluidic electrodynamics: Approach to electromagnetic propulsion[C] Aip Conf Proc. American Institute of Physics, 2009.

[59] H. B. Lawson. Spin Geometry [M]. Princeton University Press, New Jersey, 1994.